SECTION 6

Conservation of Momentum

Section Overview

Students begin by considering how traffic investigators use physics to analyze accidents. They investigate completely inelastic collisions in which the two colliding carts stick together after the collision, and how the factors of initial speed and mass affect inelastic collisions. Using their measurements, students calculate the momentum before and after the collision, and observe that momentum is conserved. They then describe and analyze both elastic and inelastic collisions by applying the law of conservation of momentum. A class discussion on the law of conservation of momentum and the different types of collisions follows. Connections are made between Newton's third law and the conservation of momentum. Students then discuss the physics concepts that traffic investigators use to analyze accidents and how they pertain to the *Chapter Challenge*. Students conclude by applying what they have learned to analyze collision situations.

Background Information

The law of conservation of momentum is fundamental in physics. It states that the momentum of a system with no external forces acting on it before a collision is equal to the momentum of the system after the collision. This can be derived from Newton's laws, and is shown in the student text in the *Active Physics Plus*, for systems composed of objects with constant nonzero mass. More information is provided about momentum in the previous section's *Background Information*.

Crucial Physics

- Momentum is a conserved vector quantity. If no external forces are acting on a system, then the momentum of a system before a collision is equal to the momentum of the system after the collision.

Learning Outcomes	Location in the Section	Evidence of Understanding
Understand and apply the law of conservation of momentum to collisions.	***Investigate*** Steps 3 and 4 ***Physics Talk*** ***What Do You Think Now?*** ***Physics Essential Questions*** ***Reflecting on the Section and the Challenge*** ***Physics to Go*** Questions 1-17	Students conclude from their measurements and analysis that momentum is conserved. Students describe and apply the law of conservation of momentum.
Measure the momentum before and after a moving mass strikes a stationary mass in a head-on collision.	***Investigate*** Steps 1-3 ***Physics Essential Questions***	Students measure the masses and speeds of two objects before and after a collision. Using their measurements, students calculate the momentum for each object. Through analysis, they conclude that momentum is conserved. Students describe how their measurements could be improved.

Section 6 Materials, Preparation, and Safety

Materials and Equipment

PLAN A		
Materials and Equipment	**Group (4 students)**	**Class**
Dynamics cart	2 per group	
Clay, modeling, lb	2 per group	
Ruler, metric, in/cm	1 per group	
Weight, slotted, 100 g	6 per group	
Stopwatch	1 per group	
Scale, electronic, 0-1500 g, 0.01 g readability		1 per class
Tape, masking		6 per group
Access to a smooth level surface*	1 per group	
Photogate probeware*		1 per class
Motion detection probeware*		1 per class
MBL or CBL technology to record probeware activity*		1 per class

*Additional items needed not supplied

PLAN B		
Materials and Equipment	**Group (4 students)**	**Class**
Dynamics cart		2 per class
Clay, modeling, lb		2 per class
Ruler, metric, in/cm		1 per class
Weight, slotted, 100 g		6 per class
Stopwatch		1 per class
AC Ticker tape timer		1 per class
Scale, electronic, 0-1500 g, 0.01 g readability		1 per class
Tape, masking		6 per group
Access to a smooth level surface*	1 per group	
Photogate probeware*		1 per class
Motion detection probeware*		1 per class
MBL or CBL technology to record probeware activity*		1 per class

*Additional items needed not supplied

Note: Time, Preparation, and Safety requirements are based on Plan A, if using Plan B, please adjust accordingly.

Time Requirement

Allow two class periods for the students to investigate the collisions.

Teacher Preparation

- Decide where students should conduct the investigation. A long level area clear of obstructions is needed for each group. To avoid carts falling, consider using the floor.
- Conduct the investigation before class to determine where your students may have difficulties. Decide if you wish to stop students at any step to have a class discussion.
- Masses of 1 and 2 kg are recommended, but certainly not required. Instead, one laboratory cart could collide with another identical laboratory cart to provide a 1:1 mass ratio, and a 2:1 (and 1:2) ratio could be obtained by loading one cart on top of another to double its mass, or any other variation.
- The colliding objects need to stick together upon colliding for the "sticky collision" to move as a single object after the collision. Clay magnets or fabric hook-and-loop fasteners are very convenient for this purpose. Placing lumps of clay on the colliding surfaces will not make the carts stick together completely, but they will almost do so.
- For an inelastic collision, the spring side of a dynamics cart should not be used.
- Tape any added masses to the dynamics carts in place to ensure they stay in the cart.
- When colliding two moving carts, caution students to carefully align the carts so an on-center collision will occur.
- Determine how your class will measure the velocity of the carts. Velocity before and after the collisions may be measured in several ways. The least-accurate measurement would be with stopwatches over a known distance due to timing error, and the carts slowing down. Tape timers attached to each cart would be more accurate, but are extremely difficult to coordinate. The best method would be to have a motion detector and associated equipment to measure the velocities

immediately before and after the collision. To help the motion detector lock onto the cart, attach a piece of cardboard about the size of an index card to each cart to improve the reflections. Be sure the area around the collision point is clear of extraneous material to prevent confusion in the motion detector.

Safety Requirements

- Students should wear closed-toed shoes.
- In all collision experiments, goggles are of extra importance.
- If this investigation is done on a lab table, take precautions so that no masses fall off the table onto a student's foot.
- Caution students to keep the cart velocities low to prevent damage to the carts or other problems.

NOTES

CHAPTER 3

Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

Learning Issue	Reference	Augmentation and Accommodations
Copying data tables	***Investigate*** Steps 1.a) and 3.a)	**Augmentation** • Students who have difficulty with fine-motor tasks, paying attention to details, and copying from one place to another will spend a lot of time copying these data tables. Verbal direction and helpful hints could be given to quicken the copying; however, in this case, it may be most effective to provide a copy of the tables for students to tape into their logs. This allows more time for data collection and analysis.
Recalling how to measure velocity	***Investigate*** Step 1.b)	**Augmentation** • If the class has not measured velocity in a week or longer, students who struggle with short- and long-term memory tasks may forget how to measure velocity. Accurate measurements are important in this section to support and reinforce students' understanding of the conservation of momentum. If the data does not show that momentum is conserved, it is very confusing for students. • Review the class's method for measuring velocity. Give students an opportunity to ask clarifying questions about measuring velocity before they begin collecting data.
Applying concepts	***Investigate*** Step 4	**Augmentation** • Organizing the 12 collision sketches on one page will make it easier for students to compare the momentums in different types of collisions. Direct students to make a chart that takes up the whole page. The chart should have three columns and seven rows. The first column would be the "Type of Collision," the second column the "Before" sketch, and the third column the "After" sketch. Provide a visual model of the table when the instructions are given. • Provide students with ideas for representing the objects and their motion. For example, a rectangle may be a sufficient representation of an object, and arrows may be the best way to represent motion. • Some students may not be able to visualize the collisions without seeing real carts crash together. Provide carts for students to collide as they are making their sketches. **Accommodation** • Provide a blank chart for students to record their sketches and tape into their logs. • Demonstrate one collision at a time for the whole class, and ask students to create their sketches after each demonstration.

Strategies for Students with Limited English-Language Proficiency

Point out new vocabulary words in context, and practice using the words as much as possible throughout the section.

analysis	reconstruct
conservation of momentum	sticky collision
crucial	tread marks
depart	tremendous
hallmark	uncertainties
optical illusion	victim

An important skill that students should learn in science is how to organize data into tables. This section presents an opportunity to practice this skill. Students should be able to distinguish between rows and columns (they often confuse the two). They should give appropriate information labels to rows, and headers to columns. Tables should be numbered and have headings. Point out examples of tables in books. Ask students how they use the information contained in the organization of a table to learn about what is presented in the table. Consider giving each group examples of one table that is well organized and one that is not, and then ask students to describe what information is missing from the poorly organized table.

Two important aspects of learning a new language are speaking and writing in that language. Some ELL students will be self-conscious about speaking in front of their peers, while others will be less reluctant to try. Be sure to encourage all ELL students to speak in class, and give them opportunities to write on the board from time to time. Experience will broaden their comfort level. Over time, the shy students will become increasingly less self-conscious about speaking in front of their classmates.

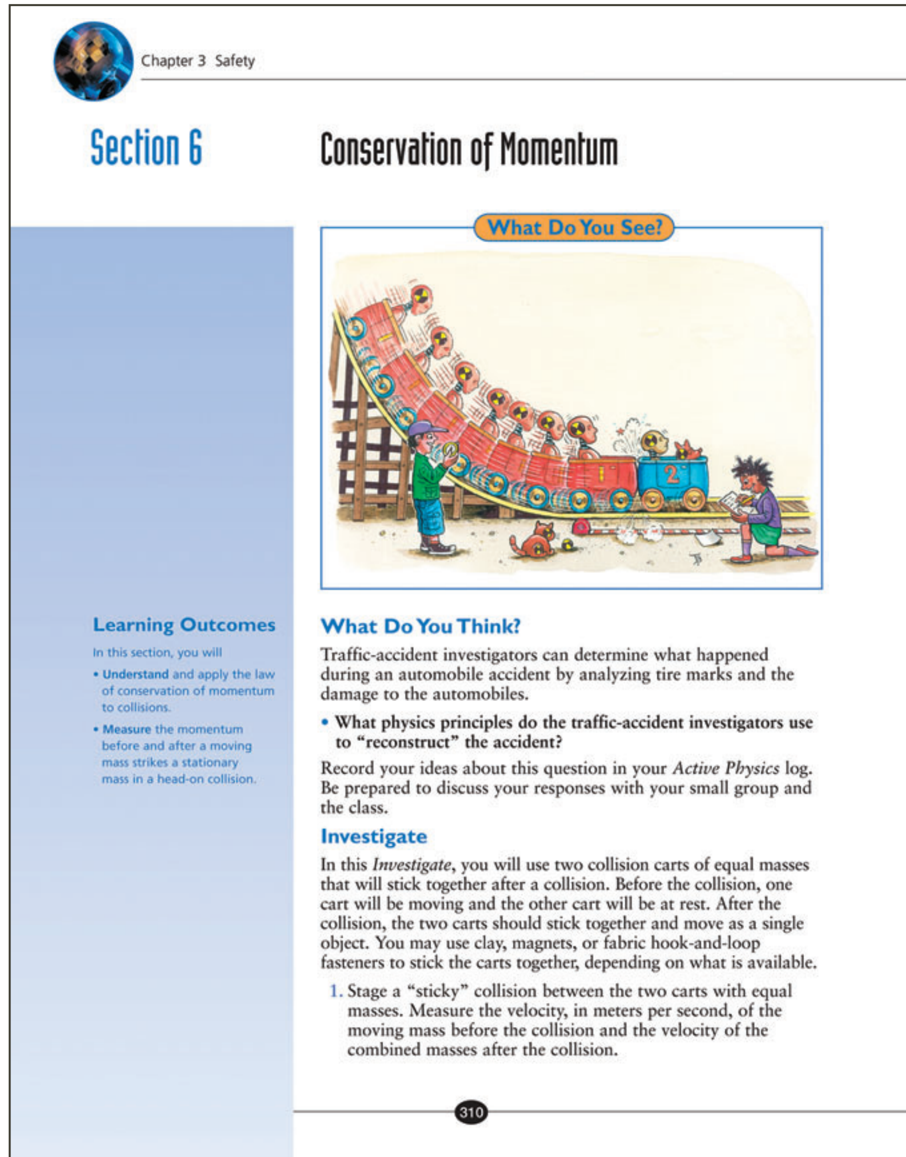
With that in mind, hold a class discussion to review *Section 6*. Call on ELL students to answer or address the bulleted items below.

- In your own words, what is the law of conservation of momentum? (The total momentum before a collision equals the total momentum after a collision, if no net external forces act on the system.)
- Is momentum a scalar quantity or a vector quantity? Explain. (The direction of momentum is important; it is the same direction as velocity.)
- What does the following equation mean, in words? $m_1v_{1\text{initial}} + m_2v_{2\text{initial}} = (m_1 + m_2)v_{\text{final}}$ (For an inelastic collision in which the objects stick together after they collide, the momentum before a collision equals the momentum after a collision.)
- Solve the equation above for v_f using these values: $m_1 = 750$ kg, $v_{1\text{initial}} = 4$ m/s, $m_2 = 250$ kg, $v_{2\text{initial}} = 8$ m/s. [$v_{\text{final}} = 5$ m/s]
- **Critical Thinking:** Do these values substituted in the equation above represent a collision between two moving vehicles or a collision between one moving vehicle and one stationary vehicle? (Two moving vehicles.) How do you know? (Neither starting momentum is zero.)

CHAPTER 3

SECTION 6

Teaching Suggestions and Sample Answers


Chapter 3 Safety

Section 6 Conservation of Momentum

What Do You See?

Learning Outcomes

In this section, you will

- **Understand** and apply the law of conservation of momentum to collisions.
- **Measure** the momentum before and after a moving mass strikes a stationary mass in a head-on collision.

What Do You Think?

Traffic-accident investigators can determine what happened during an automobile accident by analyzing tire marks and the damage to the automobiles.

- **What physics principles do the traffic-accident investigators use to "reconstruct" the accident?**

Record your ideas about this question in your *Active Physics* log. Be prepared to discuss your responses with your small group and the class.

Investigate

In this *Investigate*, you will use two collision carts of equal masses that will stick together after a collision. Before the collision, one cart will be moving and the other cart will be at rest. After the collision, the two carts should stick together and move as a single object. You may use clay, magnets, or fabric hook-and-loop fasteners to stick the carts together, depending on what is available.

1. Stage a "sticky" collision between the two carts with equal masses. Measure the velocity, in meters per second, of the moving mass before the collision and the velocity of the combined masses after the collision.

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What Do You See?

Review the illustration with the students. Consider using the overhead of the illustration as a focal point for the discussion. Elicit initial impressions of what students see in the illustration, focusing on their description of the collision. Ask students what they think is meant by conservation of momentum. Discuss how students think the illustration connects with the conservation of momentum.

What Do You Think?

Ask students to think about what traffic-accident investigators can determine by analyzing tire marks on the road and damage to the vehicles. Record students' ideas and ask students to consider what physics principles these investigators use to "reconstruct" the accident. Have students record their ideas in their log and discuss their answers with their group. Then have a class discussion on students' ideas. Record misconceptions and make sure they are addressed at appropriate times during the section. Focus on the responses that provide an opportunity for you to get the students engaged in the physics concepts presented in this chapter.

Consider emphasizing that there are no "right" answers and that all answers are acceptable. The purpose of these questions is to elicit students' prior knowledge and to get students to think about the physics involved. Ask them to refer to their answers while they are being introduced to new physics concepts. Point out that when they are aware of what they think, they will be better able to add to what they know.

You can measure the velocities with a ruler and stopwatch, with a ticker-tape timer, with a velocimeter or a computer and motion detector.

a) Prepare a data table in your log similar to the one shown below. Provide enough horizontal rows in the table to enter data for at least four collisions.

Sticky Head-on Collisions: One Object Moving before Collision

Mass of Object 1 (kg)	Mass of Object 2 (kg)	Velocity of Object 1 before Collision (m/s)	Velocity of Object 2 before Collision (m/s)	Mass of Combined Objects after Collision (kg)	Velocity of Combined Objects after Collision (m/s)
1.0	1.0		0.0	2.0	
2.0	1.0		0.0	3.0	
1.0	2.0		0.0	3.0	
			0.0		

b) Record the measured values of the velocities in the first row of the data table.

2. Stage other sticky collisions using the masses listed in the second and third rows of the data table. Then stage one or more additional collisions using other masses. Measure the velocities before and after each collision.

a) Enter the measured values in the data table.

3. Organize a table for recording the momentum of each object before and after each of the above collisions.

a) Prepare a table similar to the following example in your log.

Momentum of Object before and after Collisions
Momentum = Mass x Velocity

Before the Collision		After the Collision
Momentum of Object 1 kg (m/s)	Momentum of Object 2 kg (m/s)	Momentum of Combined Objects 1 and 2 kg (m/s)

b) Calculate the momentum of each object before and after each of the above collisions and enter each momentum value in the table.

c) Calculate and compare the total momentum before each collision to the total momentum after each collision.

d) Allowing for minor variations due to uncertainties of measurement, write in your log a general conclusion about how the momentum before a collision compares to the momentum after a collision.

4. There are a variety of collisions involving two objects. In each collision, momentum is conserved and the same equation is used. The equation gets simpler when one of the objects is at rest and has zero momentum.

What Do You Think?

A Physicist's Response

Scientists and traffic-accident analyzers can determine many things from skid marks on the street and damage done to vehicles involved in a collision. This information is related to the direction vehicles were moving in before and after the crash and the speed of the vehicles. The physics principles used to "reconstruct" an accident are many. The most important concept used in collision analysis is the law of conservation of momentum, which states that the sum of momentums of the objects involved in a collision before the collision is equal to the sum of momentums of the objects after the collision. Conservation of energy is also used, as well as Newton's laws of motion.

Students' Prior Conceptions

The accuracy of mass and speed measurements before the moving mass strikes the stationary mass and the subsequent measurements of the two moving masses, after the collision, enables students to examine the concept of conservation of momentum. Calculating the momentum of the moving mass before it strikes a stationary mass and the subsequent momentum of the masses after the collision makes it possible for students to investigate the law of conservation of momentum. In addition to measurements of mass and velocity, students must determine the physical conditions of each mass before and after the collision to ascertain if any deformation has occurred. Knowledge about kinetic energy changing to sound, light, and heat also explains to students how energy is conserved when all parts of a system are considered.

1. **Momentum is a vector.** Students do not recognize the importance of momentum when a collision involves a moving object and a stationary object. Sources of error in measurement, external unbalanced forces, and the accuracy of the model used for investigating collisions are important discussion points. You should review the concept of relative motion, encouraging students to establish the directions for the vectors with respect to a start position. It is useful to pose scaffolding questions to support inquiry-based learning.

Investigate

If you decided to have a discussion to guide the students through some of the steps in the *Investigate*, then let them know now at which step they should stop. Otherwise, have a discussion on the *Investigate* steps and let students begin and complete it.

1.

Have each group set up the materials and practice a completely sticky (inelastic) collision between a moving cart and a stationary cart.

1.a)

Students create a table in their logs to record their data.

1.b)

Students measure and record the incoming and outgoing velocities of the carts and record these values in their data table.

Teaching Tip

If masses are added to the carts to reach the 1 and 2 kg masses suggested, tape the masses to the carts to prevent sliding. Smaller masses may be used as long as the ratios are kept the same as those in the charts.

2.a)

Students repeat *Step 1* using the masses listed, and record them in their data table. Make sure students secure the masses to the carts by taping them to the carts.

3.a)

Students should create another table in their logs for the momentum of the objects before and after the collision.

3.b)

Students should calculate the momentum for each cart before the collision, and the momentum of the combined "stuck-together" carts after the collision. Students realize that momentum is mass multiplied by velocity.

3.c)

Students compare the total momentum of the incoming carts (the sum of the initial momentums of the carts) to the momentum of the combined carts after the collision.

3.d)

Students should conclude that within the uncertainties of the measurements the total momentum before the collision is equal to the total momentum after the collision.

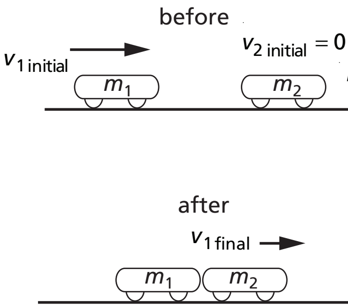
4.a)

Have students draw sketches describing the objects before and after the collisions and record the momenta they know for the six types of collisions listed. For the examples provided, the class can try setting conditions such as equal masses, or one mass being twice the mass of the other. The sign for the direction of the velocity should be included if doing sample calculations. For Collision Type 5, the carts could be moving toward each other from opposite directions, or toward each other while traveling in the same direction. Afterward, they may be moving in the opposite directions if the collision was head-on or in the same directions if the collision was from the rear.

NOTES

3-6a Blackline Master

Collision Type 1



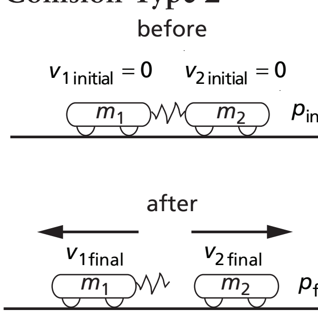
$$p_{initial} = m_1 v_{1\,initial}$$

$$v_{final} = \frac{m_1}{(m_1 + m_2)} v_{1\,initial}$$

$$p_{final} = (m_1 + m_2) v_{final}$$

Have students determine the final velocity if the two masses are equal and the initial velocity of the moving cart is 1 m/s to the right. In this collision, the final velocity of the carts stuck together after the collision is 0.5 m/s to the right.

Collision Type 2



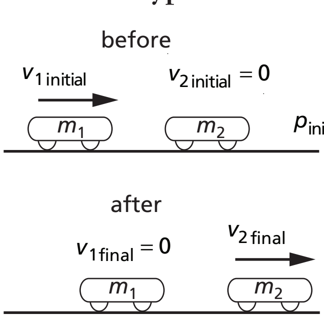
$$p_{initial} = 0$$

$$v_{2\,final} = -\frac{m_1}{m_2} v_{1\,final}$$

$$p_{final} = m_1 v_{1\,final} + m_2 v_{2\,final} = 0$$

If the two masses are equal, and the final velocity of the right side cart is 1 m/s to the right, have students determine the final velocity of the left-side cart. In this explosion, the final velocity of the cart on the left will be 1 m/s to the left.

Collision Type 3



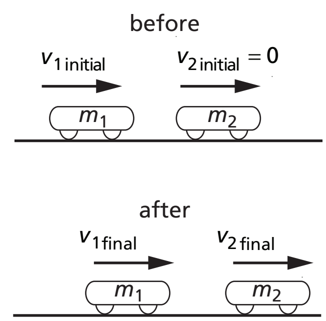
$$p_{initial} = m_1 v_{1\,initial}$$

$$v_{2\,final} = \frac{m_1}{m_2} v_{1\,initial}$$

$$p_{final} = m_2 v_{2\,final}$$

If the two masses are equal, and the initial velocity of the cart on the left is 1 m/s to the right, consider having students determine the final velocity of the cart that was initially stationary. Using the conservation of momentum, the final velocity of the cart on the right will be 1 m/s to the right, and the velocity of the cart on the left will be zero.

Collision Type 4



$$p_{initial} = m_1 v_{1\,initial}$$

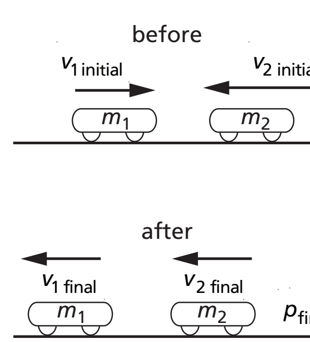
$$v_{2\,final} = \frac{m_1}{m_2}(v_{1\,initial} - v_{1\,final})$$

$$p_{final} = m_1 v_{1\,final} + m_2 v_{2\,final}$$

If the cart on the left has twice the mass of the cart on the right, and the initial velocity of the cart on the left is 1 m/s to the right, its final velocity after the collision is 0.5 m/s to the right. Have students determine the final velocity of the cart that was initially stationary. The final velocity of the cart on the right will be 1 m/s to the right.

Collision Type 5

Emphasize that for this collision, the two objects could be moving toward each other from opposite directions (a head-on collision), or they could be going in the same direction where one cart is faster than the other and hits it from behind.



$$p_{initial} = m_1 v_{1\,initial} + m_2 v_{2\,initial}$$

$$p_{final} = m_1 v_{1\,final} + m_2 v_{2\,final}$$

Consider having students determine the final velocity of the cart on the right for the following conditions. The cart on the left is twice the mass of the cart on the right. The initial velocity of the cart on the left is 0.5 m/s to the right. The initial velocity of the cart on the right is 2 m/s to the left, and its final velocity is 0.5 m/s to the left. In this case, the final velocity of the cart on the right is 0.25 m/s to the left.

CHAPTER 3

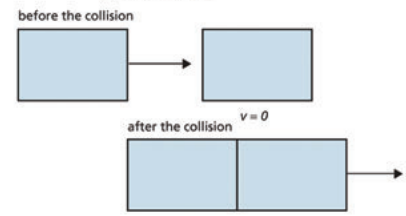
a) Draw two sketches for each collision type described at the right—one showing each object before the collision and one showing each object after the collision. By writing the momenta you know directly on the sketch, the calculations become easier. The sketch for Collision Type 1 is provided.

Collision Type 1: One moving object hits a stationary object and both stick together and move off at the same speed.

Collision Type 2: Two stationary objects explode by the release of a spring between them and move off in opposite directions.

Collision Type 3: One moving object hits a stationary object. The first object stops, and the second object moves off.

Collision Type 1



Collision Type 4: One moving object hits a stationary object, and both move off at different speeds.

Collision Type 5: Two moving objects collide, and both objects move at different speeds after the collision.

Collision Type 6: Two moving objects collide, and both objects stick together and move off at the same speed.

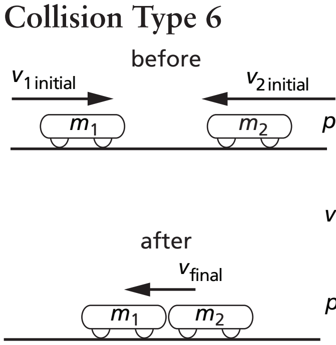
Physics Talk

THE LAW OF CONSERVATION OF MOMENTUM

In this section, you investigated a conservation principle that is a hallmark of physics—the conservation of momentum. You found evidence of the conservation of momentum when a moving cart hit a stationary cart and they stuck together. Momentum is also conserved in all the collisions you illustrated at the end of the investigation. One of these collisions (Type 2) is sometimes called an "explosion." If you add up all of the momenta *before* a collision (or explosion), you know that the sum of all the momenta after the collision will be the same. That is what physicists mean by the word "conserve." The total momentum is conserved because its value remains the same.

In the first collision of the section, a moving cart hit a stationary cart and they stuck together. The same thing happens in a traffic accident. If the momentum before a collision is 5000 kg•m/s, then the momentum after the collision must be 5000 kg•m/s. Imagine a vehicle stops at a red light. The vehicle is not moving and therefore, has a momentum equal to zero. If a second vehicle that has a momentum of 5000 kg•m/s then hits the stopped vehicle, both vehicles move off with (a combined) 5000 kg•m/s of momentum.

Collision Type 6



$$p_{\text{initial}} = m_1 v_{1\,\text{initial}} + m_2 v_{2\,\text{initial}}$$

$$v_{\text{final}} = \frac{m_1 v_{1\,\text{initial}} = m_2 v_{2\,\text{initial}}}{(m_1 + m_2)}$$

$$p_{\text{final}} = (m_1 + m_2)\, v_{\text{final}}$$

If the cart on the left is twice the mass of the cart on the right, the initial velocity of the cart on the left 1 m/s to the right, and the initial velocity of the cart on the right is 4 m/s to the left, have students determine the final velocity of the stuck-together carts after the collision. In this case, the final velocity of the stuck-together carts is 0.67 m/s to the left.

Total momentum stays the same or, in other words, total momentum is conserved.

You can write this as an equation and substitute values for the masses and velocities of the vehicles.

$$\text{Momentum BEFORE collision} = \text{Momentum AFTER collision}$$
$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_f$$
$$(1000 \text{ kg})(5 \text{ m/s}) + (1000 \text{ kg})(0 \text{ m/s}) = (1000 \text{ kg} + 1000 \text{ kg})(2.5 \text{ m/s})$$
$$5000 \text{ kg} \cdot \text{m/s} = 5000 \text{ kg} \cdot \text{m/s}$$

You add the masses "after the collision" because they stuck together and travel with the same speed. Conservation of momentum is an experimental fact. Physicists have compared momentum before and after collisions between pairs of objects ranging from railroad cars slamming together to subatomic particles impacting one another at near the speed of light. Never have any exceptions been found to the statement, "The total momentum before a collision is equal to the total momentum after the collision if no external forces act on the system." This statement is known as the **law of conservation of momentum**. In all collisions between vehicles and trucks, between protons and protons, between planets and meteors, the momentum before the collision equals the momentum after the collision.

A single cue ball hits a rack of 15 billiard balls and they all scatter. It would seem as if everything has changed. Physicists have discovered that in this collision, as in all collisions and explosions, nature does keep at least one thing from changing—the total momentum. The sum of the momentum of all of the billiard balls immediately after the collision is equal to the momentum of the original cue ball. Nature conserves momentum. Irrespective of the changes you can see, the total momentum undergoes no change whatsoever. The objects may move in new directions and with new speeds, but the momentum stays the same.

Physics Words

law of conservation of momentum: the total momentum before a collision is equal to the total momentum after the collision if no external forces act on the system.

Physics Talk

This *Physics Talk* introduces the law of conservation of momentum. Conservation of momentum is a fundamental physics concept. Emphasize the importance of this law and check students' comprehension of it.

Have a class discussion on the law of conservation of momentum using the information in the student text. Go through the examples with students and make connections with students' observations from the *Investigate*.

Describe how the momentum of a system is always conserved if no net external force is acting on the system. This is an extremely important physics concept that has never been negated. Describe how this law holds true for everything that scientists have studied, from stars to the inner makings of the atom. Discuss the special conditions for some situations. For example, for a perfectly elastic collision, kinetic energy is conserved. For all other collisions, the kinetic energy is not conserved. Discuss both sample problems in the *Student Edition* as a class.

Chapter 3 Safety

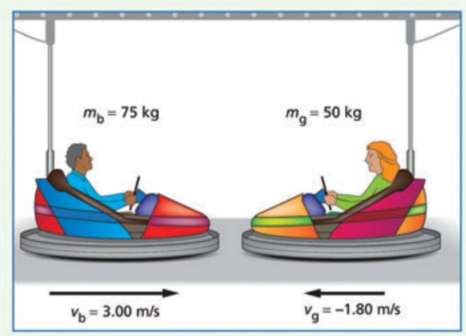
Conservation of momentum is crucial physics for analyzing any collision between objects. Traffic-accident investigators use tread marks, the positions of the automobiles, and the damage to the automobiles to understand what happened at the time of the accident. They also use the physics principle of momentum conservation to help them in their analysis.

If you know the masses and velocities of two objects before a collision, you can accurately predict the velocities after the collision. Physics allows you to predict the future!

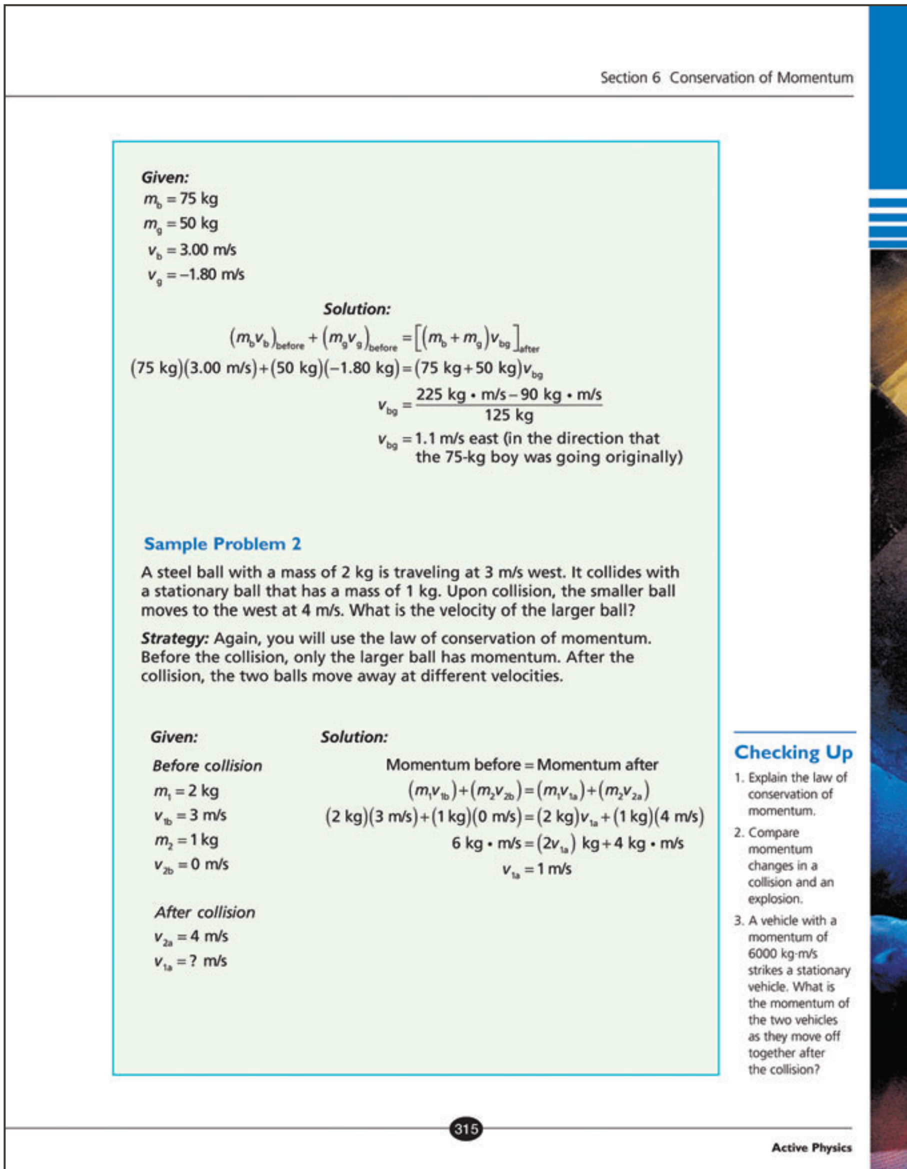
Solving conservation of momentum problems is easy. Using the definition of momentum, $p = mv$, calculate each object's momentum before the collision. Calculate each object's momentum after the collision. The total after the collision must equal the total before the collision.

Sample Problem I

A boy and a girl are riding in bumper cars at an amusement park. A 75-kg boy and car are moving to the east at 3.00 m/s toward a 50-kg girl and car who are moving toward him (west) at 1.80 m/s. If they catch up, stick together, and then move away together, what is their final velocity?



Strategy: This is a problem involving the law of conservation of momentum. The momentum of an isolated system before an interaction is equal to the momentum of the system after the interaction. As you are working through this problem, remember that the v in this expression is velocity and that it has direction as well as magnitude. Make east the positive direction, and then west will be negative.



Section 6 Conservation of Momentum

Given:

$m_b = 75$ kg

$m_g = 50$ kg

$v_b = 3.00$ m/s

$v_g = -1.80$ m/s

Solution:

$$(m_b v_b)_{before} + (m_g v_g)_{before} = [(m_b + m_g)v_{bg}]_{after}$$

$$(75 \text{ kg})(3.00 \text{ m/s}) + (50 \text{ kg})(-1.80 \text{ kg}) = (75 \text{ kg} + 50 \text{ kg})v_{bg}$$

$$v_{bg} = \frac{225 \text{ kg} \cdot \text{m/s} - 90 \text{ kg} \cdot \text{m/s}}{125 \text{ kg}}$$

$v_{bg} = 1.1$ m/s east (in the direction that the 75-kg boy was going originally)

Sample Problem 2

A steel ball with a mass of 2 kg is traveling at 3 m/s west. It collides with a stationary ball that has a mass of 1 kg. Upon collision, the smaller ball moves to the west at 4 m/s. What is the velocity of the larger ball?

Strategy: Again, you will use the law of conservation of momentum. Before the collision, only the larger ball has momentum. After the collision, the two balls move away at different velocities.

Given:

Before collision

$m_1 = 2$ kg

$v_{1b} = 3$ m/s

$m_2 = 1$ kg

$v_{2b} = 0$ m/s

After collision

$v_{2a} = 4$ m/s

$v_{1a} = ?$ m/s

Solution:

Momentum before = Momentum after

$$(m_1 v_{1b}) + (m_2 v_{2b}) = (m_1 v_{1a}) + (m_2 v_{2a})$$

$$(2 \text{ kg})(3 \text{ m/s}) + (1 \text{ kg})(0 \text{ m/s}) = (2 \text{ kg})v_{1a} + (1 \text{ kg})(4 \text{ m/s})$$

$$6 \text{ kg} \cdot \text{m/s} = (2v_{1a}) \text{ kg} + 4 \text{ kg} \cdot \text{m/s}$$

$$v_{1a} = 1 \text{ m/s}$$

Checking Up

1. Explain the law of conservation of momentum.
2. Compare momentum changes in a collision and an explosion.
3. A vehicle with a momentum of 6000 kg·m/s strikes a stationary vehicle. What is the momentum of the two vehicles as they move off together after the collision?

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Active Physics

Checking Up

1.

The law of conservation of momentum states that the total momentum of a system just before a collision is equal to the total momentum of the system just after a collision, provided that there are no net external forces acting on the system.

2.

For both a collision and an explosion, the total momentum before the event is equal to the total momentum after the event. In a collision, the total momentum of both objects just before the collision is equal to the total momentum of both objects just after the collision. Before the collision, one or both objects may be moving, having a nonzero momentum. After the collision, one or both objects may be moving, having a nonzero momentum. During an explosion, the momentum of the object before the explosion is equal to the sum of the momentums of the particles of the object after the explosion. If before the explosion the object is not moving, then the initial momentum of the system is zero. In this case, the sum of momentums of all the particles after the explosion is equal to zero.

3.

Students should apply the law of conservation of momentum to solve the problem.

$p_{\text{initial}} = p_{\text{final}}$. Because the initial momentum of the system is 6000 kg · m/s, the final momentum is also 6000 kg · m/s. Ask students what they think the final speed is compared to the initial speed. Students should realize that the final speed is less than the initial speed because the mass increased.

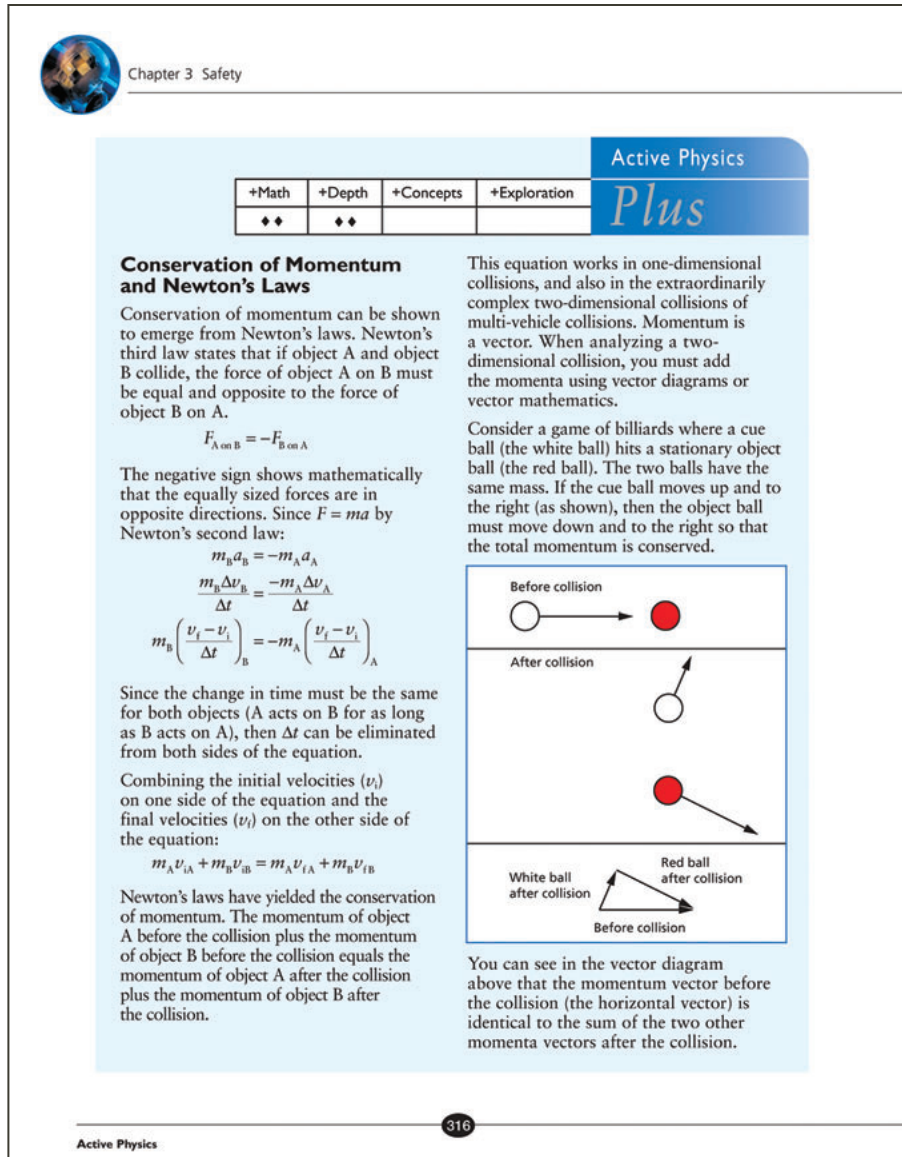
$$p_{\text{initial}} = p_{\text{final}}$$

$$m_1 v_{1\text{initial}} = (m_1 + m_2) v_{\text{final}}$$

CHAPTER 3

Active Physics Plus

This *Active Physics Plus* provides an opportunity for students to increase the depth of their understanding of conservation of momentum and how it ties in with Newton's laws. It also provides a more mathematical approach and briefly discusses two-dimensional collisions. When discussing two-dimensional collisions of equal mass, consider drawing velocity vectors for the momentum equation and show students how they add tip-to-tail to get the resultant vector equal to the initial velocity. This also clearly shows the right angle that is formed.



Chapter 3 Safety

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
◆◆	◆◆		

Conservation of Momentum and Newton's Laws

Conservation of momentum can be shown to emerge from Newton's laws. Newton's third law states that if object A and object B collide, the force of object A on B must be equal and opposite to the force of object B on A.

$$F_{\text{A on B}} = -F_{\text{B on A}}$$

The negative sign shows mathematically that the equally sized forces are in opposite directions. Since $F = ma$ by Newton's second law:

$$m_B a_B = -m_A a_A$$

$$\frac{m_B \Delta v_B}{\Delta t} = \frac{-m_A \Delta v_A}{\Delta t}$$

$$m_B \left(\frac{v_f - v_i}{\Delta t}\right)_B = -m_A \left(\frac{v_f - v_i}{\Delta t}\right)_A$$

Since the change in time must be the same for both objects (A acts on B for as long as B acts on A), then Δt can be eliminated from both sides of the equation.

Combining the initial velocities (v_i) on one side of the equation and the final velocities (v_f) on the other side of the equation:

$$m_A v_{iA} + m_B v_{iB} = m_A v_{fA} + m_B v_{fB}$$

Newton's laws have yielded the conservation of momentum. The momentum of object A before the collision plus the momentum of object B before the collision equals the momentum of object A after the collision plus the momentum of object B after the collision.

This equation works in one-dimensional collisions, and also in the extraordinarily complex two-dimensional collisions of multi-vehicle collisions. Momentum is a vector. When analyzing a two-dimensional collision, you must add the momenta using vector diagrams or vector mathematics.

Consider a game of billiards where a cue ball (the white ball) hits a stationary object ball (the red ball). The two balls have the same mass. If the cue ball moves up and to the right (as shown), then the object ball must move down and to the right so that the total momentum is conserved.

You can see in the vector diagram above that the momentum vector before the collision (the horizontal vector) is identical to the sum of the two other momenta vectors after the collision.

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Active Physics

1.

With the assumptions that momentum is conserved and 50% or half of the kinetic energy is conserved, students should calculate the following:

From the law of conservation of momentum

$p_{\text{initial}} = p_{\text{final}}$

$(1000\text{ kg})(10\text{ m/s}) =$

$(1000\text{ kg})v_{\text{car final}} + (70\text{ kg})v_{\text{person final}}$

$v_{\text{car final}} = 10\text{ m/s} - 0.07v_{\text{person final}}$

Using the condition that half of the kinetic energy is conserved

$\frac{1}{2}KE_{\text{initial}} = KE_{\text{final}}$

$\frac{1}{2}(1000\text{ kg})(10\text{ m/s})^2 =$

$\frac{1}{4}(1000\text{ kg})v^2_{\text{car final}} +$

$\frac{1}{4}(70\text{ kg})v^2_{\text{person final}}$

Substituting

$v_{\text{car final}} = 10\text{ m/s} - 0.07v_{\text{person final}}$, found from the law of conservation of momentum you have

$200{,}000\text{ J} = 1000\text{ kg} \times$

$\left(100\,\frac{\text{m}^2}{\text{s}^2} - 1.4\,\frac{\text{m}}{\text{s}}\,v_{\text{person final}} + 0.0049v^2_{\text{person final}}\right) +$

$(70\text{ kg})\,v^2_{\text{person final}}$

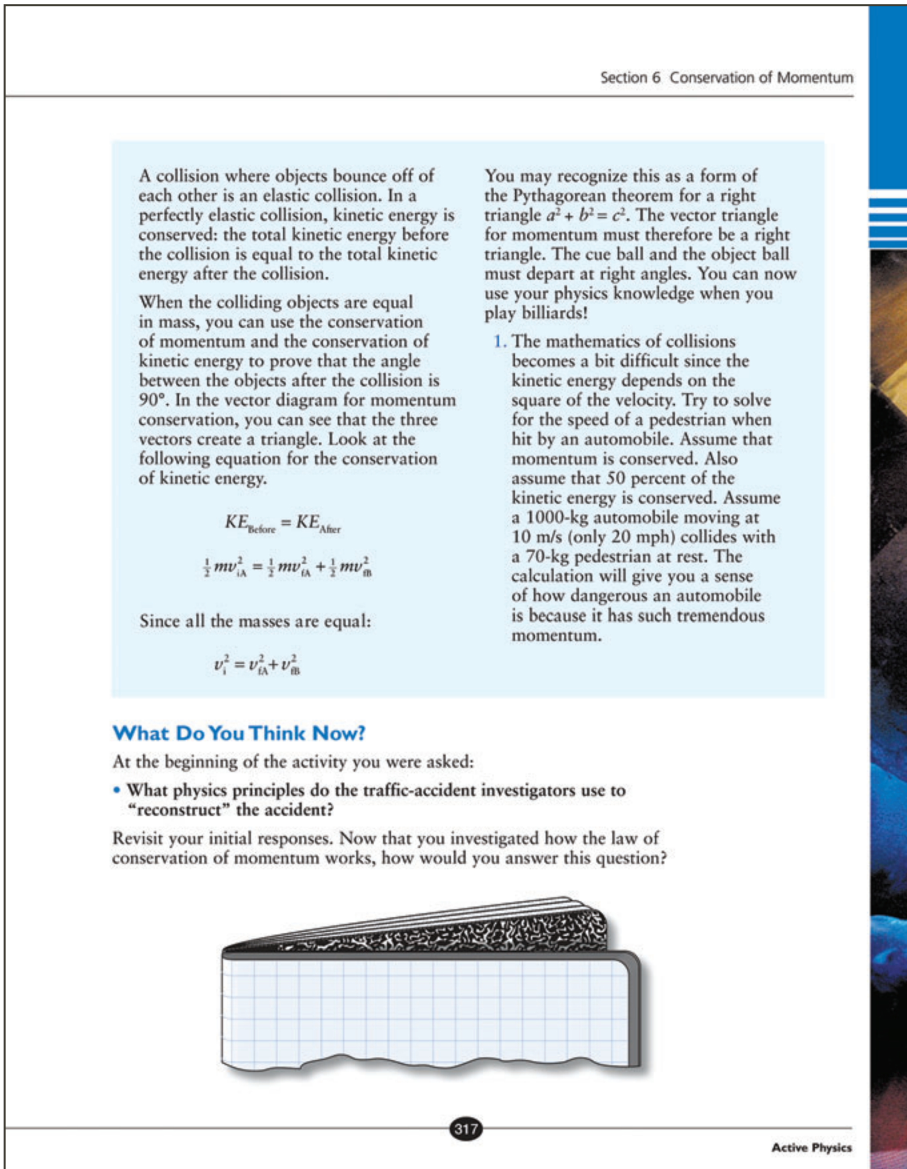
$200{,}000\text{ kg}\,\frac{\text{m}^2}{\text{s}^2} = 100{,}000\text{ kg}\,\frac{\text{m}^2}{\text{s}^2} -$

$1400\text{ kg}\,\frac{\text{m}}{\text{s}}\,v_{\text{person final}} + (4.9\text{ kg})\,v^2_{\text{person final}} +$

$(70\text{ kg})\,v^2_{\text{person final}}$

$(74.9)\,v^2_{\text{person final}} - \left(1400\,\frac{\text{m}}{\text{s}}\right)v_{\text{person final}} -$

$100{,}000\,\frac{\text{m}^2}{\text{s}^2} = 0.$



A collision where objects bounce off of each other is an elastic collision. In a perfectly elastic collision, kinetic energy is conserved: the total kinetic energy before the collision is equal to the total kinetic energy after the collision.

When the colliding objects are equal in mass, you can use the conservation of momentum and the conservation of kinetic energy to prove that the angle between the objects after the collision is 90°. In the vector diagram for momentum conservation, you can see that the three vectors create a triangle. Look at the following equation for the conservation of kinetic energy.

$$KE_{Before} = KE_{After}$$

$$\tfrac{1}{2}mv_{iA}^2 = \tfrac{1}{2}mv_{fA}^2 + \tfrac{1}{2}mv_{fB}^2$$

Since all the masses are equal:

$$v_i^2 = v_{fA}^2 + v_{fB}^2$$

You may recognize this as a form of the Pythagorean theorem for a right triangle $a^2 + b^2 = c^2$. The vector triangle for momentum must therefore be a right triangle. The cue ball and the object ball must depart at right angles. You can now use your physics knowledge when you play billiards!

1. The mathematics of collisions becomes a bit difficult since the kinetic energy depends on the square of the velocity. Try to solve for the speed of a pedestrian when hit by an automobile. Assume that momentum is conserved. Also assume that 50 percent of the kinetic energy is conserved. Assume a 1000-kg automobile moving at 10 m/s (only 20 mph) collides with a 70-kg pedestrian at rest. The calculation will give you a sense of how dangerous an automobile is because it has such tremendous momentum.

What Do You Think Now?

At the beginning of the activity you were asked:

- **What physics principles do the traffic-accident investigators use to "reconstruct" the accident?**

Revisit your initial responses. Now that you investigated how the law of conservation of momentum works, how would you answer this question?

What Do You Think Now?

Ask students to review their previous responses to the *What Do You Think?* question. Ask students how they would answer this question now and survey the class for how many students changed their ideas. Point out that scientists often change their ideas as they gather more information. Consider discussing the information in *A Physicist's Response*.

Using the quadratic equation:

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$v_{\text{person final}} = \frac{(1400\ \text{m/s}) \pm \sqrt{(-1400\ \text{m/s})^2 - 4(74.9)(-100{,}000\ \text{m}^2/\text{s}^2)}}{2(74.9)}$$

$v_{\text{person final}} = -28\ \text{m/s}$ or $47\ \text{m/s}$.

There are two possible values. From this, students should select the value that can be explained by physics. The negative value would indicate that the person went flying in the opposite direction. This does not make sense. The positive value indicates the same direction as the vehicle was originally moving in, which does make sense in physics. Therefore,

$v_{\text{person final}} = 47\ \text{m/s} = 169\ \text{km/h} = 105\ \text{mi/h}$.

Reflecting on the Section and the Challenge

Using the information in the student text, review the law of conservation of momentum and its importance. Emphasize that it always holds true for all collision types. Use the examples provided in the student text. Remind students that this concept is important to keep in mind for the *Chapter Challenge* and that they should consider these concepts as they design their prototype and in their explanations.

Chapter 3 Safety

Physics
Essential Questions

What does it mean?

What does it mean to "conserve" momentum in a collision?

How do you know?

All experiments have measurement uncertainties. How accurately were you able to measure the conservation of momentum in your activity? How could the measurements be improved?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	* Conservation laws	Good clear explanation, no more complicated than necessary

* In physics, concepts are introduced to help make complex situations appear simpler to understand. Describe in words all that happens when a cue ball hits the 15 balls at the beginning of a billiards match. Describe the same event using the concepts of conservation of momentum and conservation of energy.

Why do you care?

How would the principle of conservation of momentum influence the design of a safety system that must protect against collisions with a much more massive object?

Reflecting on the Section and the Challenge

The law of conservation of momentum is a very powerful tool for explaining collisions in traffic accidents. The law works even when one of the objects involved in a collision "bounces back," reversing the direction of its velocity and therefore, its momentum. Whether describing a collision between two people, or between a moving automobile and a tree, you can describe how the total momentum is conserved.

Physics Essential Questions

What does it mean?

To conserve momentum means that the sum of the momentums of all objects just before a collision is equal to the sum of the momentums of all objects just after the collision.

How do you know?

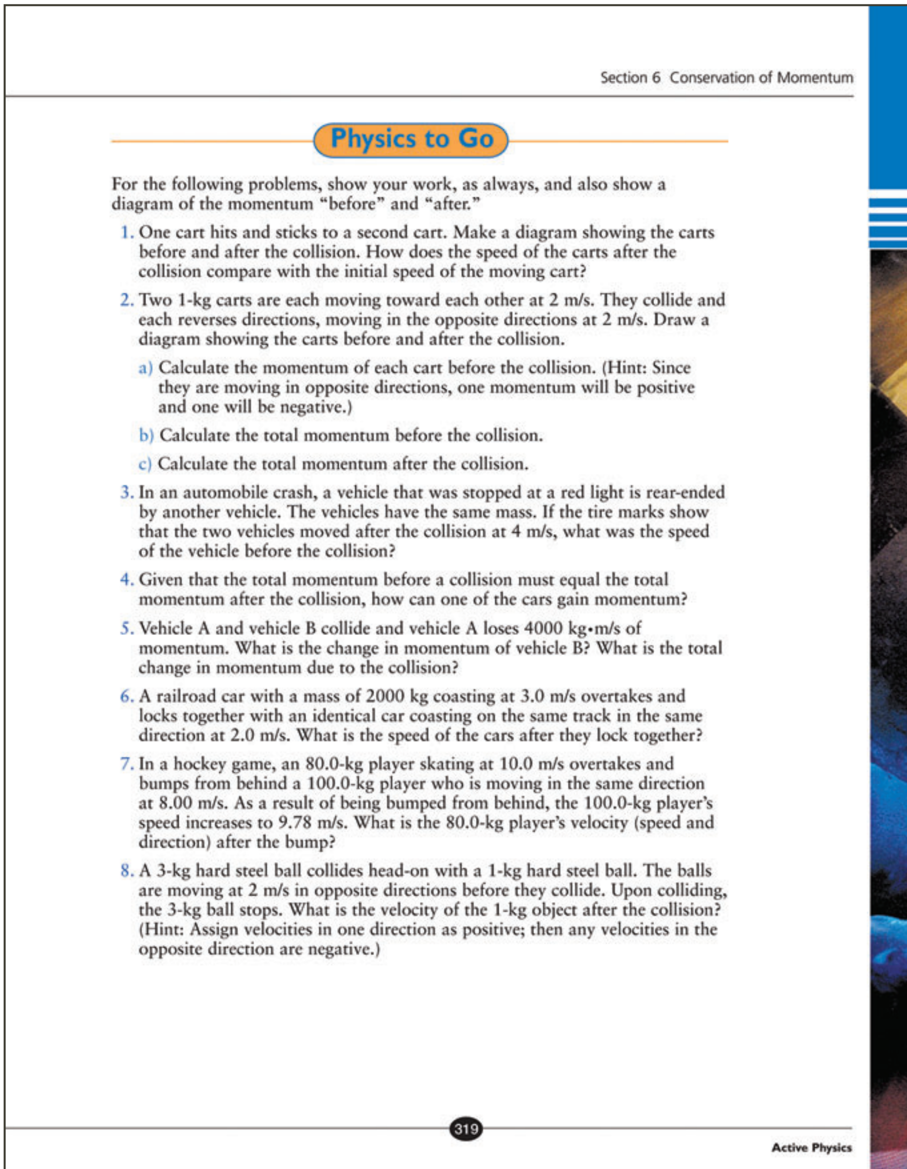
(Answer depends on student results.) The results can be improved by minimizing the friction in the carts.

Why do you believe?

The cue ball has a certain amount of energy and momentum when it strikes the 15 stationary balls. After the collision, the sum of the momentums of the 16 balls is equal to the original momentum of the cue ball. Also, the sum of the energies of the 16 balls is equal to the energy the cue ball had just before the collision if the collision is elastic. (The collision is not perfectly elastic; some energy is transferred to the surroundings as sound and heat.)

Why should you care?

A massive object, if moving, will have an enormous amount of momentum and energy that could be transferred during a collision. Designing a safety system to protect against this may require lots of padding so that when hit, the passengers experience a smaller force over a longer distance.



Section 6 Conservation of Momentum

Physics to Go

For the following problems, show your work, as always, and also show a diagram of the momentum "before" and "after."

1. One cart hits and sticks to a second cart. Make a diagram showing the carts before and after the collision. How does the speed of the carts after the collision compare with the initial speed of the moving cart?
2. Two 1-kg carts are each moving toward each other at 2 m/s. They collide and each reverses directions, moving in the opposite directions at 2 m/s. Draw a diagram showing the carts before and after the collision.
   a) Calculate the momentum of each cart before the collision. (Hint: Since they are moving in opposite directions, one momentum will be positive and one will be negative.)
   b) Calculate the total momentum before the collision.
   c) Calculate the total momentum after the collision.
3. In an automobile crash, a vehicle that was stopped at a red light is rear-ended by another vehicle. The vehicles have the same mass. If the tire marks show that the two vehicles moved after the collision at 4 m/s, what was the speed of the vehicle before the collision?
4. Given that the total momentum before a collision must equal the total momentum after the collision, how can one of the cars gain momentum?
5. Vehicle A and vehicle B collide and vehicle A loses 4000 kg•m/s of momentum. What is the change in momentum of vehicle B? What is the total change in momentum due to the collision?
6. A railroad car with a mass of 2000 kg coasting at 3.0 m/s overtakes and locks together with an identical car coasting on the same track in the same direction at 2.0 m/s. What is the speed of the cars after they lock together?
7. In a hockey game, an 80.0-kg player skating at 10.0 m/s overtakes and bumps from behind a 100.0-kg player who is moving in the same direction at 8.00 m/s. As a result of being bumped from behind, the 100.0-kg player's speed increases to 9.78 m/s. What is the 80.0-kg player's velocity (speed and direction) after the bump?
8. A 3-kg hard steel ball collides head-on with a 1-kg hard steel ball. The balls are moving at 2 m/s in opposite directions before they collide. Upon colliding, the 3-kg ball stops. What is the velocity of the 1-kg object after the collision? (Hint: Assign velocities in one direction as positive; then any velocities in the opposite direction are negative.)

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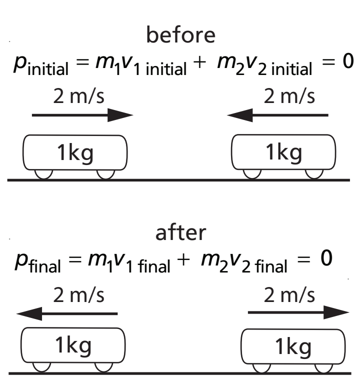
Active Physics

Physics to Go

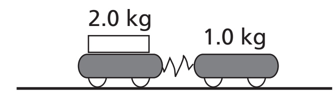
1.

Students' diagrams should look similar to the following diagram. Students should note that the final velocity of the carts after they collide and stick together is in the same direction as the initial velocity, and the final speed is less than the initial speed. Encourage students to record the momentums for the before and after situations on their diagram and to solve for the final velocity of the carts.



2.a)

Students' diagrams should look similar to the one below.



Velocity and momentum are vector quantities. This is indicated by opposite directions of the velocities having opposite signs. If the direction pointing to the right is assumed to be positive then students should calculate the following momenta:

$p_{\text{right cart initial}} = (1\text{ kg})(-2\text{ m/s}) =$ $-2\text{ kg}\cdot\text{m/s}$, where the negative sign indicates it moves to the left.

$p_{\text{left cart initial}} = (1\text{ kg})(+2\text{ m/s}) =$ $+2\text{ kg}\cdot\text{m/s}$, where the positive sign indicates it moves to the right.

2.b)

Adding the momentum of each cart before the collision to

$$p_{\text{total initial}} = 2\text{ kg}\cdot\text{m/s} - 2\text{ kg}\cdot\text{m/s} = 0$$

2.c)

The total momentum after the collision is also zero due to the conservation of momentum, and because the carts have just reversed direction.

3.

Students apply the law of conservation of momentum.

$$p_{\text{total before}} = p_{\text{total after}}$$

$$mv_{\text{initial}} + 0 = 2m(4\text{ m/s})$$

$$v_{\text{initial}} = 8\text{ m/s}$$

CHAPTER 3

4.

Students should realize that even though the total momentum before and after a collision does not change, momentum can be transferred between objects during a collision. The magnitude (how much) of each object's individual momentum can increase or decrease, and/or change direction, but the magnitude and direction of the total momentum cannot change.

5.

Students should apply the law of conservation of momentum. It is important that students realize that the change in momentum of one object must be equal in size and opposite in direction to the change in momentum of the other object.

$p_{\text{total before}} = p_{\text{total after}}$

$m_A v_{A\text{ initial}} + m_B v_{B\text{ initial}} =$

$m_A v_{A\text{ final}} + m_B v_{B\text{ final}}$

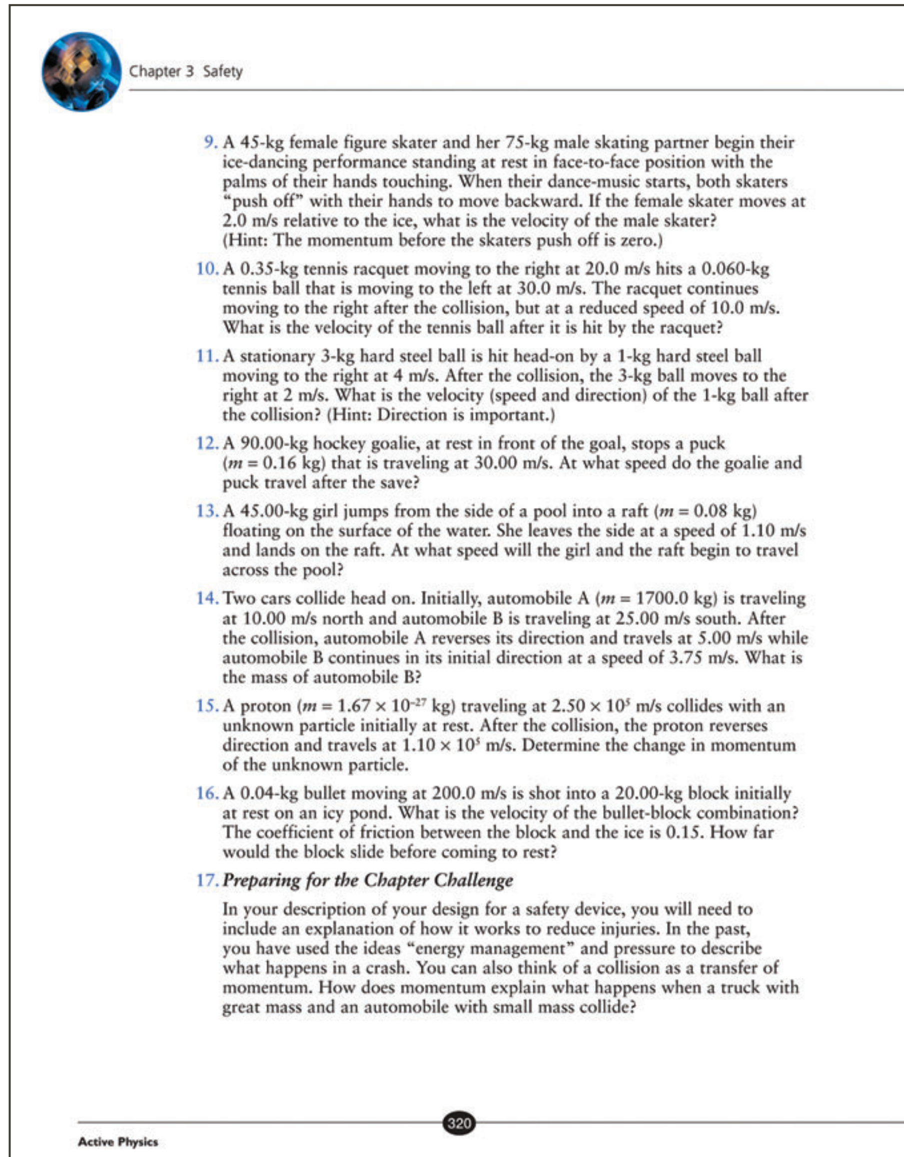
$m_A v_{A\text{ initial}} - m_A v_{A\text{ final}} =$

$m_B v_{B\text{ final}} - m_B v_{B\text{ initial}}$

$-\left(m_A v_{A\text{ final}} - m_A v_{A\text{ initial}}\right) =$

$m_B v_{B\text{ final}} - m_B v_{B\text{ initial}}$

This says that the change in momentum of vehicle B is equal and opposite to the change in momentum of vehicle A. Because vehicle A changes its momentum by 4000 kg · m/s in the "negative" direction, vehicle B must change its momentum by 4000 kg · m/s in the direction opposite to vehicle A. Students should recognize that even though the vehicles exchange momentum during the collision, the total momentum of the system consisting of both vehicles does not change. That is, the total change of momentum is zero.



Chapter 3 Safety

9. A 45-kg female figure skater and her 75-kg male skating partner begin their ice-dancing performance standing at rest in face-to-face position with the palms of their hands touching. When their dance-music starts, both skaters "push off" with their hands to move backward. If the female skater moves at 2.0 m/s relative to the ice, what is the velocity of the male skater? (Hint: The momentum before the skaters push off is zero.)

10. A 0.35-kg tennis racquet moving to the right at 20.0 m/s hits a 0.060-kg tennis ball that is moving to the left at 30.0 m/s. The racquet continues moving to the right after the collision, but at a reduced speed of 10.0 m/s. What is the velocity of the tennis ball after it is hit by the racquet?

11. A stationary 3-kg hard steel ball is hit head-on by a 1-kg hard steel ball moving to the right at 4 m/s. After the collision, the 3-kg ball moves to the right at 2 m/s. What is the velocity (speed and direction) of the 1-kg ball after the collision? (Hint: Direction is important.)

12. A 90.00-kg hockey goalie, at rest in front of the goal, stops a puck (m = 0.16 kg) that is traveling at 30.00 m/s. At what speed do the goalie and puck travel after the save?

13. A 45.00-kg girl jumps from the side of a pool into a raft (m = 0.08 kg) floating on the surface of the water. She leaves the side at a speed of 1.10 m/s and lands on the raft. At what speed will the girl and the raft begin to travel across the pool?

14. Two cars collide head on. Initially, automobile A (m = 1700.0 kg) is traveling at 10.00 m/s north and automobile B is traveling at 25.00 m/s south. After the collision, automobile A reverses its direction and travels at 5.00 m/s while automobile B continues in its initial direction at a speed of 3.75 m/s. What is the mass of automobile B?

15. A proton ($m = 1.67 \times 10^{-27}$ kg) traveling at 2.50×10^5 m/s collides with an unknown particle initially at rest. After the collision, the proton reverses direction and travels at 1.10×10^5 m/s. Determine the change in momentum of the unknown particle.

16. A 0.04-kg bullet moving at 200.0 m/s is shot into a 20.00-kg block initially at rest on an icy pond. What is the velocity of the bullet-block combination? The coefficient of friction between the block and the ice is 0.15. How far would the block slide before coming to rest?

17. ***Preparing for the Chapter Challenge***

In your description of your design for a safety device, you will need to include an explanation of how it works to reduce injuries. In the past, you have used the ideas "energy management" and pressure to describe what happens in a crash. You can also think of a collision as a transfer of momentum. How does momentum explain what happens when a truck with great mass and an automobile with small mass collide?

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Active Physics

6.

This is a completely inelastic collision or a "sticky" collision. Students should apply the conservation of momentum to calculate the unknown quantity.

$p_{\text{total before}} = p_{\text{total after}}$

$m_1 v_{1\text{initial}} + m_2 v_{2\text{initial}} = \left(m_1 + m_2\right) v_{\text{final}}$

$(2000\text{ kg})(3\text{ m/s}) +$

$(2000\text{ kg})(2.0\text{ m/s}) =$

$(2000\text{ kg} + 2000\text{ kg})v_{\text{final}}$

$10{,}000\text{ kg} \cdot \text{m/s} = (4000\text{ kg})v_{\text{final}}$

$v_{\text{final}} = 2.5\text{ m/s}$

7.

This is a completely inelastic collision. Apply the conservation of momentum to calculate the unknown quantity.

$p_{\text{total before}} = p_{\text{total after}}$

$m_1 v_{1\text{initial}} + m_2 v_{2\text{initial}} =$

$m_1 v_{1\text{final}} + m_2 v_{2\text{final}}$

$(80.0\text{ kg})(10.0\text{ m/s}) +$

$(100.0\text{ kg})(8.00\text{ m/s}) =$

$(80.0\text{ kg})v_{1\text{final}} +$

$(100.0\text{ kg})(9.78\text{ m/s})$

$1600 \text{ kg} \cdot \text{m/s} =$

$(80.0 \text{ kg})v_{1\text{final}} + (978 \text{ kg} \cdot \text{m/s})$

$622 \text{ kg} \cdot \text{m/s} = (80.0 \text{ kg})v_{1\text{final}}$

$v_{1\text{final}} = 7.78 \text{ m/s}$, in the same direction.

8.

This is a completely inelastic collision. Apply conservation of momentum to calculate the unknown quantity. The direction of travel of the 3-kg ball before the collision is assigned as positive:

$p_{\text{total before}} = p_{\text{total after}}$

$m_1 v_{1\text{initial}} + m_2 v_{2\text{initial}} =$

$m_1 v_{1\text{final}} + m_2 v_{2\text{final}}$

$(3 \text{ kg})(2 \text{ m/s}) + (1 \text{ kg})(-2 \text{ m/s}) =$

$(3 \text{ kg})(0 \text{ m/s}) + (1 \text{ kg})v_{2\text{final}}$

$4 \text{ kg} \cdot \text{m/s} = (1 \text{ kg})v_{2\text{final}}$

$4 \text{ m/s} = v_{2\text{final}}$

The 1-kg ball bounces back after the collision, moving in the opposite direction at twice the speed it had coming into the collision.

9.

Students should assign a positive direction. In the solution below, the direction of the female skater after push off is assigned as positive:

$p_{\text{total before}} = p_{\text{total after}}$

$m_1 v_{1\text{initial}} + m_2 v_{2\text{initial}} =$

$m_1 v_{1\text{final}} + m_2 v_{2\text{final}}$

$0 = (45 \text{ kg})(2 \text{ m/s}) + (75 \text{ kg})v_{2\text{final}}$

$-90 \text{ kg} \cdot \text{m/s} = (75 \text{ kg})v_{2\text{final}}$

$-1.2 \text{ m/s} = v_{2\text{final}}$

The male skater moves at 1.2 m/s in the direction opposite the female skater.

10.

In the solution below, the right direction is assigned as positive:

$p_{\text{total before}} = p_{\text{total after}}$

$m_1 v_{1\text{initial}} + m_2 v_{2\text{initial}} =$

$m_1 v_{1\text{final}} + m_2 v_{2\text{final}}$

$(0.35 \text{ kg})(20 \text{ m/s}) +$

$(0.060 \text{ kg})(-30 \text{ m/s}) =$

$(0.35 \text{ kg})(10 \text{ m/s}) + (0.060 \text{ kg})v_{2\text{final}}$

$5.2 \text{ kg} \cdot \text{m/s} =$

$3.5 \text{ kg} \cdot \text{m/s} + (0.060 \text{ kg})v_{2\text{final}}$

$28.33 \text{ m/s} = v_{2\text{final}}$

The ball moves to the right after the collision with a speed of 28.33 m/s.

11.

In the solution below, the right direction is assigned as positive:

$p_{\text{total before}} = p_{\text{total after}}$

$m_1 v_{1\text{initial}} + m_2 v_{2\text{initial}} =$

$m_1 v_{1\text{final}} + m_2 v_{2\text{final}}$

$(3 \text{ kg})(0) + (1 \text{ kg})(4 \text{ m/s}) =$

$(3 \text{ kg})(2 \text{ m/s}) + (1 \text{ kg})v_{2\text{final}}$

$4 \text{ kg} \cdot \text{m/s} = 6 \text{ kg} \cdot \text{m/s} + (1 \text{ kg})v_{2\text{final}}$

$-2 \text{ m/s} = v_{2\text{final}}$

The 1.0-kg ball moves to the left after the collision with a speed of 2 m/s.

12.

The collision described is a completely inelastic collision.

$p_{\text{total before}} = p_{\text{total after}}$

$m_1 v_{1\text{initial}} + m_2 v_{2\text{initial}} = (m_1 + m_2)v_{\text{final}}$

$(90 \text{ kg})(0) + (0.16 \text{ kg})(30.00 \text{ m/s}) =$

$(90.16 \text{ kg})v_{\text{final}}$

$0.05 \text{ m/s} = v_{\text{final}}$

The goalie and puck move in the direction of the incoming puck with a speed of 0.05 m/s.

13.

This is a completely inelastic collision.

$p_{\text{total before}} = p_{\text{total after}}$

$m_1 v_{1\text{initial}} + m_2 v_{2\text{initial}} = (m_1 + m_2)v_{\text{final}}$

$(45.00 \text{ kg})(1.10 \text{ m/s}) +$

$(0.08 \text{ kg})(0 \text{ m/s}) = (45.08 \text{ kg})v_{\text{final}}$

$1.1 \text{ m/s} = v_{\text{final}}$

The girl and raft move in the direction of the girl's jump with a speed of just less than 1.1 m/s. Because the value is rounded off, it appears from this answer that the speed is the same as that of the girl's initial jump. This is because the mass of the raft is much less than the girl's mass, so the decrease in speed is minimal.

14.

Students should indicate a positive and a negative direction. In the answer below, north is taken as positive.

$p_{\text{total before}} = p_{\text{total after}}$

$m_A v_{A\text{initial}} + m_B v_{B\text{initial}} =$

$m_A v_{A\text{final}} + m_B v_{B\text{final}}$

$(1700.0 \text{ kg})(10.00 \text{ m/s}) +$

$(m_B)(-25.00 \text{ m/s}) =$

$(1700.0 \text{ kg})(-5.00 \text{ m/s}) +$

$(m_B)(-3.75 \text{ m/s})$

$17{,}000 \text{ kg} \cdot \text{m/s} - m_B(25.00 \text{ m/s}) =$

$-8500 \text{ kg} \cdot \text{m/s} - m_B(3.75 \text{ m/s})$

$25{,}500 \text{ kg} \cdot \text{m/s} = m_B(21.25 \text{ m/s})$

$m_B = 1200 \text{ kg}$

15.

Use the conservation of total momentum to solve the change in momentum of the unknown particle.

$$p_{\text{total before}} = p_{\text{total after}}$$

$$m_1 v_{1\text{initial}} + m_2 v_{2\text{initial}} =$$
$$m_1 v_{1\text{final}} + m_2 v_{2\text{final}}$$
$$m_1 v_{1\text{initial}} - m_1 v_{1\text{final}} =$$
$$m_2 v_{2\text{final}} - m_2 v_{2\text{initial}} =$$

the change in momentum of particle 2.

$$\left(1.67\times10^{-27}\text{ kg}\right)\left(2.5\times10^{5}\text{ m/s}\right)-$$
$$\left(1.67\times10^{-27}\text{ kg}\right)\left(-1.1\times10^{5}\text{ m/s}\right)=$$
$$m_2 v_{2\text{final}} - m_2 v_{2\text{initial}}$$
$$6.01\times10^{-22}\text{ kg}\cdot\text{m/s} =$$
$$m_2 v_{2\text{final}} - m_2 v_{2\text{initial}}$$

The change in momentum of the proton is equal and opposite to the change in momentum of the unknown particle.

16.

This is a completely inelastic collision. Hence,

$$p_{\text{total before}} = p_{\text{total after}}$$
$$m_1 v_{1\text{initial}} + m_2 v_{2\text{initial}} = \left(m_1 + m_2\right) v_{\text{final}}$$
$$\left(0.04\text{ kg}\right)\left(200\text{ m/s}\right)+$$
$$\left(20.00\text{ kg}\right)\left(0\text{m/s}\right) = \left(20.04\text{ kg}\right) v_{\text{final}}$$
$$0.4\text{ m/s} = v_{\text{final}}$$

After the bullet embeds itself in the block, the block-bullet system moves with a speed of 0.4 m/s in the direction in which the bullet was traveling.

Students should calculate the frictional force acting on the block and the distance traveled by using the work-energy theorem. The only force acting on the block-bullet system after the collision and in the direction of motion is the force of friction from the ice surface acting on the block, which is given by the coefficient of friction multiplied by the mass and the acceleration due to gravity. This force does work on the block to change its initial kinetic energy to zero.

Using the work-energy theorem,

$$\Delta KE = Fd$$
$$\tfrac{1}{2} m v_{\text{final}}^2 - \tfrac{1}{2} m v_{\text{initial}}^2 = -\mu_k mgd$$

Students should realize that friction does negative work on the block-bullet system because it acts in a direction opposite to the direction of motion.

$$0 - \tfrac{1}{2}\left(20.04\text{ kg}\right)\left(0.4\text{ m/s}\right)^2 =$$
$$-\left(0.15\right)\left(20.04\text{ kg}\right)\left(9.8\text{ m/s}^2\right) d$$
$$d = 0.05\text{ m}$$

17.

Preparing for the Chapter Challenge

Student answers will vary. The descriptions should include how a collision between the truck and the automobile will involve a transfer of momentum to the automobile, but the total momentum of the truck-automobile system remains the same.

NOTES

SECTION 6 QUIZ

3-6b **Blackline Master**

1. A 1200-kg automobile is traveling at a speed of 20 m/s when it collides with a truck at rest with a mass of 10,000 kg. What is the momentum of the vehicle before the collision?

   a) 60 kg · m/s
   b) 24,000 kg · m/s
   c) 34,000 kg · m/s
   d) 200,000 kg · m/s

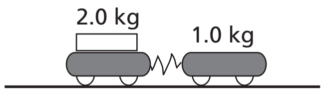
2. In *Question 1*, if the automobile and the truck stick together after the collision, what is the momentum of the combined mass?

   a) 60 kg · m/s
   b) 24,000 kg · m/s
   c) 34,000 kg · m/s
   d) 200,000 kg · m/s

3. A 2.0-kg cart moving with a speed of 3 m/s collides with a 1.0-kg cart that is at rest. After the collision, the two carts stick together. What is the velocity of the combined mass of the two carts after the collision?

   a) 1.0 m/s
   b) 2.0 m/s
   c) 3.0 m/s
   d) 1.5 m/s

4. A 2.0-kg cart and a 1.0-kg cart are at rest on a level surface with a compressed spring between them as shown. When the spring is released, if the 1.0-kg cart moves off with a speed of 3.0 m/s, what is the speed of the 2.0-kg cart?



   a) 1 m/s
   b) 2 m/s
   c) 3 m/s
   d) 1.5 m/s

5. In a circus act, a 100-kg "human cannonball" is fired from a 400-kg cannon on wheels. After the cannon is fired, which equation below describes the system?

   a) speed of the cannon + the speed of the human = 0
   b) mass of the cannon + the mass of the human = 0
   c) momentum of the cannon + the momentum of the human = 0
   d) velocity of the cannon + the velocity of the human = 0

CHAPTER 3

SECTION 6 QUIZ ANSWERS

1. b) 24,000 kg · m/s
2. b) 24,000 kg · m/s
3. b) 2.0 m/s
4. d) 1.5 m/s
5. c) momentum of the cannon + the momentum of the human = 0

NOTES

NOTES