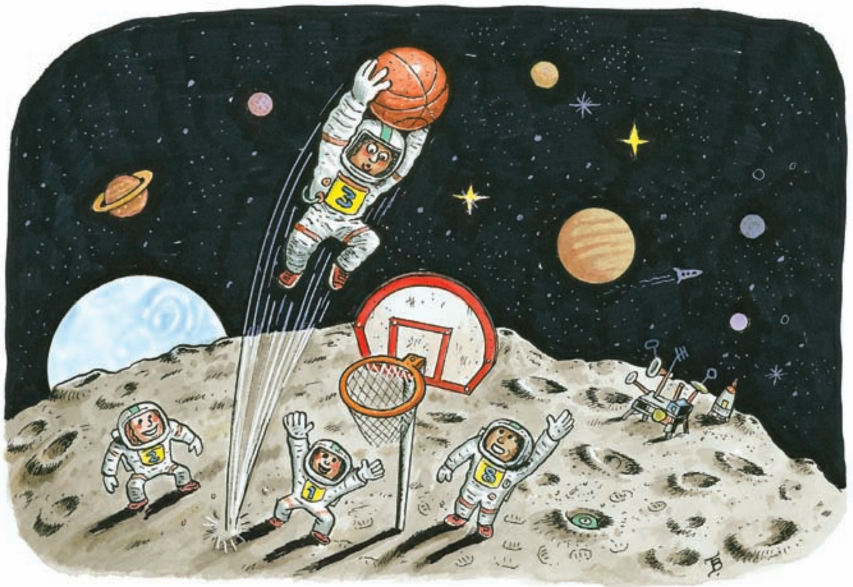
Section 5 Gravity, Work, and Energy: Jumping on the Moon

What Do You See?



Learning Outcomes

In this section, you will

- **Measure** changes in height during a vertical jump.
- **Calculate** changes in gravitational potential energy during a vertical jump.
- **Apply** conservation of energy to analysis of a vertical jump, including weight, force, height, and time of flight.
- **Make** predictions about jumping on the Moon using information gained from analyzing jumping on Earth.

What Do You Think?

Michael Jordan had a hang time of 1 s on Earth.

- **What would be a typical NBA star's hang time on the Moon?**
- **How high could the NBA star jump on the Moon? How high do you believe you could jump on the Moon?**

Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your responses with your small group and the class.

Investigate

In this *Investigate*, you will use the principles of conservation of energy and work to determine how high a person could jump on the Moon compared to that person's jump height on Earth.

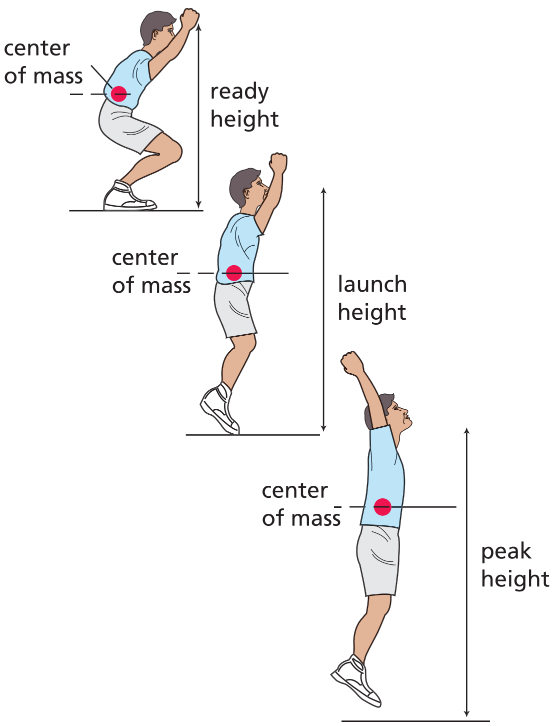
1. In an area free of obstructions, crouch and jump straight up as high as you can. Next, crouch in the same way, as if you are ready to jump, but then rise without jumping. Discuss how to answer the following questions and be prepared to share your group's responses with the class.

a) What is the source of the energy used to push your body upward in each case?

b) Why does your body leave the floor in one case, but not in the other?

2. Stand with your shoulder near a wall on which a vertical strip of paper has been mounted. Hold a marker in your hand that is near the wall. Crouch in a deep knee bend as if you are ready to jump. While in this "ready" position, raise your arm straight up and make a mark on the paper.

a) Measure and record the distance from the floor to the mark to the nearest 0.01 m and label it the "ready height."



3. Rise to your tiptoes. While in this "launch" position, raise your arm straight up and make another mark.

a) Measure and record the distance from the floor to the mark as the "launch height."

4. Crouch to the ready position and jump straight up as high as you can. Make a mark as high as you can when you are at the peak of your jump.

a) Measure and record the distance from the floor to the mark as the "peak height."

b) By subtraction, calculate and record how high you can jump. Use the equation:

Jump height = (peak height) – (launch height)

c) By subtraction, calculate and record your total change in height using the ready position as a reference and using this equation:

Total change in height = (peak height) – (ready height)

5. On a piece of paper, draw three sketches (to scale) showing the jumper in the ready position, the launch position, and the peak position using the data from your jump. In the ready position, the sketch should show the legs bent. While in the launch and peak positions, the sketch should show the legs straight.

a) Tape the sketches into your *Active Physics* log.

6. On the Moon, one would expect that the jumper would be able to go six times as high because the acceleration due to gravity is only $\frac{1}{6}$ on the Moon. This is not what really happens. Analysis shows that this factor of 6 applies to the total change in height from the ready position to the peak position (see *Physics Talk*).

a) Calculate the total change in height on the Moon by multiplying the total change in height from *Step 4.c)* by 6.

b) From the total change in height, calculate the peak height on the Moon.

7. Using the same scale as you did for your Earth data, draw three sketches (to scale) showing the jumper in the ready position, the launch position, and the peak position. The ready position and the launch position should be identical to that on Earth. The peak position should use your calculated value for the Moon.

   a) Tape the sketches into your *Active Physics* log.

Physics Talk

ANALYSIS OF JUMPING ON EARTH AND THE MOON

One of the most important ideas in physics is that under certain conditions, one form of energy can be transformed into the other form of energy, with no energy being gained or lost. An object can have **gravitational potential energy** due to its position above the surface of the planet. The object can have **kinetic energy** due to its velocity. The gravitational potential energy of the object can be transformed into kinetic energy as the object falls. The object loses gravitational potential energy (as it loses height) and gains kinetic energy (as it moves faster).

GPE represents gravitational potential energy.

$$GPE = mgh$$

where m = mass

g = acceleration due to gravity and

h = height above the ground.

since weight $= mg$

then GPE = (weight) × (height above the ground)

KE represents kinetic energy.

$$KE = \frac{1}{2}mv^2$$

where m = mass and

v = velocity

Hence, $KE = \frac{1}{2}$ (mass)(velocity squared)

Note that the units of energy are $\frac{(\text{kg} \cdot \text{m}^2)}{\text{s}^2}$. This unit is called a joule and its symbol is J.

You can analyze the jump on Earth by comparing the energy at the ready position, the launch position, and the peak position. Since the person is at rest in both the ready position and the peak position, she or he has no kinetic energy at either place. Chemical energy in your body is stored as leg muscle *PE*.

Physics Words

gravitational potential energy: the energy an object possesses as a result of its position near the surface of a planet or moon ($GPE = mgh$).

kinetic energy: the energy an object possesses because of its motion ($KE = \frac{1}{2}mv^2$).

The ready position has only leg-muscle potential energy. The peak position has only gravitational potential energy. The launch position has both kinetic energy and gravitational potential energy.

	Ready position	Launch position	Peak position
Kinetic energy *KE*	0	✔	0
Gravitational potential energy *GPE*	0	✔	✔
Leg-muscle potential energy *LPE*	✔	0	0

This analysis uses sample data for the vertical jump of a person on Earth who has a mass of 44 kg:

- Body mass = 44 kg
- Body weight on Earth = mg = 44 kg x 9.8 m/s^2 = 430 N
- Ready height (from floor to ready mark) = 1.70 m
- Launch height (from floor to launch mark) = 2.05 m
- Peak height (from floor to peak mark) = 2.65 m

You may apply this analysis to your vertical jump by substituting your personal data for the sample data, or you can assign a mass to an "unknown" person if you wish.

The following table shows all of the results of the analysis. Exactly how each result is obtained is explained in the paragraphs after the table.

	Earth	Moon
Mass (kg)	44	44
Weight (N)	430	70
Ready height (m)	1.70	1.70
Launch height (m)	2.05	2.05
Peak height (m)	2.65	7.60
Jump height (m)	0.60	5.55
Total energy (J)	410	410

The total energy of the jump is equal to the overall gain in the jumper's gravitational potential energy from the ready position to the peak position:

$$\begin{aligned}\text{Total energy} &= \text{change in gravitational potential energy}\\ &= mg\left[(\text{peak height}) - (\text{ready height})\right]\\ &= 430\ \text{N} \times (2.65\ \text{m} - 1.70\ \text{m})\\ &= 430\ \text{N} \times 0.95\ \text{m}\\ &= 410\ \text{J}\end{aligned}$$

This 410 J of energy was produced by the legs of the jumper during the push phase, while the feet were in contact with the ground.

Assuming the leg muscles work the same on the Moon, you would expect an increase of gravitational potential energy of 410 J on the Moon as well. However, the acceleration due to gravity, g_M, on the Moon is only $\frac{1}{6}$ the acceleration due to gravity on Earth ($1.6\ \text{m/s}^2$ compared to $9.8\ \text{m/s}^2$).

The Moon data would be:

- Body mass = 44 kg (identical to mass on Earth)
- Body weight $= mg = 44\ \text{kg} \times 1.6\ \text{m/s}^2 = 70\ \text{N}$ ($\frac{1}{6}$ the value on Earth)
- Ready height (from floor to ready mark) = 1.70 m (identical to the value on Earth)
- Launch height (from floor to launch mark) = 2.05 m (identical to the value on Earth)

With these data, you can compute the peak height on the Moon.

$$\begin{aligned}\text{Total energy} &= \text{change in gravitational potential energy}\\ &= mg\left[(\text{peak height}) - (\text{ready height})\right]\\ 410\ \text{J} &= 70\ \text{N} \times \left[(\text{peak height}) - 1.70\ \text{m}\right]\\ 5.9\ \text{m} &= \left[(\text{peak height}) - 1.70\ \text{m}\right]\\ \text{Peak height} &= 7.6\ \text{m}\end{aligned}$$

The jump height on Earth and Moon is found by subtracting the launch height from the peak height:

on Earth $2.65\ \text{m} - 2.05\ \text{m} = 0.60\ \text{m}$

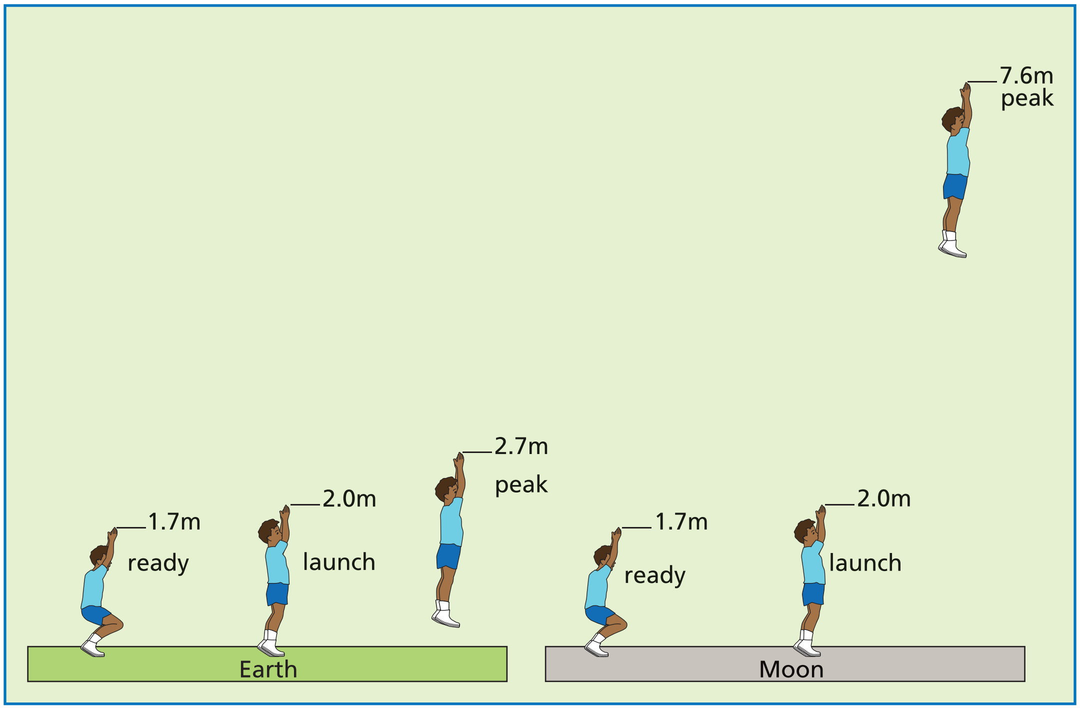
on Moon $7.6\ \text{m} - 2.05\ \text{m} = 5.55\ \text{m}$

$$\frac{5.55\ \text{m}}{0.60\ \text{m}} = 9.25$$

The ratio of these heights on the Moon and Earth is clearly more than six. A person can jump more than six times as high on the Moon as on Earth. The difference between the peak height and ready height is 0.95 m on Earth and 5.9 m on the Moon. As you can see, this is close to a factor of 6.

Checking Up

1. Before jumping, what type of energy is stored in your leg muscles when you are in the "ready" position?
2. While in the process of jumping, you go through the "ready," "launch," and "peak" positions. At which of these positions is your kinetic energy zero? At which position is it at the maximum?
3. How many times higher (greater) is the peak height of a jumper on the Moon compared to the same jumper on Earth?



Active Physics **Plus**

+Math	+Depth	+Concepts	+Exploration
◆	◆		

Kinetic Energy On The Moon

In the *Investigate,* you completed and compared the total energy of the launch position with the total energy at the peak position. Neither of these positions has kinetic energy. You can also look at the total energy of the launch position where the jumper has some gravitational potential energy and some kinetic energy. It is helpful to think about the energy during the push phase in two parts:

- The gain in gravitational potential energy of the body as it is lifted against gravity before it even leaves the ground.
- The gain in kinetic energy of the body as it is accelerated to a velocity high enough just to propel the body off of the ground.

The energy necessary to lift the body before leaving the ground (without acceleration) equals the gain in gravitational potential energy between ready distance and launch distance.

$$\begin{aligned}\text{Energy} &= mg\ [(\text{launch ht.}) - (\text{ready ht.})] \\ &= 430\ \text{N} \times (2.05\ \text{m} - 1.70\ \text{m}) \\ &= 430\ \text{N} \times 0.35\ \text{m} \\ &= 150\ \text{J}\end{aligned}$$

The kinetic energy needed to propel the body off the ground must be equal to any amount of the total energy that "remains" after subtracting the amount of energy needed to lift the body against gravity.

$$\begin{aligned} KE \text{ at launch} &= (\text{total energy}) - (\text{energy to lift}) \\ &= 410\text{ J} - 150\text{ J} \\ &= 260\text{ J} \end{aligned}$$

Of the 410 J of total energy that the muscles in the leg provide, 150 J are used to raise the body to the launch position and the remaining 260 J represents the kinetic energy for the jump.

What would happen if the person were to repeat the same jump on the Moon? What would be the person's jump height?

The person's mass, 44 kg, would remain the same on the Moon, but the person's weight would be less on the Moon:

$$\begin{aligned} \text{Weight on the Moon} &= 44\text{ kg} \times 1.6\text{ m/s}^2 \\ &= 70\text{ N} \end{aligned}$$

The energy needed just to lift the body against gravity during the push phase on the Moon is equal to gain in gravitational potential energy between the ready height and the launch height.

$$\begin{aligned} \text{Energy} &= mg\ [(\text{launch ht.}) - (\text{ready ht.})] \\ &= 70\text{ N} \times (2.05\text{ m} - 170\text{ m}) \\ &= 70\text{ N} \times 0.35\text{ m} \\ &= 25\text{ J} \end{aligned}$$

The kinetic energy needed to propel the body off of the ground on the Moon is equal to the total energy minus the energy needed just to lift the body against gravity:

$$\begin{aligned} KE \text{ at launch} &= (\text{total energy}) - (\text{energy to lift}) \\ &= 410\text{ J} - 25\text{ J} \\ &= 385\text{ J} \end{aligned}$$

Much more of the energy from the person's legs can go into propelling the body off of the ground. Although the leg muscles provided the same 410 J of energy as on Earth, only 25 J are required to lift the body on the Moon. More of the energy goes into kinetic energy allowing for a much higher jump.

The jump height can be predicted by assuming that the kinetic energy at launch is transformed into the gain in gravitational potential energy from launch to the peak of the jump. That is, at the launch position the person has some kinetic energy (385 J). But at the very top of the person's jump, the person's velocity is zero, so the person has no kinetic energy. The law of conservation of energy tells you that the 385 J of kinetic energy must be converted into 385 J of gravitational potential energy.

$$\begin{aligned} KE \text{ at launch} &= PE \text{ gained from launch to peak} \\ 385\text{ J} &= mg\ (\text{jump height}) \end{aligned}$$

Therefore:

$$\text{Jump height} = 5.47\text{ m}$$

This height is almost identical to the height you found in the *Physics Talk* section. Rounding errors in the calculations produced the difference.

Notice that the jump height on the Moon (as measured from the launch position to the peak position) is 5.47 m as compared to 0.60 m on Earth. This is a factor of nine times higher on the Moon than on Earth.

It is tempting to arrive at the conclusion that jump heights on the Moon and Earth would compare in the same way—different by a factor of six—as the accelerations due to gravity on the Moon and Earth. This analysis shows that factor is only true when you compare the change in height from the ready position to the peak position.

The equations used for analyzing the vertical jump in the above analysis are based on the assumption that a jumper applies a downward force to the ground during the jump—and also that the ground pushes with an equal and opposite upward force on the jumper—during the pre-launch phase of jumping.

Research shows that the best jumpers are able to accelerate to high speeds in a very short amount of time and are able to maintain a fairly constant force while rising from a crouch to launch position.

Not enough is known about jumping on the Moon to be sure that a jumper there would have enough time before launch to build up the muscular force assumed in this example of a Moon jump. How do these physiological changes impact the analysis?

You can now work out the general equation for predicting the jump height and hang time for a person on the Moon knowing the data from the person's vertical jump on Earth. Let the mass of the person be denoted by m. Let the launch height minus the ready height be denoted by h_1, let the peak height minus the ready height be denoted by h_2, and let the acceleration due to gravity on Earth be denoted by g_E.

1. What is the expression for the total energy generated by the person's legs during a jump?
2. What is the expression for the energy required to raise the person from the ready height to the launch height on the Moon? Use g_M for the acceleration due to gravity on the Moon.
3. What is the expression for the kinetic energy of the person as he or she leaves the ground during a vertical jump on the Moon?
4. What is the expression for the person's jump height on the Moon? Test your expression by substituting the values from the example in *Physics Talk* or the values for your own vertical jump.
5. What is the expression for the person's hang time on the Moon?

What Do You Think Now?

Michael Jordan had a hang time of 1 s on Earth.

- **What would be a typical NBA star's hang time on the Moon?**
- **How high could the NBA star jump on the Moon? How high do you believe you could jump on the Moon?**

Based on what you have learned about kinetic energy and gravitational potential energy, how would you answer these questions now?

Physics Essential Questions

What does it mean?

What is different when a person jumps on Earth compared to on the Moon?

How do you know?

What physics concepts and analysis did you use to compare jumping on Earth and on the Moon.

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	* Conservation of energy	Experimental evidence is consistent with models and theories

* Conservation of energy is a major organizing principle of physics and all science. How was the conservation of energy used in the analysis of the jump?

Why should you care?

Jumping is an integral part of many sports. For such a sport, what is going to change if it is played on the Moon?

Reflecting on the Section and the Challenge

Reflecting on energy transformations and jump heights leads to the notion that sports involving jumping would be interesting on the Moon. A human who jumps vertically should fly much higher on the Moon than on Earth. Traveling to a much greater height and returning to the ground also takes a longer time.

In a sport like gymnastics, people propel themselves with their leg muscles. In basketball, leg muscles also determine jump height and hang time. "Flying higher" will make these sports very different on the Moon. Adjustments in rules may have to be made to keep these sports fun, challenging, and competitive.

Physics to Go

1. On Earth, the top edge of a volleyball net is placed 8 ft above the ground, and a basketball hoop is 10 ft above the ground. At what heights would they need to be placed on the Moon to keep the sports equivalent in difficulty?

2. A spacesuit and life-support backpack have a combined mass of 110 kg. Wearing these reduces the height of a person's vertical jump. Calculate the reductions in the vertical jump when

   a) on the Moon.

   b) on Earth.

   Use the sample data in *Physics Talk* (mass of the person = 44 kg) in calculating your answers. The total energy of the person remains at 410 J.

3. Would jumping on a trampoline be different on the Moon than on Earth? Why or why not? If so, how?

4. How would events in gymnastics be affected if performed on the Moon? Choose an event and describe how it would be different on the Moon. Use numbers as well as descriptions.

5. What do you think will be the winning height in the high jump during the first Olympiad held on the Moon?

6. A student riding in a chair moving at constant speed throws a ball into the air and catches it when it comes back down. What would be the same and what would be different if the activity were done in exactly the same way on the Moon?

7. ***Preparing for the Chapter Challenge***

   In a basketball game, tall people have an advantage over shorter people when rebounding a missed shot because of their height. On the Moon however, basketball players would be able to jump much higher than they normally could on Earth. Could this affect the advantage taller people have when rebounding? How? Explain your reasoning using what you know about the relative jumping ability of basketball players and how their energy changes as they jump.

Inquiring Further

Acceleration due to gravity on Ceres

Some scientists have speculated that in the future, people will live on small, celestial bodies that orbit the Sun. One such body is the "dwarf planet" Ceres, which has a mass of approximately 9.5×10^{20} kg and a radius of 9.5×10^{5} m. Remembering that when gravity is reduced, your jumping ability is greatly enhanced, use what you have learned in this section to calculate the acceleration of gravity on Ceres. Determine how high you could jump if you were to live there.