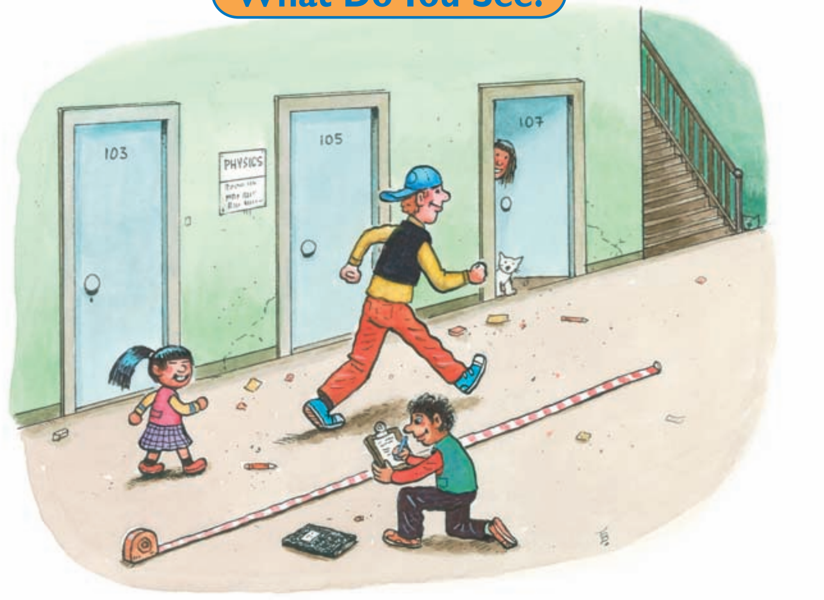
Section 2

Measurement: Errors, Accuracy, and Precision

What Do You See?



Learning Outcomes

In this section, you will

- **Calibrate** the length of a stride.
- **Measure** a distance by pacing it off and by using a meter stick.
- **Identify** sources of error in measurement.
- **Evaluate** estimates of measurements as reasonable or unreasonable.

What Do You Think?

When driving a vehicle, you often mentally measure distances and times. When investigating vehicle collisions, police officers take actual measurements at the scene. For example, the length of skid marks help officers to calculate the speed at which a vehicle was traveling.

- **Two students measure the length of the same object. One reports a length of 3 m, the other reports a length of 10 m. Has one of them made a mistake?**
- **If the students reported measurements of 3 m and 3.01 m, do you think one of them has made a mistake?**

Record your ideas about these questions in your *Active Physics* log. Explain your reasoning. Be prepared to discuss your responses with your small group and the class.

Remember to begin a new page in your *Active Physics* log each time you begin a new section. Write *Section 2 Measurement* at the top of the new page. Also record the section and page number in your *Table of Contents*.

Investigate

In this *Investigate*, you will measure a given distance by various techniques. You will have to determine which technique is best and why it is the best. You will also use estimation to decide if certain measurements are reasonable or not.

1. Your teacher and class will select and agree on a cleared distance along the floor of the cafeteria, corridor, or path around the classroom.

2. Each group will have a member pace off the distance. That is, count the number of strides it takes you to cover the marked-off distance.

   a) Record the number of strides in your log.

3. Have a group member measure the length of your stride using a meter stick. By finding the length of your stride, you are making a calibration, or a scale for a measuring instrument.

   a) Record your measurement in your log.

4. Use the number of strides you took in *Step* 2 and the length of your stride to compute the distance in meters.

   a) Record your calculations.

5. List the results of the measurements made by all the groups on the board.

   a) Do all the measurements agree? By how much do the results vary?

   b) Why do you think there are differences among the measurements made by different groups? List as many reasons for the differences in measurements as you can.

   c) Suggest a way of improving your measurements. If all groups try your method, how will the range of measurements change this time?

6. Measure the selected distance with a single meter stick. You will have to move the meter stick over and over.

   a) Record your measurement in your log.

7. List the results of the measurements made by all the groups on the board.

   a) Do all the measurements agree? By how much do the results vary?

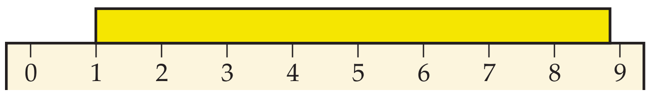
   b) Why do you think there are differences among the measurements made by different groups? List as many reasons for the differences in measurements as you can.

   c) Suggest a way of improving your measurements. If all groups try your method, what will the range of measurements be this time?

   d) What do you think would happen if each group were given a very long tape measure? List possible values the different teams may get. Do you think each group would get the exact same value?

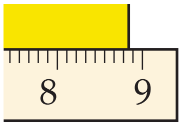
   e) Can you develop a system that will produce measurements, all of which agree exactly, or will there always be some difference in measurements? Justify your answers.

8. A difference in measurement close to a certain accepted value is called an error. Physicists identify two kinds of errors in measurement. An error that can be corrected by calculation is called a *systematic error*. For example, if you measured the length of an object starting at the 1 cm mark on a ruler instead of at the end of the ruler, you could correct your measurement by subtracting 1 cm from the final reading on the ruler.



Subtract 1 cm from the final reading.

An error that cannot be corrected by calculation is called a *random error*. No measurement is perfect. When you measure something, you make an approximation close to a certain accepted value. Random errors exist in any measurement. But you can estimate the amount of uncertainty in measurements that random errors introduce. Scientists provide an estimate of the size of the random errors in their data.



Make an approximation.

a) When measuring the hallway or class, did you have any systematic errors?

b) Estimate the size of your random errors using each technique.

9. Sometimes a precise measurement is not needed. A good estimate will do. What is a good estimate?

> Example:
>
> - Suppose one of your friends estimates that a single-serving drink container holds 5 kg (weighing about 11 lb) of liquid.
>
>   This is not a good estimate. It is unreasonable. A mass of 5 kg, or a weight of 11 lb, is about the weight of a bowling ball or a turkey. A single-serving drink weighs much less than this.

Use your common sense and prior knowledge to judge if the following measurements are reasonable. Explain your answers.

a) A college football player has a mass of 100 kg (weighing about 220 lb).

b) A high-school basketball player is 4 m (13 ft) tall.

c) Your teacher works 1440 min every day.

d) A poodle has a mass of 60 kg (about 132 lb).

e) Your classroom has a volume of 150 m^3 (about 5300 ft^3).

f) The distance across the school grounds is 1 km (about 0.6 mi).

g) On a rural road, while driving 50 mi/h (about 80 km/h), you encounter a tractor moving very slowly. You are about ¼ mi (0.4 km) away when you see that another automobile is coming toward you, travelling at 50 mi/h. Is it safe to pass the tractor?

h) While driving your pickup truck on a rural road, you approach a narrow bridge and see you will reach it at the same time as a dump truck that is coming from the opposite direction. What must you estimate in order to decide whether to stop and wait for the dump truck to cross the bridge first, or to go ahead and squeeze by the dump truck while on the bridge?

i) You are driving a motor home with bicycles standing upright in a bicycle rack mounted on the roof. A sign before the entrance to a tunnel states that the maximum height is 21 ft (6.4 m). Will your automobile make it safely through the tunnel?

Physics Talk

ERRORS IN MEASUREMENT

Random Errors

There is no exact measurement. In the *Investigate*, when you used your stride length as the measuring tool, the distance of the hallway was different for many of the groups. If you tried to improve the measurement by using a meter stick, you found that there were still differences in the measurement. Even if you had used a tape measure, there would still have been differences in your measurements.

Physicists know that all measuring tools produce **random errors**, or errors that cannot be corrected by calculating. It is the responsibility of the student scientist to record all the values of a measurement and recognize that the data will include random errors. Every time you measure the length of your desk, you might find that the measurement is different from a previous value by 0.1 cm. This difference could be in either direction (± 0.1 cm). You can use a more precise ruler and that may decrease this random error or uncertainty to only 0.05 cm (± 0.05 cm). However, the uncertainty can never be completely eliminated.

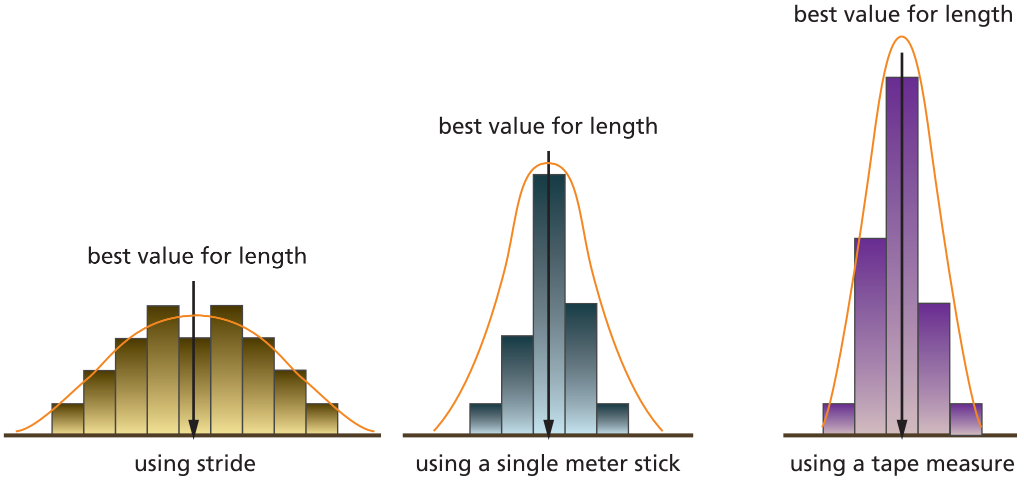
Both the measuring tool and the person doing the measuring are responsible for the uncertainty. A meter stick that has only the centimeters noted would have a greater uncertainty than a meter stick that has the millimeters noted. A meter stick that has millimeters noted may still have a large uncertainty if the person using it is not very careful in aligning the meter stick with the length being measured.

Physics Words

random error: an error that cannot be corrected by calculation.

In your measurement of the distance, you found different distributions of measurement. If you made histograms as shown below, of the length of a hallway using your stride (left figure), the meter stick (middle figure) or a tape measure (right figure), you can get a sense of the uncertainty in each type of measurement. The middle value is probably the "best guess" for the length of the room, but there will always be an uncertainty surrounding that value, as shown by the spread to the left and right of the middle value.

Length of the Hallway



Systematic Errors

There are also **systematic errors**. If you mistake a yardstick for a meter stick and report your measurement as 4 m, when in fact it is 4 yd, that is a systematic error. Every measurement you record with that yardstick will have this error. Systematic errors can be avoided or can be corrected by calculating.

Accuracy and Precision

In shooting arrows at a target, you can have **accuracy** and **precision** by getting all the arrows in the bull's-eye (left figure). You can have precision, but not accuracy by having all the arrows miss the bull's-eye by the same amount (middle figure). You can also have accuracy, but without precision by having all the arrows surrounding the bull's-eye spread out over the area (right figure). Notice that here the average position is the bull's-eye (accuracy), but not one of the arrows actually hit the bull's eye (precision).

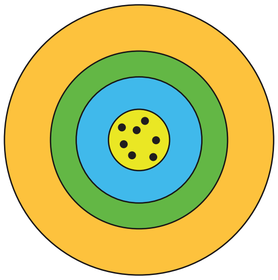
Measurements can also vary with accuracy and/or precision just as the arrows in the target.

Physics Words

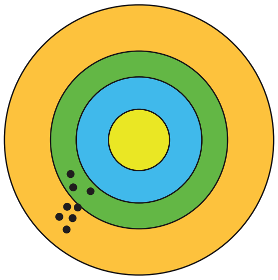
systematic error: an error produced by using the wrong tool or using the tool incorrectly for measurement and can be corrected by calculation.

accuracy: an indication of how close a series of measurements are to an accepted value.

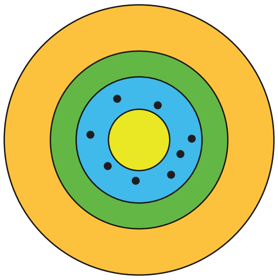
precision: an indication of the frequency with which a measurement produces the same results.



accurate and precise



precise not accurate



accurate but not precise

People do not always need the same level of precision in their measurements. One decision you must make is how precise a measurement you want. For example, in motor racing, horse racing, or Olympic skiing, time has to be measured to the thousandths or tens-of-thousandths of a second. But when a painter estimates the time required to paint a customer's house, she or he may only need to know the time within a few hours. As you increase the need for precision, the measurement becomes more difficult (and often, more expensive to make).

SI System

In *Active Physics*, you will be using the International System of Units. The units are known as SI units, abbreviated from *Le Système International d'Unités*. This is the system of units that is used by scientists. The system is based on the metric system. All units are related by some multiple of ten. There are seven base units that can be combined to measure all scientific properties. The base units that you will use in *Active Physics* are shown in the table.

Quantity	Unit	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
temperature	kelvin	K
current	ampere	A

You will also be using other units that are a combination of these base units. You will be introduced to these units when you need to use them. The best way to learn units is to use them frequently and correctly. It is not helpful to memorize lots of units.

In this section, you measured the length of a distance in meters. The meter (m) is the base unit of length. Other units that you will use for measuring and describing length are the kilometer (km), centimeter (cm), and millimeter (mm). These three units are made up of the base unit meter and a prefix.

An important feature of the metric system is that there is a single set of prefixes that relates larger and smaller units. All the prefixes are related by some power (multiple) of ten.

Prefix	Symbol	Multiple of ten by which base unit is multiplied	Example
kilo	k	$10^3 = 1000$	1 km = 1000 m 1 m = 0.001 km
centi	c	$10^{-2} = 0.01$	1 cm = 0.01 m 1 m = 100 cm
milli	m	$10^{-3} = 0.001$	1 mm = 0.001 m 1 m = 1000 mm

Driving the Roads and United States Units of Measurement

The United States does not use the metric system for everyday measurements. Distances along the road are measured in feet, yards, or miles. Speed limits are posted in miles per hour rather than kilometers per hour, as they are in many other countries.

Below are some conversion factors for length in United States measurements.

12 inches (in.) = 1 foot (ft)

3 feet = 1 yard (yd)

5280 feet (1760 yd) =
1 United States statute mile (mi)

In this chapter, *Driving the Roads,* United States measurements will be used to express distances and speeds with respect to driving and traffic. In the classroom, you will use SI units for measuring.

When obeying speed-limit signs, it is important to know what units of measurement are being used.

Checking Up

1. Explain the difference between systematic and random errors.
2. Explain why there will always be uncertainty in measurement.
3. What would the positions of arrows on a target need to be to illustrate measurements that are neither accurate nor precise?

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
◆	◆		

Precise Measurements and Olympic Records

The events in the Olympic Games depend on precise measurements of distances and times to determine who will receive a gold medal. If one skier has a time that is 3/1000 of a second better than another skier, this can be the difference between a gold and silver medal. In running and swimming, the athletes compete side by side and whoever is first wins the gold. However, the record times are compared from one Olympics to the next. If a swimmer in one Olympics beats the old record from 4 or 8 years earlier by only 1/1000 of a second, the swimmer has the new world record. But did that swimmer really swim faster than the prior record holder? Not necessarily. By following a discussion including measurements and uncertainty, you will find that the new world record holder may actually be slower than the old record holder if the time difference was only 1/1000 of a second.

Every four years, the Summer Olympics are held in a different city, and a new swimming pool must be built. The length of the swimming pool must be 50 m. You know from your investigation that every measurement has an uncertainty associated with it. When a pool is built for the Summer Olympic Games, do you think that pool's length could vary by 1 m? If so, the pool could be 49 m or 51 m in length. This is a huge difference. You can be sure that they build the pools to be closer to 50 m than 49 m.

1. What is the range of lengths for 50-m pools that have an uncertainty of ± 10 cm? ±1 cm? ± 1 mm? (For example, if the uncertainty of the pool were ± 1 m, the range of lengths would be 49–51 m.)

Suppose an Olympic pool is accurate to ± 1 cm. In one Olympics, the pool could be 49.99 m, while in another Olympics it could be 50.01 m. This does not seem to be a big difference and it seems like an accurate 50-m pool. It means that in one Olympic Games in one city, the swimmers may actually swim 50.01 m while in another Olympics in another city, the swimmers may actually swim only 49.99 m.

2. How much extra time does it take to swim 50.01 m than 49.99 m (a difference of 2 cm)? Assume a good swimmer can swim 50 m in 25 s.

The 1500-m race requires a swimmer to swim 30 lengths of the pool. If the pool in one Olympic Games is 50.01 m and in another Olympics the pool is only 49.99 m, then one swimmer will be swimming an extra 2 cm for every lap. In one Olympics, the swimmer may be swimming a total of 60 cm more than the other swimmer.

3. Estimate how long it takes to swim 60 cm. Assume a good time for the 1500-m race is 15 min.

4. In watching the Olympic Games, you hear that someone just broke the record for the 1500-m swim by 1/1000 of a second.

Explain how it is possible that this person (the new record holder) may actually be slower than the previous record holder. (Can it be that the new record holder is swimming in a shorter 50-m pool than the prior record holder?)

Comparing records for the 1500-m swim from one Olympics to the next depends on the length of the pools that the swimmers race in. The length of each pool cannot be exactly 50 m.

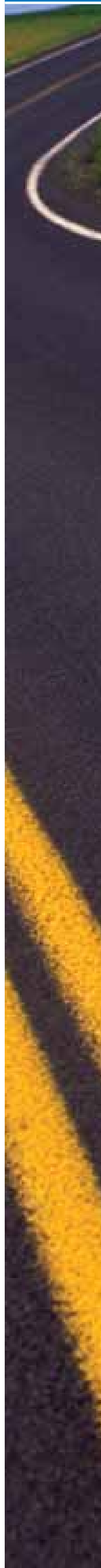
5. Write a letter to the Olympic commission addressing this issue. Include in your letter a solution to this problem that you have discovered. Including calculations of what would happen if the pools were built with an accuracy of 0.5 cm or 1 mm would make your letter more persuasive. You may also want to include something about the additional cost of making a 50-m pool this much more accurate.

What Do You Think Now?

At the beginning of this section, you were asked to think about the following:

- **Two students measure the length of the same object. One reports a length of 3 m, the other reports a length of 10 m. Has one of them made a mistake?**
- **If the students reported measurements of 3 m and 3.01 m, do you think one of them has made a mistake?**

How would you answer these questions now? Review what you have learned about random errors in measurement. How can you reduce random errors?



Physics
Essential Questions

What does it mean?

Suppose your friend mistakes a yardstick for a meter stick and measures the length of an intersection in your neighborhood. Is this error random or systematic? Which of these types of errors affect precision or accuracy?

How do you know?

Suppose you want to buy some gold jewelry. The jeweler tells you that the jewelry contains exactly 1 oz of gold. How do you know that the jeweler cannot be sure that it is exactly 1 oz?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
All physics includes measurements	Change and constancy	* Experimental evidence is consistent with models and theories

* All physics knowledge is based on experimentation. All experiments require measurements. How can you trust experiments if all measurements have uncertainties?

Why should you care?

What are the consequences of not estimating stopping distances accurately, or the width of a space between your vehicle and other vehicles while driving?

Reflecting on the Section and the Challenge

A measurement is never exact. When you make a measurement, you estimate that measurement. All measurements have systematic and random errors. An example of a systematic error might be using a measuring tape that stretches a little bit when it is pulled tightly. But if you know the amount of stretch, then you can correct the measurement using calculations. In contrast, random errors are part of any measurement process because you can only approximate a mark on a meter stick or the time on a stopwatch, with an accuracy to the closest decimal place. You can try to minimize random errors, but you cannot eliminate them entirely.

When a speed limit is 60 mi/h (about 100 km/h), you may find that sometimes you drive at 58 mi/h while other times you drive at 62 mi/h. These differences are random errors as you try to hold the speed constant. If a police officer stops you because you were driving at 75 mi/h (about 120 km/h) in a 30 mi/h zone, you will not be able to convince her that this was just an uncertainty in your measurement. Uncertainties in speeds may be something that you wish to include in your presentation or report.

Physics to Go

1. Get a meter stick and centimeter ruler. Find the length of five different-sized objects, such as a door, a tabletop, a large book, a pencil, and a stamp.
   a) Which measuring tool is best for measuring each object?
   b) Estimate the uncertainty in each measurement.
2. Count the number of strides it takes to walk around your classroom and estimate the length of each stride. Calculate the size of the room by multiplying the number of strides taken by the estimated length of each stride. Estimate your accuracy. Then check your accuracy with a meter stick.
3. Give an estimated value of something that you and your friend would agree on. Then, give an estimated value of something that you and your friend would not agree on.
4. An oil tanker is said to hold five million barrels of oil. In your estimate, how accurate is the measurement? Suppose each barrel of oil is worth $100. What is the possible uncertainty in value of the oil tanker's oil?
5. Choose five food products. How accurate are the measurements on labels?
6. Are the following estimates reasonable? Explain your answers.
   a) A 2-L bottle of soft drink is enough to serve 12 people at a meeting.
   b) A mid-sized automobile with a full tank of gas can travel from Boston to New York City without having to refuel.
7. If you are off by 1 m in measuring the width of a room, is that as much as an error as being off by 1 m in measuring the distance between your home and your school?
8. You are driving on a highway that posts a 65 mi/h (105 km/h) speed limit. The speedometer is accurate within 5 mi/h (8 km/h).
   a) What speed should you drive as shown on the speedometer to guarantee that you will not exceed the speed limit?
   b) What could a passenger in the vehicle do while you are driving to estimate how accurate the speedometer is? (Hint: The road has mile markers, and the passenger has a wristwatch that shows seconds.)
9. ***Preparing for the Chapter Challenge***

   Many accidents are caused by speeding. To limit the number of collisions, police officers give speeding tickets to drivers. If the speed limit were 30 mi/h (50 km/h) in a residential neighborhood, a person may get a ticket for driving at 40 mi/h (65 km/h). Legally, they could also get a ticket for traveling at 31 mi/h (51 km/h). Given the uncertainties in measurements (the driver has to keep the gas pedal "just right"), you may wish to mention how these uncertainties are a part of safe driving. You may wish to explain why driving 31 mi/h in a 30 mi/h zone does or does not warrant a ticket. If you do not think that 31 mi/h deserves a ticket, you will need to explain what speed should get a ticket and why.

Inquiring Further

1. **Measurement and national standards**

   The National Institute of Standards and Technology (NIST) is the nation's measurement laboratory. It provides companies and other organizations with references to use to check the accuracy of their equipment.

   What type of certainty in measurement would you expect if you were buying vegetables by the pound, gas by the gallon, or carpeting by the yard? Investigate what types of measurement standards are regulated by the government. Report to your class the certainty of the measurements that are used in industry and the marketplace.

2. **Random error and number of measurements**

   People who study the statistics of measurement have shown that, if you make N independent measurements of the same object, then the size of the random error decreases as:

$$\sqrt{\frac{1}{N}}$$

   Plot histograms, graphs that measure frequency distribution, of the measurement of desks in the classroom and see if the size of the random error decreases as you measure more and more identical desks.