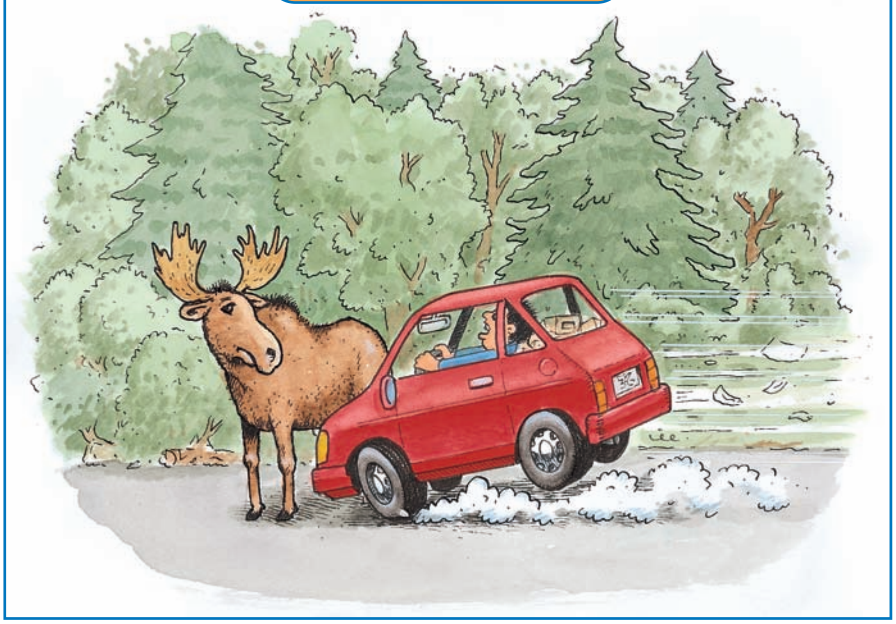
Section 5 Negative Acceleration: Braking Your Automobile

What Do You See?



Learning Outcomes

In this section, you will

- **Plan** and carry out an experiment to relate braking distance to initial speed.
- **Determine** braking distance.
- **Examine** accelerated motion.

TECH LAB
Click Here

What Do You Think?

In recent years, more than 80 percent of speed-related traffic deaths happened on secondary highways (such as two-lane, rural roads). Imagine you are driving at the speed limit on a secondary highway, and you suddenly see an animal crossing the road ahead of you. Suppose you cannot swerve to miss the animal because of trees on each side of the road.

- **What factors must you consider to determine if you will be able to stop in the distance between you and the animal to avoid hitting it?**

Record your ideas about this question in your *Active Physics* log. Be prepared to discuss your response with your small group and the class.

Investigate

In this *Investigate*, you will plan and carry out an experiment to determine the relationship between the initial speed and the braking distance of an automobile.

1. Knowing how far your automobile will travel after you have stepped on the brake pedal is important. One factor that may have an impact on braking distance is the initial speed of the automobile.

The initial speed is the speed at which you begin to apply the brakes. Braking distance is the distance required to bring the vehicle to rest once the brakes are applied. In your investigation, the initial speed will be the speed at the point at which you begin your measurement of braking distance. You will collect data to study the relationship between initial speed and braking distance.

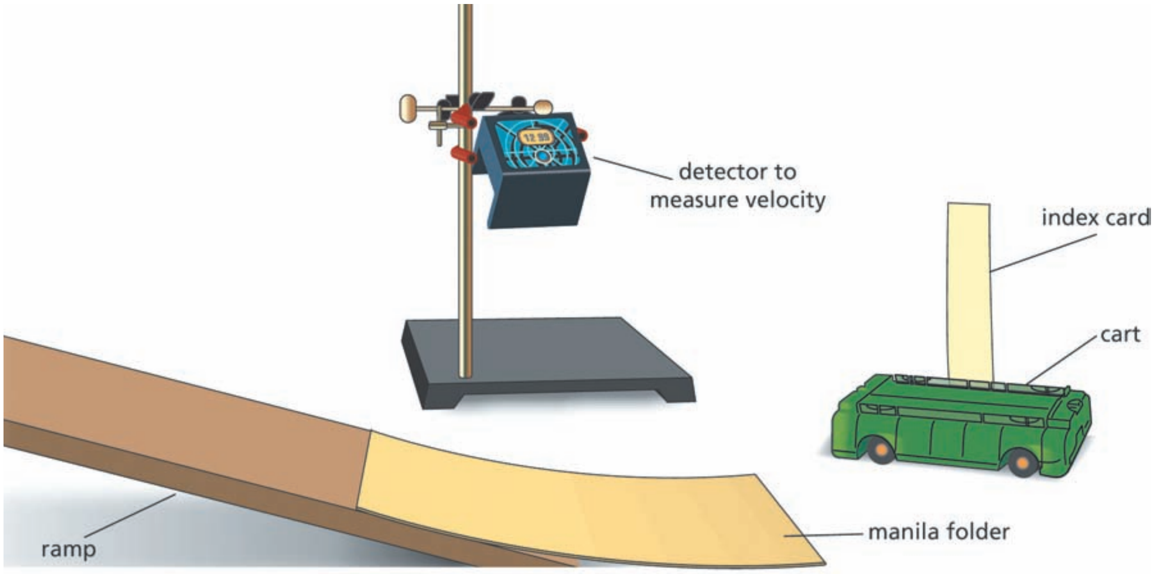
a) What would a graph of braking distance vs. initial speed look like? Sketch a graph that shows what you think the data would show. (Place the initial speed on the *x*-axis and the braking distance on the *y*-axis.) While sketching the graph, imagine what would happen to the braking distance for a slow-moving vehicle, a faster-moving vehicle, and a very fast-moving vehicle.

b) Provide an explanation for the way you sketched the graph.

2. Your teacher will provide your group with equipment similar to the equipment shown in the illustration below. Discuss with your group how you could use the equipment to study the relationship between initial speed and braking distance.

To plan your experiment, consider the following:

- How will you vary the initial speed of the cart (that is, the velocity the cart has at the bottom of the hill when the brakes are applied)?
- The cart does not really have brakes applied by a driver, but the cart will stop on its own. Friction plays the role of brakes in the cart.
- How will you determine the initial speed of the cart just before it begins braking?
- How will you measure the braking distance? (What tool should you use? Should you measure from the front or the back of the cart? How accurate will you make your measurements?)
- How many different initial speeds will your group need to examine to find a pattern?
- How many trials should you perform at each initial speed?
- What will each group member be responsible for?
- How will you organize your data?



3. After discussing these questions in your group, develop a plan for what your group will do. Your teacher may ask you to either draw a flowchart or an outline showing the steps you will take.

4. Set up your equipment and perform your experiment.

   a) Record both numerical data and observations in your *Active Physics* log.

Place the ramp on the floor in a way that does not obstruct people's ability to walk around the classroom. Do not block the emergency exit.

If you are setting up the ramp on the table, provide some means to contain the cart and prevent it from falling off the table.

5. Use the data you collected to complete the following:

   a) Draw a graph showing how the braking distance depends on the initial speed. Place the initial speed on the horizontal axis and the braking distance on the vertical axis.

   b) How does the braking distance change with initial speed?

   c) How does your graph compare to the graph you sketched in *Step 1.a*)?

   d) Compare your graph with those of other groups. What are some similarities and some differences?

   e) Does looking at the other groups' graphs make you feel more confident or less confident about your data? Explain your answer.

6. Select two values of initial speed from your graph, with one value approximately twice the value of the other. Note the braking distance which corresponds to each initial speed.

   a) What is the effect of doubling the initial speed on the distance traveled during braking?

7. Select two values of initial speed from your graph, with one value approximately three times as fast as the other. Note the braking distance which corresponds to each initial speed.

   a) What is the effect of tripling the initial speed on the distance traveled during braking?

   b) Predict how going four times faster will affect the braking distance.

8. Use the data on the sports car provided at the end of this chapter on pages 116-117 to answer the following:

   a) Where is the braking data located?

   b) The braking distance is shown for two speeds. The ratio of the two speeds is 80 mi/h : 60 mi/h. This ratio is $80/60 = 1.33$. This is an increase of 133 percent. Do you expect the ratio of the braking distances to also be in the ratio of $80/60 = 1.33$? What is the ratio of the braking distances? How does it compare with the ratio of the two speeds?

   c) How does this data correspond to what you found in your experiment?

Physics Talk

SPEED AND BRAKING DISTANCE

Negative Acceleration and Positive Acceleration

From your investigations in this section and the previous section, you observed that acceleration of an object is determined by how fast the velocity of the object changes with respect to time.

This is represented mathematically with the following equation:

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{change in time}}$$

$$a = \frac{\Delta v}{\Delta t}$$

The symbol Δ stands for "change in." This equation can also be written in the following way:

$$a = \frac{v_\mathrm{f} - v_\mathrm{i}}{\Delta t}$$

where a is acceleration,
v_i is initial velocity and
v_f is final velocity.

Physics Words

negative acceleration: a change in the velocity with respect to time of an object by decreasing speed in the positive direction or increasing speed in the negative direction.

The cart in your experiment undergoes a **negative acceleration**, because the final velocity v_f is zero when it stops, which is less than its initial velocity v_i. For an object stopping, $v_\mathrm{f} = 0$. Therefore,

$$a = \frac{0 - v_\mathrm{i}}{\Delta t} = \frac{-v_\mathrm{i}}{\Delta t}$$

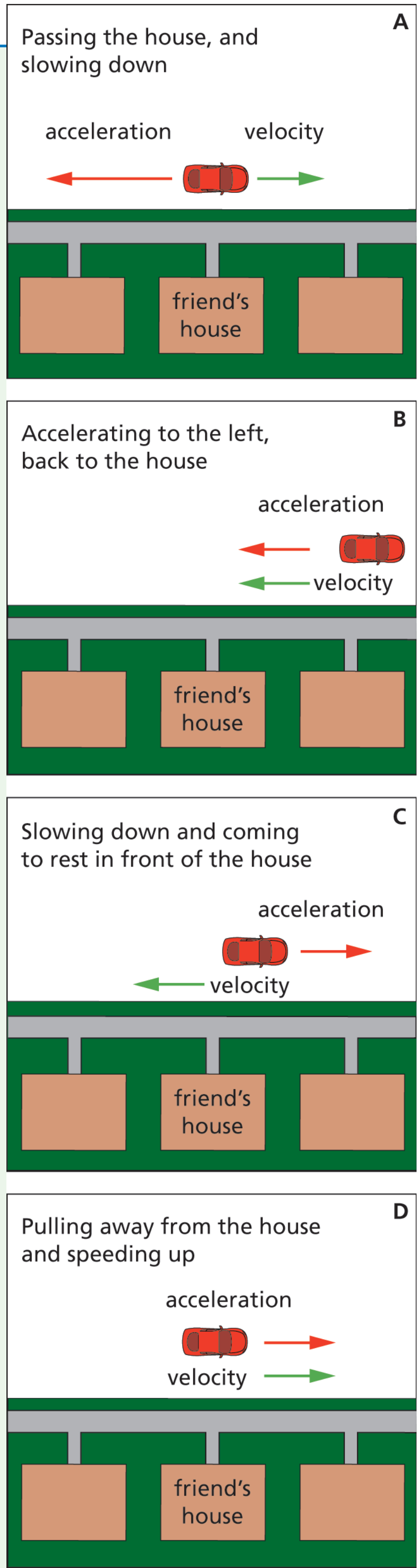
Recall that you read that sometimes people use the term acceleration to describe speeding up, and they use deceleration to describe something slowing down. In order to be clear about meanings in physics, the terms positive acceleration and negative acceleration are used. Positive represents one direction and negative represents the opposite direction. Furthermore, an object could have a negative acceleration by decreasing its speed in the positive direction or increasing its speed in the negative direction.

Imagine you are driving an automobile to pick up a friend to go to the movies. You know the street your friend lives on and the house number. You slowly drive down the street but accidentally pass your friend's house. Realizing your mistake, you slow down and stop. When you are moving forward (a positive speed), but slowing down, the automobile has an acceleration backward (a negative acceleration).

After the automobile has stopped, you check to see if it is safe, and then start to back up (with the automobile still pointing forward). When you start to increase your speed in the opposite direction you are moving backward (a negative speed), and your acceleration is also backward (a negative acceleration) until you are traveling with constant speed in reverse.

As you slow down again to stop when you are approaching the correct house, you are still moving backward (a negative speed), but you now have a forward (positive) acceleration to bring the automobile to a stop.

Finally, your friend gets into the automobile, and you now pull away going forward with a forward (positive) acceleration until you are driving with a forward (positive) constant speed to the movies.



Motion of Car	Time (s)	Velocity of car (ft/s)	Acceleration of car (ft/s^2)	Positive or Negative
Car moving forward and slowing down (Diagram A)	0	+6	$\frac{v_f - v_i}{\Delta t} = \frac{(+4)-(+6)}{1} = -2$	negative acceleration
	1	+4		
Car moving forward, slowing down and car stops	2	+2	$\frac{v_f - v_i}{\Delta t} = \frac{(0)-(+2)}{1} = -2$	negative acceleration
	3	0		
Car moving backward and speeding up (Diagram B)	4	–2	$\frac{v_f - v_i}{\Delta t} = \frac{(-4)-(-2)}{1} = -2$	negative acceleration
	5	–4		
Car moving backward and slowing down (Diagram C)	6	–6	$\frac{v_f - v_i}{\Delta t} = \frac{(-4)-(-6)}{1} = +2$	positive acceleration
	7	–4		
Car moving backward and stopping	8	–2	$\frac{v_f - v_i}{\Delta t} = \frac{(0)-(-2)}{1} = +2$	positive acceleration
	9	0		
Car moving forward and speeding up (Diagram D)	9	0	$\frac{v_f - v_i}{\Delta t} = \frac{(+2)-(0)}{1} = +2$	positive acceleration
	10	2		

In this example, you can see that a negative acceleration can sometimes decrease the speed of an automobile $(t = 0 \text{ to } t = 3)$ or increase the speed of an automobile $(t = 4 \text{ to } t = 5)$, but it always decreases the velocity of the automobile by exactly 2 ft/s every second $(\text{from} + 6 \text{ to} + 4 \text{ to} + 2 \text{ to } 0 \text{ to} - 2 \text{ to} - 4)$.

Calculating Braking Distance

Using the definition of velocity and acceleration, you can derive an equation for the braking distance when a vehicle comes to rest. The equation is shown below:

$$v_f^2 = 2ad + v_i^2$$

v_f is the final velocity of the car.

v_i is the initial velocity of the vehicle. Notice that it must be squared. This is the same as multiplying it by itself, $v_i^2 = v_i \times v_i$.

a is the acceleration. It is a negative acceleration.

d is the braking distance.

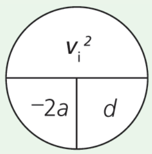
Because the final velocity of a vehicle is zero after the car comes to a stop, you may put a zero in for the final velocity, and of course zero times zero is still zero.

$$0 = 2ad + v_i^2$$

or

$$v_i^2 = -2ad$$

You can use the helpful circle to solve for any of the variables in this equation as well.



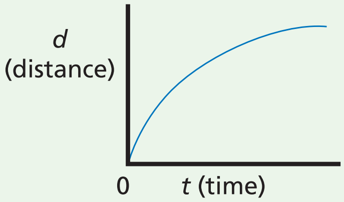
Of all the equations in your first year of physics, this one may have the greatest impact on your safety. Understanding this equation may one day even help to save your life! From this equation, you can see that if you double the initial velocity, then the braking distance *d* will have to quadruple. If you triple the initial velocity, then the braking distance *d* will be nine times as great.

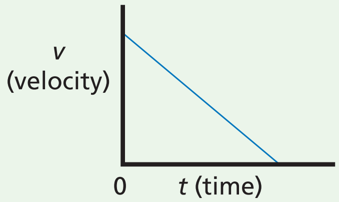
You probably found in the *Investigate* that doubling the speed increased the distance traveled while the vehicle was braking by about a factor of four and that tripling the speed increased the braking distance by about a factor of nine. Look at the data for the sports car. The speed increased by 1.33 while the braking distance increased by approximately 1.33 × 1.33 = 1.77. Experiments completed with a great deal of care, ensuring that the braking acceleration is constant between trials, find that this relationship is true.

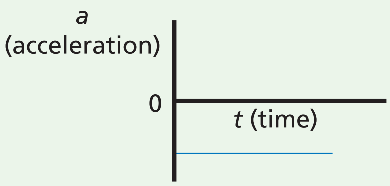
(The v^2 relationship is derived assuming constant acceleration, which is *approximately* true for real automobiles in everyday stopping situations. The $v^2 = -2ad$ equation models reality very closely, and is therefore useful for describing braking.)

How can knowledge of the v^2 relationship save many lives? If you were to decrease your speed to one third your original speed, you would need only one ninth of the braking distance. If you double the speed you do not require double the distance for the car's brakes to stop the car, but four times the distance. Decreasing your speed can save lives because of the significant effect the slower speed has on braking distance.

You have seen how equations can model the motion of an automobile braking. An automobile with a negative acceleration can also be described using graphs of distance vs. time, velocity vs. time, and acceleration vs. time as shown below.







In the velocity vs. time graph, you can see that the velocity is decreasing as the automobile comes to rest. Notice that "at rest" is equivalent to a velocity equal to 0. You should also notice that the slope of the graph is constant. This implies that the acceleration is constant since the slope of the *v-t* graph is equal to the acceleration. Finally, notice that the slope is negative (sloping downward) which implies that the acceleration is negative.

In the acceleration vs. time graph, you can see that the acceleration is constant and negative.

In the distance vs. time graph, you can see how the change in distance for a given time changes as the speed changes. Notice that the slope at the beginning times is very steep, corresponding to a large velocity. Toward the end, the slope becomes 0, corresponding to the car stopping. (Thus, the graph is a curve.)

Checking Up

1. If a vehicle is traveling at constant velocity and then comes to a sudden stop, has it undergone negative acceleration or positive acceleration? Explain your answer.
2. Explain how you know that increasing the velocity of an automobile increases the braking distance.
3. Why is the term negative acceleration used instead of deceleration?

Active Physics **Plus**

+Math	+Depth	+Concepts	+Exploration
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Motion Equations

Five motion equations can describe all the relations among position, velocity, and constant acceleration. The equations are all derived from the definitions of velocity and acceleration.

$$d = \bar{v}t$$

$$v_\mathrm{f} = at + v_\mathrm{i}$$

$$\bar{v} = \frac{v_\mathrm{f} + v_\mathrm{i}}{2}$$

$$d = \frac{1}{2}at^2 + v_\mathrm{i}t$$

$$v_\mathrm{f}^2 = 2ad + v_\mathrm{i}^2$$

The first equation is a restatement of the definition of average velocity. (The v with a bar over the top is a shorthand way of writing v_{average}.) The second equation is a restatement of the definition of acceleration. The third equation is for average velocity when there is constant acceleration. The fourth equation helps to determine distance traveled if you know the acceleration and time without the need for first finding the final velocity. The fifth equation relates the stopping distance to the acceleration and velocities without the need for calculating the time.

The fifth equation can be derived from the other four equations using algebra. Assume that the initial velocity equals zero to ease the mathematics.

You may want to try to derive the equation with acceleration not being zero, using the same approach.

These are the variables in the motion equations: d, t, a, v_f, v_i and v_{average}.

$$v_\mathrm{f} = at + v_i$$

Assuming $v_\mathrm{i} = 0$,

$$v_\mathrm{f} = at$$

Square each side

$$v_\mathrm{f}^2 = a^2t^2$$

$$v_\mathrm{f}^2 = 2a\left(\frac{1}{2}at^2\right)$$

$$v_\mathrm{f}^2 = 2ad$$

If the acceleration is constant and you are able to find or are given any three of these variables, you can use the motion equations to solve for the other two variables and completely describe the motion of the object. The object can be an automobile, an animal, a galaxy, or a cell. The motion equations describe them all.

Sample Problem I

A softball pitcher accelerates a ball from rest to a speed of 25 m/s over a distance of 1.8 m. What is the ball's acceleration?

Strategy:

Using the fifth equation of motion, derived from the other four equations using algebra, and knowing v_i, v_f, and d, you can solve for acceleration.

Given:

$$v_\mathrm{i} = 0$$

$$v_\mathrm{f} = 25 \text{ m/s}$$

$$d = 1.8 \text{ m}$$

Solution:

Knowing that $v_\mathrm{f}^2 = 2ad + v_\mathrm{f}^2$, and that $v_\mathrm{i} = 0$, the equation becomes $v_\mathrm{f}^2 = 2ad$.

$$v_f^2 = 2ad$$

$$a = \frac{v_f^2}{2d}$$

$$= \frac{(25\text{m/s})^2}{2(1.8 \text{ m})}$$

$$= \frac{25^2(\cancel{\text{m}}/\text{s})(\text{m/s})}{2(1.8 \ \cancel{\text{m}})}$$

$$= 173.6 \text{ m/s}^2 \text{ or } 170 \text{ m/s}^2$$

Therefore, the acceleration of the softball is $170\,\text{m/s}^2$.

Sample Problem 2

During an auto race, a car with a speed of 75 m/s accelerates past another car at a rate of 3.0 m/s^2 for 4.0 s. How far does the car travel during this time?

Strategy:

Knowing the car's initial velocity, time, and acceleration you can use the fourth equation

$$d = \frac{1}{2}at^2 + v_i t$$ to determine

distance traveled without the need for first finding the final velocity.

Given: $v_i = 75$ m/s

$a = 3.0$ m/s^2

$t = 4.0$ s

Solution:

Using $d = \frac{1}{2}at^2 + v_i t$ and solving for d gives

$$d = \frac{1}{2}at^2 + v_i t$$

$$d = \frac{1}{2}(3.0\frac{\text{m}}{\cancel{\text{s}}^2})(4.0 \ \cancel{\text{s}})^2 + (75\frac{\text{m}}{\cancel{\text{s}}})(4.0 \ \cancel{\text{s}})$$

$$d = 324 \text{ m or } 320 \text{ m}$$

Therefore, the car travels 320 m, or almost one quarter of a mi.

1. When a jet lands on an aircraft carrier, its speed goes from 90.0 m/s to zero in 1.5 s as it is stopped by a cable running across the aircraft carrier's deck.
   a) If the direction the jet is traveling is positive, was the jet's acceleration positive or negative?
   b) What is the jet's acceleration during the stopping process?
   c) If the jet undergoes a constant acceleration while stopping, what is the jet's average speed?
   d) How far does the jet travel along the carrier's deck while it is being brought to a stop?
2. A race is held between a sports car and a motorcycle. The sports car can accelerate at 5.0 m/s^2 and the motorcycle can accelerate at 8.0 m/s^2. The two vehicles start the race at the same time and accelerate from rest.
   a) After 5.0 s, how fast is the sports car going?
   b) After 6.0 s, what distance will the motorcycle have gone?
   c) To make the race fair, the sports car starts 50.0 m ahead of the motorcycle. If the course is 200.0 m long, which vehicle wins the race? (Hint: The vehicle that covers its distance in the least time wins.)
3. A student on a skateboard pushes off from the top of a small hill with a speed of 2.0 m/s, and then goes down the hill with a constant acceleration of 0.5 m/s^2.
   a) After traveling a distance 12.0 m, how fast is the student going?
   b) How much time does it take the student to move a distance of 21.0 m while accelerating at this rate?

Graphing Models

You have been using graphs to better understand motion. You have seen that there is a relationship among corresponding *d-t*, *v-t* and *a-t* graphs.

The slope of a *d-t* graph of an automobile is equal to the velocity of the automobile.

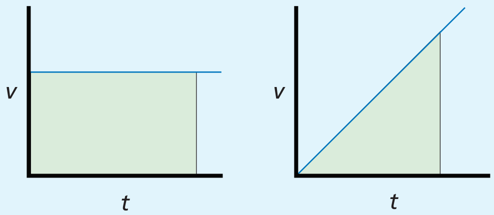
The slope of a *v-t* graph of an automobile is equal to the acceleration of the automobile.

Given a *d-t* graph, you can use this information to determine the *v-t* graph and the *a-t* graph as you have seen earlier.

The velocity vs. time graph can also tell you about the distance traveled.

In the following two velocity vs. time graphs, the shaded areas under the velocity vs. time graphs are equal to the distance traveled. This can be proven in the following way.

In the first velocity vs. time graph, the average velocity is constant, because the velocity does not change. With no change in velocity, the acceleration must be zero. The shaded area under the graph is equal to the distance traveled. The shaded area under the graph is the area of a rectangle (A = height × base). This area is (average velocity) × (time), which is the definition of distance traveled.



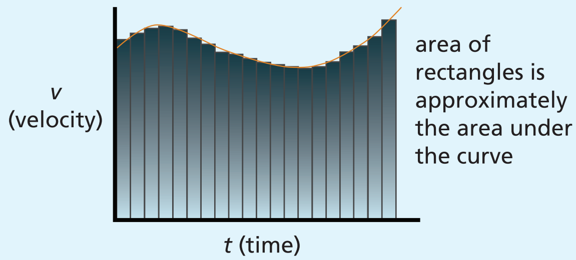
Velocity vs. Time

The second graph shows a constant acceleration. The area under the second graph is identical to the area of a triangle. The area of a triangle is ½ height × base. The base is the time. The height is the final velocity. One half the height is the average of the final velocity and the initial velocity of 0.

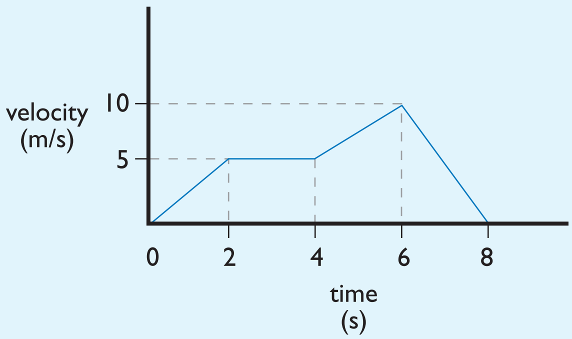
$$\frac{1}{2}\text{ height}\times\text{base}=\frac{1}{2}(\text{final velocity})\times(\text{time})$$
$$=(\text{average velocity})\times(\text{time})$$

$$\frac{1}{2}h\times b=\frac{1}{2}(v_{\mathrm{f}})\times(t)$$
$$=(v_{\text{average}})\times(t)$$

Once again, from the definition of average velocity (average velocity = distance/time), there is a way to calculate the distance traveled.



The area under a velocity vs. time graph is always equal to the distance traveled. For non-constant accelerations, the velocity vs. time graph is a curve. You can see in the diagram above how you can break a curve into a series of tiny rectangles that approximates the curve. The total area under the curve is approximately equal to the total area of all the rectangles. This is the beginning of your introduction to calculus—an advanced mathematics invented by Sir Isaac Newton, an English physicist and mathematician, to better understand physics.



For the velocity vs. time graph shown:

1. Describe the motion from $t = 0$ to $t = 8$ s.
2. Calculate the acceleration of the object for each 2 s.
3. Calculate the distance traveled for each 2 s. (Hint: For $t = 4$ to $t = 6$, the area under the curve is a trapezoid made up of both a rectangle and a triangle.)
4. Calculate the total distance traveled.

What Do You Think Now?

At the beginning of this section, you were asked the following:

- **What factors must you consider to determine if you will be able to stop in the distance between you and the animal to avoid hitting it?**

How would you answer this question now? After studying the equations of motion, how do you think velocity affects the time it takes to suddenly stop an automobile? According to the equation for braking distance, if you double an automobile's speed, what happens to the distance needed for a vehicle's brakes to bring the vehicle to a stop?

Physics
Essential Questions

What does it mean?

An automobile safety manual states that the braking distance increases with the square of the velocity of the vehicle. What does this mean? Why is this related to safe driving?

How do you know?

What evidence do you have that tripling the speed of an automobile will increase the braking distance by a factor of $3 \times 3 = 9$?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	Models	* Good, clear, explanation, no more complex than necessary

* Physics tries to use a few simply related principles to describe phenomena. Describing many different things requires a precision in language. In everyday language, you may use the words acceleration and deceleration. In physics, you use only the word acceleration. Describe the difference between positive and negative acceleration.

Why should you care?

Safe driving saves lives. How does knowing about the relationship between speed and braking distance help you to become a safe driver?

Reflecting on the Section and the Challenge

Safe driving requires the ability to stop safely. Some people think that if you triple your speed, the automobile will require triple the braking distance. You now know that it will take more than triple the braking distance – it is closer to nine times the braking distance!

You should be able to explain the importance of braking distance as it relates to speed. You should understand why slowing down is beneficial in terms of braking distance and what will happen to the required braking distance if you decrease your speed by one third.

You should always reduce your speed when driving through a school zone or a parking lot of a crowded supermarket. Slowing down decreases your braking distance and will protect unaware pedestrians.

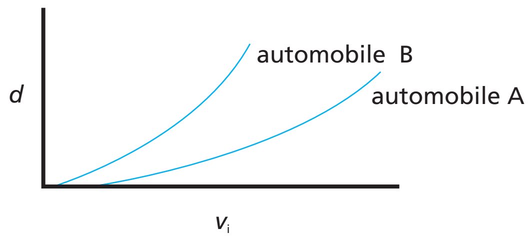
In your *Chapter Challenge*, you can now demonstrate your understanding of the relationship of speed to braking distance to the Active Driving Academy.

Physics to Go

1. A student measured the braking distance of her automobile and recorded the data in the table. Plot the data on a graph and describe the relationship that exists between initial speed and braking distance.

Initial speed	Braking distance
5 m/s	4 m
10 m/s	15 m
15 m/s	35 m
20 m/s	62 m
25 m/s	98 m
30 m/s	140 m

2. Below is a graph of the braking distances in relation to initial speed for two automobiles. Compare qualitatively (without using numbers) the braking distances when each automobile is going at a slow speed and then again at a higher speed. Which automobile is safer? Why? How did you determine what "safer" means in this question?



3. An automobile is able to stop in 20 m when traveling at 30 mi/h. How much distance will it require to stop when traveling at the following:
   a) 15 mi/h? (half of 30 mi/h)
   b) 60 mi/h? (twice 30 mi/h)
   c) 45 mi/h? (three times 15 mi/h)
   d) 75 mi/h? (five times 15 mi/h)
4. An automobile traveling at 10 m/s requires a braking distance of 30 m. If the driver requires 0.9 s reaction time, what additional distance will the automobile travel before stopping? What is the total stopping distance, including both the reaction distance and the braking distance?
5. Consult the information for the sports car at the end of this chapter. This shows the stopping distance. How far would you expect this automobile to travel until coming to rest when brakes are applied at a speed of 30 mi/h?
6. Use the information for the sedan at the end of this chapter. Find the braking distances for 50 mi/h and 25 mi/h. Draw a graph using the different braking distances. Plot the speeds on the horizontal axis and the braking distances on the vertical axis.

7. Does the braking information for the sedan include the driver's reaction time? If it does not, then how much distance is added to the total braking distance, supposing that the driver has a ½ s reaction time? Who should let the consumer know about the ½ s reaction time— the information sheet or a driver training manual?
8. Apply what you learned in this section to write a statement explaining the factors that affect stopping distance. The total stopping distance includes the distance you travel during your reaction time, plus the braking distance. What do you now know about stopping that will make you a safer driver?
9. In a perfect experiment, your data would show that the braking distance is proportional to the square of the velocity. Real data is not perfect. Describe two possible sources of error and explain how they could have impacted your results.
10. How could you revise this experiment to study better/worse braking situations? Predict how your graph might change.
11. ***Preparing for the Chapter Challenge***

    Apply what you have learned in this section to write a convincing argument against excessive speed when approaching an intersection, a traffic light, crosswalk, school zone, or any other driving situation that may require sudden braking on your part. Excessive speed means you cannot stop in the available distance if necessary. What are the consequences of approaching these situations with excessive speed?

Inquiring Further

Reconstructing an accident

Collect newspaper clippings or summarize television news reports of traffic accidents in your city or town that involved automobiles and/or motorcycles. Become an accident investigator and imagine rewinding the events leading into the accident.

- What advice might you have given to the driver(s) involved about speed, reaction time, and braking distances, that would have enabled them to avoid the accident?
- In writing, comment on whether the accident might have been prevented simply by slowing down, or whether there were other contributing factors as well (such as icy roads). If there were other factors, would they be additional reasons for reducing speed?