

# Process Economics\*

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## Nomenclature and Units

Symbol	Definition	Units	Symbol	Definition	Units
$a$	Empirical constant in general equations	Various	$f_d$	Discount factor, $(1 + i)^{-n}$	Dimensionless
$A$	Annual income or expenditure particularized by the subscript	\$/year	$f_i$	Compound-interest factor, $(1 + i)^n$	Dimensionless
$A_A$	Annual allowances against tax other than for depreciation of fixed assets	\$/year	$f_k$	Capitalized-cost factor, $f_{AP}/i$	Dimensionless
$A_D$	Annual writing down (depreciation) of fixed assets, allowable against tax	\$/year	$f_p$	Piping-cost factor defined by Eq. (9-249)	Dimensionless
(ATR)	Asset-turnover ratio defined by Eq. (9-131)	Dimensionless	$f(x)$	Distribution function of $x$ variously defined	Dimensionless
$b$	Empirical constant in general equations	Various	$F$	Future value of a sum of money	\$
$b_c$	Deviation from budgeted capacity	Dimensionless	$F_n$	Sum of $f_d$ for Years 1 to $n$	Dimensionless
$B$	Parametric constant in Eq. (9-204)	Dimensionless	$i$	Interest rate per period, usually annual, often the cost of capital	Dimensionless
$c$	Empirical constant in general equations	Various	$i_e$	Effective interest rate defined by Eq. (9-111)	Dimensionless
$c$	Cost (or income) per unit of sales or production particularized by the subscript	\$/unit	$i_m$	Minimum acceptable interest rate defined by Eq. (9-107)	Dimensionless
$c_B$	Cost of base heat supply	\$/unit	$i_r$	Entrepreneurial-risk interest rate	Dimensionless
$c_D$	Cost of heat energy delivered by a heat pump defined by Eq. (9-240)	\$/GJ	$i'$	Nominal annual interest rate	Dimensionless
$c_I$	Cost of high-grade energy supplied to the compressor of a vapor compression heat pump	\$/GJ	$I$	Value of inventory particularized by the subscript	\$
$c_L$	Cost of labor per unit of production	\$/hour	$k_n$	Constants in Eq. (9-81)	Various
$c^\circ$	Standard cost particularized by the subscript	\$/hour	$K$	Effective value of the first unit of production	\$/unit, time/unit, etc.
$C$	Cost particularized by the subscript	\$	$\ln(a)$	Logarithm to the base $e$ of $a$	Dimensionless
$C_{CT}$	Installed cost of a cooling tower	\$	$\log(a)$	Logarithm to the base 10 of $a$	Dimensionless
$C_{DS}$	Installed cost of a demineralized-water system	\$	$m$	Number of interest periods due per year	Dimensionless
$(C_{EQ})_{DEL}$	Delivered-equipment cost	\$	$m$	Number of units removed from inventory	Dimensionless
$C_K$	Capitalized cost of a fixed asset defined by Eq. (9-47)	\$	(MSF)	Measured-survival function defined by Eq. (9-106)	Dimensionless
$C_L$	Cost of land and other nondepreciable assets	\$	$n$	Number of years, units, etc.	Dimensionless
$C_{RS}$	Installed cost of a refrigeration system	\$	$N$	Slope of the learning curve defined by Eq. (9-64)	Dimensionless
$C_{RW}$	Installed cost of a river-water supply system	\$	$N$	Number of inventory orders per year	Dimensionless
$C_{WS}$	Installed cost of a water-softening system	\$	(NPV)	Net present value	\$
(CI)	Cost index as used in Eq. (9-246)	Dimensionless	$p(x)$	Probability of the variable having the value $x$	Dimensionless
(COP) <sub>A</sub>	Actual coefficient of performance of a heat pump	Dimensionless	$P$	Present value of a sum of money	\$
(CR)	Capital ratio defined by Eq. (9-134)	Year	$P_a$	Production time worked	Hour
(CRR)	Capital-rate-of-return ratio defined by Eq. (9-56)	Year	$P_b$	Budgeted production	Standard hour
(CSR)	Contribution-sales ratio defined by Eq. (9-236)	Dimensionless	$P_c$	Production efficiency defined by Eq. (9-216)	Dimensionless
$d$	Empirical constant in general equations	Various	$P_1$	Level of productive activity defined by Eq. (9-217)	Dimensionless
$d$	Symbol indicating differentiation	Dimensionless	$P_s$	Actual production rate	Standard hour
(DR)	Debt ratio defined by Eq. (9-139)	Dimensionless	$P'_s$	Book value of asset at the end of year $s'$	\$
(DCFRR)	Discounted-cash-flow rate of return	Year <sup>-1</sup>	$P_w$	Budgeted working time	Hour
$e$	Empirical constant in general equations	Various	(BBP)	Payback period defined by Eq. (9-30)	Year
$e$	Base of natural logarithms, 2.71828	Dimensionless	(PM)	Profit margin defined by Eq. (9-127)	Dimensionless
$\exp(a)$	Exponential function of $a$ , $e^a$	Dimensionless	(PSR)	Profit-sales ratio defined by Eq. (9-235)	Dimensionless
(EMIP)	Equivalent maximum investment period defined by Eq. (9-55)	Year	$q$	Quantity defining the scale of operation	Various
$f_{AF}$	Annuity future-worth factor, $i[(1 + i)^n - 1]^{-1}$	Dimensionless	$Q_D$	Process-heat-rate requirement	GJ/hour
$f_{AP}$	Annuity present-worth factor, $f_{AF}(1 + i)^n$	Dimensionless	$r$	Fraction of range of the independent variable	Dimensionless
			$R$	Production rate	Units/year
			$R^\circ$	Standard production rate	Units/year
			$R_B$	Breakeven production rate	Units/year

**Nomenclature and Units (Concluded)**

Symbol	Definition	Units	Symbol	Definition
$R_0$	Scheduled production rate	Units/year	$BOH$	Budgeted overhead
$R_s$	Sales rate	Units/year	$CF$	Cash flow after payment of tax and expenses
(ROA)	Return on assets defined by Eq. (9-129)	Dimensionless	$CI$	Cash income after payment of expenses
(ROE)	Return on equity defined by Eq. (9-130)	Dimensionless	$DCF$	Discounted cash flow
(ROI)	Return on investment defined by Eq. (9-128)	Dimensionless	$DME$	Direct manufacturing expense
$s$	Scheduled number of productive years		$FC$	Fixed capital
$s'$	Number of productive years to date		$FE$	Fixed expense
$s^\circ$	Sample standard deviation	Various	$FGE$	Fixed general expense
$S$	Scrap value of a depreciable asset	\$	FIFO	On a first-in–first-out basis
$t$	Fractional tax rate payable on adjusted income	Dimensionless	FIN	Financial-resources inventory
$t_c$	Time taken to construct plant	Years	$FME$	Fixed manufacturing expense
$t_{su}$	Time taken to start up plant	Years	$FOH$	Fixed overhead
$T$	Auxiliary variable defined by Eq. (9-92)	Various	$GE$	General expense
$U$	Size of inventory order	Units	$GP$	Gross profit
$V$	Variable cost of inventory order	\$/unit	$IME$	Indirect manufacturing expense
$W$	Power supplied at shaft of a heat pump	GJ/hour	INV	Inventory
$x$	General variable		$IO$	Inventory-orders cost
$\bar{x}$	Mean value of $x$	Various	$IT$	Income tax payable
$X$	Cumulative production from startup	Units	$IW$	Inventory working cost
$y$	Cumulative probability	Dimensionless	$L$	Labor-earnings index
$y$	Operating time of a heat pump	Hours/year	$L$	Lower-quartile value of the variable
$Y$	Cumulative average cost, production time, etc.	\$/unit, hour/unit, etc.	LIFO	Last-in–first-out basis
$Y$	Operating-labor rate in Eq. (9-204)	labor-hour/ton	max	Maximum value
$\bar{Y}$	Cumulative-average batch cost, etc.	\$/unit, etc.	$M$	Median value of the variable
$z$	Standard score defined by Eq. (9-73)	Dimensionless	$ME$	Manufacturing expense
Greek symbols			$N$	At agreed normal production rate
$\alpha$	Proportionality factor in Eq. (9-168)	Dimensionless	$NCI$	Net cash income after payment of tax
$\beta$	Proportionality factor in Eq. (9-171)	Dimensionless	$NOH$	Overhead cost at agreed normal production rate
$\beta$	Exponent in Eqs. (9-106) and (9-117)	Dimensionless	$NNP$	Net profit after payment of tax
$\delta$	Symbol indicating partial differentiation	Dimensionless	$NP$	Net profit before payment of tax
$\Delta$	Symbol indicating a difference of like quantities	Dimensionless	$OH$	Overhead cost
$\eta$	Contribution efficiency defined by Eq. (9-119)	Dimensionless	$P$	Profit
$\eta$	Margin of safety defined by Eq. (9-229)	Dimensionless	$RM$	Raw material
$\theta$	Time taken to produce a given amount of product	Hour	$s'$	In the $s'$ th productive year
$\sigma$	Population standard deviation	Various	$S$	From sales and other income
$\Sigma$	Symbol indicating a sum of like quantities	Dimensionless	$SAV$	On a simple-average basis
$\phi$	Fractional increase in production rate	Dimensionless	$ST$	Steel-price index
$\phi_p$	Parameter defined with Eq. (9-254)	Dimensionless	$SVOH$	Semivariable overhead
$\psi$	Parameter defined with Eq. (9-241)	Dimensionless	$TC$	Total capital
$\Psi$	Plant capacity in Eq. (9-204)	Tons/day	$TE$	Total expense
$\chi$	Weight of product per unit of raw material	Dimensionless	$TFE$	Total fixed expense
Subscripts			$TVE$	Total variable expense
$A$	Allowance against tax other than for capital depreciation		$U$	Utilities
$BD$	Depreciation allowance shown in company balance sheet		$U$	Upper-quartile value of the variable
$BL$	Within project boundary limits		$VE$	Variable expense
			$VGE$	Variable general expense
			$VME$	Variable manufacturing expense
			$VOH$	Variable overhead expense
			$W$	Weighted value
			$WC$	Working capital
			$WAV$	On a weighted-average basis
			$1, 2, j, n$	1st, 2d, $j$ th, $n$ th item, year, etc.

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## NOMENCLATURE

An attempt has been made to bring together most of the methods currently available for project evaluation and to present them in such a way as to make the methods amenable to modern computational techniques. To this end the practices of accountants and others have been reduced, where possible, to mathematical equations which are usually solvable with an electronic hand calculator equipped with scientific function keys. To make the equations suitable for use on high-speed computers an attempt has been made to devise a nomenclature which is suitable for machines using ALGOL, COBOL, or FORTRAN compilers. The number of letters and numbers used to define a variable has usually been limited to five. The letters are mnemonic in English wherever possible and are derived in two ways. First, when a standard accountancy phrase exists for a term, this has been abbreviated in capital letters and enclosed in parentheses, e.g., (ATR), for assets-to-turnover ratio; (DCFRR), for discounted-cash-flow rate of return. Clearly, the parentheses are omitted when the letter group is used to define the variable name for the computer. Second, a general symbol is defined for a type of variable and is modified by a mnemonic subscript, e.g., an annual cash quantity  $A_{TC}$ , annual total capital outlay, \$/year. Clearly, the symbols are written on one line when the letter group is used to define a variable name for the computer. In other cases, when well-known standard symbols exist, they have been

adopted, e.g.,  $z$  for the standard score as used in the normal distribution. Also,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  have been used to denote empirical constants and  $x$  and  $y$  to denote general variables where their use does not clash with other meanings of the same symbols.

The coverage in this section is so wide that nomenclature has sometimes proved a problem which has required the use of primes, asterisks, and other symbols not universally acceptable in the naming of computer variables. However, it is realized that each individual will program only his or her preferred methods, which will release some symbols for other uses. Also, it is not difficult to replace a forbidden symbol by an acceptable one; e.g.,  $c_{RM}$  might be rendered CARM and  $P'_S$  as PSP by using  $A$  for asterisk and  $P$  for prime. For compilers which recognize only one alphabetical case, an extra prefix can be used to distinguish between uppercase and lowercase letters, for which purpose the letters  $U$  and  $L$  have been used only in a restricted way in the nomenclature.

It is, of course, impossible to allow for all possible variations of equation requirements and machine capability, but it is hoped that the nomenclature in the table presented at the beginning of the section will prove adequate for most purposes and will be capable of logical extension to other more specialized requirements.

## INVESTMENT AND PROFITABILITY

In order to assess the profitability of projects and processes it is necessary to define precisely the various parameters.

**Annual Costs, Profits, and Cash Flows** To a large extent, accountancy is concerned with annual costs. To avoid confusion with other costs, annual costs will be referred to by the letter  $A$ .

The revenue from the annual sales of product  $A_S$ , minus the total annual cost or expense required to produce and sell the product  $A_{TE}$ , excluding any annual provision for plant depreciation, is the annual cash income  $A_{CI}$ :

$$A_{CI} = A_S - A_{TE} \quad (9-1)$$

Net annual cash income  $A_{NCI}$  is the annual cash income  $A_{CI}$ , minus the annual amount of tax  $A_{IT}$ :

$$A_{NCI} = A_{CI} - A_{IT} \tag{9-2}$$

Taxable income is  $(A_{CI} - A_D - A_A)$ , where  $A_D$  is the annual writing-down allowance and  $A_A$  is the annual amount of any other allowances. A distinction is made between the writing-down allowance permissible for the computation of tax due, the actual depreciation in value of an asset, and the book depreciation in value of that asset as shown in the company position statement. There is no necessary connection between these values unless specified by law, although the first two or all three are often assigned the same value in practice. Some governments give cash incentives to encourage companies to build plants in otherwise unattractive areas. Neither  $A_D$  nor  $A_A$  involves any expenditure of cash, since they are merely book transactions. The annual amount of tax  $A_{IT}$  is given by

$$A_{IT} = (A_{CI} - A_D - A_A)t \tag{9-3}$$

where  $t$  is the fractional tax rate. The value of  $t$  is determined by the appropriate tax authority and is subject to change. For most developed countries the value of  $t$  is about 0.35 or 35 percent.

The annual amount of tax  $A_{IT}$  included in Eq. (9-2) does not necessarily correspond to the annual cash income  $A_{CI}$  in the same year. The tax payments in Eq. (9-2) should be those actually paid in that year. In the United States, companies pay about 80 percent of the tax on estimated current-year earnings in the same year. In the United Kingdom, companies do not pay tax until at least 9 months after the end of the accounting period, which, for the most part, amounts to paying tax on the previous year's earnings. When assessing projects for different countries, engineers should acquaint themselves with the tax situation in those countries.

In modern methods of profitability assessment, cash flows are more meaningful than profits, which tend to be rather loosely defined. The net annual cash flow after tax is given by

$$A_{CF} = A_{NCI} - A_{TC} \tag{9-4}$$

where  $A_{TC}$  is the annual expenditure of capital, which is not necessarily zero after the plant has been built. For example, working capital, plant additions, or modifications may be required in future years.

The total annual expense  $A_{TE}$  required to produce and sell a product can be written as the sum of the annual general expense  $A_{GE}$  and the annual manufacturing cost or expense  $A_{ME}$ :

$$A_{TE} = A_{GE} + A_{ME} \tag{9-5}$$

Annual general expense  $A_{GE}$  arises from the following items: adminis-

tration, sales, shipping of product, advertising and marketing, technical service, research and development, and finance.

The terms gross annual profit  $A_{GP}$  and net annual profit  $A_{NP}$  are commonly used by accountants and misused by others. Normally, both  $A_{GP}$  and  $A_{NP}$  are calculated before tax is deducted. Gross annual profit  $A_{GP}$  is given by

$$A_{GP} = A_S - A_{ME} - A_{BD} \tag{9-6}$$

where  $A_{BD}$  is the balance-sheet annual depreciation charge, which is not necessarily the same as  $A_D$  used in Eq. (9-3) for tax purposes. Net annual profit  $A_{NP}$  is simply

$$A_{NP} = A_{GP} - A_{GE} \tag{9-7}$$

Equation (9-7) can also be written as

$$A_{NP} = A_{CI} - A_{BD} \tag{9-8}$$

Net annual profit after tax  $A_{NNP}$  can be written as

$$A_{NNP} = A_{NP} - A_{IT} \tag{9-9}$$

The relationships among the various annual costs given by Eqs. (9-1) through (9-9) are illustrated diagrammatically in Fig. 9-1. The top half of the diagram shows the tools of the accountant; the bottom half, those of the engineer. The net annual cash flow  $A_{CF}$ , which excludes any provision for balance-sheet depreciation  $A_{BD}$ , is used in two of the more modern methods of profitability assessment: the net-present-value (NPV) method and the discounted-cash-flow-rate-of-return (DCFRR) method. In both methods, depreciation is inherently taken care of by calculations which include capital recovery.

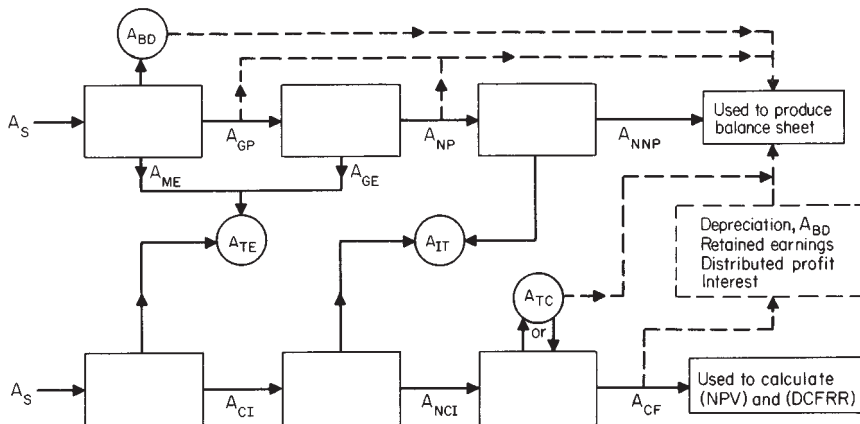
Annual general expense  $A_{GE}$  can be written as the sum of the fixed and variable general expenses:

$$A_{GE} = A_{FGE} + A_{VGE} \tag{9-10}$$

Similarly, annual manufacturing expense  $A_{ME}$  can be written as the sum of the fixed and variable manufacturing expenses:

$$A_{ME} = A_{FME} + A_{VME} \tag{9-11}$$

A variable expense is considered to be one which is directly proportional to the rate of production  $R_p$  or of sales  $R_s$ , as is most appropriate to the case under consideration. Unless the variation in finished-product inventory is large when compared with the total production over the period in question, it is usually sufficiently accurate to consider  $R_p$  and  $R_s$  to be represented by the same-numerical-value  $R$  units of sale or production per year. A fixed expense is then considered to be one which is not directly proportional to  $R$ , such as overhead charges. Fixed expenses are not necessarily constant but may be sub-



**FIG. 9-1** Relationship between annual costs, annual profits, and cash flows for a project.  $A_{BD}$  = annual depreciation allowance;  $A_{CF}$  = annual net cash flow after tax;  $A_{CI}$  = annual cash income;  $A_{GE}$  = annual general expense;  $A_{GP}$  = annual gross profit;  $A_{IT}$  = annual tax;  $A_{ME}$  = annual manufacturing cost;  $A_{NCI}$  = annual net cash income;  $A_{NNP}$  = annual net profit after taxes;  $A_{NP}$  = annual net profit;  $A_S$  = annual sales;  $A_{TC}$  = annual total cost; (DCFRR) = discounted-cash-flow rate of return; (NPV) = net present value.

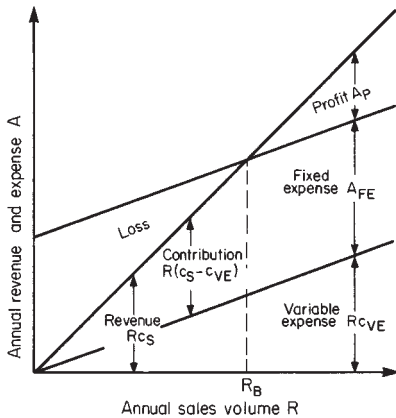


FIG. 9-2 Conventional breakeven chart.

ject to stepwise variation at different levels of production. Some authors consider such steps as included in a semivariable expense, which is less amenable to mathematical analysis than the above division of expenses.

**Contribution and Breakeven Charts** These can be used to give valuable preliminary information prior to the use of the more sophisticated and time-consuming methods based on discounted cash flow. If the sales price per unit of sales is  $c_S$  and the variable expense is  $c_{VE}$  per unit of production, Eq. (9-7) can be rewritten as

$$A_{NP} = R(c_S - c_{VE}) - A_{FE} \quad (9-12)$$

where  $R(c_S - c_{VE})$  is known as the annual contribution. The net annual profit is zero at an annual production rate

$$R_B = A_{FE}/(c_S - c_{VE}) \quad (9-13)$$

where  $R_B$  is the breakeven production rate.

Breakeven charts can be plotted in any of the three forms shown in Figs. 9-2, 9-3, and 9-4. The abscissa shown as annual sales volume  $R$  is also frequently plotted as a percentage of the designed production or sales capacity  $R_0$ . In the case of ships, aircraft, etc., it is then called the percentage utilization. The percentage margin of safety is defined as  $100(R_0 - R_B)/R_0$ .

A decrease in selling price  $c_S$  will decrease the slope of the lines in Figs. 9-2, 9-3, and 9-4 and increase the required breakeven value  $R_B$  for a given level of fixed expense  $A_{FE}$ .

**Capital Costs** The total capital cost  $C_{TC}$  of a project consists of the fixed-capital cost  $C_{FC}$  plus the working-capital cost  $C_{WC}$ , plus the cost of land and other nondepreciable costs  $C_L$ :

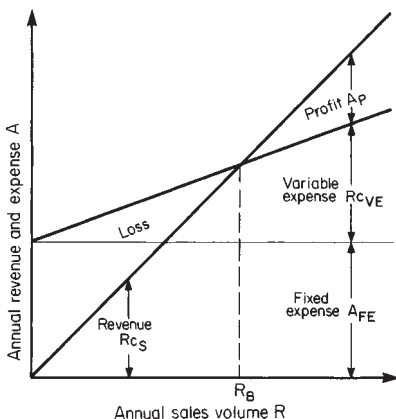


FIG. 9-3 Breakeven chart showing fixed expense as a burden cost.

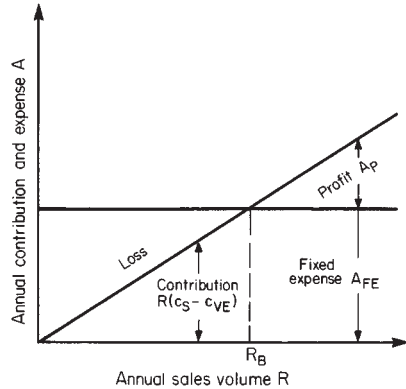


FIG. 9-4 Breakeven chart showing relationship between contribution and fixed expense.

$$C_{TC} = C_{FC} + C_{WC} + C_L \quad (9-14)$$

The project may be a complete plant, an addition to an existing plant, or a plant modification.

The working-capital cost of a process or a business normally includes the items shown in Table 9-1. Since working capital is completely recoverable at any time, in theory if not in practice, no tax allowance is made for its depreciation. Changes in working capital arising from varying trade credits or payroll or inventory levels are usually treated as a necessary business expense except when they exceed the tax debt due. If the annual income is negative, additional working capital must be provided and included in the  $A_{TC}$  for that year. The value of land and other nondepreciables often increases over the working life of the project. These are therefore not treated in the same way as other capital investments but are shown to have made a (taxable) profit or loss only when the capital is finally recovered.

Working capital may vary from a very small fraction of the total capital cost to almost the whole of the invested capital, depending on the process and the industry. For example, in jewelry-store operations, the fixed capital is very small in comparison with the working capital. On the other hand, in the chemical-process industries, the working capital is likely to be in the region of 10 to 20 percent of the value of the fixed-capital investment.

**Depreciation** The term "depreciation" is used in a number of different contexts. The most common are:

1. A tax allowance
2. A cost of operation
3. A means of building up a fund to finance plant replacement
4. A measure of falling value

In the first case, the annual taxable income is reduced by an annual depreciation charge or allowance which has the effect of reducing the annual amount of tax payable. The annual depreciation charge is merely a book transaction and does not involve any expenditure of cash. The method of determining the annual depreciation charge must be agreed to by the appropriate tax authority.

In the second case, depreciation is considered to be a manufacturing cost in the same way as labor cost or raw-materials cost. However,

TABLE 9-1 Working-Capital Costs

Raw materials for plant startup
Raw-materials, intermediate, and finished-product inventories
Cost of handling and transportation of materials to and from stores
Cost of inventory control, warehouse, associated insurance, security arrangements, etc.
Money to carry accounts receivable (i.e., credit extended to customers) less accounts payable (i.e., credit extended by suppliers)
Money to meet payrolls when starting up
Readily available cash for emergencies
Any additional cash required to operate the process or business



it is more difficult to estimate a depreciation cost per unit of product than it is to do so for labor or raw-materials costs. In the net-present-value (NPV) and discounted-cash-flow-rate-of-return (DCFRR) methods of measuring profitability, depreciation, as a cost of operation, is implicitly accounted for. (NPV) and (DCFRR) give measures of return after a project has generated sufficient income to repay, among other things, the original investment and any interest charges that the invested money would otherwise have brought into the company.

In the third case, depreciation is considered as a means of providing for plant replacement. In the rapidly changing modern chemical-process industries, many plants will never be replaced because the processes or products have become obsolete during their working life. Management should be free to invest in the most profitable projects available, and the creation of special-purpose funds may hinder this. However, it is desirable to designate a proportion of the retained income as a fund from which to finance new capital projects. These are likely to differ substantially from the projects that originally generated the income.

In the fourth case, a plant or a piece of equipment has a limited useful life. The primary reason for the decrease in value is the decrease in future life and the consequent decrease in the number of years for which income will be earned. At the end of its life, the equipment may be worth nothing, or it may have a salvage or scrap value  $S$ . Thus a fixed-capital cost  $C_{FC}$  depreciates in value during its useful life of  $s$  years by an amount that is equal to  $(C_{FC} - S)$ . The useful life is taken from the startup of the plant.

On the basis of straight-line depreciation, the average annual amount of depreciation  $A_D$  over a service life of  $s$  years is given by

$$A_D = (C_{FC} - S)/s \quad (9-15)$$

The book value after the first year  $P_1$  is given by

$$P_1 = C_{FC} - A_D \quad (9-16)$$

The book value at the end of a specified number of years  $s'$  is given by

$$P_{s'} = C_{FC} - s'A_D \quad (9-17)$$

The principal use of a particular depreciation rate is for tax purposes. The permitted annual depreciation is subtracted from the annual income before the latter is taxed. The basis for depreciation in a particular case is a matter of agreement between the taxation authority and the company, in conformity with tax laws.

Other commonly used methods of computing depreciation are the declining-balance method (also known as the fixed-percentage method) and the sum-of-years-digits method.

On the basis of declining-balance (fixed-percentage) depreciation, the book value at the end of the first year is given by

$$P_1 = C_{FC}(1 - r) \quad (9-18)$$

where  $r$  is a fraction to be agreed with the taxation authority.

The book value at the end of specified number of years  $s'$  is given by

$$P_{s'} = C_{FC}(1 - r)^{s'} \quad (9-19)$$

When the fraction  $r$  is chosen to be  $2/s$ , i.e., twice the reciprocal of the service life  $s$ , the method is called the double-declining-balance method.

The declining-balance method of depreciation allows equipment or plant to be depreciated by a greater amount during the earlier years than during the later years. This method does not allow equipment or plant to be depreciated to a zero value at the end of the service life.

On the basis of sum-of-years-digits depreciation, the annual amount of depreciation for a specified number of years  $s'$  for a plant of fixed-capital cost  $C_{FC}$ , scrap value  $S$ , and service life  $s$  is given by

$$A_{D_{s'}} = \left( \frac{s - s' + 1}{1 + 2 + 3 + \dots + s} \right) (C_{FC} - S) \quad (9-20)$$

Equation (9-20) can also be rewritten in the form

$$A_{D_{s'}} = \left[ \frac{2(s - s' + 1)}{s(s + 1)} \right] (C_{FC} - S) \quad (9-21)$$

It can be shown that the book value at the end of a particular year  $s'$  is

$$P_{s'} = 2 \left[ \frac{1 + 2 + \dots + (s - s')}{s(s + 1)} \right] (C_{FC} - S) + S \quad (9-22)$$

The sum-of-years-digits depreciation allows equipment or plant to be depreciated by a greater amount during the early years than during the later years.

A fourth method of computing depreciation (now seldom used) is the sinking-fund method. In this method, the annual depreciation  $A_D$  is the same for each year of the life of the equipment or plant. The series of equal amounts of depreciation  $A_D$ , invested at a fractional interest rate  $i$  and made at the end of each year over the life of the equipment or plant of  $s$  years, is used to build up a future sum of money equal to  $(C_{FC} - S)$ . This last is the fixed-capital cost of the equipment or plant minus its salvage or scrap value and is the total amount of depreciation during its useful life. The equation relating  $(C_{FC} - S)$  and  $A_D$  is simply the annual cost or payment equation, written either as

$$C_{FC} - S = A_D \left[ \frac{(1 + i)^s - 1}{i} \right] \quad (9-23)$$

$$\text{or} \quad C_{FC} - S = \frac{A_D}{f_{AF}} \quad (9-24)$$

where  $f_{AF}$  is the annuity future-worth factor given by

$$f_{AF} = i / [(1 + i)^s - 1]$$

In the sinking-fund method of depreciation, the effect of interest is to make the annual decrease of the book value of the equipment or plant less in the early than in the later years with consequent higher tax due in the earlier years when recovery of the capital is most important.

It is preferable not to think of annual depreciation as a contribution to a fund to replace equipment at the end of its life but as part of the difference between the revenue and the expenditure, which difference is tax-free.

Some of the preceding methods of computing depreciation are not allowed by taxation authorities in certain countries. When calculating depreciation, it is necessary to obtain details of the methods and rates permitted by the appropriate authority and to use the information provided.

Figure 9-5 shows the fall in book value with time for a piece of equipment having a fixed-capital cost of \$120,000, a useful life of 10 years, and a scrap value of \$20,000. This fall in value is calculated by using (1) straight-line depreciation, (2) double-declining depreciation, and (3) sum-of-years-digits depreciation.

### Traditional Measures of Profitability

**Rate-of-Return Methods** Although traditional rate-of-return methods have the advantage of simplicity, they can yield very misleading results. They are based on the relation

$$\text{Percent rate of return} = \frac{\text{[annual profit]}}{\text{[invested capital]}} \times 100 \quad (9-25)$$

Since different meanings are ascribed to both annual profit and invested capital in Eq. (9-25), it is important to define the terms precisely. The invested capital may refer to the original total capital investment, the depreciated investment, the average investment, the current value of the investment, or something else. The annual profit may refer to the net annual profit before tax  $A_{NP}$ , the net annual profit after tax  $A_{NNP}$ , the annual cash income before tax  $A_{CI}$ , or the annual cash income after tax  $A_{NCI}$ .

The fractional interest rate of return based on the net annual profit after tax and the original investment is

$$i = A_{NNP}/C_{TC} \quad (9-26)$$

which can be written in terms of Eq. (9-9) as

$$i = (A_{NCI}/C_{TC}) - (A_{BD}/C_{TC}) \quad (9-27)$$

where  $A_{BD}$  is the balance-sheet annual depreciation. The main disadvantage of using Eq. (9-27) is that the fractional depreciation rate



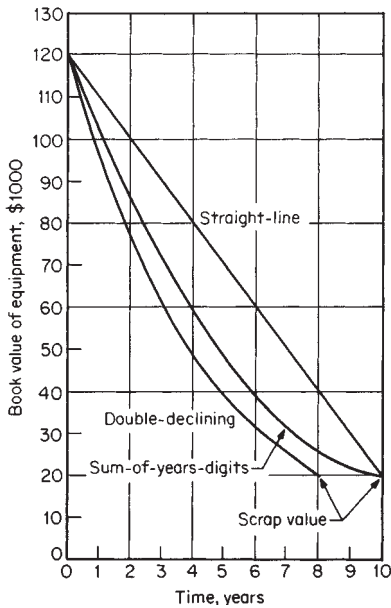


FIG. 9-5 Book value against time for various depreciation methods.

$A_{BD}/C_{TC}$  is arbitrarily assessed. Its value will affect the fractional rate of return considerably and may lead to erroneous conclusions when making comparisons between different companies. This is particularly true when making international comparisons.

Figures 9-6, 9-7, and 9-8 show the effect of the depreciation method on profit for a project described by the following data:

- Net annual cash income after tax  $A_{NCI} = \$25,500$  in each of 10 years
- Fixed-capital cost  $C_{FC} = \$120,000$

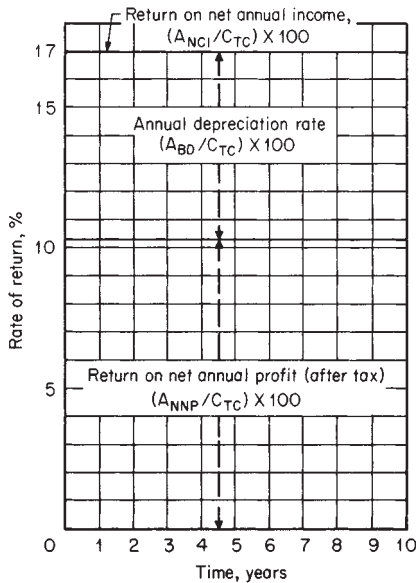


FIG. 9-6 Effect of straight-line depreciation on rate of return for a project.  $A_{BD}$  = annual depreciation allowance;  $A_{NCI}$  = annual net cash income after tax;  $A_{NNP}$  = annual net profit after payment of tax;  $C_{TC}$  = total capital cost.

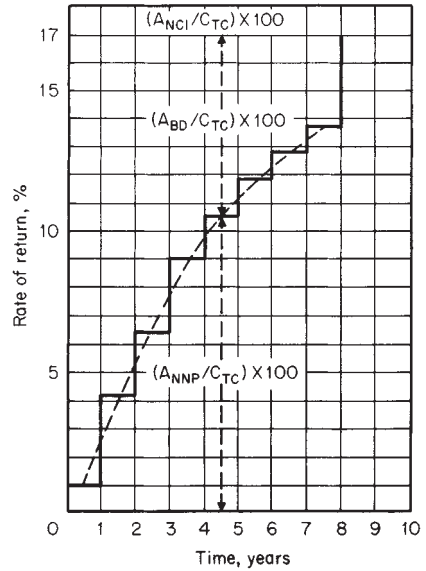


FIG. 9-7 Effect of double-declining depreciation on rate of return for a project.

- Estimated salvage value of plant items  $S = \$20,000$
- Working capital  $C_{WC} = \$10,000$
- Cost of land  $C_L = \$20,000$

In Eq. (9-27),  $i$  can be taken either on the basis of the net annual cash income for a particular year or on the basis of an average net annual cash income over the length of the life of the project. The equations corresponding to Eq. (9-26) based on depreciated and average investment are given respectively as follows:

$$i = A_{NNP} / (P_s + C_{WC} + C_L) \tag{9-28}$$

and 
$$i = 2A_{NNP} / (C_{FC} + S + 2C_{WC} + 2C_L) \tag{9-29}$$

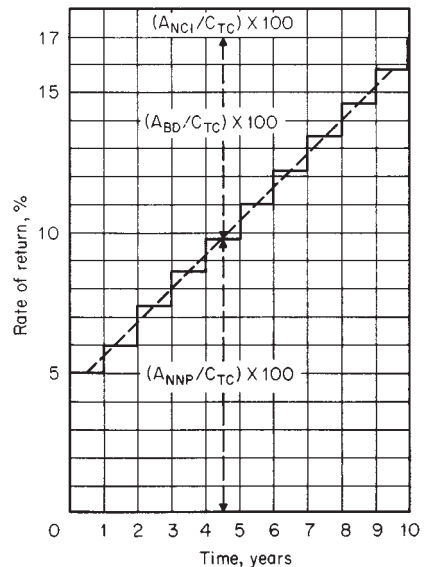


FIG. 9-8 Effect of sum-of-years-digits depreciation on rate of return for a project.

where  $P_s$  is the book value of the fixed-capital investment at the end of a particular year  $s'$ . If  $i$  is taken on the basis of average values for  $A_{NPF}$  over the length of the project, an average value for the working capital  $C_{WC}$  must be used.

In Eqs. (9-28) and (9-29), the computations are based on unchanging values of the cost of land and other nondepreciable costs  $C_L$ . This is unrealistic, since the value of land has a tendency to rise. In such circumstances, the accountancy principle of conservatism requires that the lowest valuation be adopted.

**Payback Period** Another traditional method of measuring profitability is the payback period or fixed-capital-return period. Actually, this is really a measure not of profitability but of the time it takes for cash flows to recoup the original fixed-capital expenditure.

The net annual cash flow after tax is given by

$$A_{CF} = A_{NCI} - A_{TC} \tag{9-4}$$

where  $A_{TC}$  is the annual expenditure of capital, which is not necessarily zero after the plant has been built. The payback period (BBP) is the time required for the cumulative net cash flow taken from the startup of the plant to equal the depreciable fixed-capital investment ( $C_{FC} - S$ ). It is the value of  $s'$  that satisfies

$$\sum_{s'=0}^{s'=(BBP)} A_{CF} = C_{FC} - S \tag{9-30}$$

The payback-period method takes no account of cash flows or profits received after the breakeven point has been reached. The method is based on the premise that the earlier the fixed capital is recovered, the better the project. However, this approach can be misleading.

Let us consider projects A and B, having net annual cash flows as listed in Table 9-2. Both projects have initial fixed-capital expenditures of \$100,000. On the basis of payback period, project A is the more desirable since the fixed-capital expenditure is recovered in 3 years, compared with 5 years for project B. However, project B runs for 7 years with a cumulative net cash flow of \$110,000. This is obviously more profitable than project A, which runs for only 4 years with a cumulative net cash flow of only \$10,000.

**Time Value of Money** A large part of business activity is based on money that can be loaned or borrowed. When money is loaned, there is always a risk that it may not be returned. A sum of money called interest is the inducement offered to make the risk acceptable. When money is borrowed, interest is paid for the use of the money over a period of time. Conversely, when money is loaned, interest is received.

The amount of a loan is known as the principal. The longer the period of time for which the principal is loaned, the greater the total amount of interest paid. Thus, the future worth of the money  $F$  is greater than its present worth  $P$ . The relationship between  $F$  and  $P$  depends on the type of interest used.

Table 9-3 gives examples of compound-interest factors and example compound-interest calculations.

**Simple Interest** When simple interest is used,  $F$  and  $P$  are related by

$$F = P(1 + ni) \tag{9-31}$$

where  $i$  is the fractional interest rate per period and  $n$  is the number of interest periods. Normally, the interest period is 1 year, in which case  $i$  is known as the effective interest rate.

**TABLE 9-2 Cash Flows for Two Projects**

Year	Cash flows $A_{CF}$	
	Project A	Project B
0	\$100,000	\$100,000
1	50,000	0
2	30,000	10,000
3	20,000	20,000
4	10,000	30,000
5	0	40,000
6	0	50,000
7	0	60,000
$\sum A_{CF}$	\$ 10,000	\$110,000
Payback period (BBP)	3 years	5 years

**Annual Compound Interest** It is more common to use compound interest, in which  $F$  and  $P$  are related by

$$F = P(1 + i)^n \tag{9-32}$$

or 
$$F = Pf_i \tag{9-33}$$

where the compound-interest factor  $f_i = (1 + i)^n$ . Values for compound-interest factors are readily available in tables.

The present value  $P$  of a future sum of money  $F$  is

$$P = F/(1 + i)^n \tag{9-34}$$

or 
$$F = P/f_d \tag{9-35}$$

where the discount factor  $f_d$  is

$$f_d = 1/f_i = 1/[(1 + i)^n]$$

Values for the discount factors are readily available in tables which show that it will take 7.3 years for the principal to double in amount if compounded annually at 10 percent per year and 14.2 years if compounded annually at 5 percent per year.

For the case of different annual fractional interest rates ( $i_1, i_2, \dots, i_n$  in successive years), Eq. (9-32) should be written in the form

$$F = P(1 + i_1)(1 + i_2)(1 + i_3) \dots (1 + i_n) \tag{9-36}$$

**Short-Interval Compound Interest** If interest payments become due  $m$  times per year at compound interest,  $mn$  payments are required in  $n$  years. The nominal annual interest rate  $i'$  is divided by  $m$  to give the effective interest rate per period. Hence,

$$F = P[1 + (i'/m)]^{mn} \tag{9-37}$$

It follows that the effective annual interest  $i$  is given by

$$i = [1 + (i'/m)]^m - 1 \tag{9-38}$$

The annual interest rate equivalent to a compound-interest rate of 5 percent per month (i.e.,  $i'/m = 0.05$ ) is calculated from Eq. (9-38) to be

$$i = (1 + 0.05)^{12} - 1 = 0.796, \text{ or } 79.6 \text{ percent/year}$$

**Continuous Compound Interest** As  $m$  approaches infinity, the time interval between payments becomes infinitesimally small, and in the limit Eq. (9-37) reduces to

$$F = P \exp(i'n) \tag{9-39}$$

A comparison of Eqs. (9-32) and (9-39) shows that the nominal interest rate  $i'$  on a continuous basis is related to the effective interest rate  $i$  on an annual basis by

$$\exp(i'n) = (1 + i)^n \tag{9-40}$$

Numerically, the difference between continuous and annual compounding is small. In practice, it is probably far smaller than the errors in the estimated cash-flow data. Annual compound interest conforms more closely to current acceptable accounting practice. However, the small difference between continuous and annual compounding may be significant when applied to very large sums of money.

Let us suppose that \$100 is invested at a nominal interest rate of 5 percent. We then compute the future worth of the investment after 2 years and also compute the effective annual interest rate for the following kinds of interest: (1) simple, (2) annual compound, (3) monthly compound, (4) daily compound, and (5) continuous compound. The following tabulation shows the results of the calculations, along with the appropriate equation to be used:

Interest type	Equation	Future worth $F$	Effective rate $i$ , %	Equation
1	(9-31)	\$110,000	5	(9-31)
2	(9-32)	\$110,250	5	(9-38)
3	(9-37)	\$110,495	5.117	(9-38)
4	(9-37)	\$110,516	5.1267	(9-38)
5	(9-39)	\$110,517	5.1271	(9-38)

When computing the effective annual rate for continuous compounding, the first term of Eq. (9-38),  $[1 + (i'/m)]^m$ , approaches  $e^{i'}$  as  $m$  approaches infinity.

**TABLE 9-3 Compound Interest Factors\***

(For examples demonstrating use see end of table.)

Single payment		Uniform annual series				Single payment		Uniform annual series					
Compound-amount factor	Present-worth factor	Sinking-fund factor	Capital-recovery factor	Compound-amount factor	Present-worth factor	Compound-amount factor	Present-worth factor	Sinking-fund factor	Capital-recovery factor	Compound-amount factor	Present-worth factor		
Given $P_t$ to find $F$ $(1+i)^n$	Given $F_t$ to find $P$ $\frac{1}{(1+i)^n}$	Given $F_t$ to find $A$ $\frac{i}{(1+i)^n - 1}$	Given $P_t$ to find $A$ $\frac{i(1+i)^n}{(1+i)^n - 1}$	Given $A_t$ to find $F$ $\frac{(1+i)^n - 1}{i}$	Given $A_t$ to find $P$ $\frac{(1+i)^n - 1}{i(1+i)^n}$	Given $P_t$ to find $F$ $(1+i)^n$	Given $F_t$ to find $P$ $\frac{1}{(1+i)^n}$	Given $F_t$ to find $A$ $\frac{i}{(1+i)^n - 1}$	Given $P_t$ to find $A$ $\frac{i(1+i)^n}{(1+i)^n - 1}$	Given $A_t$ to find $F$ $\frac{(1+i)^n - 1}{i}$	Given $A_t$ to find $P$ $\frac{(1+i)^n - 1}{i(1+i)^n}$	$n$	
5% Compound Interest Factors						6% Compound Interest Factors							
1	1.050	0.9524	1.00000	1.05000	1.000	0.952	1.060	0.9434	1.00000	1.06000	1.000	0.943	1
2	1.103	.9070	0.48780	0.53780	2.050	1.859	1.124	.8900	0.48544	0.54544	2.060	1.833	2
3	1.158	.8638	.31721	.36721	3.153	2.723	1.191	.8396	.31411	.37411	3.184	2.673	3
4	1.216	.8227	.23201	.28201	4.310	3.546	1.262	.7921	.22859	.28859	4.375	3.465	4
5	1.276	.7835	.18097	.23097	5.526	4.329	1.338	.7473	.17740	.23740	5.637	4.212	5
6	1.340	.7462	.14702	.19702	6.802	5.076	1.419	.7050	.14336	.20336	6.975	4.917	6
7	1.407	.7107	.12282	.17282	8.142	5.786	1.504	.6651	.11914	.17914	8.394	5.582	7
8	1.477	.6768	.10472	.15472	9.549	6.463	1.594	.6274	.10104	.16104	9.897	6.210	8
9	1.551	.6446	.09069	.14069	11.027	7.108	1.689	.5919	.08702	.14702	11.491	6.802	9
10	1.629	.6139	.07940	.12950	12.578	7.722	1.791	.5584	.07587	.13587	13.181	7.360	10
11	1.710	.5847	.07039	.12039	14.207	8.306	1.898	.5268	.06679	.12679	14.972	7.887	11
12	1.796	.5568	.06283	.11283	15.917	8.863	2.012	.4970	.05928	.11928	16.870	8.384	12
13	1.886	.5303	.05646	.10646	17.713	9.394	2.133	.4688	.05296	.11296	18.882	8.853	13
14	1.980	.5051	.05102	.10102	19.599	9.899	2.261	.4423	.04758	.10758	21.015	9.295	14
15	2.079	.4810	.04634	.09634	21.579	10.380	2.397	.4173	.04296	.10296	23.276	9.712	15
16	2.183	.4581	.04227	.09227	23.657	10.838	2.540	.3936	.03895	.09895	25.673	10.106	16
17	2.292	.4363	.03870	.08870	25.840	11.274	2.693	.3714	.03544	.09544	28.213	10.477	17
18	2.407	.4155	.03555	.08555	28.132	11.690	2.854	.3503	.03236	.09236	30.906	10.828	18
19	2.527	.3957	.03275	.08275	30.539	12.085	3.026	.3305	.02962	.08962	33.760	11.158	19
20	2.653	.3769	.03024	.08024	33.066	12.462	3.207	.3118	.02718	.08718	36.786	11.470	20
21	2.786	.3589	.02800	.07800	35.719	12.821	3.400	.2942	.02500	.08500	39.993	11.764	21
22	2.925	.3418	.02597	.07597	38.505	13.163	3.604	.2775	.02305	.08305	43.392	12.042	22
23	3.072	.3256	.02414	.07414	41.430	13.489	3.820	.2618	.02128	.08128	46.996	12.303	23
24	3.225	.3101	.02247	.07247	44.502	13.799	4.049	.2470	.01968	.07968	50.816	12.550	24
25	3.386	.2953	.02095	.07095	47.727	14.094	4.292	.2330	.01823	.07823	54.865	12.783	25
26	3.556	.2812	.01956	.06956	51.113	14.375	4.549	.2198	.01690	.07690	59.156	13.003	26
27	3.733	.2678	.01829	.06829	54.669	14.643	4.822	.2074	.01570	.07570	63.706	13.211	27
28	3.920	.2551	.01712	.06712	58.403	14.898	5.112	.1956	.01459	.07459	68.528	13.406	28
29	4.116	.2429	.01605	.06605	62.323	15.141	5.418	.1846	.01358	.07358	73.640	13.591	29
30	4.322	.2314	.01505	.06505	66.489	15.372	5.743	.1741	.01265	.07265	79.058	13.765	30
31	4.538	.2204	.01413	.06413	70.761	15.593	6.088	.1643	.01179	.07179	84.802	13.929	31
32	4.765	.2099	.01328	.06328	75.299	15.803	6.453	.1550	.01100	.07100	90.890	14.084	32
33	5.003	.1999	.01249	.06249	80.064	16.003	6.841	.1462	.01027	.07027	97.343	14.230	33
34	5.253	.1904	.01176	.06176	85.067	16.193	7.251	.1379	.00960	.06960	104.184	14.368	34
35	5.516	.1813	.01107	.06107	90.320	16.374	7.686	.1301	.00897	.06897	111.435	14.498	35
40	7.040	.1420	.00828	.05828	120.800	17.159	10.286	.0972	.00646	.06646	154.762	15.046	40
45	8.985	.1113	.00626	.05626	159.700	17.774	13.765	.0727	.00470	.06470	212.744	15.456	45
50	11.467	.0872	.00478	.05478	209.348	18.256	18.420	.0543	.00344	.06344	290.336	15.762	50
55	14.636	.0683	.00367	.05367	272.713	18.633	24.650	.0406	.00254	.06254	394.172	15.991	55
60	18.679	.0535	.00283	.05283	353.584	18.929	32.988	.0303	.00188	.06188	533.128	16.161	60
65	23.840	.0419	.00219	.05219	456.798	19.161	44.145	.0227	.00139	.06139	719.083	16.289	65
70	30.426	.0329	.00170	.05170	588.529	19.343	59.076	.0169	.00103	.06103	967.932	16.385	70
75	38.833	.0258	.00132	.05132	756.654	19.485	79.057	.0126	.00077	.06077	1,300.949	16.456	75
80	49.561	.0202	.00103	.05103	971.229	19.596	105.796	.0095	.00057	.06057	1,746.600	16.509	80
85	63.254	.0158	.00080	.05080	1,245.087	19.684	141.579	.0071	.00043	.06043	2,342.982	16.549	85
90	80.730	.0124	.00063	.05063	1,594.607	19.752	189.465	.0053	.00032	.06032	3,141.075	16.579	90
95	103.035	.0097	.00049	.05049	2,040.694	19.806	253.546	.0039	.00024	.06024	4,209.104	16.601	95
100	131.501	.0076	.00038	.05038	2,610.025	19.848	339.302	.0029	.00018	.06018	5,638.368	16.618	100

TABLE 9-3 Compound Interest Factors (Concluded)

Examples of Use of Table and Factors

*Given:* \$2500 is invested now at 5 percent.*Required:* Accumulated value in 10 years (i.e., the amount of a given principal).*Solution:*

$$F = P(1 + i)^n = \$2500 \times 1.05^{10}$$

$$\text{Compound-amount factor} = (1 + i)^n = 1.05^{10} = 1.629$$

$$F = \$2500 \times 1.629 = \$4062.50$$

*Given:* \$19,500 will be required in 5 years to replace equipment now in use.*Required:* With interest available at 3 percent, what sum must be deposited in the bank at present to provide the required capital (i.e., the principal which will amount to a given sum)?*Solution:*

$$P = F \frac{1}{(1 + i)^n} = \$19,500 \frac{1}{1.03^5}$$

$$\text{Present-worth factor} = 1/(1 + i)^n = 1/1.03^5 = 0.8626$$

$$P = \$19,500 \times 0.8626 = \$16,821$$

*Given:* \$50,000 will be required in 10 years to purchase equipment.*Required:* With interest available at 4 percent, what sum must be deposited each year to provide the required capital (i.e., the annuity which will amount to a given fund)?*Solution:*

$$A = F \frac{i}{(1 + i)^n - 1} = \$50,000 \frac{0.04}{1.04^{10} - 1}$$

$$\text{Sinking-fund factor} = \frac{i}{(1 + i)^n - 1} = \frac{0.04}{1.04^{10} - 1} = 0.08329$$

$$A = \$50,000 \times 0.08329 = \$4,164$$

*Given:* \$20,000 is invested at 10 percent interest.*Required:* Annual sum that can be withdrawn over a 20-year period (i.e., the annuity provided by a given capital).*Solution:*

$$A = P \frac{i(1 + i)^n}{(1 + i)^n - 1} = \$20,000 \frac{0.10 \times 1.10^{20}}{1.10^{20} - 1}$$

$$\text{Capital-recovery factor} = \frac{i(1 + i)^n}{(1 + i)^n - 1} = \frac{0.10 \times 1.10^{20}}{1.10^{20} - 1} = 0.11746$$

$$A = \$20,000 \times 0.11746 = \$2349.20$$

*Given:* \$500 is invested each year at 8 percent interest.*Required:* Accumulated value in 15 years (i.e., amount of an annuity).*Solution:*

$$F = A \frac{(1 + i)^n - 1}{i} = \$500 \frac{1.08^{15} - 1}{0.08}$$

$$\text{Compound-amount factor} = \frac{(1 + i)^n - 1}{i} = \frac{1.08^{15} - 1}{0.08} = 27.152$$

$$F = \$500 \times 27.152 = \$13,576$$

*Given:* \$8000 is required annually for 25 years.*Required:* Sum that must be deposited now at 6 percent interest.*Solution:*

$$P = A \frac{(1 + i)^n - 1}{i(1 + i)^n} = \$8000 \frac{1.06^{25} - 1}{0.06 \times 1.06^{25}}$$

$$\text{Present-worth factor} = \frac{(1 + i)^n - 1}{i(1 + i)^n} = \frac{1.06^{25} - 1}{0.06 \times 1.06^{25}} = 12.783$$

$$P = \$8000 \times 12.783 = \$102,264$$

\*Factors presented for two interest rates only. By using the appropriate formulas, values for other interest rates may be calculated.

**Annual Cost or Payment** A series of equal annual payments  $A$  invested at a fractional interest rate  $i$  at the end of each year over a period of  $n$  years may be used to build up a future sum of money  $F$ . These relations are given by

$$F = A \left[ \frac{(1+i)^n - 1}{i} \right] \quad (9-41)$$

or 
$$F = Af_{AF} \quad (9-42)$$

where the annuity future-worth factor is

$$f_{AF} = i / [(1+i)^n - 1]$$

Values for  $f_{AF}$  are readily available in tables.

Equation (9-41) can be combined with Eq. (9-34) to yield

$$P = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] \quad (9-43)$$

$$P = A/f_{AP} \quad (9-44)$$

where  $P$  is the present worth of the series of future equal annual payments  $A$  and the annuity present-worth factor is

$$f_{AP} = [i(1+i)^n] / [(1+i)^n - 1]$$

Values for  $f_{AP}$  are also available in tables.

Alternatively, the annual payment  $A$  required to build up a future sum of money  $F$  with a present value of  $P$  is given by

$$A = Ff_{AF} \quad (9-45)$$

$$A = Pf_{AP} \quad (9-46)$$

Equation (9-41) represents the future sum of a series of uniform annual payments that are invested at a stated interest rate over a period of years. This procedure defines an ordinary annuity. Other forms of annuities include the annuity due, in which payments are made at the beginning of the year instead of at the end; and the deferred annuity, in which the first payment is deferred for a definite number of years.

**Capitalized Cost** A piece of equipment of fixed-capital cost  $C_{FC}$  will have a finite life of  $n$  years. The capitalized cost of the equipment  $C_K$  is defined by

$$(C_K - C_{FC})(1+i)^n = C_K - S \quad (9-47)$$

$C_K$  is in excess of  $C_{FC}$  by an amount which, when compounded at an annual interest rate  $i$  for  $n$  years, will have a future worth of  $C_K$  less the salvage or scrap value  $S$ . If the renewal cost of the equipment remains constant at  $(C_{FC} - S)$  and the interest rate remains constant at  $i$ , then  $C_K$  is the amount of capital required to replace the equipment in perpetuity.

Equation (9-47) may be rewritten as

$$C_K = \left[ C_{FC} - \frac{S}{(1+i)^n} \right] \left[ \frac{(1+i)^n}{(1+i)^n - 1} \right] \quad (9-48)$$

or 
$$C_K = (C_{FC} - Sf_d)f_k \quad (9-49)$$

where  $f_d$  is the discount factor and  $f_k$ , the capitalized-cost factor, is

$$f_k = [(1+i)^n] / [(1+i)^n - 1]$$

Values for each factor are available in tables.

**Example 1: Capitalized Cost of Equipment** A piece of equipment has been installed at a cost of \$100,000 and is expected to have a working life of 10 years with a scrap value of \$20,000. Let us calculate the capitalized cost of the equipment based on an annual compound-interest rate of 5 percent.

Therefore, we substitute values into Eq. (9-48) to give

$$C_K = \left[ \$100,000 - \frac{\$20,000}{(1+0.05)^{10}} \right] \left[ \frac{(1+0.05)^{10}}{(1+0.05)^{10} - 1} \right]$$

$$C_K = [\$100,000 - (\$20,000/1.62889)](2.59009)$$

$$C_K = \$227,207$$

**Modern Measures of Profitability** An investment in a manufacturing process must earn more than the cost of capital for it to be worthwhile. The larger the additional earnings, the more profitable

the venture and the greater the justification for putting the capital at risk. A profitability estimate is an attempt to quantify the desirability of taking this risk.

The ways of assessing profitability to be considered in this section are (1) discounted-cash-flow rate of return (DCFRR), (2) net present value (NPV) based on a particular discount rate, (3) equivalent maximum investment period (EMIP), (4) interest-recovery period (IRP), and (5) discounted breakeven point (DBEP).

**Cash Flow** Let us consider a project in which  $C_{FC} = \$1,000,000$ ,  $C_{WC} = \$90,000$ , and  $C_L = \$10,000$ . Hence,  $C_{TC} = \$1,100,000$  from Eq. (9-14). If all this capital expenditure occurs in Year 0 of the project, then  $A_{TC} = \$1,100,000$  in Year 0 and  $-A_{TC} = -\$1,100,000$ . From Eq. (9-4), it is seen that any capital expenditure makes a negative contribution to the net annual cash flow  $A_{CF}$ .

Let us consider another project in which the fixed-capital expenditure is spread over 2 years, according to the following pattern:

$$C_{FC} = C_{FC0} + C_{FC1}$$

Year 0	Year 1
$C_{FC0} = \$400,000$	$C_{FC1} = \$600,000$
$C_L = 10,000$	$C_{WC} = 90,000$
$A_{TC} = 410,000$	$A_{TC} = 690,000$

In the final year of the project, the working capital and the land are recovered, which in this case cost a total of \$100,000. Thus, in the final year of the project,  $A_{TC} = -\$100,000$  and  $-A_{TC} = +\$100,000$ . From Eq. (9-4), it is seen that any capital recovery makes a positive contribution to the net annual cash flow.

During the development and construction stages of a project,  $A_{CI}$  and  $A_{JT}$  are both zero in Eqs. (9-2) and (9-4). For this period, the cash flow for the project is negative and is given by

$$A_{CF} = -A_{TC} \quad (9-50)$$

Figure 9-9 shows the cash-flow stages in a project. The expenditure during the research and development stage is normally relatively small. It will usually include some preliminary process design and a market survey. Once the decision to go ahead with the project has been taken, detailed process-engineering design will commence, and the rate of expenditure starts to increase. The rate is increased still further when equipment is purchased and construction gets under way. There is no return on the investment until the plant is started up. Even during startup, there is some additional expenditure. Once the plant is operating smoothly, an inflow of cash is established. During the early stages of a project, there may be a tax credit because of the existence of expenses without corresponding income.

**Discounted Cash Flow** The present value  $P$  of a future sum of money  $F$  is given by

$$P = Ff_d \quad (9-51)$$

where  $f_d = 1/(1+i)^n$ , the discount factor. Values for this factor are readily available in tables. For example, \$90,909 invested at an annual interest rate of 10 percent becomes \$100,000 after 1 year. Similarly, \$38,554 invested at 10 percent becomes \$100,000 after 10 years.

Thus, cash flow in the early years of a project has a greater value than the same amount in the later years of a project. Therefore, it pays to receive money as soon as possible and to delay paying out money for as long as possible.

Time is taken into account by using the annual discounted cash flow  $A_{DCF}$ , which is related to the annual cash flow  $A_{CF}$  and the discount factor  $f_d$  by

$$A_{DCF} = A_{CF}f_d \quad (9-52)$$

Thus, at the end of any year  $n$ ,

$$(A_{DCF})_n = (A_{CF})_n / (1+i)^n$$

The sum of the annual discounted cash flows over  $n$  years,  $\sum A_{DCF}$ , is known as the net present value (NPV) of the project:

$$(\text{NPV}) = \sum_0^n (A_{DCF})_n \quad (9-53)$$

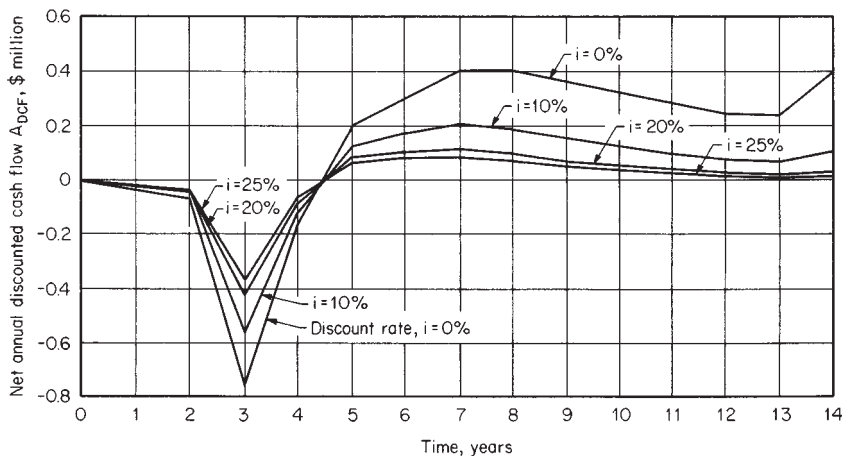


FIG. 9-9 Effect of discount rate on cash flows.

The value of (NPV) is directly dependent on the choice of the fractional interest rate *i*. An interest rate can be selected to make (NPV) = 0 after a chosen number of years. This value of *i* is found from

$$\sum_0^n (A_{DCF})_n = \frac{(A_{CF})_0}{(1+i)^0} + \frac{(A_{CF})_1}{(1+i)^1} + \dots + \frac{(A_{CF})_n}{(1+i)^n} = 0 \quad (9-54)$$

Equation (9-54) may be solved for *i* either graphically or by an iterative trial-and-error procedure. The value of *i* given by Eq. (9-54) is known as the discounted-cash-flow rate of return (DCFRR). It is also known as the profitability index, true rate of return, investor's rate of return, and interest rate of return.

**Cash-Flow Curves** Figure 9-9 shows the cash-flow stages in a project together with their discounted-cash-flow values for the data given in Table 9-4. In addition to cash-flow and discounted-cash-flow curves, it is also instructive to plot cumulative-cash-flow and cumulative-discounted-cash-flow curves. These are shown in Fig. 9-10 for the data in Table 9-4.

The cost of capital may also be considered as the interest rate at which money can be invested instead of putting it at risk in a manufacturing process. Let us consider the process data listed in Table 9-4 and plotted in Fig. 9-10. If the cost of capital is 10 percent, then the appropriate discounted-cash-flow curve in Fig. 9-10 is *abcdef*. Up to point *e*, or 8.49 years, the capital is at risk. Point *e* is the discounted breakeven point (DBEP). At this point, the manufacturing process

has paid back its capital and produced the same return as an equivalent amount of capital invested at a compound-interest rate of 10 percent. Beyond the breakeven point, the capital is no longer at risk and any cash flow above the horizontal baseline,  $\sum A_{DCF} = 0$ , is in excess of the return on an equivalent amount of capital invested at a compound-interest rate of 10 percent. Thus, the greater the area above the baseline, the more profitable the process.

When (NPV) and (DCFRR) are computed, depreciation is not considered as a separate expense. It is simply used as a permitted writing-down allowance to reduce the annual amount of tax in accordance with the rules applying in the country of earning. The tax payable is deducted in accordance with Eq. (9-2) in the year in which it is paid, which may differ from the year in which the corresponding income was earned.

A (DCFRR) of, say, 15 percent implies that 15 percent per year will be earned on the investment, in addition to which the project generates sufficient money to repay the original investment plus any interest payable on borrowed capital plus all taxes and expenses.

It is not normally possible to make a comprehensive assessment of profitability with a single number. The shape of the cumulative-cash-flow and cumulative-discounted-cash-flow curves both before and after the breakeven point is an important factor.

D. H. Allen [*Chem. Eng.*, **74**, 75-78 (July 3, 1967)] accounted for the shape of the cumulative-undiscounted-cash-flow curve up to the

TABLE 9-4 Annual Cash Flows and Discounted Cash Flows for a Project

Year	$A_{CF}$ , \$	$\sum A_{CF}$ , \$	Discounted at 10%			Discounted at 20%			Discounted at 25%		
			$f_d$	$A_{DCF}$ , \$	$\sum A_{DCF}$ , \$	$f_d$	$A_{DCF}$ , \$	$\sum A_{DCF}$ , \$	$f_d$	$A_{DCF}$ , \$	$\sum A_{DCF}$ , \$
0	-10,000	-10,000	1.00000	-10,000	-10,000	1.00000	-10,000	-10,000	1.00000	-10,000	-10,000
1	-30,000	-40,000	0.90909	-27,273	-37,273	0.83333	-25,000	-35,000	0.80000	-24,000	-34,000
2	-60,000	-100,000	0.82645	-49,587	-86,860	0.69444	-41,666	-76,666	0.64000	-38,400	-72,400
3	-750,000	-850,000	0.75131	-563,483	-650,343	0.57870	-434,025	-510,691	0.51200	-384,000	-456,400
4	-150,000	-1,000,000	0.68301	-102,452	-752,795	0.48225	-72,338	-583,029	0.40960	-61,440	-517,840
5	+200,000	-800,000	0.62092	+124,184	-628,611	0.40188	+80,376	-502,653	0.32768	+65,536	-452,304
6	+300,000	-500,000	0.56447	+169,341	-459,270	0.33490	+100,470	-402,183	0.26214	+78,642	-373,662
7	+400,000	-100,000	0.51316	+205,264	-254,006	0.27908	+111,632	-290,551	0.20972	+83,888	-289,774
8	+400,000	+300,000	0.46651	+186,604	-67,402	0.23257	+93,028	-197,523	0.16777	+67,108	-222,666
9	+360,000	+660,000	0.42410	+152,676	+85,274	0.19381	+69,772	-127,751	0.13422	+48,319	-174,347
10	+320,000	+980,000	0.38554	+123,373	+208,647	0.16151	+51,683	-76,068	0.10737	+34,358	-139,989
11	+280,000	+1,260,000	0.35049	+98,137	+306,784	0.13459	+37,685	-38,383	0.08590	+24,052	-115,937
12	+240,000	+1,500,000	0.31863	+76,471	+383,255	0.11216	+26,918	-11,465	0.06872	+16,493	-99,444
13	+240,000	+1,740,000	0.28966	+69,518	+452,773	0.09346	+22,430	+10,965	0.05498	+13,195	-86,249
14	+400,000	+2,140,000	0.26333	+105,332	+558,105	0.07789	+31,156	+42,121	0.04398	+17,592	-68,657

NOTE:  $A_{CF}$  is net annual cash flow,  $A_{DCF}$  is net annual discounted cash flow,  $f_d$  is discount factor at stated interest,  $\sum A_{CF}$  is cumulative cash flow, and  $\sum A_{DCF}$  is cumulative discounted cash flow.



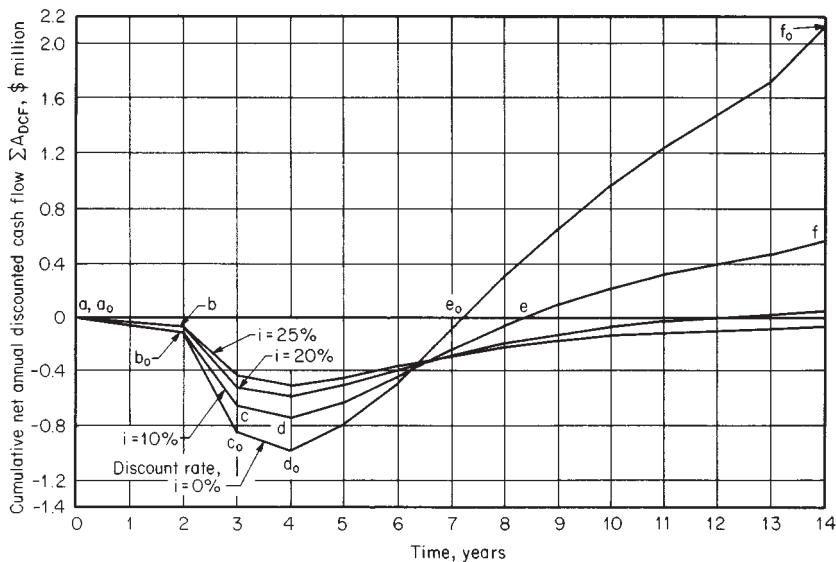


FIG. 9-10 Effect of discount rate on cumulative cash flows.

breakeven point  $e_0$  in Fig. 9-10 by using a parameter known as the equivalent maximum investment period (EMIP), which is defined as

$$(\text{EMIP}) = \frac{\text{area}(a_0 \text{ to } e_0)}{(\sum A_{CF})_{\max}} \quad \text{for } A_{CF} \leq 0 \quad (9-55)$$

where the area ( $a_0$  to  $e_0$ ) refers to the area below the horizontal baseline ( $\sum A_{CF} = 0$ ) on the cumulative-cash-flow curve in Fig. 9-10. The sum  $(\sum A_{CF})_{\max}$  is the maximum cumulative expenditure on the project, which is given by point  $d_0$  in Fig. 9-10. (EMIP) is a time in years. It is the equivalent period during which the total project debt would be outstanding if it were all incurred at one instant and all repaid at one instant. Clearly, the shorter the (EMIP), the more attractive the project.

Allen accounted for the shape of the cumulative-cash-flow curve

beyond the breakeven point by using a parameter known as the interest-recovery period (IRP). This is the time period (illustrated in Fig. 9-11) that makes the area ( $e_0$  to  $f_0$ ) above the horizontal baseline equal to the area ( $a_0$  to  $e_0$ ) below the horizontal baseline on the cumulative-cash-flow curve.

C. G. Sinclair [*Chem. Process. Eng.*, **47**, 147 (1966)] has considered similar parameters to the (EMIP) and (IRP) based on a cumulative-discounted-cash-flow curve.

Consideration of the cash-flow stages in Fig. 9-10 shows the factors that can affect the (EMIP) and (IRP). If the required capital investment is increased, it is necessary to increase the rate of income after startup for the (EMIP) to remain the same. In order to have the (EMIP) small, it is necessary to keep the research and development, design, and construction stages short.

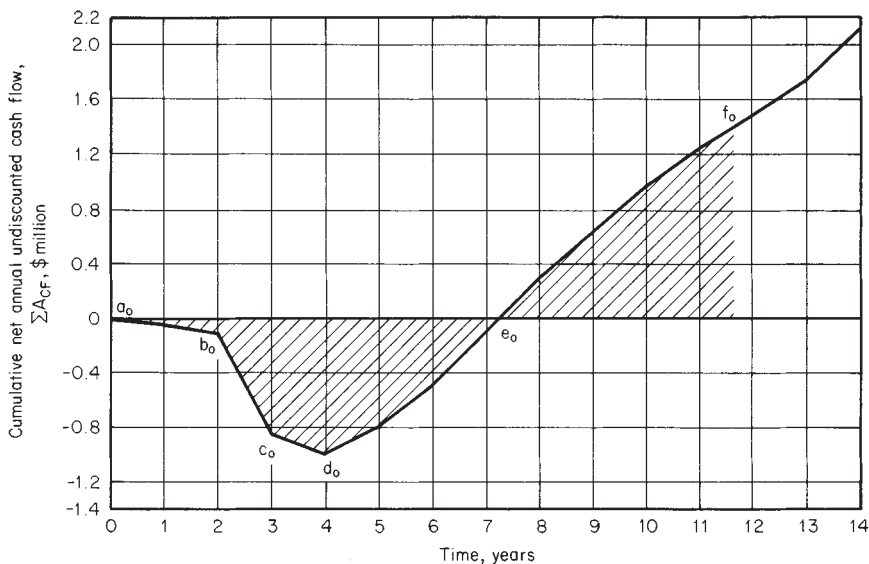


FIG. 9-11 Cumulative cash flow against time, showing interest recovery period.

**Example 2: Net Present Value for Different Depreciation Methods**

The following data describe a project. Revenue from annual sales and the total annual expense over a 10-year period are given in the first three columns of Table 9-5. The fixed-capital investment  $C_{FC}$  is \$1,000,000. Plant items have a zero salvage value. Working capital  $C_{WC}$  is \$90,000, and cost of land  $C_L$  is \$10,000. There are no tax allowances other than depreciation; i.e.,  $A_s$  is zero. The fractional tax rate  $t$  is 0.50.

We shall calculate for these data the net present value (NPV) for the following depreciation methods and discount factors:

- a. Straight-line, 10 percent
  - b. Straight-line, 20 percent
  - c. Double-declining, 10 percent
  - d. Sum-of-years-digits, 10 percent
  - e. Straight-line, 10 percent; income tax delayed for 1 year
- In addition, we shall calculate the discounted-cash-flow rate of return (DCFRR) with straight-line depreciation.

a. We begin the calculations for this example by finding the total capital cost  $C_{TC}$  for the project from Eq. (9-14). Here,  $C_{TC} = \$1,100,000$ . In Year 0, this amount is the same as the net annual capital expenditure  $A_{TC}$  and is listed in Table 9-5.

The annual rate of straight-line depreciation of the fixed-capital investment  $C_{FC}$ , from \$1,000,000 at startup to a salvage value  $S$ , of zero at the end of a productive life  $s$  of 10 years, is given by

$$A_D = (C_{FC} - S)/s$$

$$A_D = (\$1,000,000 - \$0)/10 \text{ years} = \$100,000/\text{year}$$

The annual cash income  $A_{CI}$  for Year 1, when  $A_s = \$400,000$  per year and  $A_{TE} = \$100,000$  per year, is, from Eq. (9-1), \$300,000 per year. Values for subsequent years are calculated in the same way and listed in Table 9-4.

Annual amount of tax  $A_{IT}$  for Year 1, when  $A_{CI} = \$300,000$  per year,  $A_D = \$100,000$  per year,  $A_s = \$0$  per year, and  $t = 0.5$ , is found from Eq. (9-3) to be

$$A_{IT} = [(\$300,000 - \$100,000 - \$0)/\text{year}](0.5)$$

$$= \$100,000/\text{year}$$

Values for subsequent years are calculated in the same way and listed in Table 9-4.

Net annual cash flow (after tax)  $A_{CF}$  for Year 0, when  $A_{CI} = \$0$  per year,  $A_{IT} = \$0$  per year, and  $A_{TC} = \$1,100,000$  per year, is found from Eq. (9-4) to be

$$A_{CF} = \$0/\text{year} - \$1,100,000/\text{year} = -\$1,100,000/\text{year}$$

Net annual cash flow (after tax)  $A_{CF}$  for Year 1, when  $A_{CI} = \$300,000$  per year,  $A_{TE} = \$100,000$  per year, and  $A_{TC} = \$0$  per year, is found from Eqs. (9-2) and (9-4) to be

$$A_{CF} = \$200,000/\text{year} - \$0/\text{year} = \$200,000/\text{year}$$

Values for the years up to and including Year 9 are calculated in the same way and listed in Table 9-5.

At the end of Year 10, the working capital ( $C_{WC} = \$90,000$ ) and the cost of land ( $C_L = \$10,000$ ) are recovered, so that the annual expenditure of capital  $A_{TC}$  in Year 10 is  $-\$100,000$  per year. Hence, the net annual cash flow (after tax) for Year 10 must reflect this recovery. By using Eq. (9-4),

$$A_{CF} = \$110,000/\text{year} - (-\$100,000/\text{year})$$

$$= \$210,000/\text{year}$$

The net annual discounted cash flow  $A_{DCF}$  for Year 1, when  $A_{CF} = \$200,000$  per year and  $f_d = 0.90909$  (for  $i = 10$  percent), is found from Eq. (9-52) to be

$$A_{DCF} = (\$200,000/\text{year})(0.90909) = \$181,820/\text{year}$$

Values for subsequent years are calculated in the same way and listed in Table 9-5.

The net present value (NPV) is found by summing the values of  $A_{DCF}$  for each year, as in Eq. (9-53). The net present value is found to be \$276,210, as given by the final entry in Table 9-5.

b. The same procedure is used for  $i = 20$  percent. The discount factors to be used in a table similar to Table 9-5 must be those for 20 percent. The (NPV) is found to be  $-\$151,020$ .

c. The calculations are similar to those for subexample a except that depreciation is computed by using the double-declining method of Eq. (9-19). The net present value is found to be \$288,530.

d. Again, the calculations are similar to those for subexample a except that depreciation is computed by using the sum-of-years-digits method of Eq. (9-20). The net present value is found to be \$316,610.

e. The calculations follow the same procedure as for subexample a, but the annual amount of tax  $A_{IT}$  is calculated for a particular year and then deducted from the annual cash income  $A_{CI}$  for the following year. The net present value for Year 11 is found to be \$341,980.

The discounted-cash-flow rate of return (DCFRR) can readily be obtained approximately by interpolation of the (NPV) for  $i = 10$  percent and  $i = 20$  percent:

$$(\text{DCFRR}) = 0.100 + [(\$276,210)/(0.20 - 0.10)]/[\$276,210 - (-\$151,020)]$$

$$(\text{DCFRR}) = 0.164, \text{ or } 16.4 \text{ percent}$$

The calculation of (DCFRR) usually requires a trial-and-error solution of Eq. (9-57), but rapidly convergent methods are available [N. H. Wild, *Chem. Eng.*, **83**, 153-154 (Apr. 12, 1976)]. For simplicity linear interpolation is often used.

A comparison of the (NPV) values for a 10 percent discount factor shows clearly that double-declining depreciation is more advantageous than straight-line depreciation and that sum-of-years-digits depreciation is more advantageous than the double-declining method. However, a significant advantage is obtained by delaying the payment of tax for 1 year even with straight-line depreciation.

This example is a simplified one. The cost of the working capital is assumed to be paid for in Year 0 and returned in Year 10. In practice, working capital increases with the production rate. Thus there may be an annual expenditure on working capital in a number of years subsequent to Year 0. Except in loss-making years, this is usually treated as an expense of the process. In loss-making years the cash injection for working capital is included in the  $A_{TC}$  for that year.

**Analysis of Techniques** Both the (NPV) and the (DCFRR) methods are based on discounted cash flows and in that sense are vari-

**TABLE 9-5 Annual Cash Flows, Straight-Line Depreciation, and 10 Percent Discount Factor**

Year	Before tax							After tax			
	$A_s$ , \$	$A_{TE}$ , \$	$A_{CI}$ , \$	$A_D + A_A$ , \$	$A_{CI} - A_D - A_A$ , \$	$A_{IT}$ , \$	$A_{TC}$ , \$	$A_{CF}$ , \$	$f_d$	$A_{DCF}$ , \$	(NPV), \$
0	0	0	0	0	0	0	+1,100,000	-1,100,000	1.0000	-1,100,000	-1,100,000
1	400,000	100,000	300,000	100,000	200,000	100,000	0	200,000	0.90909	181,820	-918,180
2	500,000	100,000	400,000	100,000	300,000	150,000	0	250,000	0.82645	206,610	-711,570
3	500,000	110,000	390,000	100,000	290,000	145,000	0	245,000	0.75131	184,070	-527,500
4	500,000	120,000	380,000	100,000	280,000	140,000	0	240,000	0.68301	163,920	-363,580
5	520,000	130,000	390,000	100,000	290,000	145,000	0	245,000	0.62092	152,120	-211,460
6	520,000	130,000	390,000	100,000	290,000	145,000	0	245,000	0.56447	138,300	-73,160
7	520,000	140,000	380,000	100,000	280,000	140,000	0	240,000	0.51316	123,160	+50,000
8	390,000	140,000	250,000	100,000	150,000	75,000	0	175,000	0.46651	81,640	+131,640
9	350,000	150,000	200,000	100,000	100,000	50,000	0	150,000	0.42410	63,610	+195,250
10	280,000	160,000	120,000	100,000	20,000	10,000	-100,000	210,000	0.38554	80,960	+276,210

$A_s$  = revenue from annual sales.

$A_{TE}$  = total annual expense.

$A_{CI}$  = annual cash income.

$A_D + A_A$  = annual depreciation and other tax allowances.

$A_{CI} - A_D - A_A$  = taxable income.

$A_{IT} = (A_{CI} - A_D - A_A)t$  = amount of tax at  $t = 0.5$ .

$A_{TC}$  = total annual capital expenditure.

$A_{CF} = A_{CI} - A_{IT} - A_{TC}$  = net annual cash flow.

$f_d$  = discount factor at 10%.

$A_{DCF}$  = net annual discounted cash flow.

(NPV) =  $\sum A_{DCF}$  = net present value.

ations of the same basic method. However, when ranking different projects on the basis of profitability, they can produce different results.

Discounted-cash-flow rate of return (DCFRR) has the advantage of being unique and readily understood. However, when used alone, it gives no indication of the scale of the operation. The (NPV) indicates the monetary return, but unlike that of the (DCFRR) its value depends on the base year chosen for the calculation. Additional information is needed before its significance can be appreciated. However, when a company is considering investment in a portfolio of projects, individual (NPV)s have the advantage of being additive. This is not true of (DCFRR)s.

Increasing use is being made of the capital-rate-of-return ratio (CRR), which is the net present value (NPV) divided by the maximum cumulative expenditure or maximum net outlay,  $-(\sum A_{CF})_{max}$

$$(CRR) = (NPV)/(\sum A_{CF})_{max} \quad \text{for} \quad A_{CF} \leq 0 \quad (9-56)$$

The maximum net outlay is very important, since no matter how profitable a project is, the matter is academic if the company is unable to raise the money to undertake the project.

An (NPV) or (DCFRR) estimation will be no better than the accuracy of the projected cash flows over the life of the project. Clearly, one is likely to predict cash flows more accurately for 2 or 3 years ahead than, say, for 9 or 10 years ahead. However, since the cash flows for the later years are discounted to a greater extent than the cash flows for the earlier years, the latter have less effect on the overall estimation. Nevertheless, the difficulty of predicting cash flows in later years and the inherent lack of confidence in these predictions are serious disadvantages of the (DCFRR) method. In this respect (NPV)s are more useful since they are calculated for each year of a project. Thus, a project with a favorable (NPV) in the early years is a promising one.

One way of overcoming these disadvantages of the (DCFRR) method is to make estimates of the times required to reach certain values of (DCFRR). For example, how many years will it take to reach (DCFRR)s of 10 percent, 15 percent, 20 percent per year, etc.? Although (DCFRR) trial-and-error calculations and (NPV) calculations are tedious if done manually, computer programs which are suitable for programmable pocket calculators can readily be written to make calculations easier.

It is possible for some projects to reach a stage at which repairs, replacements, etc., can exceed net earnings in a particular year. In this case the cumulative-discounted-cash-flow or net-present-value curve plotted against time has a genuine maximum.

It is important when appraising by (NPV) and (DCFRR) not to consider the past in profitability estimations. Good money should never follow bad. It is unwise to continue to put money into a project if a more profitable project exists, even though this course may involve scrapping an expensive plant. Other considerations may, however, outweigh purely financial criteria in a particular case.

No single value for a profitability estimate should be accepted without further consideration. An intelligent consideration of the cumulative-cash-flow and cumulative-discounted-cash-flow curves such as those shown in Fig. 9-10, together with experience and good judgment, is the best way of assessing the financial merit of a project.

When considering future projects, top management will most likely require the discounted-cash-flow rate of return and the payback period. However, the estimators should also supply management with the following:

- Cumulative discounted-cash-flow or (NPV) curve for a discount rate of 10 percent per year or other agreed aftertax cost of capital
- Maximum net outlay,  $(\sum A_{CF})_{max}$ , for  $A_{CF} \leq 0$
- Discounted breakeven point (DBEP)
- Plot of capital-return ratio (CRR) against time over the life of the project for a discount rate at the cost of capital
- Number of years to reach discounted-cash-flow rates of return of, say, 15 and 25 percent per year respectively

Comparisons on the basis of time can be summarized by the following:

- Duration of the project
- Breakeven point (BEP)

- Discounted breakeven point (DBEP)
- Equivalent maximum investment period (EMIP)
- Interest-recovery period (IRP)
- Payback period (PBP)

Comparisons on the basis of cash can be summarized by the following:

- Maximum cumulative expenditure on the project,  $(\sum A_{CF})_{max}$ , for  $A_{CF} \leq 0$
- Maximum discounted cumulative expenditure on the project
- Cumulative net annual cash flow  $\sum A_{CF}$
- Cumulative net annual discounted cash flow  $\sum A_{DCF}$  or net present value (NPV)
- Capitalized cost  $C_K$

Comparisons on the basis of interest can be summarized as (1) the net present value (NPV) and (2) the discounted-cash-flow rate of return (DCFRR), which from Eqs. (9-53) and (9-54) is given formally as the fractional interest rate  $i$  which satisfies the relationship

$$(NPV) = \sum_0^n (A_{DCF})_n = 0 \quad (9-57)$$

When comparing project profitability, the ranking on the basis of net present value (NPV) may differ from that on the basis of discounted-cash-flow rate of return (DCFRR). Let us consider the data for two projects:

Cost of capital	Project C	Project D
$i, \%$	(NPV), \$	(NPV), \$
4	+100,000	+62,000
8	+41,000	+28,000
12	-2,000	+10,000
16	-32,000	-4,000

These (NPV) data are plotted against the cost of capital, as shown in Fig. 9-12. The discounted-cash-flow rate of return is the value of  $i$  that satisfies Eq. (9-5). From Fig. 9-12, (NPV) = 0 at a (DCFRR) of 11.8 percent for project C and 14.7 percent for project D. Thus, on the basis of (DCFRR), project D is more profitable than project C.

The (NPV) of project C is equal to that of project D at a cost of capital  $i = 9.8$  percent. If the cost of capital is greater than 9.8 percent, project D has the higher (NPV) and is, therefore, the more profitable. If the cost of capital is less than 9.8 percent, project C has the higher (NPV) and is the more profitable.

**Benefit of Early Cash Flows** It pays to receive cash inflows as early as possible and to delay cash outflows as long as possible.

Let us consider the net annual cash flows (after tax)  $A_{CF}$  for projects E, F, and G, listed in Table 9-6. The cumulative annual cash flows  $\sum A_{CF}$  and cumulative discounted annual cash flows  $\sum A_{DCF}$ , using a discount of 10 percent for these projects, are also listed in Table 9-6. We notice that the cumulative annual cash flow for each project is +\$1000.

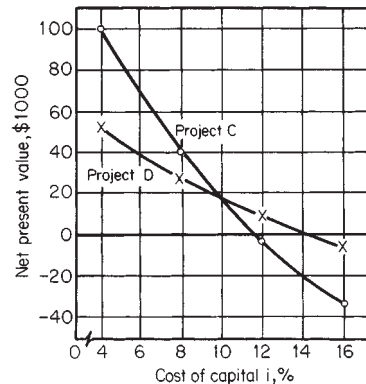


FIG. 9-12 Effect of cost of capital on net present value.

TABLE 9-6 Cash-Flow Data for Projects E, F, and G

Year	$A_{CF}$ , \$	Discounted at 10%			
		$\sum A_{CF}$ , \$	$f_d$	$A_{DCF}$ , \$	$\sum A_{DCF}$ = (NPV), \$
Project E					
0	-5000	-5000	1.0000	-5000	-5000
1	+3000	-2000	0.90909	+2727	-2273
2	+2000	0	0.82645	+1653	-620
3	+1000	+1000	0.75131	+751	+131
Project F					
0	-5000	-5000	1.0000	-5000	-5000
1	+1000	-4000	0.90909	+909	-4091
2	+2000	-2000	0.82645	+1653	-2438
3	+3000	+1000	0.75131	+2254	-184
Project G					
0	-5000	-5000	1.0000	-5000	-5000
1	+2000	-3000	0.90909	+1818	-3182
2	+2000	-1000	0.82645	+1653	-1529
3	+2000	+1000	0.75131	+1503	-26

The (DCFRR) is the discount rate that satisfies Eq. (9-57) in the final year of the project. We can approximate the (DCFRR) for each project as follows:

For project E,

$$\begin{aligned} \sum A_{CF} &= +\$1000 \text{ in Year 3 for } i = 0 \text{ percent} \\ \sum A_{DCF} &= +\$131 \text{ in Year 3 for } i = 10 \text{ percent} \\ \sum A_{DCF} &= \$0 \text{ in Year 3 for } i = (\text{DCFRR}) \end{aligned}$$

Therefore,

$$\begin{aligned} 1000/(1000 - 131) &\cong (\text{DCFRR})/10 \\ (\text{DCFRR}) &\cong 11.5 \text{ percent} \end{aligned}$$

Similarly for project F,

$$\begin{aligned} 1000/(1000 + 184) &\cong (\text{DCFRR})/10 \\ (\text{DCFRR}) &\cong 8.4 \text{ percent} \end{aligned}$$

Similarly for project G,

$$\begin{aligned} 1000/(1000 + 26) &\cong (\text{DCFRR})/10 \\ (\text{DCFRR}) &\cong 9.7 \text{ percent} \end{aligned}$$

In terms of net present value (NPV), the projects in order of merit are E, G, and F, with (NPV)s of +\$131, -\$26, and -\$184 respectively. In terms of (DCFRR), the projects in order of merit are also E, G, and F, with (DCFRR) values of 11.5 percent, 9.7 percent, and 8.4 percent respectively.

**When to Scrap an Existing Process** Let us suppose that a company invests \$50,000 in a manufacturing process that has positive net annual flows (after tax)  $A_{CF}$  of \$10,000 in each year. During the third year of operation, an alternative process becomes available. The new process would require an investment of \$40,000 but would have positive net annual cash flows (after tax) of \$20,000 in each year. The cost of capital is 10 percent, and it is estimated that a market will exist for the product for at least 6 more years. Should the company continue with the existing process (project H), or should it scrap project H and adopt the new process (project I)?

The net annual cash flows  $A_{CF}$  and cumulative discounted annual cash flow  $\sum A_{DCF}$  for a discount factor of 10 percent are listed in Table 9-7 for the two projects. At the end of Year 9, the net present values are

$$\begin{aligned} (\text{NPV}) &= +\$35,390 \text{ for project I} \\ (\text{NPV}) &= +\$7591 \text{ for project H} \end{aligned}$$

The difference is +\$27,779, which is numerically greater than the money lost by the end of Year 3 for project H. Thus project H should be scrapped, and the new project I adopted if only economic reasons need to be considered. Recovery of working capital and the cost of

TABLE 9-7 Cash-Flow Data for Projects H and I

Year	$A_{CF}$ , \$	Discounted at 10%		
		$f_d$	$A_{DCF}$ , \$	$\sum A_{DCF}$ = (NPV), \$
Project H				
0	-50,000	1.0000	-50,000	-50,000
1	+10,000	0.90909	9,091	-40,909
2	+10,000	0.82645	8,265	-32,644
3	+10,000	0.75131	7,513	-25,131
4	+10,000	0.68301	6,830	-18,301
5	+10,000	0.62092	6,209	-12,092
6	+10,000	0.56447	5,645	-6,447
7	+10,000	0.51316	5,132	-1,315
8	+10,000	0.46651	4,665	+3,350
9	+10,000	0.42410	4,241	+7,591
Project I				
3	-40,000	0.75131	-30,052	-30,052
4	+20,000	0.68301	+13,660	-16,392
5	+20,000	0.62092	+12,418	-3,974
6	+20,000	0.56447	+11,289	+7,315
7	+20,000	0.51316	+10,263	+17,578
8	+20,000	0.46651	+9,330	+26,908
9	+20,000	0.42410	+8,482	+35,390

land have been neglected since the latter is the same for each project and the former would also favor project I.

**Incremental Comparisons** A company may have the choice of, say, investing \$10,000 in project J, which will give a (DCFRR) of 16 percent, or \$7000 in project K, which will give a (DCFRR) of 18 percent. Should it spend \$10,000 on project J or spend only \$7000 on project K and invest the difference of \$3000 elsewhere?

Both projects have lives of 10 years and constant positive net annual cash flows  $A_{CF}$  of \$2069 and \$1558 for projects J and K respectively. The corresponding (NPV)s at a discount factor of 10 percent are +\$2710 and +\$2560 respectively. These data are summarized as follows:

	Project J	Project K	Project (J - K)
$A_{CF}$ , \$, in Year 0	-10,000	-7,000	-3,000
$A_{CF}$ , \$, in each of Years 1-10	+2,069	+1,558	+511
(NPV), $i = 10$ percent, \$	+2,710	+2,560	+150
(DCFRR), percent	16	18	12.4

From the difference in cash flows between the projects, the discounted-cash-flow rate of return (DCFRR) for project (J-K) can be shown as 12.4 percent. This is significantly lower than for either project J or project K. Thus, if the \$3000 can be invested to give a return greater than 12.4 percent, project K should be chosen in preference to project J.

**Comparisons on the Basis of Capitalized Cost** A machine in a process generates a positive net cash flow of \$1000. Two alternatives are available: machine L, costing \$2000, requires replacement every 4 years, and machine M, costing \$3000, requires replacement every 6 years. Neither machine has any scrap value. The cost of capital is 10 percent. Which machine is the more profitable to operate?

In this case, the lives of the machines are unequal, and the comparison is conveniently made on the basis of capitalized cost. This puts lives on the same basis, which is an infinite number of years. The net annual cash flows generated by each machine are equal.

The capitalized cost  $C_K$  of a piece of fixed-capital cost  $C_{FC}$  is the amount of capital required to ensure that the equipment may be renewed in perpetuity. For a piece of equipment with no scrap value,  $C_K$  is given by

$$C_K = C_{FC} \left[ \frac{(1+i)^n}{(1+i)^n - 1} \right] \quad (9-58)$$

For machine L,

$$C_K = (\$2000)(3.15471) = \$6309.42$$

For machine M,

$$C_K = (\$3000)(2.29607) = \$6888.21$$

Thus, machine L with the lower capitalized cost is the more profitable to operate.

**Relationship between (PBP) and (DCFRR)** For the case of a single lump-sum capital expenditure  $C_{FC}$  which generates a constant annual cash flow  $A_{CF}$  in each subsequent year, the payback period is given by the equation

$$(PBP) = C_{FC}/A_{CF} \quad (9-59)$$

if the scrap value of the capital outlay may be taken as zero.

For this simplified case the net present value (NPV) after  $n$  years with money invested at a required aftertax compound annual fractional interest rate  $i$  is given by the equation

$$(NPV) = C_{FC} - A_{CF}F_n \quad (9-60)$$

where

$$F_n = \sum_{t=1}^n \frac{1}{(1+i)^t}$$

When  $(NPV) = 0$ , the value of  $i$  given by Eq. (9-60) is the discounted-cash-flow rate of return (DCFRR), and in this case Eqs. (9-59) and (9-60) can be combined to give:

$$(PBP) = F_n \quad (9-61)$$

Figure 9-13 is a plot of Eq. (9-61) in the form of the number of years  $n$  required to reach a certain discounted-cash-flow rate of return (DCFRR) for a given payback period (PBP). The figure is a modification of plots previously published by A. G. Bates [*Hydrocarbon Process.*, **45**, 181–186 (March 1966)], C. Estrup [*Br. Chem. Eng.*, **16**, 171 (February–March 1971)], and F. A. Holland and F. A. Watson [*Process Eng. Econ.*, **1**, 293–299 (December 1976)].

In the limiting case when  $n$  approaches infinity, Eq. (9-61) can be written as

$$(DCFRR)_{\max} = 1/(PBP) \quad (9-62)$$

which means, for example, that if the payback period is 4 years, the maximum possible discounted-cash-flow rate of return which can be reached is 25 percent. The corresponding (DCFRR) for  $(PBP) = 10$  years is 10 percent.

Equations (9-59), (9-60), (9-61), and (9-62) may be used as they stand to assess expenditure on energy-conservation measures since a constant amount of energy is saved in each year subsequent to the capital outlay. However, the annual cash flows  $A_{CF}$  corresponding to the energy savings remain constant only if there is no inflation or if the money values are corrected to their purchasing power at the time of the capital expenditure.

**Sensitivity Analysis** An economic study should pinpoint the areas most susceptible to change. It is easier to predict expenses than either sales or profits. Fairly accurate estimates of capital costs and processing costs can be made. However, for the most part, errors in these estimates have a correspondingly smaller effect than changes in sales price, sales volume, and the costs of raw materials and distribution.

Sales and raw-materials prices may be affected by any of the following: discounts and allowances, availability of substitutes, contract pricing, government regulations, quality and form of the materials, and competition. Sales volume may be affected by any of the following: new uses for the product, new markets, advertising, quality, overcapacity, replacement by another product, competition, and timing of entry into the market.

Distribution costs depend on plant location, physical state of the material (whether liquid, gas, or solid), nature of the material (whether corrosive, explosive, flammable, perishable, or toxic), freight rates, and labor costs. Distribution costs may be affected by any of the following: new methods of materials handling, safety regulations, productivity agreements, wage rates, transportation systems, storage systems, quality, losses, and seasonal effects.

It is worthwhile to make tables or plot curves that show the effect of variations in costs and prices on profitability. This procedure is called **sensitivity analysis**. Its purpose is to determine to which factors the profitability of a project is most sensitive. Sensitivity analysis should always be carried out to observe the effect of departures from expected values.

For many years, companies and countries have lived with the problem of inflation, or the falling value of money. Costs—in particular, labor costs—tend to rise each year. Failure to account for this trend in predicting future cash flows can lead to serious errors and misleading profitability estimates.

Another important factor is the tendency of product prices to fall as the total national or international volume of production increases. Sales prices may fall by 20 percent for a doubling in volume or production.

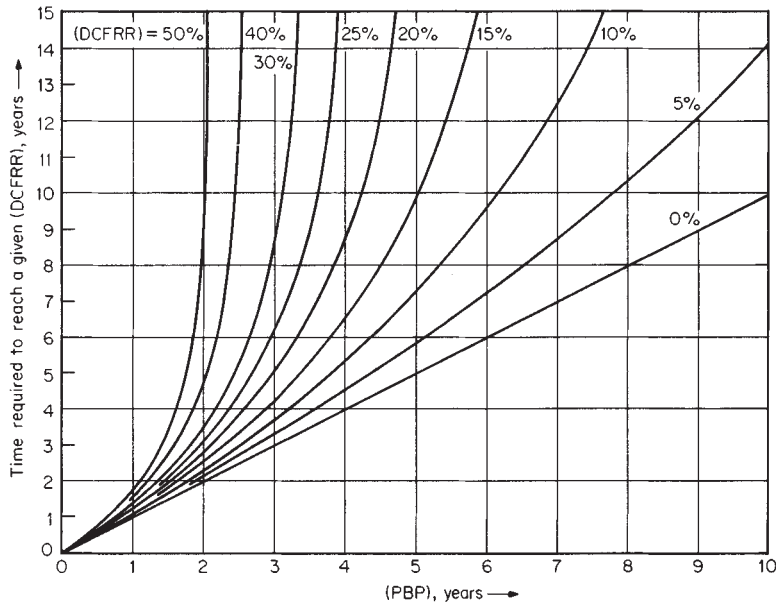


FIG. 9-13 Relationship between payback period and discounted-cash-flow rate of return.



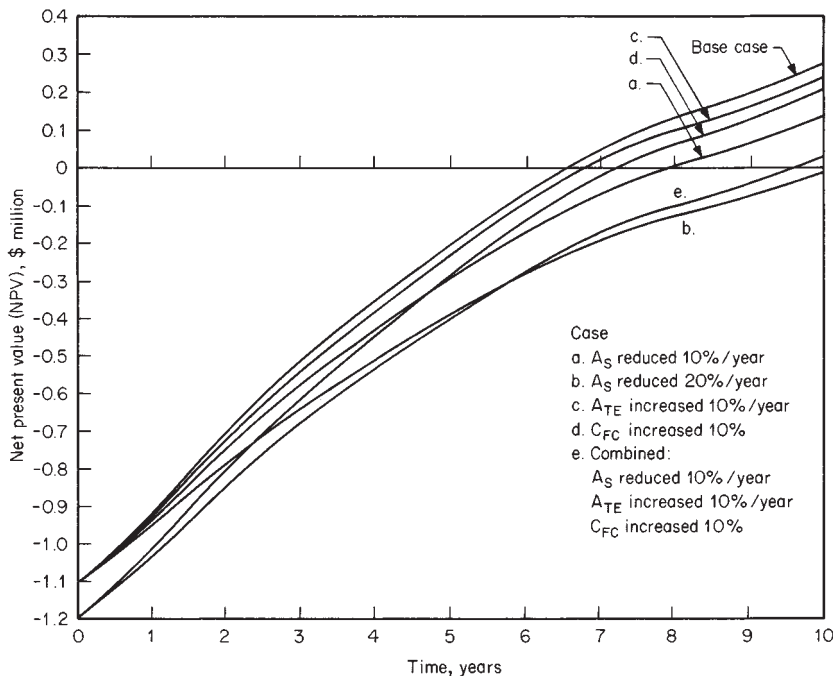


FIG. 9-14 Net present value against time, showing effect of adverse changes in cash flows.

No profitability estimate is better than the inherent accuracy of the data.

**Example 3: Sensitivity Analysis** The following data describe a project. Revenue from annual sales and total annual expense over a 10-year period are given in the first three columns of Table 9-5. The fixed-capital investment  $C_{FC}$  is \$1 million. Plant items have a zero salvage value. Working capital  $C_{WC}$  is \$90,000, and the cost of land  $C_L$  is \$10,000. There are no tax allowances other than depreciation; i.e.,  $A_A$  is zero. The fractional tax rate  $t$  is 0.50. For this project, the net present value for a 10 percent discount factor and straight-line depreciation was shown to be \$276,210 and the discounted-cash-flow rate of return to be 16.4 percent per year.

We shall use these data and the accompanying information of Table 9-5 as the base case and calculate for straight-line depreciation the net present value (NPV) with a 10 percent discount factor and the discounted-cash-flow rate of return (DCFRR) for the project with the following situations.

Case	Modification
a	Revenue $A_S$ reduced by 10 percent per year
b	Revenue $A_S$ reduced by 20 percent per year
c	Total expense $A_{TE}$ increased by 10 percent per year
d	Fixed-capital investment increased by 10 percent
e	$A_S$ reduced by 10 percent per year, $A_{TE}$ increased by 10 percent per year, and $C_{FC}$ increased by 10 percent

The results are shown in Figs. 9-14 and 9-15 and Tables 9-8 and 9-9.

**Learning Curves** It is usual to learn from experience. Consequently, the time taken to produce an article, the number of spoiled batches, the cost per unit of production, etc., tend to decrease with the number of units produced. The relationships are expressed for the ideal case by

TABLE 9-8 Annual Cash Flows, Straight-Line Depreciation, and 10 Percent Discount Factor When Revenue Is Reduced by 10 Percent per Year

Year	Base case			$\Delta A_S$ , \$	$f_d$	$\Delta A_{DCF}$ , \$	$\Delta(\text{NPV})$ , \$	Reduced (NPV), \$
	$A_S$ , \$	$A_{TE}$ , \$	(NPV), \$					
0	0	0	-1,100,000	0	1.0000	0	0	-1,100,000
1	400,000	100,000	-918,180	40,000	0.90909	18,180	18,180	-936,360
2	500,000	100,000	-711,570	50,000	0.82645	20,660	38,840	-750,410
3	500,000	110,000	-527,500	50,000	0.75131	18,780	57,620	-585,120
4	500,000	120,000	-363,580	50,000	0.68301	17,070	74,690	-438,270
5	520,000	130,000	-211,460	52,000	0.62092	16,140	90,830	-302,290
6	520,000	130,000	-73,160	52,000	0.56447	14,680	105,510	-178,670
7	520,000	140,000	+50,000	52,000	0.51316	13,340	118,850	-68,850
8	390,000	140,000	+131,640	39,000	0.46651	9,100	127,950	+3,690
9	350,000	150,000	+195,250	35,000	0.42410	7,420	135,370	+59,880
10	280,000	160,000	+276,210	28,000	0.38554	5,390	140,760	+135,450

$A_S$  = base revenue from annual sales before tax.  
 $A_{TE}$  = base total annual expense before tax.  
 (NPV) = base net present value after tax.  
 $\Delta A_S$  = decrease in annual revenue.

$f_d$  = discount factor at 10%.  
 $\Delta A_{DCF}$  = decrease in net discounted cash flow at income tax rate = 0.5.  
 $\Delta(\text{NPV}) = \sum \Delta A_{DCF}$  = decrease in net present value.  
 Reduced (NPV) =  $\sum A_{DCF}$  = reduced net present value after tax.



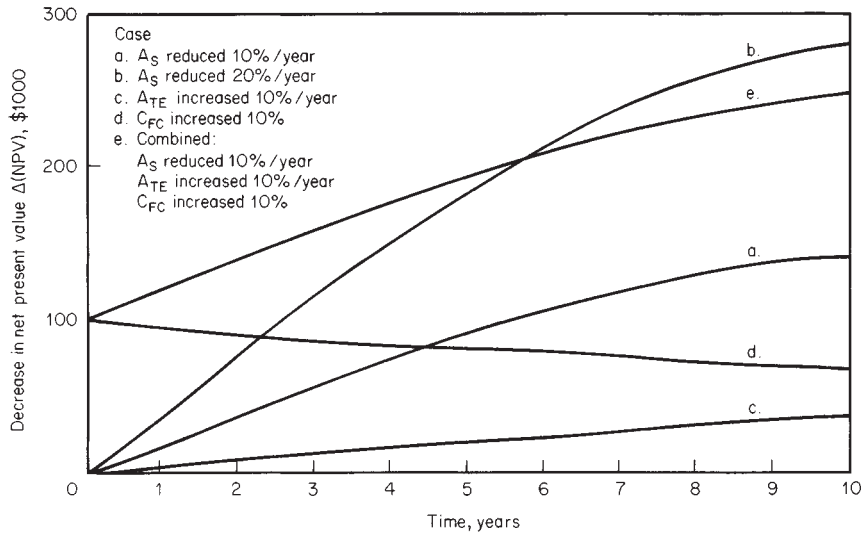


FIG. 9-15 Decrease in net present value against time resulting from adverse changes in cash flows.

$$Y = KX^N \tag{9-63}$$

where  $Y$  = cumulative-average cost, production time, etc., per unit  
 $X$  = cumulative production, units  
 $K$  = effective value of first unit produced  
 $N$  = slope of straight-line plot of  $Y$  versus  $X$  on log-log paper

The particular learning curve is usually characterized by the percentage reduction in the cumulative average value  $Y$  when the number of units  $X$  is doubled. From this definition it follows that

$$N = \log(\text{characteristic}/100)/\log 2 \tag{9-64}$$

The cost  $c_{ME}$  of the last unit of a block bringing the cumulative production to  $X$  units is, from Eq. (9-63),

$$c_{ME} = K[X^{N+1} - (X-1)^{N+1}] \tag{9-65}$$

These unit costs, or the time taken to produce the last unit, etc., may be plotted on cartesian coordinates against the number of units produced to provide a standard against which the performance of a new employee, a new machine, etc., can be judged. Figure 9-16 shows such a plot for the subsequent example.

In general, cost data will be available for multiple units. Typically, the cost of production for 1 week or of a specific order is computed and an average cost per unit obtained. This average value  $\bar{Y}$  for the batch should be plotted against the corresponding learning-curve value  $\bar{X}$  calculated by Eq. (9-66):

$$\bar{X}^N = (X_2^{N+1} - X_1^{N+1}) / (X_2 - X_1) \tag{9-66}$$

where  $X_1$  and  $X_2$  are the cumulative production before and after the batch. This form of the equation is useful when only the previous production history of the process is known, from the serial numbers or otherwise.

TABLE 9-9 Summary of Results of Sensitivity Analysis

Case	(NPV), \$ $i = 10\%$	(DCFRR), %
Base case	276,210	16.4
$A_S$ reduced 10% per year	135,450	12.9
$A_S$ reduced 20% per year	-5,330	9.8
$A_{TE}$ increased 10% per year	238,430	15.0
$C_{FC}$ increased 10%	206,890	14.0
Combined:		
$A_S$ reduced 10% per year	28,420	10.6
$A_{TE}$ increased 10% per year		
$C_{FC}$ increased 10%		

A straight line may be fitted to the  $(X, Y)$  or  $(\bar{X}, \bar{Y})$  pairs of data when plotted on log-log graph paper from which the slope  $N$  and the intercept  $\log K$  with  $\bar{X} = 1$  may be read. Alternatively, the method of least squares may be used to estimate the values of  $K$  and  $N$ , giving the best fit to the available data.

It will be noted that a value of  $N = 0$ , corresponding to a characteristic of 100 percent for the learning curve, implies that the value of  $Y$  is independent of  $X$ . This would imply that learning by experience was not possible and thus corresponds to an optimally designed process or one for which the costs are determined by external factors. Similarly, a value of  $N = -1$ , corresponding to the 50 percent learning curve, implies that the cost of production is inversely proportional to the number produced, which is absurd. Projects having characteristics less than 70 percent are impractical. Low characteristics are typical of hasty entry into a market in an attempt to preempt it. Characteristics tend to increase with experience, so that established and mature projects are likely to have characteristics around 95 percent. Characteristics close to 100 percent are unlikely to be achieved because of random factors such as changes in personnel, accidents, supply delays, etc. Figure 9-17 represents a typical practical case, from which it can be seen that the curve has a point of inflexion but eventually settles down to an approximately straight line of lower slope than that of the conventionally defined learning curve. At some point it is useful to change to the equation of this mature project line.

Significant changes in working, such as the introduction of new equipment, the influx of a large number of inexperienced workers, or a temporary reduction in skills after a long shutdown, may produce a sudden increase in all the cumulative-average curves. The simplest way to handle this, when the next accurate costing is available, is to deduct the value of  $X$  obtained from the curve from that actually achieved and to use this value as a constant correction to  $X$  until the next break in the curve is reached. If the causes of such steps recur, the size of the step can often be related to a particular cause. In such cases the estimated step change can be used for predictions until the next accurately determined values are obtained.

Applications for the learning curve are already extensive, and new uses can often be found. Care is needed in applying the techniques to ensure that it is possible for learning to take place. In projecting prices, etc., unusual items, such as the cost of the special setting up of tools or factory rearrangements, should be excluded from the production costs used to establish the learning curve. In times of inflation, costs should be corrected for the effects of inflation in the manner to be shown subsequently. Production times or spoilage rates are not affected by cost allocations or inflation and may prove to be better

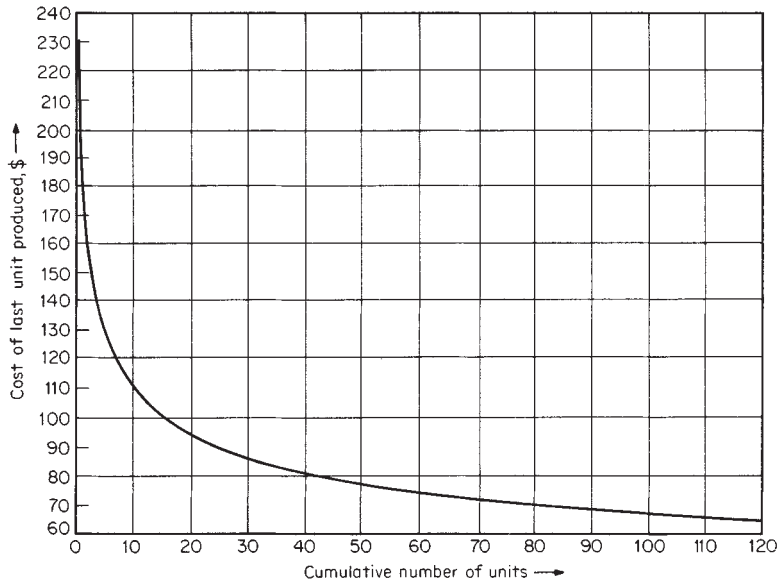


FIG. 9-16 Cartesian plot of learning curve.

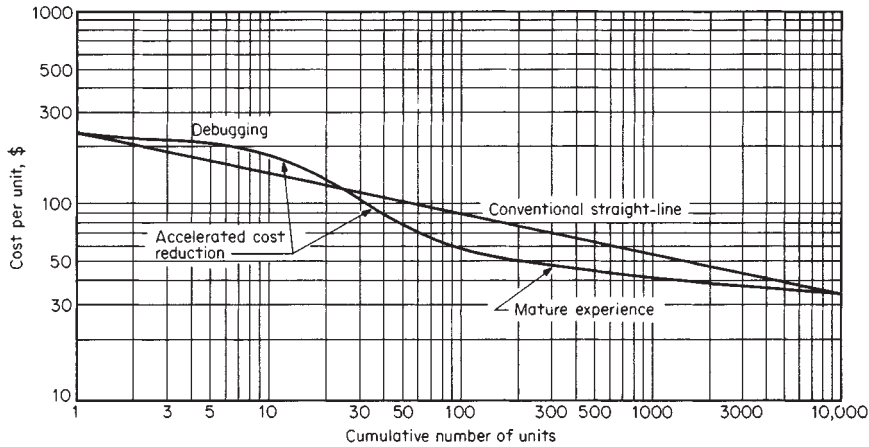


FIG. 9-17 Logarithmic plot of learning curve.

standards of performance where appropriate. However, the learning curve is often required when preparing quotations for batch production runs, particularly when competition is likely to be keen. In such cases the average cost of the production run  $\bar{Y}$  between cumulative production totals of  $X_1$  and  $X_2$  may be estimated by Eq. (9-67) when the previous cumulative-average cost  $Y_1$  is known:

$$\bar{Y} = Y_1 [(X_2/X_1)^{N+1} - 1] / [(X_2/X_1) - 1] \quad (9-67)$$

In process engineering, fractional units can often be produced so that the learning curve can be treated as being continuous. When only discrete numbers of units can be produced, the learning curve is strictly a histogram. In order to allow for this it is sufficient to increase the value of  $X$  by half a unit before applying the above equations. The difference is significant at small values of  $X$ , such as may be used for the initial estimates of  $K$  and  $N$ . As the project matures, it is better to use the equations as presented, as the cost of the first unit  $K$  is an entirely notional one. Major technological changes should, of course, be treated as the start of a new project.

R. B. Jordan (*How to Use the Learning Curve*, Materials Management Institute, Boston, 1965) discusses the uses of the learning curve

extensively and provides many tables of factors. The uses considered include estimating starting costs, determining labor requirements, establishing factory cost targets, checking employee-training progress, the make-or-buy decision, aid in purchasing negotiations, and aid in establishing a selling price.

**Example 4: Estimation of Average Cost of Incremental Units**

The cost of an initial batch of 21 units, exclusive of special tools and setting-up costs, averaged \$120 per unit. The average cost of the next batch of 80 units was \$75.81. Let us establish the learning curve implied by these data and hence estimate the probable average cost of the next 50 units. We shall establish also the unit-cost curve to be used as a control during follow-up orders.

If the batch units are capable of continuous subdivision, we proceed as follows. We substitute the given values of the cumulative-average cost  $Y$  and cumulative production  $X$  for the first batch into Eq. (9-63) to give, by taking logarithms of each side,

$$\log 120 = \log K + N \log 21$$

The cost of the first batch is  $120 \times 21 = \$2520$ , and that of the second batch is  $75.81 \times 80 = \$6065$ . The total cost of the first 101 units is therefore \$8585, with a cumulative-average unit cost of \$85. We substitute as before to give

$$\log 85 = \log K + N \log 101$$

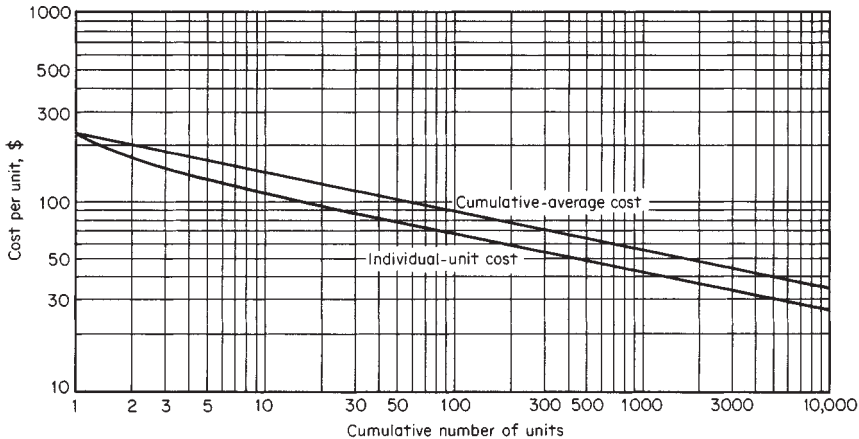


FIG. 9-18 Effect of learning on the average cost of a product.

From these equations it follows that  $K = \$234.15$  and  $N = -0.2196$ . This line is plotted in Fig. 9-18.

From Eq. (9-64) it follows that the value of the characteristic of this learning curve = 100 antilog  $(-0.2196 \log 2) = 85.0$  percent. From Eq. (9-65) the production cost of the third unit is

$$C_{ME} = (\$234.15)(3^{0.7504} - 2^{0.7504}) = \$149.70$$

Values calculated in this way are plotted in Fig. 9-16 and also in Fig. 9-18. It will be noted that after about 10 units this latter curve becomes parallel to the cumulative-average-cost curve and that the  $Y$  values are  $(N + 1)$  times those obtained from the latter curve.

Since the cumulative-average cost  $Y_2$  of the first 101 units was \$85, it follows from Eq. (9-67) that the average cost of the third batch of 50 units, bringing the cumulative total to 151, is given by

$$\begin{aligned} \bar{Y}_3 &= (\$85)[(151/101)^{0.7504} - 1]/[(151/101) - 1] \\ &= \$63.30 \text{ per unit} \end{aligned}$$

This may be used as a cost guide when quoting the order.

If the units of production may not be subdivided, the procedure is similar except that all  $X$  values are increased by 0.5 unit in establishing the curves. The results are not sufficiently different to be significant for estimation purposes.

To the above costs must be added back any unit costs omitted from those to which learning might bring improvement. These will normally include overheads and specific charges on the project such as the unit cost of special tools, jigs, etc.

**Risk and Uncertainty** Discounted-cash-flow rates of return (DCFRR) and net present values (NPV) for future projects can never be predicted absolutely because the cash-flow data for such projects are subject to uncertainty. Therefore, when stating predicted values of (DCFRR) and (NPV) for projects, it is also desirable to give a measure of confidence in the predictions.

For example, for a particular project it may be estimated that there is a 90 percent chance of the (DCFRR) being greater than 10 percent, a 50 percent chance of its being greater than 16 percent, and only a 10 percent chance of its being greater than 20 percent. Management retains the power of decision to proceed with the project or not, but the probability data provide desirable information for the decision.

The estimation of probabilities requires the use of statistics. Thus statistical methods play an increasing role in decision making.

**Predictions from Limited Data** Predictions of future sales price, sales volume, etc., are normally based on a very limited amount of data about past events. Furthermore, it would not be convenient to use the entire population of past events even if it were available. A statistic is a measure, based on limited information from a sample, that allows the corresponding parameter of the population to be estimated.

The mean value  $\bar{x}$  of a property  $x$  is a statistic based on a sample of  $n$  items defined by

$$\bar{x} = (x_1 + x_2 + x_3 + \cdots + x_n)/n \quad (9-68)$$

The mean  $\bar{x}$  is the statistic corresponding to the population parameter  $\mu$ , which is the arithmetic average of all the items in the popula-

tion. In many cases, not all the  $x$  values will be different. In such circumstances, Eq. (9-68) can be written as

$$\bar{x} = [\sum x_i f(x_i)] / [\sum f(x_i)] \quad (9-69)$$

where  $f(x_i)$  is the frequency with which a particular value  $x_i$  occurs. It is often convenient to divide the frequency of occurrence by the total number of items. In this case,  $f(x_i)$  becomes the relative frequency of occurrence of the value  $x_i$ , and  $\sum f(x_i) = 1$ .

The values of  $x$  may be either discrete or continuous. The number of sales of, say, automobiles in any one day must be an integer. If a business sells 4 automobiles, this represents all possible values of  $x$  in the range of 3.5 to 4.5.

When  $x$  represents a continuous variable quantity, it is sometimes convenient to take the total or relative frequency of occurrences within a given range of  $x$  values. These frequencies can then be plotted against the midvalues of  $x$  to form a histogram. In this case, the ordinate should be the frequency per unit of width  $x$ . This makes the area under any bar proportional to the probability that the value of  $x$  will lie in the given range. If the relative frequency is plotted as ordinate, the sum of the areas under the bars is unity.

If  $x$  is a continuous variable and the interval ranges are made smaller and smaller, a smooth curve will eventually result. The area under such a curve between  $x_1$  and  $x_2$  represents the probability that a randomly selected item will have a value of  $x$  lying in the range  $x_1$  to  $x_2$ . This is the information that is desired.

Data available from past experience can be used to generate frequency distribution curves. It is essential for a company to have an efficient commercial-intelligence system to assess market conditions.

Accuracy of sales forecasting can also be increased by a careful study of past sales records, price trends, etc. However, the uncertainty of an estimate increases the farther into the future that the estimate is projected.

Estimates of sales income and other types of forecasts are usually based on the opinions of experts. Experts should be able to estimate maximum, minimum, and most likely, or modal, values for a quantity. The modal value is not necessarily midway between the minimum and maximum values, since many distributions are skewed. An expert may be asked to estimate the probability of the occurrence of certain values on each side of the mode. When experts are questioned separately, the procedure is known as the **Delphic method**. Strictly speaking, this method requires that the opinion of each expert be assessed by a coordinator, who then feeds the results back to see if the opinions of one expert are modified by those of others. The process is repeated until agreement is reached. In practice, the procedure is too tedious to be repeated more than once.

It is useful to compare the past predictions of each expert with the results obtained in practice. This information enables the opinions to be weighted by the coordinator. When the experts work in close collaboration, it is not possible to avoid some collusion. In this case, it is

better to arrive at a single consensus opinion by a free and open discussion. This is the think-tank method. Its main disadvantage is that rank or aggressiveness might unduly weight one or more opinions.

The opinions of the experts, however obtained, provide a basis for plotting a frequency or probability distribution curve. If the relative frequency is plotted as ordinate, the total area under the curve is unity. The area under the curve between two values of the quantity is the probability that a randomly selected value will fall in the range between the two values of the quantity. These probabilities are mere estimates, and their reliability depends on the skill of the forecasters.

The estimated (DCFRR) and the estimated (NPV) are both functions of the estimated cumulative revenue from annual sales  $\sum A_s$ , the estimated cumulative total annual cost or expense  $\sum A_{TE}$ , and the estimated fixed capital cost  $C_{FC}$  of the plant. The revenue from annual sales for each year is in turn the product of the sales price and sales volume. Initially it is desirable to select those values from the distribution curves of  $\sum A_s$ ,  $\sum A_{TE}$ , and  $C_{FC}$  which enable the maximum and minimum (DCFRR) and (NPV) to be calculated.

If the maximum values of (DCFRR) and (NPV) are not acceptable to the company, the project should promptly be rejected. If the minimum values of (DCFRR) and (NPV) are acceptable, a detailed assessment should be made. If the maximum values of (DCFRR) and (NPV) are acceptable but the minimum values are not, the feasibility study should be continued.

**Mathematical Models for Distribution Curves** Mathematical models have been developed to fit the various distribution curves. It is most unlikely that any frequency distribution curve obtained in practice will exactly fit a curve plotted from any of these mathematical models. Nevertheless, the approximations are extremely useful, particularly in view of the inherent inaccuracies of practical data. The most common are the binomial, Poisson, and normal, or gaussian, distributions.

A normal distribution curve is bell-shaped (see Sec. 3). The curve obeys the relationship

$$f(x) = \frac{\exp - [(x - \mu)^2/2\sigma^2]}{\sigma(2\pi)^{0.5}} \quad (9-70)$$

where  $\sigma$  is known as the true standard deviation. The standard deviation  $s^\circ$  from a sample is given by

$$s^\circ = \left[ \frac{\sum (x_i - \bar{x})^2 f(x_i)}{\sum f(x_i) - 1} \right]^{0.5} \quad (9-71)$$

The standard deviation  $s^\circ$  for the sample corresponds to the true standard deviation  $\sigma$  for the whole population in the same way that the mean  $\bar{x}$  of the sample corresponds to the arithmetic average  $\mu$  for the whole population. Equation (9-70) can be written more compactly as

$$f(z) = [\exp (-z^2/2)] / [(2\pi)^{0.5}] \quad (9-72)$$

where the standard score  $z$  is

$$z = (x - \mu) / \sigma \quad (9-73)$$

The area under the curve of  $f(z)$  is unity if the abscissa extends from minus infinity to plus infinity. The area under the curve between  $z_1$  and  $z_2$  is the probability that a randomly selected value of  $x$  will lie in the range  $z_1$  and  $z_2$ , since this is the relative frequency with which that range of values would be represented in an infinite number of trials.

An event that will definitely occur has a probability of unity. An event that will definitely not occur has a probability of zero.

Equation (9-72) can be integrated between limits to determine the probability that a random value lies between the selected limits. Extensive tables of  $f(z)$  and the associated integral are available (see Sec. 3).

A frequency distribution curve can be used to plot a cumulative-frequency curve. This is the curve of most importance in business decisions and can be plotted from a normal frequency distribution curve (see Sec. 3). The cumulative curve represents the probability of a random value  $z$  having a value of, say,  $z_1$  or less.

If a property or variable  $c$  is a function of several other variables  $x_1, x_2$ , etc., it can be written in the form

$$c = \phi(x_1, x_2, \dots, x_n) \quad (9-74)$$

If each  $x$  is a normally distributed independent variable, then

$$\sigma_c^2 = \left( \frac{\partial c}{\partial x_1} \right)^2 \sigma_1^2 + \left( \frac{\partial c}{\partial x_2} \right)^2 \sigma_2^2 + \dots + \left( \frac{\partial c}{\partial x_n} \right)^2 \sigma_n^2 \quad (9-75)$$

where  $\sigma_c$  is the standard deviation of the variable  $c$  and  $\sigma_1, \sigma_2$ , etc., are the standard deviations of the variables  $x_1, x_2$ , etc.

Many distributions occurring in business situations are not symmetrical but skewed, and the normal distribution curve is not a good fit. However, when data are based on estimates of future trends, the accuracy of the normal approximation is usually acceptable. This is particularly the case as the number of component variables  $x_1, x_2$ , etc., in Eq. (9-74) increases. Although distributions of the individual variables ( $x_1, x_2$ , etc.) may be skewed, the distribution of the property or variable  $c$  tends to approach the normal distribution.

Let us consider an event that must have one of two outcomes. It must either occur with probability  $p_1$  or fail to occur with probability  $p_2$ . Since these are exclusive events and the probability that something will happen is unity, it follows that

$$p_1 + p_2 = 1 \quad (9-76)$$

Provided that no learning process is involved (so that the value of  $p_1$  is not influenced by previous results), the probability of  $x$  successes in  $n$  trials is given by the term containing  $p_1^n$  in the expansion of the binomial:

$$(p_1 + p_2)^n = p_1^n + \dots + \frac{n!}{x!(n-x)!} p_1^x p_2^{(n-x)} \dots + p_2^n \quad (9-77)$$

where  $x$  and  $n$  are integers and  $x!$  (read as  $x$  factorial) is the product of all integers from unity to  $x$ .

**Example 5: Probability Calculation** If a six-sided die marked with the numbers 1, 2, 3, 4, 5, and 6 is thrown, the probability that any given number will be uppermost is 1/6. If the die is thrown twice in succession, then the probability of a given sequence of numbers occurring, say, 5 followed by 6, is (1/6)(1/6) = 1/36. The chance of any particular number occurring 0, 1, 2, 3, or 4 times in four throws of the die (or in a simultaneous throw of four dice) is given by the successive terms of Eq. (9-77), expanded as

$$\begin{aligned} (5/6 + 1/6)^4 &= (1)(5/6)^4(1/6)^0 + (4)(5/6)^3(1/6)^1 + (6)(5/6)^2(1/6)^2 + (4)(5/6)(1/6)^3 \\ &\quad + (1)(5/6)^0(1/6)^4 = 0.4823 + 0.3858 + 0.1157 + 0.0154 + 0.0008 = 1 \end{aligned}$$

The distribution of the number of successes is skewed toward the low numbers. In particular, there is only a slightly better than even-money chance that any given number will occur even once in four throws. Such highly unsymmetrical distributions cannot be approximated by the normal distribution curve.

However, an increasing number of throws will result in totals that are close to the normal distribution. This fact can be used to approximate such a distribution without the enormous labor of the calculations required by the use of Eq. (9-77).

Possible values of the total of four throws of a die are integers from 4 to 24 and hence represent values in the range from 3.5 to 24.5. The mean value  $\bar{x}$  of this range is given by Eq. (9-68) as  $\bar{x} = (3.5 + 24.5)/2 = 14.0$ .

The cumulative probability of a normally distributed variable lying within 4 standard deviations of the mean is 0.49997. Therefore, it is more than 99.99 percent (0.49997/0.50000) certain that a random value will be within  $\pm 4\sigma$  from the mean. For practical purposes,  $\sigma$  may be taken as one-eighth of the range of certainty, and the standard deviation can be obtained:

$$s^\circ \cong (24.5 - 3.5)/8 = 2.625$$

From Eq. (9-73) the standard score becomes

$$z = (x - \mu) / \sigma \cong (x - \bar{x}) / s^\circ$$

For a total score of 4 (i.e.,  $x = 4$ ), the standard score is approximately  $z = (4 - 14)/2.625 = -3.81$ . Since the normal curve is symmetrical about  $z = 0$ , the height of the ordinate at  $z = -3.81$  is the same as that at  $z = +3.81$ . From tables of values of cumulative probabilities of the normal distribution, the height of the ordinate is 0.0003 in units of  $1/\sigma$ . The relative frequency of 4 occurring is thus approximately 0.0003/2.625 = 0.0001.

This concept can be used to translate Delphic or other opinions into probability distributions and hence into useful decision-making tools.

**Example 6: Calculation of Probability of Meeting a Sales Demand** A store that is open 5 days a week is to promote a new product. The manager believes that not more than 5 units will be sold in any one day, but he cannot be more precise about the probable sales pattern. Stocks are delivered once per week. What size should the first order be to give a 95 percent certainty of meeting demand?

Since the product is sold in units, the possible range of weekly sales is from -0.5 to +25.5 units. Therefore, the mean of the sales distribution will be

$$\bar{x} \cong [25.5 - (-0.5)]/2 = 13$$

The standard deviation for this example will be

$$s^\circ \cong [25.5 - (-0.5)]/8 = 3.25$$

From this, the approximate frequency distribution of daily sales can be derived by using Eqs. (9-70), (9-72), or (9-73). The desired area to the right of  $z = 0$  for the normal probability distribution curve is  $0.95 - 0.50 = 0.45$ . For this value the standard score  $z = 1.645$ .

$$x \cong 13 + (1.645)(3.25) = 18.35$$

Hence, to be 95 percent certain of meeting demand, 19 units should be purchased.

If the value of  $n$  in Eq. (9-77) is large and neither  $p_1$  nor  $p_2$  is too close to zero, the binomial distribution can be approximated by

$$z = \frac{x - np_1}{\sqrt{np_1p_2}} \quad (9-78)$$

The approximation of Eq. (9-78) is good enough for most purposes if  $np_1$  and  $np_2$  are each greater than 5.

**Example 7: Calculation of Probability of Sales** The records of a business show that never more than 1 item is sold in a day and that 2 sales per week can be expected. What is the probability of selling between 90 and 120 items in a 300-day year?

In a year consisting of 50 weeks of 6 days, the mean or expected value of the distribution is 100 items. The probability of a sale of an item on a given day is  $p_1 = 100/300 = 1/3$ , and of no sale is  $p_2 = 2/3$ . From Eq. (9-78),

$$z = \frac{x - (300)(1/3)}{\sqrt{(300)(1/3)(2/3)}} = \frac{x - 100}{8.165}$$

The integral range of 90 to 120 items contains all possible values of  $x$  from 89.5 to 120.5. For  $x = 89.5$ ,  $z = -1.286$ ; and for  $x = 120.5$ ,  $z = 2.511$ .

The cumulative probability of a standard score of 1.286 is 0.11, while that of a standard score of 2.511 is 0.99. Therefore, the probability of annual sales in the range of 90 to 120 items is  $(0.99 - 0.11) = 0.88$ , or 88 percent.

There are times when the frequency measurement is an integral number of events in a given segment of a continuum, for example, the number of automobiles passing a given point in 1 h or the number of leaks in a given length of hosepipe. In such cases, the correct frequency distribution is the Poisson distribution, in which the probability of  $x$  events per unit of a continuum occurring is given by

$$f(x) = \lambda^x e^{-\lambda} / x! \quad (9-79)$$

where  $x$  is an integer,  $e$  is the base of natural logarithms, and  $\lambda$  is a parameter of the system  $\lambda = \mu = \sigma^2$ .

As  $\lambda$  increases, the Poisson distribution approaches the normal distribution, with the relationship

$$z = (x - \lambda) / \sqrt{\lambda} \quad (9-80)$$

When the value of  $p_1$  is very close to zero in Eq. (9-77), so that the occurrence of the event is rare, the binomial distribution can be approximated by the Poisson distribution with  $\lambda = np_1$  when  $n > 50$  while  $np_1 < 5$ .

**Example 8: Calculation of Probability of Equipment Breakdown** The daily chance of a breakdown in a production line operated continuously for 300 days per year is estimated at 1 percent from past performance. Let us estimate the probability of 4 or more breakdowns in the coming year.

For  $n = 300$  and  $p_1 = 0.01$ ,  $\lambda \cong np_1 = 3 < 5$ , the probability of no breakdown is found from Eq. (9-79) to be

$$f(0) = (3)^0 e^{-3} / 0! = 1/e^3 = 0.0498$$

Similarly,

Breakdowns	Probability
1	$f(1) = (3)^1 e^{-3} / 1! = 0.1496$
2	$f(2) = (3)^2 e^{-3} / 2! = 0.2240$
3	$f(3) = (3)^3 e^{-3} / 3! = 0.2240$

Since something must happen, the probability of 4 or more breakdowns is

$$1 - 0.0498 - 0.1496 - 0.2240 - 0.2240 = 0.3522$$

A simple trial will show how much more easily the preceding calculation is carried out than direct use of Eq. (9-77).

The necessary value of  $\lambda$  may often be established as in the following example.

**Example 9: Calculation of Probability of Machine Failures**

In a production period of 100 days, 0, 1, 2, 3, and 4 machine failures occurred in a single day on 41, 37, 15, 6, and 1 occasions respectively. Let us fit a Poisson distribution to the data and estimate the maximum number of machine failures likely to occur in 1 day of a 300-day year.

The mean number of failures is found from Eq. (9-69) to be

$$\bar{x} = \frac{0(41) + 1(37) + 2(15) + 3(6) + 4(1)}{41 + 37 + 15 + 6 + 1} = 0.89$$

The standard deviation is found from Eq. (9-71):

$$s^\circ = \left[ \frac{\sum (x_i - \bar{x})^2 f(x_i)}{\sum f(x_i) - 1} \right]^{0.5}$$

The steps for calculating the numerator and denominator for this equation are tabulated as follows:

$(x_i - \bar{x})^2$	$f(x_i)$	$(x_i - \bar{x})^2 f(x_i)$
$(0 - 0.89)^2$	41	32.48
$(1 - 0.89)^2$	37	0.45
$(2 - 0.89)^2$	15	18.48
$(3 - 0.89)^2$	6	26.71
$(4 - 0.89)^2$	1	9.67
	$\sum (x_i - \bar{x})^2 f(x_i)$	$= 87.79$
	$\sum f(x_i) - 1$	$= 99$

Therefore,  $s^\circ = (87.79/99)^{0.5} = 0.9417$ .

The Poisson distribution is a good fit since

$$\lambda = \mu \cong \bar{x} = 0.8900$$

and

$$\lambda = \sigma^2 \cong (s^\circ)^2 \cong 0.8868$$

The Poisson distribution is found from Eq. (9-79) to be

$$f(x) = [(0.89)^x e^{-0.89}] / x!$$

By substituting the appropriate values of  $x$  for this example into the preceding equation, we find  $f(0) = 0.4107$ ,  $f(1) = 0.3655$ ,  $f(2) = 0.1627$ ,  $f(3) = 0.0483$ ,  $f(4) = 0.0107$ ,  $f(5) = 0.0019$ , and  $f(6) = 0.0003$ . Hence, in 300 days the expected maximum number of breakdowns in 1 day is 5 since  $(300)f(6) = 0.09$  occurrence.

In many business applications, Eq. (9-74) can be reduced to the linear relationship

$$c = k_1 x_1 + k_2 x_2 + \dots + k_n x_n \quad (9-81)$$

where the  $k$ 's are constants. Equation (9-75) then becomes

$$\sigma_c^2 = k_1^2 \sigma_1^2 + k_2^2 \sigma_2^2 + \dots + k_n^2 \sigma_n^2 \quad (9-82)$$

On the other hand, for a product function such as

$$c = x_1 x_2 \quad (9-83)$$

Eq. (9-75) can be written in the form

$$\sigma_c^2 / c^2 = \sigma_1^2 / x_1^2 + \sigma_2^2 / x_2^2 \quad (9-84)$$

The discounted-cash-flow rate of return (DCFRR) and net present value (NPV) are functions of the cumulative revenue from annual sales  $\sum A_{TE}$  and the fixed-capital cost of the plant  $C_{FC}$ , among other factors.

Equation (9-75) can be written for (DCFRR) and for (NPV) as

$$\sigma_{(\text{DCFRR})}^2 = \left[ \frac{\partial(\text{DCFRR})}{\partial \sum A_S} \right]^2 \sigma_{\sum A_S}^2 + \left[ \frac{\partial(\text{DCFRR})}{\partial \sum A_{TE}} \right]^2 \sigma_{\sum A_{TE}}^2 + \left[ \frac{\partial(\text{DCFRR})}{\partial C_{FC}} \right]^2 \sigma_{C_{FC}}^2 \quad (9-85)$$



$$\sigma_{(NPV)}^2 = \frac{\partial(NPV)}{\partial \sum A_S} \sigma_{\sum A_S}^2 + \frac{\partial(NPV)}{\partial \sum A_{TE}} \sigma_{\sum A_{TE}}^2 + \frac{\partial(NPV)}{\partial C_{FC}} \sigma_{C_{FC}}^2 \quad (9-86)$$

The revenue from annual sales  $A_S$  of a product at an annual production rate  $R$  and sales price of  $c_s$ , per unit of production is

$$A_S = R c_s \quad (9-87)$$

Equation (9-84) can be written as:

$$\sigma_{A_S}^2 = (A_S/R)^2 \sigma_R^2 + (A_S/c_s)^2 \sigma_{c_s}^2 \quad (9-88)$$

An extensive example illustrating the use of Eqs. (9-81) through (9-86) in establishing the probability of attaining a given value of the net present value or less in a particular year of a project was presented by Holland et al. [F. A. Holland, F. A. Watson, and J. K. Wilkinson, *Chem. Eng.*, **81**, 105–110 (Jan. 7, 1974)]. The result is shown in Fig. 9-19.

Decision makers often prefer to have graphs showing the probability of attaining a value greater than a given value. Such curves are easily obtained by subtracting the probability of achieving a given value or less from 100 percent. Figure 9-20 was obtained in this way and shows the probability of attaining a (DCFRR) greater than a given value.

**Monte Carlo Method** The Monte Carlo method makes use of random numbers. A digital computer can be used to generate pseudorandom numbers in the range from 0 to 1. To describe the use of random numbers, let us consider the frequency distribution curve of a particular factor, e.g., sales volume. Each value of the sales volume has a certain probability of occurrence. The cumulative probability of that value (or less) being realized is a number in the range from 0 to 1. Thus, a random number in the same range can be used to select a random value of the sales volume.

In the same way, random values of the other factors can be obtained. These can then be combined to give random values of (DCFRR) and (NPV) and, in turn, used to plot cumulative-probability curves for (DCFRR) and (NPV). The computer may be required to perform some 10,000 to 50,000 calculations.

The use of the Monte Carlo method in project appraisal was illustrated by Holland et al. [F. A. Holland, F. A. Watson, and J. K. Wilkinson, *Chem. Eng.*, **81**, 76–79 (Feb. 4, 1974)]. The cumulative-probability curves of (DCFRR) and (NPV) can never be more accurate than the opinions on which they are based, and comparable accuracy can be obtained by the use of S-shaped curves with relatively small computational effort.

**S-Shaped Curves** K. D. Tocher (*The Art of Simulation*, rev. ed., English Universities Press, London, 1967) presented a comprehen-

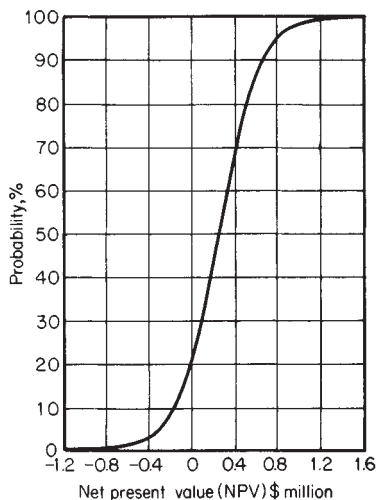


FIG. 9-19 Probability of a given net present value or less for a project.

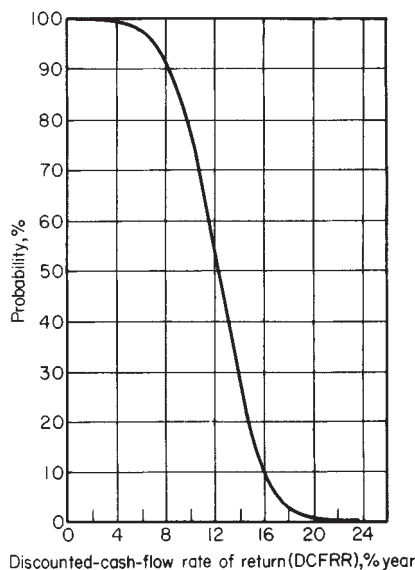


FIG. 9-20 Probability of a given discounted-cash-flow rate of return or more for a project.

sive treatment of the generation of random and pseudorandom numbers and their use in a wide range of simulated processes. He also considered sampling techniques from the various statistical distributions and the design of simulated processes. It will be noted that the cumulative distribution curves are S-shaped, and Tocher (op. cit., p. 16) recommended as a general equation for such curves

$$x = a + by + cy^2 + d(1 - y)^2 \ln y + ey^2 \ln(1 - y) \quad (9-89)$$

in which  $x$  varies from  $-\infty$  to  $+\infty$  as  $y$  varies from 0 to 1. The underlying frequency curve corresponding to Eq. (9-89) is

$$\frac{1}{p(x)} = \frac{dx}{dy} = b + 2cy + d(1 - y) \left( \frac{1 - y}{y} - 2 \ln y \right) + ey \left[ 2 \ln(1 - y) - \frac{y}{1 - y} \right] \quad (9-90)$$

If necessary, the fit can be improved by increasing the order of the polynomial part of Eq. (9-89), so that this approach provides a very flexible method of simulation of a cumulative-frequency distribution. The method can even be extended to J-shaped curves, which are characterized by a maximum frequency at  $x = 0$  and decreasing frequency for increasing values of  $x$ , by considering the reflexion of the curve in the  $y$  axis to exist. The resulting single maximum curve can then be sampled correctly by Monte Carlo methods if the vertical scale is halved and only absolute values of  $x$  are considered.

When the data do not warrant the accuracy of Eq. (9-89) or Eq. (9-90), simpler curves will usually suffice if the frequency distribution may be assumed to have a single maximum value.

Let us consider a product which is sold entirely on the basis of personal recommendation. The rate of sale will depend on the number of people who have already bought the product. Thus initially sales will increase exponentially. Eventually the market will be saturated, and only replacement purchases will be made. If the frequency curve may be assumed to be symmetrical about a single maximum value, the cumulative distribution curve is known as the logistics curve and is defined by Eq. (9-91):

$$y = c/[1 + a \exp(-bx)] \quad (9-91)$$

where  $y$  varies between zero and  $c$  as  $x$  ranges from  $-\infty$  to  $+\infty$ .



Although only three constants appear explicitly in Eq. (9-91), two further constants are implied by the choice of zero as the lower bound of  $y$  and the point of inflexion at  $y = c/2$ . The usual use of Eq. (9-91) is in sales forecasting, in which case  $y$  is sales demand and  $x$  is time. If such a curve already exists, the value of  $c$  can be read as the upper asymptote and  $a$  and  $b$  obtained by the use of an auxiliary variable  $T$  where

$$T = x_2 \text{ (at } y = r_2c) - x_1 \text{ (at } y = r_1c) \tag{9-92}$$

$$b = [\ln(1/r_1 - 1) - \ln(1/r_2 - 1)]/T \tag{9-93}$$

$$a = (1/r_1 - 1) \exp(bx_1) \tag{9-94}$$

or  $a = (1/r_2 - 1) \exp(bx_2)$

If the values of  $a$  obtained from Eq. (9-94) differ significantly, the logistics curve is not a suitable representation of the data.

**Example 10: Logistics Curve** We shall derive the logistics curve representing the cumulative-frequency distributions of the normal distribution curve defined by Eqs. (9-72) and (9-73). In this case,  $y$  varies between a cumulative probability of zero and unity as  $z$  varies from  $-\infty$  to  $+\infty$ . Since the upper bound is unity,  $c = 1$ . From Table 9-10 the area under the right-hand side of the curve between  $z = 0$  and  $z = z$  may be read. Since the frequency curve is symmetrical about the mean, this is also the area between  $z = 0$  and  $z = -z$ . Hence, the area under the frequency curve, which represents the cumulative probability, is 0.50000 at  $z = 0$  and the 80 percentile, for which the area is 0.80000, corresponds to the value  $z = 0.842$ . We substitute these values into Eqs. (9-92) through (9-94) to give

$$T = 0.842 - 0.000 = 0.842$$

$$b = [\ln(1/0.50 - 1) - \ln(1/0.80 - 1)]/0.842 = 1.6464$$

$$a = 1.0000 \text{ or } 1.00000$$

From Eq. (9-91) the corresponding logistic curve is

$$y = [1 + \exp(-1.6464z)]^{-1}$$

The cumulative-frequency function calculated from this simple expression is compared with the precise value in Table 9-10.

When a cumulative-frequency curve can be satisfactorily represented by a logistics curve, the underlying frequency curve can be obtained by differentiation of Eq. (9-91) as

$$p(x) = \frac{dy}{dx} = \frac{abc \exp(-bx)}{[1 + a \exp(-bx)]^2} \tag{9-95}$$

The probability-density function for the normal distribution curve calculated from Eq. (9-95) by using the values of  $a$ ,  $b$ , and  $c$  obtained in Example 10 is also compared with precise values in Table 9-10. In such symmetrical cases the best fit is to be expected when the median or 50 percentile  $x_M$  is used in conjunction with the lower quartile or 25 percentile  $x_L$  or with the upper quartile or 75 percentile  $x_U$ . These statistics are frequently quoted, and determination of values of  $a$ ,  $b$ , and  $c$  by using  $x_M$  with  $x_L$  and with  $x_U$  is an indication of the symmetry of the curve. When the agreement is reasonable, the mean values of  $b$  so determined should be used to calculate the corresponding value of  $a$ .

In practice most distribution curves are not symmetrical about the median but are inherently skewed. The effect of an advertising campaign is usually to increase the rate of sales in the early years. It may also increase the level of mature demand for the product, but this mature demand must be asymptotic to a finite upper limit of sales  $c$ . Such a curve is positively skewed since  $(x_M - x_L) < (x_U - x_M)$ . This situation can often be approximated by the Gompertz curve defined by Eq. (9-96):

$$\ln y = \ln c - a \exp(-bx) \tag{9-96}$$

which has its point of inflexion at  $0.3679c$ . In terms of the upper and lower quartiles and the median,

$$b = 0.8794/(x_U - x_M) \tag{9-97}$$

$$b = 0.6931/(x_M - x_L) \tag{9-98}$$

$$a = 0.6931 \exp(bx_M) \tag{9-99}$$

The suitability of the Gompertz fit to the curve can be assessed by comparing the values of  $b$  calculated from Eqs. (9-97) and (9-98), and, if suitable, the average value of  $b$  may be used in Eq. (9-99) to calculate the corresponding value of  $a$  to ensure a fit at the median and reasonable accuracy over the more important practical range within a couple of standard deviations on either side of the median.

The underlying frequency distribution curve of the Gompertz curve may be obtained by differentiation of Eq. (9-96) to give

$$p(x) = dy/dx = yab \exp(-bx) \tag{9-100}$$

The logistic and Gompertz curves are of the general shape illustrated by Fig. 9-19. They may be adapted to fit curves of the general shape illustrated by Fig. 9-20 by a little mathematical manipulation. As an example, let us consider the current ratio, the ratio of current assets to current debts, as is quoted in Dun & Bradstreet statistics. A typical value for United States industrial chemical companies might be listed as  $x_L = 1.82$ ,  $x_M = 2.59$ , and  $x_U = 3.25$ . First, we notice that  $(x_M - x_L) > (x_U - x_M)$ . This curve is, therefore, negatively skewed, or reversed S-shaped, and the logistics curve is not suitable. Nor can the Gompertz equation be used directly. However, it is clear that if the curve is drawn upside down and backward, the transformed curve will be positively skewed. Mathematically, this is equivalent to interchanging the upper and lower bounds and considering the dependent variable to be  $(c - y)$ . In the present case the quoted values represent the cumulative probabilities that the current ratio will be less than the quoted value and hence the value of  $y$  ranges between zero and unity. Hence,  $c = 1$ . In the transformed curve  $x_L = 3.25$ ,  $x_M = 2.59$ , and  $x_U = 1.82$ . Hence, from Eq. (9-97)

$$b = 0.8794/(1.82 - 2.59) = -1.1421$$

and from Eq. (9-98)

$$b = 0.6931/(2.59 - 3.25) = -1.0502$$

The variation is within 5 percent of the mean value of  $b = -1.0961$ , and the transformed curve should be sufficiently accurate for many purposes. From Eq. (9-99)

$$a = 0.6931 \exp[(-1.0961)/(2.59)] = 0.40454$$

TABLE 9-10 Data for Normal Distribution Curve

Standard score, $z$	Ordinate of normal distribution curve, $p(z)$		Area under normal curve, cumulative probability, $y = \int_0^z p(z) dz$	
	Precise	Estimated	Precise	Estimated
	0.000	0.3989	0.4116	0.0000
0.100	0.3970	0.4088	0.03983	0.04017
0.200	0.3910	0.4006	0.07926	0.08158
0.253	0.3864	0.3943	0.10000	0.10265
0.300	0.3814	0.3875	0.11791	0.12103
0.400	0.3683	0.3700	0.15542	0.15894
0.500	0.3521	0.3491	0.19146	0.19492
0.524	0.3478	0.3436	0.20000	0.20323
0.600	0.3332	0.3255	0.22575	0.22866
0.700	0.3123	0.3003	0.25804	0.25996
0.800	0.2897	0.2744	0.28814	0.28870
0.842	0.2798	0.2634	0.30000	0.30000
0.900	0.2661	0.2484	0.31594	0.31484
1.000	0.2420	0.2231	0.34134	0.33840
1.200	0.1942	0.1761	0.38493	0.37822
1.282	0.1953	0.1587	0.40000	0.39194
1.400	0.1497	0.1358	0.41234	0.40929
1.600	0.1109	0.1029	0.44520	0.43303
1.645	0.1032	0.0964	0.45000	0.43752
1.800	0.0790	0.0769	0.46407	0.45092
2.000	0.0540	0.0569	0.47725	0.46418
2.500	0.0175	0.0260	0.49379	0.48395
3.000	0.0044	0.0116	0.49865	0.49289
4.000	0.0001	0.0023	0.49997	0.49862
$\infty$	0.0000	0.0000	0.50000	0.50000

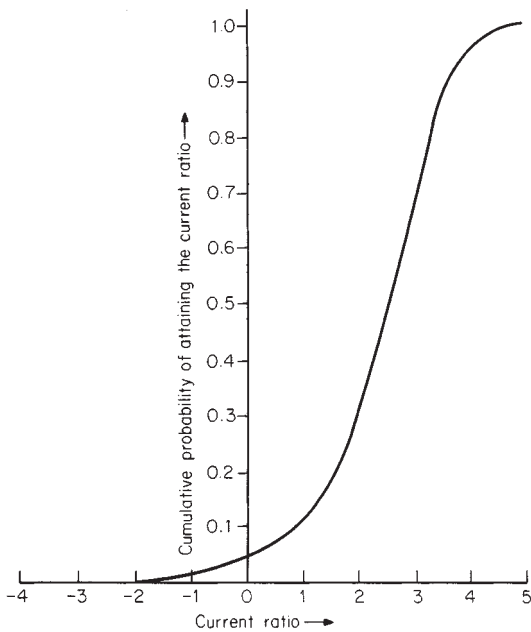


FIG. 9-21 Cumulative probability of a given current ratio.

Hence, from Eq. (9-96) the equation of the transformed curve is

$$\ln(1 - y) = -0.04054 \exp(1.0961x)$$

Since  $d(1 - y) = -dy$ , the corresponding underlying frequency distribution curve is from Eq. (9-100):

$$p(x) = +0.0444(1 - y) \exp(1.0961x)$$

Values of  $y$  and  $p(x)$  calculated from last two equations are plotted in Figs. 9-21 and 9-22 respectively.

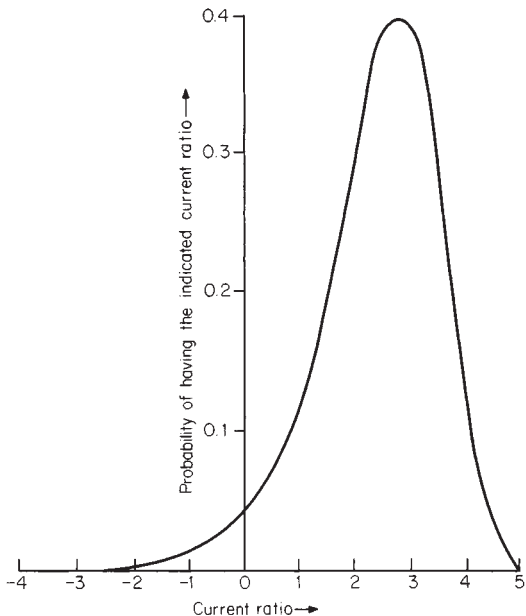


FIG. 9-22 Probability of a given current ratio.

In all such S-shaped curves the range of  $x$  is from  $-\infty$  to  $+\infty$ , so that there is always a finite possibility of negative values of  $x$  occurring. In the present case the definition of the current ratio makes values of  $x$  below zero meaningless. The error of some 4 percent in the cumulative-probability curve implied by this factor may be tolerable in a given case.

It can be shown [J. J. Molder and E. G. Rogers, *Manage. Sci.*, **15**, B-76 (1968)] that for continuous events it is possible to estimate the mean and standard deviation of a skewed distribution from estimates of a low value, a most likely or modal value, and a high value. It is suggested, since it is difficult to make very fine subjective judgments as to probabilities, that the range most likely to be accurate is that for which there is a 10 percent chance of a value less than the low value and a 10 percent chance of a value greater than the high value. These values will usually imply a skewed distribution. For the suggested 80 percent confidence level the best available estimates are

$$\bar{x} = (\text{low value}) + [(2)(\text{modal value})] + (\text{high value}) \quad (9-101)$$

$$s^\circ = [(\text{high value}) - (\text{low value})]/2.65 \quad (9-102)$$

On this basis an alternative approach to risk analysis is the parameter method [D. O. Cooper and L. B. Davidson, *Chem. Eng. Prog.*, **72**, 73-78 (November 1976)].

**Example 11: Parameter Method of Risk Analysis** Let us consider the project outlined in Table 9-5. It is estimated that the basic data represent the most likely values and that there is a 10 percent chance that  $A_S$  will be reduced by more than 20 percent or will be increased by more than 5 percent. In the same way the low and high levels at 10 percent probability for  $A_{TE}$  are considered to be 5 percent below and 25 percent above the base figures respectively. The low and high values for  $C_{FC}$  are considered to be 5 percent below and 30 percent above the base figure, while changes in other parameters are considered to be immaterial.

With a cost of capital  $i$  of 10 percent the various cash flows can be discounted and summed. Thus for the base cases  $\sum A_d f_d = \$2,815,600$ ,  $\sum A_{TE} f_d = \$754,716$ ,  $\sum A_{DF} f_d = \$614,457$ , and  $\sum C_{WC} f_d = \$61,446$ . With corporate taxes payable at 50 percent the aftertax cash flows of the first three items are  $(1 - 0.50)$  of the sums calculated above. The discounted working capital and the fixed-capital outlay are not subject to tax. These most probable values are listed and summed in Table 9-11 and, after adjustment for tax, give the modal value of the (NPV) as \$276,224.

A reduction of 20 percent in  $A$ , for each year will result in a 20 percent reduction in  $\sum A_d f_d$  below the modal value, i.e., a reduction of  $(0.20)(\$2,815,600) = \$563,120$ . The aftertax effect of this reduction on the contribution to (NPV) is  $(1 - 0.50)(\$563,120) = \$281,560$ , making the low value  $\$1,407,800 - \$281,560 = \$1,126,240$  or, more directly,  $(0.8)(\$1,407,800)$ . Other values in Table 9-11 are calculated in a similar manner.

The mean value of each of the distributions is obtained from these high, modal, and low values by the use of Eq. (9-101). If the distribution is skewed, the mean and the mode will not coincide. However, the mean values may be summed to give the mean value of the (NPV) as \$161,266. The standard deviation of each of the distributions is calculated by the use of Eq. (9-75). The fact that the (NPV) of the mean or the mode is the sum of the individual mean or modal values implies that Eq. (9-81) is appropriate with all the  $k$ 's equal to unity. Hence, by Eq. (9-81) the standard deviation of the (NPV) is the root mean square of the individual standard deviations. In the present case  $s^\circ = \$166,840$  for the (NPV).

If the resulting distribution is assumed to be normal, then the cumulative distribution curve can immediately be generated. From Table 9-10, a standard score of 4 corresponds to a probability of  $0.5 + 0.49997 = 0.99997$  and one of  $-4$  to a probability of  $0.5 - 0.49997 = 0.00003$ , virtually unity and zero respectively. From Eq. (9-73) a standard score of 4 corresponds to an (NPV) of  $\$161,266 + (4)(\$166,840) = \$828,626$  and one of  $-4$  to an (NPV) of  $\$506,094$ . Values of (NPV) corresponding to other confidence limits may be calculated in the same way and plotted to give the curve of Fig. 9-23.

TABLE 9-11 Data for Risk Analysis

Parameter	(NPV), \$/year			Mean	Standard deviation
	Low	Modal	High		
$A_S$	1,126,240	1,407,800	1,478,190	1,355,008	132,811
$A_{TE}$	-471,698	-377,358	-358,490	-396,226	-42,720
$C_{FC}$	-900,603	-692,772	-658,133	-736,070	-91,498
$C_{WC}$	-61,446	-61,446	-61,446	-61,446	0

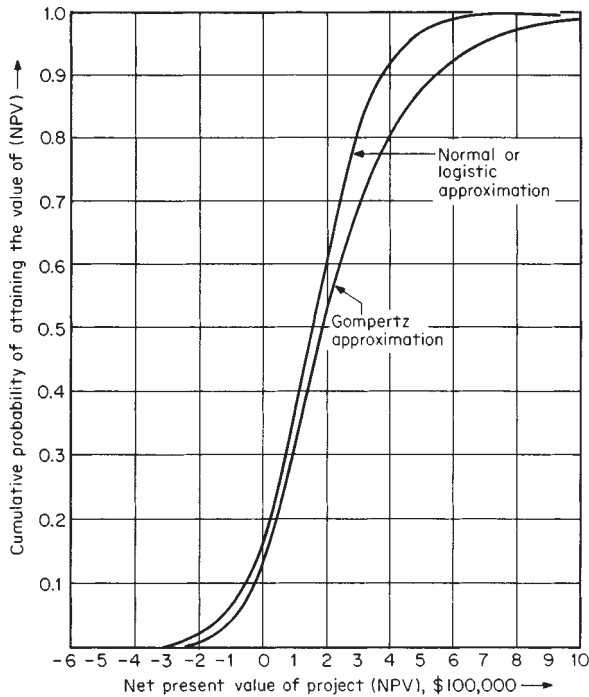


FIG. 9-23 Cumulative probability of a given net present value or less for a project showing normal and Gompertz approximations.

As has been stated, with the uncertainties attached to many business assessments of the range of various factors, the central-limit theorem implies that the assumption of a normal distribution of the main variable is sufficiently accurate provided that there are several factors contributing to that main variable. The results are as informative as most Monte Carlo estimates and have the advantage that they can be rapidly obtained without recourse to a digital computer, although a good desk calculator speeds the work. Strictly, the variables should be independent and additive. Thus it is better, for example, to treat  $(A_S - A_{TVE})$  as a single variable since both sales income and total variable expense are related to the annual rate of sales  $R$ . In such cases the standard deviation of  $A_{TVE}$  would be added to or subtracted from that of  $A_S$  before squaring to obtain the variance according as the uncertainty of the group was greater or less than that of the individual factors. Also, when a product such as  $A_S = Rc_S$  is involved, Eq. (9-84) should be used to estimate the variance rather than Eq. (9-82). When the predominant uncertainties are multiplied together, a log-normal distribution may provide a better final distribution. A similar technique may be applied to the (DCFR) provided that Eq. (9-85) is

used in place of Eq. (9-86) to estimate the overall variance of the main variable.

When the estimates are well founded, the skewness may be preserved by using a distribution such as the Gompertz. The median of that curve occurs as  $y = 0.5c$ , while the point of inflexion corresponds to the mode at  $y = c/\exp(1) = 0.3679c$ . The statistician Karl Pearson suggested as a simple measure of skewness

$$\text{Skewness} = 3(\text{mean} - \text{median})/\sigma \quad (9-103)$$

with an empirical approximation in terms of the mode given by

$$(\text{Mean} - \text{mode}) = 3(\text{mean} - \text{median}) \quad (9-104)$$

Applying these equations to the present problem,

$$(\text{Mean} - \text{mode}) = \$161,266 - \$276,224 = -\$114,958$$

$$\text{Skewness} = -\$114,958/\$166,840 = -0.6890$$

For symmetrical distributions, such as the logistic or normal, the skewness should be zero.

The Gompertz distribution requires the distribution to be positively skewed, which can be achieved by treating  $-(NPV)$  as the independent variable and  $(c - y)$  as the dependent variable. From Eq. (9-104) the median of the distribution is given approximately as

$$\text{Median} = [\$161,266 - (-\$114,958)]/3 = \$199,585$$

Substituting values into Eq. (9-96) with  $-(NPV)$  as the independent variable to give, since the range of  $y$  is zero to unity,

$$\ln(1 - 0.5) = \ln(1 - a) \exp[(-b)(-\$199,585)]$$

$$\ln(1 - 1/e) = \ln(1 - a) \exp[(-b)(-\$276,224)]$$

$$\text{whence } b = \frac{-\ln[\ln(0.5)/\ln(1 - 1/e)]}{[(-\$199,585) - (-\$276,224)]} = -5.388 \times 10^{-6}/\$$$

$$a = \frac{-\ln 0.5}{\exp[(5.388 \times 10^{-6}/\$)(-\$199,585)]} = 2.0315$$

The Gompertz curve of the distribution is then, in terms of (NPV),

$$\ln y = -2.0315 \exp[-5.388 \times 10^{-6}(\text{NPV})]$$

For the same degree of certainty as before, the minimum value of the (NPV) is likely to be

$$\ln \left[ \frac{\ln(0.00003)}{-2.0315} \right] - 5.388 \times 10^{-6} = -\$303,365$$

and the maximum of  $\$2,064,569$  calculated in the same way for  $y = 0.99997$ . Other values are calculated in the same way and are plotted as in Fig. 9-23.

**Decision Trees** In a typical decision tree, illustrated in a very simplified form by Fig. 9-24, each node represents a decision point (DP) at which one or more alternatives are available. Some quantifiable result of each alternative is chosen as a basis for comparison; for example, the net present value (NPV). A value is assigned to the probability of attaining each result, either cumulative or not as required. These may be obtained by the methods just described or otherwise. The estimates are subject to the restriction that the sum of the proba-

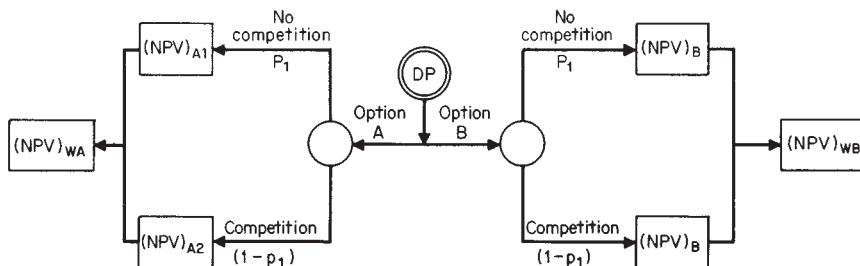


FIG. 9-24 Effect of decision-tree options on net present value.

bilities for all branches leaving each node shall be unity since some decision must be taken there.

In considering two investments, we shall let option B be a safe investment having a base net present value  $(NPV)_B$  that is independent of any competition. We shall let option A yield a net present value  $(NPV)_{A1}$  if no competition exists and  $(NPV)_{A2}$  if competition exists. We shall then let the probabilities of no competition and competition be  $p_1$  and  $p_2$  respectively. Then  $p_2$  must equal  $(1 - p_1)$ .

The expected (NPV) for option A can be written, from Eq. (9-105), which follows, as

$$(NPV)_{WA} = p_1(NPV)_{A1} + (1 - p_1)(NPV)_{A2}$$

where  $(NPV)_{WA}$  is the weighted net present value for option A based on the probabilities of encountering no competition  $p_1$  and of encountering competition  $(1 - p_1)$ .

In the same way the expected (NPV) for option B is given by

$$(NPV)_{WB} = p_1(NPV)_B + (1 - p_1)(NPV)_B = (NPV)_B$$

The gain in the expected value of option A over option B is thus

$$\Delta(NPV)_W = (NPV)_{WA} - (NPV)_{WB}$$

Let us suppose that the options represented in Fig. 9-24 were such that  $(NPV)_B = (0.5)(NPV)_{A1} = 2(NPV)_{A2}$ . Then substitution leads to

$$\begin{aligned} \Delta(NPV)_W &= [2p_1 + 0.5(1 - p_1) - 1](NPV)_B \\ &= (1.5p_1 - 0.5)(NPV)_B \end{aligned}$$

The choice is immaterial when  $\Delta(NPV)_W = 0$ , i.e., when  $p_1 = 1/3$ . If the probability of no competition is greater than  $1/3$ , option A should be chosen; otherwise option B should be chosen.

The technique is based on the methods of linear algebra and the theory of games. When the problem contains many multibranch decision points, a computer may be needed to follow all possible paths and list them in order of desirability in terms of the quantitative criterion chosen. The decision maker may then concentrate on the routes at the top of the list and choose from among them by using other, possibly subjective criteria. The technique has many uses which are well covered in an extensive literature and will not be further considered here.

**Numerical Measures of Risk** Without risk and the reward for successfully accepting risk, there would be no business activity. In estimating the probabilities of attaining various levels of net present value (NPV) and discounted-cash-flow rate of return (DCFRR), there was a spread in the possible values of (NPV) and (DCFRR). A number of methods have been suggested for assessing risks and rewards to be expected from projects.

Let us consider a proposed project in which there is a probability  $p_1$  that a net present value  $(NPV)_1$  will result, a probability  $p_2$  that  $(NPV)_2$  will result, etc. A weighted average  $(NPV)_w$ , known as the expected value, can then be calculated from

$$(NPV)_w = p_1(NPV)_1 + p_2(NPV)_2 + \dots \quad (9-105)$$

where  $p_1 + p_2 + \dots = 1.0$ .

Analogous equations may be written for other additive measures of profitability such as net profit.

**Example 12: Expected Value of Net Profit** Let us consider a contractor who stands to make a net profit of \$100,000 on a contract. The cost of preparing the bid on the contract is \$10,000. There are four competing contractors, each with a probability  $p_1 = 0.25$  of obtaining the contract. Thus, each contractor has a probability  $p_2 = 0.75$  of not obtaining the contract. Therefore, the expected value of the project is

$$0.25(\$100,000) + 0.75(-\$10,000) = \$17,500$$

In this case, the potential gain is 10 times greater than the potential loss.

If the potential loss can bankrupt the company, then decisions are not necessarily made on the basis of expected value even though the potential gain may be very high. Also, decisions are not necessarily made on the basis of expected value if the potential loss represents a relatively small amount of money to the company. Between these two extremes, expected value can be a very useful criterion, particularly for a company with a large number of projects.

A company may be considering a project with a very high potential rate of return and a low risk, but it may prove impossible to raise the money to start the project. Conversely, the company may be prepared to undertake an extremely risky project if the investment is trivial. Thus, the attitude of a company to risk depends on the circumstances.

Money does not hold the same value for each company or each individual. A dollar may keep a pauper from starvation while being a trivial amount to the person who gave it. Attempts have been made to quantify a company's attitude to money, risk, and uncertainty by asking business executives a number of questions such as the following:

"Your company has signed a business contract with potential after-tax proceeds of \$P. The probability of achieving the net gain of \$P is, say,  $p_1 = 0.75$ , and the probability of a net loss of \$P is  $p_2 = 0.25$ . If you would rather keep the contract, how much cash would you accept for your interest in it? If you would rather be released from the contract, how much cash would you pay to be released from it?"

The same questions may then be asked for different values of the probabilities  $p_1$  and  $p_2$ . The answers to these questions can give an indication of the importance to the company of \$P at various levels of risk and are used to plot the utility curve in Fig. 9-25. Positive values are the amounts of money that the company would accept in order to forgo participation. Negative values are the amounts the company would pay in order to avoid participation. Only when the utility value and the expected value (i.e., the straight line in Fig. 9-25) are the same can net present value (NPV) and discounted-cash-flow rate of return (DCFRR) be justified as investment criteria.

Since the utility curve has such a subjective basis, most companies prefer the objectivity of (NPV) and (DCFRR) over the range of the normal income and expenditure budget. Subjective methods tend to be reserved for exceptionally high risk projects.

A utility curve such as that in Fig. 9-25 is specific to a certain sum of money. The curve is likely to be different for, say,  $P = \$10,000$ . Figure 9-25 can only be used to consider projects that fall within the range of  $-\$100,000$  to  $+\$100,000$ . Other utility curves must be used to cover projects that lie outside this range.

R. O. Swalm ["Utility Theory—Insight into Risk Taking," *Harv. Bus. Rev.*, **44**, 123-136 (November-December 1966)] found that many business executives had difficulty in appreciating fine shades of odds and confined his considerations to even-money bets. He asked various executives to state what guaranteed sum of money they considered equivalent to a gamble related to the toss of a coin. If the coin fell on one side, they would win a given sum of money; if the coin fell on the other side, they would get nothing.

Swalm started by considering a sum of money equivalent to twice the maximum expenditure that the executive could authorize in 1 year. This was used to obtain a further utility. In this way, a utility curve could be sketched. Swalm chose an arbitrary utility scale based on a range of  $-120$  utiles to  $+120$  utiles. (NOTE: It is as incorrect to compare utiles by ratio as it is to imply that an object at  $30^\circ\text{C}$  is twice as hot as an object at  $15^\circ\text{C}$ .)

Swalm found that most executives are conservative in their expenditure and that the patterns of utility curves are very similar if plotted with an ordinate range of  $\pm 1$  unit. The unit, in this case, is the maximum authorized annual expenditure of the executive. Such curves may appear to differ quite widely when plotted in terms of absolute money values. The curves also show that executives tend to be more conservative when considering a loss than they do when considering a reduced gain.

**Example 13: Evaluation of Investment Priorities Using Probability Calculations** A company is considering investment in one or more of three projects, A, B, and C. We wish to evaluate the investment priorities if the probabilities of attaining various net present values (NPV) are as listed in the third column of Table 9-11. Equation (9-105) gives the expected value for  $(NPV)_w$ . Hence for project A,  $(NPV)_w$  is computed from the data in Table 9-12 and found to be

$$\begin{aligned} (NPV)_w &= 0.1(\$95,000) + 0.8(\$45,000) + 0.1(-\$75,000) \\ (NPV)_w &= \$9,500 + \$36,000 - \$7,500 \\ (NPV)_w &= \$38,000 \end{aligned}$$

Corresponding values for projects B and C are calculated in the same way and are listed in Table 9-12.

**TABLE 9-12 Comparison of Projects in Terms of Expected Value and Expected Utility**

Project	(NPV), \$	Probability, $p$	Equivalent probability of winning \$100,000	Probable utility	Expected value, \$	Expected utility, \$
A	95,000	0.1	0.80	0.080	9,500	8,000
	45,000	0.8	0.34	0.272	36,000	27,200
	-75,000	0.1	0.03	0.003	-7,500	-300
		1.0		0.355	38,000	35,500
B	50,000	0.2	0.37	0.074	10,000	7,400
	20,000	0.6	0.23	0.138	12,000	13,800
	-60,000	0.2	0.04	0.008	-12,000	-800
		1.0		0.220	10,000	22,000
C	45,000	0.1	0.35	0.035	4,500	3,500
	10,000	0.6	0.20	0.120	6,000	12,000
	-60,000	0.3	0.04	0.012	-18,000	-1,200
		1.0		0.167	-7,500	16,700

In project A, the probability  $p = 0.1$  for (NPV) = \$95,000. Figure 9-25 shows that \$95,000 is the amount of money that this company would pay for a 0.8 probability of gaining \$100,000. There is, therefore, a 0.2 probability of losing \$100,000. In this case, a probability of  $p = 0.1$  of attaining \$95,000 is equivalent to a probable utility of  $(0.1)(0.8) = 0.08$  of gaining \$100,000.

Equation (9-105) can be used to calculate the expected utility if the probabilities  $p_1, p_2, \dots$ , are replaced by the probable utilities and if the net present values (NPV)<sub>1</sub>, (NPV)<sub>2</sub>, etc., are each replaced by \$100,000. For project A, the expected utility  $U_w$  is

$$U_w = [0.1(0.8) + 0.8(0.34) + 0.1(0.03)]\$100,000$$

$$U_w = \$8,000 + \$27,200 + \$300$$

$$U_w = \$35,500$$

Corresponding values for projects B and C are calculated in the same way and are listed in Table 9-12.

The straight line in Fig. 9-25 represents the situation in which the expected value and the expected utility are equal over the range of  $-\$100,000$  to

$+\$100,000$ . In this case, decisions can be taken on the basis of the highest expected value as a routine matter. In other cases, decisions should be made on the basis of the highest expected utility. The utility curve in Fig. 9-25 represents the present attitude of management to \$100,000. This curve should be updated as the company's business position changes. In this example, the utility curve is above the straight line. This represents a tendency on the part of the company's decision makers to gamble. When it is below the straight line, the utility curve implies conservatism. The investment priorities should be to implement project A and then, if finance is available, project B.

It might appear that project C should also be considered in view of the expected utility of \$16,700. However, it is better to do nothing than to implement project C. The utility of doing nothing, which is equivalent to paying \$0, is read from Fig. 9-25 to be 0.17. This gives a corresponding probable utility of  $(1.0)(0.17)(\$100,000)$ , or \$17,000. This is a better result than investing in project C.

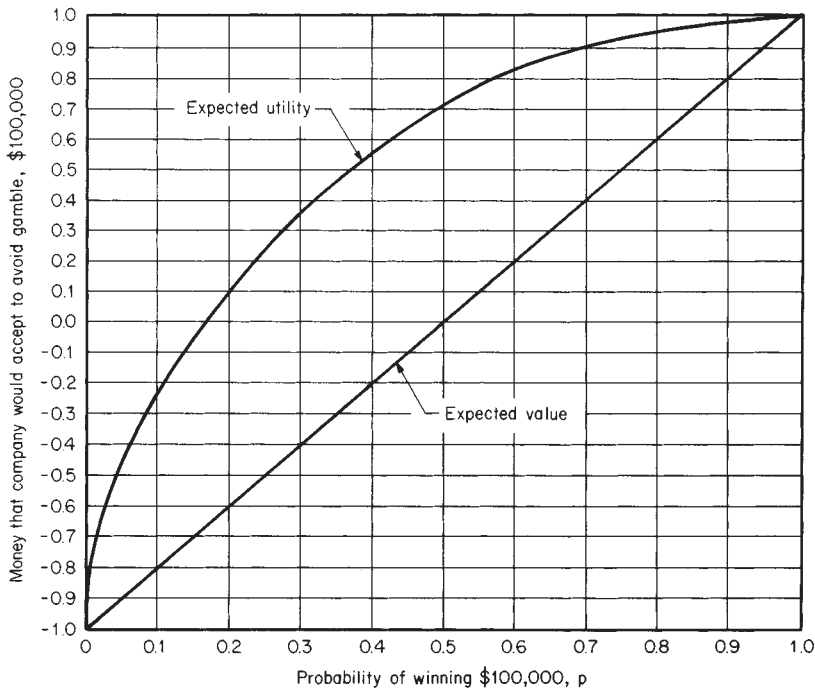
In this example, the order of priorities based on expected utilities is the same as that based on expected values. However, the order of priorities is clear-cut on the basis of expected value but much less so on the basis of expected utility.

Capital is at risk until the breakeven point has been reached. It is common practice to give consideration to the discounted breakeven point (DBEP), the time at which the (NPV) is zero when discounting at the cost of capital. At any time after the (DBEP), the project will have recovered its cost and provided a greater return on the capital than the cost of capital. It is customary for management to spread risk by diversifying the activities of a company among a portfolio of projects.

R. L. Reul [*Chem. Eng. (London)*, **238**, CE 120-125 (May 1970)] has defined a parameter, which he calls the measured-survival function (MSF), given by

$$(\text{MSF}) = 1 - (1 - p)^\beta \quad (9-106)$$

where (MSF) is the probability that a portfolio of bets with a similar strategy will at least break even, and  $\beta$  is the amount of one win divided by the amount of each bet. Reul has applied Eq. (9-106) to the research and development activities of a company. Equation (9-106) is based on the simplified assumption that a project either succeeds with probability  $p$  and achieves the expected reward or fails completely with probability  $(1 - p)$ . Therefore, (MSF) is the probability of at least


**FIG. 9-25** Utility-function plot for \$100,000.



one success when  $\beta$  similar projects are undertaken and represents a conservative measure of risk. It follows that  $\beta > 1$  and hence that  $(MSF) > p$ . Many projects may result in greater returns or have an increased probability of attaining a given return if more money is spent. Each alternative derivable result from a given project is treated as a separate risk in the portfolio.

Research and development activities do not, in themselves, produce a salable product. Thus, they cannot directly generate a return on capital outlay. A successful research and development project is one that results in an activity that earns revenue for the company. The life cycle of the revenue from an individual product may be as shown in Fig. 9-26.

This revenue has to pay not only for the successful project but for all the unsuccessful research and development activities. It is common practice to consider all R&D as a portfolio. Disbursements for R&D are relatively flexible and can be switched from less favorable to more favorable projects at short notice.

When considering individual projects,  $\beta$  should be taken as the lesser of

$$\beta = \frac{\text{expected proceeds if project is successful}}{\text{disbursement on project}}$$

or 
$$\beta = \frac{\text{total expenditure on all projects over budget period}}{\text{expenditure on project over budget period}}$$

Because the projects in a portfolio will usually have different probabilities of success and different rewards for success,  $\beta$  and  $p$  in Eq. (9-106) are conservatively estimated as follows:

$$\beta = \frac{\text{total annual proceeds if all projects are successful}}{\text{total annual disbursements on all projects}}$$

$$p = \frac{\text{total expected value of all projects}}{\text{total proceeds if all projects are successful}}$$

The expected value can be calculated from Eq. (9-105).

The relationship between  $(MSF)$ ,  $p$ , and  $1/\beta$  in Eq. (9-106) is shown graphically in Fig. 9-27. It is the responsibility of management to decide on an acceptable value of the  $(MSF)$  for its company. The value chosen will depend on the company's attitude to risk that can be quantified in the form of a utility curve such as the one shown in Fig. 9-25, from which a value of equivalent  $(MSF)$  can be obtained.

It is also the responsibility of management to estimate the probabilities for the success of individual projects after due consideration of all the data provided by the various departments. The rate of return on investment that is acceptable to management is a function of these responsibilities. Each industry has a reasonably well defined return on investment that reflects the degree of risk inherent in that industry. If management decisions are faulty, the company either will overspend or will miss opportunities.

With a disbursement of \$1000 in Year 0, the discounted breakeven point (DBEP) will be reached in 3 years at a compound-interest rate of 30 percent if the annual net profit  $A_{NP} = \$550.63$  per year. Thus, a

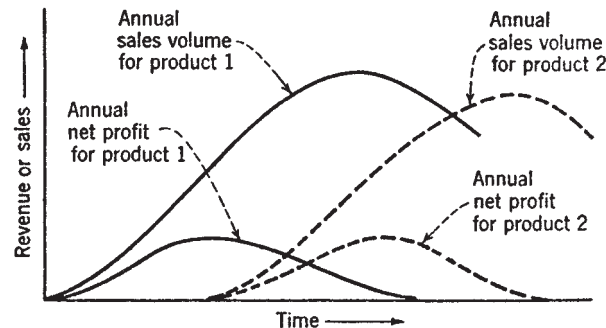


FIG. 9-26 Life cycle of products.

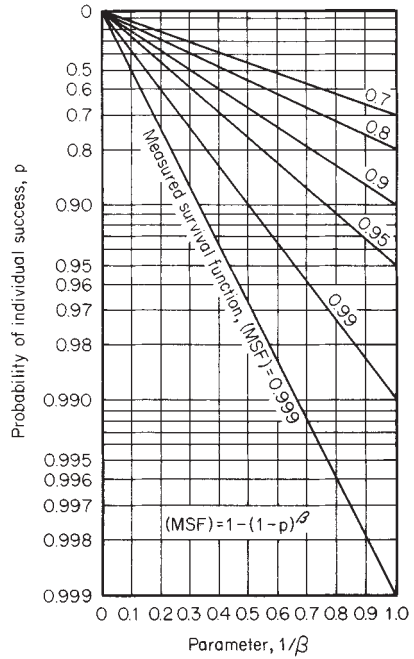


FIG. 9-27 Measured-survival-function plot.

discounted-cash-flow rate of return (DCFRR) of 30 percent corresponds to

$$\frac{1}{\beta} = \frac{\$1000}{(3 \text{ years})(\$550.63/\text{year})} = 0.61$$

For  $1/\beta = 0.61$  and an  $(MSF) = 0.999$ , the probability of individual success is read from Fig. 9-27 to be  $p = 0.985$ . Similarly, it can be deduced that if  $(MSF) = 0.999$  and  $p = 0.95$ , a (DCFRR) of 45 percent is required; if breakeven in 20 years is acceptable, then a (DCFRR) of only 10 percent is needed.

**Example 14: Estimation of Probability of a Research and Development Program Breaking Even** Details of the estimates for the current research and development program of a company are given in Table 9-13. We shall estimate the probability that this portfolio will at least break even.

The total annual proposed disbursement for R&D is \$500,000. The effective total annual income if all projects reached their anticipated income would be \$1,300,000. Therefore,

$$\beta = \$1,300,000/\$500,000 = 2.600$$

Project A has an expected value of  $(0.95)(\$500,000/\text{year}) = \$475,000/\text{year}$ ; project B has an expected value of  $(0.90)(\$400,000/\text{year}) = \$360,000/\text{year}$ ; and so on. We sum these values to obtain the total expected value of the portfolio as \$1,109,500 per year. Hence,

$$p = (\$1,109,500/\text{year})/(\$1,300,000/\text{year}) = 0.8535$$

TABLE 9-13 Example of a Portfolio of Projects for a Research and Development Program

Project	Proposed disbursement for coming year	Annual aftertax income if successful	Probability of success
A	\$125,000	\$ 500,000	0.95
B	100,000	400,000	0.90
C	100,000	125,000	0.80
D	80,000	100,000	0.75
E	50,000	60,000	0.70
F	20,000	30,000	0.65
G	20,000	70,000	0.50
H	5,000	15,000	0.20
Totals	\$500,000	\$1,400,000	



From Fig. 9-27, for a probability of success  $p = 0.8535$  and a value  $1/\beta = 1/2.600 = 0.3846$ , the (MSF) is 99.3 percent. This is the probability that this portfolio will at least break even.

Alternatively, we can substitute the values for  $p$  and  $\beta$  into Eq. (9-106) to get

$$(\text{MSF}) = 1 - (1 - 0.8535)^{2.6} = 0.9932, \text{ or } 99.32 \text{ percent}$$

The (MSF) and utility curves can be related.

**Example 15: Utility-Function Curve** Let us sketch a utility-function curve that is equivalent to the following pattern of measured-survival functions (MSF), which expresses the observed strategy of a particular manager when spending an authorized annual budget of \$1,000,000:

Case	Potential proceeds annually, \$	(MSF), %
a	Above 600,000	99.9
b	300,000–600,000	95.0
c	0–300,000	65.0
d	Losses	75.0

We shall plot the resultant curve on a utility scale of  $\pm 120$  utiles against a potential gain of  $\pm \$1,000,000$ .

The required axes range from  $-\$1,000,000$  per year to  $+\$1,000,000$  per year, and from  $-120$  utiles to  $+120$  utiles. Utiles can be compared by ratio on an absolute scale only. Hence, for purposes of calculation the axes are moved to provide a working range of \$0 per year to \$2,000,000 per year and 0 to 240 utiles as in Fig. 9-28. On these axes, a potential gain of \$600,000 per year corresponds to an absolute amount of  $(600,000 + 1,000,000) = \$1,600,000$  per year, and a potential loss of \$200,000 per year to an absolute amount of  $(-200,000 + 1,000,000) = \$800,000$  per year.

a. For annual proceeds above \$600,000 per year, (MSF) is 99.9 percent. If the certainty of an annual gain of \$600,000 has to be abandoned in an effort to obtain an annual gain of \$1,000,000, then on an absolute scale

$$\beta = (\$2,000,000/\text{year})/(\$1,600,000) = 1.2500$$

With  $1/\beta = 0.8000$  and (MSF) = 99.9 percent, we find the required probability of success by solving Eq. (9-106) for  $p$ :

$$p = 1 - (1 - 0.999)^{0.8000} = 0.996$$

The utility of an amount of money is its utility when it is certain to be obtained, multiplied by its probability of being attained. On a scale in which an absolute annual income of \$2,000,000 per year has a utility of 240 utiles, the utility of \$1,600,000 is  $(0.996)(240)$ , or 239 utiles.

b. For annual proceeds between \$300,000 and \$600,000, (MSF) = 95 percent. If the certainty of an annual gain of \$300,000 has to be abandoned to obtain an annual gain of \$600,000, then, as before,

$$1/\beta = \$1,300,000/\text{year}/\$1,600,000/\text{year} = 0.8125$$

$$p = 1 - (1 - 0.95)^{0.8125} = 0.912$$

Since, to this manager, the utility of an absolute income of \$1,600,000 is 239 utiles, the value of \$1,300,000 is  $(0.912)(239) = 218$  utiles. On the original scales, potential annual proceeds of \$300,000 have a utility of  $(218 - 120)$ , or 98 utiles.

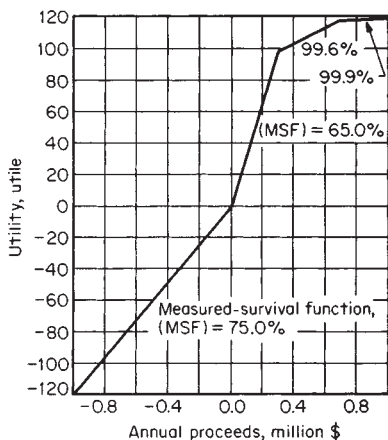


FIG. 9-28 Utility-function plot illustrating managerial strategy.

$c$  and  $d$ . Values of utility at other potential annual gains are calculated in the same way and shown graphically in Fig. 9-28.

This strategy is extremely conservative when high gains are possible but becomes less so for smaller potential gains. If potential losses are involved, the strategy is a fair one for which (NPV) would be an accurate guide for choosing alternatives.

**Insurance and Risk** In the venture-premium method of assessment, risky investments are required to yield a rate of return that adds a premium to the cost of finance. D. F. Rudd and C. C. Watson (*The Strategy of Process Engineering*, Wiley, New York, 1968, p. 91) consider this relationship:

$$i_m = i + i_r \quad (9-107)$$

where  $i$  is the cost of capital,  $i_m$  is the minimum acceptable interest rate of return on the investment, and  $i_r$  is known as the risk rate.

They suggested that each project should pay an insurance premium  $i_r$  to guarantee the expected profits. The magnitude of  $i_r$  is proportional to the amount of capital to be risked. It is also a function of the degree of risk involved. Working capital and capital for auxiliary facilities are assumed to be risk-free. Thus, the risk rate is applied only to the fraction of the capital investment likely to be lost if the project is unexpectedly terminated.

The main objection to the venture-premium method is that the assessment of the riskiness of a project may be too subjective. This could lead to the rejection of potentially attractive proposals and the acceptance of projects that merely appear to be risk-free.

Insurance is protection against risk. Commercial insurance companies minimize their own risks by covering a large number of individuals against a given risk and also by offering coverage on a wide variety of different types of risk. It is frequently quite difficult to assess the probability of success of a particular research and development project. It is much easier for an insurance company to assess its probabilities from its casualty tables.

Businesses tend to provide their own insurance cover when individual claims are likely to be a small fraction of the available capital. The cost of commercial insurance is about 30 percent higher than would be necessary to cover the same risk in one's own company. However, for low-probability, high-cost risks, most businesses prefer to insure with a commercial insurance company. Such risks include loss of plant or buildings due to fire and losses of revenue due to delays in startup or strikes.

It is also becoming necessary to insure against factors not normally considered until recently. These include possible lawsuits for polluting the environment. The cost of insurance increases the annual total expense  $A_{TE}$ . Thus, overinsurance can lead to an unnecessary decrease in profitability. The management of a company must ultimately judge its own risks.

As an example, let us calculate the required risk rate for a project that is described by the following: (1) risk strategy is equivalent to an (MSF) of 99 percent, (2) payback of risk capital is 3 years, (3) cost of capital  $i$  is 10 percent, and (4) probability of complete success of the project is estimated as 95 percent.

First, we calculate the value of  $\beta$  in Eq. (9-106). For this project, with (MSF) = 0.99 and  $p = 0.95$ ,

$$\beta = \frac{\log [1 - (\text{MSF})]}{\log (1 - p)} = \frac{\log (0.01)}{\log (0.05)} = 1.537$$

To recover this amount of capital and interest in 3 years, the average net annual cash flow  $A_{CF}$  required is

$$A_{CF} = 1.537/3 = 0.5124 \text{ (\$/year)/(\$ invested)}$$

In effect, in computing the average net annual cash flow per dollar invested, the value of  $f_{AP}$  of Eq. (9-46) has been obtained for this example. From tables of the annuity present-worth factor  $f_{AP}$  the value of the interest rate is found to be  $i_m = 0.25$  when  $f_{AP} = 0.5124$  with  $n = 3$  years.

Hence, by substituting appropriate values into Eq. (9-107) and solving for the required risk rate,

$$\begin{aligned} i_r &= i_m - i \\ &= 25 - 10 = 15 \text{ percent} \end{aligned}$$

based on the payback period of the risk capital. All capital  $C_{WC}$  is completely recoverable without risk and requires interest only at 10 percent. The unrecovered part of the risk capital  $C_{FC}$  attracts the additional risk interest rate of 15 percent, which should be reduced as the risk capital is written down.

A different view of risk is expressed in Eq. (9-108):

$$[1 + (\text{DCFRR})] = (1 + i)(1 + i') \quad (9-108)$$

The (DCFRR) represents the return on all capital invested after such capital has been paid back, together with any interest incurred by borrowing it, and after payment of all expenses, including taxes, associated with the project. It thus represents the entrepreneurial return to the company for managing the total capital employed. If the cost of capital  $i$  is set at the best risk-free use of that capital, such as the interest rate on a bank deposit or on government bonds, etc.,  $i'$  represents the increased entrepreneurial return on the capital for taking the risks involved. This is a useful concept since the probability of achieving a given (DCFRR), and hence of a particular value of  $i'$ , may be estimated by the methods detailed previously. We notice that  $i$ , as so defined, implies that all taxes and interest have been paid. Thus, \$100 deposited in a bank at a rate of 10 percent with half of the money borrowed at 15 percent and corporation tax at 40 percent would result in a risk-free income after tax of

$$[\$100(0.10/\text{year}) - (0.5)(\$100)(0.15/\text{year})](1 - 0.40) = \$1.5/\text{year}$$

The same money invested in a project with a (DCFRR) of 10 percent would, by Eq. (9-108), obtain an entrepreneurial return  $i' = 8.37$  percent on the whole investment, i.e., \$8.37/\$100. Investment of the entrepreneur's own money would only achieve an aftertax return of  $(0.1)(1 - 0.40) = 6$  percent on \$50, or \$3/\$100 of total investment. The incentive to the entrepreneur to manage the project thus corresponds to a tax-free income of \$5.37/\$100 of total investment. In practice, money is borrowed from more than one source at different interest rates and at different tax liabilities. The effective cost of capital in such cases can be obtained by an extension of the above reasoning and is treated in detail by A. J. Merrett and A. Sykes (*Capital Budgeting and Company Finance*, Longmans, London, 1966, pp. 30-48).

**Inflation** It is currently necessary to evaluate the profitability of proposed investments whose future earnings are virtually certain to be eroded by inflation. It has been common practice to ignore the effects of inflation. This is done on the reasonable grounds that predicting the market rate of interest, and thus the appropriate discount rate for future cash flows, is difficult enough without having to worry about inflation as well. But failure at least to try to predict inflation rates and take them into account can greatly distort a project's economics, especially at the double-digit rates that have been found throughout the world. It is the common experience that a given amount of money buys less and less of goods and services as time goes by. The problem is to express this experience quantitatively.

Published figures for inflation rates are based on some particular mixture of goods and services that is chosen to represent the material wants of the average citizen. If a given quantity of this specific mixture cost \$100 last year and now costs \$120, then the mix has suffered a 20 percent rate of inflation. The purchasing power of the currency (i.e., of the \$120) in respect of these goods and services has consequently fallen by a factor of  $(\$120 - 100)/\$120$ , or 16.7 percent.

Two kinds of inflation can be considered: general, or open, inflation and repressed, or differential, inflation. In the first case, all costs and

prices increase at a uniform rate. Thus, the same rate of inflation will be calculated regardless of the particular mixture of goods and services chosen. In the second case, the rate of inflation will depend on the spending spectrum of the individual or company. For instance, a given company's labor costs and material costs may inflate at different rates. To quite a large extent, inflation becomes repressed, or differential, in such fields as taxation, import control, and price restriction.

The effect of inflation on the real value of future earnings from a project should not be confused with the effect of the market rates of interest on those earnings. Strictly speaking, the market interest rate and the inflation rate are not fully independent, at least according to some economic theorists. However, they are here treated as being separate. Because of each effect, a dollar of project income next year has a smaller true value than does a dollar in hand today. The interest-rate effect could be offset because a dollar could be financially invested at the prevailing interest rate and the dollar plus interest earnings recouped in a year. By contrast, the inflation effect comes about simply because a dollar can buy more now than a year hence because of an irreversible rise in prices. The distinction is clarified in the following subsections.

**Effect of Inflation on (NPV)** When computing the (NPV) for a proposed project, error arises if the actual cash flows are simply added together instead of adjusting all the values to their purchasing power in a particular year. The reason lies in the basis of (NPV) calculations. We shall rewrite Eq. (9-57) to give

$$(\text{NPV}) = A_{CF_0} + \sum_{t=1}^n \frac{A_{CF_t}}{(1 + i)^t} \quad (9-109)$$

Equation (9-109) is valid for the case of no inflation. In the case of general inflation at a fractional rate  $i_t$ , this equation can be written in the modified form

$$(\text{NPV}) = A_{CF_0} + \sum_{t=1}^n \frac{A_{CF_t}}{(1 + i)^t(1 + i_t)^t} \quad (9-110)$$

Equation (9-110) enables all the net annual cash flows to be corrected to their purchasing power in Year 0. If the inflation rate is zero, Eq. (9-110) becomes identical with Eq. (9-109).

The following example illustrates the effect of inflation on (NPV) as well as on the taxes the company pays.

**Example 16: Effect of Inflation on Net Present Value** Let us consider a simplified project in which \$1,100,000 of capital is spent in Year 0, \$1,000,000 for fixed-capital items and \$100,000 for working capital. The fixed capital is depreciated on a straight-line basis to a book value of zero at the end of Year 5. The annual sales revenue in Years 1 through 5 is \$500,000. There is no inflation. The \$100,000 of working capital is recovered at the end of Year 5. The taxation rate is 50 percent, and the market interest rate is 10 percent. Table 9-14 lists the cash-flow data for this project, showing that the (NPV) at the end of Year 5 is \$99,326 by using Eq. (9-109).

Let us modify this example by assuming that there is a general inflation rate of 20 percent per year and that the project analyst ignores the inflation and (inappropriately) applies Eq. (9-109). The revenue and expense data for this case are shown in Table 9-15, yielding an (NPV) of \$431,269. When Eq. (9-109) is (inappropriately) used for the same example with various other rates of inflation, the resulting (NPV)s can be plotted as the upper line in Fig. 9-29.

If the inflation is correctly taken into account by applying Eq. (9-110), the results are strikingly different. By further discounting the discounted cash flows  $A_{DCF}$  of Table 9-15 by the  $f_t$  factors corresponding to an inflation rate of 20 percent before summing, it can be seen that the project actually incurs a negative (NPV) of \$208,733 in uninflated-money terms. The lower line in Fig. 9-29

TABLE 9-14 (NPV) Calculations with No Inflation

Year, $n$	Net capital expenditure, $A_{TC}$	Revenue from sales, $A_S$	Total expenses, $A_{TE}$	Cash income, $A_{CI} (= A_S - A_{TE})$	Depreciation charge, $A_D$	Taxable income, $(A_{CI} - A_D)$	Amount of tax at $t = 0.5, A_{Tt} [= (A_{CI} - A_D)t]$	Net cash flow after tax, $A_{CF} (= A_{CI} - A_{Tt})$	Discount factor at $i = 10\%, f_t [= \frac{1}{(1 + 0.1)^t}]$	Discounted net cash flow, $A_{DCF} [= A_{CF}(f_t)]$	Net present value (NPV), $(= \sum_0^n A_{DCF})$
0	\$1,100,000	0	0	0	0	0	0	-\$1,100,000	1.00000	-\$1,100,000	-\$1,100,000
1	0	\$500,000	\$100,000	\$400,000	\$200,000	\$200,000	\$100,000	300,000	0.90909	272,727	-\$27,273
2	0	500,000	100,000	400,000	200,000	200,000	100,000	300,000	0.82645	247,935	-\$79,338
3	0	500,000	100,000	400,000	200,000	200,000	100,000	300,000	0.75131	225,393	-\$53,945
4	0	500,000	100,000	400,000	200,000	200,000	100,000	300,000	0.68301	204,903	-\$149,042
5	-100,000	500,000	100,000	400,000	200,000	200,000	100,000	400,000	0.62092	248,368	+99,326

TABLE 9-15 (NPV) Calculations with Inflation Present But Not Allowed For

Year, $n$	Net capital expenditure, $A_{TC}$	Revenue from sales, $A_S$	Total expenses, $A_{TE}$	Cash income, $A_{CI}$	Depreciation charge, $A_D$	Taxable income, $(A_{CI} - A_D)$	Amount of tax at $t = 0.5, A_{IT}$	Net cash flow, $A_{CF}$	Discount factor at $i = 10\%$ , $f_d$	Discounted net cash flow, $A_{DCF}$	Net present value (NPV)
0	\$1,100,000	0	0	0	0	0	0	-\$1,100,000	1.00000	-\$1,100,000	-\$1,100,000
1	0	\$500,000	\$100,000	\$400,000	\$200,000	\$200,000	\$100,000	300,000	0.90909	272,727	-\$827,273
2	0	600,000	120,000	480,000	200,000	280,000	140,000	340,000	0.82645	280,993	-546,280
3	0	720,000	144,000	576,000	200,000	376,000	188,000	388,000	0.75131	291,508	-254,772
4	0	864,000	172,800	691,200	200,000	491,200	245,600	445,600	0.68301	304,349	+49,577
5	-100,000	1,036,800	207,360	\$29,440	200,000	629,440	314,720	614,720	0.62092	381,692	+431,269

extends the example by assuming other rates of inflation. Figure 9-29 shows that the effect of inflation, if not taken into account, is to make a project seem more profitable than it actually is.

Table 9-15 shows that the total amount of tax actually paid over the 5-year period was \$988,320. This becomes \$534,272 in uninflated-money terms when the tax for each year is corrected to its purchasing power in Year 0, using  $f_d$  factors for the 20 percent inflation rate employed for the example. Calculations for other rates of inflation can also be made, and the results plotted as in Fig. 9-30.

This confirms that although the tax paid will increase with inflation, the gain to the government is more apparent than real. It is interesting to note that although the tax paid corrected to its purchasing power in Year 0 is almost constant irrespective of the inflation rate, it does go through a maximum at an inflation rate of about 17 percent in this example.

**Effect of Inflation on (DCFRR)** A net annual cash flow  $A_{CF}$  will have a cash value of  $A_{CF}(1 + i)$  1 year later if invested at a fractional interest rate  $i$ . If there is inflation at an annual rate  $i_e$ , then an effective rate of return or interest rate  $i_e$  can be defined by the equation

$$A_{CF}(1 + i_e) = [A_{CF}(1 + i)] / (1 + i) \tag{9-111}$$

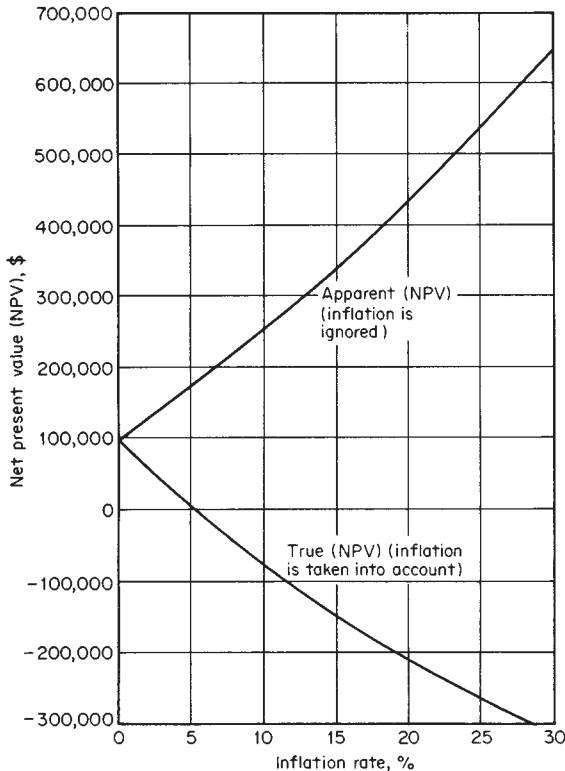


FIG. 9-29 Effect of inflation rate on net present value for a project.

which can be simplified and rewritten to give

$$i_e = i - i_e i_e \tag{9-112}$$

In the context of the discounted-cash-flow rate of return, Eq. (9-112) becomes

$$i_e = (\text{DCFRR}) - i_e - i_e i_e \tag{9-113}$$

In this equation, (DCFRR) can be viewed as the nominal discounted-cash-flow rate of return uncorrected for inflation and  $i_e$  can be thought of as the true or real discounted-cash-flow rate of return.

Instead of using Eq. (9-113), it is unfortunately common practice to try to obtain the true or effective rate of return by calculating the nominal (DCFRR), based on actual net annual cash flows uncorrected for inflation, and then subtracting the inflation rate from it as if

$$i_e = (\text{DCFRR}) - i_e \tag{9-114}$$

Equation (9-113) shows that Eq. (9-114) is only approximately true and should be used, if at all, solely for low interest rates. Let us consider the case of a nominal (DCFRR) of 5 percent and an inflation rate of 3 percent. Equation (9-14) yields an approximate effective return rate of 2 percent, compared with the real effective rate of 1.94 percent given by Eq. (9-113); i.e., there is an error of 3.1 percent. Now let us consider the case of a nominal (DCFRR) of 25 percent and an inflation rate of 23 percent. Equation (9-114) yields an approximate effective return rate of 2 percent, compared with 1.63 percent from Eq. (9-113); in this case, the error that results is 22.7 percent.

**Inflation, (DCFRR), and Payback Period** More insight into the effect of inflation on (DCFRR) calculations can be gained by considering the payback period (PBP), which is defined as the elapsed time necessary for the positive aftertax cash flows from the project to

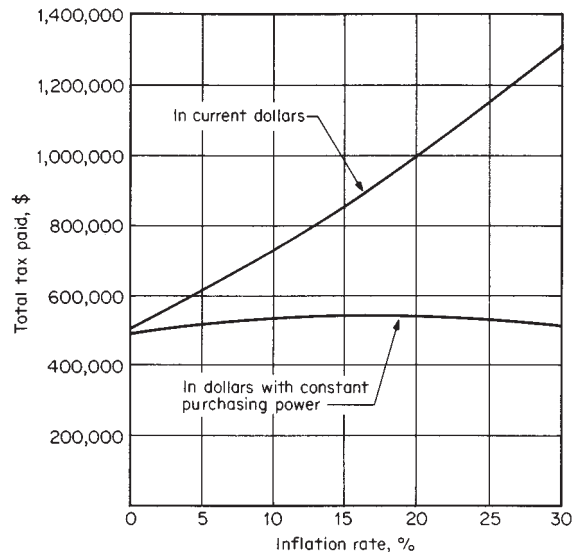


FIG. 9-30 Effect of inflation rate on taxes paid for a project.

recoup the original fixed-capital expenditure. In this definition, the cash flows are not discounted to allow for the market rate of interest or for the inflation rate, so that a project with a given (PBP) could show various values for its (DCFRR) and a given (DCFRR) could pertain to projects with various payback periods.

We shall consider the simple case of (1) a single capital expenditure made immediately before the start of production and (2) equal positive net annual cash flows  $A_{CF}$  in all the productive years of the project. For this case, Eq. (9-109) can be rewritten in terms of the payback period and the (DCFRR) as follows:

$$(PBP) = \sum_{t=1}^n \frac{1}{[1 + (DCFRR)]^t} \quad (9-115)$$

The relationship set out in Eq. (9-115) can also be viewed via a different chain of causality with (DCFRR) as a given parameter, (PBP) as the independent variable, and  $n$  as the variable whose value is being sought. Such an approach is the basis for the lines in Fig. 9-31, each of which shows the number of years of project life required to achieve an effective interest rate or a (DCFRR) of 20 percent by projects having various payback periods. The three lines differ from each other with respect to the matter of inflation.

If there is no inflation, then the middle line pertains. Because there is no inflation, the nominal (DCFRR) is equal to or identical with  $i_e$ , the real discounted-cash-flow rate of return, as can be seen from the relationship expressed in Eq. (9-113).

When inflation does exist, the relevant parameter is  $i_e$ , which is different from the nominal (DCFRR). Equation (9-113), manipulated into equivalent form,

$$(DCFRR) = (1 + i_e)(1 + i_i) - 1$$

shows that in order to achieve an  $i_e$  of 20 percent when the general inflation rate is likewise 20 percent, a project must generate a nominal (DCFRR) of 44 percent. This is the basis for the uppermost line in Fig. 9-31. Other lines pertaining to other rates of inflation could be plotted in the same way.

Let us assume that 20 percent inflation prevails but that the analyst

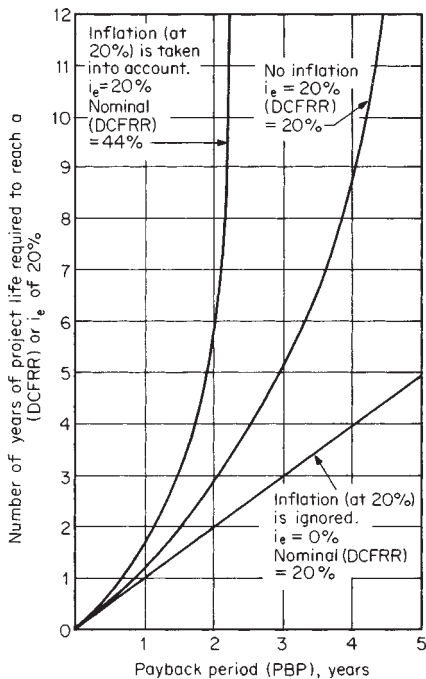


FIG. 9-31 Effect of inflation rate on the relationship between the payback period and the discounted-cash-flow rate of return.

ignores it and mistakenly takes a (DCFRR) of the project at its nominal value instead of converting it to an  $i_e$ . Equation (9-115) rearranged into the form

$$i_e = [1 + (DCFRR)] / (1 + i_i) - 1 \quad (9-116)$$

shows that with a nominal (DCFRR) of 20 percent and a general inflation rate of 20 percent, the true or effective rate of interest is zero. This is the basis for the lowest line in Fig. 9-31. Points for lines corresponding to other rates of inflation could be plotted onto that figure. Plots similar to Fig. 9-31 can be drawn for other (DCFRR) values.

Figure 9-31 shows that the elapsed time necessary to reach a nominal (DCFRR) for a given project decreases sharply with inflation. This figure, like Fig. 9-29, shows that the effect of inflation is to make a project seem more profitable than it actually is.

The magnitude of the effect comes through even more clearly in Fig. 9-32, a plot of the time needed to reach a nominal (DCFRR) of 20 percent against the inflation rate for various values of (PBP). This plot also shows that the longer the payback period, the greater the increase in apparent profitability of the project.

The true rates of return  $i_e$  can be calculated from Eq. (9-116) to be 20, 9.09, 0, and -7.69 percent respectively for general inflation rates of 0, 10, 20, and 30 percent. Thus, although the time required for a project with a payback period of 4 years to reach a nominal (DCFRR) of 20 percent is reduced from almost 9 years under conditions of no inflation to less than 3½ years for 30 percent inflation, the true rate of return that prevails for the latter condition is -7.69 percent, implying that the project loses money in real terms.

It is interesting to note that, in order to reach a real (DCFRR) or  $i_e$  of 20 percent within a reasonable project lifetime when the general inflation rate is 20 percent, it follows from Fig. 9-31 that the payback period for the project must not be much in excess of 2 years.

Although it is difficult to carry out economic-feasibility studies on projects in a time of high inflation, it is important to try to predict inflation rates and allow for them in such studies.

When different people talk about inflation, they often adopt different concepts without realizing it. The area of conceptual uncertainty

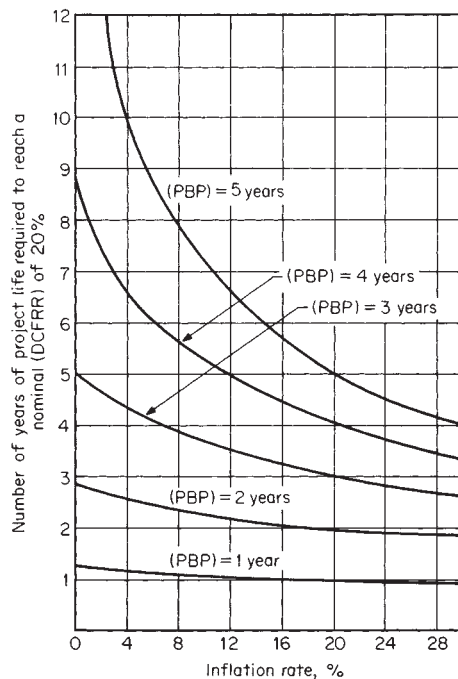
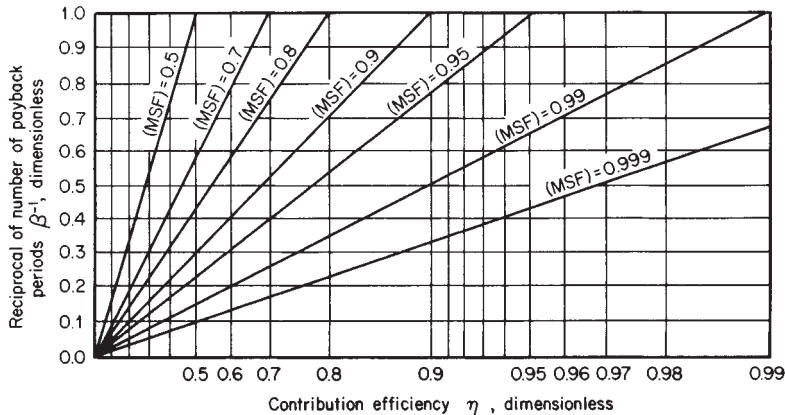


FIG. 9-32 Adverse effect of inflation for higher payback periods.





**FIG. 9-33** Relationship between measured-survival function, number of payback periods, and contribution efficiency.

can be said to lie somewhere between the upper and lower lines shown on Fig. 9-31 in most cases.

**Inflation and the (MSF)** By applying the measured-survival-function concept to manufacturing projects rather than to research and development, we can define a modified (MSF) for a given project as

$$(\text{MSF}) = 1 - (1 - \eta)^\beta \quad (9-117)$$

Here,  $\beta$  is the number of payback periods that have elapsed since the project started to generate positive net annual cash flows  $A_{CF}$  up to any given year  $n$  since project startup. It is given by

$$\beta = \frac{\sum_{n=0}^{n=n} (A_{CF})_n}{C_{FC} - S} \quad (9-118)$$

If all the net annual cash flows in Eq. (9-118) are based on their purchasing power in Year 0, then  $\beta$  is independent of inflation.

As for the contribution efficiency  $\eta$ , it is the ratio of (1) the annual profit that can actually be achieved in a given year for a given sales volume to (2) the profit that could be obtained if no repayment of capital or interest were required and all fixed-expense items were credited free to the project. It is defined by

$$\eta = [R(c_s - c_{VE}) - A_{FE}] / [R(c_s - c_{VE})] \quad (9-119)$$

where  $R$  is the annual production rate or sales volume in physical units,  $c_s$  is the sales price per unit,  $c_{VE}$  is the variable production and selling cost per unit, and  $A_{FE}$  is the annual fixed cost.

If the project gets a "free ride," i.e., if  $A_{FE}$  is zero, then  $\eta$  takes on its maximum possible value of unity. Conversely, if the project and its production rate are only at the breakeven point, then  $\eta$  becomes zero. Therefore, contribution efficiency can be regarded as a measure of the probability of success for the project.

The relationship between the number of payback periods, the contribution efficiency, and the measured-survival function as set out in Eq. (9-117) is plotted in Fig. 9-33.

The contribution efficiency defined by Eq. (9-119) may vary from year to year. In that case, Eq. (9-117) can be written in the modified form

$$(\text{MSF}) = 1 - [(1 - \eta_1)(1 - \eta_2) \cdots (1 - \eta_n)]^{\beta/n} \quad (9-120)$$

where  $\eta_1, \eta_2, \dots, \eta_n$  are the contribution efficiencies in Years 1, 2,  $\dots, n$  respectively.

As in the case of the (MSF) defined by Reul for research and development projects, it is the responsibility of management in a particular manufacturing company to decide on an acceptable level of (MSF) for

manufacturing projects. That decision reflects and helps quantify the company's attitude toward risk.

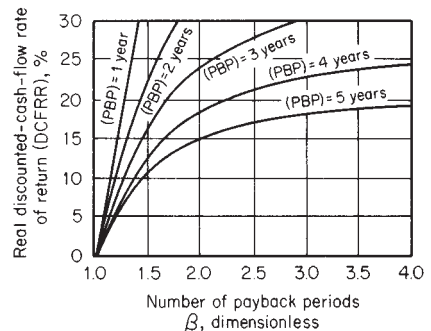
Thus, (MSF) should in practice be regarded as a given or predetermined variable, and Eq. (9-117) accordingly becomes more useful if it is rearranged. For instance, the values of contribution efficiency for a given value of (MSF) are related to the number of elapsed payback periods by

$$\eta = 1 - [1 - (\text{MSF})]^{1/\beta} \quad (9-121)$$

If the acceptable (MSF) is 0.9, this can be satisfied by a project having  $\eta = 0.9$  and  $\beta = 1$ , or a project having  $\eta = 0.684$  and  $\beta = 2$ , and so on. Once Eq. (9-121) has been used to calculate a required contribution efficiency [given the (MSF) and the expected number of payback periods of project life], Eq. (9-119) can be applied to determine the necessary selling price if  $R, c_{VE}$ , and  $A_{FE}$  are known. Similarly, Eq. (9-119) can be used to find the required production rate if  $c_s$  is known.

It is also possible to combine (MSF) considerations with evaluation of the true discounted-cash-flow rate of return (DCFRR) by using Eq. (9-62). The relationship of Eq. (9-59) is independent of inflation if all money values are based on those prevailing in the startup year. For this case, Fig. 9-34 shows the true (DCFRR) reached in a given time, expressed as the number of elapsed payback periods  $\beta$  for various values of the payback period.

Let us consider a project having a contribution efficiency of 0.684 and a payback period of 3 years. Figure 9-33 shows that when two payback periods have elapsed, a measured-survival function of 0.9 has



**FIG. 9-34** Real discounted-cash-flow rate of return against number of payback periods for various payback periods.

been attained. In addition, Fig. 9-34 shows that the discounted-cash-flow rate of return reached at that time is 24 percent.

**Effects of Differential Inflation** Inflation can be general or differential. In the first case, all costs and prices increase at a uniform rate. In the second, government controls and other factors cause the various costs and prices to inflate at different rates.

The onset of general inflation does not change the value of the contribution efficiency  $\eta$ , as can be seen from Eq. (9-119), and it does not affect the value of  $\beta$  if the cash flows in Eq. (9-118) are converted to their purchasing power in Year 0. Thus, general inflation does not cause the measured-survival function to change.

Differential inflation, on the other hand, can affect the measured-survival function. We shall assume, for instance, that the sales price per unit product  $c_s$  in Eq. (9-119) is frozen at a constant level while some or all of the production costs are allowed to rise. This causes the value of  $\eta$  to decrease; therefore, (MSF) likewise decreases, as can be seen from Eq. (9-117).

Let us consider the effect of differential inflation on the overall profitability of the project of the last example. The effect of general inflation on this project showed that the apparent profitability rises sharply, to an (NPV) of \$431,269 at a general inflation rate of 20 percent. However, when the cash flows of the (NPV) are properly corrected to their purchasing power in Year 0, the (NPV) instead becomes \$208,733.

The effect of differential inflation on this project emerges in Fig. 9-35, with all (NPV)s corrected to their purchasing power in Year 0. The top line shows (NPV) for various rates of general inflation. The bottom line shows (NPV) for the differential-inflation case in which only the costs are allowed to increase while product selling price and thus cash income remain constant from year to year. The middle line shows the effect of general inflation when the price rises are delayed by 1 year. The figure confirms that both of these situations take away from the attractiveness of the project.

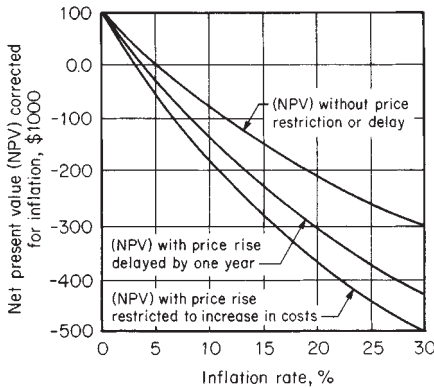


FIG. 9-35 Effect of differential inflation on inflation-corrected net present value.

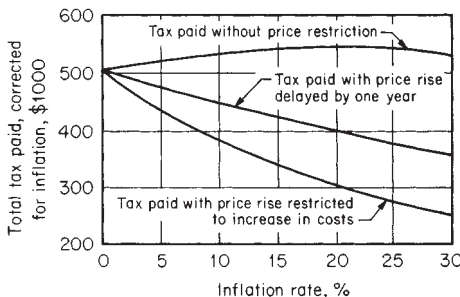


FIG. 9-36 Effect of differential inflation on inflation-corrected tax revenue.

The effect upon total taxes paid, when they are corrected to their purchasing power in Year 0, is shown in Fig. 9-36. Differential inflation not only decreases the profitability of the project to its owner but also decreases the revenue received by the taxing authority. The method of calculation is identical to that of the earlier example.

Another instance of differential inflation occurs when the prices of goods and services rise uniformly but the cost of borrowing money, the interest rate charged on a loan, does not rise.

If the fractional inflation rate is  $i_f$ , a fractional interest rate  $i_L$  on a loan can be corrected to an effective rate of interest by Eq. (9-116) with  $i_L$  substituted for (DCFRR). The effect of various amounts of loan, borrowed at various interest rates  $i_L$ , on the net present value of a particular, fairly simple project is shown in Fig. 9-37. Thus, if \$25,000 were borrowed at an interest rate of 15 percent for the project, the (NPV) would be about \$43,000 at a zero inflation rate. But if the inflation for goods and services  $i_f$  is 10 percent, the effective interest rate for that loan can be calculated from Eq. (9-116) to be only 4.55 percent. It is seen from Fig. 9-37 that this increases the (NPV) of the project to \$48,000. This confirms the economic advantage of borrowing at a fixed interest rate in a time of general inflation.

A topical aspect of differential inflation is the question of energy costs. Will the cost of a particular fuel rise or fall in relation to prices in general, and if so, what effect will this have on the economics of a project?

**Example 17: Effect of Fuel Cost on Project Economics** A process unit is heated by gas. We assume that \$100 spent on energy-conservation measures for this particular unit at the end of 1980 would save 200 therms (21.1 GJ) of gas energy in each subsequent year. If the cost of gas in 1980 is \$x per therm, the annual dollar savings at 1980 prices is \$200x. The (NPV) at the end of year  $n$  for this project is

$$(NPV) = -100 + \sum_{t=1}^n \frac{(200)x}{(1+i)^t}$$

if the appropriate discount factor is  $i$ .

This is independent of inflation provided that the cost of gas rises in line with any general rate of inflation. However, if the real cost of gas rises at a fractional annual rate  $r$  over and above the general inflation rate, it should be modified into the form

$$(NPV) = -100 + \sum_{t=1}^n \frac{(200x)(1+r)^t}{(1+i)^t}$$

This equation confirms that as the gas price rises because of inflation, the attractiveness of the conservation project also rises.

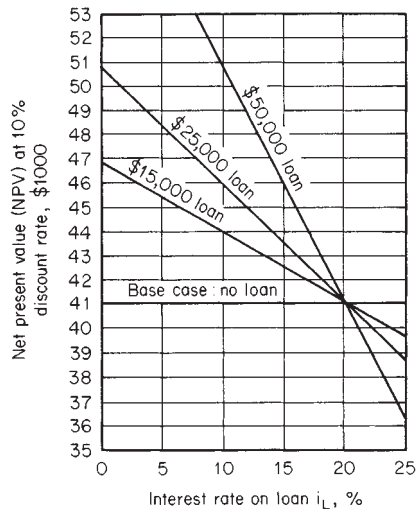


FIG. 9-37 Effect of loan interest rate on the net present value of a project.



## ACCOUNTING AND COST CONTROL

**Principles of Accounting** Accounting is the art of recording business transactions in a systematic manner. Financial statements are both the basis for and the result of management decisions. Such statements can tell managers or engineers a great deal about their company, provided that they can interpret the information correctly.

Since a fair allocation of costs requires considerable technical knowledge of operations in the chemical-process industries, a close liaison between the senior process engineers and the accountants in a company is desirable. Indeed, the success of a company depends on a combination of financial, technical, and managerial skills.

Accounting is also the language of business, and the different departments of management use it to communicate within a broad context of financial and cost terms. Engineers involved in feasibility studies and detailed process evaluations are dependent for financial information on the company accountants, especially for information on the way in which the company intends to allocate its overhead costs. It is vital that engineers correctly interpret such information and that they can, if necessary, make the accountants understand the effect of the chosen method of allocation.

The method of allocating overheads can seriously affect the assigned costs of a project and hence the apparent cash flows for that project. Since these cash flows are used to assess profitability by the net-present-value (NPV) and discounted-cash-flow-rate-of-return (DCFRR) methods, unfair allocation of overhead costs can result in a wrong choice between alternative projects.

In addition to understanding the principles of accountancy and obtaining a working knowledge of its practical techniques, engineers should be aware of possible inaccuracies of accounting information in the same way that they allow for errors in any technical data.

At first acquaintance, the language of accountancy appears illogical to most engineers. Although accountants normally express themselves in tabular form, the basis of all their practice can be simply expressed by

$$\text{Capital} = \text{assets} - \text{liabilities} \quad (9-122)$$

Equation (9-122) can alternatively be written as

$$\text{Assets} = \text{capital} + \text{liabilities} \quad (9-123)$$

Capital, often referred to as net worth, is the money value of the business, since assets are the money values of things the business owns while liabilities are the money values of the things the business owes.

Most engineers have great difficulty in thinking of capital (also known as ownership) as a liability. This is easily overcome once it is realized that a business is a legal entity in its own right, owing money to the individuals who own it. This realization is absolutely essential when considering large companies with stockholders and is used for consistency even for sole ownerships and partnerships. If an individual puts up \$10,000 capital to start a business, then that business has a liability to repay \$10,000 to the individual.

It is even more difficult to think of profit as being a liability. Profit is the increase in money value available for distribution to the owners and effectively represents the interest obtained on the capital. If the profit is not distributed, it represents an increase in capital by the normal concept of compound interest. Thus, if the individual's business makes a profit of \$5000, the liability to the individual is increased to \$15,000. With this concept in mind, Eq. (9-123) can be expanded to

$$\text{Assets} = \text{capital} + \text{liabilities} + \text{profit} \quad (9-124)$$

where the capital is considered as the cash investment in the business and is distinguished from the resultant profit in the same way that principal and interest are separated.

Profit (as referred to above) is the difference between the total cash revenue from sales and the total of all costs and other expenses incurred in making those sales. With this definition, Eq. (9-124) can be further expanded to

$$\text{Assets} + \text{expenses} = \text{capital} + \text{liabilities} + \text{revenue from sales} \quad (9-125)$$

Engineers usually have the greatest difficulty in regarding an expense as being equivalent to an asset, as is implied by Eq. (9-125). Let us consider a one-person business. We assume for a given period

a profit of \$5000 and total expenses excluding the individual's earnings of \$8000. Also we assume that the individual's labor to the business in this period is worth \$12,000. The revenue required from sales would be \$25,000. Effectively, the individual has made a personal income of \$17,000 in the period but has apportioned it to the business as \$12,000 expense for the individual's labor and \$5000 return on capital. In larger businesses, there will also be those who receive salaries but do not hold stock and, therefore, receive no profits and stockholders who receive profits but no salaries. Thus, the difference between expenses and profits is very practical.

The period covered by the published accounts of a company is usually 1 year, but the details from which these accounts are compiled are entered daily in a journal. The journal is a chronological listing of every transaction of the business, with details of the corresponding income or expenditure. For the smallest businesses, this may provide sufficient documentation, but in most cases the unsystematic nature of the journal can lead to computational errors. Therefore, the usual practice is to keep accounts that are listings of transactions related to a specific topic such as "purchase-of-oil account." This account would list the cost of each purchase of oil, together with the date of purchase, as extracted from the journal.

**Principles of Double-Entry Accounting** Many of the accounts involve both income and expenditure. The general practice is to keep accounts by the double-entry system, which may be summarized by

$$\text{Debits} = \text{credits} \quad (9-126)$$

The principle of double entry dates from the fifteenth century and is based on the premise that every transaction involves a giver and a receiver of value. Double entry requires that each transaction be entered into two accounts, the convention being that the account of the giver is credited and the account of the receiver is debited with the same amount of money, as noted in the journal. For convenience, each account is divided centrally, and the debit items are entered on the right-hand side. It is also usual to provide a cross-reference to the journal entry so that errors and omissions can be checked.

Let us consider the purchase of \$50,000 worth of plant equipment by company A, paid for by check. The accounting entries are: debit the plant-equipment account \$50,000, and credit the bank account \$50,000. The plant-equipment account is then said to have a debit balance of \$50,000, and the bank account a credit balance of \$50,000, if these happen to be the only entries.

If company A then sells \$100,000 worth of product that is paid for by check, the accounting entries are: credit the sales account \$100,000, and debit the bank account \$100,000. The bank account will now have a debit balance of  $(\$100,000 - \$50,000) = \$50,000$ , and the sales account a credit balance of \$100,000, if this happens to be the only sale to date in the accounting period.

In principle, the debiting and crediting of accounts are relatively straightforward. However, a great deal of practice is essential in order to achieve proficiency. Although it is not at all necessary for engineers to compete with professional accountants in this field, engineers should appreciate what accountants do and why they do it.

Of the accounts considered in the preceding illustrations, the plant-equipment and bank accounts are asset accounts, and the sales account is a liability account. To increase an asset, debit the asset account; to increase a liability, credit the liability account. Conversely, to decrease an asset, credit the asset account; to decrease a liability, debit the liability account.

**Closing the Books** At the end of the accounting period, the individual accounts are closed by balancing each in accordance with Eq. (9-126). The balances are transferred either to the balance sheet in the case of capital expenditure or to the income statement in the case of revenue expenditure. An alternative name for the balance sheet is the position statement; the income statement is also called the trading and profit-and-loss account.

The purpose of capital expenditure, such as the purchase of a piece of plant equipment for \$50,000, is to earn future revenue. In contrast, the purpose of revenue expenditure is to maintain existing business.

**TABLE 9-16 Income Statement for ABC Company**

Revenue		
Sales revenue	\$1,900,000	
Other revenue	100,000	
		\$2,000,000
Expenses		
Raw materials	953,000	
Wages	185,000	
Utilities	44,000	
Depreciation	68,000	
Other expenses	376,000	
Income taxes	194,000	
		1,820,000
<b>Net profit (after tax)</b>		<b>\$ 180,000</b>

Revenue expenditure includes the direct material costs and direct labor costs incurred in the manufacture of a product, together with the associated overheads that include maintenance of the plant. Since these expenses are debits, the debit balance for a given accounting period is obtained by adding up the debit balances from each individual expenditure account. Similarly, since revenues from sales and other income are credits, the credit balance for a given accounting period is obtained by adding up the credit balances from each individual income or revenue account.

To ascertain profit or loss (calculated as income minus expenditure for a given accounting period), income and expenditure must be matched. For example, any rent paid in advance beyond the current accounting period should not be included in the profit or loss calculation. Similarly, goods sold but not yet paid for in a given accounting period should not be included in the revenue total for that period.

An income statement such as the one shown in Table 9-16 is used to obtain the profit or loss for a given period. The debit and credit balances of all the accounts that do not represent expenditure or income for a given accounting period are entered as assets and liabilities in a balance sheet such as that shown in Table 9-17.

There is no rigid format for either the income statement or the balance sheet. Tables 9-16 and 9-17 show common layouts for the income statement and balance sheet respectively, but these are not the only forms. For example, vertical balance sheets, with the assets listed above the liabilities and equity, are also popular.

Some expenditures are partly capital and partly revenue. For example, repair and improvement work may be done on a plant simultaneously. In this case, the repair work should be classified as revenue expenditure and the plant-improvement work as capital expenditure.

**Accounting Concepts and Conventions** Accounting is based on the following concepts: (1) money measurement, (2) business entity, (3) going concern, (4) cost, and (5) matching.

**Concept 1.** "Money measurement" means that only those facts that can be represented in monetary terms are recorded. The balance sheet and income statement for a company give no indication as to what might happen in the future. The company may be about to be successfully sued for a large sum of money, or a competitor may be launching a new product that will seriously reduce future sales of the company's products.

**Concept 2.** "Business entity" means that accounts are kept for the company quite independently of the people who may own the company. For example, if an individual puts an additional \$10,000 into a one-person business, the accounts show that the business is \$10,000 richer. They do not show that the individual's personal wealth has been depleted by \$10,000.

**Concept 3.** "Going concern" means that the accounting is based on the premise that the business will continue indefinitely. It is most unlikely that the values of the assets shown in the balance sheet are what the assets would realize if sold. No attempt is made in normal accounting to measure the value of the business to a potential buyer.

**Concept 4.** "Cost" means that the assets are normally shown in the balance sheet at cost price together with their subsequent depreciation. Some assets such as land may be considerably more valuable than when originally purchased, but no indication of this is given in the balance sheet. However, some governments now require a note giving the current estimated value of the land.

**Concept 5.** "Matching" means that the revenue in a given accounting period should correspond to the expenses for that accounting period.

Accounting is also based on the following conventions: (1) materiality, (2) conservatism, or prudence, and (3) consistency.

Materiality deals with determining whether certain expenditures will have a significant effect on a company's accounting procedures. This is a matter of judgment that is to be made by each company. Obviously, the purchase of a vehicle is a material item, but writing paper or tools for maintenance are less obvious. Although such items may last well beyond the current accounting period, it may not be worth the accounting effort to treat them as material items. Some companies will treat a particular item as capital; other companies, as expenditure. Clearly, the purchase of a piece of equipment costing, say, \$1000, will be regarded as less material by a giant company than by a small one.

Conservatism, or prudence, means monetary values that tend to understate rather than overstate the profit are taken.

Consistency means that accounting items are normally treated in

**TABLE 9-17 Balance Sheet for XYZ Company**

Assets (thousands of dollars)		Liabilities and stockholders' equity (thousands of dollars)	
<b>Current assets</b>		<b>Current liabilities</b>	
Cash	\$ 38,893	Notes payable	\$ 34,507
Notes and accounts receivable	110,740	Accounts payable and accrued liabilities	106,433
Inventories:		Accrued taxes	7,264
Finished products	17,396	Total current liabilities	148,204
Work in process	56,690		
Raw materials and supplies (at cost)	35,790	<b>Long-term liabilities</b>	67,677
Total inventories	109,876	Deferred income taxes	13,225
Total current assets	259,509	Other deferred credits	2,307
<b>Investments and long-term receivables (at cost)</b>	94,000	<b>Stockholders' equity</b>	
<b>Property, plant, and equipment (at cost)</b>		Common stock, \$20 par value	
Land	6,110	Shares authorized, 7,750,000	
Buildings	63,848	Shares issued, 4,794,450	95,889
Machinery and equipment	106,185	Capital in excess of par value of common stock	31,798
	176,143	Retained earnings	101,492
Less accumulated depreciation	75,163	Total stockholders' equity	229,179
Net property, plant and equipment	100,980		
Prepaid and deferred, charges	6,094		
Total assets	\$460,592	Total liabilities and stockholders' equity	\$460,592

the same way over an indefinite number of years. For example, an individual item would not be treated as an expenditure during one year and as a capital item during the next year without good reason being given.

**Balance Sheet** The balance sheet, also called the position statement, presents an accounting view of the financial status of a company at a particular point in time. A typical balance sheet is shown in Table 9-17. Although a balance sheet has two sides that balance, it is not part of the double-entry system. In fact, it is not an account but rather a statement listing all the assets of a company and the various claims against these assets on the last day of the accounting period. The assets must be equal to the claims against them at all times. Those who have claims against the assets are the owners (stockholders in a business corporation) and the people to whom the company owes money. In the case of the latter, the company is said to have liabilities to its creditors. The total claim against the assets is often labeled "liabilities and owners' equity."

Assets are classified as current or fixed, and liabilities as current or long-term. Fixed assets are material items that have a relatively long life and normally include land, buildings, plant, vehicles, etc. They are held for the specific purpose of earning revenue and are not for sale in the normal course of business. Current assets include cash and those items that can be fairly easily converted into cash, such as raw-materials inventories, etc. In contrast to fixed assets, current assets are acquired for the specific purpose of conversion into cash in the normal course of business. However, what is regarded as a fixed asset by one type of company might be regarded as a current asset by another. For example, a chemical company would normally classify its vehicles as a fixed asset. However, a company whose primary business was to sell vehicles would classify them as a current asset.

Similarly, the distinction between current and long-term liabilities is also not clear-cut. Current liabilities include accounts payable (money owed to creditors), taxes payable, dividends payable, etc., if due within a year. Long-term liabilities include deferred income taxes, bonds, notes, etc., that do not have to be paid within a year. The owners' equity includes the par, or face, value of the capital received from stockholders and any retained earnings. The balance sheet shows only the nominal value and not the current or real value of this capital.

A balance sheet includes items that are not regarded as assets or liabilities in normal language, such as expenditures carried forward and accumulated profits.

Accountants regard assets as resources that have not yet been used up. Assets are normally shown on the balance sheet at cost minus accumulated depreciation. In this sense, the depreciation charge for an accounting period is the means of converting a part of an asset into a current expenditure that is then listed as an expense in the income statement.

Let us consider plant equipment costing \$1 million and purchased on Jan. 1, 1988. Table 9-18 shows the provision for the depreciation account for 1988, 1989, and 1990 for straight-line depreciation, assuming a service life of 10 years and zero scrap value. The credit entries of \$100,000 for the depreciation in each year are balanced by the depreciation charge of \$100,000 debited to the income statement (or trading and profit-and-loss account) in each year. Table 9-19 shows the corre-

**TABLE 9-18 Provision for Depreciation of Plant-Equipment Account**

1988			
Jan. 1 Balance brought down			0
Dec. 31 Balance carried down	<u>\$100,000</u>	Dec. 31 Debited to income statement	<u>\$100,000</u>
1989			
Jan. 1 Balance brought down			\$100,000
Dec. 31 Balance carried down	<u>\$200,000</u>	Dec. 31 Debited to income statement	<u>100,000</u>
	<u>\$200,000</u>		<u>\$200,000</u>
1990			
Jan. 1 Balance brought down			\$200,000
Dec. 31 Balance carried down	<u>\$300,000</u>	Dec. 31 Debited to income statement	<u>100,000</u>
	<u>\$300,000</u>		<u>\$300,000</u>

**TABLE 9-19 Balance-Sheet Entries**

<b>As of Dec. 31, 1988</b>			
Plant equipment at cost		\$1,000,000	
Less depreciation to date		<u>100,000</u>	
			\$900,000
<b>As of Dec. 31, 1989</b>			
Plant equipment at cost		\$1,000,000	
Less depreciation to date		<u>200,000</u>	
			\$800,000
<b>As of Dec. 31, 1990</b>			
Plant equipment at cost		\$1,000,000	
Less depreciation to date		<u>300,000</u>	
			\$700,000

sponding entries in the balance sheets for the years 1988, 1989, and 1990. Entries for subsequent years are made in the same way.

A balance sheet is true only for one particular point in time; it tells nothing about the trends in a company. However, by comparing balance sheets for successive years, management can follow changes in the various items. If the observed trend is undesirable, management can take corrective action. Since the accounting period of 1 year is long for most businesses, it is usual to draw up balance sheets at more frequent intervals for control purposes. These may be less formal than those issued annually to the stockholders. In general, balance sheets are less useful to management than are income statements.

**Income Statement** Income statements range from the very simple presentation shown in Table 9-16 to the more informative and more complex presentation shown in Table 9-20. The income statement shows the revenue and the corresponding expenses that were incurred to earn that revenue over a period of time. It is the most obvious measure of the efficiency of a business. Although published income statements are normally for 1-year periods, many companies use monthly income statements for internal purposes.

Income statements are very useful tools to assist management in controlling a business and planning for the future. Since management needs to follow the trends of the normal expenses, extraordinary expenses such as those incurred as a result of a major fire or flood should be shown separately.

If revenue and expenses are not properly matched, an understatement or an overstatement of profit may occur. If raw materials were previously purchased at a lower cost than their current cost, profit will be overstated. Any overstatement of profit will mean that more tax will be paid.

One of the most important items in an income statement is depreciation expense. Although depreciation should not be thought of as a means to build up a fund to replace plant, it nevertheless does enable money to be retained in the business by reducing the profit available for distribution to stockholders. It is of course a duty of both accountants and management to see that sufficient money is retained in the business to replace assets and to invest such money in other processes or outside investment.

A further duty of accountants and management is to ensure that the company always has sufficient working capital to enable it to carry on its business.

**Types of Accountancy** The traditional work of accountants has been to prepare balance sheets and income statements. Nowadays, accountants are becoming increasingly concerned with forward planning. Modern accountancy can roughly be divided into two branches, financial accountancy and management or cost accountancy.

Financial accountancy is concerned with stewardship. This involves the preparation of balance sheets and income statements that represent the interest of stockholders and are consistent with existing legal requirements. Taxation is an important element of financial accounting.

Management accounting is concerned with decision making and control. This is the branch of accountancy closest to the interest of most process engineers. Management accounting is concerned with standard costing, budgetary control, and investment decisions.

Accounting statements present only facts that can be expressed in financial terms. They do not indicate whether a company is developing new products that will ensure a sound business future. A company

**TABLE 9-20 Income Statement for a Mature Year for a New Chemical Product, Produced at 10 Million lb/Year**

		Unit values, cents/lb	% sales revenue, %
Revenue from annual sales $A_S$	\$2,000,000	20.00	100.0
Direct manufacturing expense $A_{DME}$			
Raw materials	\$ 884,000	8.84	44.2
Catalysts and solvents	69,000	0.69	3.4
Operating labor	102,000	1.02	5.1
Operating supervision	20,000	0.20	1.0
Utilities	22,000	0.22	1.1
Operating maintenance	21,000	0.21	1.1
Operating supplies	4,000	0.04	0.2
Royalties and patents	10,000	0.10	0.5
Total $A_{DME}$	\$1,132,000	<u>11.32</u>	<u>56.6</u>
Indirect manufacturing expense $A_{IME}$			
Payroll overhead	28,000	0.28	1.4
Central laboratory	10,000	0.10	0.5
General plant overhead	52,000	0.52	2.6
Packaging and storage	22,000	0.22	1.1
Property taxes	14,000	0.14	0.7
Insurance	6,000	0.06	0.3
Total $A_{IME}$	\$132,000	<u>1.32</u>	<u>6.6</u>
Total manufacturing expense (excluding depreciation) $A_{ME}$	\$1,264,000	12.64	63.2
Depreciation $A_{BD}$	68,000	0.68	3.4
Other expenses			
Administration	74,000	0.74	3.7
Sales and shipping	124,000	1.24	6.2
Advertising and marketing	40,000	0.40	2.0
Technical service	10,000	0.10	0.5
Research and development	60,000	0.60	3.0
Total other expenses	308,000	<u>3.08</u>	<u>15.4</u>
Total expense $A_{TE}$	\$1,640,000	16.40	82.0
Net annual profit $A_{NP}$	\$360,000	3.60	18.0

may have impressive current financial statements and yet be heading for bankruptcy in a few years' time if provision is not being made for the introduction of sufficient new products or services.

### Financing Assets by Equity and Debt

**Financial Ratios** Probably the most commonly mentioned ratio is the profit margin (PM), defined as

$$(PM) = \frac{\text{net annual profit}}{\text{revenue from annual sales}} 100 \quad (9-127)$$

Another common ratio is the return on investment (ROI), defined as

$$(ROI) = \frac{\text{net annual profit}}{\text{investment}} 100 \quad (9-128)$$

In both Eq. (9-127) and Eq. (9-128), the net annual profit can be either before or after tax. It can also include interest and dividends receivable, etc.

Obviously, the net annual profit must be clearly defined before comparisons are made with other companies. Similarly the term "investment" in Eq. (9-128) can have a variety of meanings. The two most common ones (used when assessing the profitability of companies as opposed to projects) are total assets and owners' equity or capital employed. In the first case, Eq. (9-128) can be written as

$$(ROA) = \frac{\text{net annual profit}}{\text{total assets}} 100 \quad (9-129)$$

where (ROA) is called the return on assets. In the second case, Eq. (9-128) can be written as

$$(ROE) = \frac{\text{net annual profit}}{\text{stockholders' equity}} 100 \quad (9-130)$$

where (ROE) is the return on equity.

Asset-turnover ratio (ATR) is a commonly used measure of company performance, defined as

$$(ATR) = \frac{\text{revenue from annual sales}}{\text{total assets}} 100 \quad (9-131)$$

A comparison between Eqs. (9-127), (9-129), and (9-131) shows that

$$(ROA) = (ATR)(PM) \quad (9-132)$$

Thus (ROA) can be improved by increasing either (ATR) or (PM). A variation of Eq. (9-131) is the fixed-asset turnover ratio (FATR), defined as

$$(FATR) = \frac{\text{revenue from annual sales}}{\text{fixed assets}} 100 \quad (9-133)$$

Clearly, (FATR) is of less value than (ATR) when applied to companies that use relatively large amounts of working capital. The (FATR) is the inverse of the capital ratio (CR) for single projects. (CR) is defined as

$$(CR) = C_{FC}/A_S \quad (9-134)$$

where  $C_{FC}$  is the fixed-capital cost for a green-fields (grass-roots) site and  $A_S$  is the revenue from annual sales.

The fixed assets in Eq. (9-133) and those included in the total assets in Eqs. (9-129) and (9-131) are usually taken at their written-down, or book, value, which may differ significantly from their market value. This is one disadvantage in using Eqs. (9-129), (9-131), and (9-133).

The revenue from annual sales referred to in Eqs. (9-127), (9-131), and (9-132) is normally taken to be the gross turnover, which includes intergroup sales. However, intergroup sales are eliminated in consolidated or group accounts. Again, revenue from annual sales must be clearly defined before comparisons are made with other companies.

Let us consider the simplified balance-sheet or position statement shown in Table 9-21. Essentially, total assets are related to liabilities and stockholders' equity by

$$\text{Total assets} = \text{stockholders' equity} + \text{total debt} \quad (9-135)$$

Equation (9-135) can also be written as

$$\text{Stockholders' equity} = \text{total assets} - \text{total debt} \quad (9-136)$$

Equations (9-130) and (9-136) can be combined to give

$$(ROE) = \frac{\text{net annual profit}}{\text{total assets} - \text{total debt}} 100 \quad (9-137)$$

Equation (9-137) can also be written to include a quantity called the debt ratio (DR), which gives

$$(ROE) = \frac{\text{net annual profit}}{\text{total assets}} \left[ \frac{100}{1 - (DR)} \right] \quad (9-138)$$

**TABLE 9-21 Simplified Balance Sheets for Companies X and Y**

X Company balance sheet			
	Total debt		0
	Stockholders' equity		\$100,000
Total assets	<u>\$100,000</u>	Total liabilities and stockholders' equity	<u>\$100,000</u>
Y Company balance sheet			
	Total debt		\$ 50,000
	Stockholders' equity		\$ 50,000
Total assets	<u>\$100,000</u>	Total liabilities and stockholders' equity	<u>\$100,000</u>



where (DR) is the debt ratio as given by

$$(DR) = \frac{\text{total debt}}{\text{total assets}} \tag{9-139}$$

Return on assets (ROA) can be related to the return on equity (ROE) by combining Eqs. (9-129) and (9-138):

$$(ROA) = (ROE)/[1 - (DR)] \tag{9-140}$$

(ROE) can also be related to the asset-turnover ratio (ATR) and the profit margin (PM) by combining Eqs. (9-132) and (9-140):

$$(ROE) = [(ATR)(PM)]/[1 - (DR)] \tag{9-141}$$

**Financing by Debt, or Leverage** The debt ratio (DR) is also known as the leverage, or gearing, ratio. Highly levered companies have a high proportion of debt to total assets. At first glance, it may appear that the use of leverage is a simple way of increasing the return on equity (ROE). However, interest charges have to be paid on the debt. Whether leverage is a good thing or not will depend on exactly what the interest charges are in relation to the return on assets and the return on equity.

Let us consider the simplified balance sheets of two companies, X and Y, shown in Table 9-21. Companies X and Y have a debt, or leverage, ratio of zero and 0.5 respectively. Let us assume that the debt is of the debenture type for tax purposes and that the interest rate is 10 percent per annum. The return on equity (ROE) after tax is given in Table 9-22 for companies X and Y for various values of net annual profit  $A_{NP}$  before tax.  $A_{NNP}$  is the net annual profit after tax. The data for Table 9-21 are plotted in Fig. 9-38. This figure shows that leverage has no effect on the (ROE) when the interest rate charged for the borrowed money is equal to the return on assets (ROA) before tax. Leverage provides increased (ROE) values when the (ROA) is greater than the interest rate charged for the borrowed money and decreased (ROE) values when it is less.

The greater the debt, or leverage, ratio (DR), the more sensitive the (ROE) is to a change in (ROA) and the steeper the slope of the line in Fig. 9-38. Dividends to stockholders are paid out of the net annual profit after tax  $A_{NNP}$ , from which the (ROE) after tax in Fig. 9-38 is calculated. Thus, the higher the leverage, the greater the financial risk to the stockholder. This risk is not the same as the business risk of the company, which is a function of its overall prospects in its particular industry. Leverage increases the return to the stockholders when the (ROA) is higher than the interest rate on debt and decreases the return when the (ROA) is lower than the interest rate.

Whether the assets of a company are financed largely by stockholders' equity (also called net worth), or largely by debt, or by some combination of the two depends on a number of factors. If sales do not fluctuate, a company is in a good position to pay the fixed interest charges on debt. This is also the case if the revenue from sales is steadily increasing. In this case, any new common stock issued by the company is likely to command a good price, and it also increases the attractiveness of equity financing.

The attitude of management is also an important factor in deter-

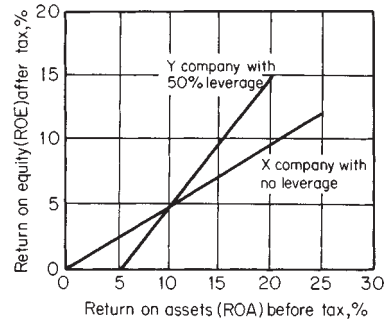


FIG. 9-38 Effect of leverage on the return on equity.

mining how much debt financing is used. In a small firm in which management owns most of the equity, management may be very reluctant to issue further amounts of common stock that would lead to a dilution of its control. Furthermore, if management has great confidence in future prospects, it will wish to ensure the maximum return for itself. In contrast, the equity in a large company is widely distributed, and the issue of further amounts of common stock has little effect on the control of the company.

The difference between equity financing and debt financing is not always clear-cut. For example, preferred stock can be classified as stockholders' equity or debt, depending on who is doing the financial analysis.

**Equity Financing** Typically, the company balance sheet will show the stockholders' equity and list the preferred stock, common stock, and retained earnings as in Table 9-23.

The issue of common stock is the basic method of financing a company. Common stockholders take the ultimate risk in a business because they have no right to a return on their investment. However, they have the right to elect the directors of the company, who in turn are responsible for the management of the business. Stockholders are likely to vote the board of directors out if adequate dividends are not paid. Usually the liability of stockholders is limited to the nominal, or par, value of their stock, and hence they can lose only what they have already paid for the stock. If the liability is not limited by law, the personal assets of the stockholders are at risk in the event of company bankruptcy, in proportion to the amount of stock held.

Preferred stock is often used as an alternative to debt when companies do not wish to issue additional common stock or to incur the fixed interest charges required to finance debt. Preferred stockholders are not normally allowed to vote for the board of directors. They have the right to receive fixed amounts of dividends before common stockholders are paid any dividends. However, a company does not have to pay dividends. The board of directors may decide to pay small or no dividends in a particular year. Holders of cumulative preferred stock

TABLE 9-22 Return on Equity after Tax for Companies X and Y

(ROA) before tax	5%	10%	15%	20%
X company				
$A_{NP}$	\$5,000	\$10,000	\$15,000	\$20,000
Less tax at 50%	(\$2,500)	(\$ 5,000)	(\$ 7,500)	(\$10,000)
$A_{NNP}$	\$2,500	\$ 5,000	\$ 7,500	\$10,000
(ROE) after tax	2½%	5%	7½%	10%
Y company				
$A_{NP}$ before interest	\$5,000	\$10,000	\$15,000	\$20,000
Less interest	(\$5,000)	(\$ 5,000)	(\$ 5,000)	(\$ 5,000)
$A_{NP}$ after interest	0	\$ 5,000	\$10,000	\$15,000
Less tax at 50%	0	(\$ 2,500)	(\$ 5,000)	(\$ 7,500)
$A_{NNP}$	0	\$ 2,500	\$ 5,000	\$ 7,500
(ROE) after tax	0	5%	10%	15%

TABLE 9-23 Stockholders' Equity as Shown in Section of a Company's Balance Sheet

Preferred stock, par value \$100 per share		
Authorized 2000 issued and outstanding	1,500	\$ 150,000
Less discount on preferred stock		(10,000)
Preferred-stock equity		\$ 140,000
Common stock, par value \$10 per share		
Authorized and issued 100,000 shares	\$1,000,000	
Amount paid in excess of par	100,000	
Retained earnings		\$1,100,000
		100,000
Common-stock equity		\$1,200,000
Total stockholders' equity		\$1,340,000

are entitled to receive compensation for the previous underpayment of dividends when the company again pays dividends.

Common stockholders have a right to the residual assets of a company in the event of dissolution or liquidation but only after all the creditors and then any liabilities to the preferred stockholders have been paid. The larger the proportion of debt financing in a company, the smaller the amount the common stockholders are likely to receive if the company is liquidated.

Common stockholders normally have a preemptive right to the first option to purchase any additional issues of common stock. This prevents management from using an additional issue of common stock to override the control exercised by existing stockholders. Preemptive rights also protect existing stockholders from having the value of their shares decreased by such dilution, since the same net earnings would be spread over more units of stock.

Let us consider the very simplified case of a company with 100,000 shares of common stock, each with a market value of \$10, giving a total market value of \$1,000,000. If a further 50,000 shares are sold at \$4 each, the total market value of the 150,000 shares is \$1,200,000, or \$8 each. This means that the new stockholders have gained at the expense of the original ones. The preemptive right is designed to prevent this. In practice, the situation is rather more complex than is indicated here.

Both common and preferred stocks normally have a par, or nominal, value. In the case of common stock, the market value at the time of issue usually differs from the par value. Stock can be issued either at a premium or at a discount, depending on prevailing economic conditions and the strength of the company. The difference between the actual amount paid and the par value is listed in the stockholders' equity section of the balance sheet, as shown in Table 9-23. The issuance of stock at a premium or a discount is done to protect existing stockholders.

In the case of preferred stock, the par value has more meaning than with common stock, since it is the amount due preferred stockholders if the company goes into liquidation, provided that this is a condition of issue.

The advantage of using common stock to finance assets is that it does not incur fixed interest charges. Furthermore, there is no maturity date, as there is with all loans and most preference issues. Common stock can often be issued more easily than debt can be financed. However, the flotation costs of common stock can be quite high, especially when stock values are depressed, so that large discounts for the stock are needed to induce purchase.

Stockholders' equity in a company is made up of the capital contributed by the stockholders and the capital generated from retained earnings. The presence of retained earnings on a balance sheet, as shown in Table 9-23, does not necessarily mean that they are matched by an equal amount of cash. In fact, there may be little or no cash available. The retained earnings shown on a balance sheet may be largely fictitious. For example, the assets on a balance sheet may be worth less than shown by at least the value of the retained earnings.

**Purchase and Sale of Equities** Stockholders usually require an adequate return on their investment, and the quoted price of the stock reflects the consensus opinion of investors as to the current health of the company. Purchases or sales are normally made through stockbrokers.

Most stock transactions are completed through organized security exchanges on which the stock is listed. Such exchanges have physical existence in the form of buildings located in different regions of the country. Each exchange has members who are often the nominated representatives of large brokerage firms having offices in various cities. These offices are in constant telephone and telegraph communication with the members at the exchange, passing on requests to buy or sell specified stocks. Since brokers live by commissions and charges on transactions, they attempt to match such requests either directly or by dealings with other brokers. In the United Kingdom, brokers must deal through an independent "jobber," similar in function to a specialist broker, who quotes a low price for sales and a higher one for purchases before the jobber knows whether the broker is buying or selling. The difference represents the jobber's margin, or "turn." If requests to buy exceed offers for sale, the price of the stock rises until someone is tempted to sell. Conversely, if an excess of stock is offered for sale, the price is likely to fall.

It is an advantage to a company to be listed on a stock exchange since its investors can more easily sell their stock if they decide to do so. This increased liquidity makes investors more willing to accept a lower rate of return, which effectively lowers the cost of capital to the company.

Because dealings in the stock of a listed company are published, a healthy company engenders confidence that makes it easier to obtain other forms of finance. In the absence of a regular market, stock transactions are necessarily infrequent, and prices are liable to wide fluctuation, which may make creditors wary and possibly lead to bankruptcy proceedings. Such dealings are usually referred to as "over-the-counter" and are confined to the relatively few specialist brokers who hold inventories of such stock and are prepared to "make a market" in them or are limited to private transactions.

**Retained Earnings** Much confusion is caused by the practice of dividing retained earnings under various headings such as reserve for replacement of plant, reserve for contingencies, etc. This procedure also restricts the flexibility of management in expenditure decisions.

The amount of retained earnings shown on a balance sheet should not be taken as a measure of the amount of future dividends that the company is likely to pay. A contract may exist that specifies a minimum balance of retained earnings, which is then not available for dividends until bonds issued by the company have been retired.

Dividends can be paid either as cash or in the form of an additional issue of stock. A stock split is really a stock dividend, and both are used to reduce the price of stock when management considers that it is too high. A stock dividend is essentially a transfer of retained earnings to the common-stock account and makes the amount transferred unavailable for future dividends. A stock dividend may be used in place of a cash dividend when a company is short of cash.

**Debt Financing** In practice, debt financing covers a variety of fixed-income securities, both long-term and short-term. The most common forms of long-term debt are bonds, mortgages, and debentures.

A bond is simply a long-term promissory note. It is a contract established between borrower and lender in a document called an indenture. A bond indenture includes a detailed description of assets that are pledged, together with any protective clauses and provisions for redemption. A trustee is appointed to look after the interest of the bondholders. The trustee is normally a commercial bank. Bonds may be issued with a call provision that enables a company to redeem its bonds at any date earlier than scheduled. Obviously, this would be an advantage to a company in times of falling interest rates. However, a company has to pay more than the par value of the bond for this privilege. The additional amount is called the bond premium.

Sometimes a company uses a sinking fund to retire a bond. A series of equal annual payments  $A$ , invested at a fractional interest rate  $i$  and made at the end of each year over a period of  $n$  years, is equivalent to a sum of money of present value  $P$ , given by

$$A = Pf_{AP} \quad (9-46)$$

where  $f_{AP}$  is the annuity present-worth factor, which is

$$f_{AP} = [i(1+i)^n]/[(1+i)^n - 1]$$

A company may use a sinking fund in a variety of ways, but the simplest is to pay a fixed amount  $A$  at the end of each year to buy and retire bonds until after  $n$  years all the bonds have been retired. This annual payment may prove a significant strain on the resources of a company. Failure to make the payment could result in bankruptcy. In the case of income bonds, a company is required only to pay interest when it earns it.

A mortgage is a bond in which specific real assets are pledged as security. A senior mortgage has a prior claim on assets. A junior mortgage is normally a second mortgage on the residual value of the assets. A blanket mortgage is a pledge on all real property owned by a company.

A debenture is an unsecured bond. Strong companies are in a better position to issue debentures than weak companies since they have less need to pledge specific assets. Debenture holders are really general creditors. Subordinated debenture holders have claims on assets only after the claims of certain other claimants have been met. The issue of subordinated debentures provides a tax advantage for a com-



TABLE 9-24 Comparative Ratios for Selected United States Industry Groups for 1992\*

Industry	Agricultural chemicals	Paints and allied products	Petroleum refining	Plastic materials and resins	Soap and other detergents
Number of companies Ratio	2391	238	98	234	66
Current assets Current debt	2.0	2.8	1.3	1.8	2.2
Net profit $\times$ 100, % net sales	1.9	1.8	1.5	1.5	3.8
Net profit $\times$ 100, % net worth	9.3	6.8	6.7	14.3	16.3
Net sales Net working capital	9.3	5.2	15.8	9.9	5.9
Collection period, days	27.8	39.6	34.8	39.3	44.7
Net sales Inventory	11.4	8.1	14.6	11.9	8.9
Fixed assets $\times$ 100, % Net worth	43.0	30.8	131.5	20.0	42.3
Current debt $\times$ 100, % Net worth	52.1	42.0	68.4	96.8	46.9
Total debt $\times$ 100, % Net worth	71.9	66.2	147.7	111.0	61.4
Current debt $\times$ 100, % Inventory	116.1	89.0	193.7	143.6	119.4

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pany compared with the issue of preferred stock because the interest payable is a tax-deductible expense.

A financial analyst looking at a company from a potential common stockholder's point of view is likely to classify preferred stock as debt. In contrast, bondholders and general creditors are likely to regard preferred stock as additional equity. Since preferred stock is a hybrid type of security, it may be issued by a company whose management is divided over the question of whether to use equity or debt to finance additional assets. However, preferred stock does have the disadvantage that the dividends are not allowed as a tax-deductible expense.

**Comparative Company Data** Table 9-24 gives comparative company data that have been compiled by Dun & Bradstreet for various types of processing industries. The median value for each ratio is given.

Row 1 in Table 9-24 is the

$$\text{Current ratio} = \frac{\text{current assets}}{\text{current liabilities}} \quad (9-142)$$

Compare 
$$\text{Quick ratio} = \frac{\text{liquid assets}}{\text{current liabilities}} \quad (9-143)$$

Row 2 in Table 9-24 is the profit margin (PM) of Eq. (9-127). In this case, the net profit referred to is the net annual profit after tax and depreciation  $A_{NTP}$ . The net sales is the revenue from annual sales  $A_S$  after deductions for returns, allowances, and discounts for gross sales.

Row 3 in Table 9-24 is the return on equity (ROE) of Eq. (9-130). In this case, the net worth is the tangible net worth representing the sum of the preferred and common stocks and the surplus and undistributed profits or retained earnings, less any intangible items such as goodwill, etc.

Row 7 in Table (9-24) is the

$$\text{Average collection period} = \frac{\text{average value of accounts receivable}}{\text{revenue from sales per day}} \quad (9-144)$$

The funded debt (referred to in row 14) consists of mortgages,

bonds, debentures, serial notes, or other obligations with maturity of more than 1 year from the statement date.

Robert Morris Associates also compiles extensive comparative company data for various industries. In addition to ratios similar to the Dun & Bradstreet ratios shown in Table 9-24, Robert Morris Associates gives very useful breakdowns of assets and liabilities for various industries. Table 9-25 shows a breakdown of assets and liabilities for United States manufacturers of industrial inorganic chemicals.

**Application of Overall Company Ratios** The various ratios for a hypothetical company are listed in Table 9-26. The balance sheet shown in Table 9-27 has been built up from the ratios in Table 9-26 in terms of the revenue from net annual sales  $A_S$ .

Let us calculate the following values for the right-hand side of the balance sheet as follows:

From ratio 5

$$\text{Net worth} = A_S/2.50 = 0.4 A_S$$

From ratio 11

$$\text{Total debt} = (0.4 A_S)(0.65) = 0.26 A_S$$

From ratio 10

$$\begin{aligned} \text{Current debt} &= (0.4 A_S)(0.35) = 0.14 A_S \\ \text{Long-term debt} &= \text{total debt} - \text{current debt} \\ \text{Long-term debt} &= 0.26 A_S - 0.14 A_S = 0.12 A_S \end{aligned}$$

We calculate the following values for the left-hand side of the balance sheet:

From ratio 99-45

$$\text{Fixed assets} = (0.4 A_S)(0.74) = 0.29 A_S$$

From ratio 1

$$\text{Current assets} = (0.14 A_S)(2.60) = 0.36 A_S$$

From ratio 8

$$\text{Inventory} = A_S/7.14 = 0.14 A_S$$

From ratio 7

$$\text{Accounts receivable} = (A_S/365)/(61) = 0.167 A_S$$

**TABLE 9-25 Typical Balance Sheet for a United States Manufacturer of Industrial Inorganic Chemicals**

Assets	%	Liabilities	%
Cash	4.3	Short-term due to banks	5.6
Marketable securities	2.2	Due to trade	14.5
Net receivables	26.2	Income taxes	3.1
Net inventory	24.6	Current maturities long-term debt	2.1
All other current assets	0.9	All other current liabilities	5.0
Total current assets	58.3	Total current debt	30.3
Fixed assets	36.3	Noncurrent debt unsubordinated	18.3
All other noncurrent assets	5.4	Total unsubordinated debt	48.6
		Subordinated debt	3.1
		Tangible net worth	48.3
Total assets	100.0	Total liabilities and stockholders' equity	100.0

Abridged from *Annual Statement Studies*, 1973 ed., copyright 1973 by Robert Morris Associates, Philadelphia.

Cash and short-term investments = total current assets  
- (inventory + accounts receivable)

Cash and short-term investments =  $0.364 A_S - 0.307 A_S = 0.057 A_S$

In addition to the data for the balance sheet, we calculate the net annual profit (after tax), i.e., ratio 2, to be  $A_{NVP} = 0.04 A_S$ .

**TABLE 9-26 Ratios for a Typical Industrial Chemical Company**

No.	Ratio
1	$\frac{\text{Current assets}}{\text{Current debt}} = 2.60$
2	$\left( \frac{\text{Net profit}}{\text{Net sales}} \right) 100 = 4.00$
3	$\left( \frac{\text{Net profit}}{\text{Net worth}} \right) 100 = 10.0$
4	$\left( \frac{\text{Net profit}}{\text{Net working capital}} \right) 100 = 18.18$
5	$\frac{\text{Net sales}}{\text{Net worth}} = 2.50$
6	$\frac{\text{Net sales}}{\text{Net working capital}} = 4.50$
7	Collection period = $\frac{\text{accounts receivable}}{\text{sales per day}} = 61$ days
8	$\frac{\text{Net sales}}{\text{Inventory}} = 7.14$
9	$\left( \frac{\text{Fixed assets}}{\text{Net worth}} \right) 100 = 74.00$
10	$\left( \frac{\text{Current debt}}{\text{Net worth}} \right) 100 = 35.00$
11	$\left( \frac{\text{Total debt}}{\text{Net worth}} \right) 100 = 65.00$
12	$\left( \frac{\text{Inventory}}{\text{Net working capital}} \right) 100 = 63.00$
13	$\left( \frac{\text{Current debt}}{\text{Inventory}} \right) 100 = 100.00$
14	$\left( \frac{\text{Funded debt}}{\text{Net working capital}} \right) 100 = 76.50$

In practice, the ratios are obtained from the information published in the balance sheet. The advantage of the above presentation is that it relates everything to the revenue from net annual sales and hence underlines the importance of sales.

Careful study of the ratios can produce many inferences as to the health of the company. For example, the leverage, or debt, ratio (DR) for this example is

$$(\text{DR}) = \frac{\text{total debt}}{\text{total assets}} = \frac{0.260 A_S}{0.660 A_S} = 0.40$$

This value is quite low and does not present any problems of control by debtors, such as can arise when (DR) is greater than 1.

From Table 9-27 we calculate the ratio for

$$\frac{\text{Current debt}}{\text{Cash + short-term investments}} = \frac{0.140 A_S}{0.057 A_S} = 2.45$$

Therefore, requests for early repayment by more than 40 percent of the debtors could be met. Hence, no liquidity problems are likely to arise, and advantage can be taken of discounts for early payment. Also, the current debt could be met by sale of the inventory, which takes  $(0.140 A_S/A_S)(365)$ , or 51 days. The quick ratio is  $1/2.45 = 0.407$ .

If it is assumed that current debtors are due for payment within 61 days, the same time as that allowed to creditors, no bankruptcy petitions are likely.

The profit of 10 percent, indicated by ratio 3 in Table 9-26, will be reduced by any dividend due to preferred stockholders, because such payments are not part of fixed-debt expenses; the residue is shared among the ordinary stockholders. If all the long-term debts were in redeemable 6 percent preferred shares, then (from ratio 3) the net annual profit (after tax) is  $A_{NVP} = 0.10(0.40 A_S)$ , or  $0.04 A_S$ . Interest due on preferred shares is  $0.06(0.12 A_S)$ , or  $0.0072 A_S$ . Therefore, the earnings for the ordinary shares are

$$(0.04 A_S - 0.0072 A_S)/(0.4 A_S - 0.12 A_S) = 0.1171$$

This value corresponds to 11.71 cents per dollar of common stockholders' equity.

If it is assumed that available interest rates offered by banks, government, etc., for no-risk investment of capital are 10 percent, then the maximum economic market price of \$100 stock units in this hypothetical company is about \$117. If all the debt is in bonds, etc., earnings on ordinary stock would be 10 cents per dollar of net worth, and the maximum economic price of the stock would be about \$100 unless stock prices were expected to rise.

Other ratios can easily be deduced from those listed. For example, the return on assets (ROA) and the asset-turnover ratio (ATR) are

$$(\text{ROA}) = \frac{\text{net annual profit}}{\text{total assets}} 100 = \frac{0.04 A_S}{0.66 A_S} = 6.06$$

$$(\text{ATR}) = \frac{\text{revenue from annual sales}}{\text{total assets}} 100$$

$$(\text{ATR}) = (A_S/0.66 A_S) 100 = 151.5$$

**TABLE 9-27 Balance Sheet for a Typical Industrial Chemical Company, Dec. 31, 1991**

Assets		Liabilities and stockholders' equity	
Current assets		Liabilities	
Cash	0.057 $A_3$	Current debt	0.140 $A_3$
Accounts receivable	0.167 $A_3$	Long-term debt	0.120 $A_3$
Inventory	0.140 $A_3$		
Total current assets	0.364 $A_3$	Total debt	0.260 $A_3$
Fixed assets	0.296 $A_3$	Net worth	0.400 $A_3$
Total assets	0.660 $A_3$	Total liabilities and stockholders' equity	0.660 $A_3$

**Cost of Capital** The value of the interest rate of return used in calculating the net present value (NPV) of a project is usually referred to as the cost of capital. It is not a constant value since it depends on the financial structure of the company, the policy of the company toward a particular project, the local method of assessing taxation, and, in some cases, the measure of risk associated with the particular project. The last-named factor is best dealt with by calculating the entrepreneur's risk allowance inherent in the project  $i'_r$  from Eq. (9-108), written in the form

$$i'_r = [1 + (\text{DCFRR})]/(1 + i) - 1$$

where  $i$  is the cost of capital exclusive of the risk allowance. The value of  $i'_r$  should be compared with the probability of exceeding or of failing to achieve an (NPV) of zero when using that value of  $i$ . The decision to proceed can then be made with a full knowledge of the odds against success. The decision can be related to the company attitude to budgets of the relevant size by the use of probable utilities, as has already been discussed. Cash flows used in calculating (NPV) and (DCFRR) should, of course, be corrected for the anticipated rates of inflation, preferably to the time when the utility curve was obtained. This is important since inflation is likely to have a distorting effect on utility curves obtained at different times. This may be due to an unconscious wish to protect against inflation by achieving higher rewards while assigning less importance to any losses incurred, thus tending toward a gambling outlook.

In the absence of a risk allowance the cost of capital becomes a financial computation based on sources of funds and company policy. As such it will usually be presented as a figure specified for use in a particular appraisal and is therefore of little concern to the project assessor. However, the following résumé indicates the kinds of factors to be considered.

In most companies the objective of company policy is to maximize the financial return to the equity stockholders. This is not invariably the case, since a young company will often plow back an unusually large proportion of its profits to encourage growth. Also, it is increasingly the case that projects are undertaken to restore or preserve an environmental amenity or to bring work into a particular locality. In such circumstances a low value of the cost of capital might be assigned to the project. In many government projects a limited loss is acceptable, in which case the value of  $i$  would be negative.

When the objective is to maximize the aftertax return to the stockholders, a balance must be struck between the proportion of aftertax company profits which are retained to permit growth of the company assets and the proportion which are distributed to provide an income for the stockholders in the form of dividends. The latter will usually be subject to personal income taxes, sometimes at higher-than-normal rates. The growth potential should be reflected in an increased value of the stock as quoted on the stock exchange. Such growth may result in the imposition of capital gains or inheritance taxes. The selection of the right proportion of earnings to be retained is crucial since this affects the appeal of the company to investors and hence its credit worthiness in the eyes of creditors. The optimum split is influenced by the type of investor since institutional tax rates and exemptions often differ from those applied to private investors. It is for this reason that the optimum split is sensitive to local taxation policy.

Most companies can maintain a given level of business only by continuous reinvestment in plant and equipment. If company growth is

required, additional investment is essential. In general, a company has only three sources of new money, namely, cash received from the sale of newly issued shares, retained earnings, and debt capital of all kinds including deferred taxes. In certain circumstances cash grants may be forthcoming from government sources. Each of these sources has its own effective rate of interest, and it is the weighted average of these rates which constitutes the cost of capital exclusive of risk allowance.

There is no interest payable directly on equity stocks, but there is a concealed rate expected by investors. Without the expectation of a certain return on their investment they would not invest in a new issue, nor would they retain existing holdings of stock. The sale of stocks on the stock exchange does not affect the cash holding of the company, but new issues must be at prices lower than existing values quoted on the exchange unless great confidence exists that the new money will produce an increased income greatly in excess of the reduction in earnings per share caused by the new issue. Stock carrying a fixed interest rate normally has the interest treated as an allowable expense before tax in the same way as a bank overdraft, which is a relatively short-term source of debt. Deferred taxes carry an interest rate which, like an overdraft, is normally compounded daily at a nominal annual rate but naturally is not an allowable expense for tax purposes. Cash owing on outstanding bills carries a notional rate of interest since in many cases prompt settlement of bills would attract a cash discount.

**Example 18: Risk-Free Cost of Capital** A company requires an investment of \$100,000 in new plant to maintain its present sales. Let us determine the current cost of capital to the company and the risk-free cost of capital that it should assign to the plant-replacement project, given the following data.

Company assets: from stock sales	\$ 300,000
from retained earnings	200,000
as bills due	100,000
as deferred taxes	200,000
as bank overdraft	200,000
Total assets	\$1,000,000
Current annual income	\$ 200,000

Bills are due on monthly account with a 2 percent discount for cash. Overdraft and deferred-tax interest are compounded daily at nominal annual interest rates of 15 and 9 percent respectively. Corporation tax, capital gains tax, and personal income tax rates are 50, 40, and 30 percent respectively. The current rate of inflation is at 8 percent per year. The traditional return expected by investors is 7 percent per year net of all taxes in real terms.

The interest-rate equivalent of the cash discounts is 2 percent per month, since this discount could be obtained every month if payment were to be made at the beginning of the month rather than, as at present, at its end. Since the bills are settled monthly, the notional interest is paid monthly and should not be compounded. The discount is equivalent to 12 monthly simple-interest payments per year. Hence, from Eq. (9-31) the effective annual interest rate on discounts =  $(12)(0.02) = 0.24 = 24$  percent. It would, therefore, be a good use of surplus cash to reduce this debt as quickly as possible. This would require cash equivalent to one-sixth of the annual bills due, or \$16,700, to be available. It can, therefore, be assumed that this level of liquidity is not available for capital projects, either as working capital to reduce the debt or for fixed-capital projects. Further, since the new project will not increase sales, it cannot generate further debt of this kind. Hence, this source is not available to capitalize the new project.

Since the overdraft is payable daily at a nominal annual interest rate of 15 percent, it follows from Eq. (9-38) that the effective annual interest rate on overdraft =  $(1 + 0.15/365)^{365} - 1 = 16.18$  percent. Similarly, the effective annual interest rate on deferred tax =  $(1 + 0.09/365)^{365} - 1 = 9.42$  percent.

The new plant will not increase sales and will therefore not increase the tax debt, so that this source is not available to capitalize the project. An increase in overdraft may be available, subject to a maximum imposed by the acceptable gearing of the company.

Since the liquidity of the company is so low, it is possible that it is already extended to its maximum debt, in which case the gearing

$$\frac{\text{Total equity}}{\text{Total debt}} = \frac{\$300,000 + \$200,000}{\$100,000 + \$200,000 + \$200,000} = 1.00$$

Since neither increased bills due nor increased tax debt is available to finance the new project, this implies that the required \$100,000 of new capital will be available as \$50,000 from increased overdraft and \$50,000 from increased equity. The effective interest rate on the equity involved must therefore be calculated.

Equity is available from two sources. First, the company can sell new stock which, if in the form of ordinary shares, carries no interest payment. Although this course appears cheap, its use for projects which do not increase earnings, at least to a compensatory level, is usually inadvisable. This leaves retained earnings as the most likely source of equity for the present project.

Equity holders require a real return on their outlay, which they assume to be at the stock-market price if this differs from the face value of the stock, of 7 percent net of all taxes. Retained earnings attract a 40 percent capital gains tax; hence the actual interest rate required on distribution forgone is  $7/(1 - 0.40) = 11.67$  percent. This is in real terms and at a time of 8 percent inflation rate must be increased in cash terms to  $(1 + 0.1167)/(1.08) - 1 = 20.60$  percent.

In the same way the effective interest rate on distributed earnings, on which an income tax of 30 percent is payable, may be calculated to be  $[0.07/(1 - 0.30) + 1]/(1.08) - 1 = 18.80$  percent. If the shares are currently valued at par by the stock exchange, this would require distribution, on the \$300,000 of issued equity, of  $(\$300,000)(0.188) = \$56,400$ . This amount is required after corporation tax has been paid on the earnings of \$200,000. Thus the earnings which can be retained are  $(\$200,000)(1 - 0.50) - \$56,400 = \$43,600$ . This is close to the \$50,000 required. If the company were well regarded on the stock exchange, a slight reduction in distributed dividends might not reduce share values since the purpose is to maintain future earnings. However, it would be possible for share values to be reduced by up to  $(\$56,400 - \$50,000)/\$56,400 = 0.1135 = 11.35$  percent. If this happened, the effective interest rate on the retained earnings should be increased to  $20.60/(1 - 0.1135) = 23.24$  percent. These rates are net of corporation tax, which is the correct form for use in (NPV) calculations.

The interest rate due on deferred tax is also net of corporation tax at 9.42 percent. The interest payable on overdrafts is an expense fully allowable against tax, so that the effective aftertax rate is reduced to  $(16.18)(1 - 0.50) = 8.09$  percent. Similarly, as the advantage forgone on the discounts would tend to increase company profits and hence tax due, the effective aftertax gain is reduced to  $(24)(1 - 0.50) = 12.00$  percent.

The present cost of capital to the company is the weighted-average interest payable on the various sources of funds on an after-corporation-tax basis. This is readily calculable to be at least  $[(\$300,000)(0.00) + (\$200,000)(20.60) + (\$100,000)(12.00) + (\$200,000)(9.42) + (\$200,000)(8.09)]/(\$1,000,000) = 8.82$  percent. The cost of capital to the new project, with only two sources, should be  $[(\$50,000)(20.60) + (\$50,000)(8.09)]/(\$100,000) = 14.35$  percent.

Since this project is essential if current production is to be maintained, many companies would assess the cost of capital at somewhere near the lower value. Values of cost of capital in the region of 10 percent are to be expected in developed countries at the present time.

We notice in particular that inflation does not affect quoted interest rates when assessing present values of cost of capital. It must, however, be taken into account in assessing the interest rate on the dividend which will be expected by investors.

As has been stated, it is alternatively possible to assign to the cost of capital the best risk-free return available on the money. The assessment then proceeds as discussed in connection with Eq. (9-108).

**Management and Cost Accounting** In any given time period, cost may be divided into expired and unexpired cost. An expired cost is an expense; an unexpired cost is an asset. This division is the basis for income statements and balance sheets.

Cost accounting is the name traditionally given to accounting for manufacturing costs. The manufacturing cost of a product is traditionally taken as the sum of the costs for (1) direct materials, (2) direct labor, (3) manufacturing overheads, and (4) administration, selling, and finance.

Two methods are in general use in accounting for manufacturing costs, absorption costing and marginal costing. In absorption costing, which is the traditional method, all manufacturing overhead costs are included in the cost of sales. In marginal costing only variable manufacturing overhead costs are included in the cost of sales. Marginal costing is more valuable than absorption costing in decision making.

However, it is sometimes quite difficult to separate costs and particularly manufacturing overhead costs into fixed and variable components. In the long term virtually all costs are variable. The difference between the two methods assumes great importance in inventory evaluation. In cost accounting, costs are identified with cost centers. These are accounting devices which may or may not have a physical existence. In the simplest case of a plant manufacturing a single product, the entire plant may be the cost center.

In practice there are two major classifications of cost accounting systems, job costing and process costing. In the former, costs are collected for each job or batch irrespective of the accounting period. This system is normally used in construction work. Process costing is normally used in continuous and semicontinuous processes. Costs are collected for a specific accounting period.

**Allocation of Overheads** How overheads are allocated can affect the total cost of a product and, hence, the estimated future cash flows for a project. Since these cash flows are used in the net-present-value (NPV) and discounted-cash-flow-rate-of-return (DCFRR) methods for estimating profitability, erroneous allocations could result in the wrong choice of project.

The modern trend is for overhead costs to become an increasing proportion of total product costs. This results from the ever-greater sophistication of process plants. Therefore, it is highly desirable that chemical engineers should have some say in the allocation of overheads and that this should not be left entirely to accountants.

Direct costs are those that can be directly charged to a single product. The most obvious direct cost is for raw materials, of which the quantity consumed is directly proportional to the amount of product manufactured. Direct process labor is also considered to be a direct cost.

However, many costs cannot be directly charged to an individual product. These so-called indirect, burden, or overhead costs range from the lighting and heating required for the plant and offices to the cafeteria and medical facilities provided. When several products are made in a plant, it becomes increasingly difficult to allocate overheads correctly among the various products.

A number of different methods are commonly used to estimate the amount of overhead to be allocated to an individual product. These methods are necessary because accountancy costs become prohibitive for charging all costs directly to an individual product. Unfortunately, there is always an arbitrary element inherent in the process of allocation.

Overheads in the chemical-process industries are commonly calculated as a percentage of (1) direct materials cost, (2) direct labor cost, or (3) prime or direct costs. Other methods of allocating overheads are on the basis of (1) plant area, (2) number of employees, (3) capital value, and (4) electric power.

These listings do not include all the methods in use. The validity of a particular method depends on the process and the industry. An inappropriate method can lead to misleading and even absurd results.

Let us consider the manufacture of metal ornaments. The processing cost, exclusive of material, may vary very little for a wide range of materials. However, the direct materials cost will be much greater for precious than for base metals. In this case, an overhead allocation on the basis of direct material costs could be very misleading, while one based on direct labor cost could be quite accurate.

Problems can also arise when allocating overheads on the basis of direct labor cost. Let us consider a company that evaluates overheads at 125 percent of direct labor cost. A process plant employs seven operators, each with a direct cost of \$10,000 per budget period. As a result of a works-study exercise, it is found that the plant can operate satisfactorily with six operators. The actual cost saving is likely to be far nearer to the direct labor savings of \$10,000 per period than to the calculated saving of  $\$10,000 + \$10,000(125/100) = \$22,500$  per period. The \$22,500 calculated saving is the direct labor cost plus overheads taken as 125 percent of the direct labor cost.

A thorough analysis should be made before production is stopped on a product that is losing money. Although direct costs of the discontinued product will be saved, overheads are not eliminated, as might be inferred from taking overheads as a percentage of direct material, direct labor, or prime costs. The plant is still there, together with its



associated costs for interest charges, insurance, painting, some maintenance, etc. Continued production can still make a useful contribution to such overheads. This contribution is lost if production is stopped and must then be borne by other products.

Problems can arise with each of the methods used for allocating overheads. Two process plants may occupy similar areas yet have vastly different material or labor costs. Problems can also arise with an individual plant that can be used to make different products, as Example 19 will show.

**Example 19: Overhead in Two Different Projects** Let us consider a plant that can make either product A or product B. At normal capacity, the overhead cost is known to be \$2.50 per unit. Product A has a direct materials cost of \$8 per unit and a direct labor cost of \$2 per unit. For simplicity, the prime cost is here taken as the sum of these two costs, i.e., \$10 per unit.

In Table 9-28, a correct overhead cost of \$2.50 per unit at normal capacity is calculated by taking either 31.25 percent of the direct materials cost, 125 percent of the direct labor cost, or 25 percent of the prime cost. All these methods give a total cost of \$12.50 per unit and a profit of \$1.50 per unit for a selling price of \$14 per unit.

The alternative, product B, has a direct materials cost of \$6 per unit and a direct labor cost of \$4 per unit. In Table 9-28 overhead costs of \$1.875 per unit, \$5 per unit, and \$2.50 per unit are calculated by taking 31.25 percent of direct materials cost, 125 percent of direct labor cost, and 25 percent of prime cost respectively. Total costs are \$11.875 per unit, \$15 per unit, and \$12.50 per unit respectively and profits of \$2.125 per unit, -\$1 per unit, and \$1.50 per unit respectively, for a selling price of \$14 per unit.

An alternative to allocating overheads by using a single method is to classify the various overheads into groups and to use the most appropriate allocation for each group. For example, depreciation would be allocated on the basis of capital cost, while indirect labor might be allocated either on the basis of direct labor cost or on the number of employees. Clearly, this alternative method is more complex, increases the associated accountancy costs, and is prone to misinterpretation and possibly abuse.

**Inventory Evaluation and Control**

**Inventory Effect on Cash Income and Profit** When the annual production rate is equal to the annual sales volume, the revenue from annual sales  $A_S$  is

$$A_S = R c_S \tag{9-87}$$

where  $R$  is the production rate, units per year, and  $c_S$  is the sales price per unit. In this case, the annual cash income  $A_{CI}$  and the net annual profit  $A_{NP}$  before tax are given respectively from Eqs. (9-1), (9-8), and (9-12) by

$$A_{CI} = R(c_S - c_{TVE}) - A_{TFF} \tag{9-145}$$

$$A_{NP} = R(c_S - c_{TVE}) - A_{TFF} - A_{BD} \tag{9-146}$$

where  $c_{TVE}$  is the total variable expense per unit of production,  $A_{TFF}$  is the total annual fixed expense required to produce and sell a product but excluding any annual provision for plant depreciation, and  $A_{BD}$  is the balance-sheet annual depreciation charge.

However, in a given accounting period the sales volume may differ from the volume of production. In this case, the inventory of finished product  $I_1$  at the beginning of the accounting period will differ from

that at the end,  $I_2$ , and Eqs. (9-145) and (9-146) need to be written in modified form as

$$A_{CI} = R(c_S - c_{TVE}) - A_{TFF} + (I_1 - I_2) \tag{9-147}$$

$$A_{NP} = R(c_S - c_{TVE}) - (A_{TFF} + A_{BD}) + (I_1 - I_2) \tag{9-148}$$

If the annual sales volume exceeds the annual production rate  $R$  by an amount  $\Delta R$ , then Eqs. (9-147) and (9-148) can be written as

$$A_{CI} = R(c_S - c_{TVE}) - A_{TFF} + \Delta R(c_S - c_{INV}) \tag{9-149}$$

$$A_{NP} = R(c_S - c_{TVE}) - (A_{TFF} + A_{BD}) + \Delta R(c_S - c_{INV}) \tag{9-150}$$

where  $c_{INV}$  is the value per unit of inventory of the finished product.

Clearly, the value of  $c_{INV}$  affects both the annual cash income and the net annual profit. Since annual cash incomes are the basic data for (NPV) and (DCFRR) methods of estimating profitability, the actual value per unit of inventory is of direct importance for chemical engineers engaged in economic assessments.

Let us divide Eq. (9-150) by Eq. (9-87) to give

$$\frac{A_{NP}}{A_S} = \left(1 - \frac{c_{TVE}}{c_S}\right) - \frac{(A_{TFF} + A_{BD})}{A_S} + \left(\frac{\Delta R}{R}\right) \left(1 - \frac{c_{INV}}{c_S}\right) \tag{9-151}$$

$$\frac{A_{NP}}{A_S} = (CSR) - \frac{(A_{TFF} + A_{BD})}{A_S} + \left(\frac{\Delta R}{R}\right) \left(1 - \frac{c_{INV}}{c_S}\right) \tag{9-152}$$

where  $(CSR) = 1 - (c_{TVE}/c_S)$  is the contribution to the sales-price ratio. When the ratio of net annual profit to revenue from annual sales  $A_{NP}/A_S$  is expressed as a percentage it is known as the profit margin.

The terms  $(CSR)$  and  $(A_{TFF} + A_{BD})/A_S$  have a similar order of magnitude in chemical processing. For example,  $(CSR)$  values of 0.1 to 0.4 are typical, and these are quite close to a value of 0.2 that is common in general chemical processing for  $(A_{TFF} + A_{BD})/A_S$ . Thus, the profit margin is very sensitive to the value  $c_{INV}$  per unit of inventory.

For example, let us consider that both  $(CSR)$  and  $(A_{TFF} + A_{BD})/A_S$  are equal to 0.3. In this case, Eq. (9-152) can be written as

$$A_{NP}/A_S = (\Delta R/R)(1 - c_{INV}/c_S) \tag{9-153}$$

If the sales volume exceeds the annual production rate by 10 percent and the inventory is valued at the sales price, then Eq. (9-153) shows that the profit margin is  $(A_{NP}/A_S)100 = 0$  percent. If the inventory is valued at the total variable cost, then the profit margin  $(A_{NP}/A_S)100 = (0.1)(1 - 0.7)(100) = 3$  percent. Hence, the value of the inventory is of vital importance.

Unfortunately, there is no universally accepted method for valuing inventory. The value of  $c_{INV}$  per unit can be taken on any of the following bases:

1. Direct material plus direct labor cost.
2. Direct material plus direct labor plus other direct production expenses. (This is the total variable production cost.)
3. Total variable production cost plus fixed production overhead cost.
4. Total variable cost. (This includes both production and general expenses.)
5. Total cost. (This includes variable and fixed production and general expenses.)

Methods 1, 2, and 4 are termed direct costing, variable costing, and marginal costing respectively. Although direct costing is being increas-

**TABLE 9-28 Profits of Products with Different Methods of Overhead Allocation**

	Product A—basis of overhead allocation			Product B—basis of overhead allocation		
	31.25% of direct materials cost, \$/unit	125% of direct labor cost, \$/unit	25% of prime cost, \$/unit	31.25% of direct materials cost, \$/unit	125% of direct labor cost, \$/unit	25% of prime cost, \$/unit
Direct materials cost	8.00	8.00	8.00	6.000	6.000	6.000
Direct labor cost	2.00	2.00	2.00	4.000	4.000	4.000
Prime or direct cost	10.00	10.00	10.00	10.000	10.000	10.000
Overhead cost	2.50	2.50	2.50	1.875	5.000	2.500
Total cost	12.50	12.50	12.50	11.875	15.000	12.500
Selling price	14.00	14.00	14.00	14.000	14.000	14.000
Profit/(loss)	1.50	1.50	1.50	2.125	(1.000)	1.500

ingly used for internal accounting and control purposes, it is not acceptable to tax authorities as a basis for calculating profit. Tax authorities and most accountants favor method 3, which is known as absorption costing. We have already seen that this method has the disadvantage that the fixed-overhead cost per unit is determined for a particular normal production rate. If the production rate exceeds the normal, there is overabsorption of fixed overheads. Conversely, if the production rate falls below the normal, there is underabsorption of fixed overheads. Method 5 is known as full-absorption costing.

Most people agree that general expenses incurred in administration, selling, distribution, etc., should not be included in the cost of inventory. In fact, many feel that no costs should be absorbed before they have been incurred. In general, method 2 is favored by engineers and method 3 by most accountants. However, the accountancy convention is to value at either cost or market value, whichever is the lower. In the methods considered, either actual or standard costs can be used. Note that method 3 shows a higher profit than method 2 when sales volume exceeds the production rate and a lower profit when the production rate exceeds sales volume.

The total profits (before tax) over the life of a project are independent of the method used to value inventory. Over a project life of  $n$  years, Eq. (9-148) can be written as

$$\sum_0^n A_{NP} = \sum_0^n R(c_s - c_{TVE}) - \sum_0^n (A_{TFE} + A_{BD}) + \sum_0^n (I_m - I_{m+1}) \quad (9-154)$$

where the last term in Eq. (9-154) becomes  $\sum_0^n (I_m - I_{m+1}) = I_0 - I_{n+1}$ . Since there will be no material in inventory in the year before the project starts or in the year after it terminates,  $I_0 = I_{n+1} = 0$ . Hence, total profits do not depend on individual values for  $I$ .

However, the annual profit  $A_{NP}$  (before tax) does depend on the value of the inventory. Since the tax payable in any individual year is based on  $A_{NP}$ , the net annual profit  $A_{NNP}$  (after tax) is also dependent on the method chosen for valuing inventory. Frequently, a particular method for valuing inventory is chosen to delay payment of tax as long as it is legally possible to do so.

So far, only the inventory of finished product has been considered. There are also inventories of raw materials and work in process, i.e., partially processed materials or intermediate products, to be considered. It is necessary to modify Eqs. (9-147) through (9-154) accordingly to take these inventories into account.

**Effect of Raw-Materials Prices** Raw materials for the chemical-process industries are subject to relatively wide variations in price. These effects on profits will now be considered.

When the price of raw materials varies from week to week, not all the units in storage will have been purchased at the same price. Let us consider  $\chi$  units in storage at the start of the inventory period, purchased at a price  $c_1$  per unit. Additional quantities  $\chi_2, \chi_3, \dots$ , are purchased at prices  $c_2, c_3, \dots$ , per unit respectively until finally  $\chi_n$  units are purchased at the latest price of  $c_n$  per unit at the end of the inventory period. The total value of the inventory  $C_{INV}$  at the end of the inventory period (in the absence of any withdrawal) is given by

$$C_{INV} = \sum_1^n \chi_j c_j \quad (9-155)$$

The value of the inventory  $I$  at any given time depends on the values ascribed to the units withdrawn from inventory. There are five methods for valuing inventory: (1) FIFO (first-in–first-out), (2) LIFO (last-in–first-out), (3) average cost, (4) standard cost, and (5) market value.

In the FIFO method, the units taken out of storage are valued at their purchase price beginning with the earliest item purchased. If a number of units  $m$  are removed from inventory during the period, the total cost of these items on a FIFO basis is given by:

$$C_{FIFO} = \sum_1^p \chi_j c_j + \left(m - \sum_1^p \chi_j\right) c_{p+1} \quad (9-156)$$

providing that the value of  $m$  satisfies

$$m < \sum_0^n \chi_j$$

and where  $p$  is the largest integer, such that

$$\sum_1^p \chi_j < m$$

Hence, the value of the inventory,  $I_{FIFO}$  at any given time is

$$I_{FIFO} = C_I - C_{FIFO} \quad (9-157)$$

In terms of Eqs. (9-155) and (9-156), Eq. (9-157) can be written as

$$I_{FIFO} = \sum_1^n \chi_j c_j - \sum_1^p \chi_j c_j - \left(m - \sum_1^p \chi_j\right) c_{p+1} \quad (9-158)$$

In the LIFO method, the  $m$  units taken out of storage are valued at their purchase price, beginning with the latest item purchased. In a similar manner, the value of the material  $C_{LIFO}$  taken out of inventory is given by

$$C_{LIFO} = \sum_p^n \chi_j c_j + \left(m - \sum_p^n \chi_j\right) c_{p-1} \quad (9-159)$$

where  $p$  is the smallest integer, such that

$$\sum_p^n \chi_j \leq m$$

Hence, the value of the inventory  $I_{LIFO}$  at any given time is

$$I_{LIFO} = C_I - C_{LIFO} \quad (9-160)$$

In terms of Eqs. (9-155) and (9-159), Eq. (9-160) can be written

$$I_{LIFO} = \sum_1^n \chi_j c_j - \sum_p^n \chi_j c_j - \left(m - \sum_p^n \chi_j\right) c_{p-1} \quad (9-161)$$

**Example 20: Inventory Computation** Let us consider 10 successive batches of raw materials, of 1000 units, purchased in a time of rising prices in which  $c_1 = \$0.10$  per unit,  $c_2 = \$0.11$  per unit, etc., as listed in Table 9-29. The total cost of the purchases in the inventory is found from Eq. (9-155) to be \$1450.

Let us calculate the value of the raw-materials inventory after, say, 5500 units have been withdrawn from inventory, first by using FIFO and then by using LIFO.

We substitute the appropriate quantities into Eq. (9-158) for FIFO, keeping in mind that  $p = 5, n = 10$ , and  $m = 5500$ , to get

$$I_{FIFO} = \$1450 - \$600 - (5500 - 5000)(\$0.15) \\ I_{FIFO} = \$775$$

In a similar manner, we substitute values into Eq. (9-161) for LIFO (except that  $p$  is now 6) to get

$$I_{LIFO} = \$1450 - \$850 - (5500 - 5000)(\$0.14) \\ I_{LIFO} = \$530$$

Thus, in a time of rising raw-materials prices, the FIFO method gives a higher value for the remaining inventory than will LIFO. In a time of falling prices, the FIFO method will give a lower value for the remaining inventory than will LIFO.

**Average-Cost Basis for Inventory** Either a simple average or a weighted average can be used to value inventory cost.

Using the simple-average method, the value  $C_{SAV}$  of the material taken out of inventory is given by

$$C_{SAV} = m(c_1 + c_n)/2 \quad (9-162)$$

**TABLE 9-29 Costs of Inventory with Rising Prices**

$p$	$x_j$ , units	$c_j$ , \$/unit	$x_j c_j$ , \$	$\sum_1^p x_j$ , units	$\sum_1^p x_j c_j$ , \$	$\sum_p^n x_j$ , units	$\sum_p^n x_j c_j$ , \$
1	1,000	0.10	100	1,000	100	10,000	1,450
2	1,000	0.11	110	2,000	210	9,000	1,350
3	1,000	0.12	120	3,000	330	8,000	1,240
4	1,000	0.13	130	4,000	460	7,000	1,120
5	1,000	0.14	140	5,000	600	6,000	990
6	1,000	0.15	150	6,000	750	5,000	850
7	1,000	0.16	160	7,000	910	4,000	700
8	1,000	0.17	170	8,000	1,080	3,000	540
9	1,000	0.18	180	9,000	1,260	2,000	370
10	1,000	0.19	190	10,000	1,450	1,000	190



The value of the inventory  $I_{SAV}$  at any given time is

$$I_{SAV} = C_I - C_{SAV} \tag{9-163}$$

In terms of Eqs. (9-155) and (9-162), Eq. (9-163) can be written as

$$I_{SAV} = \sum_1^n \chi_j c_j - [m(c_1 + c_n)/2] \tag{9-164}$$

We shall consider the value of the inventory after 5500 units have been withdrawn, using the data listed in Table 9-29. On the basis of a simple average, the materials withdrawn are priced at  $(\$0.10 + \$0.19)/2 = \$0.145$  per unit. Since the total cost of the purchases in the raw-materials inventory is found from Eq. (9-155) to be \$1450, the value of the inventory after 5500 units have been withdrawn is calculated from Eq. (9-164) to be

$$I_{SAV} = \$1450 - (5500)(\$0.145) = \$652.50$$

For the weighted-average method, the value of the material  $C_{WAV}$  taken out of inventory is given by

$$C_{WAV} = m \sum_1^n \chi_j c_j / \sum_1^n \chi_j \tag{9-165}$$

$$I_{WAV} = C_I - C_{WAV} \tag{9-166}$$

In terms of Eqs. (9-153) and (9-164), Eq. (9-166) can be written

$$I_{WAV} = \sum_1^n \chi_j c_j \left[ 1 - \left( m / \sum_1^n \chi_j \right) \right] \tag{9-167}$$

Let us use Eq. (9-167) to value the inventory, after 5500 units have been withdrawn, by employing the data of Table 9-29.

$$I_{WAV} = \$1450 [1 - (5500/10,000)] = \$652.50$$

For this example, the values of  $I_{SAV}$  and  $I_{WAV}$  are the same because the batches purchased are of equal size and the prices are linearly progressive. This is a combination rarely found in practice.

A more realistic example, in which the buyer seeks to purchase at the lowest price, is provided by the data of Table 9-30. The quantities bought vary according to price, but some may have been made at high prices to maintain production.

On the basis of Table 9-30, when 5500 items have been removed from inventory, the value of the inventory by using the FIFO, LIFO, simple-average, and weighted average methods respectively is

$$I_{FIFO} = \$1175 - \$560 - (5500 - 5200)(\$0.10)$$

$$I_{FIFO} = \$573.00$$

$$I_{LIFO} = \$1175 - \$645 - (5500 - 5000)(\$0.10)$$

$$I_{LIFO} = \$480.00$$

$$I_{SAV} = \$1175 - 5500 [(\$0.10 + \$0.15)/2]$$

$$I_{SAV} = \$487.50$$

$$I_{WAV} = \$1175 [1 - (5000/10,000)]$$

$$I_{WAV} = \$528.75$$

**Pros and Cons of Inventory Valuation** In the standard-price method of inventory valuation, all materials are taken out at the same

price. In addition to simplicity, the method has the advantage that the efficiency of raw-materials purchase is constantly checked.

In both the average-cost and the standard-cost methods of valuing inventory, materials are not charged out at actual cost. Thus, the amount of profit or loss for the period may be varied by the method chosen to value the inventory. For this reason, accountants usually insist that the method of inventory valuation be consistent from period to period. This causes inertia but does not prevent a change of method when it can be justified. In such cases, it is usual to inform stockholders of the change because the influence on declared profits can be large.

Unfortunately, there is no right or wrong way to value inventory, although certain methods are not allowed in certain countries for tax-assessment purposes. For example, LIFO is not allowed in the United Kingdom. As a general rule, the method used should be the one that gives the lowest tax liability. However, it is generally accepted that consistency is also a virtue in inventory valuation.

It is important to realize that the method used to value inventory for cost accounting purposes is not necessarily the one used to draw up the balance sheet and financial accounts. In this case, inventory is valued either at the cost given by another method or at the market value, whichever is lower.

**Inventory Control** The optimum size of inventory depends on the type of industry and on the skills available to the individual company. Inventories are high in the tobacco industry and low in perishable-foods businesses. The larger the inventory, the larger the warehousing and associated costs. These costs include insurance, taxes, depreciation, handling and security charges, etc., and can be taken as roughly proportional to the value of the inventory  $I$ . The annual cost  $A_{IW}$  of maintaining an inventory of value  $I$  is given by

$$A_{IW} = \alpha I \tag{9-168}$$

where  $\alpha$  is the proportionality factor which is of the order of 0.25 for many industries.

In contrast, some costs can be reduced as a result of larger inventories. For example, larger discounts can be obtained on bulk purchases and deliveries. In addition, larger inventories reduce the risk of losing sales and goodwill through interruptions to production and consequently running out of stocks of finished goods.

The cost of placing an order for materials is partly fixed and partly variable. The annual cost of ordering  $A_{IO}$  is given by

$$A_{IO} = FN + VR \tag{9-169}$$

where  $F$  is the fixed cost per order,  $N$  is the number of orders per year,  $V$  is the variable cost of ordering per unit of production, and  $R$  is the annual production rate. Although the administrative cost of placing an order will be more or less fixed, the shipping costs are proportional to the size of the order and, hence, for the year are proportional to the annual production rate. The total annual cost of inventory  $A_I$  is the sum of Eqs. (9-168) and (9-169):

$$A_I = \alpha I + FN + VR \tag{9-170}$$

For a given number of orders per year  $N$ , the value of the inventory is proportional to the magnitude of the average individual order  $U$ . Hence, Eq. (9-170) can be written as

$$A_I = \beta U + (FR/U) + VR \tag{9-171}$$

where  $\beta$  is a proportionality factor and the number of orders per year  $N$  has been written as  $R/U$ .

By differentiating Eq. (9-171) with respect to  $U$ , the optimum size of order can be estimated. The differential is:

$$dA_I/dU = \beta - (FR/U^2) \tag{9-172}$$

Setting the right-hand side of Eq. (9-172) equal to zero yields

$$U = \sqrt{FR/\beta} \tag{9-173}$$

where  $U$  is now the optimum size of order.

A number of models have been developed to enable managers to handle inventories in the most profitable manner. These models can be applied to other elements of working capital, such as cash.

**TABLE 9-30 Costs of Inventory with Fluctuating Prices**

$p$	$x_j$ , units	$c_j$ , \$/unit	$x_j c_j$ , \$	$\sum_1^p x_j$ , units	$\sum_1^p x_j c_j$ , \$	$\sum_p^n x_j$ , units	$\sum_p^n x_j c_j$ , \$
1	1,000	0.10	100	1,000	100	10,000	1,175
2	1,500	0.11	165	2,500	265	9,000	1,075
3	500	0.13	65	3,000	330	7,500	910
4	2,000	0.10	200	5,000	530	7,000	845
5	200	0.15	30	5,200	560	5,000	645
6	1,000	0.14	140	6,200	700	4,800	615
7	2,500	0.11	275	8,700	975	3,800	475
8	300	0.15	45	9,000	1,020	1,300	200
9	100	0.20	20	9,100	1,040	1,000	155
10	900	0.15	135	10,000	1,175	900	135

**Working Capital** The amount and disposition of working capital and the efficiency of its use determine the immediate prospects for future growth in a company. The bulk of managerial effort in a company is directly or indirectly concerned with the manipulation of working capital. Insufficient or misused working capital is the commonest cause of business failure.

Engineers concerned with cost estimations tend to make estimates of the fixed-capital cost of a project, leaving considerations of working capital to the accountants. Although the estimation of fixed-capital cost is more straightforward from an engineering point of view, the estimation of working capital is of vital importance both for an individual project and for the company as a whole.

Working capital can range from about 10 percent to almost 100 percent of the invested capital, depending on the industry, and is an important factor in the profitability index of a business. For this reason, it is best to compare the performance of an individual company with that of others that are as similar as possible.

Gross working capital is normally defined as total current assets, while net working capital is current assets minus current liabilities. Current assets normally amount to more than half of the total assets of a company. When accountants wish to emphasize working capital in a company, they present the balance sheet in the vertical form, as shown in Table 9-31. In this table, the stockholders' equity of \$91,650 funds the sum of \$38,650 (net working capital) and the sum of \$53,000 (fixed assets less the long-term loan). The flow of working capital is diagrammatically illustrated in Fig. 9-39.

The necessary working capital varies with sales volume or production rate. For example, sales and hence accounts receivable will double for a doubling in sales volume. In addition, an increase in sales volume normally requires increased inventories of raw materials, work in progress, and finished goods, all of which tie up capital. An increase in sales volume may lead to a relative shortage of working capital. In turn, this may mean that accounts payable cannot be paid in time and that valuable cash discounts may be lost or interest and penalty charges incurred. Creditors may take legal action to obtain payments and thereby put an additional strain on the current assets of the company to provide legal fees. Often, such action leads other creditors to take similar steps, which may lead a fundamentally sound company into bankruptcy.

A shortage of cash may prevent a company from taking advantage of large discounts available for bulk purchase of raw materials. The importance of the availability of adequate cash or near cash can be seen by considering an account payable within 28 days, with a 2 percent discount allowed if paid within 7 days. If cash is not available to pay the account within 7 days, this is then equivalent to paying 2 percent interest on the money for the remaining 21-day period, or an annual compound-interest rate of more than 41 percent.

Adequate cash and a history of prompt payment of accounts strengthen the credit standing of a company and make it easier to obtain bank loans, etc. A company needs additional cash as a contingency against fires, floods, strikes, etc., as well as for additional adver-

**TABLE 9-31 Balance Sheet for BCD Company, Dec. 31, 1991**

<b>Current assets</b>		
Cash	\$14,575	
Accounts receivable	35,575	
Inventories		
Raw materials	6,000	
Work in process	3,750	
Finished product	10,000	
Gross working capital		\$69,900
<b>Current liabilities</b>		
Accounts payable	25,000	
Notes payable	3,000	
Bank loans	3,000	
Accruals payable	250	
		31,250
Net working capital		\$38,650
<b>Fixed assets</b>		
Long-term loan	75,000	
	22,000	
		\$53,000
<b>Stockholders' equity</b>		
		\$91,650

tising required to counteract the activities of competitors. This additional money is normally held as interest-bearing investments that can be turned into cash on short notice.

**Working-Capital Ratios** Financial analysts make extensive use of ratios in assessing the economic health of a company. For evaluating the ability of a company to successfully maintain and develop its immediate business activities, analysts apply a current ratio and a quick (or acid-test) ratio, as given by

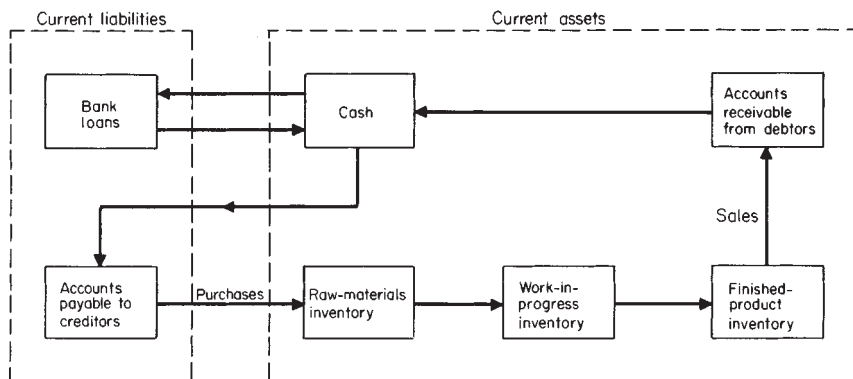
$$\text{Current ratio} = \frac{\text{current assets}}{\text{current liabilities}} \quad (9-142)$$

$$\text{Quick ratio} = \frac{\text{liquid assets}}{\text{current liabilities}} \quad (9-143)$$

Liquid assets are those that can be realized almost immediately, such as cash, accounts receivable, and marketable securities. Although inventories are current assets, they must not be regarded as liquid assets because they cannot usually be converted into cash without winding up the business.

Although a high current ratio is desirable, this may be achieved by having unnecessarily high inventories that bring no profit except when commodity prices are rising rapidly. The quick ratio is less misleading in this respect.

Good management practice will hold inventories at the lowest possible levels consistent with customer satisfaction and efficient plant operation. Excessive inventories are unproductive and are an investment having little or no rate of return. Excessive inventories should be maintained only when supplies are erratic or rising in price. Management should normally aim for a high inventory-turnover ratio, as given by:



**FIG. 9-39** Flow of working capital showing relationship between current liabilities and current assets.

$$\text{Inventory-turnover ratio} = \frac{\text{revenue from annual sales}}{\text{average value of inventory}} \quad (9-174)$$

Similarly, good management practice is to hold accounts receivable at a low level and to have a high accounts-receivable-turnover ratio, as given by

$$\begin{aligned} \text{Accounts-receivable-turnover ratio} \\ = \frac{\text{revenue from annual sales}}{\text{average value of accounts receivable}} \quad (9-175) \end{aligned}$$

Alternatively, it is good practice to have a low average collection period, as given by

$$\begin{aligned} \text{Average collection period} \\ = \frac{\text{average value of accounts receivable}}{\text{revenue from sales per day}} \quad (9-144) \end{aligned}$$

Equation (9-144) gives the number of days during which sales are tied up in receivables.

An accounts-receivable-turnover ratio of 12 is considered fairly good for a manufacturing company. This implies an average collection period of about 1 month. The price obtained for the goods should include an allowance for interest (otherwise obtainable) on the money tied up for such a period.

Since ratios, like balance sheets, refer to a particular point in time, they have a limited use unless they are compared with previous values. A study of ratio trends indicates whether or not a company is approaching a working-capital or a liquidity crisis and may enable management to compare the performance of the company with that of competitors.

**Funds Statement** A typical funds statement is shown in Table 9-32. It displays the change in net working capital and can be obtained from a statement of changes in working capital, such as the one shown in Table 9-33. A funds statement shows where the cash came from and how it was used.

Changes in working capital  $\Delta C_{WC}$  in an annual accounting period can be represented by

$$\Delta C_{WC} = A_S - A_{TE} - \sum \Delta C_{FC} - \sum \Delta C_L + \sum \Delta C_{FIN} \quad (9-176)$$

where  $A_S$  is the revenue from annual sales of a product,  $A_{TE}$  is the total cost or expense required to produce and sell the product but excluding an annual provision for plant depreciation,  $\sum \Delta C_{FC}$  is the sum of the changes in depreciable fixed assets,  $\sum \Delta C_L$  is the sum of the changes in nondepreciable fixed assets, and  $\sum \Delta C_{FIN}$  is the sum of the

**TABLE 9-32 Funds Statement for Year Ending Dec. 31, 1991**

Sources of funds			
Cash income from operations			\$15,000
New finance			10,000
Sale of plant equipment			2,000
Total sources of funds			\$27,000
Applications of funds			
Purchase of land	\$3,000		
Cash dividend on stock		9,000	
Total applications of funds			\$12,000
Increase in net working capital			\$15,000

**TABLE 9-33 Statement of Changes in Working Capital**

	Jan. 1, 1991	Jan. 1, 1992	Change
Cash	\$ 80,000	\$ 94,000	+\$14,000
Accounts receivable	20,000	35,000	+15,000
Inventories	30,000	25,000	-5,000
Total current assets	\$130,000	\$154,000	+\$24,000
Current liabilities	(\$ 60,000)	(\$ 69,000)	(\$ 9,000)
Net working capital	\$ 70,000	\$ 85,000	+\$15,000
Current ratio	2.17	2.23	
Quick ratio	1.67	1.87	

changes in financial resources such as loans, bonds, preferred stock, common stock, etc.

Equation (9-176) can also be written as

$$\Delta C_{WC} = A_{CI} - \sum \Delta C_{FC} - \sum \Delta C_L + \sum \Delta C_{FIN} \quad (9-177)$$

where  $A_{CI}$  is the annual cash income, which is the main source of funds for most companies. In this case, the annual cash income excludes all noncash expenses such as the balance-sheet annual depreciation charge  $A_{BD}$ , which is purely a book transaction.

A positive value of any term in Eq. (9-177) implies an increase in working capital, and a negative value a decrease. For example, the sale of fixed assets such as plant, buildings, land, etc., is a source of cash, and the purchase of fixed assets uses up cash. Similarly, an increase in financial resources in the form of loans and stock and bond issues is a source of cash, and a decrease in financial resources in the form of repayment of loans, retirement of stocks and bonds, and the payment of cash dividends uses up cash. (Note that a stock dividend as opposed to a cash dividend does not use up cash.)

The relation between (1) net annual cash flow  $A_{CF}$  after tax for individual projects, (2) the annual amount of tax  $A_{IT}$ , and (3) the annual expenditure of capital  $A_{YC}$  is

$$A_{CF} = A_{CI} - A_{IT} - A_{YC} \quad (9-178)$$

Equation (9-178) for a single project is really analogous to Eq. (9-177) for a company.

An income statement or profit-and-loss account gives the net annual profit  $A_{NP}$  before tax. In order to assess the annual cash income  $A_{CI}$  as a source of funds from the value of the net annual profit  $A_{NP}$  given in the income statement, it is necessary to add back all noncash expenses such as the balance-sheet annual depreciation charge  $A_{BD}$ . This practice sometimes erroneously suggests that depreciation is a source of funds, whereas cash income is the only source of funds.

Although  $A_{BD}$  does not affect working capital in any way, the annual depreciation charge  $A_D$  does affect the annual amount of tax  $A_{IT}$  given by

$$A_{IT} = (A_{CI} - A_D - A_A)t \quad (9-3)$$

where  $t$  is the fractional tax rate and  $A_A$  is the annual amount of any other allowances. Thus, the net annual cash income is affected by depreciation allowances, as follows:

$$A_{NCI} = A_{CI} - A_{IT} \quad (9-2)$$

In this sense, depreciation makes working capital available by reducing the cash outflow for taxes.

When a fixed asset is sold at a price that differs from its book value, an accounting gain or loss is recorded. This gain or loss does not affect working capital, which has simply been increased by the amount of cash received from the sale. For example, if an item of plant is sold for \$40,000 (whatever its book value), the increase in working capital resulting from the sale is \$40,000. If the book value of the plant had been \$50,000, the accounting loss of \$10,000 on the sale would be included as an expense when calculating the cash income. This \$10,000 must be added back to the cash income in order to get the cash income excluding noncash expenses as required for  $A_{CI}$ . Conversely, any accounting gain on the sale of a fixed asset must be subtracted if it has been included in the cash income.

Book values of fixed assets are determined by the balance-sheet annual depreciation charges  $A_{BD}$ , which do not affect working capital. Although the accounting gain or loss on the sale of a fixed asset is based on its book value, working capital is not affected by depreciation assessments.

Transactions that change the character of the net working capital but do not affect its value occur in a company. For example, a cash payment of \$10,000 for accounts payable reduces both the current asset of cash by \$10,000 and the current liability of accounts payable by \$10,000, leaving the net working capital unchanged. However, this transaction affects both the current and the quick ratios.

After the balance sheet and the income statement or profit-and-loss account, the funds statement is generally regarded as the most important financial document. However, many financial managers regard a statement showing changes in cash as being of equal importance.

As with balance sheets and income statements, there is no rigid format for funds statements. These vary in the amount of detail given and

**TABLE 9-34 Statement of Changes in Working Capital and Sources and Applications of Funds for BCD Company**

	Balance sheets		Change in funds	
	Dec. 31, 1991	Dec. 31, 1992	Applications	Sources
Cash	\$ 14,575	\$ 13,000		\$ 1,575
Accounts receivable	35,575	36,000	\$ 425	
Inventories	19,750	20,750	1,000	
Total current assets	\$ 69,900	\$ 69,750		
Accounts payable	\$ 25,000	\$ 27,000		2,000
Notes payable	3,000	2,500	500	
Bank loans	3,000	2,000	1,000	
Accruals payable	250	300		50
Total current liabilities	\$ 31,250	\$ 31,800		
Net working capital	\$ 38,650	\$ 37,950		
Fixed assets	75,000	85,000	10,000	
Net assets	\$113,650	\$122,950		
Long-term loan	\$ 22,000	\$ 22,000		
Stockholders' equity	\$ 91,650	\$100,950		9,300
			\$12,925	\$12,925

also in their layout. Funds statements help management in planning for the future, for example, in timing and making financial provision for future expenditures.

A typical management accountant's statement for changes in working capital and sources and applications of funds is shown in Table 9-34. This is based on the following relation: an increase in application of funds equals an increase in sources of funds. The relation can also be expressed as follows: an increase in assets plus a decrease in liabilities equals an increase in liabilities plus a decrease in assets.

In Table 9-34, the cash income for the year has been absorbed in the increase in stockholders' equity, which consists of issued stock plus retained profits or income.

**General Considerations** Many people regard the management of working capital as essentially a cash-flow problem. Certainly, expansion involves increased investment in fixed assets and in the various items that comprise the current assets, all of which require cash. If this leads to a shortage of cash so that a company cannot pay its liabilities, then a situation called overtrading results, and the company may ultimately be forced into liquidation. Therefore, it is essential that a company have access to readily available cash or sources of short-term financing, preferably without having to specify a particular asset as collateral, since such collateral can only be pledged to an agreed realizable value that is much less than its true value.

Accounts payable, also called trade credit, are the major source of short-term financing. Accounts payable normally amount to about 40 percent of the current liabilities of a manufacturing company. Such short-term financing is relatively expensive when available discounts are lost.

The second most important source of short-term financing is notes payable from commercial banks. Banks normally require a borrower to maintain a compensating balance. For example, if a company requires a loan of \$100,000, it must borrow more than this, say, \$120,000 (on which it pays interest), in order to maintain a minimum checking-account balance of \$20,000. Commercial banks also provide a wide variety of other services that can be of great help to companies in temporary financial difficulties.

The third most important source of short-term financing is the commercial paper, or promissory notes, of large companies. This is the

cheapest form of finance. However, the amount of money available is a function of the excess liquidity of the large companies at a given time, and the money may have to be repaid at relatively short notice.

Sometimes, fixed assets are purchased via short-term loans, which can lead to liquidity problems. For the most part, fixed assets should be financed from long-term or permanent capital such as stocks or bonds. The proven ability of management to handle working capital efficiently will put a company in a better position to obtain such long-term capital when required, because the confidence of bankers and stockholders will have been obtained.

**Budgets and Cost Control** R. J. Bull (*Accounting in Business*, 2d ed., Butterworth, London, 1972, p. 163) defined a budget as a comprehensive and coordinated plan, expressed in monetary terms, directing and controlling the resources and trading activities of an enterprise for some specified period in the future. A budget is not a forecast. A forecast is an estimate of the future which may or may not be attained. A budget is an overall objective based on a forecast. In addition, a budget defines the detailed objectives to be achieved by various levels of management in an organization. Since the achievement of these objectives requires the complete cooperation of management, the targets set must be realistic in terms of resources and past performance.

A comparison of actual with budgeted results can be used as the basis for control at the company, departmental, plant, or project level. In addition, a continuing record of performance should be maintained to provide the data for the preparation of further budgets.

Since company accounts are normally published annually, 1 year is commonly taken as a budget period. However, budget periods can vary widely depending on the nature of the operation. For example, a sales budget may be for a period of, say, 3 or 6 months, while the budget period for the installation of, say, a nuclear power station would extend over many years.

The basic objectives of budgets are planning and control. The first step is to determine the limiting factor. For example, budgeted sales cannot exceed the maximum productive capacity of the available plant. Since all the activities in the plan are interrelated, the extent of the plan is determined by the limiting factor.

After the plan is put into operation, actual progress is monitored against established standards. These data may subsequently lead to the plan's being modified in order to achieve the objectives more effectively.

R. Pilcher (*Appraisal and Control of Project Costs*, McGraw-Hill, Maidenhead, England, 1973, p. 233) stated the main purposes of a cost control system to be:

1. To provide immediate warning of uneconomic operations in both the long and the short terms
2. To provide the relevant feedback, carefully qualified in detail by all the conditions under which the work has been carried out, to the estimator who is responsible for establishing the standards in the past and in the future
3. To provide data to assist in the valuation of those variations that will arise during the course of the work
4. To promote cost consciousness
5. To summarize progress

Budgeted income statements are identical in form to actual income statements. However, the budgeted numbers are objectives rather than achievements. Budgetary models based on mathematical equations are increasingly being used. These may be used to determine rapidly the effect of changes in variables. Variance analysis is discussed in the treatment of manufacturing-cost estimation.

## MANUFACTURING-COST ESTIMATION

The annual manufacturing cost or expense  $A_{ME}$  can be written as the sum of the direct manufacturing or prime cost  $A_{DME}$  and the indirect manufacturing or overhead cost  $A_{IME}$ :

$$A_{ME} = A_{DME} + A_{IME} \quad (9-179)$$

The determination of direct or prime costs is more straightforward than the determination of indirect or overhead costs. When more than one product is involved, the question arises as to the correct distribution of overhead costs between the various products.

In addition to fixed and variable costs there are mixed or semivari-



able costs. These have a fixed and a variable element. The variable element can vary with production linearly, stepwise, or in a curvilinear manner. However, it is a convenient simplification to divide all manufacturing costs into fixed and linearly variable costs.

**General Considerations** Manufacturing costs are best considered in the context of the manufacturing, trading, and profit-and-loss accounts. Typical examples of these are shown in Tables 9-35, 9-36, and 9-37, respectively. These are based on the conventional accountancy period of 1 year.

The gross annual profit  $A_{CP}$  in Table 9-36 is dependent on the balance-sheet annual depreciation charge  $A_{BD}$ , which is not necessarily the same as the depreciation allowance used for tax purposes. Since  $A_{BD}$  is arbitrarily chosen, it can be used to make the gross annual profit  $A_{CP}$  high or low according to the company policy.

The gross annual profit  $A_{CP}$  is also dependent on the method of valuing the inventory. For example, raw materials may have been purchased at the beginning of the accounting year at, say, 9 cents per kilogram. The purchase price may have risen to, say, 12 cents per kilogram at the end of the accounting year. If valuation of the inventory is made at the higher purchase price, the production cost is lower and the gross profit higher than if the valuation is made at the lower purchase price. Although in this case the profit looks better, a higher tax is payable.

**TABLE 9-35 Manufacturing Account**

Inventory of raw materials, Jan. 1, 1991	\$ 35,000	
Add purchases	870,000	
Add carriage inward	25,000	
	<u>930,000</u>	
Less inventory of raw materials Dec. 31, 1991	51,000	
Cost of raw materials consumed	\$ 879,000	
Direct wages	122,000	
Direct utilities	22,000	
Other direct expenses	<u>104,000</u>	
Prime cost or direct manufacturing expense, $A_{DME}$	\$1,127,000	\$1,127,000
Payroll overhead	28,000	
General plant overhead	52,000	
Other indirect expenses	<u>52,000</u>	
Indirect manufacturing expense, $A_{IME}$	\$ 132,000	
Depreciation, $A_{BD}$	68,000	
	\$ 200,000	200,000
		<u>\$1,327,000</u>
Add work in progress, Jan. 1, 1991		30,000
		<u>\$1,357,000</u>
Less work in progress, Dec. 31, 1991		35,000
Production cost of products		<u>\$1,322,000</u>

**TABLE 9-36 Trading Account**

Inventory of finished products, Jan. 1, 1991	\$ 200,000
Add production cost of products	<u>1,322,000</u>
	\$1,522,000
Less inventory of finished products, Dec. 31, 1991	190,000
	<u>\$1,332,000</u>
Gross profit	668,000
Sales	<u>\$2,000,000</u>

**TABLE 9-37 Profit-and-Loss Account**

Administration	\$ 74,000	
Sales and shipping	124,000	
Advertising and marketing	40,000	
Technical service	10,000	
Research and development	60,000	
	<u>\$308,000</u>	\$308,000
Net profit before taxes		360,000
Gross profit		<u>\$668,000</u>

The profit is also dependent on the method of valuing the work in progress. We shall consider the manufacture of 100,000 kg of product with a prime or direct manufacturing cost of 10 cents per kilogram and an additional indirect manufacturing expense of 5 cents per kilogram. We assume that 90,000 kg of the product is sold and 10,000 kg is stored in inventory. The value of the inventory is \$1000 on the basis of prime or direct cost and \$1500 when the indirect manufacturing expense is included. It is still a controversial question as to whether manufacturing overheads should be absorbed in the cost. The latter is known as absorption costing and is the traditional accounting method. Direct costing is being increasingly used and is particularly favored by engineers.

The annual cash income  $A_{CI}$  before tax, in terms of the revenue from annual sales  $A_S$  of a product and the components of the total annual cost or expense required to produce and sell the product (excluding any allowance for plant depreciation), is expressed by

$$A_{CI} = [A_S - (A_{VGE} + A_{VME})] - (A_{FGE} + A_{FME}) \quad (9-180)$$

where  $A_{VGE}$  and  $A_{VME}$  are the annual variable general and variable manufacturing expenses respectively and  $A_{FGE}$  and  $A_{FME}$  are the annual fixed general and fixed manufacturing expenses respectively.

Revenue from annual sales is

$$A_S = R c_S \quad (9-181)$$

where  $R$  is the production rate and  $c_S$  the sales price per unit of production.

Similarly,  $A_{VGE}$  can be taken as proportional to the annual production rate  $R$  and the variable general expense per unit of production  $c_{VGE}$ :

$$A_{VGE} = R c_{VGE} \quad (9-182)$$

Likewise, for  $A_{VME}$ ,

$$A_{VME} = R c_{VME} \quad (9-183)$$

In practice, annual direct variable costs such as raw materials, utilities, etc., are not always proportional to the production rate.

Substituting Eqs. (9-182) and (9-183) into the first term on the right of Eq. (9-180) yields

$$A_S - (A_{VGE} + A_{VME}) = R [c_S - (c_{VGE} + c_{VME})] \quad (9-184)$$

In Eq. (9-184), the term  $[c_S - (c_{VGE} + c_{VME})]$  is the contribution to cash income.

For simultaneous production of more than one product, Eq. (9-184) can be written as

$$A_S - (A_{VGE} + A_{VME}) = R_1 [(c_S)_1 - [(c_{VGE})_1 + (c_{VME})_1]] + R_2 [(c_S)_2 - [(c_{VGE})_2 + (c_{VME})_2]] + \dots \quad (9-185)$$

where the subscripts 1, 2, etc., refer to the various coproducts.

The contribution to cash income made by a particular product depends on the method of accounting. Widely different values for the contribution can be calculated by using different methods of assigning manufacturing expenses.

The cost of each item in a cost estimate should be presented in such a way that the estimate can be modified and updated at any time in the future when revised data become available.

Published data and shortcut estimating methods can be used to calculate the approximate manufacturing cost of a new product. However, most companies have extensive data on various items of cost such as overheads, property taxes, etc. These data should be used whenever possible to give the estimate that is most valid for a particular company.

For a new product, the ratio of manufacturing expense to sales price  $c_{ME}/c_S$  should be compared with the ratio of total manufacturing cost or expense to sales revenue for the company as a whole. If the ratio  $c_{ME}/c_S$  is less than or equal to the ratio for the company, then the proposed sales price appears to be reasonable and the product is probably commercially viable. This comparison is, of course, used only as an approximate guide in preliminary assessments.

**One Main Product Plus By-Products** We shall let one unit of raw material yield  $\chi_1, \chi_2$ , etc., weights of products 1, 2, etc., respectively. The variable general expense per unit of raw material will be



$c_{VGE}$  and the variable manufacturing expense per unit of raw material will be  $c_{VME}$ . The unit of raw material may be either a single material or a mixture of several components. I. Leibson and G. A. Trischman [*Chem. Eng.*, **78**, 69–74 (May 31, 1971)] showed the effects on manufacturing expenses of alternate feedstocks having different compositions and costs for producing the same products.

If product 1 is the main product, then this carries all the variable expenses, so that

$$(c_{VGE})_1 = c_{VGE}/\chi_1 \quad (9-186)$$

$$(c_{VME})_1 = c_{VME}/\chi_1 \quad (9-187)$$

where  $(c_{VGE})_1$  and  $(c_{VME})_1$  are the variable general and variable manufacturing expenses respectively per unit of product 1.

For by-product 2, etc.,  $(c_{VGE})_2 = 0$  and  $(c_{VME})_2 = 0$ , etc. For this case, Eq. (9-184) becomes

$$A_S - (A_{VGE} + A_{VME}) = R_1 [(c_s)_1 - (c_{VGE} + c_{VME})/\chi_1] + R_2(c_s)_2 + \dots \quad (9-188)$$

If  $R$  is the annual rate at which the raw material is consumed, then

$$R_1 = \chi_1 R \quad (9-189)$$

$$R_2 = \chi_2 R \quad (9-190)$$

In terms of one unit of raw material, Eq. (9-185) can be combined with Eq. (9-189) and (9-190) and written as

$$[A_S - (A_{VGE} + A_{VME})]/R = \chi_1(c_s)_1 - (c_{VGE} + c_{VME}) + \chi_2(c_s)_2 + \dots \quad (9-191)$$

Equation (9-191) gives the contribution of products 1, 2, etc., per unit of raw material. In this particular case, the contribution of the main product per unit weight of raw material is  $\chi_1(c_s)_1 - (c_{VGE} + c_{VME})$ . The contribution of product 2 is  $\chi_2(c_s)_2$ , i.e., its selling price per unit weight of raw material.

**Two Main Products**

**By Weight** We shall let one unit of raw material yield  $\chi_1$  and  $\chi_2$  weights of products 1 and 2 respectively. The variable general expense per unit of raw material will be  $c_{VGE}$  and the variable manufacturing expense per unit of raw material  $c_{VME}$ . In practice, it is rare for  $\chi_1 + \chi_2$  to be exactly unity.

If the variable expenses are shared by weight, then

$$(c_{VGE})_1 = \chi_1 c_{VGE} / [(\chi_1 + \chi_2)\chi_1] \quad (9-192)$$

$$(c_{VME})_1 = \chi_1 c_{VME} / [(\chi_1 + \chi_2)\chi_1] \quad (9-193)$$

$$(c_{VGE})_2 = \chi_2 c_{VGE} / [(\chi_1 + \chi_2)\chi_2] \quad (9-194)$$

$$(c_{VME})_2 = \chi_2 c_{VME} / [(\chi_1 + \chi_2)\chi_2] \quad (9-195)$$

where  $(c_{VGE})_1$  and  $(c_{VME})_1$  are the variable general and variable manufacturing expenses per unit of product 1 respectively and  $(c_{VGE})_2$  and  $(c_{VME})_2$  are the comparable expenses for product 2.

For this example, Eq. (9-185) becomes

$$A_S - (A_{VGE} + A_{VME}) = R_1 \left[ (c_s)_1 - \frac{(c_{VGE} + c_{VME})}{(\chi_1 + \chi_2)} \right] + R_2 \left[ (c_s)_2 - \frac{(c_{VGE} + c_{VME})}{(\chi_1 + \chi_2)} \right] \quad (9-196)$$

In Eqs. (9-192) through (9-196) the sum of  $\chi_1$  and  $\chi_2$  may be equal to or less than 1. The above analysis can be extended for any number of coproducts.

In terms of one unit of raw material, Eq. (9-196) can be combined with Eqs. (9-189) and (9-190) and written as

$$\frac{A_S - (A_{VGE} + A_{VME})}{R} = \chi_1 \left[ (c_s)_1 - \frac{(c_{VGE} + c_{VME})}{(\chi_1 + \chi_2)} \right] + \chi_2 \left[ (c_s)_2 - \frac{(c_{VGE} + c_{VME})}{(\chi_1 + \chi_2)} \right] \quad (9-197)$$

Equation (9-197) gives the contribution of products 1 and 2 per unit weight of raw material.

**By Value** We shall let one unit of raw material yield  $\chi_1$  and  $\chi_2$  weights of products 1 and 2 respectively, with values of  $\chi_1(c_s)_1$  and  $\chi_2(c_s)_2$  respectively;  $(c_s)_1$  and  $(c_s)_2$  are the sales prices of products 1 and 2 per unit of production.

We shall let the variable general expense per unit of raw material be  $c_{VGE}$  and the variable manufacturing expense per unit of raw material be  $c_{VME}$ . If the variable expenses are shared by value then

$$(c_{VGE})_1 = \frac{\chi_1(c_s)_1 c_{VGE}}{[\chi_1(c_s)_1 + \chi_2(c_s)_2]\chi_1} \quad (9-198)$$

$$(c_{VME})_1 = \frac{\chi_1(c_s)_1 c_{VME}}{[\chi_1(c_s)_1 + \chi_2(c_s)_2]\chi_1} \quad (9-199)$$

$$(c_{VGE})_2 = \frac{\chi_2(c_s)_2 c_{VGE}}{[\chi_1(c_s)_1 + \chi_2(c_s)_2]\chi_2} \quad (9-200)$$

$$(c_{VME})_2 = \frac{\chi_2(c_s)_2 c_{VME}}{[\chi_1(c_s)_1 + \chi_2(c_s)_2]\chi_2} \quad (9-201)$$

where  $(c_{VGE})_1$  and  $(c_{VME})_1$  are the variable general and variable manufacturing expenses per unit of product 1 respectively and  $(c_{VGE})_2$  and  $(c_{VME})_2$  are the comparable expenses for product 2.

For this case, Eq. (9-185) becomes

$$A_S - (A_{VGE} + A_{VME}) = R_1 \left\{ (c_s)_1 - \frac{(c_s)_1(c_{VGE} + c_{VME})}{[\chi_1(c_s)_1 + \chi_2(c_s)_2]} \right\} + R_2 \left\{ (c_s)_2 - \frac{(c_s)_2(c_{VGE} + c_{VME})}{[\chi_1(c_s)_1 + \chi_2(c_s)_2]} \right\} \quad (9-202)$$

This analysis can be extended for any number of coproducts.

In terms of one unit of raw material, Eq. (9-202) can be combined with Eqs. (9-189) and (9-190) and written as

$$\frac{A_S - (A_{VGE} + A_{VME})}{R} = \chi_1 \left\{ (c_s)_1 - \frac{(c_s)_1(c_{VGE} + c_{VME})}{[\chi_1(c_s)_1 + \chi_2(c_s)_2]} \right\} + \chi_2 \left\{ (c_s)_2 - \frac{(c_s)_2(c_{VGE} + c_{VME})}{[\chi_1(c_s)_1 + \chi_2(c_s)_2]} \right\} \quad (9-203)$$

Equation (9-203) gives the contribution of products 1 and 2 per unit weight of raw material.

**Example 21: Calculation of Contributions to Income for Multiple Products** One kilogram of raw material is used to manufacture  $\chi_1 = 0.32$  kg of product 1 and  $\chi_2 = 0.64$  kg of product 2. The balance of the raw material goes to waste. Product 1 sells at  $(c_s)_1 = 40$  cents per kilogram, and product 2 sells at  $(c_s)_2 = 12$  cents per kilogram. The variable general and variable manufacturing expenses, including raw materials,  $c_{VGE} + c_{VME}$ , total 10 cents per kilogram.

Let us calculate (a) the total contribution to cash income per kilogram of raw material and (b) the individual contribution of each product to cash income per kilogram of raw material for the following:

Case	Condition
1	Product 1 as the main product charged for all the variable expenses
2	Product 2 as the main product charged for all the variable expenses
3	Products 1 and 2 sharing the variable expenses on the basis of weight
4	Products 1 and 2 sharing the variable expenses on the basis of value

**Case 1.** The total contribution per kilogram of raw material is obtained by substituting the appropriate values into Eq. (9-191). For these conditions,

$$\begin{aligned} \chi_1(c_s)_1 + \chi_2(c_s)_2 - (c_{VGE} + c_{VME}) &= 0.32(40 \text{ cents/kg}) \\ &\quad + 0.64 (12 \text{ cents/kg}) - 10 \text{ cents/kg} \\ &= 12.8 \text{ cents/kg} + 7.68 \text{ cents/kg} \\ &\quad - 10 \text{ cents/kg} \\ &= 10.48 \text{ cents/kg} \end{aligned}$$

The individual contribution of product 1 as the main product is 12.8 cents – 10 cents = 2.8 cents per kilogram of raw material. The individual contribution of product 2 as the by-product is 7.68 cents per kilogram of raw material.

**Case 2.** The total contribution when product 2 is the main product is also obtained from Eq. (9-191) and as in Case 1 is found to be 10.48 cents per kilogram.

The individual contribution of product 2 as the main product is 7.68 cents - 10 cents = -2.32 cents per kilogram of raw material. The individual contribution of product 1 as the by-product is 12.8 cents per kilogram of raw material.

**Case 3.** When products 1 and 2 share the variable expenses on the basis of weight, the total contribution per kilogram of raw material is found by substituting the unit costs into Eq. (9-197). Values for each term are

$$\chi_1(c_s)_1 = 12.8 \text{ cents/kg} \quad (\text{same as Case 1})$$

$$\chi_2(c_s)_2 = 7.68 \text{ cents/kg} \quad (\text{same as Case 1})$$

$$[\chi_1/(\chi_1 + \chi_2)]_{(CVGE + CVME)} = (0.32/0.96)(10 \text{ cents/kg}) = 3.34 \text{ cents/kg}$$

$$[\chi_2/(\chi_1 + \chi_2)]_{(CVGE + CVME)} = (0.64/0.96)(10 \text{ cents/kg}) = 6.66 \text{ cents/kg}$$

Therefore, the total contribution becomes 12.8 + 7.68 - 3.34 - 6.66 = 10.48 cents per kilogram.

The individual contribution of product 1 is 12.8 cents - 3.34 cents = 9.46 cents per kilogram of raw material. The individual contribution of product 2 is 7.68 cents - 6.66 cents = 1.02 cents per kilogram of raw material.

**Case 4.** When products 1 and 2 share the variable expenses on the basis of value, the total contribution per kilogram of raw material is found by substituting the unit costs into Eq. (9-203). Values of each term are

$$\chi_1(c_s)_1 = 12.8 \text{ cents/kg} \quad (\text{same as Case 1})$$

$$\chi_2(c_s)_2 = 7.68 \text{ cents/kg} \quad (\text{same as Case 1})$$

$$\frac{\chi_1(c_s)_1(CVGE + CVME)}{\chi_1(c_s)_1 + \chi_2(c_s)_2} = \frac{(12.8)(10)}{12.8 + 7.68} = 6.24 \text{ cents/kg}$$

$$\frac{\chi_2(c_s)_2(CVGE + CVME)}{\chi_1(c_s)_1 + \chi_2(c_s)_2} = \frac{(7.68)(10)}{12.8 + 7.68} = 3.76 \text{ cents/kg}$$

Therefore, the total contribution becomes 12.8 + 7.68 - 6.24 - 3.76 = 10.48 cents per kilogram.

The individual contribution of product 1 is found as 12.8 cents - 6.24 cents = 6.56 cents per kilogram of raw material. The individual contribution of product 2 then becomes 7.68 cents - 3.76 cents = 3.92 cents per kilogram of raw material.

**Direct Manufacturing Costs** Direct manufacturing costs include raw materials, operating labor, utilities, and some miscellaneous items. A summary of the characteristics of each follows.

**Raw Materials** The cost of raw materials is normally the largest item of expense in the manufacturing cost of a product. The quantities of raw materials consumed can be calculated from material balances.

Material costs are conveniently presented in tables that give the following: name of material, form and grade, method of delivery, unit of measure, cost per unit, source of cost, annual consumption, annual cost, fractional consumption per unit of production, and cost per unit of production.

Net consumption of materials should be used for catalysts, solvents, filter aids, etc., that may have a recovery value. Current prices of chemicals are published in various trade journals. However, quotations from suppliers should be used whenever possible.

It may be possible for a company to negotiate the purchase of a material at a cost per unit that is significantly lower than the current published price. This is particularly true if large quantities are involved. Thus, estimates should be presented for both minimum and maximum costs. Price trends, availability, and quality are other factors that should be considered. A knowledge of price trends is particularly important for a product that a company may not manufacture for several years.

The yield in a chemical reaction determines the quantities of materials in the material balance. Assumed yields are used to obtain approximate exploratory estimates. In this case, possible ranges should be given. Firmer estimates require yields based on laboratory or, preferably, pilot-plant work.

**Operating Labor** The cost of operating labor is the second largest item of expense in the manufacturing cost. Labor requirements for a process can be estimated from an intelligent study of the equipment flow sheet, paying careful attention to the various primary process steps such as fractionation, filtration, etc. The hourly wage rate should be that currently paid in the company. Once the number of persons required per shift has been estimated for a particular production rate, the annual labor cost and the labor cost per unit of production can be estimated.

H. E. Wessel [*Chem. Eng.*, 59, 209-210 (July 1952)] made a study of the operating-labor requirements in the United States chemical

industry and presented the data as a plot of labor-hours per ton per processing step versus plant capacity in tons per day. These data can be represented by:

$$\log_{10} Y = 0.783 \log_{10} \chi + 1.252 + B \quad (9-204)$$

where  $Y$  is the operating-labor-hours per ton per processing step;  $\chi$  is plant capacity, tons per day; and  $B$  is a constant having values of 0.132 when multiple units are used to increase capacity or when the process is completely batch, of 0 for the average chemical-processing plant, and of -0.167 for large, highly automated plants or plants concerned with fluid processing.

Wessel's data for the United States chemical industry refer to the short ton equal to 2000 lb, or 907.2 kg. Labor requirements are higher in countries with lower productivities.

The approximate cost of supervision for operating labor is equivalent to 10 percent of the labor cost for simple operations and 25 percent for complex operations.

**Utilities** These include steam, cooling water, process water, electricity, fuel, compressed air, and refrigeration. The consumption of utilities can be estimated from the material and energy balances for the process, together with the equipment flow sheet.

Let us consider a cooler in the equipment flow sheet. The required rate of heat removal is known from the balances, and the rate of cooling water can be calculated once the inlet and outlet temperatures of the water have been specified. The calculation of the consumption of other utilities is also straightforward. Allowances should be made for wastage.

The current cost per unit for each utility is usually well known in a company. Thus, the annual cost for utilities and the utilities cost per unit of production can be estimated. The latter is normally much smaller than the raw-materials and labor costs. However, a great deal more work is involved in calculating the utilities cost than for any other item in the manufacturing cost.

Unfortunately, there are no satisfactory shortcut methods for doing this. When the utilities cost is relatively small, it may be possible to make an intelligent guess on the basis of known costs for similar processes in the company. Alternatively, published data for the consumption of utilities per unit of production for various processes may be used.

**Miscellaneous Direct Costs** Estimates for the cost of maintenance and repairs, operating supplies, royalties, and patents are best based on company records for similar processes. A rough average value for the annual cost of maintenance is 6 percent of the capital cost of the plant. This percentage can vary from 2 to 10 percent, depending on the severity of plant operation. Approximately half of the maintenance costs are for materials and half for labor. Royalty and patents costs are in the order of 1 to 5 percent of the sales price of the product.

**Indirect Manufacturing Costs** Estimates for the cost of payroll overhead, control laboratory, general plant overhead, packaging, and storage facilities are best based on company records for similar processes.

Payroll overhead includes the cost of pensions, holidays, sick pay, etc., and is normally between 15 and 20 percent of the operating-labor cost. Laboratory work is required for product quality control, and its cost is approximately 10 to 20 percent of the operating-labor cost.

Plant overhead includes the cost of medical, safety, recreational, effluent-disposal, and warehousing facilities, etc. In general, the larger the plant, the lower the overhead per unit of production. Plant-overhead costs can vary between 15 and 150 percent of the operating-labor cost. Packaging costs depend on the physical and chemical nature of the product as well as on its use and value. The cost of packaging is as high as one-third of the selling price for soaps and pharmaceuticals.

**Rapid Manufacturing-Cost Estimates** Fixed manufacturing costs are a function of the fixed-capital investment and are independent of the production rate of the plant. Property taxes or rates depend on location. They may be taken as 2 percent of the fixed-capital cost of the plant in the absence of specific data. The cost of insurance depends on both location and the hazardous nature of the materials handled. This cost is normally of the order of 1 percent of the fixed-capital cost of the plant.

The manufacturing cost of a product is the sum of the processing or conversion cost and the cost of raw materials. The processing cost can be roughly broken down into three parts: investment-related cost, labor-related cost, and utility cost.

Companies usually include in the charge for overhead the following items: operating supplies, supervision, indirect payroll expenses, plant protection, plant office, general plant overhead, and control laboratory. This overhead charge is frequently taken as an equivalent percentage of the direct labor cost.

The percentage is best obtained from company records. Although it can vary over a wide range, a reasonable value is 125 percent. In this case, the labor-related cost would be 2.25 times the direct labor cost. For a 6000-h year and  $N$  persons per shift earning  $c_L$  \$ per hour, the annual labor-related cost would be  $13,500(c_L N)$ .

Let us consider a plant of fixed-capital cost  $C_{FC}$ . If the annual property taxes are taken as 0.02  $C_{FC}$ , insurance as 0.01  $C_{FC}$ , and maintenance as 0.06  $C_{FC}$ , the annual investment-related cost would be 0.09  $C_{FC}$ . Annual utilities cost is  $A_U$ . The annual processing cost  $A_p$  can be represented by

$$A_p = \eta_1 C_{FC} + \eta_2 c_L N + A_U \tag{9-205}$$

where the factors  $\eta_1$  and  $\eta_2$  can be obtained from the data available in a particular company and have the dimensions of year<sup>-1</sup> and hours per year respectively.

Substituting the information previously given into Eq. (9-205) yields the relationship

$$A_p = 0.09 C_{FC} + 13,500 c_L N + A_U \tag{9-206}$$

Equation (9-206) represents very closely the manufacturing costs of a particular company and is typical of the coefficients to be expected.

The annual processing cost  $A_{p2}$  for a similar plant of a different size designed for an annual production rate  $R_2$  can be approximately calculated from an equation of the form

$$A_{p2} = \eta_1 C_{FC1} (R_2/R_1)^{0.7} + \eta_2 c_L N_1 (R_2/R_1)^{0.25} + A_{U1} (R_2/R_1) \tag{9-207}$$

(F. A. Holland, F. A. Watson, and J. K. Wilkinson, *Introduction to Process Economics*, 2d ed. Wiley, London and New York, 1983, p. 158).

The processing cost per unit of production for a plant with an annual production  $R_2$  can be approximately calculated from

$$\frac{A_{p2}}{R_2} = \frac{100}{R_2} \left[ \eta_1 C_{FC} \left( \frac{R_2}{R_1} \right)^{0.7} + \eta_2 c_L N_1 \left( \frac{R_2}{R_1} \right)^{0.25} + A_{U1} \left( \frac{R_2}{R_1} \right) \right] \tag{9-208}$$

where  $A_{p2}/R_2$  is in cents per kilogram.

Equation (9-208) can be used to compute data for plots such as Fig. 9-40, which shows the decrease in processing cost per unit of production  $A_{p2}/R_2$  with increasing plant size.

**Manufacturing Cost as a Basis for Product Pricing** Pricing on the basis of cost plus a fair profit has the disadvantage of ignoring demand. The modern approach is to price on the basis of market research. However, the classic cost-plus-fair-profit approach can still give useful complementary information. This can be done by any of the following three methods:

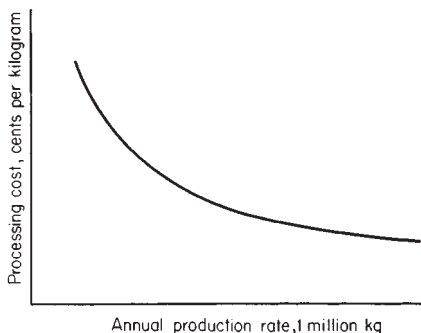


FIG. 9-40 Decrease in processing cost against increase in plant size.

1. Absorption pricing
2. Rate-of-return pricing
3. Marginal pricing

The gross annual profit  $A_{CP}$  for a product is given by

$$A_{CP} = A_S - A_{ME} - A_{BD} \tag{9-6}$$

where  $A_S$  is the revenue from annual sales,  $A_{ME}$  the annual manufacturing cost, and  $A_{BD}$  the balance-sheet annual depreciation charge.

Equation (9-6) can also be rewritten in the form

$$A_S = A_{CP} + A_{VME} + A_{FME} + A_{BD} \tag{9-209}$$

where  $A_{VME}$  and  $A_{FME}$  are the annual variable and fixed manufacturing costs or expenses respectively.

Equation (9-209) can also be rewritten as

$$c_s = c_{VME} + (A_{CP}/R) + (A_{FME} + A_{BD})/R \tag{9-210}$$

where  $R$  is the annual sales volume taken as equal to the annual production rate,  $c_s$  is the sales price per unit of production, and  $c_{VME}$  is the variable manufacturing expense per unit of production.

Absorption pricing is based on a normal annual production rate  $R$ . The gross profit per unit  $A_{CP}/R$  is taken as a fixed percentage  $\chi$  of the fixed plus variable manufacturing costs given by the equation

$$\frac{A_{CP}}{R} = \left( \frac{\chi}{100} \right) \left[ c_{VME} + \left( \frac{A_{FME} + A_{BD}}{R} \right) \right] \tag{9-211}$$

We combine Eqs. (9-210) and (9-211) to give

$$c_s = \left( \frac{100 + \chi}{100} \right) \left[ c_{VME} + \left( \frac{A_{FME} + A_{BD}}{R} \right) \right] \tag{9-212}$$

Equations (9-210), (9-211), and (9-212) are based on a fixed normal annual production rate  $R$ .

Let us consider a change in annual production rate to  $R + \Delta R$ . In order to maintain the gross profit per unit as  $A_{CP}/R$ , the sales price per unit of production would need to be  $c_s - \Delta c_s$ . For this case Eq. (9-210) can be written in the modified form

$$c_s - \Delta c_s = c_{VME} + (A_{CP}/R) + [(A_{FME} + A_{BD})/(R + \Delta R)] \tag{9-213}$$

We subtract Eq. (9-213) from Eq. (9-212) to give

$$\Delta c_s = \frac{\Delta R}{R} \left( \frac{A_{FME} + A_{BD}}{R + \Delta R} \right) \tag{9-214}$$

Equation (9-214) gives the overpricing  $\Delta c_s$  per unit of production for an increase in annual production rate  $\Delta R$ . Equation (9-214) also gives the underpricing  $\Delta c_s$  per unit of production for a decrease in annual production rate  $\Delta R$ . In the first case the fixed costs or overheads are said to be overabsorbed and in the second case underabsorbed.

Absorption pricing is rigid and arbitrary and may result in business being turned away if the fixed sales price  $c_s$  cannot be obtained even though the business may give a useful contribution to fixed costs.

Rate-of-return pricing is a modified form of absorption pricing. It is based on the equation

$$\frac{A_{CP}}{A_{ME}} = \left( \frac{C_{TC}}{A_{ME}} \right) \left( \frac{A_{CP}}{C_{TC}} \right) \tag{9-215}$$

where  $C_{TC}$  is the total capital employed. Equation (9-215) can also be written as

$$\text{Percentage markup on cost} = (\text{capital-turnover ratio})(\text{projected rate of return on capital})$$

The percentage markup on cost is calculated for a known capital-turnover ratio and a desired rate of return on capital. As with absorption pricing, the percentage markup on manufacturing cost per unit of production is calculated for a normal annual production rate. If this production rate is exceeded, the rate of return on capital will be higher than projected because of the decrease in unit cost. Conversely, if the production rate is lower than normal, the rate of return on capital will be lower than projected because of the increase in unit cost. For production rates both higher and lower than the normal production rate, the percentage markup is based on the normal unit cost. Thus the method is strictly valid only for the normal production rate.

With marginal pricing a company chooses its selling prices so as to maximize the total contribution  $\sum R(c_s - c_{VME})$  from its various products. The method is particularly useful for large multiproduct companies with extensive existing facilities since it is the marginal cost that must be considered as the base case when entering into competition with another company. The lowest acceptable price for a product is that which gives the lowest worthwhile contribution to fixed costs. Marginal pricing enables a company to develop a more aggressive pricing policy than when using absorption or rate-of-return pricing.

Absorption, rate-of-return, and marginal pricing have been considered here on the basis of manufacturing cost. Total cost, which is the sum of manufacturing and general costs, can also be considered as the basis. In this case the appropriate profit to consider is the net annual profit rather than the gross annual profit.

**Standard Costs for Budgetary Control** For convenience and simplicity, we shall consider the total cost of a manufactured product to be the sum of the material, labor, and overhead costs. Standard costs are those that have been predetermined and budgeted for the manufacture of a given amount of product in a given time. The deviation of the actual cost from the standard cost is called the variance. It is far easier to make comparisons between periods by using variances than by using actual production data. The different variances for material, labor, and overhead costs are listed in Table 9-38.

Standard costing is extensively used in budgetary-control systems. Criteria for the establishment of standards range from the maximum possible under ideal conditions to those expected under normal conditions. Past or historical costs are not always the best basis for setting up standards because past performance may have been unnecessarily inefficient.

**Static and Flexible Budgets** Overhead cost can significantly affect the profitability of a project and is the only cost outside the control of the project manager. The project is expected to contribute a definite amount toward the expenses of the company and will be charged this amount even if the production rate is zero. This is the fixed component of the overhead cost and will include directly allocable costs such as depreciation and a proportion of general costs such as office salaries and heating.

Other nonproduction costs such as indirect labor may vary linearly with the production rate and represent the variable component of the overhead. Costs that are neither fixed nor variable but occur in dis-

TABLE 9-39 Flexible Budget for Overhead Costs

Overhead cost, \$/month	Production, 1 million lb/month				
	8	9	10	11	12
Fixed	40,000	40,000	40,000	40,000	40,000
Variable	40,000	45,000	50,000	55,000	60,000
Semivariable	40,000	40,000	60,000	60,000	90,000
Total	120,000	125,000	150,000	155,000	190,000

crete steps at various production levels (such as supervisory labor) are the semivariable component of the overhead cost. It is an easy matter to determine these various components for various production rates and list them as shown in Table 9-39.

Two types of overhead budget are currently in use. The static (often referred to as the fixed) budgeted overhead cost is related to the standard budgeted production rate. The flexible budgeted overhead cost is that shown as the total cost in Table 9-39. Values for intermediate production rates are often obtained by interpolation. This is justifiable only when semivariable costs are a negligible part of overhead costs.

Flexible budgeting is more widely used than static budgeting despite certain logical difficulties. This is so because production in many cases is seasonal and the use of a static production norm might distort evaluation of performance. Variances are the difference between the actual costs expended and the budgeted costs expected. Variances are unfavorable if positive and favorable if negative. Any variance should be explained and, if necessary, controlled; the largest variance should be considered first.

Let us consider the overhead-cost data for Table 9-39 with 10 million kg per month as the standard production rate. The static budgeted overhead is then \$150,000 per month, or 1.5 cents per kilogram. We assume that the actual overhead is \$186,000 for a month in which 12 million kg was produced. Then, the static budgeted overhead cost would be 12 million(1.5), or \$180,000 per month. Therefore, the variance is \$186,000 - \$180,000 = +\$6000, which is unfavorable because \$6000 more was spent than was anticipated.

From Table 9-39 we find that the flexible budgeted overhead cost for a production rate of 12 million kg per month is \$190,000. The corresponding variance is \$186,000 minus \$190,000, or -\$4,000, which is favorable because \$4,000 less was spent than was anticipated. Thus, the use of flexible budgeting makes this particular performance look better without changing either the production rate or a single cost of the planned budget.

**The Standard Hour** The standard hour can be defined as the number of units of output expected to be produced in 1 h. It is often used as a measure of output rate by cost accountants.

Let us consider a batch processing unit that can produce either 1000 kg of product A in a cycle time of 5 h or 900 kg of product B in a cycle time of 3 h. Thus, for this processing unit a standard hour is 200 kg of product A or 300 kg of product B. In a budget period of, say, 1000 h, it is possible to produce 200,000 kg of product A, or 300,000 kg of product B, or any appropriate combination of the two products.

For example, let us assume that production requirements are twice as great for product A as for product B, i.e., a ratio of 600 kg, or 3 standard hours, of product A to 300 kg, or 1 standard hour, of product B. On this basis, for a budget period of 1000 h, 750 standard hours [(3/4)1000] would be used to produce 750(200) = 150,000 kg of product A, and 250 standard hours [(1/4)1,000] would be used to produce 250(300) = 75,000 kg of product B.

Production efficiency  $P_e$  can be calculated from

$$P_e = (P_s/P_a)100 \tag{9-216}$$

where  $P_s$  is the actual production rate in standard hours and  $P_a$  is the actual hours worked.

The level of production activity  $P_\ell$  can be calculated from

$$P_\ell = (P_s/P_b)100 \tag{9-217}$$

where  $P_b$  is the budgeted production in standard hours.

The deviation from budgeted capacity  $b_c$  can be calculated from

$$b_c = (P_w/P_w)100 \tag{9-218}$$

where  $P_w$  is the budgeted number of working hours.

TABLE 9-38 Variances

Revenue from sales $Rc_s - R^*c_s^*$	Sales price $R(c_s - c_s^*)$	Sales volume $c_s^*(R - R^*)$	
Profit $Rc_{NP} - R^*c_{NP}^*$	Unit profit $R(c_{NP} - c_{NP}^*)$	Quantity $c_{NP}^*(R - R^*)$	
Total cost $Rc_{TE} - R^*c_{TE}^*$	Unit cost $R(c_{TE} - c_{TE}^*)$	Production rate $c_{TE}^*(R - R^*)$	
Direct material cost $Rc_{RM} - R^*c_{RM}^*$	Material price $R(c_{RM} - c_{RM}^*)$	Material usage $c_{RM}^*(R - R^*)$	
Direct labor cost $\theta c_L - \theta^*c_L^*$	Wage rate $\theta(c_L - c_L^*)$	Labor efficiency $c_L^*(\theta - \theta^*)$	
Overhead cost $C_{OH} - \theta^*c_{NOH}^*$	Budgeted cost $(C_{OH} - \theta c_{BOH})$	Volume $\theta(c_{BOH} - c_{NOH}^*)$	Efficiency $c_{NOH}^*(\theta - \theta^*)$

$c_{BOH}$  = Flexible budgeted overhead cost, \$/h.

$c_L$  = Actual labor cost, \$/h.

$c_{NOH}$  = Standard overhead cost based on normal production rate, \$/h.

$c_{NP}$  = Actual net profit before tax, \$/unit.

$C_{OH}$  = Actual overhead cost, \$/period.

$c_{RM}$  = Actual raw-materials cost, \$/unit.

$c_s$  = Actual selling price, \$/unit.

$c_{TE}$  = Actual total cost, \$/unit.

$R$  = Actual quantity, units/period.

$R^*$  = Standard quantity, units/period.

$\theta$  = Actual time to produce a given quantity, h.

$\theta^*$  = Standard time to produce a given quantity, h.

$c_{NP} = c_s - c_{TE}$ .

NOTE: The asterisk on all items not otherwise defined indicates the standard cost for that item.



TABLE 9-40 Sales, Profits, and Manufacturing Costs

Component	Actual, \$/period	Standard, \$/period	Variance, \$/period
Direct materials cost	325,080	350,000	-24,920
Direct labor cost	55,001	51,600	+3,401
Overhead cost	75,000	68,370	+6,630
Manufacturing cost	455,081	469,970	-14,889
Revenue from sales	588,240	570,000	+18,240
Gross profit	133,159	100,030	+33,129

**Actual Costs versus Standard Costs** Let us consider the sales, profits, and manufacturing-cost data in Table 9-40. The gross profit is \$33,129 per period better than expected. Clearly, there is less incentive to investigate overall costs when the profit variance is favorable than if the profit were less than expected. However, standard costing enables an objective analysis of the data, whether good or bad, to be made.

The individual variances in Table 9-40 show that the increased profit is due to reduced material costs, which affect manufacturing costs to a greater extent than increased labor and overhead costs. The individual variances also show that to an even greater extent the increased profit is due to the increase in sales revenue.

Clearly, management will wish to investigate both labor and overhead costs for any inefficiencies and to ascertain the reasons for the improved sales revenue. If necessary, the standard values can be revised.

We notice that profit is obtained as the difference between two large cash sums and that variances of some 3 percent in manufacturing costs and sales revenue have resulted in a variance of some 33 percent in gross profit.

Table 9-40 is a very simplified presentation. In a full standard costing system, the direct material, direct labor, and overhead variances are broken down into component parts to enable an even closer look at the operation. Standard costing is an invaluable aid to management for controlling a business.

**Variances: Direct Material Cost** Since the variance in direct material cost  $\Delta(RC_{RM})$  is the difference between actual cost and standard cost,

$$\Delta(RC_{RM}) = RC_{RM} - R^{\circ}c_{RM}^{\circ} \quad (9-219)$$

where  $R$  is the actual quantity and  $R^{\circ}$  is the standard quantity, units per period,  $c_{RM}$  is the actual price, and  $c_{RM}^{\circ}$  is the standard price, \$ per unit.

Equation (9-219) can be written in an expanded form:

$$\Delta(RC_{RM}) = R(c_{RM} - c_{RM}^{\circ}) + c_{RM}^{\circ}(R - R^{\circ}) \quad (9-220)$$

where  $R(c_{RM} - c_{RM}^{\circ})$ , known as the direct-material-price variance, is the actual quantity multiplied by the deviation in unit price, and  $c_{RM}^{\circ}(R - R^{\circ})$ , known as the direct-material-usage variance, is the standard unit price multiplied by the deviation in quantity.

By using the data of Table 9-41, let us calculate the direct materials cost and the standard direct materials cost as

$$RC_{RM} = 1,806,000(0.18) = \$325,080/\text{period}$$

$$R^{\circ}c_{RM}^{\circ} = 1,750,000(0.20) = \$350,000/\text{period}$$

From Eq. (9-219) we calculate the direct-material-cost variance as  $-\$24,920$  per period. This variance is favorable. However, by using the relations of Eq. (9-220) we calculate a direct-material-price variance

TABLE 9-41 Cost Data for Problems

Factor	Actual value	Standard value
Raw-materials cost, \$/unit	$c_{RM} = 0.18$	$c_{RM}^{\circ} = 0.20$
Direct labor cost, \$/unit	$c_L = 8.45$	$c_L^{\circ} = 8.00$
Production rate, units/period	$R = 1,806,000$	$R^{\circ} = 1,750,000$
Production time h/period	$\theta = 6509$	$\theta_N^{\circ} = 6250$
Fixed and semivariable overhead cost, \$/period		$C_{FOH}^{\circ} = 60,000$
Variable overhead cost, \$/h		$C_{VOH}^{\circ} = 1.00$
Overhead cost, \$/period	$C_{OH} = 75,000$	

of  $(1,806,000)(0.18 - 0.20) = -\$36,120$  per period. This variance is favorable. Likewise, we calculate a direct-material-usage variance of  $(0.20)(1,806,000 - 1,750,000) = \$11,200$  per period. This variance is unfavorable.

In this case, the favorable direct-material-cost variance was achieved because of a lower unit price despite an inefficient material usage, which needs to be investigated. (There is no room for complacency since the lower unit price may well be temporary.)

In the case of mixtures of raw materials, the direct-material-usage variance can be further subdivided into (1) a direct-material-mixture variance and (2) a direct-material-yield variance. The former is due to the difference between the actual and standard mixture compositions, and the latter to the difference between the actual and standard yields. Here, the standard yield is the output expected from the standard input of material. The yield variance denotes the extent of loss of material. The direct-material-mixture variance can be illustrated by Example 22.

**Example 22: Direct-Material-Mixture Variance** A standard mixture of 100 units of material contains 70 percent of material A at \$0.08 per unit and 30 percent of material B at \$0.12 per unit. The standard mixture cost is  $70(0.08) + 30(0.12) = \$9.20$ .

Now let us consider a mixture of 100 units containing 75 percent of material A and 25 percent of material B. The cost of this mixture at standard prices is  $75(0.08) + 25(0.12) = \$9.00$ . The direct-material-mixture variance is  $\$9.00 - \$9.20 = -\$0.20$  and is favorable. The favorable variance has been brought about by using more of the lower-priced material A and less of the higher-priced material B.

The direct-material-yield variance is illustrated as follows. Let us assume that the standard mixture (cost \$9.20 for 100 units) has a standard loss of 20 percent, making the cost \$9.20 for 80 units, or \$0.115 per unit of output. Now let us consider the actual loss to be 30 percent, leaving 70 units of output for each 100 units of input. The direct-material-yield variance is  $0.115(80 - 70) = \$1.15$  and is unfavorable.

**Variances: Direct Labor Cost** Since the variance in direct labor cost  $\Delta(\theta c_L)$  is the difference between actual cost and standard cost,

$$\Delta(\theta c_L) = \theta c_L - \theta^{\circ} c_L^{\circ} \quad (9-221)$$

where  $c_L$  is the actual pay or wage rate, \$ per hour;  $c_L^{\circ}$  is the standard pay or wage rate, \$ per hour;  $\theta$  is the actual time taken to produce a given quantity of product in a given period, hours; and  $\theta^{\circ}$  is the standard time taken to produce a given quantity of product in a given period, hours.

Equation (9-221) can also be written in expanded form:

$$\Delta(\theta c_L) = \theta(c_L - c_L^{\circ}) + c_L^{\circ}(\theta - \theta^{\circ}) \quad (9-222)$$

where  $\theta(c_L - c_L^{\circ})$ , known as the direct pay, or wage-rate, variance, is the actual time taken to produce a given output multiplied by the deviation in wage rate, and  $c_L^{\circ}(\theta - \theta^{\circ})$ , known as the direct-labor-efficiency variance, is the standard wage rate multiplied by the deviation in time taken to produce a given output.

By using the data of Table 9-41, we calculate that 1 standard hour corresponds to

$$(1,750,000/6250) = 280 \text{ units/standard hour}$$

Standard time to actual production is

$$\theta^{\circ} = (1,806,000/280) = 6450 \text{ standard hours/period}$$

Direct labor cost is

$$\theta c_L = 6509(8.45) = \$55,001/\text{period}$$

Standard direct labor cost is

$$\theta^{\circ} c_L^{\circ} = 6450(8.00) = \$51,600/\text{period}$$

From Eq. (9-219) we calculate the direct-labor-cost variance as \$3401 per period. This variance is unfavorable. However, by using the relations of Eq. (9-220), we calculate a direct pay, or wage-rate, variance of  $\theta(c_L - c_L^{\circ}) = 6509(8.45 - 8.00) = \$2929$  per period. This direct pay variance is unfavorable. Likewise, we calculate the direct-labor-efficiency variance as  $c_L^{\circ}(\theta - \theta^{\circ}) = 8.00(6509 - 6450) = \$472$  per period. This variance is also unfavorable.



For this example, the adverse direct-labor cost variance of \$3401 is due to both a higher wage rate per hour and a higher number of labor-hours.

The direct-labor-cost variance can, if necessary, be broken down into a direct-labor-idle-time variance in addition to the direct-wage-rate and direct-labor-efficiency variances. The direct-labor-idle-time variance is simply the number of idle labor-hours in the period multiplied by the standard wage rate. This is rarely relevant to the conditions existing in process plants except when maintenance is involved.

**Variances: Overhead Cost** The variance in overhead cost  $\Delta C_{OH}$  is the difference between actual overhead cost and static standard overhead cost,

$$\Delta C_{OH} = C_{OH} - \theta^{\circ} c_{NOH}^{\circ} \quad (9-223)$$

where  $C_{OH}$  is the actual overhead cost incurred in a given period, \$ per period;  $c_{NOH}^{\circ}$  is the static standard overhead cost based on the normal production rate, \$ per hour;  $\theta$  is the actual time taken to produce a given quantity of product in a given period; and  $\theta^{\circ}$  is the standard time taken to produce a given quantity of product in a given period, hours.

Equation (9-223) can also be written in expanded form

$$\Delta C_{OH} = (C_{OH} - \theta c_{BOH}) + \theta(c_{BOH} - c_{NOH}^{\circ}) + c_{NOH}^{\circ}(\theta - \theta^{\circ}) \quad (9-224)$$

where  $c_{BOH}$  is the flexible budgeted overhead cost at the actual production rate or operating capacity.

In Eq. (9-223),  $(C_{OH} - \theta c_{BOH})$  is known as the budgeted overhead-cost variance,  $\theta(c_{BOH} - c_{NOH}^{\circ})$  as the overhead-volume variance, and  $c_{NOH}^{\circ}(\theta - \theta^{\circ})$  as the overhead-efficiency variance. The last is analogous to the labor-efficiency variance and is the standard overhead rate multiplied by the deviation in time taken to produce a given output.

Also in Eq. (9-224),  $c_{BOH}$  is simply the flexible budgeted overhead cost in dollars per hour for the actual production rate, and the overhead-volume variance  $\theta(c_{BOH} - c_{NOH}^{\circ})$  is the actual time taken to produce a given output multiplied by the difference between the flexible budgeted overhead cost and the standard overhead cost in dollars per hour. The budgeted overhead-cost variance  $(C_{OH} - \theta c_{BOH})$  is the difference between the actual overhead cost and the actual time (in hours) required to produce the given output multiplied by the flexible budgeted overhead cost (in dollars per hour).

We shall write the fixed overhead cost for the budget period as  $C_{FOH}^{\circ}$ , the semivariable overhead cost as  $C_{SVOH}^{\circ}$ , and the standard hours to produce the agreed normal production as  $\theta_N^{\circ}$ . The standard overhead cost at the agreed normal production rate can then be calculated from

$$c_{NOH}^{\circ} = [(C_{FOH}^{\circ} + C_{SVOH}^{\circ})/\theta_N^{\circ}] + c_{VOH}^{\circ} \quad (9-225)$$

where  $c_{VOH}^{\circ}$  is the standard variable overhead cost, \$ per hour.

For production rates that differ from the agreed normal rate, the flexible budgeted overhead cost is given by

$$c_{BOH} = [(C_{FOH}^{\circ} + C_{SVOH}^{\circ})/\theta] + c_{VOH}^{\circ} \quad (9-226)$$

where  $\theta$  is the actual hours taken to produce a given amount of product.

For production rates lower than normal, the fixed overheads are underused, and the flexible budgeted overhead cost  $c_{BOH}$  is greater than the standard overhead cost  $c_{NOH}^{\circ}$ . For production rates higher than normal,  $c_{BOH}$  is less than  $c_{NOH}^{\circ}$ .

It is common practice in cost accountancy to treat the standard semivariable cost  $C_{SVOH}^{\circ}$  at the normal production rate as part of the standard fixed cost. In this case, Eqs. (9-225) and (9-226) can be written respectively as

$$c_{NOH}^{\circ} = (C_{FOH}^{\circ}/\theta_N^{\circ}) + c_{VOH}^{\circ} \quad (9-227)$$

$$c_{BOH} = (C_{FOH}^{\circ}/\theta) + c_{VOH}^{\circ} \quad (9-228)$$

By using the data of Table 9-40 in Eq. (9-227), we calculate the standard overhead cost to be

$$c_{NOH}^{\circ} = (60,000/6250) + 1.00 = \$10.60/h$$

From Eq. (9-228) and the data in Table 9-40 we calculate the flexible budgeted overhead cost to be

$$c_{BOH} = (60,000/6509) + 1.00 = \$10.22/h$$

By substituting into Eq. (9-223), we calculate the overhead cost variance:

$$\Delta C_{OH} = 75,000 - 6450(10.60) = \$6630$$

This variance is unfavorable. (Note that standard time for actual production was previously calculated to be 6450 h per period.)

The overhead cost variance comprises (1) budgeted overhead-cost variance, (2) overhead-volume variance, and (3) overhead-efficiency variance. The calculations for each follow:

$$c_{OH} - \theta c_{BOH} = 75,000 - 6509(10.22) = \$8478$$

Budgeted overhead-cost variance is positive and, therefore, unfavorable.

$$\theta(c_{BOH} - c_{NOH}^{\circ}) = 6509(10.22 - 10.60) = -\$2473$$

Overhead volume variance is negative and favorable.

$$c_{NOH}^{\circ}(\theta - \theta^{\circ}) = 10.60(6509 - 6450) = \$625$$

Overhead efficiency variance is positive and unfavorable.

The total variances in each category are listed in Table 9-40.

Chemical engineers usually make detailed evaluations of costs rather than evaluations for profits or sales. However, the latter can be analyzed in a similar manner to costs by using the equations shown in Table 9-38. For this purpose, the sign convention will be reversed because an increase in sales or profits would be considered favorable, whereas an increase in cost would be considered unfavorable. The equations can be applied to both batch and continuous processes.

Budgets can be used for both forward planning and control. Variances show managers what their costs should have been and how near they came to meeting budgeted values. Managers will be able to assess, over a number of budget periods, the rate of improvement in performance in their areas of responsibility. A good budgetary system not only should provide detailed information and an appraisal of performance but also should motivate people to improve performance.

**Contribution Analysis** Contribution analysis can be used to make rapid assessments of the effect of changes in manufacturing costs on profitability. A dimensionless contribution efficiency  $\eta$  can be defined by rewriting Eq. (9-12) in the form

$$\eta = \frac{R(c_S - c_{VE}) - A_{FE}}{R(c_S - c_{VE})} \quad (9-229)$$

[F. A. Holland and F. A. Watson, *Eng. Process Econ.*, **1**, 135-143 (1976)].

This represents the ratio of the net annual profit  $A_{NP}$  actually achieved divided by the profit which could be obtained if no repayment of capital or interest were required and all fixed-expense items were credited free to the project. The contribution efficiency  $\eta$  is also the profit per unit of contribution. A value for  $\eta$  of unity would be obtained for a very high production rate  $R$  whether  $c_S$  is greater or less than  $c_{VE}$ . For the unusual case of  $c_S$  being equal to  $c_{VE}$ , the value of  $\eta$  would become negatively infinite for a finite annual fixed expense  $A_{FE}$  or positively infinite if  $A_{FE}$  became negative because of excessive subsidy of expenses. However, for most projects which are intended to pay their own expenses and taxes,  $A_{NP}$  must be positive, and hence  $c_S$  is usually greater than  $c_{VE}$ , so that  $\eta$  will normally have values in the range of zero to unity. For projects which are not intended to make a profit but are provided for their social or amenity value, the aim should be to bring the value of  $\eta$  as near to zero as possible.

The breakeven production rate  $R_B$  is defined by Eq. (9-13) as the production rate at which the project makes neither a profit nor a loss. Equation (9-13) and (9-229) can be combined to give

$$\eta = (R - R_B)/R \quad (9-230)$$

which shows that the contribution efficiency  $\eta$  is a function of the production rate  $R$  and that  $\eta$  has the value zero when the production rate is the breakeven production rate  $R_B$ . For all real projects  $R_B$  will be positively finite while  $R$  cannot be less than zero, and hence the practical range of  $\eta$  is from negative infinity to unity.

At first glance it might appear that it is desirable to have a value of  $\eta$  as near to unity as possible. However, this is not necessarily so. Reference to Eq. (9-229) will show that if the unit contribution  $(c_S - c_{VE})$

is positive, as it must be if the project is ever to make a profit, a value of unity for the contribution efficiency  $\eta$  implies a negligible value of the annual fixed expense  $A_{FE}$  when compared with the annual contribution  $R(c_s - c_{VE})$ . In such cases either  $R(c_s - c_{VE})$  is very large, thus attracting competition in spite of high capital charges, or the fixed expenses are very low, so that it is easy for many small competitors to enter the market. The result of such competition often leads to a rapid reduction in sales price  $c_s$ . Ball-point pens and electronic calculators were both drastically reduced in price as a result of competitors entering the market.

Since the variable expense per unit of production  $c_{VE}$  is by definition independent of production rate, the unit contribution  $(c_s - c_{VE})$  and hence the value of  $\eta$  will be reduced. On the other hand, a very large company may be in a stronger and more stable position with a modest contribution efficiency and relatively high fixed costs which will deter competitors from entering the market and thereby depressing the sales price. In this argument it is implicit that  $c_s$  is also independent of output from the project under consideration. In many cases this will be the case since if many small buyers can choose from many alternative producers, the individual producer cannot adjust the price to suit its output, while at the opposite extreme a group of producers that are in a position to make such adjustments are also likely to attract the attention of antitrust legislation.

It is of interest to be able to examine the effect of changes in productivity on the profitability of projects. Historically, labor costs have been regarded as variable costs, implying that if workers doubled their output their net wages also doubled. This may have been the case for some piecework rates, but it is generally not true today. It is not normally possible to reduce the work force in step with falling demand or to recruit and train labor in step with increasing demand. In general it is better to consider labor as a fixed cost, with any part of a production bonus which is truly proportional to output included in the variable expense  $c_{VE}$ . If the annual fixed expense  $A_{FE}$  varies significantly with production rate  $R$  owing to this factor, then the breakeven chart will consist of curves, the simplicity of the method is lost, and it will be assumed that a particular change in productivity agreements implies a step change in  $A_{FE}$  and/or in  $c_{VE}$ . Let us consider an increase in productivity for the same fixed labor cost, with other fixed costs remaining the same. We shall let the original production rate  $R$  be increased by an increase in productivity by a fraction  $\phi$ . Therefore,

$$\Delta R = R\phi \quad (9-231)$$

where  $\Delta R$  is the increase in sales volume or production rate. By substitution into Eq. (9-229), the resulting increase in profit  $\Delta A_{NP}$  is given by

$$A_{NP} + \Delta A_{NP} = A_s + \Delta A_s - (A_{VE} + \Delta A_{VE}) - (A_{FE} + 0) \quad (9-232)$$

We shall subtract  $A_{NP}$  from Eq. (9-232) to give

$$\Delta A_{NP} = \Delta A_s - \Delta A_{VE} = \Delta R(c_s - c_{VE}) \quad (9-233)$$

It follows from Eqs. (9-12), (9-229), and (9-233) that

$$\Delta A_{NP}/A_{NP} = \phi/\eta \quad (9-234)$$

Thus a change of productivity  $\phi$  of 10 percent will result in a 10 percent increase in profit when  $\eta = 1$  and a very large increase in profit when  $\eta$  is close to zero. If  $\eta$  is negative, increased productivity reduces the profit (or increases the loss).

Equation (9-234) illustrates the enormous influence that go-slow tactics can have on the profitability of companies and processes which have low contribution efficiencies, since a slowdown has little effect on  $A_{FE}$ . It is sometimes the case that in different countries productivity per worker varies considerably in similar industries. When poor productivity is not the result of technical or capital inadequacy, it should be possible to increase profitability without a proportionate increase in  $A_{FE}$ .

Breakeven charts present a snapshot of the present situation by means of graphs which are generally drawn in the manner shown in Figs. 9-2, 9-3, and 9-4. Since the lines are straight, this implies that  $c_s$ ,  $c_{VE}$ , and  $A_{FE}$  will remain constant over the range of variation of  $R$ , which is of interest. The values would be based on the production rate currently achieved (or scheduled), since all the data are available from

the financial analysis of current production, so that  $c_s$  would be  $(A_s)_0/R_0$ , and so on.

Two well-known ratios used by financial analysts are the profit-to-sales ratio (PSR) and the contribution-to-sales ratio (CSR). The profit-to-sales ratio is simply Eq. (9-127) for profit margin (PM) rewritten as the ratio

$$(\text{PSR}) = A_{NP}/Rc_s \quad (9-235)$$

The contribution-to-sales ratio

$$(\text{CSR}) = (A_s - A_{VE})/A_s = (c_s - c_{VE})/c_s \quad (9-236)$$

We substitute these values into Eq. (9-229) to give

$$\eta = (\text{PSR})/(\text{CSR}) \quad (9-237)$$

The contribution efficiency at the scheduled output  $\eta_0$  is given by substituting the value of the scheduled output into Eq. (9-229) to give

$$\eta_0 = \frac{R_0(c_s - c_{VE}) - A_{FE}}{R_0(c_s - c_{VE})}$$

The characteristic shape of a given breakeven chart of the type represented by Figs. (9-2), (9-3), and (9-4) can be defined by the two ratios (CSR) and  $\eta_0$ , while the scale of the project can be defined by a single annual cost such as  $A_{FE}$ . This information may be used for the rapid investigation of the likely effect on current profits obtainable by changes in various factors such as prices, expenses, and throughput. It should be noted that this technique is not intended to replace discounted methods of investment appraisal but to provide a rapid assessment of the probable effect of changes in current conditions. If the current profitability is always maximized, then the discounted-cash-flow present value will always be made as great as conditions in a changing world will permit.

**Valuation of Recycled Heat Energy** The rising cost of energy is having an inflationary effect on manufacturing costs. One obvious way to reduce energy costs is to recycle heat energy whenever possible [S. A. K. El-Meniawy, F. A. Watson, and F. A. Holland, *Indian Chem. Eng.*, **22** (July–September 1980)].

Heat pumps are particularly suitable for recycling heat energy in the chemical-process industries. For the outlay of an additional fixed-capital expenditure  $C_{FC}$  on a heat-pump system, a considerable reduction in the annual heating cost can be effected.

Let us consider a process unit requiring heat at the rate of  $Q_D$  GJ/h operating for  $y$  h in a year. We shall let the unit cost of this base heating requirement be  $c_B$  \$ per gigajoule. Therefore the annual heating cost for this unit is  $Q_D y c_B$  \$ per year.

We then consider the use of a heat pump to supply this heat so that

$$Q_D = W(\text{COP})_A \quad (9-238)$$

where  $W$  is the rate of energy input to the compressor in gigajoules per hour and  $(\text{COP})_A$  is the actual coefficient of performance of the heat pump.

When interest charges are involved, the fixed-capital expenditure on a heat-pump system  $C_{FC}$  can be related to an annual cost  $A_{FC}$  for the estimated life of the heat pump in years by the equation

$$A_{FC} = C_{FC} f_{AP} \quad (9-239)$$

where  $f_{AP} = [i(1+i)^n]/[(1+i)^n - 1]$ , the annuity present-worth factor, and  $i$  is the fractional interest rate per year payable on the borrowed money.

A given value for  $A_{FC}$  enables a cost in, say, dollars per gigajoule, to be assigned to the heat energy made available by a heat pump.

We shall consider a heat-pump system which operates for  $y$  h/year and consumes  $W$  GJ/h of high-grade energy to drive the compressor. We shall let the unit cost of the input energy to the compressor be  $c_I$  \$ per gigajoule. The annual amount of heat delivered by the heat pump is  $Q_{DJ}$ , which in terms of Eq. (9-238) can be written as  $W(\text{COP})_{Ay}$  in gigajoules per year. The annual cost of this delivered heat, neglecting any maintenance cost, is  $(Wc_I y + A_{FC})$  in dollars per year. Therefore the unit cost  $c_D$  of the heat energy delivered by a compressor-driven pump is

$$c_D = (Wc_I y + A_{FC})/[W(\text{COP})_{Ay}] \text{ $/GJ} \quad (9-240)$$

The ratio of the unit costs of the delivered heat energy and the input energy can be obtained by combining Eqs. (9-239) and (9-240) to give

$$(c_D/c_I) = [(1 + \psi)/(COP)_A] \quad (9-241)$$

where  $\psi = (C_{FC}/W)(f_{AP}/y c_I)$ .  $\psi$  is a dimensionless parameter which contains cost and usage data for a particular heat pump; it should have as low a value as possible in order to minimize the unit cost of the delivered heat  $c_D$ . The cost  $C_{FC}/W$  is the fixed-capital cost per unit of input energy in  $\$ (\text{GJh}^{-1})^{-1}$ , and this should also be as low as possible consistent with a long life and good reliability for the heat pump. Clearly  $y$ , the number of operating hours per year, should approach as closely as possible to the maximum value of 8760 h for a 365-day year.

Since all costs refer to a given year, Eqs. (9-239), (9-240), and (9-241) are independent of inflation.

The annual cost of heat delivered by the heat pump is  $Q_D y c_D$ , where the unit cost  $c_D$  is given by Eq. (9-240). Therefore, the annual saving on heating costs in dollars per year is  $Q_D y (c_B - c_D)$ , which can also be written in terms of Eq. (9-238) as  $W(COP)_A y (c_B - c_D)$ .

The payback period in years for a heat-pump system is the additional fixed-capital cost  $C_{FC}$  divided by the annual saving on heating costs. This can be written as

$$(\text{PBP}) = \left( \frac{C_{FC}}{W} \right) \left[ \frac{1}{(COP)_A y (c_B - c_D)} \right] \quad (9-242)$$

which can also be written in terms of Eq. (9-241) as

$$(\text{PBP}) = \left( \frac{C_{FC}}{W} \right) \left\{ \frac{1}{y[(COP)_A c_B - c_I(1 + \psi)]} \right\} \quad (9-243)$$

For the special case of the unit cost of the input energy to the compressor being the same as the unit cost of the base heat supply, i.e.,  $c_B = c_I$ , Eq. (9-243) simplifies to

$$(\text{PBP}) = \left( \frac{C_{FC}}{W} \right) \frac{1}{y c_I [(COP)_A - (1 + \psi)]} \quad (9-244)$$

Equation (9-244) can be used to calculate the payback period when electricity, oil, or gas, etc., is used to drive the compressor and also to provide the base heating.

Equation (9-244) shows that, to have a low payback period (PBP),  $C_{FC}/W$  and  $\psi$  should be small and  $y$ ,  $(COP)_A$ , and  $c_I$  large. Clearly as the unit cost of input energy  $c_I$  increases, the economics of heat pumps becomes more favorable.

The value of  $\psi$  will for most cases be less than 0.2 and with the right application may well be less than 0.1. Values for the annuity present-worth factor will in most cases be less than 0.15.

Since 1 bbl (0.159 m<sup>3</sup>) of oil is normally quoted as having a thermal-energy value of 6.12 GJ, a world oil price of, say, US\$40 per barrel is equivalent to US\$6.54 per gigajoule.

For simplicity, we substitute  $c_I = \text{US}\$6\frac{2}{3}$  per gigajoule and a conservative value of  $[(COP)_A - (1 + \psi)] = 3$  into Eq. (9-244) to give

$$(\text{PBP}) = (C_{FC}/W)(1/20y) \quad (9-245)$$

where (PBP) is the payback period in years,  $y$  is the operating hours per year, and  $(C_{FC}/W)$  is the fixed-capital cost in US\$  $(\text{GJh}^{-1})^{-1}$  of primary-energy input.

Equation (9-245) shows that in this particular case the fixed-capital cost per unit of input energy  $(C_{FC}/W)$  must not exceed \$160,000  $(\text{GJh}^{-1})^{-1}$ , or \$576 per kilowatt, to have a 1-year payback period if the heat pump is operational for 8000 h/year. For this case the corresponding value of  $\psi$  is about 0.12 for a heat pump with an operating life of 10 years purchased with money borrowed at a 10 percent rate of interest.

Equation (9-245) also shows that the fixed-capital cost per unit of energy input  $(C_{FC}/W)$  must not exceed \$40,000  $(\text{GJh}^{-1})^{-1}$ , or \$144 per kilowatt, to have a 1-year payback period if the heat pump is operational for only 2000 h/year. For this case the corresponding value of  $\psi$  is also about 0.12 for a heat pump with an operating life of 10 years purchased with money borrowed at a 10 percent rate of interest.

## FIXED-CAPITAL-COST ESTIMATION

**Total Capital Cost** The installed cost of the fixed-capital investment  $C_{FC}$  is obviously an essential item which must be forecast before an investment decision can be made. It forms part of the total capital investment  $C_{TC}$ , defined by Eq. (9-14). The fixed-capital investment is usually regarded as the capital needed to provide all the depreciable facilities. It is sometimes divided into two classes by defining battery limits and auxiliary facilities for the project. The boundary for battery limits includes all manufacturing equipment but excludes administrative offices, storage areas, utilities, and other essential and nonessential auxiliary facilities.

**Cost Indices** The value of money will change because of inflation and deflation. Hence cost data can be accurate only at the time when they are obtained and soon go out of date. Data from cost records of equipment and projects purchased in the past may be converted to present-day values by means of a cost index. The present cost of the item is found by multiplying the historical cost by the ratio of the present cost index divided by the index applicable at the previous date. Ideally each cost item affected by inflation should be forecast separately. Labor costs, construction costs, raw-materials and energy prices, and product prices all change at different rates. Composite indices are derived by adding weighted fractions of the component indices. Most cost indices represent national averages, and local values may differ considerably.

Table 9-42 presents information on some cost indices for the United States. *Engineering News-Record* updates its construction-cost index in March, June, September, and December. The *Oil and Gas Journal* gives the Nelson-Farrar refinery indices in the first issue of each quarter. The *Chemical Engineering* plant-cost index and Marshall and Swift equipment-cost index are given in each issue of the publication *Chemical Engineering*. Derivation of the base values is referred to in the respective publications.

Table 9-43 is based on the method suggested by J. Cran [*Eng. Process Econ.*, **2**, 89-90 (1977)]. He showed that reasonably accurate plant-cost indices for various countries could be derived by using two component indices in the equation

$$(CI)_P = 0.327(CI)_{ST} + 0.673(CI)_L \quad (9-246)$$

where  $(CI)_{ST}$  is the steel-price index and  $(CI)_L$  the earnings index for labor in the particular country. Most of the data required can be obtained from the *United Nations Monthly Bulletin of Statistics* or the Organization for Economic Cooperation and Development (OECD) annual review of the iron and steel industry. In Table 9-43, the plant-cost indices have been brought to a common base of 1980 = 100. The values given do not relate costs in one country to those in another country, as this involves many complex and difficult problems. However, the table indicates the inflationary trends in plant costs since 1980 for each of the countries listed.

**Types and Accuracy of Estimates** Capital-cost estimates may be required for a variety of reasons, among others to enable feasibility studies to be carried out, to enable a manufacturing company to select from alternative investments, to assist in selection from alternative designs, to provide information for planning the appropriation of capital, and to enable a contractor to bid on a new project. It is therefore essential to achieve the greatest accuracy of estimation with a minimum expenditure of time and money.

Two simple rules are invaluable in aiding the production of consistently accurate estimates:

1. Check the completeness of the project scope.
2. Reduce the effect of bias by using statistically proven methods of estimation based on experience.

Estimates which are lower than actual project costs are often the result of sizable omissions of equipment, services, or auxiliary facilities

TABLE 9-42 Cost Indices for the United States

Year	Price indices*			Construction cost indices			
	Consumer	Producer	Industrial chemicals	Chemical equipment†	Process plants‡	Petroleum refinery§	General construction¶
1980	100	100	100	660	261	286	303
1981	110	108	113	721	297	315	330
1982	117	113	106	746	314	340	357
1983	121	114	103	761	317	357	380
1984	126	115	102	780	323	370	387
1985	131	115	100	790	325	374	392
1986	133	114	97	798	318	380	401
1987	138	117	102	814	324	391	412
1988	144	122	106	852	343	406	422
1989	151	127	108	895	355	417	429
1990	159	132	111	915	358	427	441
1991	165	132	111	931	361	436	452
1992	170	133	110	943	358	445	470
1993	175	136	111	967	359	453	489

\*Bureau of Labor Statistics, 1980 = 100.

†Marshall & Swift, *Chem. Eng.*, 1926 = 100.‡*Chem. Eng.*, plant index, 1957–1959 = 100.§Nelson-Farrar refinery index, *Oil Gas J.*, 1967 = 100.¶*Eng. News Rec.*, 1967 = 100.

rather than of errors in pricing or estimation methods. To avoid this, the use of a checklist of items involved in a new project as given in Table 9-44 can be invaluable.

The first stage toward producing an accurate estimate is to use a standard cost code for all construction projects. Table 9-45 shows a suitable numerical cost code, and Table 9-46 shows a typical alphabetical-numerical code. The cost-code system can be used throughout the estimating and construction stages for the collection of cost data by manual or computer methods. There are numerous types of fixed-capital-cost estimates, but in 1958 the American Association of Cost Engineers defined five types as follows:

1. *Order-of-magnitude estimate (ratio estimate)*. Rule-of-thumb method based on cost data for previous similar types of plant; probable error within 10 to 50 percent.

2. *Study estimate (factored estimate)*. Better than order-of-magnitude; requires knowledge of major items of equipment; used for feasibility surveys; probable error up to 30 percent.

3. *Preliminary estimate (budget-authorization estimate)*. Requires more detailed information than study estimate; probable error up to 20 percent.

4. *Definitive estimate (project-control estimate)*. Based on considerable data prior to preparation of completed drawings and specifications; probable error within 10 percent.

5. *Detailed estimate (firm or contractor's estimate)*. Requires completed drawings, specifications, and site surveys; probable error within 5 percent.

Greater accuracy of estimation may be achieved, within limits, by the expenditure of more time and money. The greater the accuracy required, the greater the time and effort needed to obtain the design and cost data prior to making the estimate.

W. R. Park investigated the cost and accuracy of estimates for a project with a total cost of \$1 million as shown in Fig. 9-41 (*Cost Engi-*

*neering Analysis*, Wiley, New York, 1973, p. 133). Table 9-47 shows typical average costs for producing estimates [adapted from A. Pikulik and H. E. Diaz, *Chem. Eng.*, **84**, 106–122 (Oct. 10, 1977)].

### Rapid Estimations

**Ratio Methods** J. E. Haselbarth [*Chem. Eng.*, **74**, 214–215 (Dec. 4, 1967)] published data giving the total capital investment per unit of annual production capacity  $C_{TC}/R$ . Table 9-48 lists data for many processes involving production units constructed on a previously developed site. Plants built on a green-field site would cost about 30 to 40 percent more, but enlargements of an existing plant would cost about 20 to 30 percent less than the values given in Table 9-48. Total fixed-capital investments for installations within the battery limits are given in Table 9-48. These refer to North American values corresponding to a Marshall and Swift index of 1000 and are adapted from the data of D. R. Woods [*Financial Decision Making in the Process Industry*, Prentice Hall, Englewood Cliffs, NJ, 1975, pp. 288–290]. L. Lynn and R. F. Howland [*Chem. Eng.*, **67**, 131–136 (Feb. 8, 1960)] studied the capital ratios for 17 process industries and summarized data for more than 1000 processes. The capital ratio (CR) for a plant erected on a green-field site is defined as the ratio of the fixed-capital investment  $C_{FC}$  to the annual sales revenue  $A_S$ :

$$(CR) = C_{FC}/A_S \quad (9-247)$$

However, Lynn and Howland included in the fixed-capital cost not only money invested in production and storage facilities but also that invested in land, research and development costs, and any auxiliary facilities necessary to support the process. Typical values of capital ratios for the year 1958 are listed in Table 9-49.

Both of the preceding methods are relatively inaccurate and can be used only for rough screening. They have the advantage that an esti-

TABLE 9-43 International Plant-Cost Indices\*

Year	Australia	Belgium	Canada	Denmark	France	Germany	Italy	Japan	Netherlands	Spain	Sweden	United Kingdom	United States
1985	151	134	137	143	171	121	201	112	119	190	151	148	124
1986	160	135	142	148	178	124	206	111	121	207	162	156	122
1987	170	138	146	159	186	128	217	110	123	222	171	166	123
1988	180	141	153	163	191	133	231	115	124	233	183	182	131
1989	193	147	160	177	200	137	253	122	126	249	199	200	137
1990	203	152	166	184	204	142	260	126	130	267	215	216	136
1991	208	161	173	193	210	150	269	127	133	283	227	232	138
1992	215	167	174	199	217	160	286	127	140	303	232	241	136
1993	218	171	180	198	223	162	292	129	142	315	244	240	136

\*From *Process Eng.*, London, U.K. (monthly). Annual averages for each year (1980 = 100).



**TABLE 9-44 Checklist of Items for Fixed-Capital-Cost Estimates**

<p>Land:</p> <ul style="list-style-type: none"> <li>Surveys</li> <li>Fees</li> <li>Property cost</li> </ul> <p>Site development:</p> <ul style="list-style-type: none"> <li>Site clearing</li> <li>Grading</li> <li>Roads, access and on-site</li> <li>Walkways</li> <li>Railroads</li> <li>Fence</li> <li>Parking areas</li> <li>Other paved areas</li> <li>Wharves and piers</li> <li>Recreational facilities</li> <li>Landscaping</li> </ul> <p>Process buildings:</p> <p>(List as required) Include in each as required substructure, superstructures, platforms, supports, stairways, ladders, access ways, cranes, monorails, hoists, elevators</p> <p>Auxiliary buildings:</p> <ul style="list-style-type: none"> <li>Administration and office</li> <li>Medical or dispensary</li> <li>Cafeteria</li> <li>Garage</li> <li>Product warehouse(s)</li> <li>Parts or stores warehouse</li> <li>Maintenance shops—electric, piping, sheet metal, machine, welding, carpenters, instrument</li> <li>Guard and safety</li> <li>Hose houses</li> <li>Change houses</li> <li>Smoking stations (in hazardous plants)</li> <li>Personnel building</li> <li>Shipping office and platforms</li> <li>Research laboratory</li> <li>Control laboratories</li> </ul> <p>Building services:</p> <ul style="list-style-type: none"> <li>Plumbing</li> <li>Heating</li> <li>Ventilation</li> <li>Dust collection</li> <li>Air conditioning</li> <li>Sprinkler systems</li> <li>Elevators, escalators</li> <li>Building lighting</li> <li>Telephones</li> <li>Fire alarm</li> <li>Paging</li> <li>Intercommunication systems</li> <li>Painting</li> </ul> <p>Process equipment:</p> <p>(List carefully from checked flow sheets)</p> <p>Nonprocess equipment:</p> <ul style="list-style-type: none"> <li>Office furniture and equipment</li> <li>Cafeteria equipment</li> <li>Safety and medical equipment</li> <li>Shop equipment</li> <li>Automotive heavy maintenance and yard material-handling equipment</li> <li>Laboratory equipment</li> <li>Lockers and locker-room benches</li> <li>Garage equipment</li> <li>Shelves, bins, pallets, hand trucks</li> <li>Housekeeping equipment</li> <li>Fire extinguishers, hoses, fire engines</li> </ul> <p>Process appurtenances:</p> <ul style="list-style-type: none"> <li>Piping—carbon steel, alloy, cast iron, lead-lined, aluminum, copper, asbestos-cement, ceramic, plastic, rubber, reinforced concrete</li> <li>Pipe hangers, fittings, valves</li> <li>Insulation—piping, equipment</li> <li>Instruments</li> <li>Instrument panels</li> <li>Electrical—panels, switches, motors, conduit, wire, fittings, feeders, grounding, instrument and control wiring</li> </ul>	<p>Utilities:</p> <ul style="list-style-type: none"> <li>Boiler plant</li> <li>Incinerator</li> <li>Ash disposal</li> <li>Boiler feed-water treatment</li> <li>Electric generation</li> <li>Electrical substations</li> <li>Refrigeration plant</li> <li>Air plant</li> <li>Wells</li> <li>River intake</li> <li>Primary water treatment—filtration, coagulation, aeration</li> <li>Secondary water treatment—deionization, demineralization, pH and hardness control</li> <li>Cooling towers</li> <li>Water storage</li> <li>Effluent outfall</li> <li>Process-waste sewers</li> <li>Process-waste pumping stations</li> <li>Sanitary-waste sewers</li> <li>Sanitary-waste pumping stations</li> <li>Impounders, collection basins</li> <li>Waste treatment, including gases</li> <li>Storm sewers</li> </ul> <p>Yard distribution and facilities (outside battery limits):</p> <ul style="list-style-type: none"> <li>Process pipe lines—steam, condensate, water, gas, fuel oil, air, fire, instrument, and electric lines</li> <li>Raw-material and finished-product handling equipment—elevators, hoists, conveyors, airveyors, cranes</li> <li>Raw-material and finished-product storage—tanks, spheres, drums, bins, silos</li> <li>Fuel receiving, blending, and storage</li> <li>Product loading stations</li> <li>Track and truck scales</li> </ul> <p>Miscellaneous:</p> <ul style="list-style-type: none"> <li>Demolition and alteration work</li> <li>Catalysts</li> <li>Chemicals (initial charge only)</li> <li>Spare parts and noninstalled equipment spares</li> <li>Surplus equipment, supplies and equipment allowance</li> <li>Equipment rentals (for construction)</li> <li>Premium time (for construction)</li> <li>Inflation cost allowance</li> <li>Freight charges</li> <li>Taxes and insurance</li> <li>Duties</li> <li>Allowance for modifications and extra construction work during startup</li> </ul> <p>Engineering costs:</p> <ul style="list-style-type: none"> <li>Administrative</li> <li>Process, project, and general engineering</li> <li>Drafting</li> <li>Cost engineering</li> <li>Procurement, expediting, and inspection</li> <li>Travel and living expense</li> <li>Reproductions</li> <li>Communications</li> <li>Scale model</li> <li>Outside architect and engineering fees</li> </ul> <p>Construction expense:</p> <ul style="list-style-type: none"> <li>Construction, operation, and maintenance of temporary sheds, offices, roads, parking lots, railroads, electrical, piping, communication, and fencing</li> <li>Construction tools and equipment</li> <li>Warehouse personnel and expense</li> <li>Construction supervision</li> <li>Accounting and timekeeping</li> <li>Purchasing, expediting, and traffic</li> <li>Safety and medical</li> <li>Guards and watchmen</li> <li>Travel and transportation allowance for craft labor</li> <li>Fringe benefits</li> <li>Housekeeping</li> <li>Weather protection</li> <li>Permits, special licenses, field tests</li> <li>Rental of off-site space</li> <li>Contractor's home office expense and fees</li> <li>Taxes and insurance, interest</li> </ul>
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**TABLE 9-45 Standard Cost Code (Numerical) Summary**

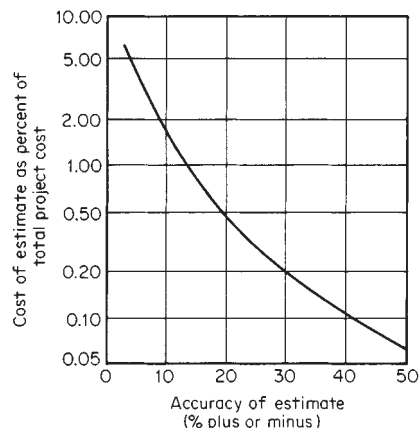
Code Group	Subdivision
<b>Direct costs:</b>	
0-349	Process section (process equipment listed in flow-sheet sequence 1, 2, 3, 4, 5, . . . , N, preceded by process identification number)
350-399	Site development
400-449	Process buildings
450-489	Auxiliary buildings
490-499	Nonprocess equipment
500-599	Process appurtenances (piping, insulation, instrumentation, electrical, etc.)
600-699	Utilities and yard services (boiler plant, refrigeration, compressed air, water supply and treatment, effluents, fire protection, yard piping, yard electrical, yard materials handling, raw and finished-product storage)
700-729	Substructures
730-749	Superstructures
750	Painting
760-769	Building services
770-799	Demolition and alteration to existing structure
800-819	Surplus equipment, supplies and materials, royalty payments
820-830	Design modifications, construction modifications, and extra work during startup
<b>Indirect costs:</b>	
850-874	Home office engineering
875-889	Architect's and engineer's charges and fees
900-969	Construction expenses
970-994	Taxes and insurance
995	Contractor's home office expenses
996	Contractor's fees

mate can be made in a few minutes, and they do not require design work or process flow sheets.

**Step Count Methods** These methods, used for order-of-magnitude estimates, are based on the definition of the functional units required to carry out the process. A functional unit is a signifi-

**TABLE 9-46 Standard Cost Code (Alphabetical-Numerical)**

A	Site work and foundations
B	Buildings (less foundations)
C	Steel structures and platforms (other than buildings)
D	Heat exchangers
E	Fractionating towers
F	Tanks and drums
G	Pumps and pump drivers
H	Compressors and blowers
J	Reactors and converters
K	Grinding, crushing, and classifying equipment
L	Materials-handling equipment
M	Fired heaters
N	Catalysts and chemicals
O	Laboratory equipment
P	Piping
Q	Instruments and controls
R	Electrical
S	Insulation and painting
T	Utility equipment (boilers, generators, refrigeration)
U	Plant and building accessories (railroads, fence, etc.)
V	Laboratory equipment
W	Safety equipment
X	Warehouse spares
100	Equipment and materials delivered and installed (A through X)
200	Sales and use taxes
300	Temporary facilities
320	Construction equipment, tools, and supplies
340	Construction supervision
360	Field office expense
380	Warehousing expense
400	Payroll taxes and insurance
500	Home office engineering costs
600	Procurement costs
700	Resident engineering
800	Royalty payments
900	Engineering administrative overhead and profit
950	Constructor's administrative overhead and profit

**FIG. 9-41** Relationship between cost and accuracy of cost estimations.

cant process step, including all the equipment and ancillaries to operate. The sum of the costs of the functional units gives an estimate of the total capital cost of the plant. Usually a functional unit is a unit operation, a unit process, or a separation involving energy transfer, moving parts, and possibly a high level of internals.

Pumping and heat exchange form part of a functional unit. "In-process" storage is ignored, but large storage requirements for raw materials, intermediates, or products are usually estimated separately. Sometimes there are difficulties in the identification and definition of the functional units involved in a process.

For a particular process, the capital cost per functional unit is given by

$$C_F = f(q, T, p, M_c, CI) \quad (9-248)$$

where  $q$  is capacity,  $T$  is operating temperature,  $p$  is pressure,  $M_c$  is a materials of construction factor, and  $CI$  is a relevant cost index.

Typical methods are those of F. C. Zevnik and R. L. Buchanan [*Chem. Eng. Progr.*, **59**, 70-77 (Feb. 1963)] and J. H. Taylor [*Eng. & Proc. Econ.*, **2**, 259-267, 1977]. The former is mainly a graphical method of estimating the cost per functional unit ( $C_F$ ) based on the capacity, the maximum pressure, the maximum temperature, and the materials of construction. The Taylor method requires the determination of the **costliness index**, which is dependent on the complexity of the process. A simpler method was suggested by S. R. Timms (M.Phil. thesis, Aston University, England, 1980) to give the battery limits cost for gas phase processes only in U.S. dollars with a Marshall and Swift index of 1000. The simple equation is

$$C_{BL} = 11,800Nq^{0.615} \quad (9-249a)$$

Taking into account materials of construction, temperature, and pressure effects, this becomes

$$C_{BL} = 5500Nq^{0.639}M_c(T_{max})^{0.066}(p_{max})^{-0.016} \quad (9-249b)$$

where  $N$  is the number of functional units;  $q$  is the capacity (tons/y);  $M_c$  is the materials of construction factor: 1.0 for carbon steel, 1.15 for

**TABLE 9-47 Typical Average Costs for Making Estimates (1990)\***

Type of estimate	Cost of project		
	Less than \$2 million	\$2 to \$10 million	\$10 to \$100 million
Order of magnitude estimate	\$ 3,000	\$ 6,000	\$ 13,000
Study estimate	20,000	40,000	80,000
Preliminary estimate	50,000	80,000	130,000
Definitive estimate	80,000	160,000	320,000
Detailed estimate	200,000	520,000	1,000,000

\*Adapted from A. Pikulik and H. E. Diaz, "Cost Estimating for Major Process Equipment." *Chem. Eng.*, **84**, 106-122 (Oct. 10, 1977).

TABLE 9-48 Capital-Cost Data for Processing Plants\*

Product	Process route	Size, 1000 metric tons/year	Approximate cost† \$ × 10 <sup>6</sup>	Total capital cost (\$) metric ton/yr of product	Size range 1000 metric tons/year	Exponent n‡
Chemical plants						
Acetaldehyde	Ethylene	50	13.7	274	20–150	0.70
Acetic acid	Methanol/CO	10	6.6	660	3–30	0.68
Acetone	Propylene	100	35.0	350	30–300	0.45
Ammonia	Steam reforming	100	26.0	260	30–300	0.70
Ammonium nitrate	Ammonia/nitric acid	100	6.0	60	30–300	0.65
Butanol	Propylene/CO/H <sub>2</sub> O	50	44.0	880	20–150	0.40
Chlorine	Electrolysis of NaCl	50	31.0	620	20–150	0.45
Ethylene	Refinery gases	50	14.4	288	20–150	0.83
Ethylene oxide	Ethylene/O <sub>2</sub>	50	55.0	1100	20–150	0.78
Formaldehyde (37%)	Methanol	10	17.7	1770	3–30	0.55
Glycol	Ethylene/Cl <sub>2</sub>	5	16.6	3320	2–20	0.75
Hydrofluoric acid	Hydrogen fluoride/H <sub>2</sub> O	10	8.8	880	3–30	0.68
Methanol	CO <sub>2</sub> /natural gas/steam	60	14.4	240	20–200	0.60
Nitric acid (conc.)	Ammonia oxidation	100	6.6	66	30–300	0.60
Phosphoric acid	Calcium phosphate/H <sub>2</sub> SO <sub>4</sub>	5	3.3	660	2–20	0.60
Polyethylene (high density)	Ethylene	5	17.7	3540	2–20	0.65
Propylene	Refinery gases	10	3.3	330	3–30	0.70
Sulfuric acid	Sulfur	100	3.3	33	30–300	0.65
Urea	Ammonia/CO <sub>2</sub>	60	8.8	147	20–200	0.70
Refinery units						
Alkylation (H <sub>2</sub> SO <sub>4</sub> )	Catalytic	10	21.0	2100	3–30	0.60
Coking (delayed)	Thermal	10	28.8	2880	3–30	0.38
Coking (fluid)	Thermal	10	17.7	1770	3–30	0.42
Cracking (fluid)	Catalytic	10	17.7	1770	3–30	0.70
Cracking	Thermal	10	5.5	550	3–30	0.70
Distillation (atm)	65% vaporized	100	35.4	354	30–300	0.90
Distillation (vac.)	65% vaporized	100	21.0	210	30–300	0.70
Hydrotreating	Catalytic desulfurization	10	3.3	330	3–30	0.65
Reforming	Catalytic	10	32.0	3200	3–30	0.60
Polymerization	Catalytic	10	5.5	550	3–30	0.58

\*Adapted from M. S. Peters and K. D. Timmerhaus, "Plant Design and Economics for Chemical Engineers", 4th ed., McGraw-Hill, New York, 1991.

†All costs are approximate U.S.A. values with M & S = 1000, assuming 330 operating days per year.

‡Exponents apply roughly for threefold capacity ratio extending either way from the plant size given.

low-grade stainless steel, 1.2 for medium-grade stainless steel, 1.3 for high-grade stainless steel;  $T_{max}$  is the maximum process temperature (K); and  $p_{max}$  is the maximum process pressure (bars).

For liquid and/or solid handling, A. V. Bridgwater and F. D. G. Bosson (*Proceedings of the 6th International Cost Engineering Conference*, Mexico, October, 1980) suggested the following equations:

$$C_{BL} = 2160N(q/s)^{0.675} \quad (9-250)$$

for plant capacities ( $q$ ) above 60,000 tons/y, where  $C_{BL}$  is the capital cost for battery limits in U.S. dollars (M & S = 1,000);  $N$  is the number of functional units; and  $s$  is the reactor conversion (weight of desired

reactor product/weight of reactor input). Thus,  $q/s$  represents the feed throughput in tons per year. For plant capacities below 60,000 tons per year, the equation becomes:

$$C_{BL} = 189,300N(q/s)^{0.30} \quad (9-251)$$

In general, the step count method of estimation can be applied to any special situation to derive a model equation for that particular industry or group of processes.

**Exponential Methods** Rapid capital-cost estimates can be made by using capacity-ratio exponents based on existing cost data of a company or drawn from published correlations.

If the cost of a piece of equipment or plant of size or capacity  $q_1$  is  $C_1$ , then the cost of a similar piece of equipment or plant of size or capacity  $q_2$  can be calculated from

$$C_2 = C_1(q_2/q_1)^n \quad (9-252)$$

where the value of the exponent  $n$  depends on the type of equipment or plant. Cost indices should be used to bring the cost data to a common year. Table 9-48 gives typical values of  $n$  for various processes along with the cost of a plant of given capacity at a particular time and the capacity range of applicability. For process plants, capacity is expressed in terms of annual production capacity in metric tons per year.

Exponential cost correlations have been developed for individual items of equipment. Care must be taken in determining whether the cost of the equipment has been expressed as free on board (FOB), delivered (DEL), or installed (INST), as this is not always clearly stated. In many cases the cost must be correlated in terms of parameters related to capacity such as surface area for heat exchangers or power for grinding equipment. There are four main sources of error in such cost correlations:

1. Oversimplification by correlating the cost of equipment in terms of a single variable

TABLE 9-49 Capital Ratios for Process Industries

Industry	Capital ratio,* 1958
Chemicals, general	2.02
Carbon black	3.98
Explosives	1.64
Glass	1.46
Fibers, synthetic	3.44
Foodstuffs, processed	0.66
Inorganics, heavy	2.24
Nonferrous metals	3.31
Petroleum	3.08
Pharmaceuticals	0.92
Pigments, paints, and inks	1.04
Pulp and paper	2.01
Resins and plastics	1.90
Rubber	1.04
Soap and detergents	0.69
Steel	2.78
Sulfur	1.97
Average	2.01

\*Capital ratio = (fixed-capital investment)/(annual sales revenue).

2. Representation of data by using a simple exponential relationship
3. Failure to include the effects of technological improvements
4. Errors incurred because of special circumstances

Table 9-50 gives typical values of the exponent  $n$  for many types of equipment. Prices are North American with a Marshall and Swift index of 1000, mainly for carbon steel equipment.

**Factor Estimations** Most factor methods for estimating the total installed cost of a process plant are based on a combination of materials, labor, and overhead cost components. These can be conveniently grouped as

1. Cost of major items of equipment
2. Cost of complete installation of equipment
3. Auxiliary equipment to make the process work
4. Engineering and field expenses
5. Contractor's fees and contingencies

A great variety of factors are in use, depending on the time available and the accuracy expected. Normally the input information required is the base cost. Determination of this cost usually requires a knowledge of equipment sizes, probably using mass and energy balances for the proposed process.

**Equipment or Base Cost** The total cost of the main-plant items is generally used as the base cost. Again, care must be taken with equipment costs which may be quoted as installed (INST), delivered to site (DEL), or free on board the delivery vehicle at the place of manufacture or other specified location (FOB).

**Base equipment** includes all equipment within the battery limits whose cost is as significant as the cost of a pump. For example, storage tanks, knockout drums, accumulators, heat exchangers, and pumps are classed as main-plant items (MPI). Early in the development of the process-flow diagram, it is advisable to increase the estimated (MPI) cost by 10 to 20 percent to allow for later additions. When the scope of the process has been well defined, (MPI) costs should be increased by 1 to 10 percent.

For order-of-magnitude estimates the cost of equipment delivered ( $C_{EQ}$ )<sub>DEL</sub> varies approximately from 1.1 to 1.25 times the FOB cost ( $C_{EQ}$ )<sub>FOB</sub>. The factor would be at the lower end of the range for domestic purchases and at the higher end for imports. Installation costs include unpacking, mounting, and connecting up to existing auxiliaries or utilities. The cost of equipment installed ( $C_{EQ}$ )<sub>INST</sub> varies with type and size but generally ranges from 1.4 to 2.2 times the delivered-equipment cost ( $C_{EQ}$ )<sub>DEL</sub>.

**Single-factor methods** collect the various items of expenditure into one factor, which is usually used to multiply the total cost of delivered equipment  $\sum (C_{EQ})_{DEL}$  to give the fixed-capital cost for plant within the battery limits:

$$(C_{FC})_{BL} = f \sum (C_{EQ})_{DEL} \quad (9-253)$$

Typical values for single factors  $f$  for battery-limit-plant costs (for a carbon steel plant including auxiliaries but not land) are as follows:

Solids processing (S)	3.8
Solids-fluid processing (S-F)	4.1
Fluid processing (F)	4.8

Thus the factors vary with the type of processing, although the boundaries between the classifications are not clear-cut and considerable judgment is required in selection of the correct factor.

**Multiple-factor methods** include the cost contributions for each given activity, which can be added together to give an overall factor. This factor can be used to multiply the total cost of delivered equipment  $\sum (C_{EQ})_{DEL}$  to produce an estimate of the total fixed-capital investment either for grass-roots or for battery-limit plants. The costs may be divided into four groups:

1. Cost of plant within battery limits
2. Cost of auxiliaries
3. Cost of engineering and field expenses
4. Cost of contractor's fees plus contingency allowance

Table 9-51 gives typical values of such factors for carbon steel installations taken from the data of D. R. Woods (*Financial Decision Making in the Process Industry*, Prentice Hall, Englewood Cliffs, NJ, 1975, p. 184). Auxiliaries and site preparation are given as factors of the delivered-equipment cost in Table 9-51, whereas C. A. Miller [*Chem.*

*Eng.*, **72**, 226–236 (Sept. 13, 1965)] expresses auxiliary costs as factors of the battery-limit (BL) cost. Table 9-52 gives the factors from the breakdown of Miller, which is more detailed than that of Woods.

**Example 23: Estimation of Total Installed Cost of a Plant** Let us estimate the total installed cost for a grass-roots plant producing an organic chemical (S-F process) on a continuous basis. We assume that the total cost of delivered equipment  $\sum (C_{EQ})_{DEL}$  is \$1 million and use suitable factors from Table 9-51.

The estimated values for the various contributions are given in Table 9-53, resulting in an estimate of \$4,280,000 for the total fixed-capital investment, including a contingency factor.

A multiple-factor method for predesign cost estimating has been put forward by D. H. Allen and R. C. Page [*Chem. Eng.*, **82**, 142–150 (Mar. 3, 1975)] for fluid-type plants (F) that include some vapor processing. The method requires the following input information:

1. Plant flow sheet giving main-plant items and process streams
2. Total process-stream input per year
3. Extreme temperature and pressure conditions, if any
4. Materials of construction for main-plant items
5. Operating phases for each main-plant item
6. Expectation of any unusually high or low, direct or indirect initial costs

By means of 12 procedural steps involving the input information, several equations, graphs, and tables, the total cost of delivered equipment  $\sum (C_{EQ})_{DEL}$  is estimated. This is then converted into a grass-roots investment estimate by dividing  $\sum (C_{EQ})_{DEL}$  by a single factor ranging from 15 to 30 percent (average value, 21 percent). The method is rapid and is claimed to be accurate within –20 to +25 percent, but it has only been tested by using data published in the literature for eight plants.

#### **Multiple-Factor Methods That Separate Materials and Labor**

These methods have become increasingly popular. While they are similar to the preceding methods, labor and materials costs are considered separately. Hence it is possible to allow for variations in efficiency and labor costs in different localities or countries. H. C. Bauman (*Fundamentals of Cost Engineering in the Chemical Industry*, Van Nostrand Reinhold, New York, 1964, p. 295) divides most of the components of Table 9-51 into material and labor components, quoting the data as ranges and medians of the percentage of the total fixed-capital investment. In Table 9-54, Bauman's data have been converted to factors of the delivered-equipment cost for a grass-roots installation.

A study has been made by A. V. Bridgwater [*Chem. Eng.*, **86**, 119–121 (Nov. 5, 1979)] of the geographical variations in capital costs. He concluded that because of trade and competition basic equipment costs do not vary significantly in the industrialized countries of the western world. The main differences in construction costs at various international locations are due to variations in labor costs and productivity, the use of specialized equipment, and sundry local factors. Table 9-55 gives location factors for the construction of chemical plants of similar function in various countries (1993 values). The factors have been corrected by Bridgwater for location variations in labor costs and efficiency and converted at the average value of the exchange rate.

**Factor Methods Using the Modular Approach** These are methods used for estimating the cost of major-equipment units and have been proposed by several authors. Perhaps the most comprehensive is the method suggested by K. M. Guthrie [*Chem. Eng.*, **76**, 114–142 (Mar. 24, 1969)]. Table 9-56 gives average factors for major-equipment items based on a ( $C_{EQ}$ )<sub>FOB</sub> cost for carbon steel units. To the FOB cost of the item is added, by means of factors, the total materials cost to complete the module  $M$ . Erection and setting costs  $L$  are added as a factor or calculated from the  $L/M$  cost ratio to give  $M + L = X$ , the direct module cost. Indirect costs, such as freight, taxes, insurance, engineering, and field expense, are added to ( $M + L$ ) to give the total module cost. This excludes contingency allowances, contractor's fees, auxiliaries, site development, land, and industrial buildings, which may have to be added when applicable. The factors in Table 9-56 were based on mid-1968 prices for a United States Gulf Coast location.

TABLE 9-50 Typical Exponents for Equipment Cost versus Capacity

Equipment	Size	Unit	Approximate cost, \$000	Size range	Exponent
Agitator, turbine, top entry, open, FOB	10 (7.5)	hp (kW)	7.0	2-30 (1.5-22.4)	0.45
Agitator, turbine, top entry, closed, FOB	10 (7.5)	hp (kW)	10.7	2-200 (1.5-150)	0.56
Blower, centrifugal, 4 lbf/in <sup>2</sup> (27.6 kN/m <sup>2</sup> ), DEL, excluding motor	10 (4.72)	10 <sup>3</sup> sft <sup>3</sup> /min (sm <sup>3</sup> /s)	67	0.5-150 (0.24-71)	0.60
Cone crusher, FOB, crusher only	100 (74.6)	hp (kW)	130	30-300 (22.4-224)	0.92
Jaw crusher, FOB, excluding motor	10 (7.5)	hp (kW)	34	1-60 (0.75-44.7)	0.65
Jaw crusher, FOB, excluding motor	100 (74.6)	hp (kW)	284	60-400 (44.7-300)	0.81
Centrifugal pump, C/S, FOB, excluding motor	10 (7.5)	hp (kW)	1.6	0.5-40 (0.37-30)	0.30
Centrifugal pump, C/S, FOB, excluding motor	100 (74.6)	hp (kW)	4.4	40-400 (30-300)	0.67
Conveyor, belt, C/S, FOB, excluding motor	100 (9.3)	ft <sup>2</sup> (m <sup>2</sup> )	6.7	60-200 (5.6-18.6)	0.50
Conveyor, screw, C/S, DEL, excluding motor	70 (540)	ft × m diameter (m × mm diameter)	10	50-100 (390-780)	0.46
Centrifuge, automatic batch, horizontal, C/S, FOB	20 (1.86)	Filter area, ft <sup>2</sup> (m <sup>2</sup> )	100	7-80 (0.65-7.43)	0.65
Compressor, reciprocating, <1000 lbf/in <sup>2</sup> , FOB, including motor	300 (224)	hp (kW)	133	1-20000 (0.75-1490)	0.84
Crystallizer, forced circulation, C/S, FOB	100 (91)	ton/day (Mg/day)	283	10-1000 (9.1-970)	0.59
Dryer, drum, C/S, FOB, excluding motor	100 (9.3)	ft <sup>2</sup> (m <sup>2</sup> )	73	10-400 (0.9-37)	0.52
Dryer, vacuum, shelf, C/S, FOB, excluding trays, vacuum equipment	100 (9.3)	ft <sup>2</sup> (m <sup>2</sup> )	17	15-1000 (1.4-93)	0.56
Dust collector, cloth, shaker type, FOB, including motors	10 <sup>4</sup> (4.7)	sft <sup>3</sup> /min (m <sup>3</sup> /s)	17	10 <sup>3</sup> -5 × 10 <sup>4</sup> (0.47-23.6)	0.79
Dust collector, multicyclones, FOB	10 <sup>4</sup> (4.7)	sft <sup>3</sup> /min (m <sup>3</sup> /s)	7	10 <sup>3</sup> -1.5 × 10 <sup>5</sup> (0.47-70.8)	0.66
Electrostatic precipitator, FOB	2 × 10 <sup>5</sup> (94)	ft <sup>3</sup> /min at 40°C (m <sup>3</sup> /s)	77	10 <sup>3</sup> -8 × 10 <sup>4</sup> (0.47-73.8)	0.39
Ejector, single-stage, 100 psig, steam, FOB	3 (10 <sup>-2</sup> )	lb/h (air/mmHg absolute)	2.7	8 × 10 <sup>4</sup> -10 <sup>6</sup> (37.8-472)	0.81
Ejector, two-stage, FOB, including condenser, piping	1 (3.4 × 10 <sup>-3</sup> )	[kg/h/(N/m <sup>2</sup> )]	6.3	0.2-30 (6.8 × 10 <sup>-1</sup> )	0.50
Ejector, multistage, FOB, including condenser, piping	10 (3.4 × 10 <sup>-2</sup> )	[kg/h/(N/m <sup>2</sup> )]	16.7	0.2-100 (6.8 × 10 <sup>-1</sup> -0.34)	0.43
Filter, vertical-pressure leaf, C/S, DEL	100 (9.3)	ft <sup>2</sup> (m <sup>2</sup> )	17	30-1500 (2.8-140)	0.57
Filter, plate and frame, C/S, DEL	100 (9.3)	ft <sup>2</sup> (m <sup>2</sup> )	5.7	10-1000 (0.9-93)	0.55
Filter, vacuum rotary drum, C/S, FOB, including motor	100 (9.3)	ft <sup>2</sup> (m <sup>2</sup> )	63.3	10-1500 (0.9-140)	0.48
Heat exchanger, shell-tube, floating head, C/S, DEL; fixed tube × 0.85; U tube × 0.87; kettle × 1.35	1000 (93)	ft <sup>2</sup> (m <sup>2</sup> )	21.7	20-20000 (1.9-1860)	0.59
Heat exchanger, thermal screw, C/S, FOB, excluding motor	100 (9.3)	ft <sup>2</sup> (m <sup>2</sup> )	33	10-400 (0.9-37)	0.78
Kettle, jacketed, glass-lined, FOB	100 (0.38)	U.S. gal (m <sup>3</sup> )	53	50-1000 (0.2-3.8)	0.48
Motors, ac induction, wound rotor, TEFC, FOB	10 (7.5)	hp (kW)	12.3	10-25 (7.5-18.6)	0.56
Motors, ac induction, wound rotor, TEFC, FOB	70 (52)	hp (kW)	19.3	25-200 (18.6-149)	0.77
Piping, typical straight run, C/S, FOB, \$/ft					
Installed: \$/ft × 6 to 7	6 (152)	Nominal diameter in (mm)	0.0093	1-24 (25-610)	1.33
Complex network: FOB \$/ft × 2					
Installed: \$/ft × 13					
Pressure vessel horizontal drum (150 psig), C/S	1000 (3.8)	U.S. gal (m <sup>3</sup> )	6.3	100-80000 (0.4-302)	0.62
Jacketed reactors, including mixer, FOB	100 (0.38)	U.S. gal (m <sup>3</sup> )	9.3	10-4000 (0.04-15.1)	0.53
Refrigeration, packaged mechanical, INST	100 (351.7)	U.S. tons (kW)	133	10-1000 (35.2-3520)	0.73
Screen, vibrating, single-deck, DEL, including motor	500 (46)	ft <sup>2</sup> (m <sup>2</sup> )	10	150-700 (14-65)	0.62
Stack, carbon steel		ft (m)	—	20-150 (6.1-45.7)	1.00
Tanks: atm, horizontal cylinder, C/S, FOB	1000 (3.8)	U.S. gal (m <sup>3</sup> )	4.7	100-40000 (0.4-151)	0.57
Vertical cylinder, C/S, FOB	1000 (3.8)	U.S. gal (m <sup>3</sup> )	3.3	100-20000 (0.4-76)	0.30
Vertical jacketed, C/S, FOB	1000 (3.8)	U.S. gal (m <sup>3</sup> )	15	70-1500 (0.26-5.7)	0.57
Vertical agitated, C/S, FOB, including motor	1000 (3.8)	U.S. gal (m <sup>3</sup> )	12.3	100-20000 (0.4-76)	0.50
Towers, distillation including internals, INST	4000 (trays)	$\left(\frac{\text{feed, lb/year}}{10^6}\right)^{0.65}$	3300	300-30000	1.00

NOTE: All costs are North American values with M & S = 1000.

**TABLE 9-51 Factors to Convert Delivered-Equipment Costs into Fixed-Capital Investment**

Details	Grass-roots plants			Battery-limit installations		
	Solids processing	Solids-fluid processing	Fluid processing	Solids processing	Solids-fluid processing	Fluid processing
Equipment, delivered	1.00	1.00	1.00	1.00	1.00	1.00
Installed	0.19-0.23	0.39-0.43	0.76	0.45	0.39	0.27-0.47
Piping	0.07-0.23	0.30-0.39	0.33	0.16	0.31	0.66-1.20
Structural steel foundations, reinforced concrete			0.28			0-0.13
Electrical	0.13-0.25	0.08-0.17	0.09	0.10	0.10	0.09-0.11
Instruments	0.03-0.12	0.13	0.13	0.09	0.13	
Battery-limits building and service	0.33-0.50	0.26-0.35	0.45	0.25	0.39	0.18-0.34
Excavation and site preparation	0.03-0.18	0.08-0.22		0.13	0.10	0.10
Auxiliaries	0.14-0.30	0.48-0.55	Included above	0.40	0.55	0.70
Total physical plant	2.37	2.97	3.04	2.58	2.97	3.50
Field expense	0.10-0.12	0.35-0.43		0.39	0.34	0.41
Engineering		0.35-0.43	0.41	0.33	0.32	0.33
Direct plant costs	2.48	3.73	3.45	3.30	3.63	4.24
Contractor's fees, overhead, profit	0.30-0.33	0.09-0.17	0.17	0.17	0.18	0.21
Contingency	0.26	0.39	0.36	0.34	0.36	0.42
<i>C<sub>FC</sub></i> : total fixed-capital investment	3.06	4.27	3.98	3.81	4.17	4.87

**TABLE 9-52 Factor Method of Miller (Based on Delivered-Equipment Costs = 100)\***

		Battery-limit costs (range of factors in percent of basic equipment); average unit cost of main-plant item (MPI)						
		Under \$9000	\$9000 to \$15,000	\$15,000 to \$21,000	\$21,000 to \$30,000	\$30,000 to \$39,000	\$39,000 to \$51,000	\$51,000
Field erection of basic equipment	High percentage of equipment involving high field labor	23/18	21/17	19.5/16	18.5/15	17.5/14.2	16.5/13.5	15.5/13
	Average (mild steel) equipment	18/12.5	17/11.5	16/10.8	15/10	14.2/9.2	13.5/8.5	13/8
	High percentage of corrosion materials and other high-unit-cost equipment involving little field erection	12.5/7.5	11.5/6.7	10.8/6	10/5.5	9.2/5.2	8.5/5	8/4.8
Equipment foundations and structural supports	High: predominance of compressors or mild steel equipment requiring heavy foundations			17/12	15/10	14/9	12/8	10.5/6
	Average: for mild steel fabricated-equipment solids			12.5/7	11/6	9.5/5	8/4	7/3
	Average: for predominance of alloy and other high-unit-price fabricated equipment	7/3	8/3	8.5/3	7.5/3	6.5/2.5	5.5/2	4.5/1.5
	Low: equipment more or less sitting on floor	5/0	4/0	3/0	2.5/0	2/0	1.5/0	1/0
Piping, including ductwork but excluding insulation	Piling or rock excavation			Increase above values by 25 to 100%				
	High: gases and liquids, petrochemicals, plants with substantial ductwork	105/65	90/58	80/48	70/40	58/34	50/30	42/25
	Average for chemical plants: liquids, electrolytic plants	65/33	58/27	48/22	40/16	34/12	30/10	25/9
Insulation of equipment only	Liquids and solids	33/13	27/10	22/8	16/6	12/5	10/4	9/3
	Low: solids	13/5	10/4	8/3	6/2	5/1	4/0	3/0
	Very high: substantial mild steel equipment requiring lagging and very low temperatures	13/10	11.5/8.5	10/7.4	9/6.2	7.8/5.3	6.8/4.5	5.8/3.5
	High: substantial equipment requiring lagging and high temperatures (petrochemicals)	10.3/7.5	9/6.3	7.8/5.2	6.7/4.2	5.7/3.4	4.7/3.8	4.8/2.5
Insulation of piping only	Average for chemical plants	7.8/3.4	6.5/2.6	5.5/2.1	4.5/1.7	3.6/1.4	2.9/1.1	2.2/8
	Low	3.5/0	2.7/0	2.2/0	1.8/0	1.5/0	1.2/0	1/0
	Very high: substantial mild steel piping requiring lagging and very low temperatures	22/16	19/13	16/11	14/9	12/7	9/5	6/3.5
	High: substantial piping requiring lagging and high temperatures (petrochemicals)	18/14	15/12	13/10	11/8	9/6	7/4	4.5/2.5
	Average for chemical plants	16/12	14/10	12/8	10/6	8/4	6/2	4/2
Low	14/8	12/6	10/5	8/4	6/3	4/2	2/1	



**TABLE 9-52 Factor Method of Miller (Based on Delivered-Equipment Costs = 100) (Concluded)**

		Battery-limit costs (range of factors in percent of basic equipment); average unit cost of main-plant item (MPI)						
		Under \$9000	\$9000 to \$15,000	\$15,000 to \$21,000	\$21,000 to \$30,000	\$30,000 to \$39,000	\$39,000 to \$51,000	\$51,000
All electrical except building, lighting, and instrumentation	Electrolytic plants, including rectification equipment		55/42	50/38	45/33	40/30	35/26	
	Plants with mild steel equipment, heavy drives, solids	26/17	22.5/15	19.5/12.5	17/10	14/8.5	12/7	10/6
	Plants with alloy or high-unit-cost equipment, chemical and petrochemical plants	18/9.5	15.5/8.5	13/6.5	11/5.5	9/4.5	7.3/3.5	6/2.5
	Substantial instrumentation, central control panels, petrochemicals		58/31	46/24	37/18	29/13	23/10	18/7
Instrumentation*	Miscellaneous chemical plants		32/13	26/10	20/7	15/5	11/3	8/2
	Little instrumentation, solids		21/9	17/7	13/5	10/3	7/2	5/1
Miscellaneous, including site preparation, painting, and other items not accounted for above	Top of range—large complicated processes; bottom of range—smaller, simple processes	Range for all values of basic equipment is 6 to 1%						
Buildings—architectural and structural, excludes building services†		Building evaluation when most process units are located inside buildings						
		High, brick and steel		Medium		Economical		Evaluation
Quality of construction		+4		+2		0		
		Very high unit cost equipment		Mostly alloy steel		Mixed materials		Costly carbon steel
Type of equipment		-3		-2		-1		0
		Very high		Intermediate		Atmosphere		
Operations pressures		-2		-1		0		
		cost equipment, chemical		Building class = algebraic sum =				
		Average unit cost of MPI						
Building class		Under \$9000	\$9000 to \$15,000	\$15,000 to \$21,000	\$21,000 to \$30,000	\$30,000 to \$39,000	\$39,000 to \$51,000	\$51,000
Cost of process	+2	92/68	82/61	74/56	67/49	59/44	52/39	46/33
Units inside buildings	+1 to -1	72/49	62/43	56/38	51/33	45/29	41/26	36/21
	-2	50/37	44/33	40/29	35/25	30/21	27/18	23/15
Open-air plants with minor buildings		37/16	32/13	28/11	24/8	20/6	17/4	14/2
Building services‡		High		Normal	Low			
Compressed air for general service only		4		1½	0.5			
Electric lighting		18		9	5			
Sprinklers		10		6	3			
Plumbing		20		12	3			
Heating		25		16	8			
Ventilation:								
Without air conditioning		18		8	0			
With air conditioning		45		35	25			
Total overall average§		85		55	20			

The above factors apply to those items normally classified as building services. They do not include (1) services located outside the building such as substations, outside sewers, and outside water lines, all of which are considered to be outside the battery limit as well as outside the building; and (2) process services.

\*Courtesy C. A. Miller of Canadian Industries Ltd. and the American Association of Cost Engineers.

NOTE: The average unit cost of the main-plant items is the total cost of the MPI divided by the total number of items. Figures include up to 3 percent for BL outside lighting, which is not covered in building services.

†Total instrumentation cost does not vary a great deal with size and hence is not readily calculated as a percentage of basic equipment. This is particularly true for distillation systems. If in doubt, detailed estimates should be made.

‡When building specifications and dimensions are known, a high-speed building-cost estimate is recommended, especially if buildings are a significant item of cost. If a separate estimate is not possible, evaluate the buildings as shown before selecting the factors.

§The following factors are for battery-limit (process) buildings only and are expressed in percentage of the building architectural and structural cost. They are not related to the basic equipment cost.

¶The totals provide the ranges for the type of building involved and are useful when individual service requirements are not known. Note that the overall averages are not the sum of the individual columns.

TABLE 9-53 Estimate Using Factors from Table 9-51

Details (solids-fluid, grass-roots plant)	Factor assumed	Cost, \$	Percentage of total
Equipment, delivered	1.00	1,000,000	23.4
Installed	0.41	410,000	9.6
Piping	0.34	340,000	8.0
Electrical	0.13	130,000	3.0
Instruments	0.13	130,000	3.0
Battery-limit building and service	0.30	300,000	7.0
Excavation and site preparation	0.15	150,000	3.5
Auxiliaries	0.52	520,000	12.2
Total physical plant	2.98	2,980,000	69.7
Field expense	0.39	390,000	9.1
Engineering	0.39	390,000	9.1
Direct plant costs	3.76	3,760,000	87.9
Contractor's fees, overhead, profit	0.13	130,000	3.0
Contingency	0.39	390,000	9.1
Total fixed-capital investment	4.28	4,280,000	100.0

TABLE 9-54 Typical Factors with Separation of Materials and Labor\*

	Total factor	Materials factor	Labor factor
Equipment delivered	1.00		
Installation	0.09		0.09
Instruments installed	0.13	0.09	0.04
Piping	0.29	0.155	0.135
Foundations and steel	0.18	0.08	0.10
Insulation painting	0.11	0.025	0.085
Electrical	0.18	0.06	0.12
Battery-limit building	0.21	0.13	0.08
Site preparation	0.08		
Auxiliaries	0.55		
Physical-plant cost	2.82		
Engineering and home office	0.31	0.01	0.30
Field expense	0.43	0.30	0.13
Direct plant cost	3.56		
Contractor's fees	0.17		
Contingency	0.39		
Fixed-capital cost	4.12		

\*Based on the data of H. C. Bauman, *Fundamentals of Cost Engineering in the Chemical Industry*, Van Nostrand Reinhold, New York, 1964, p. 295, for essentially carbon steel equipment.

A. Pikulik and H. E. Diaz [*Chem. Eng.*, **84**, 106–122 (Oct. 10, 1977)] presented a graphical method for estimating the fabricated cost of distillation columns and pressure vessels, storage tanks, fired heaters, pumps and drivers, compressors and drivers, and vacuum equipment.

TABLE 9-56 Factors for Individual Items\*

Details	Exchangers			Vessels		Pump and driver	Compressor and driver	Tanks
	Furnaces	Shell and tube	Air-cooled	Vertical	Horizontal			
FOB equipment	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Piping	0.18	0.46	0.18	0.61	0.42	0.30	0.21	
Concrete	0.10	0.05	0.02	0.10	0.06	0.04	0.12	
Steel		0.03		0.08				
Instruments	0.04	0.10	0.05	0.12	0.06	0.03	0.08	
Electrical	0.02	0.02	0.12	0.05	0.05	0.31	0.16	
Insulation		0.05		0.08	0.05	0.03	0.03	
Paint			0.01	0.01	0.01	0.01	0.01	
Total materials = $M$	1.34	1.71	1.38	2.05	1.65	1.72	1.61	1.20
Erection and setting ( $L$ )	0.30	0.63	0.38	0.95	0.59	0.70	0.58	0.13
$X$ , excluding site preparation and auxiliaries ( $M + L$ )	1.64	2.34	1.76	3.00	2.24	2.42	2.19	1.33
Freight, insurance, taxes, engineering, home office, construction		0.08		0.08	0.08	0.08	0.08	0.08
Overhead or field expense	0.60	0.95	0.70	1.12	0.92	0.97	0.97	
Total module factor	2.24	3.37	2.46	4.20	3.24	3.47	3.24	1.41

\*From K. M. Guthrie, *Chem. Eng.*, **76**, 114–142 (Mar. 24, 1969). Based on FOB equipment cost = 100 (carbon steel).

TABLE 9-55 Location Factors for Chemical Plants of Similar Functions (1993 Values)

Location	Factor (United States = 1.0)
Australia	1.04
Austria	0.85
Belgium	0.70
Canada	1.14
Central Africa	1.51
Central America	1.20
Denmark	0.85
Finland	0.88
France	0.73
Germany	0.76
Greece	0.80
India	
imported element	0.80
indigenous element	0.25
Ireland	0.70
Italy	0.79
Japan	1.46
Malaysia	0.42
Middle East	0.84
New Zealand	1.27
North Africa	
imported element	0.65
indigenous element	0.44
Norway	0.92
Portugal	1.00
South Africa	0.90
South America	1.36
Spain	0.83
Sweden	0.75
Switzerland	0.94
Turkey	0.80
United Kingdom	0.76
United States	1.00

NOTE:

1. Increase a factor by 10 percent for each 1000 mi or part of 1000 mi that the new plant is distant from a major manufacturing or import center, or both.
2. When materials or labor, or both, are obtained from more than a single source, prorate the appropriate factors.
3. Investment incentives have been ignored.

**Equipment Costs** The cost of delivered equipment forms the basis of most methods of estimating the fixed-capital cost. The equipment required can usually be divided into (1) processing equipment, (2) equipment for handling and storage of raw materials, and (3) finished products handling and storage equipment.

Quotations for equipment costs from fabricators or suppliers are the most accurate. Therefore most companies base their costs on (1) quotations from fabricators, (2) past purchase records updated with

appropriate cost indices, and (3) exponential methods of adjusting prices for capacity changes.

A large number of graphs for estimating the costs of various types of process equipment and auxiliaries are presented in the excellent text by M. S. Peters and K. D. Timmerhaus (*Plant Design and Economics for Chemical Engineers*, 4th ed., McGraw-Hill, New York, 1991, Chaps. 14–16) at January 1990 prices. Information on process equipment costs also appears, from time to time, in various journals (see Table 9-50). Although these published cost data are extremely useful for making rapid estimates, no published data can compete with the detailed and usually confidential cost records of large companies.

**Piping Estimation** The cost of fabrication and installation of process-plant piping appears to range from 18 to 61 percent of the FOB equipment cost as indicated in Table 9-56. This would normally represent about 7 to 15 percent of the installed plant cost and is obviously a significant item. The various available piping-estimation methods are as follows:

1. Detailed pricing from piping drawings
2. Guthrie method
3. Dickson  $N$  method
4. Pricing by weight of specific types of pipe
5. Price estimation by cost per joint
6. Pricing as a factor of equipment cost
7. Pricing as a factor of total plant installed cost

The first five methods are applicable only after rigorous circuit analysis and when piping layouts and isometric drawings or scale models are available for quantity takeoff (e.g., pipe size, length, and specification, flanges and valve count, etc.).

Guthrie's method [K. M. Guthrie, *Chem. Eng.*, **76**, 201–216 (Apr. 14, 1969)] is mainly graphical, using average mid-1968 costs for a United States Gulf Coast location.

The Dickson  $N$  method [R. A. Dickson, *Chem. Eng.*, **54**, 121–123 (November 1947)] is a variation of the detailed price takeoff. Various circuits for each type of pipe are completely priced for a base size. Another chart gives an  $N$  factor for all other pipe sizes. Multiplying the cost of the circuit for the base size by the appropriate  $N$  factor yields the estimated cost of the new circuit of the desired pipe size. The method depends for its accuracy on periodic repricing of the base-size circuits in order to keep the base charts up to date.

Estimating by weight requires virtually complete takeoff, including weight calculations and a full record of past costs on this basis. Its only advantage lies in the time saved in the detailed estimates of the cost of piping components.

Estimating by cost per joint depends on the accumulation of past data, analyzed and conveniently correlated for use. The main advantage of the method lies in the fact that good engineering flow sheets can be used for the estimation.

Figure 9-42 is a plot of the number of labor-hours of field erection time per joint against the nominal pipe size of shop-fabricated carbon steel and low-alloy pipe. The unit of work measurement used in this

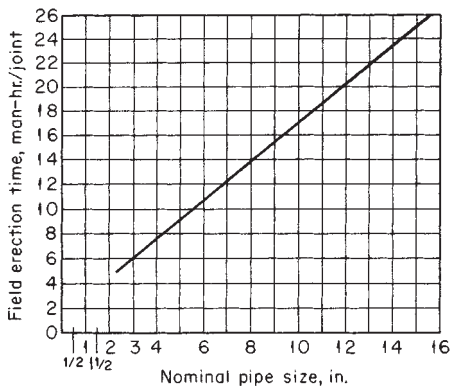


FIG. 9-42 Labor-hours required to erect large quantities of shop-fabricated steel and low-alloy piping.

TABLE 9-57 Components of Total Installed Piping Cost

Material	Pipe, valves, fittings, nuts, bolts, gaskets, and hangers
Labor	Cut, erect, align, fit, bolt, thread or weld, and test
Indirect costs	Handle and haul, store, scaffold, lost time, tools and rentals, contractor's overhead and profit
Factors applicable to labor-hours	Craft rate, productivity, height, and complexity
Crafts involved in piping erection	Pipefitters, laborers, carpenters, warehouse workers, teamsters, and operating engineers

method is the pipe joint, requiring two joints for couplings and valves, three for tees, etc., as most of the labor-hours involved in pipe erection are expended in making connections. The additional costs of handling, suspending, and placing lengths of pipe in position are included in the chart.

It should be noted that Fig. 9-42 gives labor-hours only. Material costs must be obtained by price takeoff from drawings on which all valves and instrument connections are shown. Pipe lengths and fittings are taken off by referring to the equipment-layout plan and elevation drawing. The graph of Fig. 9-42 can be updated by using actual costs for a specific job, in which case the labor cost per joint represents a total labor cost including all the factors applicable to labor shown in Table 9-57. It should be possible to analyze statistically uniform data from a number of complete jobs to determine the value of each factor for various project locations.

Methods 6 and 7 are simpler procedures, using factors for estimating piping costs when neither flow sheets nor detailed piping drawings are available. Tables 9-51, 9-52, 9-54, and 9-56 include typical values of piping factors based on total equipment cost, delivered or FOB, as indicated in the particular table. These methods require some degree of judgment in selecting the appropriate factor, based on experience gained by comparing piping costs for similar previously installed process plants.

A rough method of estimating the piping factor as a percentage of the total delivered cost of major process equipment (excluding instruments and electrical items) was presented by E. S. Sokullu in the form

$$f_p = 11\phi_p^{1.6} \quad (9-254)$$

where  $\phi_p$  = (number of actual pipes on flow diagram)/(number of major process equipment units) [*Chem. Eng.*, **76**, 148–150 (Feb. 10, 1969)].

The equipment-unit method would appear to give more accuracy than the preceding methods, particularly for unfamiliar process arrangements. It requires the accumulation of piping costs for various sizes of main-plant items such as pumps, heat exchangers, evaporators, tanks, and columns. Basically it is assumed that piping designs for specific items are similar for most projects. Statistical analysis of such data shows good agreement with the more detailed takeoff pricing methods. Since for most processes the length of pipe used is a small proportion of the total piping cost, the assumption of an average length of piping per main-plant item, based on actual costs for several previous jobs, should give sufficient accuracy. Correction for escalation of costs can be carried out by using a single cost index, unlike methods 1 to 5.

Most of the factorial methods of estimation given previously, with the exception of the method of Allen and Page (loc. cit.), tend to estimate costs which are based on carbon steel equipment or installations. Table 9-58 gives typical multiplying factors for converting carbon steel costs to equivalent-alloy costs for a few items of equipment. (Adapted from *A Guide to Capital Cost Estimating*, 3d ed., Institution of Chemical Engineers, Rugby, England, 1988, p. 70.)

**Electrical and Instrumentation Estimation** These costs usually range from 4 to 10 percent of the total installed plant cost, with a median value of about 7.5 percent. As with piping estimation, the process design must be almost completed before detailed drawings and specifications can be prepared for estimating purposes. However, actual electrical costs can be up to 100 percent higher than estimated costs, and so it is important to attempt to maintain the accuracy range within reasonable limits.

**TABLE 9-58 Typical Factors for Converting Carbon Steel Cost to Equivalent-Alloy Costs**

Material	Pumps, etc.	Other equipment
All carbon steel	1.00	1.00
Stainless steel, Type 410	1.43	2.00
Stainless steel, Type 304	1.70	2.80
Stainless steel, Type 316	1.80	2.90
Stainless steel, Type 310	2.00	3.33
Rubber-lined steel	1.43	1.25
Bronze	1.54	
Monel	3.33	

Material	Heat exchangers
Carbon steel shell and tubes	1.00
Carbon steel shell, aluminum tubes	1.25
Carbon steel shell, monel tubes	2.08
Carbon steel shell, 304 stainless tubes	1.67
304 stainless steel shell and tubes	2.86

During the design stages, frequent changes in the type and sizes of equipment lead to delays in establishing electrical requirements. Hence it is very difficult to obtain a detailed estimate of the cost of the electrical part of the project. For order-of-magnitude and study estimates, an appropriate factor in the range 4 to 10 percent of the total installed plant cost can be used. However, for budget-authorization or preliminary estimates requiring an accuracy within 5 percent more accurate methods are necessary.

The methods available for electrical estimates are as follows:

1. Detailed takeoff
2. Factored electrical cost as a percentage of total installed plant cost for specific types of plant
3. Unit pricing

The detailed-takeoff method can rarely if ever be used. When detailed drawings are available, costs may be estimated by pricing materials and components from suppliers' catalogs or, for special items, from quotations. Handbooks are available which give typical values of the labor-hours required to perform units of installation work, such as installation of switches, starters, motors, conduit wiring, and push buttons of various sizes, for both hazardous and nonhazardous areas. Labor rates can be obtained from various government statistical sources or elsewhere. For the United States the National Electrical Contractors Association publishes an excellent manual of electrical costs. From the complete plans and specifications, the estimator can take off materials, estimate the labor cost, apply appropriate factors for labor efficiency, productivity, and local conditions, and achieve good results.

The factor estimate, if based on tested actual data, gives good results in the study estimate and often proves adequate at the preliminary estimate stage. It is essential to accumulate from past experience data showing actual electrical costs (1) as a percentage of total installed plant cost and (2) as a percentage of installed equipment costs. Studies of electrical installations for more than 100 plants (H. C. Bauman, *Fundamentals of Cost Engineering in the Chemical Industry*, Van Nostrand Reinhold, New York, 1964, p. 134) showed electrical costs ranging from about 4 to 11 percent of total plant cost, with a median for battery-limit process plants of 7.5 percent. The corresponding range based on installed equipment costs was 15 to 40 percent, with a median of 26 percent. Thus, it appears that there is a better correlation between electrical costs and total installed plant cost than with installed equipment costs. Table 9-59, taken from Bauman's data, gives typical values of electrical costs as a percentage of total installed plant cost. Cost ranges for installed instrumentation costs are also included in Table 9-59, as these would form part of electrical costs. The ranges of values are rather wide, depending on the degree of automatic control required.

Electrical costs involve four main components: (1) power wiring, (2) lighting, (3) transformation and service, and (4) instrument and control wiring. A breakdown of these component costs as a percentage of total electrical cost is given in Table 9-60.

**TABLE 9-59 Electrical and Instrumentation as Percentage of Total Installed Plant Cost**

Type of plant	Electrical (process and service)		Instrumentation	
	Range, %	Median, %	Range, %	Median, %
Solids plants	3.7-10.7	5.4 %	0.3-6.0	0.8
Grass-roots process	4.0-7.9	5.9 %	1.9-4.3	3.2
Battery-limit process	4.3-10.1	7.5 %	0.1-7.9	3.7

**TABLE 9-60 Component Electrical Costs as Percentage of Total Electrical Cost**

Component	Range, %	Median, %
Power wiring	25-50	40
Lighting	7-25	12
Transformation and service	9-65	40
Instrument-control wiring	3-8	5

The unit-cost method can give a quick and accurate estimation, provided it is based on accumulated data from many jobs on various types of plant. The actual data are analyzed to provide unit-cost information for electrical components as follows:

1. Total installed cost per motor
2. Total installed cost per lighting outlet by type
3. Total installed cost of receptacles by type (incandescent, fluorescent, etc.)
4. Total installed cost for each wired instrumentation point
5. Total installed cost for each unit of transformation
6. Total installed cost per lineal foot of distribution by type (overhead bare and insulated, underground)
7. Total installed cost of each interlock point

Each unit cost contains all the costs involved in the installation of that unit. For motors installed costs include the starter, conduit, wire, and a proportionate share of the service panelboard and busbars. The motor cost is not included since this will be part of the equipment cost. In the case of lighting, the installed cost includes the lighting fixtures, the conduit and wire, and a proportional share of the lighting panelboard and service switching costs.

**Auxiliaries Estimation** Chemical-plant auxiliaries normally include all structures, equipment, and services which are not directly involved in the process. Within this broad range there are two major classifications, utilities and service facilities.

The typical cost range for auxiliaries is from 20 to 40 percent of the total installed plant cost. For a small continuous-process plant making a single product, the cost of auxiliaries would lie in the lower part of the range, while for large multiprocess grass-roots plants the factor would tend to be near the upper limit of the range.

**Auxiliary Buildings** Typical variations in the cost of auxiliaries for a variety of process plants are given in Table 9-61. The widest variation is shown for auxiliary buildings, which is not surprising in view of the many types and quality of materials and the wide variation in methods of construction. For example, amenities buildings such as offices, cafeteria, first-aid rooms, gatehouses, and control rooms would necessitate fairly expensive brick and plaster-wall construction. On the other hand, services buildings such as substations, switch rooms, and pump or compressor houses would cost about 5 to 10 percent less. Provision of air conditioning, furniture, and equipment for cafeteria, laboratory, and office buildings would add about 50 percent to the basic cost of the building.

**Steam-Generating Facilities** These form the second largest investment item for chemical-plant auxiliary equipment. Variations in capacity, location indoors or outdoors, the type of fuel used, pressure and temperature levels, and the type of process served have an important effect on actual cost as well as on cost relative to other auxiliary items. Package boiler installations can be purchased as shop-built units which are assembled, piped, and wired ready to be erected on the owner's foundations. They are available in units up to about 136,000 kg/h (300,000 lb/h), although units larger than about 45,360

**TABLE 9-61 Typical Ranges of Auxiliary Facilities as Percentage of Total Installed Plant Cost**

	Range, %	Median, %
Auxiliary buildings	3-9	5.0
Steam generation	2.6-6	3.0
Refrigeration, including distribution	1-3	2.0
Water supply, cooling, and pumping	0.4-3.7	1.8
Finished-product storage	0.7-2.4	1.5
Process-waste systems	0.4-1.8	1.1
Raw-materials storage	0.3-3.2	1.1
Steam distribution	0.2-2	1.0
Electrical distribution	0.4-2.1	1.0
Air compressor and distribution	0.2-3.0	1.0
Water distribution	0.1-2	0.9
Fire protection system	0.3-1.0	0.7
Water treatment	0.2-1.1	0.6
Railroads	0.3-0.9	0.6
Roads and walks	0.2-1.2	0.6
Gas supply and distribution	0.2-0.4	0.3
Sanitary-waste disposal	0.1-0.4	0.3
Communications	0.1-0.3	0.2
Yard and fence lighting	0.1-0.3	0.2

kg/h (100,000 lb/h) may be available only on a semierected basis. It is usually necessary to obtain firm price quotations that take into account all the factors involved. Housing the boiler installations in buildings will generally increase the cost by about 7 to 9 percent per kilogram-hour (15 to 20 percent per pound-hour) of steam-generating capacity over the cost for outdoor installations or for installations in existing buildings.

For most chemical plants, process steam is used at pressures of 1.825 MN/m<sup>2</sup> (250 psig), saturated or lower. When combined heat and power generation is economically justified, the steam may be generated at about 5.96 MN/m<sup>2</sup> (850 psig) appropriately superheated and used to drive back-pressure steam turbines passing out process steam at the required pressure level.

**Electricity** A reliable and adequate electricity supply is usually available through government or private enterprises. Owing to the increasing cost of purchased electricity, many companies have installed combined heating and power (CHP) generation systems. A cogeneration plant may (1) be owned and operated by the industrial user or the utility, (2) serve or be isolated from one or more industrial users, or (3) form an integral part of the local utility grid. Typical costs for generating steam range from \$7.90 to \$9.50 per 1000 kg (\$3.60 to \$4.30 per 1000 lb) at 3550 kPa (500 psig); \$3.70 to \$7.70 per 1000 kg (\$1.70 to \$3.50 per 1000 lb) at 790 kPa (100 psig); and \$2.00 to \$3.70 per 1000 kg (\$0.90 to \$1.70 per 1000 lb) for exhaust steam. The above costs apply for a Marshall and Swift index of 1000.

For most plants, electric distribution systems start at the power company's service point on the plant's property. The choice of an electrical system will depend on many factors relating to the particular project. A wide range of items, from switchgear, transformers, and motor control centers to cabling, earthing, and lighting, are required. Normally, competitive quotations would be obtained to take an estimate beyond the study stage.

Various types of overhead and underground distribution systems may be used depending on local conditions. Generally, an overhead system will incur only about 30 percent of the cost of an underground distribution system.

**Water Systems** These systems usually form the third highest cost item in chemical-plant auxiliaries, with cooling towers representing the largest part of the investment. Although the installed cost increases with the terminal temperature range, an approximate cost correlation is given by

$$C_{CT} = 100q^{0.57} \quad (9-255)$$

where  $C_{CT}$  is the installed cost of the cooling tower in United States dollars for a Marshall and Swift (M & S) index of 1000 and  $q$  is the capacity in United States gallons per minute over the range from (1)(10<sup>3</sup>) to (1)(10<sup>5</sup>) U.S. gal/min.

River-water pumping and filtering installations can be approximately correlated by

$$C_{RW} = 0.65q^{0.81} \quad (9-256)$$

where  $C_{RW}$  is the installed cost of the river-water system in United States dollars for an M & S index of 1000 and  $q$  is the capacity over the range from (4)(10<sup>5</sup>) to (1)(10<sup>7</sup>) U.S. gal/day.

Similarly, installed costs of water-softening systems can be correlated in United States dollars (M & S = 1000) as follows:

$$C_{WS} = 1380q^{0.44} \quad (9-257)$$

over the range of capacity from (3)(10<sup>7</sup>) to (1)(10<sup>9</sup>) U.S. gal/day and of demineralizing systems by

$$C_{DS} = 0.17q^{1.9} \quad (9-258)$$

over the range of  $q$  values from (1)(10<sup>4</sup>) to (4)(10<sup>5</sup>) U.S. gal/day. Actual water-treatment costs may vary widely from the above, depending on the quality of the water, the percentage of dissolved solids, and the total hardness.

**Refrigeration Systems** These systems are being used increasingly in chemical processing. Installed costs of packaged mechanical units in United States dollars (M & S = 1000) can be approximately correlated by

$$C_{RS} = 4630q^{0.73} \quad (9-259)$$

where  $q$  is the capacity in tons of refrigeration over the range from 10 to 1000 tons. One ton of refrigeration is equivalent to a rate of heat removal of 3.517 kW (12,000 Btu/h).

**Roads and Walkways** The cost of roads and walkways in chemical plants is difficult to estimate, since these vary with type of construction and thickness of applied cover. Some typical unit costs for roads are as follows: For 305-mm (12-in) gravel base covered with 76-mm (3-in) asphalt, the cost is \$17.10 per square meter (\$14.30 per square yard); for a reinforced concrete slab with a 152-mm (6-in) sub-base, the cost is from \$28.40 to \$35.10 per square meter (\$23.80 to \$29.30 per square yard), depending on the thickness of concrete (for M & S = 1000).

Installed costs for railroads, including switches and frogs, can be roughly estimated as follows (for M & S = 1000):

Linear meters	Linear feet	\$/meter	\$/foot
152-305	500-1000	230.00	70.00
305-915	1000-3000	210.00	64.00
915-3050	3000-10000	200.00	61.00
Above 3050	Above 10000	187.00	57.00

Usually the cost of roads and walkways amounts to 0.2 to 1.2 percent of the fixed capital cost with a typical value of 0.6 percent. Similarly, railroads cost 0.3 to 0.9 percent of the fixed capital cost, having an average value of 0.6 percent.

**Use of Computers in Cost Estimation** A large part of estimation consists of the collection and storage of data obtained from records of actual plant costs. The data then must be correlated and updated and the required information rapidly retrieved for use in further cost estimations. A comprehensive survey (C. J. Liddle and A. M. Gerrard, *The Application of Computers to Capital Cost Estimation*, Institution of Chemical Engineers, London, 1975, pp. 6-17) suggests that large chemical manufacturers, equipment vendors, and some contractors are using the computer increasingly for data retrieval, followed by simple correlation and the application of factorial methods to cost estimation.

In the case of equipment vendors, the computer's contribution appears to be particularly worthwhile owing to the elimination of estimating errors in producing price quotations. Several companies have developed an automated quotation system to overcome delay and inaccuracy in estimating and bidding. Such systems appear to have been developed by firms already possessing significant computing facilities, since the cost of computer time is small compared with the cost of the computer. Qualitatively, operating costs for an automated quotation system appear to be about half of those of a manual system



of price quotation. The methods of estimation used are based on the manual methods described previously.

Several of the larger chemical manufacturers, particularly those in the petrochemicals field, have developed computer packages based on manual methods of factorial estimating. Usually the input data consists of the cost of each main-plant item (MPI) obtained from quotations or historical records. The program then estimates the costs of erection, piping, instrumentation, electricals, civil engineering, and lagging for each (MPI) in turn by adding a series of factors. These account for the complexity of the process and the constructional difficulties for each (MPI) to produce an estimate of the overall plant cost. It is obviously necessary to introduce appropriate inflation indices to bring the estimated costs up to date.

For process plants, it is often possible to use these cost-estimation programs with a design or flow-sheet program to optimize on a particular component or even over the whole plant (*A Guide to Capital Cost Estimating*, 3d ed. Institution of Chemical Engineers, Rugby, England, 1988, pp. 48–49). However, it must be remembered that optimization is expensive on computer time, although there appear to be no data available on the cost effectiveness of the computer in this area. It is also possible to incorporate the capital-cost estimate in an investment evaluation involving forecasts of expenses and revenue from sales. Thus, by means of the computer design and costing can be brought together. There is an immediate feedback of information, resulting in improved design and lower costs. In some types of plants, costing data can be fed in as a subroutine to the design programs. All these possibilities assume that the total cost of using the computer is not unreasonable.

**Startup Costs** Startup problems can reduce aftertax earnings during the early years, the most serious effect being to delay the startup of production, causing a loss of earnings. An accurate estimate of startup time and cost can help in (1) predicting the availability of new products, (2) planning market entry, and (3) estimating the overall profitability because of more accurate cash-flow forecasts and (NPV) calculations [R. P. Feldman, *Chem. Eng.*, **76**, 87–90 (Nov. 3, 1969)].

Startup costs are defined as the total of those costs directly related to bringing a new production facility into operation. They should not include the costs of entering or expanding a business. Hence startup costs include the following:

1. All expenses due to changes in process and equipment after completion of construction but excluding those due to changes in project scope
  2. All labor costs after completing construction, especially those incurred in checking the functioning of equipment
  3. All costs incurred during the startup period but excluding normal operating expenses
  4. Expenses for training plant personnel even if incurred before startup has officially begun
  5. All research and development costs incurred during startup
- The following expenses should *not* be included:
1. Marketing costs
  2. Expense of training sales representatives
  3. Penalties for shipping outside optimum freight areas
  4. Costs associated with starting a new company
  5. Lost sales unless there is a contract with a penalty
  6. Profit lost due to timing

Startup time may be defined as the time span between the end of construction and the beginning of normal operation. Hence it should start when the contractor finishes the whole plant or a specified section of it to enable comparisons to be made with other startup times. It is usual to define "normal" operation as (1) operations at a certain percentage of design capacity, (2) a specified number of days of continuous operation, or (3) the capability of making products of a specified purity.

It is essential for project and production management to agree beforehand on the definition to be applied. Obtaining agreement on the definition of "normal" operation is important since (1) it sets a target for field personnel, (2) it ensures that everyone is striving for the same target, (3) it permits comparisons with other plants, and (4) it determines a cutoff point for completion of startup. It may be neces-

sary to wait until the plant is running well to obtain the actual total cost of startup.

For control purposes it is advisable to estimate startup cost and time beforehand and then try to stay within the estimates. The general parameters which can be used to estimate startup cost  $C_{SU}$ , which are usually between 2 and 20 percent of the battery-limit fixed-capital cost, are as follows:

1. Direct fixed-capital cost for plant (battery-limit capital),  $(C_{FC})_{BL}$
2. Newness of process and technology,  $b$
3. Newness of type and size of equipment,  $c$
4. Labor quality and quantity,  $d$
5. Interplant dependency,  $e$

Hence startup cost may be expressed as

$$C_{SU} = (C_{FC})_{BL}[0.10 + b + c + d + ne] \quad (9-260)$$

When applied to large air-separation and ammonia plants (1000 to 1400 metric tons/day), the following values for the parameters can be used:

- $b = 0.05$  for a radically new process  
 $= 0.02$  for a relatively new process  
 $= -0.02$  for an old process

- $c = 0.07$  if radically new  
 $= 0.04$  if very new  
 $= 0.02$  if relatively new  
 $= -0.03$  if old

- $d = 0.04$  if labor is in very short supply  
 $= 0.02$  if labor is in short supply  
 $= -0.1$  if labor is in surplus supply

- $e = 0.04$  if plant is very dependent on another  
 $= 0.02$  if moderately dependent on another  
 $= -0.02$  if independent

and

$n =$  number of plants or sections making up the process chain

Startup time  $t_{SU}$  for these plants may be estimated from construction time  $t_C$  by developing an equation similar to Eq. (9-260):

$$t_{SU} = t_C(0.15 + b + c + d + ne) \quad (9-261)$$

For the same type of plant the values of the parameters are

- $b = 0.15$  for a radically new process  
 $= 0.05$  for a relatively new process  
 $= -0.01$  for an old process

- $c = 0.15$  if radically new  
 $= 0.08$  if very new  
 $= 0.05$  if relatively new  
 $= -0.01$  if old

- $d = 0.15$  if labor is in very short supply  
 $= 0.05$  if labor is in short supply  
 $= -0.01$  if labor is in surplus supply

- $e = 0.25$  if plant is very dependent on another  
 $= 0.10$  if moderately dependent on another  
 $= -0.02$  if independent

and

$n =$  number of plants or sections making up the process chain

It should be noted that these values are based on previous experience with certain types of plants, but appropriate values which apply to other processes and locations could be selected.

**Construction Time** The duration of construction is difficult to estimate owing to the large number of variables involved. In general, estimates of construction time tend to be overoptimistic, especially for

larger projects. Usually projects costing less than \$5 million at 1993 prices can be completed in 10 to 18 months, while those costing more than \$10 million may take from 18 to 42 months to complete. Delays of up to 12 months behind schedule are quite possible, particularly when there are labor problems. As mentioned previously, such delays will usually result in increased construction costs. Often, a more serious effect is loss of earnings resulting from a delayed startup. Both of these factors increase the payback period and reduce the attainable net present value and discounted-cash-flow rate of return of the project.

**Project Control** Having made a good estimate of the capital cost and the expected construction time, it is essential to introduce an effective system for controlling expenditure of time and money during construction. Good capital-cost control can cut down expenditures even when the definitive estimate is not very accurate. It is most important for management to receive early warnings if overruns in expenditure or time are likely to occur.

Effective cost control should start from the beginning of the project at the research and development stage and continue through the design and estimating stages to initial operation of the plant [J. W. Hackney and K. K. Humphreys (eds.), *Control and Management of Capital Projects*, 2d ed., McGraw-Hill, New York, 1991]. The stages discussed here are the later steps after authorization of funds and during project construction. After the purchasing department has placed the orders for equipment and materials, speed and efficiency during the construction stage is most important in ensuring the financial success of the project. Field expenditure during construction can amount to 30 to 60 percent of the fixed-capital cost and includes the costs of all labor, installed equipment, and materials together with associated process piping, electrical instrumentation, and insulation. Construction therefore requires efficient execution and prompt feedback of progress information, necessitating a good cost-control system.

Figure 9-43 shows the flow of information needed for cost control. The chart assumes a definitive estimate which has been linked to a standard code of accounts. As construction proceeds, up-to-date cost-control reports are supplied to the field cost engineer. From the home office the engineer receives monthly reports of engineering and drafting labor-hours used and money expended, together with a list of drawings and specifications completed up to that time. Monthly expenditures and current commitments come in coded detail from the job ledgers of the accountants. Timekeepers' records give details of craft and nonmanual labor-hour expenditures. Quantities of equipment and material held on site are reported daily by quantity surveyors to the construction superintendent. All purchase orders are posted in the ledgers as current commitments, whether they are placed at the home office or in the field, and an up-to-date warehouse inventory is maintained.

At the end of each month, the field cost engineer collects all current information on a detailed cost report form. As these are actual costs, they can be used to estimate future job costs to completion. Daily reports of unit-cost progress for concrete, excavation, masonry, steel, piping, and electrical work, etc., are then used to predict possible overruns or underruns for the various items. Analysis and comparison with the original estimate point out trouble spots for early attention. If an item is running into difficulty, it is red-flagged to the resident and project engineers for remedial action.

In practice, the existence of a tight cost-control system tends to spread a cost consciousness among the personnel involved in the project. Such an awareness, even in construction-equipment maintenance and job housekeeping, can lead to efficient cost control throughout.

Cost reports should be brief but informative, preferably in summary form. They should report expenditures and commitments, estimated costs to complete, and expected overruns or underruns of the authorized budget for each important item of cost. Brief notes should emphasize significant deviations from predicted cost. Any large, persistent overrun should have already been investigated and reported to the project and construction managers for immediate attention. If an expected overrun cannot be avoided, the current summary cost report should serve as justification for a request for additional funds.

When organized efficiently, the cost-control system should require no more paperwork than for the normal construction procedure. The cost of cost control appears to vary between 0.2 and 0.5 percent of the total project value. Proper use of the normal records available for craft-labor time, warehouse-inventory control, and the usual accounting purposes should be adequate. The savings achieved by good cost control should far exceed the additional costs of operating the system. Additional details on the technique are given by H. C. Bauman (*Fundamentals of Cost Engineering in the Chemical Industry*, Van Nostrand Reinhold, New York, 1964, pp. 190-196).

Scheduling construction to ensure that the project is completed in the shortest possible time is an essential part of project control. The project-control estimate defines to a large extent the construction-time schedule. It is then possible to prepare a master schedule from the control estimate by carefully sequencing and synchronizing the installation work according to past experience. Drawings are usually completed in predictable order owing to the dependence of certain designs on preceding work. The normal order of completion of drawings and specifications is (1) site work, (2) substructures, (3) equipment and building superstructures, (4) equipment layouts, (5) piping, (6) insulation, (7) instrumentation, and (8) electrical work.

Detailed planning and scheduling then involve establishing the items of work required and determining the correct sequence of work and the number of persons required to perform each item of work.

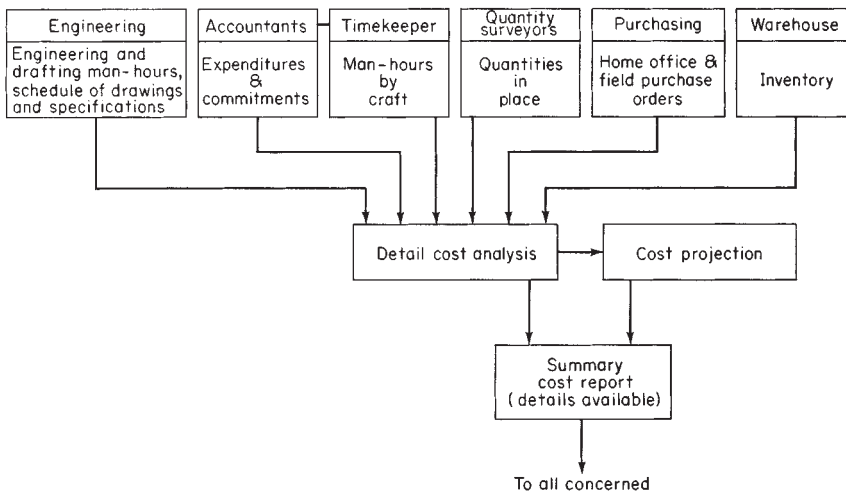


FIG. 9-43 Information flow for cost control.

From this information it is possible to prepare bar charts by using a 4-week month and noting on the chart the interdependence of the various functions. The starting time for each class of work is fixed on the chart, and the duration is calculated from the labor-hours allocated to that work from the control estimate. Work should progress smoothly as time elapses, but the operations must be linked by the order of necessary precedence. Starting times for the various items of work will be staggered as drawings are released and also to smooth out labor requirements.

Sketching the bar chart is commenced by inserting the arrival dates of key items, observing any necessary precedence. Estimates of the duration of erection time can be made to obtain the starting date for process piping. Since the supporting structure must be in place when the key item arrives, it is possible to work back along the bar chart to the preparation of the foundations. From the complete bar chart, built up in a similar manner, a tentative startup date can be set after allowing a few weeks for tidying up bits and pieces. Some activities can be speeded up, but it is necessary to estimate the increased cost of so doing.

Actual progress made with construction work can be indicated on the bar chart by filling in the open bars according to the percentage toward completion. Comparison of the actual progress bar for the whole project with the cumulative labor-hour curve indicates whether the job is ahead of schedule or not. If corrective action is required, effort should be concentrated on the key or critical items.

Large projects will usually require network analysis using the critical-path method (CPM) or program evaluation and review technique (PERT) in the planning, scheduling, and progress-control stages. Examples of bar charts and a fuller description of network analysis are given by J. W. Hackney and K. K. Humphreys (eds.), *Control and Management of Capital Projects*, 2d ed., McGraw-Hill, New York, 1991. A detailed treatment of the use of PERT and CPM techniques as applied to contract bidding strategy and to project control is given by L. A. Swanson and H. L. Pazer (*Pertsim: Text and Simulation*, International Textbook, Scranton, Pa., 1969), who present a hand simulation technique based on probabilistic methods.

**Overseas Construction Costs** Although Table 9-55 gives location factors for the construction of chemical plants of similar function in various countries at 1993 values, these may vary differentially over a period of time owing to local changes in labor costs and productivity. Hence, it is often necessary to estimate the various components of overseas construction costs separately. Equipment and material prices will depend on local labor costs and the availability of raw materials. If the basic materials have to be imported, costs in the source area become important and import duties and freight charges must be added.

**Equipment and material** normally amount to about 40 to 45 percent of the costs of a typical chemical plant. In general, equipment and material costs are slightly cheaper in European countries and Japan, whereas in Mexico and Canada they are nearer the United States average.

**Construction labor** makes up about 20 to 35 percent of total costs for a chemical plant. Table 9-62 compares average 1994 hourly rates

**TABLE 9-62 Comparative National Labor Costs**

Hourly labor costs	\$	Relative to U.S.
Germany	25.35	1.53
Switzerland	22.49	1.36
Belgium	20.84	1.25
Netherlands	19.82	1.20
Japan	19.05	1.15
Austria	19.02	1.15
United States	16.58	1.00
France	16.05	0.97
Italy	15.69	0.95
Britain	12.90	0.78
Ireland	11.94	0.72
Spain	11.36	0.69
Greece	6.29	0.38
Portugal	4.70	0.28

NOTE: Figures include fringe benefits.

SOURCE: Adapted from Economist Intelligence Unit January 1994.

for various types of construction labor in several countries with those for the United States.

**Fringe benefits** are known in countries other than the United States as "social charges"; they vary considerably in degree of coverage from country to country. Typical allowances in these countries include family benefits based on number of children, health service, maternity benefits, disability allowances, grants for funeral expenses, old-age and war pensions, unemployment benefits, and pension schemes. Additional fringe benefits may include paid holidays, starting allowances for new workers, relocation grants, severance pay, profit sharing, production bonuses, special gratuities, and sometimes housing allowances. It is essential to investigate the local situation thoroughly to determine the benefits payable and the additional cost on the basic hourly wage rate.

**Labor productivity** is very much dependent on the health and well-being of the workers and also on the availability of labor-saving tools and construction equipment. The frequency of strikes, holidays, slowdowns, and political unrest will also depress productivity. Closed-shop practices or demarcation disputes will also affect the productivity of labor. The use of standard equipment, parts, and methods tends to improve productivity.

In a particular country, productivity will depend largely on the number of hours worked per week. Production will increase with the number of hours worked during the week, but as more overtime is worked, fatigue will produce a falloff in productivity.

In the United States, construction craft labor usually work a normal 40 h/week. The United Kingdom operates a 40-h schedule, although there is strong pressure to reduce this to 38 h/week. European countries tend to work a normal 40 h/week, and some Far Eastern countries may work up to 45 h/week.

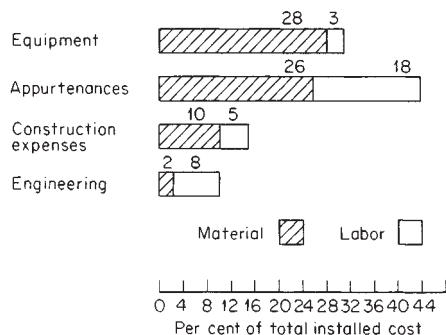
Productivity of local craft labor also depends on the use and availability of modern mechanical tools and construction equipment. Normally, the low cost of labor in certain countries tends to cut out the purchase or hire of sophisticated labor-saving equipment and to encourage the employment of large pools of labor, particularly in developing countries such as India, Pakistan, southeast Asian countries, and many African countries. In turn, this usually leads to higher construction costs. The use of labor-saving equipment is prevalent in Canada, western Europe, Japan, and, to an increasing extent, the Middle East.

**Complete Plant Costs** It is difficult to compare costs of domestic and overseas plants owing to the wide variation in types of plants and sizes and the rapid changes in technology. Useful data are scarce, and the following comparisons must be used with caution and then only for order-of-magnitude estimates of fixed-capital costs.

The method uses a breakdown of costs for a typical chemical plant installed in the United States, as shown in Fig. 9-44. Costs of equipment, appurtenances, construction, and engineering with material and labor separate are given as a percentage of total installed United States costs. The four components of cost are defined as follows:

**Equipment** includes all prefabricated machines, appliances, or systems such as tanks, heat exchangers, pumps, motors, switchgear, and boilers.

**Appurtenances** are auxiliary items which cover materials, such as pipes, valves, fittings, conduit, wire, tubing, and insulation.



**FIG. 9-44** Typical breakdown of chemical-plant costs by major component.

**Construction expense** includes the cost of construction equipment, tools, sheds, railroad trackage, road materials, welding machines, scaffolding, and timber, which are all used in construction but do not form a permanent part of the plant.

**Engineering** is mainly labor but has a small component cost which can be classified with equipment and materials, such as tools, paper, pencils, and reproduction costs.

In total, labor amounts to 34 percent and material to 66 percent of total installed costs.

Table 9-63 uses the data of Fig. 9-44 to compare the relative fixed-capital costs for plant construction in other countries with those for the United States. The relative cost ratios were developed from data similar to those in Table 9-62. Labor ratios were corrected for the different local rates and hours per working week, job duration, and degree of mechanization available in other countries. Some of these factors are difficult to estimate, and the final "total" ratios give a reasonable order-of-magnitude value for relative construction costs for equivalent plants in the countries indicated.

**TABLE 9-63 Relative Plant Construction Costs in Various Countries Compared with the United States**

Country	Equipment	Material	Labor	Engineering	Total
United States	0.28	0.38	0.26	0.08	1.00
England	0.26	0.41	0.18	0.05	0.90
Italy	0.22	0.32	0.31	0.05	0.90
Mexico	0.26	0.35	0.35	0.04	1.00
Australia	0.38	0.54	0.29	0.09	1.30
Canada	0.32	0.44	0.31	0.08	1.15
France	0.27	0.40	0.22	0.06	0.95
Germany	0.26	0.36	0.32	0.06	1.00
Japan	0.29	0.35	0.22	0.04	0.90

The choice of an overseas manufacturing site involves the consideration of many political and economic factors in addition to costs. Table 9-64 gives a list of 92 items which should be taken into account when choosing a plant location for manufacturing abroad.

**TABLE 9-64 Factors in Choosing a Foreign Manufacturing Site**

<p>Economic factors</p> <ul style="list-style-type: none"> <li>Size of GNP and rate of growth</li> <li>Is there a working development plan?</li> <li>Resistance to recession</li> <li>Relative dependence on imports and exports</li> <li>Foreign-exchange position</li> <li>Balance-of-payments outlook</li> <li>Stability of currency; convertibility</li> <li>Remittance and repatriation regulations</li> <li>Balance of economy (industry-agriculture-trade)</li> <li>Size of market for your products; rate of growth</li> <li>Size of population; rate of growth</li> <li>Per capita income; rate of growth</li> <li>Income distribution</li> <li>Current or prospective membership in a customs union</li> <li>Price levels; rate of inflation</li> </ul> <p>Political factors</p> <ul style="list-style-type: none"> <li>Stability of government; its form</li> <li>Presence or absence of class antagonism</li> <li>Special political, ethnic, and social problems</li> <li>Attitude toward private and foreign investment</li> <li>Acceptability of United States investment by government</li> <li>Acceptability of United States investment by customers and competitors</li> <li>Presence or absence of nationalization threat</li> <li>Presence or absence of state industries</li> <li>Do state industries receive favored treatment?</li> <li>Concentration of influence in small groups</li> <li>Treaty of friendship or establishment with United States?</li> </ul> <p>Government factors</p> <ul style="list-style-type: none"> <li>Are fiscal and monetary policies sound?</li> <li>Freedom from bureaucratic red tape</li> <li>Fairness and honesty of administrative procedures</li> <li>Degree of anteforeign or anti-United States discrimination</li> <li>Fairness of courts</li> <li>Clear and modern corporate investment laws</li> <li>Patentability of your products</li> <li>Presence or absence of price controls</li> <li>Restrictions on 100 percent United States or foreign ownership</li> </ul> <p>Geographic factors</p> <ul style="list-style-type: none"> <li>Efficiency of transport (railways, waterways, highways)</li> <li>Port facilities</li> <li>Free ports, free zones, bonded warehouses</li> <li>Proximity of site to export markets</li> <li>Proximity of site to suppliers, customers</li> <li>Proximity to raw-material sources</li> <li>Existing supporting industry</li> <li>Availability of local raw materials</li> <li>Availability of power, water, gas</li> <li>Reliability of utilities</li> <li>Waste-disposal facilities</li> <li>Can exports be easily made?</li> <li>Can imports be easily made?</li> </ul>	<p>Geographic factors (<i>Continued</i>)</p> <ul style="list-style-type: none"> <li>Are plant sites readily available?</li> <li>Cost of suitable land</li> </ul> <p>Labor factors</p> <ul style="list-style-type: none"> <li>Availability of English-speaking managerial, technical, office personnel</li> <li>Availability of skilled labor</li> <li>Availability of semiskilled and unskilled labor</li> <li>Level of worker productivity</li> <li>Training facilities</li> <li>Outlook for increase in labor supply</li> <li>Degree of skill and discipline at all levels</li> <li>Tranquility of labor relations</li> <li>Presence or absence of militant or Communist-dominated unions</li> <li>Degree of labor voice in management</li> <li>Freedom to hire and fire</li> <li>Compulsory and voluntary fringe benefits</li> <li>Social security taxes</li> <li>Total cost, including fringes, compared with alternative sites</li> <li>Compulsory or customary profit sharing</li> </ul> <p>Tax factors</p> <ul style="list-style-type: none"> <li>Tax rates (corporate and personal income, capital, withholding, turnover, excise, payroll, capital gains, customs, other indirect and local taxes)</li> <li>General tax morality</li> <li>Fairness and incorruptibility of tax authorities</li> <li>Long-term trend for taxes</li> <li>Taxation of export income and income earned abroad</li> <li>Tax incentives for new businesses</li> <li>Depreciation rates</li> <li>Tax-loss carry-forward and carry-back</li> <li>Joint tax treaties</li> <li>Duty and tax drawbacks when imported goods are exported</li> <li>Availability of tariff protection</li> </ul> <p>Capital-sources factors</p> <ul style="list-style-type: none"> <li>Availability of local capital</li> <li>Costs of local borrowing</li> <li>Normal terms for local borrowing</li> <li>Availability of convertible currencies locally</li> <li>Modern banking system</li> <li>Government credit aids to new businesses</li> <li>Availability and cost of export financing, insurance</li> <li>Do United States or European capital sources favor loans here?</li> </ul> <p>Business factors</p> <ul style="list-style-type: none"> <li>Availability of United States government investment insurance</li> <li>General business morality</li> <li>State of marketing and distribution system</li> <li>Are administrative procedures simple and effective?</li> <li>Normal profit margins in general, in your industry</li> <li>Competitive situation in your industry; is it cartelized?</li> <li>What are antitrust and restrictive practices laws, and do they conflict with United States laws?</li> <li>Availability of amenities for United States expatriate executives and families</li> </ul>
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