Section 8 Modeling Human Motion: Bounding on the Moon

What Do You See?

Learning Outcomes

In this section, you will

- **Apply** a cylinder as a model of a human leg acting as a pendulum during walking.
- **Measure** the amount of time for a human leg to swing forward and back as a human walks on Earth.
- **Predict** the amount of time for a human leg to swing forward as a human walks on the Moon.
- **Explain** why it is not possible to walk unaffected on the Moon.
- **Discover** how the period of a pendulum depends on its length, mass, and angle of swing.

What Do You Think?

Neil Armstrong was the first human to set foot on the Moon.

- **Why do astronauts "bound" instead of walk or run on the Moon?**
- **Would running events in track and field be different if they were held on the Moon, even indoors?**

Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your responses with your small group and the class.

Investigate

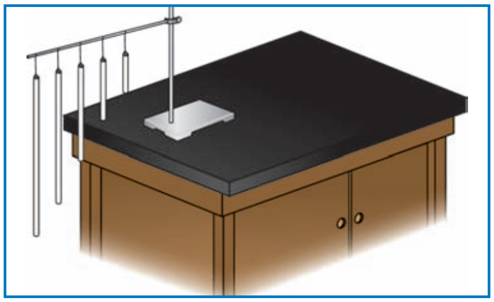
1. Observe the *Active Physics* video of astronauts "walking" on the Moon. Record answers to the following in your log book:

   a) Compare how the astronauts use their legs and feet to move across the surface of the Moon to how legs and feet are used in habitual walking.

   b) Why do you think astronauts use their legs and feet that way to walk or run on the Moon?

2. Use a set of cylinders of various lengths as pendulums. Each cylinder has a hole at one end to allow it to be hung as shown in the diagram. The length of the string should be as short as possible while still allowing the cylinder to swing freely. If string is unavailable, you may choose to slip the drilled end of the cylinder directly on to the cross arm. This will allow the cylinder to swing freely. Measure the length of each cylinder in centimeters (to the nearest 0.1 cm) from the pivot point where the string is tied to the horizontal bar to the bottom edge of the cylinder.

   a) Record the lengths of the cylinders (in centimeters) in a column in your *Active Physics* log. Leave room for several more columns of data to the right of this first one.



3. Choose one of the longer cylinders, pull it aside about ½ way up, and allow it to swing as a pendulum. Use a stopwatch to measure the *period* of this pendulum. (The period of a pendulum is the time, in seconds, for the pendulum to complete one full swing over and back. The symbol for period is "T" and it is capitalized to show that it is not the usual time, but the specific time for one full swing to take place.) A good way to measure the period accurately is to measure the time to complete 10 swings over and back and then divide the measurement by 10.

   a) Record the measurement of this period in your *Active Physics* log in a separate column next to the length of the cylinder.

4. Using the same cylinder, pull it aside about ¼ of the way up, and allow it to swing. Measure the period of the pendulum.

   a) Record it in your *Active Physics* log. Does it differ from the period measured in *Step 3*? If so, keep this in mind as you make measurements.

5. Measure the period of each of the cylinders using the method of *Step 3*.

   a) Record the period of each cylinder, in seconds, in the column next to the lengths of the cylinders.

   b) Plot a graph of period versus length for the cylindrical pendulums. Plot period, T, (in seconds) on the vertical axis, and length (in centimeters) on the horizontal axis. Mark the data points on your plot and sketch a smooth line that has an even distribution of data points around it. Observe carefully to decide whether the line should be straight or curved.

6. Now you will examine two types of walking, stiff-legged and the more usual style.

Observe a member of your group as he or she walks stiff-legged (without bending the knees). This can look pretty funny. Notice how after one foot hits the ground, that the opposite leg, trailing behind, swings forward as a pendulum before it is used for the next step. Also notice that a human leg is similar in shape to a cylinder. It is suspended at the top from the hip joint. The person does not use much muscular force to swing the leg forward because the force of gravity helps swing the leg forward. Therefore, the forward swing of a stiff human leg can be modeled by the cylinders used above.

Observe a member of the group walking more normally. Notice that the leg bends at the knee, making the motion more complicated. Now observe a group member walking again, focusing on the lower leg, from the center of the knee down. Maybe this part of the leg can be modeled by a swinging cylinder.

7. Measure the length of the lower leg (in centimeters), from the knee to the bottom of the foot, for each member of your group.

a) Create a table in your *Active Physics* log to record the names and lengths.

b) As each member of your group walks in a normal way, other members should use stopwatches to measure the time for the person's lower leg to swing forward during one stride. For accuracy, take the average of several measurements. Since the forward swing of the lower leg is only ½ of its period, double the measurement and record that as the period of each person's lower leg in the table in your *Active Physics* log.

c) Create a graph of period versus length to find out how well a cylindrical pendulum models the forward swing of your lower leg. Explain whether a cylindrical pendulum is a reasonably good model of a person's lower leg while walking.

8. Use the following equation to calculate the predicted periods of each cylindrical pendulum for which you made measurements (see *Physics Talk* for the source of this equation):

$$T = 5.1\sqrt{\frac{L}{g}}$$

Here L is the length of the cylinder (in centimeters) and g is the acceleration due to gravity ($9.8\ \text{m/s}^2 = 980\ \text{cm/s}^2$). Include units of measurement when you do the calculations to be sure that the answer is in seconds. Divide the work among members of your group and then share the results.

a) Add a column to the data table in your *Active Physics* log, and record the values predicted by the equation. Write a comment comparing the measured and predicted periods.

b) Use the same equation to calculate the period of your lower leg. Share results within your group, enter the data in your log, and compare the predicted results to the measured values. Write a comment in your log comparing them.

9. Since acceleration due to gravity on the Moon is ⅙ the acceleration due to the gravity on Earth, the pendulum and your lower leg will require more time to swing. You might expect that the leg would take 6 times longer to swing since gravity is 6 times smaller on the Moon. Notice that the "g" in the equation for the period is within the square root sign. This informs you that the leg will not take 6 times as long to swing but $\sqrt{6}$ or 2.5 times as long.

a) Multiply the time for the forward swing of your lower leg (half of the period) by 2.5 to find how much time it would take your lower leg to swing forward on the Moon. Try to walk with the "swing time" that your lower leg would have when powered by the Moon's gravity. You could ask a group member to give you time signals to help you get it right.

Physics Talk

PENDULUMS AND GRAVITY

Your leg swings back and forth as you walk. If your leg had one long bone, you could model this with a long rod swinging. The leg is more complicated. Analyzing your walking requires you to carefully observe the motion of your foot, ankle, lower leg, knee, and upper leg as you walk. In this section, you tried to find a model that is simpler to analyze and may provide insights into why your leg moves the way it does. In the investigation, a solid pendulum was used as a model. You found that the **period** of this pendulum varies with the length of the pendulum. This variation is similar to the variation with a simple pendulum — a mass hanging from a string.

Physicists have developed equations to predict the period of many kinds of pendulums. The "simple pendulum" (a ball hanging on a string) has a period:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where T is the period,
L is the distance from the point of suspension to the center of the ball, and
g is the acceleration due to gravity.

The equation for the period of a cylindrical pendulum of the kind you have been using in the *Investigate* is

$$T = 2\pi\sqrt{\frac{2L}{3g}}$$

$$\text{or } T = 5.1\sqrt{\frac{L}{g}}$$

Notice that the equations show that the periods of both kinds of pendulums are directly proportional to the square root of the length and are inversely proportional to the square root of the acceleration due to gravity. This explains why, for example, small children with short legs have such quick strides. The equations also predict that pendula and human legs swinging as pendula would behave differently on the Moon than on Earth due to the reduced effect of gravity on the Moon. Since the Moon's gravity is known to be $\frac{1}{6}$ of Earth's gravity, the equations can be adjusted to predict the periods of pendulums on the Moon by substituting $\frac{g}{6}$ for g. Therefore, the period of a cylindrical pendulum on the Moon should be not 6 times longer but $\sqrt{6}$ or 2.5 times longer.

Physics Words

period: the time required to complete one cycle (usual symbol is T).

Checking Up

1. What factors determine the period of a cylindrical pendulum?
2. Why do children seem to have such quick strides?
3. Why do humans walking on the Moon seem to have a much slower stride than humans walking on Earth?
4. What do astronauts do on the Moon to overcome the slow stride that the Moon induces?

$$T = 5.1\sqrt{\frac{L}{\left(\frac{g}{6}\right)}}$$

$$\text{or } T = 2.5\left(5.1\sqrt{\frac{L}{g}}\right)$$

The above equation shows that the period of a cylindrical pendulum is about 2.5 times greater on the Moon than on Earth. Perhaps astronauts do not walk normally on the Moon because they cannot. The Moon's gravity does not assist the swing of the leg enough to allow normal walking with normal rhythm on the Moon.

As you swing your legs in this investigation, gravity assists the movement. As the astronaut swings his leg, gravity does not assist nearly as much. The leg moves forward at a much slower rate and the astronaut walks in a different way than on Earth. Many astronauts, while traveling on the Moon, decide not to move by walking and swinging their legs but by jumping from one location to another.

Active Physics *Plus*

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The Period of a Pendulum

The period of a simple pendulum (a mass hanging from a long string) is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where T is the period (time) for one complete motion back and forth, L is the length of the pendulum and g is the acceleration due to gravity — $9.8\,\text{m/s}^2$ on Earth and $1.6\ \text{m/s}^2$ on the Moon.

The period of a compound pendulum (a cylindrical mass hanging from a tiny hook) is

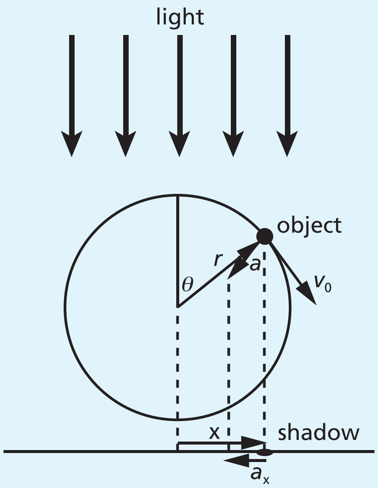
$$T = 2\pi\sqrt{\left(\tfrac{2}{3}\right)\frac{L}{g}}$$

where T is the period (time) for one complete motion back and forth, L is the length of the pendulum and g is the acceleration due to gravity ($9.8\,\text{m/s}^2$ on Earth and $1.6\,\text{m/s}^2$ on the Moon).

1. Determine mathematically what length rod and what length of simple pendulum will have the same period. Check the result experimentally by placing the two pendulums side by side and swinging them.

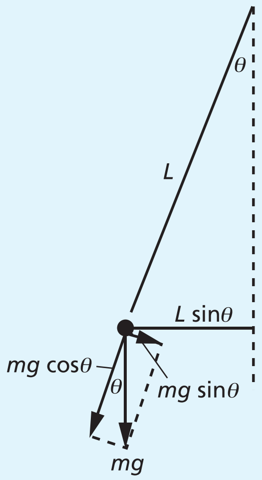
The motion of a pendulum can be simplified if you approximate it as a back-and-forth horizontal motion rather than the arc of a circle. This is the case when the amplitude of the motion is very small. This idealized motion is called simple harmonic motion, and occurs when the force on the object is always proportional to the distance the object is from its equilibrium position and always directed toward the equilibrium position. This means the force switches direction when it passes through the equilibrium position.

This relationship between force and distance is represented by the equation $F = -kx$, where k is the proportionality constant. If the force $-kx$ is set equal to the mass m times the acceleration, a (Newton's second law), then it follows that the ratio of the acceleration to the distance for simple harmonic motion is a negative constant equal to $-k/m$, $ma = -kx$.



The motion of an object traveling around a circle of radius r with a constant velocity v_o (called uniform circular motion) is related to simple harmonic motion. Imagine that there is a light to one side of the circle and the shadow of the object falls on a screen as shown in the figure. The shadow moves back-and-forth as the object goes around the circle.

2. Compare the velocity of the shadow with the velocity of the pendulum bob (the mass).
   a) Where is the speed the greatest? Where is the speed equal to zero?
   b) How can you calculate the period of the shadow from the circular motion of the object?
3. Use a projector and a pendulum. As you rotate the pendulum in a horizontal circle, report on your observations of the motion of the shadow.



As seen above, the force on the mass when the string makes an angle θ with the vertical is equal to $mg \sin\theta$. This is because the force of gravity (weight) can be broken into two components, one along the line of the string and the other perpendicular to it. The component of the weight $mg \cos\theta$ is balanced by the force of the string on the mass. That's why the mass does not accelerate along the line of the string and instead only accelerates along the line perpendicular to the line of the string (the arc of the swing). If the angle θ is small, then the force $mg \sin\theta$ is nearly horizontal. The distance the mass is from the center position is approximately equal to $L \sin\theta$ and directly opposite to the force. Therefore, the spring constant is

$$k = \frac{-F}{x} = \frac{-(-mg\sin\theta)}{L\sin\theta} = \frac{mg}{L}$$

The period then is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}} = 2\pi\sqrt{\frac{L}{g}}$$

It is important to keep in mind that this relationship is approximate. The closer the pendulum is to a string of very low mass with a significant mass of very small physical size at its end, and the smaller the amplitude of the motion, the more accurate this relationship is.

What Do You Think Now?

At the beginning of this section, you were asked the following questions:

- **Why do astronauts "bound" instead of walk or run on the Moon?**
- **Would running events in track and field be different if they were held on the Moon, even indoors?**

Based on what you have learned in your investigation, how would you answer these questions now? Write your answers in your *Active Physics* log.

Physics Essential Questions

What does it mean?

How does the period of a swinging cylinder or simple pendulum depend on its length? Does it depend on the acceleration due to gravity at its location? Can a pendulum be used to measure the acceleration due to gravity?

How do you know?

What did you observe about how the period of a swinging cylinder or simple pendulum depended on its length? What did you infer from the equation about how it is affected by the acceleration due to gravity?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	* Models	Experimental evidence is consistent with models and theories

* Physicists use models to explain a wide variety of phenomenon. Why was a solid rod used as a model for the leg rather than a mass on a long string?

Why should you care?

Your model of the lower leg should have revealed that it swings on the Moon with a much longer period than it swings on Earth. Because of this, every sport in which people move using their legs is going to be drastically different on the Moon. For the sport you are proposing, how will it be affected?

Reflecting on the Section and the Challenge

There is a problem with walking on the Moon, and perhaps the same problem would extend to running on the Moon. Your legs will swing more slowly on the Moon. The period of the natural swing will be 2.5 times longer on the Moon. However, when running, does the runner simply allow the leg to swing forward? This swing delay, if it happens to runners also, could have serious implications for many sports on the Moon, unless "bounding" like astronauts is an acceptable substitute for walking or running. It probably can't even be said that a good runner on Earth would necessarily be a good "bounder" on the Moon because different muscles and skills are used. Maybe an Olympic champion who finished first in the 100-m dash would finish last in the "100-m bound" on the Moon! The time is nearing to write your proposal, so it's time to sort out the possibilities for sports on the Moon.

Physics to Go

1. The period of a "simple pendulum," a small massive object hanging from a string, is given by the equation $T = 2\pi\sqrt{(L/g)}$, where T is the time for the pendulum to swing once over and back, L is the distance from the point of suspension of the string to the center of mass of the object, and g is the acceleration due to gravity. Make a simple pendulum, let it swing, and see if the equation works. In using the equation, make sure the distance units of L and g are both measured in meters and m/s^2.
2. Describe how difficulty with walking or running on the Moon would affect at least one sport of your choice.
3. How would walking and running be affected on a planet that has an acceleration due to gravity greater than g on Earth?
4. How long would a simple pendulum need to be to have a period of 1.0 s ? Make one and see if it works. (Hint: Solve for L in the equation $T = 2\pi\sqrt{(L/g)}$ or try different values of L and calculate T.)
5. Pendulums were used as the mechanical basis for making the first accurate clocks. What is it about the period of pendulums, even as they swing less and less, that makes them good for clocks?
6. You also use your arms as pendulums when you walk. Do you think you use your arms for a reason? Why or why not?
7. Why do you "shorten" the length of your arms by bending at the elbows when you are running?
8. Obtain data for a small child's leg swing. Does it fit the data on your graph?

9. Using the equation for a pendulum, answer the following questions:

   a) How does the period of a swinging cylinder or simple pendulum depend on its length?

   b) How does the period of a swinging cylinder or simple pendulum depend on its mass?

   c) How does the period of a swinging cylinder or simple pendulum depend on the acceleration due to gravity?

   d) Can a pendulum be used to measure the acceleration due to gravity?

10. ***Preparing for the Chapter Challenge***

    On Earth, many competitive sports consist of people running from one place to another while something else is occurring. A tennis player chases an opponent's shot, a baseball player runs the bases when the ball is hit, and so on. How could the game of tennis be modified so that an athlete playing on the Moon with a much slower stride could still return an opponent's shot in time?

Inquiring Further

Factors affecting the period of a pendulum

The equations for both simple and cylindrical pendulums presented in *Physics Talk* make no mention of mass or amplitude (swing distance) as variables that may affect the period. Do you think it is true that such characteristics do not affect the period? Design experiments to test the effects of these and other properties of pendulums on the period and report your procedures and results. If you measure data and graph it, be certain the scales for both axes start at the zero point.