Section 3 Sounds in Strings Revisited

What Do You See?

Learning Outcomes

In this section, you will

- **Calculate** the wavelength of a standing wave on a string.
- **Organize** data in a table.
- **Describe** how the pitch of the sound produced by a vibrating string depends on the wave speed, wavelength, and frequency of the waves on the string.

What Do You Think?

You investigated how the pitch of a vibrating string depends on the length of the string and the tightness (tension) of the string. How is the length of the vibrating string related to the wavelength of the standing wave set up on the string?

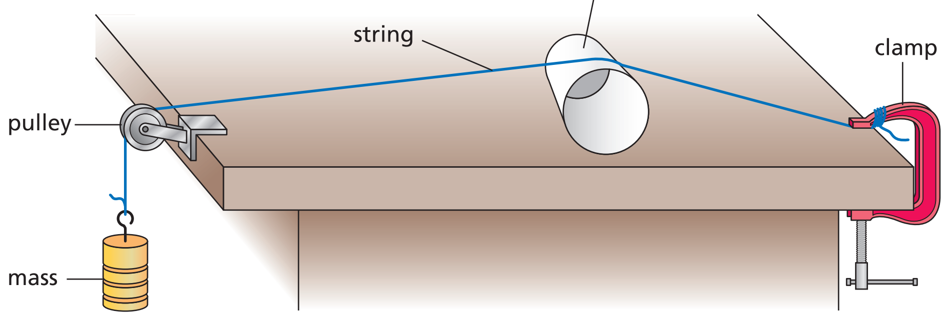
- **Why does the pitch change when you change the tension in the string?**

Record your ideas about this question in your *Active Physics* log. Be prepared to discuss your responses with your small group and with your class.

Investigate

1. Carefully mount a pulley over one end of a table, as you did in the *Investigate* in the first section. Securely tie one end of a string to the clamp on the other end of the table.

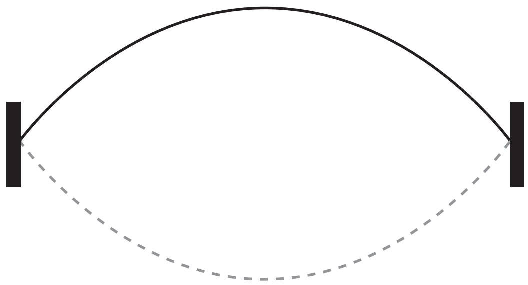
Be sure to wear impact goggles while doing the experiment.



2. Tie the other end of the string around a 500-g mass. Extend the string over the pulley. Place a plastic or styrene-foam cup under the string near the clamp, so the string can vibrate without hitting the table, as shown in the diagram above. You can adjust the length of the vibrating string by sliding the cup back and forth.
3. Hang one 500-g mass on the string. Pluck the string, listen to the sound, and observe the string vibrate.

   The vibrating string is producing a standing wave. When you pluck the string, it does not move at the ends. Just as you could transmit a standing wave on the coiled spring, you can produce a standing-wave pattern on the string with the two fixed ends being nodes and the center being an antinode. It is more difficult to see the vibration of the antinode on the string because the string vibrates so quickly.

4. Measure the length of your vibrating section of string, and calculate the wavelength of the vibration. The length of the string equals ½ of the total wavelength (½ λ).
   a) Record the length of the vibrating string and the wavelength in a table in your log.



5. Shorten the vibrating part of the string.
   a) Record how the pitch (frequency) of the sound changed.
   b) Record the new length of the vibrating string and the new wavelength of the standing wave for this shortened string.
6. Repeat *Step 5* for two new lengths.
7. Look at the data in your table.
   a) In your log, make a general statement about what happens to the wavelength as you change the length of the string.

b) Make a second general statement about what happens to the pitch or frequency of the sound as you change the length of the string.

8. In the first section, you changed the tension in the string by adding weights and observed a change in pitch—the greater the tension, the higher the pitch. Since the length of the string stayed the same, the wavelength must have also stayed the same.

a) What wave property changed to make the frequency higher? (Hint: Recall the wave equation, $v = f\lambda$. If the wavelength remains the same and the frequency changes, what else needs to change?)

9. You can explore the relationship among wave speed, wavelength, and wave frequency with the following investigation. You may have to do this investigation in the hall or outside on the sidewalk or athletic field.

Place small pieces of masking tape about 30 cm apart on the floor. Cover a distance of about 10 m in a straight line. Now walk, stepping on each piece of tape by taking one step each second. Your "frequency" is one step per second and your "wavelength" is 30 cm (the distance between pieces of tape).

To help you perform this task, have a member of your group call out the time using a stopwatch.

a) Time your overall travel and then calculate your speed by dividing the total distance traveled by the time elapsed. Compare that result to what you find from multiplying wavelength times frequency.

10. Now change your "wavelength" by stepping on every other piece of tape. Keep the same frequency (one step per second).

a) Again, compare your speed for the trip with the result obtained by multiplying wavelength by frequency.

11. Now change your frequency by taking one step every two seconds. Make your wavelength 30 cm.

a) Again, compare your speed for the trip with the result of multiplying wavelength by frequency.

12. Make up your own combination of frequency and wavelength and see how they affect your speed.

a) Record your findings in your *Active Physics* log.

Physics Talk

WAVELENGTH, WAVE SPEED, AND FREQUENCY

Frequency and Wavelength

The vibrating string producing the sound is actually setting up a standing wave between its endpoints. The length of the string determines the wavelength of this standing wave. If the string is 40 cm, then the wavelength of the lowest-frequency standing wave is 80 cm. The length of the string is always ½ the wavelength of the lowest-frequency standing wave.

The pitch that you hear is related to the frequency of the wave. The higher the pitch, the higher the frequency. You expressed this with a mathematical equation.

To get a higher frequency, you have to shorten the string or generate a smaller wavelength. Recall the wave equation:

Wave speed = wave frequency × wavelength

In symbolic form,

$$v = f\lambda$$

$$\begin{aligned} \text{where } v &= \text{speed,} \\ f &= \text{frequency, and} \\ \lambda &= \text{wavelength.} \end{aligned}$$

You can rewrite this equation to solve for frequency. Divide both sides of the equation by the wavelength:

$$f = \frac{v}{\lambda}$$

Now analyze the equation and ensure that it is consistent with your experimental findings in the *Investigate*. When you shortened the length of the string, you also shortened the wavelength of the standing wave. You found that this shorter wavelength increased the pitch. An increase in pitch corresponds to an increase in frequency. When you shortened the wavelength of the string's standing wave, you increased the frequency.

The equation above shows this as well. When the wavelength gets shorter, the denominator on the right side of the equation gets smaller. When the denominator gets smaller, the value of the fraction gets larger (as long as the numerator stays the same). For example, $\frac{1}{100}$ is much smaller than $\frac{1}{2}$. If the fraction on the right side of the equation gets larger, then the left side of the equation also gets larger. In the equation, $f = v/\lambda$, the frequency gets larger.

This is what your experiment demonstrated. The shorter the wavelength is, the higher the frequency. This is called an **inverse relationship**. In an inverse relationship, decreasing one variable increases the other variable or vice versa. In this instance, decreasing the wavelength increases the frequency and pitch.

decreasing the wavelength increases the frequency and pitch

Physics Words

inverse relationship: a relationship in which decreasing one variable increases the other variable or vice versa.

Tension and Thickness of a String and Frequency

How are tension and pitch related? In the first section of this chapter, you also found that changing the tension in the string by adding masses would change the frequency or pitch of the sound. When you increased the tension, you did not change the wavelength of the standing wave.

You can use the equation $f = v/\lambda$ to help you understand what happened.

Since the wavelength did not change, and the frequency increased, you must conclude that the speed of the wave increased.

The increased tension in the string means that a portion of the string that is displaced to the side will feel a larger force pulling it back toward its rest position. An increase in tension produces a larger force. A larger force will produce a greater acceleration on that portion of the string that is displaced and make it vibrate faster. This vibration makes the disturbance travel more quickly down the string.

In the equation, $f = {}^{v}\!/_{\lambda}$, increasing the wave speed increases the value of the right side of the equation. This corresponds to an increase of the left side of the equation or an increase in frequency. This a **direct relationship.** Increasing one variable also increases the other variable. In this case, the wave speed increases and therefore the frequency or pitch must increase.

Physics Words

direct relationship: a relationship in which increasing one variable increases the other variable or decreasing one variable also decreases the other variable.

increasing the wave speed increases the frequency and pitch

How does increasing the thickness of the string lead to a different wave speed in the string? You know from looking at guitars or violins or the inside of pianos that the thick strings produce the lower frequency sounds. The increased mass in the string means that a portion of the string that is displaced to the side will require an increased force to pull it back toward its rest position. This force is the tension produced in the string. For a given amount of tension force, a heavier mass will have a smaller acceleration on the displaced portion of the string and hence, it will move more slowly back and forth. The decrease in acceleration will make the disturbance travel more slowly down the string.

Because the tension changes the wave speed, a weaker tension would mean a wave travels more slowly on the spring.

In the equation, $f = {}^{v}\!/_{\lambda}$, decreasing the wave speed decreases the value of the right side of the equation. This corresponds to a decrease of the left side of the equation or a decrease in frequency.

Is There an Equation?

Standing waves occur when the length of the coiled spring or string and the wavelength have a particular relationship. The length of the coiled spring must equal $\frac{1}{2}$ wavelength, 1 wavelength, $\frac{3}{2}$ wavelengths, 2 wavelengths, and so on. Mathematically, this can be stated as

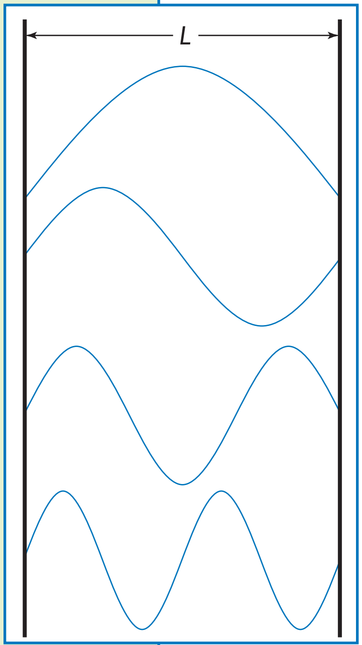
$$L = \frac{n\lambda}{2}$$

where L is the length of the coiled spring,

λ is the wavelength, and

n is a number (1, 2, 3…).

The number n is the number of antinodes in the standing wave pattern.



Sample Problem 1

You and your partner sit on the floor and stretch out a coiled spring to a length of 3.5 m. You shake the coiled spring so that the pattern has one antinode between the two of you. Your partner measures the time for 10 vibrations and finds that it takes 24.0 s for the coiled spring to make 10 vibrations.

a) What is the wavelength of this wave?

Strategy: Draw a sketch of the wave you made. It should look like the first wave pattern in the diagram. It is one-half of a full cycle of the wave. This is the maximum wavelength for a standing wave on this length of coiled spring. You can use the equation that shows the relationship between the length of the coiled spring and the wavelength.

Given:

$L = 3.5 \text{ m}$

$n = 1$

Solution:

$$L = \frac{n\lambda}{2}$$

Rearrange the equation to solve for λ.

$$\lambda = \frac{2L}{n} = \frac{2(3.5 \text{ m})}{1} = 7.0 \text{ m}$$

Notice that the wavelength is twice the length of the coiled spring.

b) What is the period of vibration of the wave?

Strategy: The period is the amount of time for one vibration. You have the amount of time for 10 vibrations.

Solution:

$$T = \frac{\text{time for 10 vibrations}}{\text{10 vibrations}} = \frac{24.0 \text{ s}}{10} = 2.4 \text{ s}$$

c) What is the frequency of this standing wave?

Strategy: The frequency represents the number of vibrations per second. It is the reciprocal of the period.

Given:

$T = 2.4 \text{ s}$

Solution:

$$f = \frac{\text{number of vibrations}}{\text{time}} \text{ or } f = \frac{1}{T}$$

$$= \frac{1}{2.4 \text{ s}}$$

$$= 0.42 \text{ vibrations per second}$$

$$= 0.42 \text{ Hz}$$

d) Determine the speed of the wave you have generated on the coiled spring.

Strategy: The speed of the wave may be found by multiplying the frequency times the wavelength.

Given:

$f = 0.42 \text{ Hz or } 0.42 \text{ s}^{-1} \left(\frac{1}{\text{s}}\right)$

$\lambda = 7.0 \text{ m}$

Solution:

$$v = f\lambda$$

$$= 0.42\left(\frac{1}{\text{s}}\right) \text{x } 7.0 \text{ m}$$

$$= 2.94 \text{ m/s or } 2.9 \text{ m/s}$$

Sample Problem 2

You stretch out a coiled spring to a length of 4.0 m, and your partner generates a pulse that takes 1.2 s to go from one end of the coiled spring to the other. What is the speed of the wave on the coiled spring?

Strategy: Use the equation for speed.

Given:

$\Delta d = 4.0 \text{ m}$

$\Delta t = 1.2 \text{ s}$

Solution:

$$v = \frac{\Delta d}{\Delta t} = \frac{4.0 \text{ m}}{1.2 \text{ s}}$$

$$= 3.3 \text{ m/s}$$

Checking Up

1. How does decreasing the wavelength increase the frequency of a wave? Explain, using an equation that relates the two variables of frequency and wavelength to wave speed.
2. How is the tension of a string related to its pitch?
3. Explain how tension relates to wave speed.
4. What is the equation that relates the length of a coiled spring and the wavelengths of the standing waves that can be produced on the spring?

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
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Is There an Equation?

A vibrating string sets up a standing wave. The frequency of the sound can be increased by shortening the string length, by increasing the tension, or by using a thinner string (thereby decreasing the mass). You can derive an equation that combines all of these relationships.

$$f = \frac{v}{\lambda}$$

Since the wavelength is ½ the length of the string: $\lambda = 2L$

$$f = \frac{v}{2L}$$

From other studies, physicists have found

$$v = \sqrt{\frac{T}{m/L}}$$

where m = mass of one string,
L = length of the string, and
T = tension of the string.

By combining these equations:

$$f = \frac{\sqrt{\frac{T}{m/L}}}{2L} = \sqrt{\frac{T}{4mL}}$$

1. A standard acoustic guitar has six strings. For the highest pitch notes, a thin string is used. For the lowest notes, a thick string is used. For the intermediate pitch notes, a medium thickness string is used. The string tension is controlled by the tuning pegs at the end of the guitar. To keep the neck of the guitar from bending to one side or the other, you want to have the tension in all of the strings about the same. Using what you learned in this section, explain why the guitar designer decided to use thick strings for the low notes and thin strings for the high notes.

What Do You Think Now?

At the beginning of this section you were asked the following:

- **Why does the pitch change when you change the tension in the string?**

Use what you learned in this section to explain how the length of a vibrating string is related to the wavelength of a standing wave on the string, how the pitch of a vibrating string changes when you change the tension of the string, and how the thickness of the string affects the velocity and the frequency of a standing wave on a string.

Physics Essential Questions

What does it mean?

When you plucked a string instrument, you set up a standing wave on the string. What is a standing wave?

How do you know?

Physicists want to know how the quantities like wave speed depend on the properties of the medium in which the wave is traveling. How does the speed of a wave on a string depend on the string's length and its tension? How does the wave speed depend on the thickness of the string? Describe the evidence you have for this from your experiments.

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Waves and interactions	Models	* Experimental evidence is consistent with models and theories

* Physicists develop models and mathematical relationships that apply to many different situations. Waves and their interactions are big ideas in physics and the same models of waves can explain sound, water, light, and waves on a string. Explain how what you learned in this section about string vibrations illustrates the general relationships that link wavelength, wave frequency, and wave speed.

Why should you care?

Physicists are always looking for general principles that apply to many different situations. You care about relationships connecting wavelength, wave speed, and wave frequency because you are interested in music and how you can generate sounds on string instruments for the *Chapter Challenge*. You may also care because you or your friends play string instruments in a band or an orchestra. Give some examples of where different kinds (different materials) of vibrating strings or cords show up in everyday life. How will what you learned in this section help you create musical instruments for your challenge?

Reflecting on the Section and the Challenge

In this section, you related your observations of the pitch of vibrating strings in *Section 1* to the wave vocabulary developed in the previous section. You learned that the length of the vibrating string determines the wavelength of the standing wave. The tension in the string determines the wave speed. Together, these effects determine the wave frequency, which is what determines the pitch of the sound that you hear. That's the physics of string instruments!

If you wanted to create a string or multi-string instrument for your show, you would now know how to adjust the length and tension and mass of the string to produce the notes you want. If you were to make such a string instrument, you could explain how you change the pitch by referring to the results of this section.

Physics to Go

1. Tell how changing the tension of a vibrating string changes the frequency of the wave produced.
2. Tell how changing the length of a vibrating string changes the wavelength of the standing wave in the string.
3. How would you change both the tension and the length of a vibrating string and keep the frequency the same?
4. Suppose you changed both the length and the tension of a vibrating string at the same time. What would happen to the sound in terms of wavelength and frequency?
5. For the guitar, tell how a performer changes the frequency of vibration of the strings to tune the instrument.
6. A guitar has six strings of the same length. The thickness or mass of the strings is different and each string has a different pitch and frequency. Explain why the mass of the string affects the frequency of the wave. (Hint: Think about how force, mass, and acceleration are related.)
7. ***Preparing for the Chapter Challenge***

   Design a string instrument that you may consider using in your sound and light show. Provide the explanation that will meet the requirements of the challenge. You will want to describe how the string forms a standing wave, the wavelength of that standing wave, and how wavelength relates to the frequency and pitch you hear. Use the rubric to grade yourself on this part of the challenge.

Inquiring Further

Investigate frequency using a frequency meter

1. Set up the vibrating string as you did in the *Investigate*. This time, you will measure the frequency of the sound. Set up a frequency meter on your computer. (A free frequency-counter program for your computer can be found on the Internet.) Pick up the sound with a microphone. Investigate how changing the length of the string changes the frequency of the sound. Sketch a graph to describe the relationship.
2. Set up the vibrating string, computer, and microphone as you did in *Step 1*. This time, investigate how changing the string tension changes the frequency of the sound. Sketch a graph to describe the relationship.
3. Set up vibrating strings of differing thicknesses and investigate how the mass of the string changes the frequency of the sound. Does the wave speed remain the same in all of the strings? Use the frequency obtained from your frequency meter and the wavelength that you measured in this section to calculate the wave speed. Is the wave speed slower in thick, heavy strings, than in thin, lighter strings under the same tension?