Section 8 Work and Power: Getting to the Top

TECH LAB

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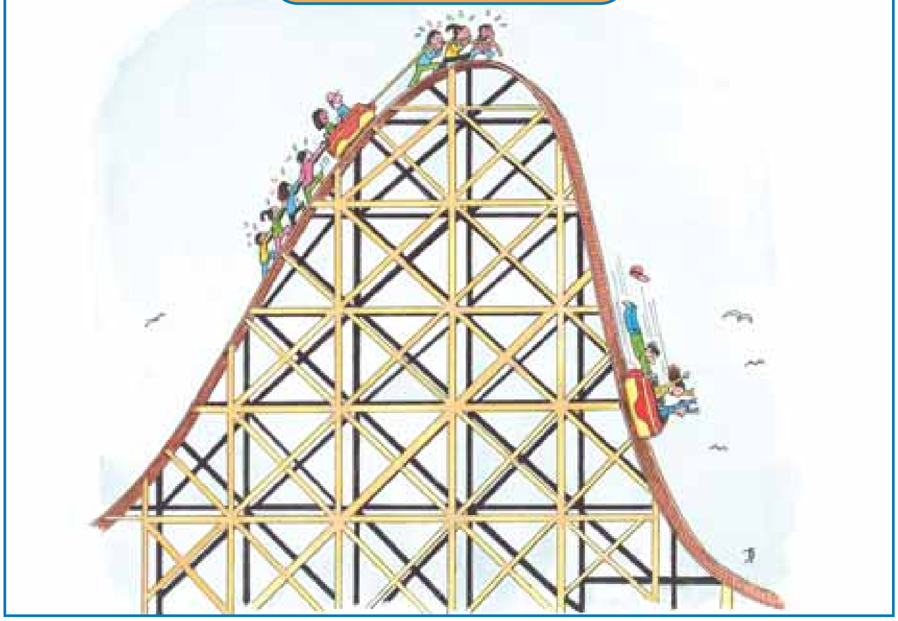
Learning Outcomes

In this section, you will

- **Calculate** and compare the product of force and distance for lifting an object up a ramp to the same height for different angles of the ramp.
- **Define** work in terms of force, *F*, and displacement, *d*, in the direction of the force.
- **Explain** the relationship between work, gravitational potential energy, and spring potential energy.
- **Define** power as the rate of doing work and the units of power as watts.

What Do You See?



What do You Think?

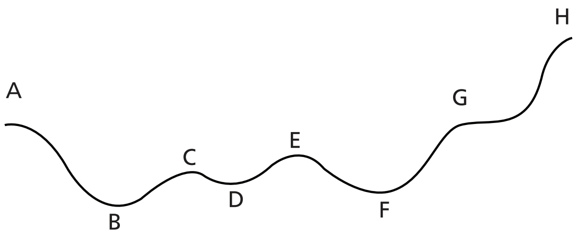
The greatest drop for a roller coaster is 125 m (400 ft). The roller coaster must be pulled up to that height to get the ride started.

- **Does it take more energy to pull the roller coaster up a steep incline than a gentle incline?**
- **Why is it more difficult to walk up a steep incline than a gentle incline?**

Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your response with your small group and the class.

Investigate

1. The roller coaster at the top of the hill is ready to go. It goes up and down the hills and around the curves without any energy input. An idealized roller coaster would keep going forever. The car on the track pictured will go from point A to B to C to D to E to F to G. It will then reverse and go from G to F to E to D to C to B to A. It will then begin the trip again. Of course, in a real system some of the kinetic energy and gravitational potential energy will be converted to other forms of energy. Friction turns the mechanical energy into thermal energy, sound energy, and so on, and the roller coaster carts will eventually stop.



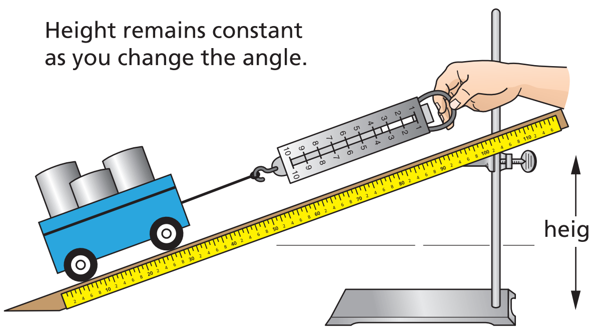
a) Why do you think that the roller coaster cannot scale a higher hill than the one from which it began?

2. You will now investigate the force required to lift a roller-coaster cart to a certain height. You will use a cart and a ramp in your classroom. You can pull the cart to the top of the ramp with the use of a spring scale. The spring scale will indicate the force required to pull the cart.

A meter stick can be used to record the distance that the cart moved along the ramp. You can then vary the distance along the ramp by varying the ramp angle *while keeping the height to which you raise the cart the same*.

The ramp should be measured from the bottom of the ramp to the height of the support for the ramp.

Be careful when lifting and lowering the weight, as it could cause injury if it were to fall. Make smooth, unhurried movements.



a) Create a data table in which you can record the force required to pull the cart up the ramp the four different distances at the four different angles.

b) Measure the force required to pull the cart up the ramp to a specified height at constant speed. Reminder: You must always pull the cart to the same height and parallel to the ramp. How will you determine if the speed is constant?

c) Measure the distance that the cart travels along the ramp from the bottom to the specified height.

3. Complete your investigation.

a) Record your information in your data table.

b) What conclusion can you reach about the force required to move the cart and the distance the cart moves along the ramp to attain a specific height?

4. Any time you take measurements, there is some uncertainty in the measurement. When you weigh yourself on a scale, the weight reading may be off by a little bit. If the scale reads 143 lb, you may actually weigh 143 lb and a few ounces. The scale does not give you an exact measurement. No measurement is ever exact.

a) What are the uncertainties in your measurements of distance? Could your measurement of distance be off by as much as 3 cm? Could your measurement of distance be off by as much as 1 cm? What is the largest amount that your distance measurement may be off? Write down this value with the notation ± to signify that you may have been under or over by that amount. For instance, if you think that your distance measurement could have been off by 2 cm, you would write this as ±2 cm.

b) Record the uncertainties in your measurements of force by noting the accuracy of your spring-scale reading. If the uncertainty is 0.1 N, record this as ±0.1 N. These uncertainties will be important when you analyze your data in the *Investigate*.

5. Another way that you can get the cart to the top of the incline is to lift it vertically. Use the spring scale to lift the cart vertically.

a) Record the force required to lift the cart vertically and the height that you lifted it.

6. There are other ways that the cart could be lifted to the top of the incline. For example, you could have an electric motor pull the cart to the top. Brainstorm and generate a list of at least three ways in which the cart could be brought to the top of the incline. (Brainstorming allows for all ideas to be included, even those that appear silly or impractical.)

a) Record your ideas in your *Active Physics* log.

7. Did you find that when the distance the cart travels to reach height h increases, the required force decreases? When one quantity increases and a second quantity decreases, this is referred to as an "inverse relation." If F is one quantity and d is the other quantity, one inverse relation can be described mathematically by the equation $F \bullet d = c$ where c is a constant. In the following table are some F and d values forming an inverse relation where $F \bullet d = 12$.

F	d	F•d=c
1	12	12
2	6	12
3	4	12
4	3	12
6	2	12
12	1	12

a) Make a graph of F (vertical axis) versus d (horizontal axis) to show the relationship for the inverse relation $F \bullet d = 12$ with this sample data.

8. Create a graph for the data from your experiment.

a) In the equation $F \bullet d = 12$, the product of the F and d values always equals 12. Does the product of the force and distance in your experiment always equal a certain value? Make the calculations and record the results on the side of your chart.

b) Why would the values in your experiment not be expected to be exactly the same?

c) For one pair of force and distance, repeat the calculation force × distance with the force value raised by the force uncertainty and the distance value increased by the distance uncertainty.

d) Now calculate the difference between the original product and the new product in *c)*. That difference is a good estimate of the uncertainty to be associated with the results of your calculations in *Step* 6.

e) Do your results agree if you take this uncertainty into account? For example, one product might be 6.1 N•m, while another product is 6.3 N•m. If the uncertainty estimated for the product is 0.3 N•m, you see that the difference between the two products is less than the uncertainty, and you can say that "the results agree within the experimental uncertainty."

Physics Talk

WORK

The roller coaster must get to the top of the first hill to begin the ride. In the *Investigate*, you moved a cart to the top of an inclined ramp by applying a force with the spring scale over a certain distance and you find that the product of the force you applied and the distance through which it acted is the same, regardless of the slope of the ramp. In physics, the product of force multiplied by distance is called **work**. The work done by a force F on an object as the object undergoes a displacement parallel to the force is defined through the following equation:

Work = force (parallel to the displacement) × displacement

$$W = F \bullet d$$

where F is the part of the force parallel to the displacement and

d is the displacement.

Note that the definition of work involves only that part of the force that is in the same direction or opposite direction to the displacement.

In this *Investigate*, the spring scale pulled the lab cart up the incline and the force was in the same direction as the displacement. You found that the product of force times displacement (work) was the same regardless of the angle of the incline. The force was larger for a steeper incline, but the distance along the incline was smaller. The product of the force and distance moved along the ramp was always the same. That quantity was the work that was done by the spring scale on the cart. The work done by a force on an object is a measure of the energy transferred to the object. In the case of the cart, the gravitational potential energy (*GPE*) of the cart increased as a result of the work done by the spring scale. Recall that gravitational potential energy is energy of position relative to the surface of Earth or of an identified surface such as a table or a floor. Pulling the cart up the ramp changed the elevation of the cart and increased its gravitational potential energy.

To bring the roller coaster to the top of its first hill, work must be applied to the roller-coaster system. The work will increase the energy of the roller-coaster system. The work to lift the roller coaster up the ramp to a certain height is identical to the work to lift it vertically to that height. When you lift it vertically, the force required is about equal in magnitude to the weight of the cart. The vertical displacement is the height that it must be lifted.

$$W = F \bullet d$$
$$= \text{weight} \times \text{height}$$
$$= mgh$$

Physics Words

work: the product of displacement and the force in the direction of the displacement; the energy transferred to an object.

The work done on the roller coaster is *mgh*. This is equal to the change in gravitational potential energy, *GPE*, of the roller coaster.

Sample Problem I

A lab cart that weighs 300 N is lifted to the top of an incline 2 m above the ground.

a) What is the work done on the cart by the force that lifted the cart?

b) How much force would be required to lift the same cart to the same height using a 10-m long inclined ramp?

a) ***Strategy:*** The force required to lift the cart at constant velocity is equal in magnitude to its weight. The displacement is the height that the cart was lifted. The force and the displacement are both in the vertical direction.

Given:

$F = 300\text{ N}$

$d = 2\text{ m}$

Solution:

$$W = F \bullet d$$
$$W = (300\text{ N})(2\text{ m})$$
$$= 600\text{ N}\bullet\text{m}$$
$$= 600\text{ J}$$

b) ***Strategy:*** The work required to lift the cart would be identical since the cart began at the same height and ended at the same height. Since you know the new displacement, you can find the new force.

Given:

$W = 600\text{ J}$

$d = 10\text{ m}$

Solution:

$$W = F \bullet d$$
$$F = \frac{W}{d}$$
$$= \frac{600\text{ N}\bullet\cancel{\text{m}}}{10\ \cancel{\text{m}}}$$
$$= 60\text{ N}$$

By using the ramp, you need a force of only 60 N to slide the cart up the ramp, instead of the 300 N to lift it. That is why truckers use a ramp when loading a truck. The ramp is considered to be a simple machine. The same work is done, but with much less force. Of course, the force must be applied over a longer distance because the energy transfer is the same.

More Roller-Coaster Energy

The roller-coaster car is usually raised with electrical energy supplied by a motor. Electrical energy can be calculated by measuring the voltage, current, and time. Creating steam to push it up the incline could also have raised the roller-coaster cart. In this method, the heat energy can also be calculated. In all of these methods, work is done by the spring, by the electricity, or by the heat. The roller-coaster system gains that amount of energy. The roller coaster has increased its *GPE* by that amount.

In any system, the total energy remains the same. This is an organizing principle of physics and is referred to as the conservation of energy.

Although you treat the roller-coaster's energy as primarily *KE* and *GPE*, real roller coasters have some energy transferred to other forms such as heat energy and sound energy. There is work done by friction and work done by air resistance. This work removes *KE* and *GPE* from the roller coaster. The work done by friction, for instance, becomes heat energy that is dissipated into the air surrounding the roller coaster.

Braking the Roller Coaster

Your roller coaster must have a means of stopping the cars at the end of the ride. Normally, brakes stop the coaster cars. The brakes use friction to convert the *KE* of the car's motion into thermal energy. The brakes might fail; so you need to have a back-up mechanism to stop the coaster cars. One way to do this is to have a large spring that the car can compress. As the spring is compressed, the *KE* of the cars is stored as spring potential energy. The expression for spring potential energy is $\frac{1}{2}kx^2$, where x is the distance that the springs are stretched or compressed.

Power

Sometimes it is important to know how fast work is done. In this *Investigate*, you pulled the lab cart up the incline. You could have pulled it up with a variety of speeds. To take the time into account, you divide the work done by the time elapsed. The result is called **power**. The definition of power is

$$\text{power} = \frac{\text{work done}}{\text{time elapsed}}$$

$$P = \frac{W}{\Delta t}$$

Note that the scientific definition of power is different from the ordinary usage of the word in sentences like, "I have power over you," or "She has lots of political power."

Physics Words

power: the work done divided by the time elapsed; the speed at which work is done and energy is transferred.

Sample Problem 2

Tomas runs up the stairs in 24 s. His weight is 700 N and the height of the stairs is 10 m.

a) What is the work done by Tomas to get to the top of the stairs?

b) How much power must Tomas supply?

Given: $F = 700\ \text{N}$

$d = 10\ \text{m}$

Strategy: You can use the definition for work and power to solve this problem. Since it is assumed that Tomas goes up the stairs at a constant speed, the force acting upward on Tomas must equal his weight.

Solution:

a) $W = F \bullet d$
$= (700\ \text{N})(10\ \text{m})$
$= 7000\ \text{J}$

b) $P = \dfrac{W}{\Delta t}$
$= \dfrac{7000\ \text{J}}{24\ \text{s}}$
$= 290\ \text{J/s or } 300\ \text{J/s}$

Notice that the unit for power is joules per second, which is given the name **watts**. You are familiar with the power ratings of light bulbs in watts. You may have heard of horsepower as another unit for power. One horsepower is the energy output of a horse over a specific time. One horsepower is approximately 750 W (watts). A "one horsepower electric motor" uses electrical energy at a rate of about 750 W.

Sometimes the letter W is used for watts. Be sure not to confuse this W with the *W* used for work.

Physics Words

watt: the SI unit for power; 1 W = 1 J/s.

Checking Up

1. When a spring scale is used to do work pulling a cart to the top of an incline, where has the energy gone when the cart is at rest at the top?
2. Where does the roller coaster get its gravitational potential energy when it is at the top of the first hill?
3. Why do truckers use a ramp when loading a truck if the work required is the same with or without a ramp?
4. When the brakes stop a roller coaster, what happens to the coaster's kinetic energy?
5. What is the unit for power?

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
◆	◆	◆	

Direction of Force in Work Done

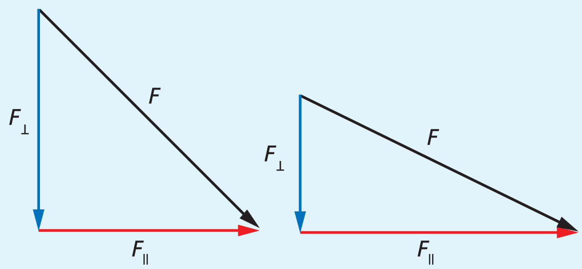
It may seem that the force would always be in the same direction as the displacement. This is not always the case. Consider a push lawn mower. The push lawn mower has no motor. It moves because someone pushes it.

The force is applied along the handle of the lawn mower. The displacement of the lawn mower is the distance along the ground. The force and the displacement are not in the same direction, but there is some work done. That is, there is some energy transfer to the mower.

Although the entire force is not in the same direction as the displacement, some of the force is in the same direction as the displacement.

That part of the force is called $F_{\parallel}$. The symbol $\parallel$ stands for parallel. This force is parallel to the displacement

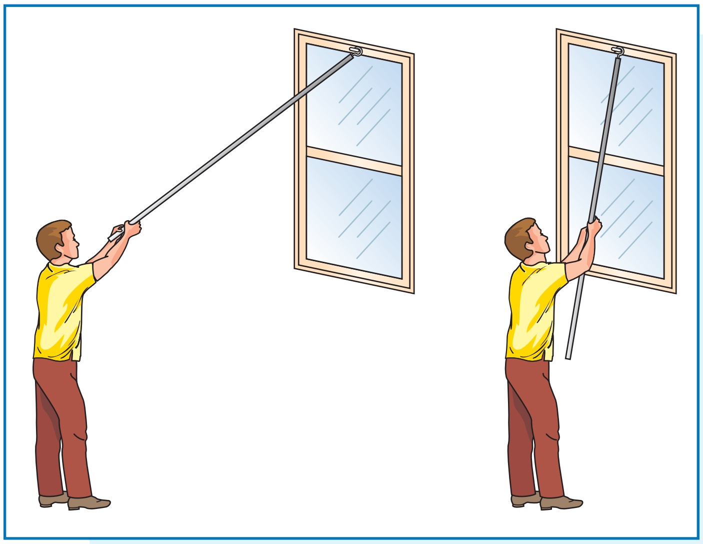
The force along the handle can be broken into its two vector components by finding the horizontal and vertical forces that, when added together, would be identical to the original vector. In the diagram on the left, the horizontal and vertical component forces are approximately equal in size. The symbol $\perp$ stands for perpendicular. This force is perpendicular to the displacement.



In the diagram on the right, the horizontal vector is much larger than the vertical component. Most of the force applied to the handle is now in the same direction as the displacement. Even though the total force is identical (note that the length of the force vector is the same in each diagram), the horizontal component is larger as the angle between the handle and the ground gets smaller. The same total force and the same displacement, but more work is done when the horizontal component is greater.

Why then don't you push the lawn mower with a small angle? Although more work would be done, it would hurt your back. Therefore, you sacrifice some work in order to make using the lawnmower more comfortable.

1. A student is asked to use a window pole to slide a window up. If the window moves the same distance up, is the work applied equal in the two cases shown on the next page? Is the force applied equal in the two cases?



2. A child is seated in a cart. Explain why it is easier to pull the child with a longer rope? (Hint: Draw two diagrams — one for a short rope and one for a long rope.)

Spring Energy

Recall the pop-up toy investigation. In that *Investigate*, the potential energy stored in the spring was converted into the kinetic energy of the toy. How much energy was stored when the toy spring was compressed? The force of a spring that obeys Hooke's law is $F = kx$. The force is not constant but changes as the stretch or compression of the spring changes. The spring force is zero when the spring is not compressed at all and a maximum value of kx when the spring is compressed the maximum distance x. If you compress the spring a distance x, then the average force that you exert on the spring will be $\frac{1}{2}kx$.

For an ideal spring, the force that compresses the spring must be equal to the force that the spring pushes back with. The work done on the spring is

$$W = F_{\parallel} d$$

$$= \left(\frac{1}{2}kx\right)x$$

$$= \frac{1}{2}kx^2$$

The same expression applies if the spring is stretched by the distance x.

The work done on the spring is equal to the potential energy stored in the spring (*SPE*).

1. Could you get your roller coaster to the top of its first hill using the energy stored in a (very large) spring? What would be the advantages and disadvantages of starting the roller coaster that way?

What Do You Think Now?

At the beginning of this section, you were asked the following:

- **Does it take more energy to pull the roller coaster up a steep incline than a gentle incline?**
- **Why is it more difficult to walk up a steep incline than a gentle incline?**

Revise your answers to these questions using the concepts of work and energy. Relate your answers to the *Investigate* activities. Discuss your revisions with other students in your group.

Physics

Essential Questions

What does it mean?

Lifting the roller coaster to the top of the first hill requires work. Work is the crucial concept in this section. Explain what work means as a scientific concept.

How do you know?

Physicists prefer to express concepts in mathematical form. The concept of work involves the product of force and displacement as long as the two are parallel to each other. Describe the evidence from this section that shows that it is the product of force and displacement in the direction parallel to the force that is important for work.

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Force and motion	Conservation laws	* Optimal prediction and explanation

* Conservation of energy is an organizing principle of physics. In a roller coaster, losses in gravitational potential energy (*GPE*) produce gains in kinetic energy (*KE*). Work by an external force can add to or remove from energy to the roller coaster. Give an example of a force doing work on the coaster to either increase or decrease the total energy of the coaster.

Why should you care?

Scientists often use everyday words in ways that don't agree with everyday usage. This precision in language is crucial to the communication of scientific ideas. Compare some examples of the scientific meaning of work with the common use of the work that shows up in everyday life. How will what you learned about work in this section help you with your challenge?

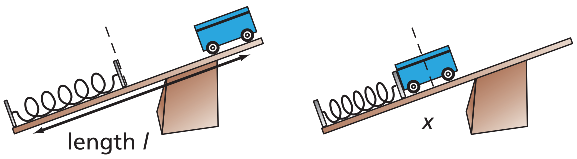
Reflecting on the Section and the Challenge

A roller-coaster ride always begins with a slow, suspenseful ride to the top of the first hill. On the way up, the roller coaster is designed to shake a bit and to make a few extra noises in order to add to the drama. The roller coaster is gaining gravitational potential energy (*GPE)* on the way up. The motor is performing work on the roller coaster cart. Work is a precisely defined term in physics: $W = F \cdot d$. The work supplied by the motor increases the energy of the roller coaster. At the top of the incline, the motor is disengaged and the roller coaster is on its own. There is some work by friction with the air and track that removes energy from the roller coaster. At the end of the ride, the brakes are applied and negative work is done because force and distance are in opposite directions. The kinetic energy of the roller coaster cars is converted to thermal (heat) energy by the brakes.

In designing your roller coaster, you will have to include a motor to lift the roller coaster. You will have to decide on the slope of the track going up and the time you want the ride to take to get to the top of the first hill. Work and energy will be useful ways of describing what is needed in your design. You will also want to know how fast this work is done. For that you will use the concept of power where power = (work done) ÷ (time elapsed).

Physics to Go

1. A lab cart starts at the top of the incline. It slides down the incline a distance l and comes to rest after compressing a spring a distance x.



   a) Compare the *GPE* of the cart at the top of the incline and at the bottom.

   b) How much work was done on the cart by the force of gravity (the cart's weight) as the cart went from the top to the bottom?

   c) How much work was done on the cart by the spring as the spring was compressed?

   d) What is the spring's *SPE* when it is compressed by the distance x?

   e) Describe the total energy of the cart just before it hits the spring.

   f) At which point does the cart begin to slow down?

2. Calculate the work done in the following situations:

   a) A waiter applies a force of 150 N to hold a tray filled with plates on his shoulder. He then moves 7 m toward the kitchen door. What is the work done on the tray by the waiter?

   b) A bowler lifts a 60-N bowling ball from the rack to his chest, a vertical distance of 0.5 m. What is the work done on the bowling ball by the bowler?

   c) A girl pulls her sled up a hill. The length of the hill is 40 m and the pulling force required was 75 N. What is the work done by the girl on the sled if she pulls the rope on the sled while the string is parallel to the hill?

   d) The weight of a dumbbell is 500 N. It is lifted over a body-builder's head, a distance of 0.7 m. What is the work done by the body-builder on the dumbbell?

3. Why are you told to conserve energy if the conservation of energy tells you that energy is always conserved? Create a better way of saying "conserve energy."

4. If you were to fill the lab cart you used in the *Investigate* with clay to represent the people in the roller coaster, what would have changed in the experiment?

5. An electric motor lifts a roller-coaster car that weighs 10,000 N to the top of the first hill that is 20 m above the ground. To add suspense, the ride up takes 150 s.

   a) Calculate the work done by the motor.

   b) Calculate the power of the motor.

6. ***Preparing for the Chapter Challenge***
   In the Terminator Express roller coaster, describe one trip of the coaster car around the ramp in terms of work and energy.

Inquiring Further

Power from an electric motor

Have the lab cart pulled up to the top of the incline with a motor. Measure the energy of the motor using voltage, current, and time. Compare the energy from the motor with the increase in *GPE* of the cart.