## Section 4

## Projectile Motion on he Moon



## Learning Outcomes

In this section, you will

- Apply the acceleration due to gravity on Earth to projectile motion on Earth.
- Apply this understanding to describe the acceleration due to gravity on the Moon to projectile motion on the Moon.
- Design a mathematical model and a physical model of the trajectory of a projectile on the Moon.


## What Do You Think?

A baseball has $1 / 6$ the weight on the Moon as on Earth, but a baseball's mass on the Moon is the same as on Earth.

- Can a batter hit or a player throw a baseball faster on the Moon than on Earth?
- Can a batter hit or a player throw a baseball farther on the Moon than on Earth?
- If your answer to either question is yes, how much faster or farther?
Record your ideas about these questions in your Active Physics log. Be prepared to discuss your responses with your small group and the class.


## Investigate

In this Investigate, you will set up a scale drawing to calculate the range and maximum height achieved for a projectile launched with the same velocity when it is on Earth and on the Moon.

1. Use the following instructions to produce a $1 / 10$ scale drawing, that is, a drawing $1 / 10$ of the actual size, of a trajectory model of a projectile (the path an object you throw will take)
launched at a speed of $4.0 \mathrm{~m} / \mathrm{s}$. Work with members of your group.
a) On a standard-size sheet of paper (about 22 cm by 28 cm ) as shown reduced in size below, mark a starting point 2 cm above and 2 cm to the right of the bottom-left corner of the paper. From the starting point, draw two straight lines entirely across the sheet, one horizontal and another inclined at an angle of $30^{\circ}$. Add the title shown in the sketch.

D) The horizontal line represents ground level, and the inclined line represents the path that a projectile launched from the starting point at a $30^{\circ}$ angle would follow if there were no gravity. Measuring from the starting point, mark points at $4.0-\mathrm{cm}$ intervals on the inclined line. Since the launch speed is $4.0 \mathrm{~m} / \mathrm{s}(400 \mathrm{~cm} / \mathrm{s})$, the projectile would travel 40 cm every tenth of a second. This model is $1 / 10$ scale, so 4.0 cm is $1 / 10$ of the actual distance $(40 \mathrm{~cm})$ that the projectile would travel in 0.10 s . The marked points represent the position of the projectile every 0.10 s for a zerogravity condition. Begin by labeling the starting point as 0.00 s , label successive points on the inclined line as $0.10 \mathrm{~s}, 0.20 \mathrm{~s}, 0.30 \mathrm{~s}$, and so on.
©c) Also mark points at $2.0-\mathrm{cm}$ intervals on the horizontal line. Begin by labeling the starting point as 0.00 m .

Mark successive points on the horizontal line as $0.20 \mathrm{~cm}, 0.40 \mathrm{~cm}$, 0.60 cm , and so on. These points represent distance along the ground, scaled, of course, by a factor of 10 from real-world distances.
\d) Use the equation

$$
d=\frac{1}{2} g t^{2}
$$

(where $g=980 \mathrm{~cm} / \mathrm{s}^{2}$ instead of the usual $9.8 \mathrm{~m} / \mathrm{s}^{2}$ ) to calculate the total distance an object on Earth would fall. Start from rest and then determine the distance fallen in $0.10 \mathrm{~s}, 0.20 \mathrm{~s} \ldots 0.60 \mathrm{~s}$. Draw three columns in your Active Physics log to make a table. Enter the time in seconds in the first column, the distance fallen in the second column, and then divide each fall distance by 10 to fit the $1 / 10$ scale of the drawing.
De) Next, draw a line vertically downward from each marked point on the inclined line to show the projectile's position at that time. For example, the line at the point labeled 0.10 s should extend 0.49 cm (or 4.9 mm ) downward from the inclined line because 4.9 cm divided by 10 equals 0.49 cm .
Sf) The bottom ends of the vertical fall lines represent the projectile's position at 0.10 s intervals during its flight. Connect the bottom ends of the lines with a smooth curve to show the shape of the trajectory and label the curve "Trajectory on Earth."
gg) Use the distance scale established on the horizontal line to measure, to the nearest 0.10 m , the projectile's real-world maximum height above ground level, and the horizontal range of the projectile before striking the ground. Record the maximum height and range on the drawing.

Dh) Use the time scale established on the inclined line to measure, to the nearest 0.010 s , the projectile's time of flight. Record the time of flight on the drawing.
2. You will now draw the trajectory that would result if the projectile were launched at the same speed and in the same direction on the Moon.
دa) Use the same equation

$$
d=\frac{1}{2} g t^{2}
$$

to calculate the total distance an object on the Moon falls, starting from rest, in $0.10 \mathrm{~s}, 0.20 \mathrm{~s}, 0.30 \mathrm{~s}$, and so on. The value of acceleration to use in the equation is the acceleration due to gravity on the Moon, $1.6 \mathrm{~m} / \mathrm{s}^{2}$, or $160 \mathrm{~cm} / \mathrm{s}^{2}$.

Prepare a table in your Active Physics log, making it similar to the table for Earth distances fallen, to show the calculated value of the total distance of fall at the end of each 0.10 s of flight on the Moon. Enter the time in seconds in the first column, the distance fallen in the second column, and then divide each fall distance by 10 to fit the $1 / 10$ scale of the drawing. Draw the trajectory for the projectile on the Moon in a similar manner as the trajectory you drew on Earth.
b) Draw a vertical line downward from each marked point on the inclined line to show the projectile's position at that time on the Moon. For example, the line at the point labeled 0.30 s should extend 0.72 cm , or 7.2 mm , downward from the inclined line. This line, and others, will need to be drawn on top of or immediately next to the lines drawn earlier for fall distances on Earth.


Extend the size of the paper to be able to show the entire trajectory on the Moon as shown in the sketch. Tape the sheet of paper containing your drawing to the lower left-hand corner of a sheet of wrapping paper approximately 46 cm high and 91 cm wide.
دc) The bottom ends of the vertical fall lines represent the projectile's position at 0.10 s intervals during its flight on the Moon. Connect the bottom ends of the lines with a smooth curve to show the shape of the trajectory and label the curve "Trajectory on the Moon."
d) Use the distance and time scales on the drawing to measure the projectile's maximum height, range, and time of flight on the Moon. Record the values on the drawing. Fold and save your drawing.
3. Create a table to show the above measurements of the maximum heights, ranges, and times of flight of a projectile launched with equal initial velocities on Earth and the Moon to complete the calculations below.
دa) Max height of projectile on Earth Max height of projectile on the Moon Range of projectile on the Moon
Dc) Time of flight on Earth

Time of flight on the Moon
d) Show your work and discuss it in your Active Physics log.
4. Write a summary of the effects of the Moon's " $\frac{1}{6} g$ " on the maximum height, range, and time of flight of a projectile launched on the Moon as compared to the same projectile launched at the same speed and angle of elevation on Earth.
دa) Record your summary in your Active Physics log.

## Physics Talk

## PROJECTILES ON THE MOON

In the Investigate for this section, you drew a scaled-down version of an object's path, launched at an angle on the Moon. To do this, you assigned the projectile an initial speed of $4 \mathrm{~m} / \mathrm{s}$ at an angle of $30^{\circ}$ to the horizontal. To plot the object's trajectory, you calculated how far the object would fall from a line drawn at the $30^{\circ}$ angle at one-tenth second intervals.
This plotting procedure relies on the principle that the motion of a projectile is composed of two independent motions: a constant-velocity portion and an accelerated-motion portion. The line ascending at $30^{\circ}$ represents the path the projectile would take if there were no gravity to affect its path. Without gravity, the projectile would follow Newton's first law and continue in motion in a straight line with constant speed.
When gravity exists, it causes an object to accelerate toward the surface of the planet, and the distance the object would fall is given by the equation $d=\frac{1}{2} g t^{2}$.For a planet that has an acceleration due to gravity of $2 \mathrm{~m} / \mathrm{s}^{2}$ (close to that of the Moon), the object would fall a distance of 1 m by the end of the first second, a total of 4 m by the end of second number two, a total of 9 m by the end of second three, and so on. When these two motions are combined, the trajectory or path of the projectile is established. The trajectory is a parabola because gravity's downward acceleration is constant.
The operation described above can be used to find the position of a projectile at all points along its path at any time. To plot the trajectory in the more familiar $x, y$ coordinates, only a slight modification of your procedure is required. If the velocity of the projectile at the launch angle is broken down into individual vertical and horizontal motions, each of these can be plotted separately. To find how high the object would travel during any time interval in the absence of gravity, you can just use the familiar equation

$$
v_{y}=\frac{\Delta d_{y}}{\Delta t} \text { or } d_{y}=v_{y} \Delta t
$$

where $v_{y}$ is the vertical component of the launch velocity, and $d_{y}$ is the height at any time.
To find the height when gravity is included, add the vertical height without gravity to the calculated fall due to gravity for that time. When these two vertical motions are added, the vertical or " $y$ " coordinate has been found.

$$
d_{y}=\frac{1}{2} g t^{2}+v_{y} t
$$

The acceleration is negative because it is in the opposite direction of $v_{y}$.

The projectile's horizontal position at any time can be found from the equation

$$
v_{\mathrm{x}}=\frac{\Delta d_{\mathrm{x}}}{\Delta t} \text { or } d_{\mathrm{x}}=v_{\mathrm{x}} \Delta t
$$

where $v_{\mathrm{x}}$ is the horizontal component of velocity and
$d_{x}$ is the horizontal distance at any time.
Nothing needs to be added to the horizontal position, since the force of gravity works only in the vertical direction. The path of a projectile affected only by gravity (no air resistance) has a specific curved shape. This curve is referred to as a parabola. You drew a parabolic path on Earth and the Moon in the Investigate.
The vertical and horizontal components of the launch velocity can be found either by graphical methods (measuring) or by calculation. To find the velocities graphically, draw a right triangle with an angle to the horizontal the same as the launch velocity. Make the length of the hypotenuse equal to a scaled down version of the launch velocity. When you use a ruler to measure the size of the vertical and horizontal sides of the triangle, you have found the scaled down size of the vertical and horizontal launch velocities. You then scale the velocities up to the correct values.
To calculate the vertical and horizontal launch velocities, trigonometry can be used.
Knowing these velocities and using this method allows you to solve numerous problems in trajectories.

## Sample Problem

A field-goal kicker in a football game on the Moon kicks the ball at an angle of $53^{\circ}$ to the horizontal with a speed of $10.0 \mathrm{~m} / \mathrm{s}$. Will the football clear the crossbar 3.0 m above the ground if the goal post is 54 m away?
Strategy: First, measure the vertical and horizontal components of the velocity by setting up a scale diagram similar to the
 one on the right. The measured vertical and horizontal velocities then will be $v_{\mathrm{y}}=8 \mathrm{~m} / \mathrm{s}$ and $v_{\mathrm{x}}=6 \mathrm{~m} / \mathrm{s}$, respectively.

$$
\begin{array}{ll}
\text { Given: } g=1.6 \mathrm{~m} / \mathrm{s}^{2} & \begin{array}{l}
v_{y}=8 \mathrm{~m} / \mathrm{s} \\
v_{\mathrm{x}}
\end{array}=6 \mathrm{~m} / \mathrm{s}
\end{array}
$$

## Solution:

The time it takes the ball to travel the 54 m horizontally to the goal post is found using the horizontal velocity equation

$$
v_{\mathrm{x}}=\frac{\Delta d_{\mathrm{x}}}{\Delta t}
$$

## Solving for $t$ gives

$$
\begin{aligned}
\Delta t & =\frac{\Delta d_{\mathrm{x}}}{v_{\mathrm{x}}} \\
& =\frac{54 \mathrm{\mu h}}{6 \mathrm{ph} / \mathrm{s}} \\
& =9 \mathrm{~s}
\end{aligned}
$$

To find how high the ball is above the ground 9 s after being kicked, find the vertical distance the ball would travel without gravity and then subtract the fall due to gravity.

$$
\begin{aligned}
\Delta d_{y} & =\left(\Delta v_{y}\right) \Delta t \\
& =(8 \mathrm{~m} / \mathrm{s})(9 \mathrm{~s}) \\
& =72 \mathrm{~m}
\end{aligned}
$$

The fall distance due to gravity is found from

$$
\begin{aligned}
d & =\frac{1}{2} g t^{2} \\
& =\frac{1}{2}\left(1.6 \mathrm{~m} / \mathrm{s}^{2}\right)(9 \mathrm{~s})^{2} \\
& =\frac{1}{2}\left(1.6 \mathrm{~m} / \mathrm{s}^{\prime 2}\right)(9 \$)\left(9 s^{\prime}\right) \\
d & =65 \mathrm{~m}
\end{aligned}
$$

Combining the two vertical distances gives 72 m of rise minus 65 m of fall or 7 m . The ball is 7 m above the ground, and easily clears the crossbar.

## Checking Up

1. What is the path of a projectile without gravity? Why does it follow this path?
2. What is the shape of a projectile's path with gravity?
3. What two motions should be combined to find the vertical position of a projectile at any time?
4. What must be known to find the horizontal position of a projectile at any time during its flight?
5. How can you obtain the vertical and horizontal components of a projectile's velocity?

## Active Physics

| +Math | +Depth | +Conc |
| :---: | :---: | :---: |
| -* | * | - |

## Equations for Projectile Motion

If you have studied projectile motion prior to this section, you know some of the relationships between position, velocity, and acceleration. You may recall
that the motion in the horizontal direction ( $x$ direction) is independent of the motion in the vertical direction ( $y$ direction). These relationships are summarized in equations for the horizontal and vertical motions on the following page.

All quantities are considered positive if they are directed upward. The object is assumed to have started at the position $(0,0)$.
$v_{\mathrm{x}}=v_{\mathrm{x} 0} \quad x=v_{\mathrm{x} 0} t$
$v_{\mathrm{y}}=v_{\mathrm{y} 0}+g t \quad y=v_{\mathrm{y} 0} t+\frac{1}{2} g t^{2}$
In these equations, $x$ and $y$ are the horizontal and vertical positions, $v_{\mathrm{x} 0}$ and $v_{\mathrm{y} 0}$ are the horizontal and vertical components of the velocity at time $=0$. Here, $g$ is the acceleration due to gravity, and $t$ is the time.

Notice that the horizontal component of the velocity is constant and equal to its value at time $=0$. Finally, $g$ is a negative number since the acceleration due to gravity is down $\left(-9.8 \mathrm{~m} / \mathrm{s}^{2}\right.$ on Earth and $-1.6 \mathrm{~m} / \mathrm{s}^{2}$ on the Moon).
In analyzing projectile motion where the object starts at some height and returns to the same height, there is an easy way to use these relationships. Note that the motion from the start up to the point of maximum height is the same as the motion from the maximum height down to the finish except that one is the reverse of the other. The important point is that the time it takes for the projectile to reach its maximum height is the same as the time it takes for it to descend from its maximum height to the finish. Find the time it takes to reach its maximum height first, and use this result to find other quantities of interest.
As the projectile goes up, the vertical component of its velocity $v_{y}$ decreases. At the same time the horizontal component of its velocity $v_{\mathrm{x}}$ remains constant. At some point in time $v_{y}$ decreases to zero, then through zero, and finally
becoming negative. When $v_{y}$ is positive, the projectile is rising, and when $v_{\mathrm{y}}$ is negative, the projectile is falling. The point at which $v_{y}$ is zero is the point of maximum height of the projectile. Setting $v_{\mathrm{y}}=0$ yields

$$
0=v_{\mathrm{y} 0}+g t_{\max } \quad \text { or } \quad t_{\max }=\frac{v_{\mathrm{y} 0}}{-g} .
$$

The total time the projectile is in the air $t_{\text {total }}$ is twice the time it takes to reach maximum height, so

$$
t_{\text {total }}=2 t_{\max }=\frac{2 v_{\mathrm{y} 0}}{-g} .
$$

From this result, it is easy to see that if two projectiles on Earth and on the Moon start off with the same value of $v_{\mathrm{y} 0}$, the time in flight is six times longer on the Moon because the value for the acceleration due to gravity $g$ is one sixth as large on the Moon.

1. Did your results for the two trajectories you constructed in the Investigate agree with this?
Finding the maximum height is not difficult now that you know the time it takes the projectile to reach its maximum height. Simply substitute $t_{\text {max }}$ into the equation for $y\left(\right.$ let $\left.y_{0}=0\right)$ and see if you can get the equation

$$
y_{\max }=\frac{v_{y 0}^{2}}{-2 g}
$$

2. How should the maximum heights of a projectile launched with the same value of $v_{\mathrm{y} 0}$ on Earth and the Moon compare? Compare this with your constructed trajectories.

The range of the projectile (denoted $R$ or $x_{\text {max }}$ ) can be determined by finding the horizontal position $x$ at time equal to $t_{\text {total }}$. Try this for yourself. The result is

$$
R=x_{\max }=\frac{2 v_{\mathrm{x} 0} v_{\mathrm{y} 0}}{-g}
$$

3. How should the ranges of projectiles on Earth and on the Moon compare if they start with the same values for
both of the two components of the velocity? Is this what you found in the Investigate?
4. Question 3 in Physics to Go states that if a projectile is launched at $45^{\circ}$, the range is the velocity squared divided by the acceleration due to gravity (considered positive). See if you can derive this result starting with the more general equation for range given above.

## What Do You Think Now?

At the beginning of this section you were asked the following questions:

- Can a batter hit or a player throw a baseball faster on the Moon than on Earth?
- Can a batter hit or a player throw a baseball farther on the Moon than on Earth?
- If your answer to either question is yes, how much faster or farther?

Based on what you have learned about projectile motion, how would you answer these questions now? Record your answers in your Active Physics log.


## Physics Essential Questions

What does it mean?
How do the trajectories of the same projectile launched identically on Earth and on the Moon differ?

How do you know?
What analysis did you perform that supports your previous answer? Be specific and use values in your answer.

Why do you believe?

| Connects with Other Physics Content | Fits with Big Ideas in Science | Meets Physics Requirements |
| :--- | :--- | :---: |
| Forces and motion | Change and constancy | * Good, clear explanation, no more <br> complex than necessary |

* The laws of physics are identical on Earth and the Moon, but the acceleration due to gravity and the path of a baseball may not be. Explain how this can be so.


## Why should you care?

Projectiles are a part of many sports. The human body is the projectile in some sports. What is going to be different for projectile sports when they are played on the Moon as compared to when they are played on Earth?

## Reflecting on the Section and the Challenge

This section clearly demonstrates that some sports may be, quite literally, "out of sight" on the Moon. For example, a $300-\mathrm{m}$ golf drive on Earth should translate into an $1800-\mathrm{m}$ drive on the Moon. That is almost 2 km (over a mile). Could a golf ball be found after such a drive? Probably not. To a tall person on the Moon, the horizon is only about 2.5 km away because the Moon is much smaller than Earth. On Earth, that same person would be able to see the horizon about 5 km away.

Adapting "Earth sports" to the Moon is not as simple as you may have imagined initially. A proposal to play golf on the Moon with no adjustments would, without doubt, "not fly" with NASA.
Consider a baseball hit to the outfield in a Moon stadium. It might take so long for the ball to fall that everyone would be bored. Any sport involving projectile motion will need careful analysis to see if it is feasible to be used on the Moon. Adaptations of the sport may require you to speed up the game. How you do that will depend on your imagination and creativity.

## Physics to Go

1. Due to the increased time and distance of travel of a projectile, discuss potential adjustments you need to make to play each of the following sports on the Moon:
a) football
b) gymnastics
c) trapeze
d) baseball
2. A typical sports arena on Earth has a playing field $120-\mathrm{m}$ long and $100-\mathrm{m}$ wide surrounded by tiered seats for spectators. World-class shot-put athletes throw the steel shot 23 m on Earth. Explain whether the spectators would be safe if a shot-put event were held in a stadium of similar size on the Moon.
3. The maximum range of a projectile launched at ground level occurs when the launch angle is $45^{\circ}$. Physicists have shown that the range of a projectile launched at $45^{\circ}$ is given by the equation $R=v^{2} / g$, where $R$ is the range, $v$ is the launch speed, and $g$ is the acceleration due to gravity on the planet or moon where the projectile is launched. How would this equation be useful for estimating the size of facilities needed for sports on the Moon?
4. If a golf ball were hit at a speed of $40 \mathrm{~m} / \mathrm{s}$ at a launch angle of $45^{\circ}$ on the Moon, what would be its range? (Hint: Use the equation $R=v^{2} / g$, where $R$ is the range, $v$ is the launch speed, and $g$ is the acceleration due to gravity on the Moon.)
5. Since the Moon's gravity is weaker than that on Earth, and since projectiles near the surface of the Moon do not experience air resistance, is it possible for an object thrown straight upward from the surface of the Moon to "escape" the Moon's gravity, never to fall back down to the surface of the Moon? Write a brief statement about your thoughts on this possibility.
6. You have found that the path of a trajectory from the ground to the ground requires six times the distance and six times the time. In basketball, the ball does not start at the ground and the hoop is not on the ground. You can, however, predict the trajectory in the following way:
a) Draw a person 1.8 m tall and a hoop some distance away that is 3.5 m high.
b) Draw a parabola that shows the ball moving from the player's head up and down through the hoop.
c) On the same diagram, draw a horizontal line at the original location of the ball. Extend the basketball's path through the basket until it crosses this line.
d) The basketball on the Moon will travel six times higher from this line and six times further along this line. Approximate the high point of the ball and where it will hit this line. Draw the complete path on the Moon.
e) How will you adapt the game of basketball given this new insight into how the ball moves on the Moon?

## 7. Preparing for the Chapter Challenge

You have seen in this section that the range of a projectile on the Moon is greatly increased. Some of your classmates may suggest that the way to reduce the range of the projectiles is to increase their mass. Using what you know about how objects of different masses are affected by gravity, explain why this will not solve the problem once the projectiles are in the air with equal speeds. But use your knowledge of the principle of inertia to explain why increasing the mass might lead to projectiles with reduced velocity and thus, reduced range.

## Inquiring Further

## Orbital velocity

You have seen that as celestial bodies get smaller, their gravity becomes weaker, and this weaker gravity leads to increased range. During this investigation, it was assumed that gravity was acting straight down off a flat surface. Real planets are curved. When the curvature is taken into account, is it possible for the range of the projectile to increase so much that the object never comes down? Look up Newton's canon to investigate what happens when the speed of a projectile is increased on a spherical body. Find a reference for the term "orbital velocity" and relate that to the speed required for a projectile never to return to the surface once launched.


