SECTION 5

The Range of Projectiles: The Shot Put

Section Overview

In this section, students create and compare mathematical and physical models of a projectile's motion to improve performance in events where trajectories occur. Students first measure the acceleration due to gravity, using one of two methods chosen by the teacher, then use this value to calculate an object's position and speed at any time after it enters a state of free fall. In addition, they use this knowledge for a quantitative treatment of the horizontal (constant speed) and vertical (free fall) components of the motion of a projectile. They model the trajectory of a projectile and find some of the variables affecting the height and range of a projectile. Through the *Investigate*, they learn to apply mathematical and physical models to analyze the speed of a falling object. In the mathematical model, students calculate the average speed of an object dropped vertically, using the value of the acceleration due to gravity. In the physical model, they construct a representation of the path of a projectile that is launched at an angle where its velocity has both a horizontal and a vertical component.

Background Information

Acceleration Due to Gravity: All objects in a state of free fall, regardless of mass, shape, or other characteristics, experience a uniform (constant) acceleration, g, of approximately 10 m/s² in the downward direction. (Falling objects which experience significant air resistance are not considered to be in a state of free fall.) Earth exerts an amount of downward gravitational pull on objects at or near its surface that is proportional to the mass of each object. The outcome of this is that all objects experience the same amount of force per unit of mass and, therefore, the same acceleration. For example, Earth's pull on a 2-kg rock is twice as much as the pull on a 1-kg rock; the result is that both accelerate at the same rate during free fall. Active Physics "rounds off" g to 10 m/s², which is within two percent of the average value, about 9.81 m/s² on our planet. The acceleration due to gravity depends on the radial distance from the center of Earth; therefore, the value of g varies with location by as much as 0.04 m/s^2 due to terrain differences (mountains, valleys), local rock composition, and the fact that Earth is not a perfect sphere (the equatorial diameter is slightly greater than the polar diameter). Some argue that local variations in g, if taken into account, would sometimes affect performance in a track and field event. For example, a shot-put result at a location with a slightly higher value of g may actually be better than a longer throw made at a location having a lower value of g. Physicists have determined that variations in acceleration due to gravity sometimes could make a difference in records in field events: distance, speed, acceleration, and time of free fall would all be affected by this variation. This section follows arguments originally presented by Galileo regarding free fall.

Galileo hypothesized that free fall involved constant or uniform acceleration. From the definition of constant acceleration, $a = \Delta v / \Delta t$, he reasoned that the speed attained by an object falling from a "rest start" at any time during its fall would be $v = \Delta v = a(\Delta t)$. (The speed, v, at the end of a time of fall, Δt , would be equal to the change in speed, Δv , because the initial speed for a rest start is zero.) Galileo further reasoned that, because the speed of a falling object increases uniformly (or at a constant rate) with time (his hypothesis), the average speed for a time of fall, Δt , would be half of the speed attained at the end of a time of fall. Therefore, Average speed at the end of a time interval,

$$v_{\text{average}} = \Delta v/2 = a(\Delta t)/2.$$

Finally, Galileo reasoned that the distance of fall and the end of a time of fall, Δt , would be, simply, the average speed multiplied by the time of fall. Therefore, fall distance at the end of time interval, *t*, is

$$d = \frac{a(\Delta t)}{2} \times \Delta t = \frac{1}{2} a \Delta t^2.$$

The above reasoning was used as the strategy for developing the table in *Step 2* of the *Investigate* for this section as an extension of the above for your understanding as the teacher. Galileo was limited in his ability to test his assumptions and reasoning. He did not have adequate instruments to measure acceleration and speed directly, so he devised a test of his thinking which involved what he could measure. From the last line in the above derivation, $d = \frac{1}{2} a \Delta t^2$, he solved acceleration as follows:

$$a = \frac{2a}{\Delta t^2}$$

Rolling a sphere down a ramp to "dilute" the effect of gravity (he assumed that a sphere rolling down a ramp also had constant acceleration, but less of it than an object in free fall) he used a crude timing device to measure the amounts of time for the sphere to roll, starting from rest, several measured distances along the ramp. Upon substituting pairs of distance and time measurements into the relation $a = 2d/\Delta t^2$, he obtained a fixed value for the right-hand side of the relation, proving that the acceleration indeed is constant.

The Trajectory of a Projectile

The steps used to develop the model of a trajectory in this section provide a fine quantitative example of the independence of the vertical and horizontal components of the motion of a projectile. While it is not expected that students will predict the range, maximum height, time of flight, and other parameters of a projectile in terms of launch speed and direction, the following equations will prepare you to do so, if needed. For a projectile launched horizontally at speed v from height, h, time of flight, $t = \sqrt{(2h/g)}$, and range (horizontal distance), $R = vt = v\sqrt{(2h/g)}$. For the general case of a projectile launched from ground level at speed v at an angle θ above the horizontal, and traveling over flat ground the following relationships apply:

speed in the horizontal direction is $v_x = v \cos \theta$ (remains constant)

initial speed in the vertical direction is $v_{y} = v \sin \theta$

speed in the vertical direction at time, *t*, is $v_{y} = (v \sin \theta) - gt$

horizontal position at time, t, is $x = (v \cos \theta)t$

vertical position at time, *t*, is $y = (v \sin \theta)t - \frac{1}{2}gt^2$

time to reach maximum height is $t_{\text{max}} = (v \sin \theta)/g$

total time of flight is $t = 2t_{max}$

range (horizontal distance) is $R = (v^2 \sin 2\theta)/g$

Note: In the final of the above equations that sin 2θ (and, therefore, also the range) has a maximum value of 1 when $\theta = 45^{\circ}$, ($2\theta = 90^{\circ}$). This expression is only true for a projectile returning to the level from which it was launched. If the projectile lands higher or lower than the launch position (for example in the shot put), a more complex relationship is required to predict the optimal angle and range.

Crucial Physics

- Projectiles travel in parabolas (if we ignore the effects of air resistance).
- The horizontal motion and vertical motion of a trajectory are independent of one another.
- A projectile will travel the greatest distance if the angle of release is 45°. At angles less than 45°, the projectile has a greater horizontal speed but does not stay in the air as long. At angles greater than 45°, the projectile stays in the air a longer time, but has a smaller horizontal velocity.
- The acceleration due to gravity can be determined experimentally and is found to be equal to 9.8 m/s^2 .
- Mathematical models of projectiles:
 - When the acceleration is constant, the average speed during an interval of time is equal to half the sum of the initial and final velocities for the time interval. If the object is at rest at the beginning of the time interval, the average speed is half the velocity at the end of the time interval.
 - When the acceleration is constant, the distance traveled during an interval of time is equal to the average velocity during the time interval multiplied by the interval of time. If the object is at rest at the beginning of the time interval, the distance traveled is equal to half the velocity at the end of the interval of time multiplied by the interval of time.
 - If the acceleration is constant, the acceleration equals the velocity at the end of the time interval minus the velocity at the beginning of the time interval divided by the time interval. If the object is at rest at the beginning of the time interval, the acceleration is equal to the velocity at the end of the time interval divided by the time interval.
 - If the acceleration is constant and the object starts from rest, then the distance traveled over a time interval is equal to half the acceleration multiplied by the square of the time interval.

Learning Outcomes	Location in the Section	Evidence of Understanding
Measure the acceleration due to gravity.	<i>Investigate</i> Step 1	Students measure acceleration due to gravity, <i>g</i> , by using a "picket fence" and a photogate timer attached to a computer. The teacher could also ask students to use an alternative method using a ticker-tape timer and a mass.
Calculate the speed attained by an object that has fallen freely from rest.	<i>Investigate</i> Step 2.b)	Students calculate and record the speed of a falling object at every 0.10 s of its fall. Students complete a table using an example in their <i>Active Physics</i> textbook.
Identify the relationship between the average speed of an object that has fallen freely from rest and the final speed attained by the object.	<i>Investigate</i> Step 2.c)	Students calculate the average speed of a falling object by finding the average of the initial and final speeds. Since the initial speed is zero, the average speed comes out to one-half of the final speed.
Calculate the distance traveled by an object that has fallen freely from rest.	<i>Investigate</i> Step 2.c)	Students calculate and record the distance an object falls at the end of each 0.10 s of its fall by using the equation, distance = (Average speed × time).
Use mathematical models of free fall and uniform speed to construct a physical model of the trajectory of a projectile.	<i>Investigate</i> Step 3	Students put together two identical string and mass assemblies, with the string length equal to the distance of fall and the time of fall assigned to each group. The string separation is determined by the horizontal speed of the projectile.
Use the motion of a real projectile to test a physical model of projectile motion.	<i>Investigate</i> Steps 7-10	A student volunteer throws a tennis ball horizontally to match the curve connecting the position of assembly masses on the strings.
Use a physical model of projectile motion to infer the effects of launch speed and launch angle on the range of a projectile.	<i>Investigate</i> Steps 11.a), 11.d)	Students rest the end of the stick corresponding to 0.0 s on the tray at the bottom of the chalkboard and incline it at angles of 30° , 40° , 60° , and 90° . They then predict the greatest range of a projectile.

Section 5 Materials, Preparation, and Safety

Materials and Equipment

PLAN A				
Materials and Equipment	Group (4 students)	Class		
Timer, ticker tape, AC	1 per group			
Scissors	1 per group			
Meter stick, wood	1 per group			
Ring stand, large	1 per group			
Clamp, extension	1 per group			
Holder, right angle, cast iron	1 per group			
Calculator, basic	1 per group			
Washer, 3/4 in. (outside diameter) x 5/16 in. (inside diameter)	2 per group			
Weight, slotted, 100 g	10 per group			
Ball, tennis		1 per class		
Model, trajectory		1 per class		
Weight, fishing, small		7 per class		
Timer, photogate (with electromagnet)		1 per class		
Ticker tape, roll	1 per group			
Notes, sticky, pad, 3 in. x 3 in.		3 per class		
String, cotton, ball		1 per class		
Chalkboard*		1 per class		
Chalk*		1 per class		
Probeware, photogate*		1 per class		
Fence, picket*		1 per class		
MBL or CBL Technology (to record probeware activity)*		1 per class		
Ladder*		1 per class		

*Additional items needed not supplied

PLAN	В	
Materials and Equipment	Group (4 students)	Class
Timer, ticker tape, AC		1 per class
Scissors		1 per class
Meter stick, wood		1 per class
Ring stand, large		1 per class
Clamp, extension		1 per class
Holder, right angle, cast iron		1 per class
Calculator, basic	1 per group	
Washer, 3/4 in. (outside diameter) x 5/16 in. (inside diameter)	2 per group	
Weight, slotted, 100 g		10 per class
Ball, tennis		1 per class
Model, trajectory		1 per class
Weight, fishing, small		7 per class
Timer, photogate (with electromagnet)		1 per class
Ticker tape, roll		1 per class
Notes, sticky, pad, 3 in. x 3 in.		3 per class
String, cotton, ball		1 per class
Chalkboard*		1 per class
Chalk*		1 per class
Probeware, photogate*		1 per class
Fence, picket*		1 per class
MBL or CBL Technology (to record probeware activity)*		1 per class
Ladder*		1 per class

*Additional items needed not supplied

Note: Time, Preparation, and Safety requirements are based on Plan A, if using Plan B, please adjust accordingly.

Time Requirements

This Investigate requires two class periods or 80 min.

Teacher Preparation

- Assemble the materials required for the student groups to measure the acceleration due to gravity. The material required will depend upon the method chosen. If you wish to use the ticker-tape timer method, you will need masses to attach to the ticker tape to pull it vertically downward through the timer. Regardless of the method you choose, try several runs yourself prior to the students using the equipment to ensure it is working properly.
- Prepare the "portable" version of the mass and string assembly by attaching masses and strings of the same length as those students will be assembling on the board. These should be attached to a 2.4-m long piece of wood or similar device so that it may be tilted at an angle. If the 2.4-m length proves prohibitive to use in your classroom at the angles suggested, make plans to have the students try to match the trajectories outside or in an area with a higher ceiling.

Safety Requirements

• No particular safety precautions are required for this activity.

NOTES

Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

Learning Issue	Reference	Augmentation and Accommodations
Understanding vocabulary	What Do You Think?	 Augmentation Students with memory and focus issues may struggle to remember the meaning of words including trajectory, projectile, range, and launch. If students drew pictures to represent these words in <i>Section 4</i>, ask them to refer back to these pictures before recording their ideas. If students did not draw pictures in <i>Section 4</i>, remind them what these words mean before they record their ideas.
Measuring the acceleration due to gravity	<i>Investigate</i> Step 1	 Augmentation Students may need the teacher to model the method the class will use to measure the acceleration due to gravity, especially if students are asked to use the second method with the ticker-tape timer. This method requires students to do calculations they learned earlier in the chapter, and students may not remember how to solve for velocity and acceleration using the ticker tape. Accommodation Model this activity as a whole-group demonstration and ask for volunteers to help with calculations.
Creating a physical model as a group	<i>Investigate</i> Steps 3-8	 Augmentation Encourage students to initiate tasks and stay focused, because in this part of the <i>Investigate</i> the learning of the whole class depends on everyone doing their task. Provide pairs of students with a meter stick, string, mass, and some masking tape to construct their string-and-mass assemblies. Then tell students they have 4-7 minutes to construct the string-and-mass assembly. Pair students strategically to include at least one student who can focus on a task and one who is proficient with measurement. Accommodation Provide students with string-and-mass assemblies that are already constructed and direct them to hang the assemblies on the appropriate pin.
Reading comprehension	Physics Talk	 Augmentation Instruct students to create a graphic organizer or Venn diagram that names the two kinds of motion involved in projectile motion. Then ask pairs of students to list factors that affect each of these motions. Accommodation Provide students with a list of factors and ask them to sort the factors according to which type of motion they affect.
Understanding key concepts	<i>Physics Talk</i> <i>Physics to Go</i> Steps 1-3 and 8	 Augmentation This diagram of trajectories at different angles summarizes many of the key concepts and vocabulary words about projectiles. Provide students with a copy of the diagram and the statements below it. Then instruct students to use different-colored highlighters to make visual connections between the statements below the diagram and the trajectories in the diagram. Model the first example by highlighting the "45° launch angle" statement in yellow and then tracing that trajectory with the yellow highlighter. Accommodation Provide students with serious visual motor or graphomotor issues with a copy of the diagram and statements that have already been color-coded.

Learning Issue	Reference	Augmentation and Accommodations
Reading comprehension	<i>Reflecting on the Section and the Challenge</i>	 Augmentation The previous augmentation and accommodation will help students with reading comprehension issues to understand the summary in this reflection. Students can also use their highlighted diagram to assist them with the voice-over challenge.

Strategies for Students with Limited English-Language Proficiency

Learning Issue	Reference	Augmentation
Following complex procedures	Investigate	The steps in this <i>Investigate</i> run 4 pages. Break down the <i>Investigate</i> into smaller chunks that allow students to comprehend each portion of the activity before moving on to the next one. This approach will allow students to get comfortable following the procedures outlined within each step, to grasp the math, and to internalize new concepts introduced. Lead a brief class discussion after each step to allow students the opportunity to demonstrate acquired knowledge and understanding, and to allow you to correct misunderstandings.
Understanding scientific concepts	Physics Talk Physics Essential Questions	The scientific meaning of the word "model" can be difficult for all students to grasp. Now that students have worked with both a mathematical model and a physical model, explain that in science, a model is anything that accurately represents what we know of how the natural world behaves. This concept is fundamental to an understanding of how scientists work. Revisit the concept of "model" when students answer the "Why do you believe?" question.
Comprehension	Physics Talk	The shape of a parabola can be difficult to understand. All students may benefit from looking at and holding a parabola shape. You may wish to cut a few conic sections out of a cone-shaped paper party hat and pass them around for students to examine. Point out that the parabolas are the shapes that are "open," whereas the circles and ellipses are closed shapes.
Vocabulary comprehension	Active Physics Plus	ELL students and some other students may have difficulty inferring the meaning of "displacement." Point to the root words "place" and "displace," which give a clue to its meaning. Some students may be familiar with the concept of an object displacing water when it is submerged. Tell students that displacement means how far an object has moved from one place to another. Contrast position and displacement. Position is a scalar, and with this term there is no implicit history of how an object arrived at its position. Displacement, on the other hand, is a vector, and its definition implies specifying an initial position and a final position.
Vocabulary comprehension Comprehension	<i>Active Physics Plus</i> Sample Problem	ELL students may struggle with the word "irrespective" in <i>Step 1</i> . Allow them adequate time to think through the possible meaning. Then provide additional guidance if necessary. You may suggest they think back to the meaning of "independent" in <i>Section 4</i> .
		Collaborate with the students' math teachers to determine what level of comprehension students have obtained for working with square roots.

SECTION 5

Teaching Suggestions and Sample Answers

What Do You See?

The What Do You See? illustration should give you ample opportunity to hook students' attention to a sport in which guessing the trajectory of a projectile becomes significant to winning or losing. You might want to ask students how familiar they are with soccer and how the illustration makes a connection with the topic. Soccer is a popular game and most students will respond instantly to the illustration. Ask them to use their knowledge of soccer in interpreting the images. Specific questions based on how soccer players are successful in striking goals will guide students towards more meaningful responses.

Chapter 2 Physics in Action Section 5 The Range of Projectiles: The Shot Put What Do You See? Y1 X1 92 0.1X2 Learning Outcomes In this section, you will • Measure the acceleration What Do You Think? due to gravity. A world record in the men's shot put of 23.12 m was set • Calculate the speed attained by an object that has fallen freely from rest. by Randy Barnes of the United States in 1990. In the women's javelin throw, Osleidys Menendez of Cuba broke the world record at 71.70 m in 2005. Identify the relationship between the average speed of an object that has fallen freely · Describe the trajectories of projectiles launched from the ground at various angles. from rest and the final speed attained by the object. · Describe how a greater launch speed of a projectile might Calculate the distance traveled change the range when the launch angle is the same. by an object that has fallen freely from rest. Record your ideas in your Active Physics log. Be prepared to discuss your responses with your small group and the class. • Use mathematical m of free fall and uniform speed to construct a physical model of the trajectory of a projectile Investigate In this *Investigate*, you will measure the acceleration due to gravity. You will then use a mathematical model to construct a • Use the motion of a real projectile to test a physical model of projectile motion physical model of the trajectory of a projectile. Finally, you will use a real projectile to test the physical model. • Use a physical model of projectile motion to infer the effects of launch speed 1. Your teacher will provide you with a method of measuring the acceleration caused by Earth's gravity for objects in a condition and launch angle on the range of free fall. of a projectile

Students' Prior Conceptions

Students need to use mathematical models of free fall and of uniform speed to construct a physical model of the trajectory of a projectile. They then need to analyze the motion of a real projectile to test this physical model. Measuring and analyzing real data encourages students to align prior conceptions to accepted theory and to understand how the constant force of gravity and the launch angle act to determine the range of the projectile.

- 1. Students do not understand that the force of gravity is constant as it acts close to the surface of Earth. This force is proportional to the constant acceleration of a projectile, not to its velocity. The speed and vertical velocity of a projectile change during the flight while the vertical acceleration remains constant. The speed of a projectile is constant in the horizontal direction and the horizontal acceleration is zero in the absence of air resistance.
- 2. Students confound constant forces with constant motion. This preconception may continue from *Sections 1* and 2. The same is true for the following preconception.
- 3. As soon as the propelling force is removed from a projectile, it slows down or stops.
- 4. Gravity simultaneously affects both perpendicular components of projectile motion. Forces that act perpendicular to projectiles affect trajectories in different ways. Students do not intuitively understand this concept. It is important for students to measure the acceleration due to gravity and to calculate speeds at various points in the trajectory of a projectile in order to confirm that a projectile exhibits constant motion in the horizontal direction of its flight and accelerated motion in the vertical direction.

You might want to ask students what specific intent the artist could have in making the ball bounce over another player's head and why the range of the soccer ball is shown by dotted lines. Check if students know what a projectile is and whether they are familiar with the shot put. Encourage students to discuss their ideas without hesitating to think if they could be trivial or irrelevant. Discuss their answers and streamline their curiosity toward the What Do You Think? questions. Students should know that they will have a chance to build on their initial impressions, once they have investigated the

concepts presented in this section in more detail.

What Do You Think?

These questions are designed to bring forth a variety of responses. It is important for students to know that you are not looking for correct answers, but only for a discussion of ideas. This is the time when you can find out what your students know about concepts related to this section. Most students will have a limited knowledge of terms like "free fall" and "launch angle." Ask them to record their answers in their *Active Physics* logs.

What Do You Think?

A Physicist's Response

Generally, for a fixed launch velocity (speed and angle), the range will be greater for a higher launch point (as when launching from a tower) and less for a lower launch point (as when launching toward a rising hill). The optimum range across level ground is attained for a 45°-launch angle. The range of a projectile across level ground is governed by the equation: $R = (v^2 \sin 2\theta)/q$ where R is the range, v is the initial speed, θ is the launch angle measured from the horizontal, and g is the acceleration due to gravity. (It is not suggested that this equation be presented to students with the intent of mastery, if at all.)

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Investigate

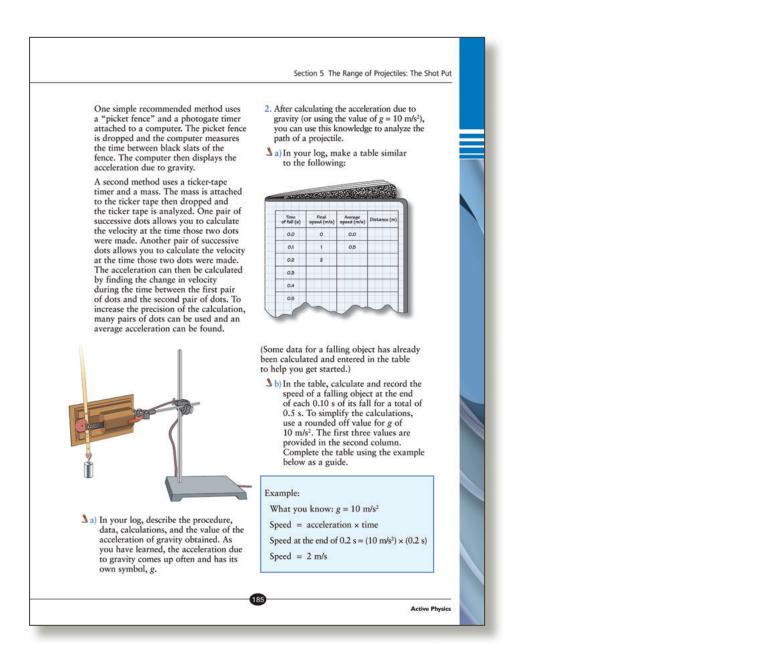
1.a)

Students may find the procedure difficult if they are not familiar with the ticker-tape timer. The students might initially suggest the activity could be done in a manner similar to that of *Section 2*, where the tape is cut into six-dot strips. Allow the students to drop a mass pulling the tape in order to discover that the time it takes the mass to fall will not provide sufficient dots for an extensive evaluation when the tape is cut into strips this size. Ask the students what size is appropriate, and they may suggest three- or two-dot strips. If the mass falls to the floor from a height of 1 m, there should be approximately 30 dots, or 10 three-dot strips, which should prove sufficient. Remind the students that a three-dot strip would represent ¹/₂ the time of a six-dot strip. From the above, the students should be able to obtain the data and do the calculations for finding acceleration due to gravity. Ask the students why the measurement is not done with a meter stick and stopwatch.

Teaching Tip

For the first part of this activity, two methods to measure the acceleration of gravity are detailed. In addition, the use of a motion detector and computer to make a graph of velocity vs. time for a falling object works well. The slope of the line then represents the acceleration. If no equipment, such as a motion detector or those mentioned in the activity, is available, the students may use a stopwatch to time the fall of something (e.g., a softball) over a distance of two meters several times and to obtain the average time. The students can then use the same procedure as discussed in Step 2 to calculate the final velocity of the ball when it strikes the ground. To calculate the ball's acceleration, the students can use the formula for acceleration:

 $a = \frac{v_{\rm f} - v_{\rm i}}{t}$



2.a)-b)

The completed table of calculated values of speed, average speed, and distance at 0.10 s intervals for an object falling from rest is shown to the right. (See chart. For clarity, significant digits have not been used.) Since g is limited to one significant figure, 10 m/s^2 , the tabled values calculated using g also should be limited to one significant figure. Strict adherence would suggest that the

fall distance of 1.25 m should be rounded to 1 m, but it perhaps is not advisable to distract students with that detail at this time – use your judgment on whether or not to bring this up.

Time of Fall (s)	Speed at End of Fall (m/s)	Average Speed (m/s)	Distance (m)
0.0	0	0.0	0.00
0.1	1	0.5	0.05
0.2	2	1.0	0.20
0.3	3	1.5	0.45
0.4	4	2.0	0.80
0.5	5	2.5	1.25

2.c)

See data table on previous page for values. Ask the students if the velocity increases in regular increments.

2.d)

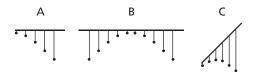
See data table on the previous page for values. Ask the students if the distance increases in regular increments.

3.-4.

Students will assemble the stringand-mass assemblies assigned by the teacher from the data table. The students should make two identical copies of their assigned lengths of fall, one to be used in *Step 6* and one for *Step 8*.

5.

From the amount of space you have for the hanging weights, determine the horizontal distance between pins and tell this distance to the students. Here is a schematic drawing of what the string-and-mass assembly should look like in the three cases.



<u>5.a)</u>

If the horizontal distance between pins is 40 cm, the horizontal speed is 400 cm/s or 4 m/s. Ask: What is the force on this object you are modeling traveling horizontally with constant speed?

6)

Students mark end of string assembly on chalkboard.



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- S c) When speeds are changing at a constant rate, then the average speed during a time interval is the average of the speeds at the beginning and the end of the time interval. Calculate and record the average speed for each time interval in the table. The falling object's speed has increased uniformly from zero to the final speed. In each time interval, the average speed will be the average of zero and the final speed reached at the end of each 0.10 s of falling. This average speed will come out to one half of the final speed.
- Example: Average speed =

 $\frac{\text{zero + speed at the end of time interval}}{2}$ Average speed during 0.2 s of fall = $\frac{(0 \text{ m/s} + 2 \text{ m/s})}{2}$ = 1 m/s
Complete the third column of the chart.

d) Calculate and record the distance the object has fallen at the end of each 0.10 s of its fall. To do this, use the familiar equation: Distance = average speed × time.

Example:

The average speed during 0.2 s of falling is 1 m/s. Distance = average speed × time = $(1 \text{ m/s}) \times (0.2 \text{ s})$

= 0.2 m

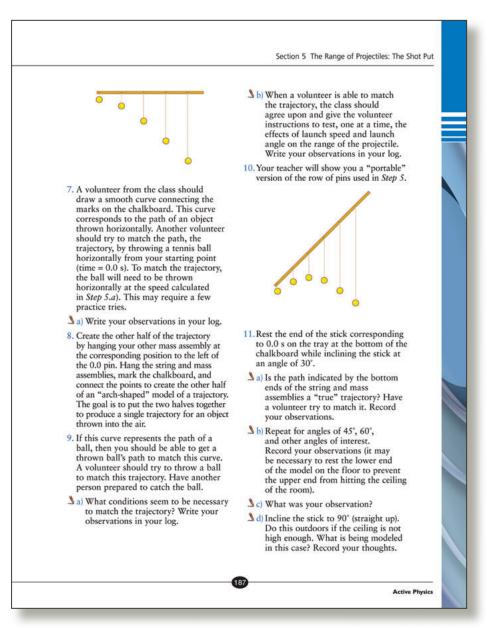
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3. The table you have completed is a mathematical model of an object falling freely from rest. Now you will change the mathematical model into a physical model. Your teacher will assign your group a particular row in the data table providing information about the falling object.

Assemble two identical string and mass assemblies, as shown in the diagram, with an assembly length equal to the distance of fall assigned to your group.

- 4. Label the mass showing your group's name and the time of fall.
- 5. Your teacher will place a horizontal row of pins or tape labeled 0.0 s, 0.1 s, 0.2 s, and so on, along the top edge of a chalkboard in your classroom. The times noted on the labels correspond to the instants for which you calculated distances of fall in the table. The horizontal spacing of the pins is a model of the positions an object would have every 0.10 s if it traveled along the horizontal row of pins at a constant speed.
- (1) Calculate the horizontal speed by dividing the distance traveled during each 0.1-s time interval by 0.1 s. (Dividing a number by 0.1 is equivalent to multiplying the number by 10.) Show your calculation and the result in your log.
- 6. Hang one of your string and mass assemblies from the pin corresponding to the time assigned to your group. Place a small mark on the chalkboard at the bottom end of the string and mass assembly.

Active Physics



7.a)

Students record their observations about a volunteer drawing the curve on the chalkboard and then throwing the ball. The curve on the board and the trajectory of the ball should be parabolas.

8.

The students will assemble the mirror half of the trajectory model to look like diagram B on the previous page.

9.a)

Students throw the ball upward with the angle and speed necessary to duplicate the trajectory in *Step 6*. (Note: This will not be the same speed as in *Step 6*).

9.b)

Students record their observations for a number of different angles and speeds, as suggested by the students.

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10.

Teacher activity. You will display portable model of stick and string model of the "Trajectory of a Projectile" with the addition of the string added for 0.6 s that is 1.8 m long.

Teaching Tip

To make the trajectory seem more life like, you can have the students attach tennis or golf balls on the ends of the string at the correct lengths, making what appears to be a time-lapse photograph of the ball's motion as it covers the trajectory.

<u>11.a)</u>

Yes. All are "true trajectories."

11.b)

Students record the height and range of the trajectories for different angles.

11.c)

45°

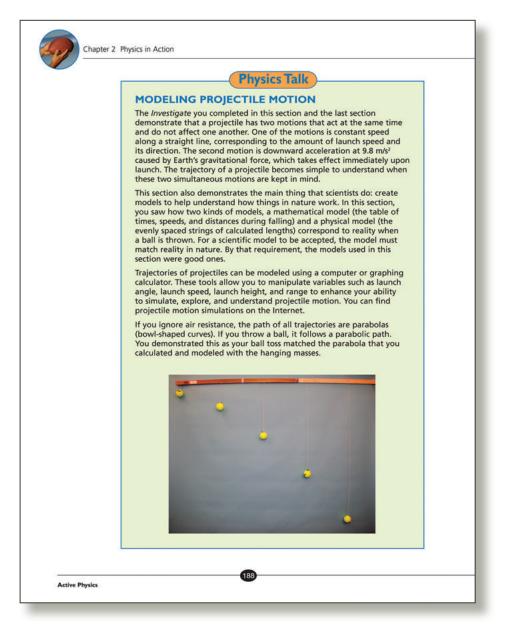
<u>11.d)</u>

A ball is thrown straight up. Students should be encouraged to realize that the model came from two motions, horizontal motion with a constant velocity and vertical motion due to the force of gravity. That this model agrees with actual projectile motion means that it can be used to calculate what happens during projectile motion.

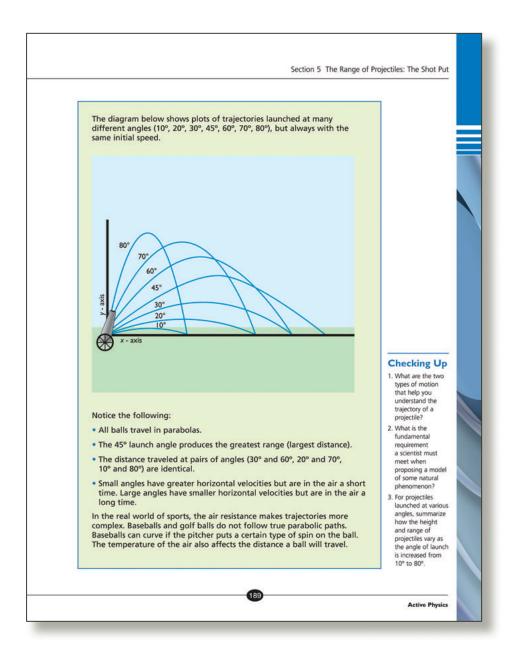
Physics Talk

It is important to emphasize to students that the path of a projectile has a horizontal as well as a vertical component, both independent of each other. Have them sketch the position of a projectile at regular time intervals to identify these two components of projectile motion. Ask them to label the motion that would be constant speed and the motion that would be downward acceleration.

Students should have a clear grasp of why models are helpful in explaining natural phenomena. You might want to take specific examples from the *Investigate* and ask them how models help in demonstrating the motion of a projectile. Encourage them to use the Internet as a resource for understanding projectile motion. Internet simulations allow students to manipulate variables such as the speed, angle, and height of a launch. Ask them to simulate the path of a projectile at different launch angles by using a computer or graphing calculator. Draw their attention to the diagram in their Active Physics textbook to visually reinforce the trajectory of a projectile launched at different angles. Students should summarize their investigations by noting that the symmetry of the projectile's path around a 45°-launch angle means that the projectiles launched at supplementary angles will have the same horizontal range. Supplementary angles result in different times of flight and vertical height of the projectiles at the peak of the trajectory.



The *Physics Talk* ends with the caveat that our model, though useful, still falls short of making accurate predictions in a world of air resistance.



Checking Up

1.

The two types of motion that help you understand the trajectory of a projectile are vertical and horizontal motion.

2.

The horizontal speed does not change during the projectile's flight because the vertical descent due to the force of gravity hasn't started.

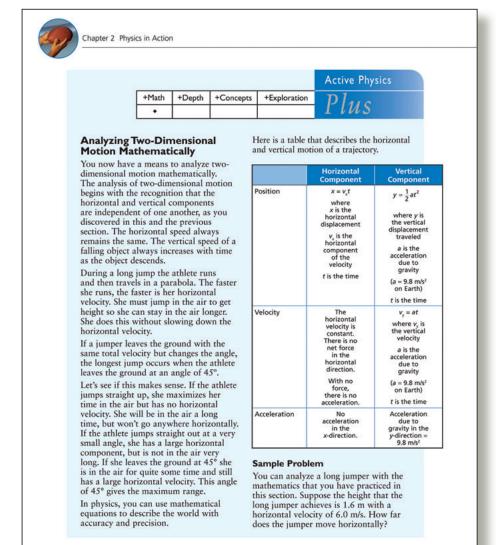
3.

A projectile launched at an angle that comes back to the same level has its greatest range when launched at 45°.

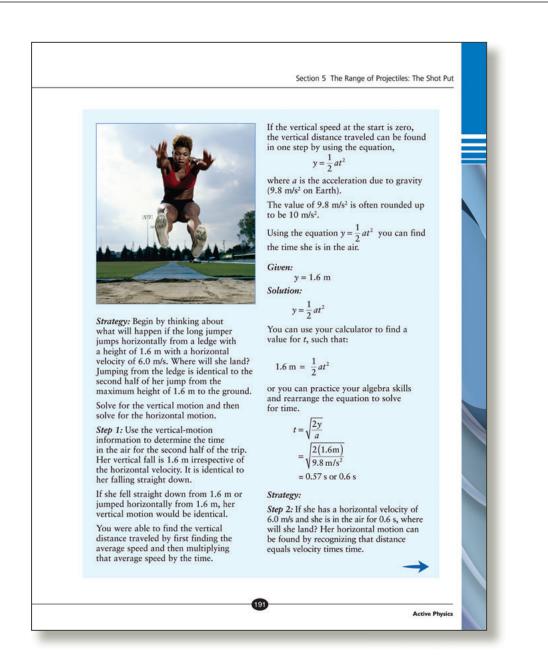


Active Physics Plus

Students study sample problems that analyze the horizontal and vertical motion of a long jumper and then calculate the horizontal distance covered by the athlete. They also solve problems to see how far a long jumper and a ball travel horizontally after achieving the maximum height at a certain horizontal velocity.



Active Physics



CHAPTER 2

1.

The long jumper is in the air for 1.18 s.

$$t = \sqrt{\frac{2(1.7 \text{ m})}{9.8 \text{ m/s}^2}} = 0.59 \text{ s, or}$$

 $2t = 1.18 \text{ s}$

The distance the long jumper achieves is the distance she travels in 1.18 s, which is d = (7.0 m/s)(1.18 s) = 8.3 m.

2.

To calculate the range of the ball, you must know both the vertical and horizontal velocities. The horizontal velocity is given as 45 m/s. To determine the vertical velocity from the height, use

$$v_{\rm f}^2 = v_{\rm i}^2 + 2ad.$$

knowing that the final vertical velocity at the peak is zero gives

$$0^{2} = v_{i}^{2} + 2(-9.8 \text{ m/s}^{2})(1.5 \text{ m})$$

 $v_{i} = 5.4 \text{ m/s}.$

With an initial vertical speed of 5.4 m/s, the ball will take 5.4/9.8 s to reach the peak, and an equal amount of time to return to the ground, giving t = 1.1 s.

The horizontal distance traveled will be the horizontal velocity multiplied by the time of flight or

 $d = v_r t = 49.8 \text{ m}.$

What Do You Think Now?

This is a good time to return to the *What Do You Think?* questions and have a student read them aloud. You will find your students more informed as



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Chapter 2 Physics in Action
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Solution:
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x = v_x t
= (6.0 m/s)(0.6 s)
= 3.6 m
The jumper moves horizontally 3.6 m
```

on the way down for the second half of the trip. Strateev:

Step 3: If a long jumper achieves a height of 1.6 m and has a horizontal velocity of 6.0 m/s for the second half of the trip, then her horizontal distance is twice the value of the distance for the entire trip, since she moves horizontally on the way up as well. (Remember modeling the other part of the motion in the *Investigate.*)

Solution:
$$x_{\text{total}} = 2(3.6 \text{ m})$$

1. Calculate how far horizontally a long jumper travels if she achieves a height of 1.7 m and a horizontal velocity of 7.0 m/s.

You can solve lots of problems by analyzing half of the motion like this. You can calculate the path of a football or baseball or golf ball. The calculations will not apply to real-life situations as well as you might expect because of the effects of air resistance.



The path of a golf ball should be a parabola. Air resistance changes the shape. A baseball should also travel in a parabola, but when the pitcher puts a certain type of spin on it, the air resistance allows it to curve and therefore change the calculated path of our model.

2. Calculate how far a ball will travel horizontally if the ball reaches a high of 1.5 m above the ground and is thrown at a horizontal velocity of 45.0 m/s. Assume that the ball is caught at the same height it is thrown and that there is no air resistance.

What Do You Think Now?

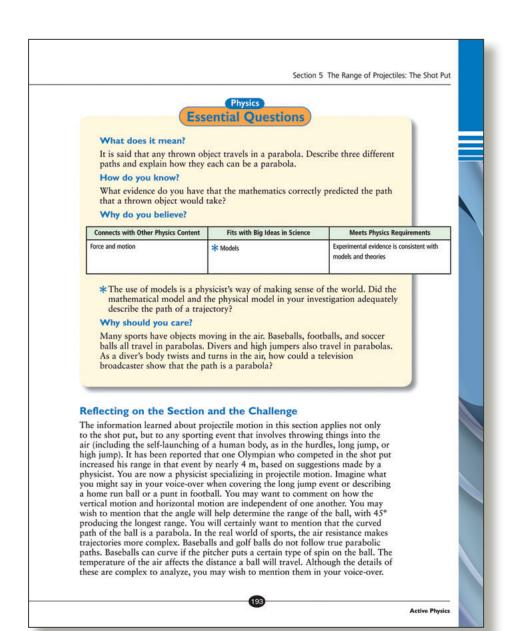
Active Physics

At the beginning of this section, you were asked the following:

- Describe the trajectories of projectiles launched from the ground at various angles.
- Describe how a greater launch speed of a projectile might change the range when the launch angle is the same.

You can use evidence from the mathematical model and the physical model of this section to describe the path of the object and to describe how the angle of the trajectory determines the distance the object travels.

they begin to give their responses. Ask them to refer to the answers they recorded previously in their log books. You might want to emphasize how their grasp of projectile motion increased with their hands-on investigations, and subsequent connections they made in the *Physics Talk*. Expect them to use new terms and concepts introduced in this section with ease. A good way to improve the comfort level in using new terms is to use those terms frequently, yourself, in discussions, so that the unfamiliar is put into different contexts and becomes familiar. A term such as *projectile* may require repeated usage.



Reflecting on the Section and the Challenge

Giving students the time to reflect and ponder is essential to their application of new knowledge. Draw their attention to how an athlete would find the physics behind projectile motion useful, and invite them to discuss how they too might use their present understanding in sports events. This is an opportunity for them to incorporate new information into their Chapter Challenge, and make a rough draft of what might be said in a voice-over when describing a sport that involves projectile motion. Also have students reflect on how air resistance might change the course of a trajectory or spinner might put a spin on the ball to change the distance the ball travels, as mentioned in the Active *Physics* textbook.

CHAPTER 2

Physics Essential Questions

What does it mean?

A line drive in baseball; a pop fly in baseball; an outfield hit in baseball. All are parabolas but they all reach different heights.

How do you know?

We showed mathematically that the path would be a parabola. We then tried to toss objects in any shape other than a parabola, but were unsuccessful.

Why do you believe?

The mathematical model predicted a parabola and our investigation of the path showed it to be a parabola.

Why should you care?

A television broadcaster could show how the path of the center of the diver is still a parabola.

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Physics to Go

The greatest range is provided by an angle of 45°. This angle provides a lot of time in the air coupled with a lot of horizontal speed, but it does not provide a maximum of either one of these variables.

2.a)

More time

2.b)

Less time

3.a)

The complement of the angle: 60°

3.b)

75°

4.

The horizontal running speed of a long jumper is much greater than the initial vertical speed that the jumper can attain; therefore, the angle is far less than 45°.

5.

Carl Lewis can run fast, which is half of the requirement for a good long jumper. Apparently, he also jumps well vertically.

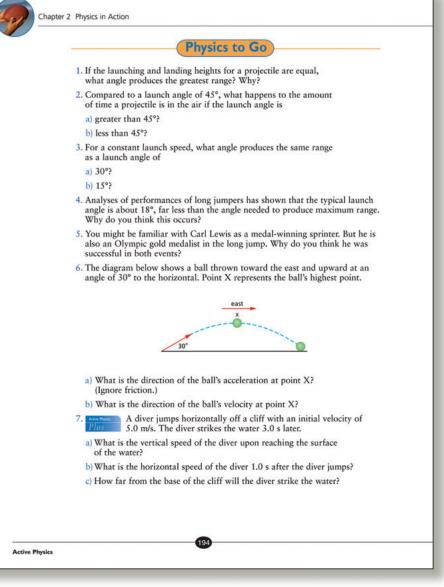
6.a)

The acceleration at point X is the acceleration due to gravity. Its direction is vertically down.

6.b)

The velocity at point X is horizontal. At the highest point, the ball is neither moving up nor moving down.





7.a)

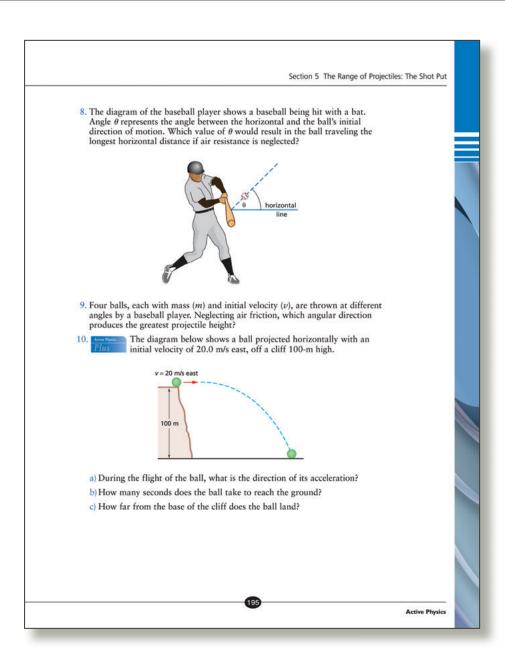
The vertical motion of the horizontally diving person will be identical to the vertical motion of a dropped ball. $v_{y} = gt = (9.8 \text{ m/s}^{2})(3.0 \text{ s}) =$ 29.4 m/s (or 30 m/s if using $g = 10 \text{ m/s}^2$).

7.b)

It remains 5.0 m/s. The horizontal speed does not change because there is no force in the horizontal direction.

7.c)

 $x = v_{x} t = (5.0 \text{ m/s})(3 \text{ s}) = 15 \text{ m}$



8.

The ball thrown the closest to 45° will travel the farthest. The ones thrown at 35° and 55° will travel identical distances. The 55° ball has less horizontal velocity but is in the air longer than the 35° ball. The ball thrown with the highest angle reaches the greatest height. If it could be thrown 90° (straight up) it would reach the greatest height.

9.

The angle for the longest horizontal distance is 45° when the object leaves the ground and when it returns to the ground. Because the baseball is 1 m above the ground, the optimum angle will be a very small amount less than 45° , as the ball already has some vertical displacement.

10.a)

The acceleration is the acceleration due to gravity. Its direction is vertically down.

10.b)

The ball will take the same amount of time to reach the ground as an object dropped from 100 m.

$$d = \frac{1}{2}gt^2$$

$$100 \text{ m} = \frac{1}{2} (10 \text{ m/s}^2) t^2$$
$$t = \sqrt{\frac{200 \text{ m}}{10 \text{ m/s}^2}} = 4.5 \text{ s}$$

10.c)

90 m

SECTION 5 QUIZ



For all the questions in this quiz, air resistance is considered to be zero.

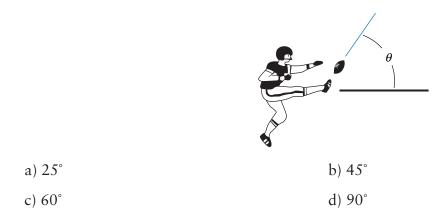
- 1. A 4.0 kg rock and a 1.0 kg stone fall freely from rest from a height of 100 m. After they fall for 2.0 s, the ratio of the rock's speed to the stone's speed is
 - a) 1:1. b) 2:1.
 - c) 1:2. d) 4:1.
- 2. A stone with an initial velocity of zero is dropped from a bridge above a river. After 3 s, the stone strikes the water below the bridge. How fast is the stone traveling when it strikes the water?

a) 10 m/s	b) 20 m/s
c) 30 m/s	d) 45 m/s

3. How far will the stone have fallen in these 3 s?

a) 10 m	b) 20 m
c) 30 m	d) 45 m

4. The diagram below shows a football being kicked. Angle θ represents the angle between the horizontal and the ball's initial direction of motion. Which value of θ would result in the ball traveling the longest distance?



- 5. As the ball is hit harder at the same angle, its acceleration during its flight after leaving the club will
 - a) decrease.
 - b) increase.
 - c) remain the same.

SECTION 5 QUIZ ANSWERS

- a) All objects fall at the same rate regardless of their mass if there is no air resistance.
- 2 c) Objects that are falling freely under the influence of gravity have an acceleration of 10 m/s². After 3 s of fall the object's speed will have increased to 30 m/s.
- 3 d) After 3 s of fall, when the rock started from rest, its average speed would be 15 m/s. The distance traveled in 3 s with an average speed of 15 m/s is 45 m.
- b) The angle that would provide the greatest range is 45° when the object returns to the same height.
- 5 c) The acceleration of any object in free fall near Earth's surface is 10 m/s², and does not depend upon the object's velocity.

NOTES