SECTION 2

Measurement: Errors, Accuracy, and Precision

Section Overview

In this section, students study the different sources of errors within measurements. They select an area in the school, away from traffic, and estimate its distance by taking strides along the length of that area. Each stride length is then measured and multiplied by the total number of strides taken to cover the selected distance. The differences in student calculations are recorded each time students measure the distance using the stride method. All measurements are then listed on the board and the sources of error are discussed as random or systematic. The discussion of random and systematic errors leads to the understanding of uncertainty in measurements, which in turn helps students to identify the difference between accuracy and precision.

Background Information

The measurement of a quantity is a comparison to a standard. A meaningful measurement contains both a numerical value and the unit. In the 1790s, the French Academy of Sciences established the first standard unit of length, the meter. Prior to that, different people used different units of length, which varied from place to place. The standard meter was one ten-millionth of the distance from the Earth's equator to either pole, and was represented by a platinum rod. About 100 years later, the meter was defined more precisely as the distance between two engraved marks on a particular bar of platinum-iridium alloy. One of these bars was designated the International Standard and was kept at the International Bureau of Weights and Measures, near Paris, and the others were housed in scientific laboratories all over the world. By 1960, the meter was redefined as 1/1,650,763.73 wavelengths of an orange light emitted by the gas krypton-86. In 1983, the meter was redefined again as the length of a path traveled by light in a vacuum during 1/299,792,458 of a second. The most important system of units today is the Systeme International, or SI. The standard of length is the meter (m); of mass, the kilogram (kg); and of time, the second (s).

Random errors arise because every measurement is uncertain. Any measurement has limited accuracy, and users are unable to read an instrument beyond some fraction of the smallest unit shown. Good measurements include some estimate of the fraction of the smallest unit, and should state the estimated uncertainty. The uncertainty of a measurement is the range within which the actual value is likely to fall, compared to the measured value. The convention for stating uncertainty is to write ± and the smallest measurement that can be made with the instrument after the measurement itself. For example, a length measured as 5.4 cm is approximately written 5.4 cm ± 0.1 cm. Often, the approximation is not written, but understood. The percent error is found by dividing the difference between the measured value and the accepted value by the accepted value, and then multiplying the quotient by 100%.

In addition to their use in measurement, numbers that are defined or counted are used in physics. There is no uncertainty in these numbers. The number of sides of a triangle, for example, is a defined number. The number of pennies in a jar is a counted number.

Crucial Physics

- All measurements have uncertainties or random errors.
  - Some variation in measurement is inevitable.
  - No measurement is exact.
- Repeated measurements can vary in accuracy and precision.
- Random errors can be attributed to the measurement and/or the measuring instrument.

Learning Outcomes	Location in the Section	Evidence of Understanding
Calibrate the length of a stride.	***Investigate*** Step 3	Students measure the length of their stride using a meter stick and record the measurement in their log.
Measure a distance by pacing it off and by using a meter stick.	***Investigate*** Steps 2, 3, and 4	Students pace off the selected distance and record it in their logs. Then students multiply the measured length of one stride by the total number of strides.
Identify sources of error in measurement.	***Investigate*** Step 7.b)	Students list sources of error and give reasons for the sources of error.
Evaluate estimates of measurements as reasonable or unreasonable.	***Investigate*** Step 9)	Students estimate measurements as reasonable or unreasonable.

NOTES

Section 2 Materials, Preparation, and Safety

Materials and Equipment

PLAN A

Materials and Equipment	Group (4 students)	Class
Tape measure, wind-up	1 per group	
Meter stick, wood	1 per group	
Tape, masking		1 per class
Access to a clear highway*		1 per class

*Additional items needed not supplied

PLAN B

Materials and Equipment	Group (4 students)	Class
Tape measure, wind-up	1 per group	
Meter stick, wood	1 per group	
Tape, masking		1 per class
Access to a clear highway*		1 per class

*Additional items needed not supplied

Note: Time, Preparation, and Safety requirements are based on Plan A, if using Plan B, please adjust accordingly.

Time Requirement

To complete the experiment, 1.5 class periods or 60 minutes are required.

Teacher Preparation

- Notify other teachers in the section of the building where the students will be making measurements in the hallway.
- Mark off the length you wish the students to measure during the *Investigate*. If masking tape is used, be certain that the tape is removed at the end of the class.
- Inform students the day before the *Investigate* that they should wear appropriate clothing because they will be making measurements on the floor.

Safety Requirements

While working on the floor, students doing the measuring must be careful not to have their hands stepped upon by other students.

- All equipment must be picked up and safely returned to the classroom, prior to any change of class that would have non-class students in the halls.
- Students should be cautioned about rulers or other objects on the floor that might present hazards.

NOTES

Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

Learning Issue	Reference	Augmentation and Accommodations
Visualizing and estimating measurements	***What Do You Think?***	**Augmentation** • Students with visual-spatial issues often have a difficult time visualizing measurements. Physically show students the difference between 3 m and 10 m and between 3 m and 3.1 m.
Following directions and recording data	***Investigate*** Steps 1-8	**Augmentation** • Students must record data accurately during each step of the *Investigate* to be able to compare measurements and to analyze the sources of error. In each group, assign one person to read directions and one to record data. **Accommodation** • Provide students with an observation chart to record the required data. • Pair students strategically in groups of two to increase their focus on a task.
Making scientific measurements	***Investigate*** Steps 2-4	**Augmentation** • Ask a student to model one or two strides and then show your students how to measure the stride with a meter stick. • Group students intentionally to compensate for measurement difficulties. **Accommodation** • Provide one-on-one or small-group support for students who are really struggling at the beginning of this *Investigate*.
Comparing data	***Investigate*** Step 7	**Augmentation** • Some students struggle to compare numerical data. *Step 7* requires students to compare data and make some generalization about measurement. Have students complete *Step 7* in their logs independently. Then check in with each group to see what ideas they recorded, or facilitate a whole-group discussion to assess if students are making reasonable generalizations.
Estimating	***Investigate*** Step 9	**Augmentation** • Students may not have the prior knowledge or experiences to make reasonable estimations. Students can complete *Step 9* in their logs independently and then compare their answers with a partner. • Tactile and kinesthetic learners may need to manipulate real-life objects for making comparisons. For example, provide 5-kg masses or mark off a distance that is 10 m long, and allow students to move around the room, manipulating objects to estimate measurements.
Reading comprehension	***Physics Talk***	**Augmentation** • Model how to create a Venn diagram to compare these two types of errors. Place the similarities where the circles overlap, and the differences outside. Provide one or two similarities or differences to help students get started. Students may struggle with this tool the first few times, but Venn diagrams will help them to compare concepts and organize information, especially as they become proficient in learning this tool. **Accommodation** • Although the reading is clear and informative, some students may require direct instruction to understand random errors, systematic errors, accuracy, and precision. • Provide students with opportunities to use these words frequently.

Strategies for Students with Limited English-Language Proficiency

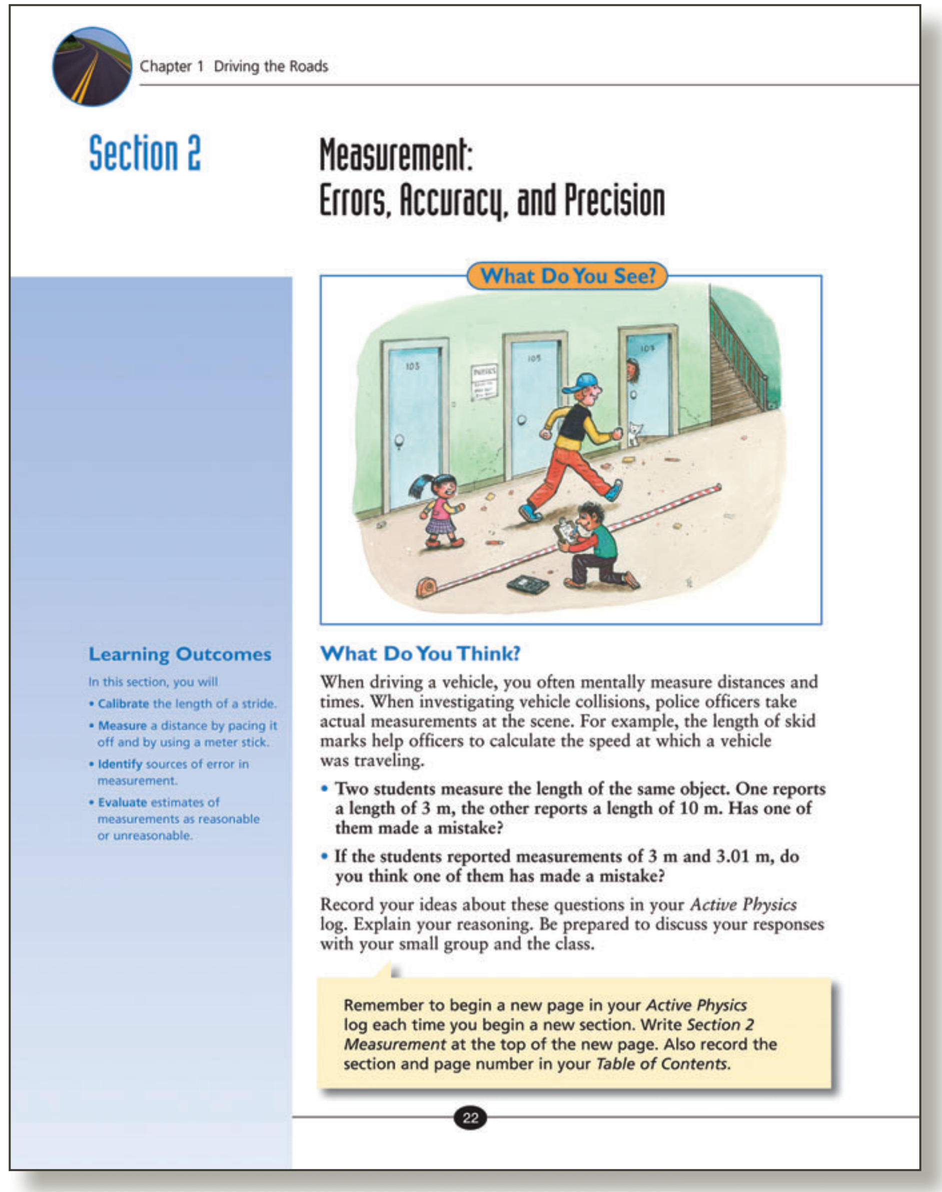
Learning Issue	Reference	Augmentation
Understanding concepts Vocabulary comprehension	***What Do You Think?***	Hold a class discussion on what it means to "mentally measure" distances and times. Include "estimation" in your discussion. Students may not know the term "skid marks." Explain how tires can leave rubber on the road when a driver brakes forcefully.
Vocabulary comprehension	***Investigate*** Step 2	You may need to explain the terms "pace" and "stride." A demonstration should help.
Comprehension	***Investigate*** Step 5.b)	Encourage ELL students to share their lists with other students. Have a brief discussion about the reasons for differences in measurements to check student understanding.
Understanding concepts	***Investigate*** Step 8	Discuss "error," "systematic error," and "random error," and clear up any misunderstandings in meaning. To fully understand "random error," students also need to understand "approximation" in context — close to the actual value, but not exact.
Comprehension	***Investigate*** Step 9	ELL students may be unfamiliar with the term "common sense" (good judgment). Discuss and check student understanding of the term.
Understanding prefixes	***Physics Talk*** SI System	Help ELL students use the chart to determine the meaning of the prefixes "kilo-" (thousand), "centi-" (hundredth), and "milli-" (thousandth). Be sure they grasp the difference between "thousand" and "thousandth." To check understanding, have students put these prefixes in front of "meter" and then order the units from smallest to largest. For more practice, repeat this exercise with "gram."
Demonstrating higher-order thinking	***Physics Essential Questions*** Why do you believe?	To give ELL students speaking experience, invite them to explain their thoughts on the question, "How can you trust experiments if all measurements have uncertainties?" Their answers will give you the opportunity to check understanding.

Two important aspects of learning a new language are speaking and writing in that language. Some ELL students will be self-conscious and shy about speaking in front of their peers, while others will be less reluctant to try. Be sure to encourage all ELL students to speak in class, and give them opportunities to write on the board from time to time. Experience will broaden their comfort level.

- To encourage class discussion, make a handout with five targets on a piece of paper. Put dots on one target to represent "accurate," on another to represent "precise," and on another to represent "accurate and precise." Put random dots as distractors on the other two targets. Have students label the targets as "accurate," "precise," and "accurate and precise." Draw the same diagrams on the board and have a volunteer come to the board to label the diagrams. Encourage students to discuss any differences of opinion concerning which term is correctly identified with each target.

SECTION 2

Teaching Suggestions and Sample Answers


Chapter 1 Driving the Roads

Section 2

Measurement: Errors, Accuracy, and Precision

What Do You See?

Learning Outcomes

In this section, you will

- **Calibrate** the length of a stride.
- **Measure** a distance by pacing it off and by using a meter stick.
- **Identify** sources of error in measurement.
- **Evaluate** estimates of measurements as reasonable or unreasonable.

What Do You Think?

When driving a vehicle, you often mentally measure distances and times. When investigating vehicle collisions, police officers take actual measurements at the scene. For example, the length of skid marks help officers to calculate the speed at which a vehicle was traveling.

- **Two students measure the length of the same object. One reports a length of 3 m, the other reports a length of 10 m. Has one of them made a mistake?**
- **If the students reported measurements of 3 m and 3.01 m, do you think one of them has made a mistake?**

Record your ideas about these questions in your *Active Physics* log. Explain your reasoning. Be prepared to discuss your responses with your small group and the class.

Remember to begin a new page in your *Active Physics* log each time you begin a new section. Write *Section 2 Measurement* at the top of the new page. Also record the section and page number in your *Table of Contents.*

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What Do You See?

Student reactions will vary. They might state that they see two people walking and another person writing something down. The purpose of this illustration is to ask them what they think people are doing. You should allow them to share their ideas in a discussion as they will have a chance to return to the illustration after completing the *Investigate*.

All responses are valid; however, be alert to misconceptions that tend to distract students from a constructive response that directs them toward the topic they are about to study.

Students' Prior Conceptions

Making estimations is a valuable technique in science. Many scientists are known for their ability to estimate results of observations. Naturally, these scientists also seek the most precise results through experimentation. *Section 2* opens the door for the teacher to discuss the value of evaluating estimates of measurements as reasonable or unreasonable.

1. **You can only measure to the smallest unit shown on the measuring device.** This section presents you with the opportunity to encourage students to evaluate their data using the questions "How do you know?" and "Why do you believe?" as they attempt to form reasonable answers from their investigation. The *Elicit*, *Evaluate*, and *Explain* cycles of learning apply well in this section.

2. **You should start at the end of the measuring device when measuring distance.** The human factors involved in this section of measuring and estimating can lead students to recognize how precision can be affected by faulty or worn measuring tools. Students can see the value of employing accurate tools to eliminate sources of error in measurement. Although it is not a standard prior conception, students often confuse the significance of quantitative values with errors in their reading of the tools, rather than an implicit limitation of the tool. It might be helpful for you to talk about percent error in this section.

What Do You Think?

The *What Do You Think?* section is designed to initiate discussion and engage students. You might want to emphasize that there are no "right" answers and that all answers are acceptable. The purpose of these questions is to elicit students' prior knowledge. Ask them to refer to their answers while discovering new physics concepts. Pointing out to them to refer to prior knowledge will help them increase or alter what they already know. It will also help them realize how the scientific process works.

What Do You Think?

A Physicist's Response

- If one person measures 3 m and the other 10 m, both of them cannot be approximately right. A difference in measurement that differs by a factor of three would indicate a serious error. For a measurement of this size, greater accuracy is expected. In some cases, when measuring extremely large or small distances, errors of this proportion would be acceptable. However, for a commonly used measurement of this size, this error would be excessive.
- A difference of 0.1 m out of 3 m is only about a 3 percent difference. Both may be approximately right. A difference in measurement of this size could be acceptable.

NOTES

Investigate

1.

You should choose the distance the students measure. Many schools have either terrazzo blocks (3 ft on a side) or tile squares (1 ft on a side). These may make a convenient measuring device for the students, in which case you should choose a length that does not coincide exactly with the end of one of these markers. However, students should make their measurements using strides.

2.a)

Students record the number of strides. They may be confused when they reach the end and are confronted with a distance that is not a full stride. Tell them to estimate this distance to a reasonable fraction (for example, one-half, one-third, and so on) of a stride.

3.a)

Students find the length of their stride and record it in their logs. Stride lengths will vary greatly, from about 0.4 m to almost a meter for taller students.

Teaching Tip

Students often make the mistake of measuring their stride from the front of one foot to the back of the other. Make certain that students measure from front to front or back to back.

4.

You may choose to set parameters for the students so they will know if they made an error. For example, if the room is 10 m long, you could tell the students that it is between 5 m and 15 m to allow those who are confused to realize when they have made a serious mistake.

4.a)

Students record their calculations.

5.a)

With all the students' measurements on the board, the average measurement should be fairly close to the accepted value.

5.b)

Some reasons for varying results in students' measurements would be inconsistency of the stride, incorrect stride measurement mentioned in *Step* 3, or poor estimation of the fractional stride at the end. Ask if the difference in stride length is compensated for in any manner. Some students should realize the different stride lengths would mean a different number of steps and should be accounted for by the multiplication factor.

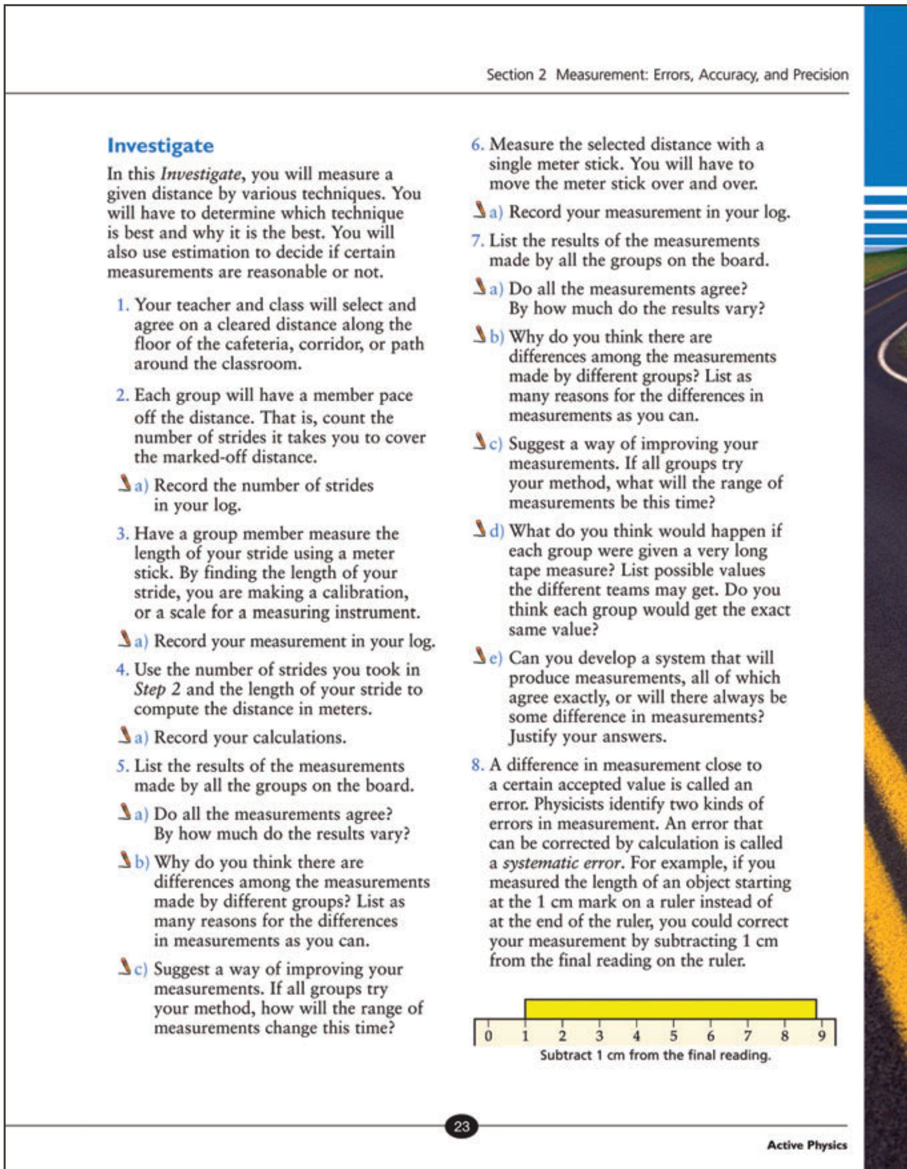
5.c)

The students may suggest reporting the measurement several times and averaging the results. They may also suggest the reasons mentioned in *Step 5.b)*. If all students use one method, the range of measurements should be less severe.

6.a)

Students record the measurement. They may find this method does not work well, unless done carefully. You could also give one group a meter stick without metal end caps with 1 cm cut off at the end. The group that uses this meter stick will be shorter than the rest. This can set up your discussion on systematic vs. random errors in *Step 7.b)*.

NOTES



Investigate

In this *Investigate*, you will measure a given distance by various techniques. You will have to determine which technique is best and why it is the best. You will also use estimation to decide if certain measurements are reasonable or not.

1. Your teacher and class will select and agree on a cleared distance along the floor of the cafeteria, corridor, or path around the classroom.

2. Each group will have a member pace off the distance. That is, count the number of strides it takes you to cover the marked-off distance.

   a) Record the number of strides in your log.

3. Have a group member measure the length of your stride using a meter stick. By finding the length of your stride, you are making a calibration, or a scale for a measuring instrument.

   a) Record your measurement in your log.

4. Use the number of strides you took in *Step* 2 and the length of your stride to compute the distance in meters.

   a) Record your calculations.

5. List the results of the measurements made by all the groups on the board.

   a) Do all the measurements agree? By how much do the results vary?

   b) Why do you think there are differences among the measurements made by different groups? List as many reasons for the differences in measurements as you can.

   c) Suggest a way of improving your measurements. If all groups try your method, how will the range of measurements change this time?

6. Measure the selected distance with a single meter stick. You will have to move the meter stick over and over.

   a) Record your measurement in your log.

7. List the results of the measurements made by all the groups on the board.

   a) Do all the measurements agree? By how much do the results vary?

   b) Why do you think there are differences among the measurements made by different groups? List as many reasons for the differences in measurements as you can.

   c) Suggest a way of improving your measurements. If all groups try your method, what will the range of measurements be this time?

   d) What do you think would happen if each group were given a very long tape measure? List possible values the different teams may get. Do you think each group would get the exact same value?

   e) Can you develop a system that will produce measurements, all of which agree exactly, or will there always be some difference in measurements? Justify your answers.

8. A difference in measurement close to a certain accepted value is called an error. Physicists identify two kinds of errors in measurement. An error that can be corrected by calculation is called a *systematic error*. For example, if you measured the length of an object starting at the 1 cm mark on a ruler instead of at the end of the ruler, you could correct your measurement by subtracting 1 cm from the final reading on the ruler.

Subtract 1 cm from the final reading.

7.a)

The measurements made by the various groups should now be much closer to one another, unless a group was given the altered meter stick, which should have the largest difference.

7.b)

In measuring the room, there will be some variance about an average. This may be due to inaccurate readings of the meter stick, careless end-to-end placement of meter sticks, not measuring in a straight line, or variations in the length of the meter stick.

7.c)

Suggestions may include making repeated measurements and averaging, using two meter sticks rather than one, so the position on the floor can be accurately determined when moving a meter stick, and possibly drawing a line on the floor to show a path perpendicular to the ends to get the straightest path from one end to the other. If all the groups used these methods, the lengths measured by each group should be closer to each other, reducing both the variance between the readings, and the error size.

7.d)

Tape-measure values should correlate even closer than meterstick values. If each group used a tape measure, variations should be 1–2 cm, assuming the tape measures were identical. This error would be primarily due to not pulling the tape tight when making a measurement.

7.e)

By now the students should start to realize there will never be exact agreement between data sets. "Exact" means accurate to an infinite number of decimal places, and one can never measure anything to an infinite number of decimal places.

Teaching Tip

One estimate of the "size" of the error in measuring with an instrument (such as a meter stick) that employs a graduated scale is to round to the nearest half of the smallest interval on the scale ("least count" estimate of uncertainty).

8.a)

Students' errors would be predominantly random errors. However, if the students initially measured the stride length incorrectly, then used the stride to measure the length, this would be equivalent to having a faulty measuring device, and therefore, a systematic error.

8.b)

The measurement of the room using the stride length would most likely be accurate to approximately 0.5 m. Using the meter sticks to measure the length would probably lead to an error of several centimeters, and the tape measure should be accurate to about 1 cm.

9.a)

100 kg is about 220 lbs (1 kg equals 2.2 lb), so that's a good estimate for a football player.

9.b)

4 m is over 12 ft.

9.c)

1440 min = 24 h. You may feel as if you work 1440 minutes per day!

9.d)

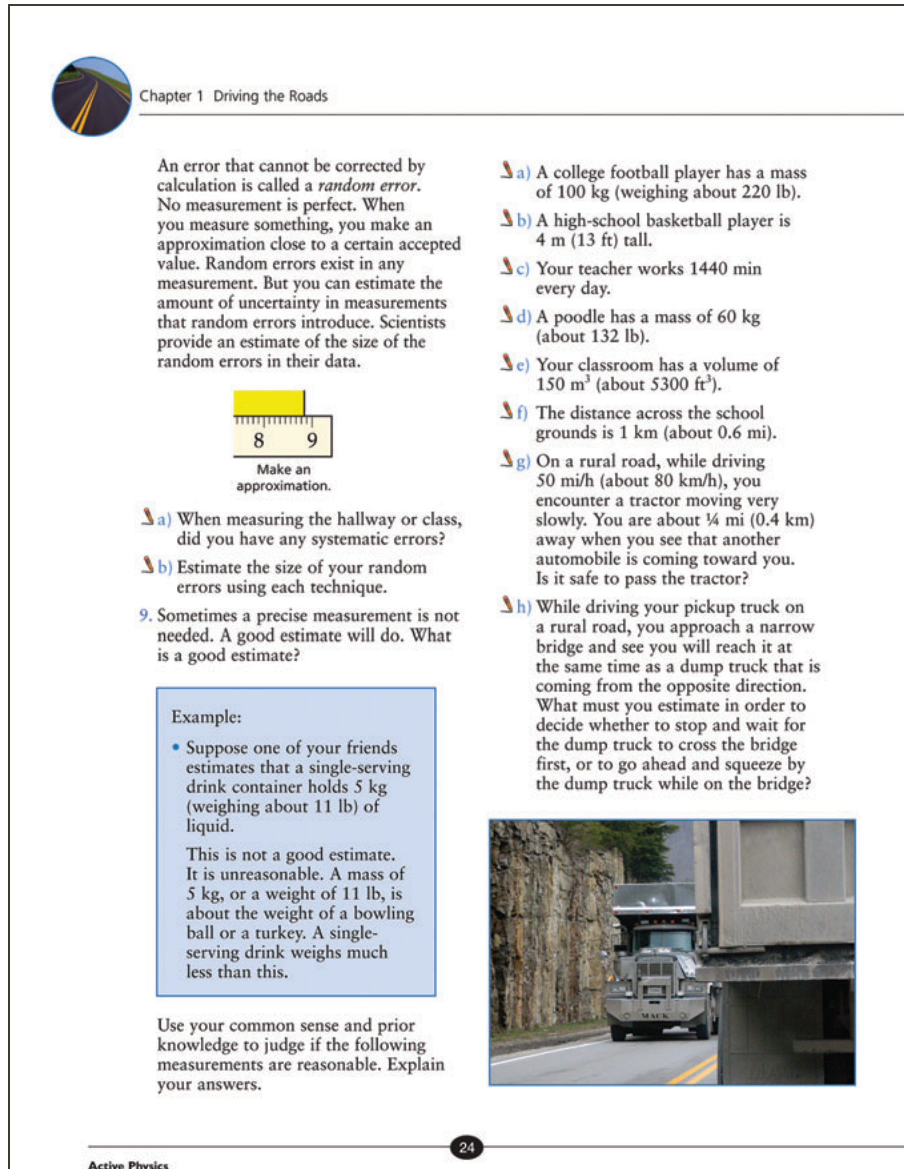
A 20-kg poodle would weigh about 44 lbs, or just over half the weight of a college football player. Some large poodles may come close.

9.e)

If necessary, have the students measure out 1 m^3, then do the estimate.

9.f)

1 km is about 6/10 mi.



Chapter 1 Driving the Roads

An error that cannot be corrected by calculation is called a *random error*. No measurement is perfect. When you measure something, you make an approximation close to a certain accepted value. Random errors exist in any measurement. But you can estimate the amount of uncertainty in measurements that random errors introduce. Scientists provide an estimate of the size of the random errors in their data.

8 9

Make an approximation.

a) When measuring the hallway or class, did you have any systematic errors?

b) Estimate the size of your random errors using each technique.

9. Sometimes a precise measurement is not needed. A good estimate will do. What is a good estimate?

Example:

- Suppose one of your friends estimates that a single-serving drink container holds 5 kg (weighing about 11 lb) of liquid.

  This is not a good estimate. It is unreasonable. A mass of 5 kg, or a weight of 11 lb, is about the weight of a bowling ball or a turkey. A single-serving drink weighs much less than this.

Use your common sense and prior knowledge to judge if the following measurements are reasonable. Explain your answers.

a) A college football player has a mass of 100 kg (weighing about 220 lb).

b) A high-school basketball player is 4 m (13 ft) tall.

c) Your teacher works 1440 min every day.

d) A poodle has a mass of 60 kg (about 132 lb).

e) Your classroom has a volume of 150 m^3 (about 5300 ft^3).

f) The distance across the school grounds is 1 km (about 0.6 mi).

g) On a rural road, while driving 50 mi/h (about 80 km/h), you encounter a tractor moving very slowly. You are about ¼ mi (0.4 km) away when you see that another automobile is coming toward you. Is it safe to pass the tractor?

h) While driving your pickup truck on a rural road, you approach a narrow bridge and see you will reach it at the same time as a dump truck that is coming from the opposite direction. What must you estimate in order to decide whether to stop and wait for the dump truck to cross the bridge first, or to go ahead and squeeze by the dump truck while on the bridge?

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9.g)

Assume the oncoming vehicle approaching you is also traveling at 50 mi/h (80 km/h). The speed at which the two vehicles are approaching each other is then 160 km/h. The passing distance available is 0.40 km. Using $v = d/t$, and solving for time gives you $t = d/v$. The time available to pass the slow-moving tractor (assume a very low speed, so that the distance the tractor moves during the passing process is negligible) is $t = 0.40\ \text{km}/(160\ \text{km/h}) = 2.5 \times 10^{-3}\ \text{h}$. Converting hours to seconds, $(2.5 \times 10^{-3}\ \text{h})(3600\ \text{s/h}) = 9\ \text{s}$. A 9 s passing time would be sufficient to pass a short vehicle that is moving very slowly. If the tractor is 8 m long and has a velocity of 5 m/s (approx. 10 mi/h), it would only move forward to a distance of 45 m


Section 2 Measurement: Errors, Accuracy, and Precision

i) You are driving a motor home with bicycles standing upright in a bicycle rack mounted on the roof. A sign before the entrance to a tunnel states that the maximum height is 21 ft (6.4 m). Will your automobile make it safely through the tunnel?

Physics Talk

ERRORS IN MEASUREMENT

Random Errors

There is no exact measurement. In the *Investigate*, when you used your stride length as the measuring tool, the distance of the hallway was different for many of the groups. If you tried to improve the measurement by using a meter stick, you found that there were still differences in the measurement. Even if you had used a tape measure, there would still have been differences in your measurements.

Physicists know that all measuring tools produce **random errors**, or errors that cannot be corrected by calculating. It is the responsibility of the student scientist to record all the values of a measurement and recognize that the data will include random errors. Every time you measure the length of your desk, you might find that the measurement is different from a previous value by 0.1 cm. This difference could be in either direction (± 0.1 cm). You can use a more precise ruler and that may decrease this random error or uncertainty to only 0.05 cm (± 0.05 cm). However, the uncertainty can never be completely eliminated.

Both the measuring tool and the person doing the measuring are responsible for the uncertainty. A meter stick that has only the centimeters noted would have a greater uncertainty than a meter stick that has the millimeters noted. A meter stick that has millimeters noted may still have a large uncertainty if the person using it is not very careful in aligning the meter stick with the length being measured.

Physics Words
random error: an error that cannot be corrected by calculation.

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Teaching Tip

Students have difficulty making estimates in metric units due to their unfamiliarity. Emphasize that a kilogram is about 2 lb, a liter is about a quart, and a meter is slightly longer that a yard to help them visualize the metric equivalents.

during the passing time of 9 s. The passing vehicle has a passing distance of 200 m (one-half the 400-m separation), giving it 150 m of distance to pull ahead of the tractor, prior to a collision.

9.h)

You will have to estimate the width of the bridge and compare that to your estimate of the sum of the widths of your pickup and the dump truck. Leave enough space so you and the oncoming truck do not exchange paint. Allow the space for mirrors.

9.i)

Because most bicycles are less than 4 ft tall at the handlebars, allowing for 2 ft for the roof rack means that the motor home would have to be taller than 15 ft to crash the bikes.

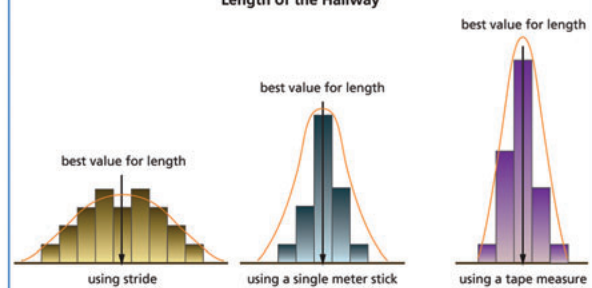
Physics Talk

Students learn that no measurement is perfect. All measurements have errors. These are classified as random and systematic. The *Physics Talk* builds on the *Investigate* where students measured selected distances and recorded their measurements to determine if they were the same or different. You might want to emphasize at this point that differences in measurements are due to an uncertainty that will always be present. Students should be able to clearly distinguish between accurate and precise measurements.

You can enhance their learning experience with a visual focus, drawing students' attention to the bull's-eye diagrams that will reinforce the difference between accuracy and precision. In this section, as students are also introduced to the SI system, it would be useful if they had a table of metric units and the corresponding symbols in their *Active Physics* logs to provide a quick reference.

Chapter 1 Driving the Roads

In your measurement of the distance, you found different distributions of measurement. If you made histograms as shown below, of the length of a hallway using your stride (left figure), the meter stick (middle figure) or a tape measure (right figure), you can get a sense of the uncertainty in each type of measurement. The middle value is probably the “best guess” for the length of the room, but there will always be an uncertainty surrounding that value, as shown by the spread to the left and right of the middle value.



Physics Words

systematic error: an error produced by using the wrong tool or using the tool incorrectly for measurement and can be corrected by calculation.

accuracy: an indication of how close a series of measurements are to an accepted value.

precision: an indication of the frequency with which a measurement produces the same results.

Systematic Errors

There are also **systematic errors.** If you mistake a yardstick for a meter stick and report your measurement as 4 m, when in fact it is 4 yd, that is a systematic error. Every measurement you record with that yardstick will have this error. Systematic errors can be avoided or can be corrected by calculating.

Accuracy and Precision

In shooting arrows at a target, you can have **accuracy** and **precision** by getting all the arrows in the bull’s-eye (left figure). You can have precision, but not accuracy by having all the arrows miss the bull’s-eye by the same amount (middle figure). You can also have accuracy, but without precision by having all the arrows surrounding the bull’s-eye spread out over the area (right figure). Notice that here the average position is the bull’s-eye (accuracy), but not one of the arrows actually hit the bull’s eye (precision).

Measurements can also vary with accuracy and/or precision just as the arrows in the target.

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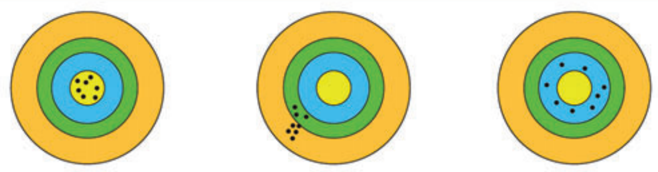
1-2a Blackline Master

CHAPTER 1



People do not always need the same level of precision in their measurements. One decision you must make is how precise a measurement you want. For example, in motor racing, horse racing, or Olympic skiing, time has to be measured to the thousandths or tens-of-thousandths of a second. But when a painter estimates the time required to paint a customer's house, she or he may only need to know the time within a few hours. As you increase the need for precision, the measurement becomes more difficult (and often, more expensive to make).

SI System

In *Active Physics*, you will be using the International System of Units. The units are known as SI units, abbreviated from *Le Système International d'Unités*. This is the system of units that is used by scientists. The system is based on the metric system. All units are related by some multiple of ten. There are seven base units that can be combined to measure all scientific properties. The base units that you will use in *Active Physics* are shown in the table.

Quantity	Unit	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
temperature	kelvin	K
current	ampere	A

You will also be using other units that are a combination of these base units. You will be introduced to these units when you need to use them. The best way to learn units is to use them frequently and correctly. It is not helpful to memorize lots of units.

Checking Up

1.

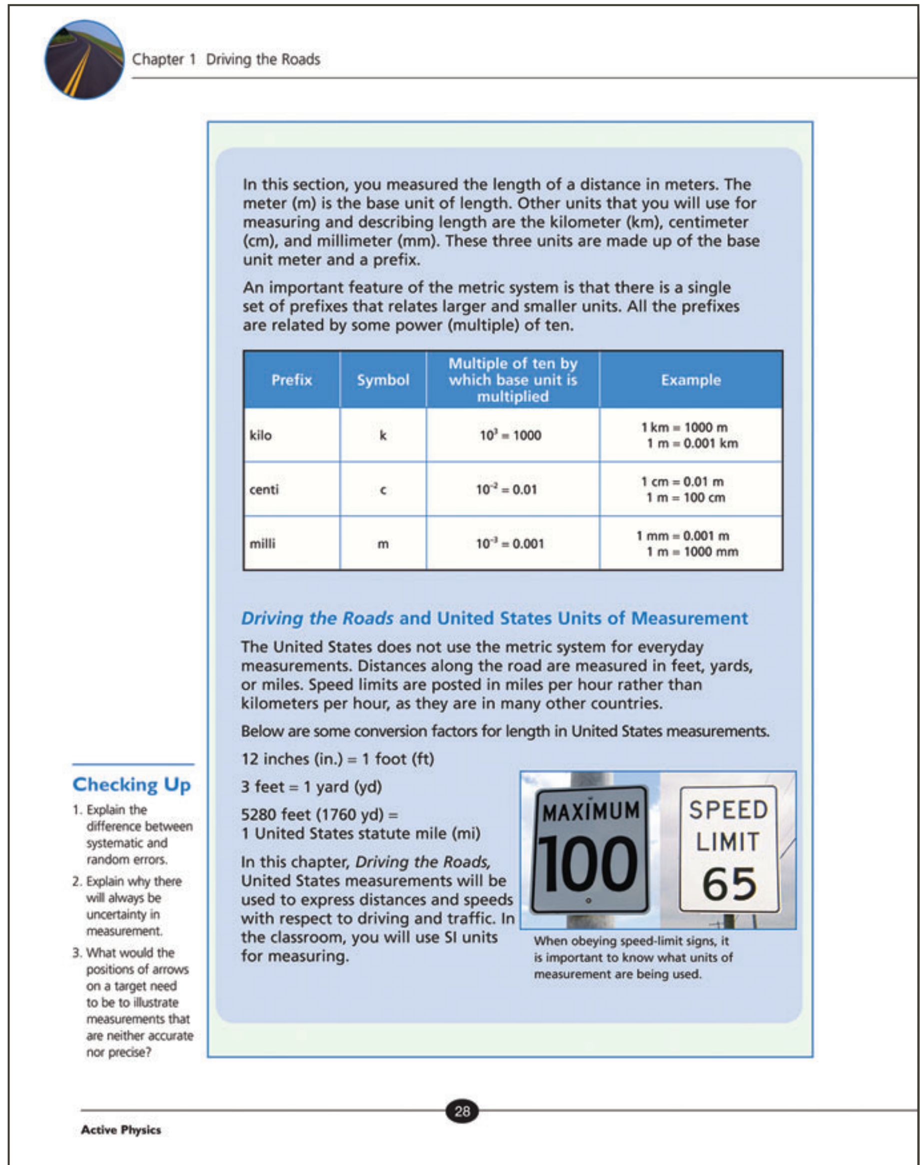
A systematic error can be corrected by calculation while a random error cannot be corrected by calculation. If you weigh an object with a balance that is not set to the 0-kg mark, but is instead at 1 kg, then you can correct the error by moving the pointer to the 0-kg mark. However, if an object is weighed by different people and the readings are determined to be very close to each other, then the uncertainty in measurement is said to contain a random error.

2.

Uncertainty in measurement can never be eliminated because all measurements are made around a certain value thought to be true. The difference between the true value and the actual value produces an uncertainty in the measurement.

3.

The arrows would be spread out unevenly around the target, illustrating that they were neither accurate nor precise.



In this section, you measured the length of a distance in meters. The meter (m) is the base unit of length. Other units that you will use for measuring and describing length are the kilometer (km), centimeter (cm), and millimeter (mm). These three units are made up of the base unit meter and a prefix.

An important feature of the metric system is that there is a single set of prefixes that relates larger and smaller units. All the prefixes are related by some power (multiple) of ten.

Prefix	Symbol	Multiple of ten by which base unit is multiplied	Example
kilo	k	$10^3 = 1000$	1 km = 1000 m 1 m = 0.001 km
centi	c	$10^{-2} = 0.01$	1 cm = 0.01 m 1 m = 100 cm
milli	m	$10^{-3} = 0.001$	1 mm = 0.001 m 1 m = 1000 mm

Driving the Roads and United States Units of Measurement

The United States does not use the metric system for everyday measurements. Distances along the road are measured in feet, yards, or miles. Speed limits are posted in miles per hour rather than kilometers per hour, as they are in many other countries.

Below are some conversion factors for length in United States measurements.

12 inches (in.) = 1 foot (ft)

3 feet = 1 yard (yd)

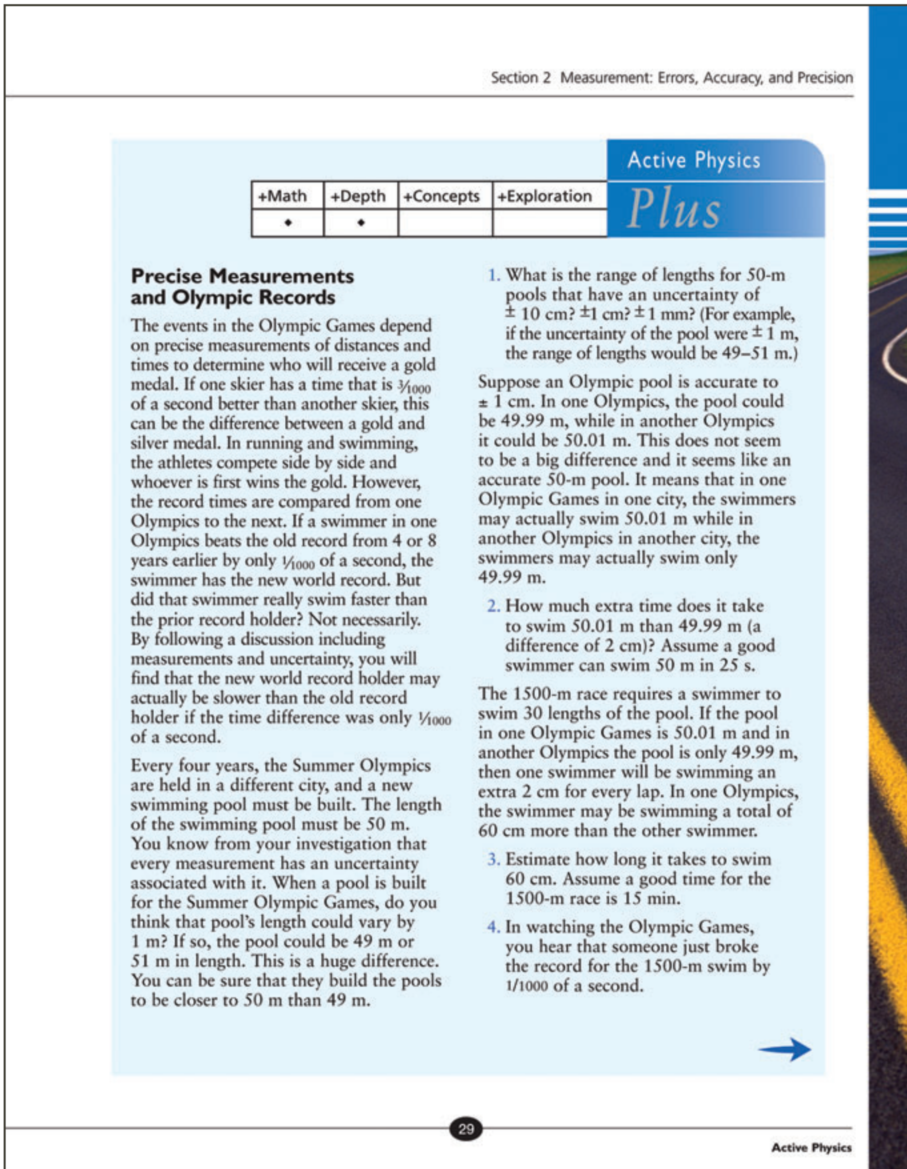
5280 feet (1760 yd) = 1 United States statute mile (mi)

In this chapter, *Driving the Roads,* United States measurements will be used to express distances and speeds with respect to driving and traffic. In the classroom, you will use SI units for measuring.

When obeying speed-limit signs, it is important to know what units of measurement are being used.

Checking Up

1. Explain the difference between systematic and random errors.
2. Explain why there will always be uncertainty in measurement.
3. What would the positions of arrows on a target need to be to illustrate measurements that are neither accurate nor precise?



Section 2 Measurement: Errors, Accuracy, and Precision

Active Physics *Plus*

+Math	+Depth	+Concepts	+Exploration
•	•		

Precise Measurements and Olympic Records

The events in the Olympic Games depend on precise measurements of distances and times to determine who will receive a gold medal. If one skier has a time that is 1/1000 of a second better than another skier, this can be the difference between a gold and silver medal. In running and swimming, the athletes compete side by side and whoever is first wins the gold. However, the record times are compared from one Olympics to the next. If a swimmer in one Olympics beats the old record from 4 or 8 years earlier by only 1/1000 of a second, the swimmer has the new world record. But did that swimmer really swim faster than the prior record holder? Not necessarily. By following a discussion including measurements and uncertainty, you will find that the new world record holder may actually be slower than the old record holder if the time difference was only 1/1000 of a second.

Every four years, the Summer Olympics are held in a different city, and a new swimming pool must be built. The length of the swimming pool must be 50 m. You know from your investigation that every measurement has an uncertainty associated with it. When a pool is built for the Summer Olympic Games, do you think that pool's length could vary by 1 m? If so, the pool could be 49 m or 51 m in length. This is a huge difference. You can be sure that they build the pools to be closer to 50 m than 49 m.

1. What is the range of lengths for 50-m pools that have an uncertainty of ± 10 cm? ±1 cm? ±1 mm? (For example, if the uncertainty of the pool were ± 1 m, the range of lengths would be 49–51 m.)

Suppose an Olympic pool is accurate to ± 1 cm. In one Olympics, the pool could be 49.99 m, while in another Olympics it could be 50.01 m. This does not seem to be a big difference and it seems like an accurate 50-m pool. It means that in one Olympic Games in one city, the swimmers may actually swim 50.01 m while in another Olympics in another city, the swimmers may actually swim only 49.99 m.

2. How much extra time does it take to swim 50.01 m than 49.99 m (a difference of 2 cm)? Assume a good swimmer can swim 50 m in 25 s.

The 1500-m race requires a swimmer to swim 30 lengths of the pool. If the pool in one Olympic Games is 50.01 m and in another Olympics the pool is only 49.99 m, then one swimmer will be swimming an extra 2 cm for every lap. In one Olympics, the swimmer may be swimming a total of 60 cm more than the other swimmer.

3. Estimate how long it takes to swim 60 cm. Assume a good time for the 1500-m race is 15 min.

4. In watching the Olympic Games, you hear that someone just broke the record for the 1500-m swim by 1/1000 of a second.

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Active Physics

Active Physics Plus

This *Active Physics Plus* provides an opportunity for students to understand the importance of making precise measurements. Before students begin to answer questions, have them make a list of the reasons that they think are significant to making precise measurements. Ask them how the degree of uncertainty would matter in decisive factors that influence events. Each math problem they solve will emphasize the need for precision in measurements.

1.

The range of uncertainties for these pools would be 49.9 to 50.1 m, 49.99 to 50.01 m, and 49.999 to 50.001 m.

2.

The time to swim an extra 2 cm can be determined from the velocity equation $v = d/t$. Solving for t gives $t = d/v$ so the extra time would be $t = (0.02\text{ m})/(2\text{ m/s}) = 0.01\text{ s}$.

3.

The speed for a 1500 m race in 15 min (900 s) would be $v = d/t = (1500\text{ m})/(900\text{ s}) = 1.67\text{ m/s}$. Using this gives $t = d/v = 0.60\text{ m}/(1.67\text{ m/s}) = 0.36\text{ s}$.

4.

If the pool is short by 1 cm in length, the difference in time would be 0.18 s (half the time for a 2-cm difference), which is 18 hundredths of a second. The swimmer might actually be 0.17 s more than the world record for a pool of the proper length.

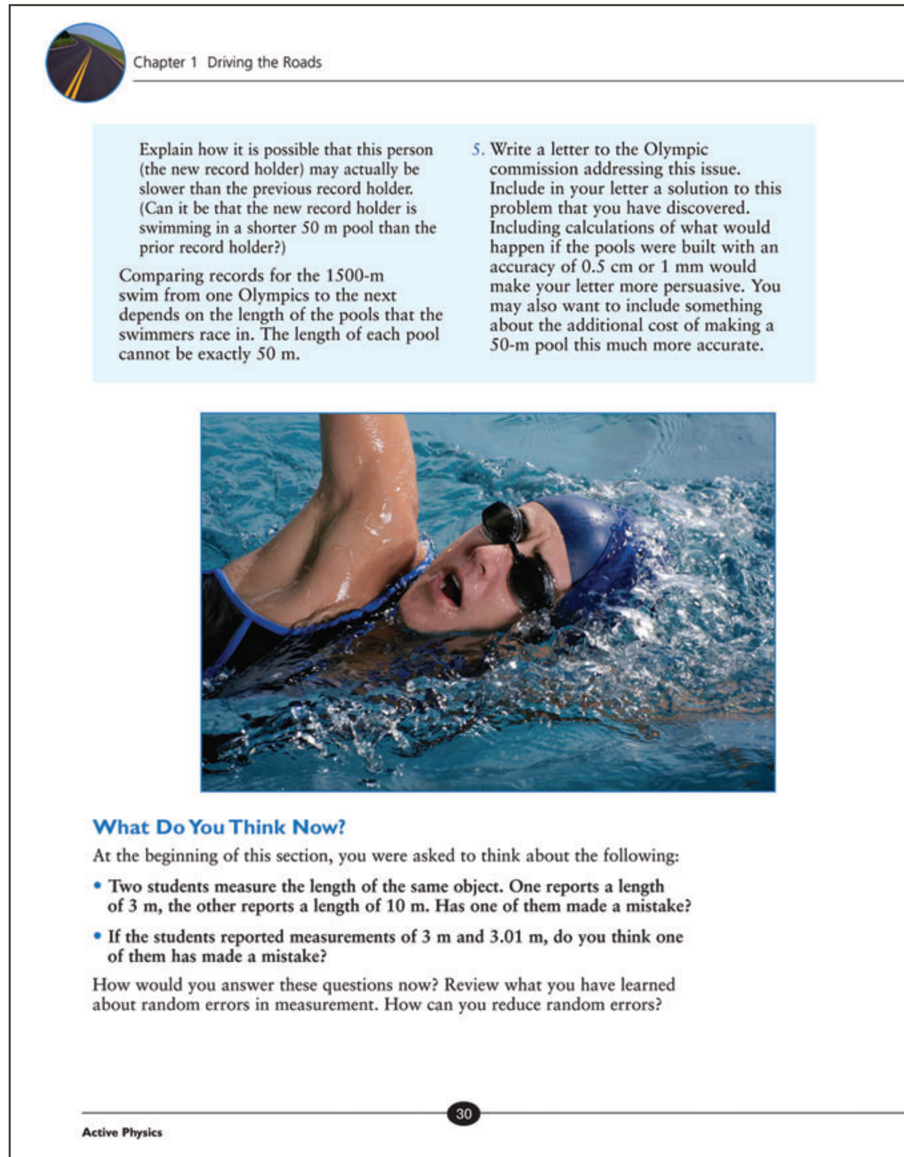
5.

Students' letters should indicate what they have discovered in the analysis above from the questions they have answered to explain their case.

CHAPTER 1

What Do You Think Now?

Ask students to review their previous answers. This is an opportunity to identify relevant answers during the discussion and provide them with *A Physicist's Response*. Students should by now have a clear understanding of the sources of systematic and random errors and should be able to explain the difference between these types of errors. You might want to ask them to review the *Physics Talk* before they begin to answer the question on how they could reduce random errors.



Explain how it is possible that this person (the new record holder) may actually be slower than the previous record holder. (Can it be that the new record holder is swimming in a shorter 50 m pool than the prior record holder?)

Comparing records for the 1500-m swim from one Olympics to the next depends on the length of the pools that the swimmers race in. The length of each pool cannot be exactly 50 m.

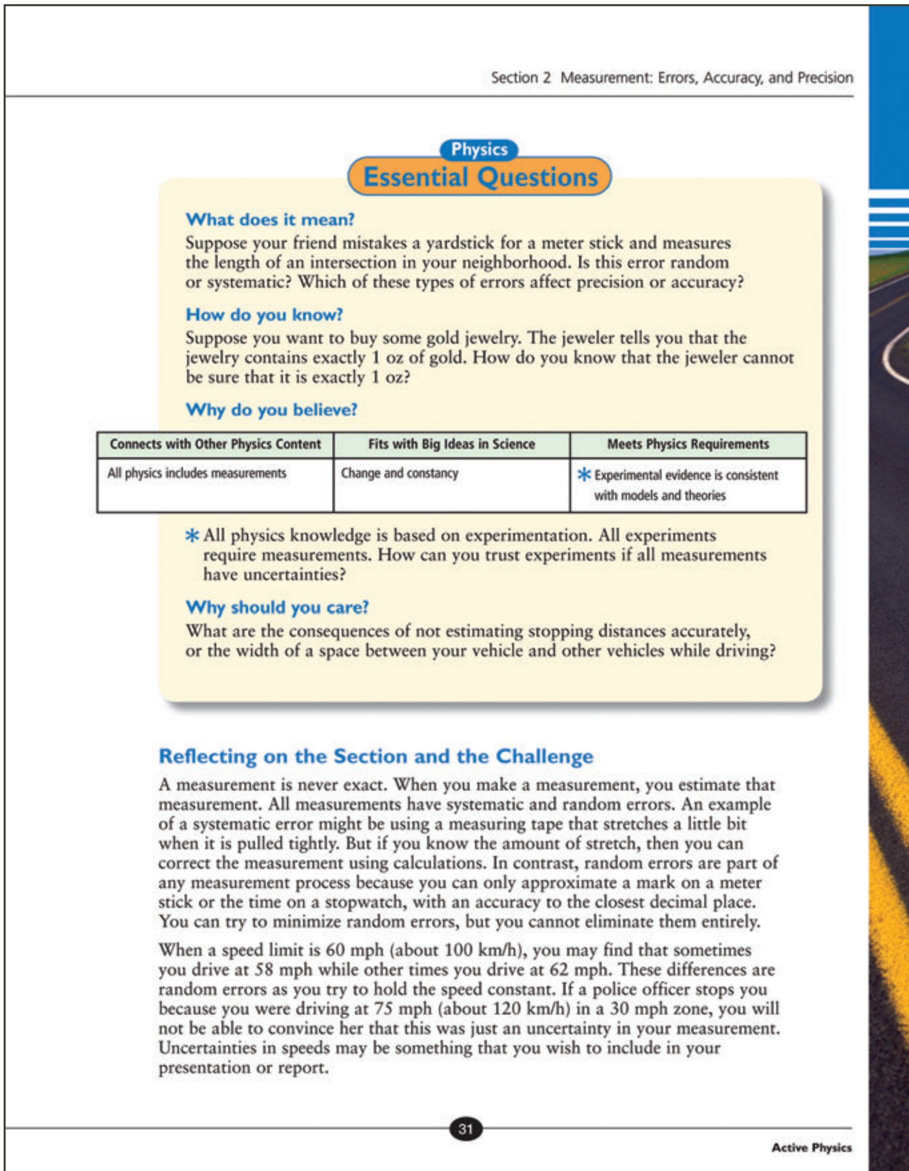
5. Write a letter to the Olympic commission addressing this issue. Include in your letter a solution to this problem that you have discovered. Including calculations of what would happen if the pools were built with an accuracy of 0.5 cm or 1 mm would make your letter more persuasive. You may also want to include something about the additional cost of making a 50-m pool this much more accurate.

What Do You Think Now?

At the beginning of this section, you were asked to think about the following:

- **Two students measure the length of the same object. One reports a length of 3 m, the other reports a length of 10 m. Has one of them made a mistake?**
- **If the students reported measurements of 3 m and 3.01 m, do you think one of them has made a mistake?**

How would you answer these questions now? Review what you have learned about random errors in measurement. How can you reduce random errors?


Section 2 Measurement: Errors, Accuracy, and Precision

Physics
Essential Questions

What does it mean?

Suppose your friend mistakes a yardstick for a meter stick and measures the length of an intersection in your neighborhood. Is this error random or systematic? Which of these types of errors affect precision or accuracy?

How do you know?

Suppose you want to buy some gold jewelry. The jeweler tells you that the jewelry contains exactly 1 oz of gold. How do you know that the jeweler cannot be sure that it is exactly 1 oz?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
All physics includes measurements	Change and constancy	* Experimental evidence is consistent with models and theories

* All physics knowledge is based on experimentation. All experiments require measurements. How can you trust experiments if all measurements have uncertainties?

Why should you care?

What are the consequences of not estimating stopping distances accurately, or the width of a space between your vehicle and other vehicles while driving?

Reflecting on the Section and the Challenge

A measurement is never exact. When you make a measurement, you estimate that measurement. All measurements have systematic and random errors. An example of a systematic error might be using a measuring tape that stretches a little bit when it is pulled tightly. But if you know the amount of stretch, then you can correct the measurement using calculations. In contrast, random errors are part of any measurement process because you can only approximate a mark on a meter stick or the time on a stopwatch, with an accuracy to the closest decimal place. You can try to minimize random errors, but you cannot eliminate them entirely.

When a speed limit is 60 mph (about 100 km/h), you may find that sometimes you drive at 58 mph while other times you drive at 62 mph. These differences are random errors as you try to hold the speed constant. If a police officer stops you because you were driving at 75 mph (about 120 km/h) in a 30 mph zone, you will not be able to convince her that this was just an uncertainty in your measurement. Uncertainties in speeds may be something that you wish to include in your presentation or report.

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Active Physics

Reflecting on the Section and the Challenge

The concepts students learn in this section establish the presence of random errors in measurements. Because random errors can never be eliminated, they provide a basis for deviating from the accepted value by a small margin. A good example might be variations in the speedometers of vehicles for leeway in speed limits. The students can use this knowledge of random errors to develop their *Chapter Challenge*. They should be able to put forth the connection between driving at a safe speed and reading the speedometer with accuracy.

Physics Essential Questions

What does it mean?

Because a yardstick is smaller than a meter stick, the measured length of the intersection will be incorrect. This is a systematic error and can be corrected once you realize that you used a yardstick instead of a meter stick. Systematic errors affect accuracy. Random errors affect precision.

How do you know?

Every measurement has some uncertainty. There is no such thing as an exact measurement.

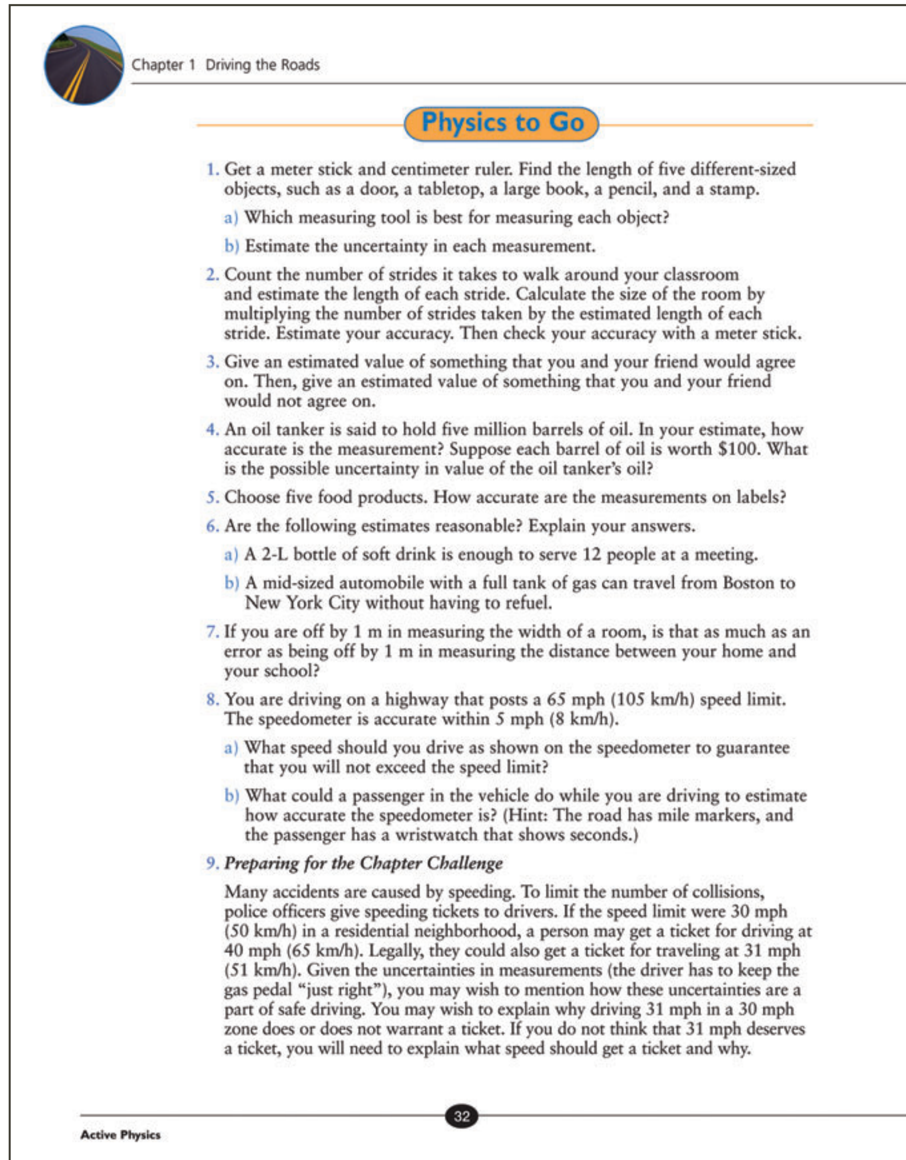
Why do you believe?

The uncertainties in measurements place a limitation on what you may be able to conclude. However, you can draw conclusions within these limitations. For example, you may not know your weight to the nearest ounce, but you may know it to the nearest pound. From that knowledge, you can know whether you need to change your diet.

Why should you care?

If you estimate distances poorly, you may find yourself hitting the vehicle in front of you.

Physics to Go



Physics to Go

1. Get a meter stick and centimeter ruler. Find the length of five different-sized objects, such as a door, a tabletop, a large book, a pencil, and a stamp.
   a) Which measuring tool is best for measuring each object?
   b) Estimate the uncertainty in each measurement.
2. Count the number of strides it takes to walk around your classroom and estimate the length of each stride. Calculate the size of the room by multiplying the number of strides taken by the estimated length of each stride. Estimate your accuracy. Then check your accuracy with a meter stick.
3. Give an estimated value of something that you and your friend would agree on. Then, give an estimated value of something that you and your friend would not agree on.
4. An oil tanker is said to hold five million barrels of oil. In your estimate, how accurate is the measurement? Suppose each barrel of oil is worth $100. What is the possible uncertainty in value of the oil tanker's oil?
5. Choose five food products. How accurate are the measurements on labels?
6. Are the following estimates reasonable? Explain your answers.
   a) A 2-L bottle of soft drink is enough to serve 12 people at a meeting.
   b) A mid-sized automobile with a full tank of gas can travel from Boston to New York City without having to refuel.
7. If you are off by 1 m in measuring the width of a room, is that as much an error as being off by 1 m in measuring the distance between your home and your school?
8. You are driving on a highway that posts a 65 mph (105 km/h) speed limit. The speedometer is accurate within 5 mph (8 km/h).
   a) What speed should you drive as shown on the speedometer to guarantee that you will not exceed the speed limit?
   b) What could a passenger in the vehicle do while you are driving to estimate how accurate the speedometer is? (Hint: The road has mile markers, and the passenger has a wristwatch that shows seconds.)
9. ***Preparing for the Chapter Challenge***

   Many accidents are caused by speeding. To limit the number of collisions, police officers give speeding tickets to drivers. If the speed limit were 30 mph (50 km/h) in a residential neighborhood, a person may get a ticket for driving at 40 mph (65 km/h). Legally, they could also get a ticket for traveling at 31 mph (51 km/h). Given the uncertainties in measurements (the driver has to keep the gas pedal "just right"), you may wish to mention how these uncertainties are a part of safe driving. You may wish to explain why driving 31 mph in a 30 mph zone does or does not warrant a ticket. If you do not think that 31 mph deserves a ticket, you will need to explain what speed should get a ticket and why.

1.a)

Use a meter stick for measuring objects that are several meters long; use the ruler for objects that are a few centimeters long. The large book could be measured with either tool.

1.b)

The uncertainty will depend on the smallest markings on the ruler being used.

2.

If the stride is well-calibrated, the students should give an answer within a few centimeters (or tens of centimeters) of the meter-stick method.

3.

As an example, students might agree on the estimated length of a football field at 100 yd, but disagree on the length of a semi-tractor trailer.

4.

Five million barrels is a rough estimate. That probably means "more than four million, nine hundred thousand but less than five million, one hundred thousand."

5.

You may choose to have the students answer this question as part of an in class assignment because it will require them to measure the mass (weight) of the food and compare them with the labeled weight. This will require a balance, and the values and accuracy will depend upon the food choices. Dried foods such as spaghetti are preferable to liquids in cans. Dried food that can be weighed in the box with the weight of the box subtracted will yield the most accurate results. Expect difference from the labeled amounts due to imprecision of the machines filling the boxes, but the errors should be both higher and lower than the listed weight.

6.a)

Two liters is slightly more than 2 qt, so each person would have about 1/6 of a qt. This is a small amount, but reasonable.

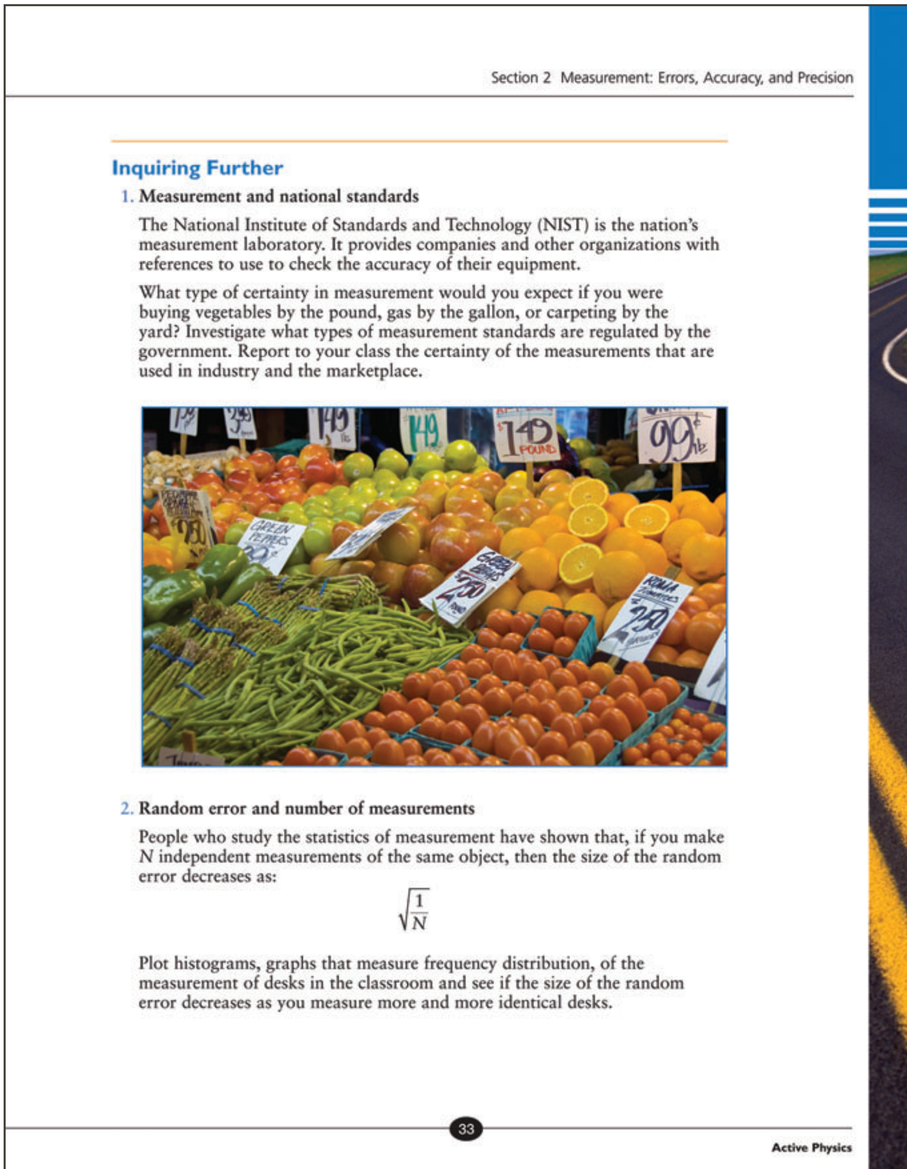
6.b)

The distance from New York City to Boston is about 300 mi. A mid-sized vehicle might get 25 mi to the gallon, making its gasoline usage around 12 gal. Most mid-sized vehicles would have a 16-gal tank.

7.

Having the students justify their answers is more important than the answers themselves.

CHAPTER 1


Section 2 Measurement: Errors, Accuracy, and Precision

Inquiring Further

1. **Measurement and national standards**

The National Institute of Standards and Technology (NIST) is the nation's measurement laboratory. It provides companies and other organizations with references to use to check the accuracy of their equipment.

What type of certainty in measurement would you expect if you were buying vegetables by the pound, gas by the gallon, or carpeting by the yard? Investigate what types of measurement standards are regulated by the government. Report to your class the certainty of the measurements that are used in industry and the marketplace.

2. **Random error and number of measurements**

People who study the statistics of measurement have shown that, if you make *N* independent measurements of the same object, then the size of the random error decreases as:

$$\sqrt{\frac{1}{N}}$$

Plot histograms, graphs that measure frequency distribution, of the measurement of desks in the classroom and see if the size of the random error decreases as you measure more and more identical desks.

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Active Physics

8.a)

60 mi/h might really be 65 mi/h, so to be safe, you should drive 60 mi/h.

8.b)

The passenger may watch the mile markers and mark the time when they are passed. A driver traveling at 60 mi/h would travel 1 mi in 60 s (60 mi/h is 60 mi in 60 min), so if the vehicle is traveling 60 mi/h, the next mile marker should be passed in exactly 60 s.

9.

Preparing for the Chapter Challenge

Students may wish to discuss the problems with maintaining an exact speed such as 30 mi/h. Automobile speedometers are legislated to an accuracy of ± 5 percent, but may not actually be that accurate. An accuracy of 5 percent means that at 30 mi/h, the vehicle's true speed is between 28.5 and 31.5 mi/h. If the automobile has tires that are larger than the manufacturer's specifications, the error could be even greater. For reasons such as these, police officers often allow a leeway of ± 5 mph, or for a 30-mi/h speed limit a 15 percent deviation from the speed limit. Allowing for the accuracy of the speedometer, anyone driving at 32 mi/h or higher in a 30-mi/h zone should be liable for a ticket. A driver who spends too much time watching the speedometer to maintain the exact speed limit risks not paying enough attention to other aspects of safe driving. Driving below the speed limit allows the driver to stay within the law and pay more attention to the road.

Inquiring Further

1. Measurement and national standards

Students will have difficulty finding standards for the measurement of volumetric quantities, such as gasoline, because the volume of gas dispensed by a pump varies with temperature. Gasoline pumps are difficult to monitor for a number of reasons, but the industry claims to be accurate to within 1 percent. Carpets likewise will stretch to a certain degree, making exact measurements difficult. Again, accuracy within a few percent is expected. Measurements of the weight of fruits and vegetables are determined by the accuracy of the scales used, and state rules vary as to what is acceptable.

2. Random error and number of measurements

To do this investigation, you may want students to look up "standard deviation" as a measure of random error prior to their making a series of measurements.

SECTION 2 QUIZ

1-2b Blackline Master

1. The approximate mass of a high school student is

a) 1.0 kg. b) 5.0 kg.

c) 65 kg. d) 250 kg.

2. The approximate length of a baseball bat is

a) 1 cm. b) 1 m.

c) 1 km. d) 1 mm.

3. The maximum time allowed for this quiz is approximately

a) 300 s. b) 2000 s.

c) 3000 s. d) 10,000 s.

4.Three students measure the length of their classroom and get the following values:

Student A – 6.54 m Student B – 6.57 m Student C – 6.52 m

Because all three students received three different measurements, this is an example of

a) an uncertainty error. b) a predictable error.

c) a systematic error. d) a random error.

5. Students in a class measure their stride, and use that measurement to determine the length of a school's soccer field. As a result, each group arrives at a different value for the length of the field. The way to achieve the most accurate value for the length of the field would be to

a) get the class to agree on using one group's value.

b) pick the highest and the lowest values and average them.

c) average all the values of each group.

d) have everyone measure again, and then find out which number is repeated.

SECTION 2 QUIZ ANSWERS

1. c) A high school student might have a weight of 140 lbs. At 2.2 lbs/kg, this is approximately 65 kg. A mass of 5 kg would be 12 lbs. A mass of 250 kg would be 550 lbs.

2. b) A baseball bat is approximately 36 in. long, and a meter stick is about 39 in. long. A length of 1 cm is about $\frac{1}{2}$ in., a millimeter is even smaller, and a kilometer is about 3000 ft long.

3. a) The maximum time allowed for the quiz is 300 s, which is about 5 min. A time of 2000 s is about 35 min (too long for a 5-question quiz) and the other times are even longer.

4. d) Measurements that vary around the correct value would be an example of random error. If all the measurements were on the same side of the accepted value (either higher or lower), the error might be systematic. The other answers are made-up terms.

5. c) Agreeing on one group's value does not improve the accuracy, although choosing the highest and lowest values may work. If most of the values were high and one value was much lower, it would most likely lead to a false result. Having a repeating value does not mean that this is anywhere near the correct value, because it may be due to a systematic error.