Section 3 Spring Potential Energy: More Energy

What Do You See?

TECH LAB
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Learning Outcomes

In this section, you will

- **Calculate** the kinetic energy of a pop-up toy.
- **Calculate** the spring potential energy from the conservation of energy.
- **Calculate** the spring potential energy by using an equation.
- **Relate** spring potential energy with conservation of mechanical energy using an equation.
- **Recognize** the general nature of the conservation of energy as it involves heat, sound, chemical, and other forms of energy.

What Do You Think?

The concept of a "lift hill" for a roller coaster was developed in 1885. This was the initial hill that began a roller coaster ride. A chain or a cable often pulled the train up to the top of this hill.

- **How does the roller coaster today get up to its highest point?**
- **Does it cost more to lift the roller coaster if it is full of people?**

Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your responses with your small group and the class.

Investigate

Little pop-up toys are fun for all ages. You press the plunger, place it on a table, and "pop!" it flies into the air. In this inquiry investigation, you will determine the kinetic energy, *KE*, of the pop-up toy when it leaves the ground.

> ⚠ Be sure to use the toy safely, always placing it firmly on the table before releasing it and keeping your face and the faces of your classmates away from where the toy may jump or pop. Eye protection must be worn by everyone during this experiment.

1. Play with the toy to get a sense of how high it jumps.
   - a) What is the approximate height of a jump?
   - b) How consistent is the pop-up toy from one jump to the next?

2. Discuss ideas with your group to identify two distinct methods you can use to determine the *KE* when the pop-up toy leaves the table. One method will use the velocimeter. The second method will use a meter stick to measure the height of the jump.
   - a) Record your two methods in your *Active Physics* log. Since another team may want to understand what you have done, be quite careful to list all the steps. Indicate how all measurements are completed, and what is recorded or calculated.
3. Conduct the investigation into *KE* using both of your methods.
   - a) Record your results. If you changed your procedure during the experiment, you should also record any changes here. These modifications are similar to the *Process* step of your *Engineering Design Cycle*.
   - b) Compare the *KE* determinations from the two methods.
4. Measure the mass of the pop-up toy using a balance. Tape some coins to the top of the pop-up toy in order to approximately double its mass. The mass of a nickel is approximately 5 g. You can probably come close to doubling the mass of the pop-up toy by adding nickels.
5. Repeat the investigation and find the *KE* of the pop-up toy as it leaves the ground. Be sure the coins are taped securely on the toy. Retape after every two or three trials.
   - a) Why do you think that the heavier pop-up toy behaved differently? Use the terms *GPE* and *KE* in your explanation.

Remember to wear eye protection during this *Investigate* and to have team members step back before the toy is released.

6. Answer the following questions in your *Active Physics* log:
   - a) What is the *KE* and *GPE* of the toy when it sits on the table?
   - b) What happens to the *GPE* and *KE* of the toy as it rises from the table?
   - c) If the total energy of the toy is conserved, where does the *KE* and *GPE* come from as it rises?
   - d) Where is the toy when its *KE* is greatest and where is it when its *GPE* is greatest?
7. The pop-up toy had both *KE* and *GPE* as it rose above the table. While the toy was sitting on the table, it also had *spring potential energy*, *SPE*. This *SPE* was converted to *KE* when the toy leaped off the table. The *KE* then became increasing *GPE* and decreasing *KE* as the pop-up toy ascended and slowed down. Using the concept of conservation of energy from the last section, you notice that before popping up, the energy of the toy was all *SPE*. Just after popping up, it was all *KE*. When reaching the highest point, the energy was all *GPE*.

The total energy at all other points was the same as the total *SPE* before popping. The total *KE* just after popping or the total *GPE* at its peak also equals the spring potential energy before the toy pops. You can show this in a table. Total energy is conserved, but you now have spring potential energy, *SPE,* as another form of energy in addition to *GPE* and *KE*.

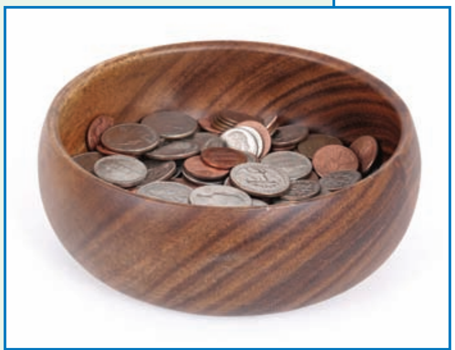
a) Complete the table in your log with other reasonable values for *SPE*, *KE*, *GPE* and the sum in the respective columns.

b) Draw an energy bar chart like the one in the *Physics Talk* in the previous section, but now including *SPE* as well as *KE* and *GPE*.

Position above table (m)	SPE (J)	KE (J)	GPE (J)	SPE + KE + GPE (J)
At rest on table: height = 0 m	20	0	0	20
Just after popping: height = 0 m	0	20	0	20
At peak: height = 0.30 m	0	0	20	20
1/2 way up: height - 0.15 m			10	
With the spring only partially open: height = 0 m				
Some other position: height = ? m				

Physics Talk

CONSERVATION OF ENERGY



Kaitlyn, Hannah, and Nicole share an apartment. Hannah keeps a bowl by the door filled with quarters that she can use for the washer and dryer at the laundromat. On Tuesday, Hannah counts her money and finds that she has 24 quarters, or \$6.00 in quarters. This is just the right amount for her laundry on Saturday. On Wednesday morning, Nicole comes rushing up to the apartment because she needs some quarters for the parking meter. She takes three quarters from the bowl and replaces them with six dimes and three nickels. The total money in the bowl is still \$6.00. On Wednesday afternoon, Kaitlyn needs to buy a fifty-cent newspaper from the machine that takes all coins but pennies. Kaitlyn takes two quarters from Hannah's bowl and replaces these coins with fifty pennies. The total in the bowl is still \$6.00.

Wednesday night, Hannah comes home and notices that her bowl is filled with quarters, pennies, nickels, and dimes. She knows that it still adds up to the \$6.00 that was there in the morning, but also knows that she cannot do her laundry unless all the money is in quarters. Her roommates agree to exchange all the coins with quarters the next day.

The money in the bowl could represent the energy in a system. The total amount of energy may have been 600 J. As the coins in the bowl change from quarters to dimes and nickels to pennies and back to quarters, the energy in the system can vary from kinetic energy to gravitational potential energy to **spring potential energy** in any combination. *(Note*: In Chapter 2, the term *EPE* (elastic potential energy) was used. Bungee cords, trampolines, and bent poles in pole vaulting all have elastic potential energy (*EPE*). The best approximation for *EPE* is that these behave like springs. In this chapter, we refer to *SPE*, or spring potential energy.)

Physics Words

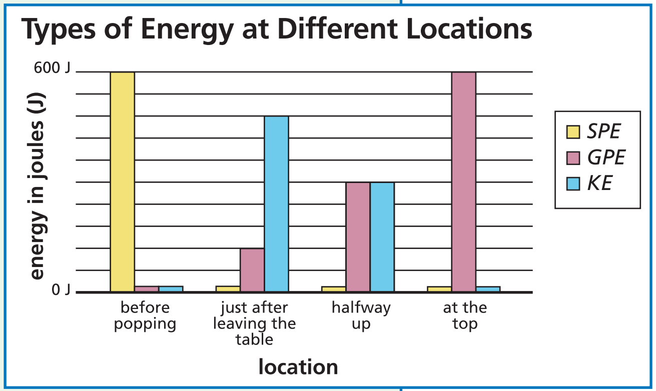
spring potential energy: the energy stored in a spring due to its compression or stretch.

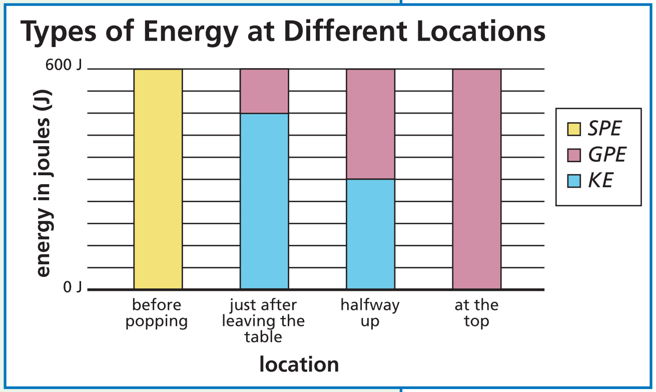
If Kaitlyn had taken the two quarters and not replaced them with pennies, then the total money would be less. The loss in money due to Kaitlyn would have resulted in that money being somewhere else. In some systems, energy is also lost. A bouncing ball does not get to the same height in each successive bounce. Some of the energy of the ball becomes sound energy and heat energy. These can be measured and will indicate that some energy left the system but did not disappear. In the pop-up toy and the roller coaster, the total energy can be *GPE*, *KE*, or *SPE*, but the sum of the energies must always be the same.

As you followed the changes in Hannah's bowl of money, you knew that there were ways to measure the total amount of money. Fifty pennies is identical in value to two quarters. Scientists look for all the energies in a system. There is electrical energy, light energy, nuclear energy, sound energy, heat (thermal) energy, chemical energy, and others. Each one is able to be calculated using measurements. All the energies are measured in joules. If you take into account all forms of energy, the total number of joules must always remain the same. The total energy is conserved.

In this section, you were able to observe how the spring potential energy of the pop-up toy became the kinetic energy of the pop-up toy, which then became the gravitational potential energy of the pop-up toy. A graph of the three types of energy at different locations in the top diagram to the right shows that each type of energy changes in value, but that the total energy remains the same. The total energy can best be shown in a stacked bar chart of the same data in the bottom diagram to the right.

In this *Investigate*, you were also able to observe how the pop-up toy's behavior changed when you doubled its mass.





The larger mass pop-up toy did not go as high as the original, lower-mass pop-up toy. Both pop-up toys had identical spring potential energies. Since "before popping" the *SPE* represents the total energy, both pop-up toys had identical total energies as well. The graphs shown on the previous page could describe the heavier pop-up toy as well. The less massive and more massive pop-up toys can have the same *GPE* if and only if the more massive pop-up toys do not go as high. Since $GPE = mgh$, the larger mass has a smaller height and the smaller mass has a larger height.

In a real roller coaster, the roller coaster has all its energy as *GPE* (gravitational potential energy) as it sits on the highest hill. Most of this energy becomes *KE* (kinetic energy) as the roller coaster is released. Some small amount of the energy is converted to thermal energy and a smaller part to sound energy.

Where does the roller coaster get all of that *GPE* that drives the rest of the ride? Something has to pull the roller coaster up to the top of the hill. The energy to pull the roller coaster is usually electric. The electrical energy comes from a power plant (that burns oil, gas, coal, or uses nuclear energy or water's potential energy) or from a local generator that may use gasoline.

After the cars are pulled to the top of the hill, the total *GPE* and *KE* of the roller coaster remains the same except for losses due to thermal and sound energy. At the end of the ride, the *KE* is converted to thermal energy as the brakes bring the cars to a halt. If the brakes fail, there may be a large spring that will stop the car as the car compresses it.

Calculating Spring Potential Energy

In this section, you extended the conservation of energy principle to include the spring potential energy. It is possible to calculate the spring potential energy.

The equation for spring potential energy is

$$SPE = \frac{1}{2}kx^2,$$

where k is the spring constant and

x is the amount of stretch or compression of the spring.

A spring that is difficult to compress or stretch will have a large spring constant (k). That spring will "pack" more *SPE* for an identical compression than a spring that is easy to compress.

The total energy of a spring toy that can jump into the air is the sum of the *SPE*, the *GPE*, and the *KE*. Once the spring is compressed, the sum of these three energies, *GPE*, *KE*, and *SPE* must remain constant.

$$GPE + KE + SPE = \text{constant}$$

$$mgh + \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

Sample Problem

A spring pop-up toy with a mass of 0.02 kg reaches a maximum height of 0.50 m. The compression length of the spring is 0.03 m. Find the following:

a) *GPE* at the top,

b) *SPE* before the "pop,"

c) the spring constant, *k*, and

d) *KE* at the moment the spring toy leaves the ground.

Strategy:
You can use the equation for *GPE* to calculate the *GPE* at the top because you are given the mass of the toy and the maximum height. Using the law of conservation of energy, you know that the *GPE* at the top must equal the *SPE* before the pop and the *KE* at the moment the spring toy leaves the ground.

Given:
mass (m) = 0.02 kg
height (h) = 0.50 m
compression length (x) = 0.03 m
Use $g = 9.8\ \text{m/s}^2$.

Solution:

a) $$\begin{aligned} GPE &= mgh \\ &= (0.02\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.50\ \text{m}) \\ &= 0.098\ \text{J} \end{aligned}$$

b) $$\begin{aligned} SPE &= GPE \\ SPE &= 0.098\ \text{J} \end{aligned}$$

c) $$SPE = \frac{1}{2}kx^2$$

$$\begin{aligned} k &= \frac{2(SPE)}{x^2} \\ &= \frac{2(0.098\ \text{J})}{(0.03\ \text{m})^2}\left(\text{Since J} = \text{N}\bullet\text{m, note that the units are } \frac{\text{N}\bullet\cancel{\text{m}}}{\text{m}^{\cancel{2}}}.\right) \\ &= 217.78\ \frac{\text{N}}{\text{m}} \\ &= 218\ \frac{\text{N}}{\text{m}} \end{aligned}$$

d) $$\begin{aligned} KE &= SPE \\ KE &= 0.098\ \text{J} \end{aligned}$$

(The *KE* is actually a bit less than this because the toy is 0.03 m above the table when it pops. You can calculate the *GPE* at this height to find the exact *KE*.)

Checking Up

1. What happens to the spring potential energy of a "pop-up" toy after it leaps off the table?
2. A "pop-up" toy has 2 J of spring potential energy before popping. How much kinetic energy will the toy have just after leaving the table?
3. A "pop-up" toy has 2 J of spring potential energy before popping. How much gravitational potential energy will it have at the top?
4. What two factors determine the amount of spring potential energy that is stored in a spring?

Active Physics

Plus

+Math	+Depth	+Concepts	+Exploration
◆◆	◆		

Using Algebra to Derive an Equation for Height

You can use algebra to find out how high the pop-up toy should go.

1. When the spring is completely compressed, but before the toy has popped, what is the *KE* of the toy? What is the equation for the toy's *SPE*?
2. As usual, choose *GPE* to be 0 when the spring is completely compressed. Write an algebraic expression for the total energy = *KE* + *GPE* + *SPE* when the spring is totally compressed but before the toy pops.
3. After the toy pops, it will shoot upward. When it gets to its highest point, what is its *KE*? What is its *SPE* at that point? Explain how you arrived at your answers.
4. If you denote the height of the high point above the starting point as h, write an algebraic expression for the *GPE* at the high point.
5. Write an algebraic expression for the total energy = *KE* + *GPE* + *SPE* at the high point.
6. If you assume that the total energy stays the same from before popping to the high point, you may equate the expression from *Question 2.* with the expression from *Question 5.* Solve the resulting expression for h and show that

   $h = \dfrac{\frac{1}{2}kx^2}{mg}$ where x is the original amount of spring compression.

What Do You Think Now?

At the beginning of this section, you were asked the following:

- **How does the roller coaster today get up to its highest point?**
- **Does it cost more to lift the roller coaster if it is full of people?**

How would your answers to these questions vary, based on what you learned from your investigation? A roller coaster is not lifted up by a spring but by cables and electricity. Will more electrical energy be required to lift a heavier roller coaster? The experiment that you conducted with the pop-up toy when its mass was increased can provide insight into this question. How did the height of the pop-up toy change when the mass changed?

Physics

Essential Questions

What does it mean?

The principle of conservation of energy states that energy may change its form, but the total energy for a system stays the same. Write a short description of a situation in which the energy changes form, but where the total amount of energy stays the same.

How do you know?

All principles of science can be checked with quantitative measurements. Write a short explanation of how your investigation with the pop-up toy with different masses attached illustrates the conservation of energy.

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	* Conservation laws	Experimental evidence is consistent with models and theories

* Conservation laws are a major organizing principle of physics. Energy can appear in many forms, but the total energy is always conserved. To understand the behavior of the pop-up toy, you had to include the potential energy of the compressed spring, *SPE*, and the kinetic energy, *KE*, and the gravitational potential energy, *GPE*. Compare the pop-up toy with a child on a trampoline and with a roller coaster in order to demonstrate how the conservation of energy can be used to describe each situation.

Why should you care?

The conservation of energy helps you understand many phenomena in the world around you. How will what you learned about the conservation of energy in this section help you with your roller-coaster design challenge?

Reflecting on the Section and the Challenge

There are other energies — heat, sound, chemical, and so on. In your analysis of the roller coaster, you may decide to ignore heat and sound, but you had better mention this in your report. In the actual construction, it will be important to take into account that a small amount of mechanical energy (*KE* plus *GPE*) is being dissipated (lost to other forms of energy).

The roller coaster uses electrical energy to get the cars to the top of the hill. This is similar to using the chemical energy of your body to compress the pop-up toy so that you can watch it jump.

Once the energy is in the spring of the pop-up toy, the *SPE* can become *KE*, which becomes *GPE*. In the same way, once the cars are on top of the hill, the *GPE* can become *KE*.

Describing the energy transformations will be a good way to describe the physics of your design of the roller coaster.

You may want to add a spring at the end of the ride to stop a "run-away" roller coaster.

Physics to Go

1. Complete the table with other reasonable values for *SPE*, *KE*, *GPE* for the pop-up toy. In the last column, fill in the sum in the respective columns.

Position above table (m)	SPE (J)	KE (J)	GPE (J)	SPE + KE + GPE (J)
At rest on table: height = 0 m	25			
Just after popping: height = 0 m				
At peak: height = 0.60 m				
1/2 the way up: height = 0.30 m				
With the spring only partially opened: height = 0 m				
Some other position: height = ? m				

2. Draw an energy bar chart for the situation described in *Question 1*. Include bars for *SPE*, *KE*, and *GPE*. Write a brief description of how the energy changes from one form to another during different parts of the pop-up toy's motion.
3. How would the table values in *Question 1* change if some extra mass were attached to the pop-up toy?
4. You throw a ball into the air and catch it on the way down. Beginning with the chemical energy in your muscles, describe the energy transformations of the ball.
5. Why can the second hill of the roller coaster not be higher than the first hill?
6. Why does the roller coaster not continue forever and go back and forth and up and down the hills over and over again?
7. A roller coaster of mass 300 kg ascends to a height of 15 m. How much electrical energy was required to raise the cars to this height?

8. A roller-coaster car has a mass of 400 kg and a speed of 15 m/s.

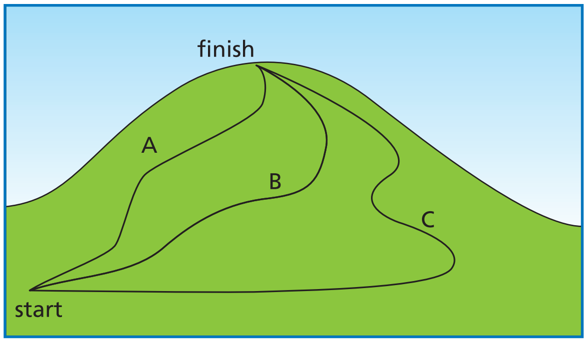
   a) What is the *KE* of the roller-coaster car?

   b) What will be the *GPE* of this roller-coaster car at its highest point, where *KE* = 0 at that point?

   c) How high can the roller-coaster car go with this much energy?

9. A ball is thrown upward from Earth's surface. While the ball is rising, is its gravitational potential energy increasing, decreasing, or remaining the same?

10. Three people of equal mass climb a mountain using paths A, B, and C shown in the diagram. Along which path(s) does a person gain the greatest amount of gravitational potential energy from start to finish: A only, B only, C only, or is the gain the same along all paths?



11. In an experiment similar to your toy, the mass of the spring toy was 0.020 kg. The height that the toy rose to was 0.40 m. The initial speed of the spring toy as measured by the velocimeter was 2.7 m/s.

    a) Do the *GPE* and the *KE* both give approximately the same values?

    b) What is the *SPE* before the toy pops?

    c) What height would you expect the pop-up toy to reach if its mass were tripled?

12. A roller coaster begins at a height of 18 m. The mass of the roller coaster and passengers is 300 kg. When the roller coaster reaches the bottom, its brakes fail. An emergency spring must bring the coaster to rest.

    a) What must be the spring constant of this spring if it will be compressed by 4 m?

    b) How much will the spring compress if an additional 100 kg of people are aboard?

13. An umbrella has an automatic opening mechanism. When the umbrella is closed, a spring is compressed. The spring constant is 40 N/m and the spring is compressed 0.3 m. What is the *KE* of the umbrella when it begins to open?