

SOME MATHEMATICAL OPERATIONS

In chemistry we frequently use very large or very small numbers. Such numbers are conveniently expressed in *scientific*, or *exponential*, notation.

A-1 SCIENTIFIC NOTATION

In scientific notation, a number is expressed as the *product of two numbers*. By convention, the first number, called the *digit term*, is between 1 and 10. The second number, called the *exponential term*, is an integer power of 10. Some examples follow.

$$\begin{array}{ll}
 10000 = 1 \times 10^4 & 24327 = 2.4327 \times 10^4 \\
 1000 = 1 \times 10^3 & 7958 = 7.958 \times 10^3 \\
 100 = 1 \times 10^2 & 594 = 5.94 \times 10^2 \\
 10 = 1 \times 10^1 & 98 = 9.8 \times 10^1 \\
 1 = 1 \times 10^0 & \\
 1/10 = 0.1 = 1 \times 10^{-1} & 0.32 = 3.2 \times 10^{-1} \\
 1/100 = 0.01 = 1 \times 10^{-2} & 0.067 = 6.7 \times 10^{-2} \\
 1/1000 = 0.001 = 1 \times 10^{-3} & 0.0049 = 4.9 \times 10^{-3} \\
 1/10000 = 0.0001 = 1 \times 10^{-4} & 0.00017 = 1.7 \times 10^{-4}
 \end{array}$$

Recall that, by definition, (any base)⁰ = 1.

The exponent of 10 is the number of places the decimal point must be shifted to give the number in long form. A *positive exponent* indicates that the decimal point is *shifted right* that number of places. A *negative exponent* indicates that the decimal point is *shifted left*. When numbers are written in *standard scientific notation*, there is one nonzero digit to the left of the decimal point.

$$\begin{aligned}
 7.3 \times 10^3 &= 73 \times 10^2 &= 730 \times 10^1 &= 7300 \\
 4.36 \times 10^{-2} &= 0.436 \times 10^{-1} &= 0.0436 \\
 0.00862 &= 0.0862 \times 10^{-1} &= 0.862 \times 10^{-2} &= 8.62 \times 10^{-3}
 \end{aligned}$$

In scientific notation the digit term indicates the number of significant figures in the number. The exponential term merely locates the decimal point and does not represent significant figures.

Addition and Subtraction

In addition and subtraction all numbers are converted to the same power of 10, and the digit terms are added or subtracted.

$$\begin{aligned}
 (4.21 \times 10^{-3}) + (1.4 \times 10^{-4}) &= (4.21 \times 10^{-3}) + (0.14 \times 10^{-3}) = \underline{4.35 \times 10^{-3}} \\
 (8.97 \times 10^4) - (2.31 \times 10^3) &= (8.97 \times 10^4) - (0.231 \times 10^4) = \underline{8.74 \times 10^4}
 \end{aligned}$$

Multiplication

The digit terms are multiplied in the usual way, the exponents are added algebraically, and the product is written with one nonzero digit to the left of the decimal.

Two significant figures in answer.

$$(4.7 \times 10^7)(1.6 \times 10^2) = (4.7)(1.6) \times 10^{7+2} = 7.52 \times 10^9 = \underline{7.5 \times 10^9}$$

Two significant figures in answer.

$$(8.3 \times 10^4)(9.3 \times 10^{-9}) = (8.3)(9.3) \times 10^{4-9} = 77.19 \times 10^{-5} = \underline{7.7 \times 10^{-4}}$$

Division

The digit term of the numerator is divided by the digit term of the denominator, the exponents are subtracted algebraically, and the quotient is written with one nonzero digit to the left of the decimal.

Three significant figures in answer.

$$\frac{8.4 \times 10^7}{2.0 \times 10^3} = \frac{8.4}{2.0} \times 10^{7-3} = \underline{4.2 \times 10^4}$$

$$\frac{3.81 \times 10^9}{8.412 \times 10^{-3}} = \frac{3.81}{8.412} \times 10^{[9-(-3)]} = 0.45292 \times 10^{12} = \underline{4.53 \times 10^{11}}$$

Powers of Exponentials

The digit term is raised to the indicated power, and the exponent is multiplied by the number that indicates the power.

$$(1.2 \times 10^3)^2 = (1.2)^2 \times 10^{3 \times 2} = 1.44 \times 10^6 = \underline{1.4 \times 10^6}$$

$$(3.0 \times 10^{-3})^4 = (3.0)^4 \times 10^{-3 \times 4} = 81 \times 10^{-12} = \underline{8.1 \times 10^{-11}}$$

These instructions are applicable to most calculators. If your calculator has other notation, consult your calculator's instruction booklet.

Electronic Calculators To square a number: (1) enter the number and (2) touch the (x^2) button.

$$(7.3)^2 = 53.29 = \underline{53} \quad (\text{two sig. figs.})$$

To raise a number y to power x : (1) enter the number; (2) touch the (y^x) button; (3) enter the power; and (4) touch the (=) button.

$$(7.3)^4 = 2839.8241 = \underline{2.8 \times 10^3} \quad (\text{two sig. figs.})$$

$$(7.30 \times 10^2)^5 = 2.0730716 \times 10^{14} = \underline{2.07 \times 10^{14}} \quad (\text{three sig. figs.})$$

Roots of Exponentials

The exponent must be divisible by the desired root if a calculator is not used. The root of the digit term is extracted in the usual way, and the exponent is divided by the desired root.

$$\sqrt{2.5 \times 10^5} = \sqrt{25 \times 10^4} = \sqrt{25} \times \sqrt{10^4} = \underline{5.0 \times 10^2}$$

$$\sqrt[3]{2.7 \times 10^{-8}} = \sqrt[3]{27 \times 10^{-9}} = \sqrt[3]{27} \times \sqrt[3]{10^{-9}} = \underline{3.0 \times 10^{-3}}$$

Electronic Calculators To extract the square root of a number: (1) enter the number and (2) touch the (\sqrt{x}) button.

$$\sqrt{23} = 4.7958315 = \underline{4.8} \quad (\text{two sig. figs.})$$

On some models, this function is performed by the $\sqrt[y]{x}$ button.

To extract some other root: (1) enter the number y ; (2) touch the (INV) and then the (y^x) button; (3) enter the root to be extracted, x ; and (4) touch the (=) button.

A-2 LOGARITHMS

The logarithm of a number is the power to which a base must be raised to obtain the number. Two types of logarithms are frequently used in chemistry: (1) common logarithms (abbreviated log), whose base is 10, and (2) natural logarithms (abbreviated ln), whose base is $e = 2.71828$. The general properties of logarithms are the same no matter what base is used. Many equations in science were derived by the use of calculus, and these often involve natural (base e) logarithms. The relationship between $\log x$ and $\ln x$ is as follows.

$$\ln x = 2.303 \log x$$

$$\ln 10 = 2.303$$

Finding Logarithms The common logarithm of a number is the power to which 10 must be raised to obtain the number. The number 10 must be raised to the third power to equal 1000. Therefore, the logarithm of 1000 is 3, written $\log 1000 = 3$. Some examples follow.

Number	Exponential Expression	Logarithm
1000	10^3	3
100	10^2	2
10	10^1	1
1	10^0	0
$1/10 = 0.1$	10^{-1}	-1
$1/100 = 0.01$	10^{-2}	-2
$1/1000 = 0.001$	10^{-3}	-3

To obtain the logarithm of a number other than an integral power of 10, you must use either an electronic calculator or a logarithm table. On most calculators, you do this by (1) entering the number and then (2) pressing the (log) button.

$$\log 7.39 = 0.8686444 = \underline{0.869}$$

$$\log 7.39 \times 10^3 = 3.8686 = \underline{3.869}$$

$$\log 7.39 \times 10^{-3} = -2.1314 = \underline{-2.131}$$

The number to the left of the decimal point in a logarithm is called the *characteristic*, and the number to the right of the decimal point is called the *mantissa*. The characteristic only locates the decimal point of the number, so it is usually not included when counting significant figures. The mantissa has as many significant figures as the number whose log was found.

To obtain the natural logarithm of a number on an electronic calculator, (1) enter the number and (2) press the (ln) or ($\ln x$) button.

$$\ln 4.45 = 1.4929041 = \underline{1.493}$$

$$\ln 1.27 \times 10^3 = 7.1468 = \underline{7.147}$$

Finding Antilogarithms Sometimes we know the logarithm of a number and must find the number. This is called finding the *antilogarithm* (or *inverse logarithm*). To do this on a calculator, we (1) enter the value of the log; (2) press the (INV) button; and (3) press the (log) button.

$$\log x = 6.131; \quad \text{so } x = \text{inverse log of } 6.131 = \underline{1.352 \times 10^6}$$

$$\log x = -1.562; \quad \text{so } x = \text{inverse log of } -1.562 = \underline{2.74 \times 10^{-2}}$$

On some calculators, the inverse log is found as follows:

1. enter the value of the log
2. press the (2ndF) (second function) button
3. press (10^x)

On some calculators, the inverse natural logarithm is found as follows:

1. enter the value of the ln
2. press the (2ndF) (second function) button
3. press (e^x)

To find the inverse natural logarithm, we (1) enter the value of the ln; (2) press the (INV) button; and (3) press the (ln) or (ln x) button.

$$\ln x = 3.552; \quad \text{so } x = \text{inverse ln of } 3.552 = \underline{3.49 \times 10^1}$$

$$\ln x = -1.248; \quad \text{so } x = \text{inverse ln of } -1.248 = \underline{2.87 \times 10^{-1}}$$

Calculations Involving Logarithms

Because logarithms are exponents, operations involving them follow the same rules as the use of exponents. The following relationships are useful.

$$\log xy = \log x + \log y \quad \text{or} \quad \ln xy = \ln x + \ln y$$

$$\log \frac{x}{y} = \log x - \log y \quad \text{or} \quad \ln \frac{x}{y} = \ln x - \ln y$$

$$\log x^y = y \log x \quad \text{or} \quad \ln x^y = y \ln x$$

$$\log \sqrt[y]{x} = \log x^{1/y} = \frac{1}{y} \log x \quad \text{or} \quad \ln \sqrt[y]{x} = \ln x^{1/y} = \frac{1}{y} \ln x$$

A-3 QUADRATIC EQUATIONS

Algebraic expressions of the form

$$ax^2 + bx + c = 0$$

are called **quadratic equations**. Each of the constant terms (a , b , and c) may be either positive or negative. All quadratic equations may be solved by the **quadratic formula**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If we wish to solve the quadratic equation $3x^2 - 4x - 8 = 0$, we use $a = 3$, $b = -4$, and $c = -8$. Substitution of these values into the quadratic formula gives

$$\begin{aligned} x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)} = \frac{4 \pm \sqrt{16 + 96}}{6} \\ &= \frac{4 \pm \sqrt{112}}{6} = \frac{4 \pm 10.6}{6} \end{aligned}$$

The two roots of this quadratic equation are

$$x = 2.4 \quad \text{and} \quad x = -1.1$$

As you construct and solve quadratic equations based on the observed behavior of matter, you must decide which root has physical significance. Examination of the *equation that defines x* always gives clues about possible values for x . In this way you can tell which is extraneous (has no physical significance). Negative roots are often extraneous.

When you have solved a quadratic equation, you should always check the values you obtained by substitution into the original equation. In the preceding example we obtained $x = 2.4$ and $x = -1.1$. Substitution of these values into the original quadratic equation, $3x^2 - 4x - 8 = 0$, shows that both roots are correct. Such substitutions often do not give a perfect check because some round-off error has been introduced.