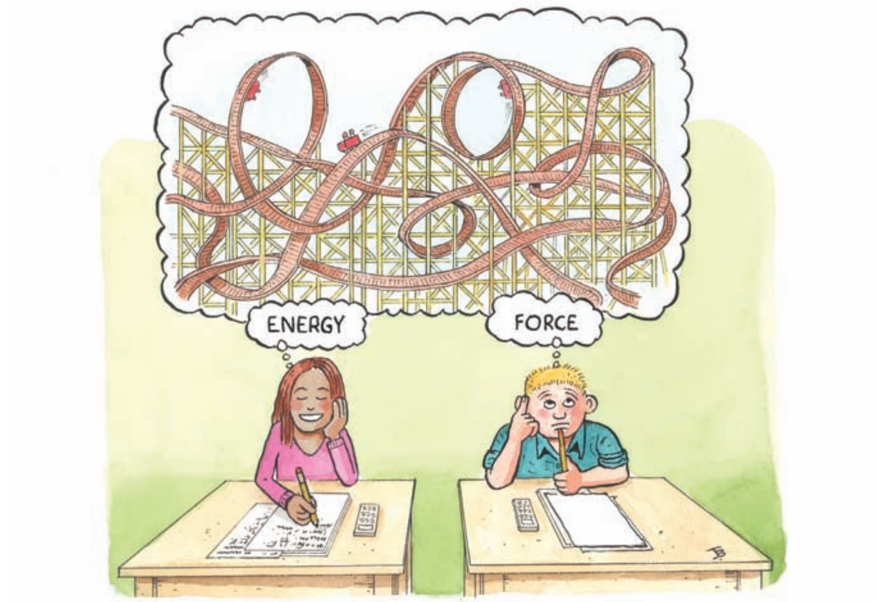
Section 9 Force and Energy: Different Insights

What Do You See?



Learning Outcomes

In this section, you will

- **Describe** instances in which two cars will attain the same speed but require different times to reach those speeds.
- **Recognize** that force is described by vectors and energy is described by scalars.
- **Explain** how force and energy considerations provide different insights into roller-coaster rides.
- **Discover** whether energy or force considerations are more appropriate for analyzing aspects of roller-coaster rides.

What Do You Think?

"The Snake" roller coaster stays at ground level throughout the ride. The passengers move left, then right, then left again.

- **Which parts of The Snake will be the most thrilling?**
- **If the speed of The Snake always remains the same, why will it still be fun?**

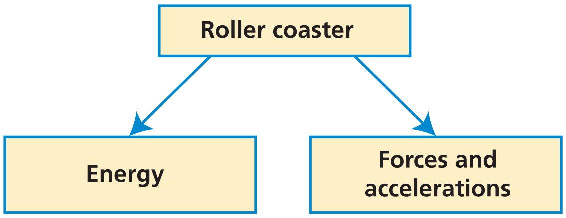
Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your response with your small group and the class.

Investigate

Part A: Energy and Forces in a Roller Coaster

1. Your study of roller coasters has actually taken two turns. You have investigated energy changes in roller coasters. You have also investigated forces and accelerations in roller coasters.

   a) Copy this beginning of a concept map into your log.



A concept map is a way to organize your thoughts. It serves as a good review of what you have learned. Creating a concept map often helps you increase your understanding. On a set of note-sized pieces of paper (or sticky-note paper), write down at least four things you know about energy and how it relates to roller coasters. Each note should have one concept only. (Review *Sections* 2 and *3* for assistance.)

2. Sort the concepts into a map that connects the concepts in a logical fashion. Add these concepts to your log.

3. On a new set of note-sized pieces of paper (or sticky-note paper), write down at least four things you know about forces and accelerations and how they relate to roller coasters. Each note should have one concept only. (Review *Sections 1, 4, 5* and *6* for assistance.)

4. Sort the concepts into a map. Add these concepts to your log.

5. The left half of your map reminds you of the relationships between energy concepts. The right half of your map reminds you of the relationships among force and acceleration concepts.

   a) Is there a bridge between these two sides of the map? Describe how energy is related to forces and accelerations. (Review *Section 7* for assistance.)

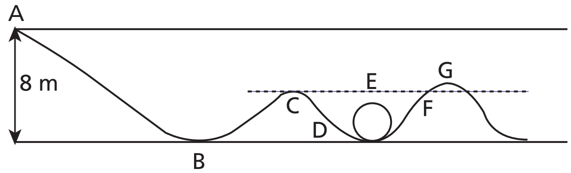
You use both energy and force approaches to understand roller coasters because they both provide you with valuable information. Sometimes it is easier to look at a roller coaster as an energy ride, while other times it is best to look at a roller coaster as a force ride. As you become more comfortable with physics, you will become better at matching what you want to know with the energy or the force approach. Sometimes you need both and sometimes they are redundant.

6. In the roller coaster below, the initial height of the roller coaster is given.

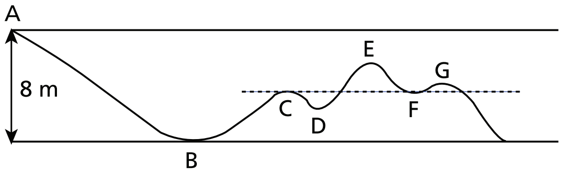
   a) At which two points does the roller coaster have the same speed if friction is negligible?

   b) How did you determine your answer? Write down your approach in your log.

   c) At which point would the roller-coaster car experience the largest contact (normal) force from the track? Write down in your log how you arrived at an answer.



7. Describe how the new roller coaster shown below is different from the roller coaster in Step 6.



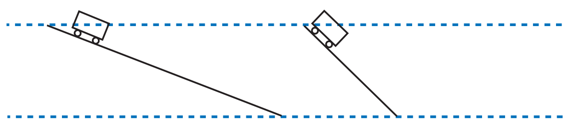
   a) In this roller coaster, at which two points does the roller coaster have the same speed if friction is insignificant?

   b) How did you determine your answer? Write down your approach in your log.

8. In either roller coaster, part of the track could have been replaced with horizontal track indicated by the dotted line.

   a) Why would the flat track be less fun than the roller-coaster track?

9. Look at the following diagram.



   a) Using energy principles, predict which cart would have the greater speed when it reaches the bottom.

b) Predict which car will get to the bottom in the least time. On what did you base your response? Record your explanation in your log.

Part B: Using Vectors to Describe a Path

1. Your teacher will give you a penny or a piece of wrapped candy.

a) Record the date stamped on the penny or write your initials on the candy wrapper. Hide the penny or piece of candy somewhere in the room.

b) Provide a set of detailed instructions to allow another student to find your penny or piece of candy if they start at your desk.

2. Exchange directions and try to find your partner's penny or piece of candy.

a) Did their instructions include how far you have to walk?

b) Did their instructions include any changes in direction (left turns or right turns)?

c) Did their instructions include reaching up or down?

d) Rewrite the instructions so that each instruction describes how far the person should move in meters and in which direction.

e) Compare this new set of directions with your first set. What advantages and disadvantages does each set have?

Physics Talk

ADDING SCALARS AND ADDING VECTORS

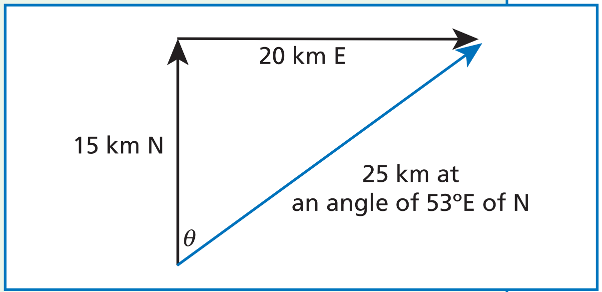
You can walk 30 m east. You can ride at 60 mph toward Mexico. Both descriptions include a number and a direction. Both are vectors. There are some descriptions that include a number, but no direction. There are 26 students in the classroom. The temperature is 18°C. Physicists have found that whether a number has a direction or not is an extremely important distinction. You can understand the world better if you recognize which quantities can have directions and deal with them accordingly.

It is fairly obvious that some quantities, like force, always have directions. Some quantities, like your age, never have direction. There are some quantities, like how fast you are traveling, that can include direction. Your car can be traveling at 30 mph or you can describe the car traveling at 30 mph north.

Recall that a quantity with both a number (often referred to as magnitude) and a direction is called a vector. A quantity with a number and no direction is a scalar.

Scalars are easy to add, subtract, multiply, and divide. If you walk 15 km and then walk another 20 km, the total distance traveled is 35 km. After walking 35 km, you know how tired you will be and how worn your shoes will be. This scalar quantity is called distance. Traveling from New York to Florida, your average speed might be 50 mph. This takes into account the total distance traveled and the total time, but does not take into account any turns you made. Speed is also a scalar.

Displacement is described by a vector. You may walk 15 km north and then walk another 20 km east; both displacements have a magnitude (the distance traveled) and a direction. Your total displacement is only 25 km. To add vectors, you must draw them and use vector addition. In this case, when the two vectors are perpendicular to each other, vector addition is an application of the Pythagorean theorem. You can also draw the diagram to scale and measure the distance and the angle. The distance can be measured using a ruler and the scale of the diagram. Using a protractor, you find the angle is close to 53° east of north (east of the north direction).



Energy – A Scalar Quantity

Energy is a scalar and addition of scalars is simple. As you explored in earlier activities, the roller-coaster ride may have *GPE* (gravitational potential energy) and *KE* (kinetic energy). It may have used electrical energy to lift the roller coaster to the top of the first hill. All energies can be calculated, and they are all measured in the same units, joules. To find the total energy at any place or at any time, you just add up all the energies. This is what makes the roller-coaster analysis using energies so powerful. After the roller coaster begins moving downhill, the sum of *GPE* and *KE* remains the same. The roller coaster begins with *GPE* and as the coaster car moves, the *GPE* converts to *KE* as the roller coaster picks up speed and then converts the energy back to *GPE* as the cart goes higher and loses speed. Whatever the energy of the roller coaster is at the beginning of the ride, that is the energy at all times as long as friction is not significant. If two points on the roller-coaster ride have the same height, then they must have the same *GPE*. If they have the same *GPE*, then they also have identical *KE*. It doesn't matter what the cart did between the two points. It may have gone up, down, or in a loop-the-loop, but the *KE* will be the same at all points a specified distance above the ground.

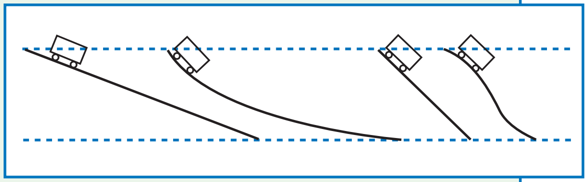
In this *Investigate*, you looked at a roller coaster in *Step 6*. The speeds of the coaster carts are the same at points C and F. Both points C and F have the same height and therefore have the same *GPE*. Since all points on the roller coaster have the same total mechanical energy (*GPE* + *KE*) then both points must have the same *KE*. The same *KE* implies the same speed ($KE = \frac{1}{2}mv^2$).

In the roller coaster in *Step 7* of this section, the speeds of the coaster cars were still the same at points C and F even though the track changed between C and F.

In roller-coaster physics, energy considerations tell you three things:

- The total mechanical energy (*GPE* + *KE*) is the same at every point (as long as friction is not significant or motors do not add energy).
- The *GPE* depends only on the height from a reference position (*GPE* = *mgh*) since the mass and the gravitational force remain the same.
- If two points on a roller coaster have the same height, the roller coaster is moving at the same speed at those two points.

Energy considerations are path independent. You can look at the energy at one point and compare it to the energy at a later point. The energy will remain the same. It does not matter what happens between the places that are of interest.



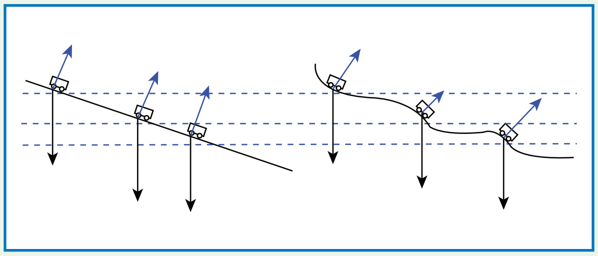
In the four roller-coaster sections shown in the diagram, the coaster cars begin at the top with zero *KE* and 20,000 J of *GPE*. When they reach the bottom, all will have the same *KE* (kinetic energy). This means that they will all have the same speed. To find this *KE* or speed, you only have to look at the beginning point and the final point. The path does not affect the final speed since you are not considering friction as a factor.

Force – A Vector Quantity

Although the roller-coaster cars all get to the bottom with the same speed, they do not get there in the same time. To find the time, you would have to look at the forces and this becomes a vector problem. In all tracks, the force of gravity is always down. The normal contact force between the track and the carts is always perpendicular to the track.

The straight tracks are the easiest to analyze. The force of gravity and the normal force remain in fixed directions. You move down the incline and go faster and faster. The steeper the slope, the larger the gravitational force down the incline and the quicker you get to the bottom. It is a big acceleration for a short time and you reach the maximum speed. On a small incline, there is a small resultant force down the incline. It is a small acceleration for a long time, but you reach the same maximum speed.

The inclines with shifting directions add to the thrill. Your speed changes as you move to different heights. As you move closer to the ground, your speed increases. The normal force (the force of the track on the carts) and the contact force of the cart on you are always changing direction. This causes you to accelerate in lots of different directions. The changes in the acceleration (both in size and in direction) give you that bouncy feeling and the thrill of the roller coaster. The diagram on the next page shows the gravitational force and normal forces at different points on a roller coaster.



- On the straight incline, the gravitational force and the normal force remain in fixed directions. The car has an acceleration that is constant in magnitude and direction.
- On the curved incline, the normal force changes direction (it must be perpendicular to the incline) and changes in magnitude. The cart has an acceleration that changes both in magnitude and in direction. This provides big thrills.
- The speeds of the carts are identical on the two inclines at the points shown. When the heights above the ground are the same, the *GPE* is the same. If the *GPE* is the same and the total energy is the same, the *KE* is the same. If the *KE* is the same, then the speed is the same.

When to Consider Force and When to Consider Energy

The mathematics of energy conservation requires simple addition. The mathematics of forces and accelerations requires vector addition. When the roller coaster looks complex, with lots of curves, physicists think of energy first because of the ease of using simple addition rather than vector addition.

When asked about how much time something will take, physicists think about forces and accelerations because acceleration is the change in velocity with respect to time.

Force and energy are related. The force of gravity does work on the roller coaster and increases its *KE*. Changes in energy always require work by a force. Work is a force applied over a distance $\left(W = F \bullet d\right)$.
The only external force doing work on the roller coaster once it is moving downward is gravity. There is positive work on the roller coaster since work increases the *KE* of the coaster carts. The normal force never does any work since it is always perpendicular to the displacement. No part of the normal force is ever in the direction the roller-coaster cart is moving.

Checking Up

1. What process is needed to add vector quantities?
2. Is energy a vector or a scalar? Is force a vector or a scalar?
3. For roller coasters, what three things do energy considerations tell you about the coaster at different points?
4. Does the energy of the roller coaster depend upon the path the roller coaster takes?
5. What is required to provide a change in the energy of a roller coaster?

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Using the Pythagorean Theorem

1. A roller coaster at a 25° incline makes a sharp right turn as it descends the hill. The velocity of the roller-coaster cart is 5.0 m/s south before the turn. After the turn, the velocity of the roller-coaster cart is 12.0 m/s west but it is also pointing downward at an angle of 25°. Ignore the downward angle.

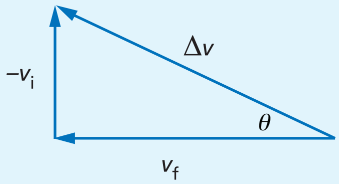
   a) Determine the change in velocity of the roller coaster using a vector diagram. Recall that $v_f - v_i$ is identical to $v_f + (-v_i)$.

   b) Determine the magnitude of the change in velocity of the roller coaster using the Pythagorean theorem. (Hint: Use your vector diagram of the two velocity vectors and the change in velocity vector.)

   c) Use a protractor to determine the direction of the change in velocity vector. Express your answer as an angle relative to the direction south.

2. How would your answer to *Question 1* change if you took into account the downward angle of the incline?

3. Just as you can find the length (magnitude) of the change in velocity vector mathematically using the Pythagorean theorem, you can also find the angle. You may have learned in mathematics that the tangent function of an angle in a right triangle is the ratio of the length of the side opposite the angle to the length of the side adjacent to the angle. The inverse tangent button on the calculator, often labeled "$\tan^{-1}$" will tell you the angle if you know the lengths of the sides. For the velocities given in *Question 1*, you can find the angle for the change in velocity vector. First, divide the side opposite the angle (= 5) by the side adjacent to the angle (= 12). By pushing the "inverse tan" button, the calculator will provide the angle of 23°.



4. Displacement is described by a vector. Suppose that you walk 15 km north and then walk another 20 km east; both displacements have a magnitude (the distance traveled) and a direction. In this case, your net displacement from your starting point is only 25 km.

   a) Draw a vector diagram for this situation. Draw the displacement vectors carefully to scale and determine the magnitude (length) and direction (angle) of the net displacement vector from the diagram.

   b) Use the Pythagorean theorem to find the length of the net-displacement vector.

   c) Use the tangent function to find the angle for the net-displacement vector.

   d) Compare the results of *b*) and *c*) with the results found from your vector diagram.

What Do You Think Now?

At the beginning of this section, you were asked the following:

- **Which parts of The Snake will be the most thrilling?**
- **If the speed of The Snake always remains the same, why will it still be fun?**

Review and, if necessary, revise the answers to these questions in terms of forces acting on the riders at various parts of the ride. The Snake roller coaster stays at ground level throughout the ride. The passengers move left, then right, then left again. Discuss your revisions with other students in your group.

Physics Essential Questions

What does it mean?

Scientists introduce concepts like force and energy that help you understand many different phenomena. The crucial concepts in this section are force and energy. Explain what force means and what energy means and how they are different.

How do you know?

Science often provides several ways of understanding a given situation. Describe an example of some aspect of a roller-coaster ride in this section where force is a useful concept. Describe an example of some aspect of a roller-coaster ride in this section where energy is a useful concept.

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	Conservation laws	* Good, clear explanation, no more complex than necessary

* For concepts to be useful in science they should apply to many different situations. Force and energy are important concepts in all areas of science. Describe some examples from other *Active Physics* units where force and energy play a role. Explain how force or energy, or both, help you understand what is going on in those examples.

Why should you care?

Both force and energy help you understand many different situations. Give some examples of where energy shows up in everyday life. (Hint: A calorie is a unit of energy; a kilowatt-hour is a unit of energy.)

Give some examples of where force shows up in everyday life. (Hint: A pound is a unit of force; a ton is a unit of force.)

How will what you learned in this section about force and energy help you with your challenge?

Reflecting on the Section and the Challenge

The thrill of the roller coaster comes from the changing velocities. You can analyze the changes in speed using energy considerations. Energy is a scalar. *GPE* can be easily calculated at every point on the roller coaster. Once you know the *GPE*, you can find the *KE* and then determine how fast the roller coaster moves. Understanding the mathematics of energy is as simple as 3 + 4 = 7. Energies add with simple arithmetic just like all scalars.

You can also analyze the thrills of changing velocities by noting the forces acting on the roller coaster. Forces are described by vectors. Vectors have both magnitude and direction. When more than one force acts on a roller coaster (e.g., the gravitational force and the normal force), you have to add forces using vector arithmetic. You can always do this with a vector diagram. When the forces are perpendicular, you can readily use mathematics and the Pythagorean theorem to find magnitude, and a protractor or more mathematics to determine the angle and direction.

Designing a roller coaster requires you to know how fast it will be going at each point along the path. You can use energy considerations to determine this.

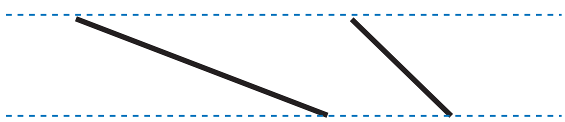
You will also have to know how large the forces are because you will need to figure out the strength of the materials needed to provide the forces by the track. If too large a force is applied, the track may break. Adding the forces can provide you with this information.

You will also have to know the accelerations of the passengers. Too large of an acceleration or a change in acceleration and the riders may get sick or become unconscious. Newton's second law relating forces and accelerations ($F_{net} = ma$) can help you with this.

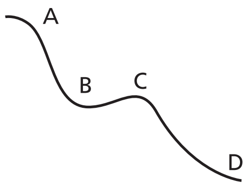
Making an exciting roller coaster requires changes in forces. The whips and turns and the ups and downs will change the speeds, the accelerations, and the forces on the passengers.

Physics to Go

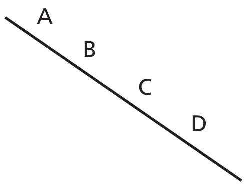
1. A roller coaster makes a sharp right turn. The velocity of the roller coaster car is 5.0 m/s south before the turn and 5.0 m/s west after the turn.
   a) Determine the change in velocity of the roller coaster cart using a vector diagram.
   b) Determine the change in velocity of the roller coaster cart using the Pythagorean theorem. You can figure out the angle in this case from the vector diagram.
2. All roller coasters that begin at the same height have the same speeds at the bottom. Explain why these two roller-coaster tracks provide the same change in speed when a cart goes from the top to the bottom.



3. Identify the following as vectors or scalars:
   a) distance
   b) displacement
   c) speed
   d) velocity
   e) acceleration
   f) force
   g) kinetic energy
   h) potential energy
   i) work
4. Which of the following statements are about vectors and which are about scalars?
   a) Mark traveled 30 km.
   b) Maia's weight (the force of gravity on her) is 600 N.
   c) The roller-coaster car had a kinetic energy of 1200 J.
   d) The cart was traveling at 30 m/s toward the center of town.
5. Sometimes it is easier to look at a roller coaster as an energy ride, while other times it is best to look at a roller coaster as a force ride. Give an example of each approach.
6. a) Draw the forces acting on roller coaster 1 at points A, B, C, and D
   b) Draw the forces acting on roller coaster 2 at points A, B, C, and D.
   c) Why is it easier to use forces to analyze roller coaster 2?

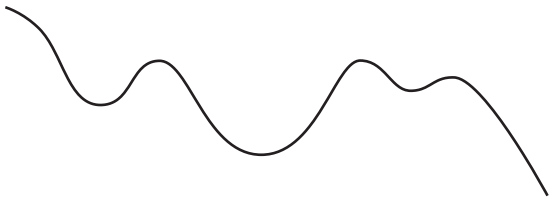


roller coaster 1



roller coaster 2

7. a) Label three points on roller coaster 3 that have the same *GPE*.
   b) Compare the total energies of the roller coaster at these points.
   c) Compare the *KE* of the roller coaster at these points.
   d) Why are you able to ignore the other points of the roller coaster when comparing the *GPE* and *KE* and total energy?



roller coaster 3