SECTION 10

Safety is Required but Thrills Are Desired

Section Overview

Students investigate the criteria required for roller-coaster safety and estimate the conditions under which the roller-coaster ride would be closed to the public.

They explore safe accelerations for loops with different radii and other positions of the ride, and find the centripetal force required to keep the object of a given mass moving in a circle. Students learn how to produce thrills by changing the speed of the roller coaster, the shape of the curve, and describing visual effects that add to the thrills. They investigate the forces acting on the roller-coaster cars and the tracks to determine the need for strong materials. Students then analyze how accelerations greater than 4 *g* affect passenger safety and why during free fall a person feels weightless. This section also explores the psychological effects of roller-coaster accidents and what happens when rides are not properly designed.

Background Information

The mathematics associated with roller coasters at this level of treatment include calculation of velocities at different heights of the roller coaster using the conservation of energy. These velocities can then be used to determine if the centripetal accelerations around curves on any radius are deemed too large for safety. A conservative value of four times the acceleration due to gravity $\left(4 \times 9.8 \text{ m/s}^2 = \text{approximately } 40 \text{ m/s}^2\right)$ is used as the maximum allowable acceleration.

At the top of a vertical loop in the roller coaster, the downward acceleration will be at least 1 *g*—the acceleration due to the force of gravity. This acceleration is the minimum centripetal acceleration at the top of the loop. From this minimum acceleration, you can calculate the minimum speed that a coaster must have to maintain circular motion and not leave the track. Any speed greater than this minimum speed will require a larger centripetal acceleration. This additional acceleration can be provided by the normal force of the track pushing down on the coaster.

An additional safety concern of a roller coaster is that the track be strong enough to provide the forces required to sustain the car during the accelerations. When the coaster car is traveling in a circle, this force is dependent on the mass of the roller-coaster car, the radius of the circle and the car's speed. It is therefore necessary to estimate the maximum weight of the people that will ride in the roller coaster in designing a safe ride. Roller coasters are made more fun by using optical illusions and sounds during the ride to heighten the thrill. For example, it may appear that the roller coaster is about to hit a wall or plunge into water. Sounds are used to make the roller coaster sound as if it may be broken during the long, slow uphill ride to the top.

Crucial Physics

- Humans become unconscious at accelerations greater than 9 *g* due to the blood leaving the brain. Some humans may black out when experiencing accelerations of only 5 *g* or greater.
- Centripetal force is the force pointing toward the center of a curved path that causes an object to follow a curved path. When the path is circular and the object moves at a constant speed, then the centripetal force is given by:

  $$F_c = \frac{mv^2}{r},$$

  where F_c is the centripetal force, m is the mass of the object, v is the speed, and r is the radius of the circle.
- Centripetal acceleration is the acceleration directed toward the center of a curved path an object is traveling on and is given by:

  $$a_c = \frac{v^2}{r}.$$

Learning Outcomes	Location in the Section	Evidence of Understanding
Calculate the speed of the roller coaster at different positions, using conservation of energy.	***Investigate*** Step 1 ***Physics to Go*** Questions 2-5	Students calculate the speed of a roller coaster at different locations and solve problems using conservation of energy to determine a safe acceleration.
Calculate the acceleration of the roller coaster at turns.	***Investigate*** Step 2	Students locate where accelerations occur and calculate accelerations at the turns at roller-coaster turns to determine if the roller coaster has a safe speed to make the turn.
Determine if the acceleration is below 4 *g* for safety.	***Physics Talk***	Students adjust speed or circle radii to limit the acceleration of a coaster to meet safety requirements.
Determine if the speed at the top of the loop is sufficient for safety concerns.	***Physics Talk***	Students solve sample problems by changing the radius of the loop to determine if the acceleration is large enough for the roller coaster to complete the loop.
Construct sounds and scenery to enhance the thrills of a roller-coaster ride.	***Investigate*** Step 5	Students describe three visual effects that use scenery to add to the thrills of a roller-coaster ride.

Section 10 Materials, Preparation, and Safety

Materials and Equipment

Not Applicable

Plan A and B

Time Requirements

- Allow one class period or 40 minutes for this section's *Investigate*.

Teacher preparation

- No special preparation is required.

Safety

- There are no specific safety issues for this *Investigate*.

Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

Learning Issue	Reference	Augmentation and Accommodations
Analyzing concepts qualitatively using a formula	***Physics Talk*** ***Physics to Go*** Question 8	**Augmentation** • Students often struggle to attach concrete meaning to symbols without known values, and they also have a difficult time understanding fractional relationships. The formula, $a = v^2/r$, includes a numerator that is squared and a denominator. Students must understand that velocity has a direct relationship with acceleration and affects the acceleration more because it is squared. Also, the radius has an inverse relationship with acceleration. • Explain the meaning of direct and inverse mathematical relationships. Ask students if they can think of any other examples of direct or inverse relationships from physics this year. • Use the quantitative sample problems to provide evidence for students who need examples to understand the larger concept (inductive learners).
Calculating answers for multi-step problems	***Active Physics Plus*** Questions 1-6 ***Physics to Go*** Questions 2-7	**Augmentation** • Some students get overwhelmed by multi-step problems and often panic. Being organized and following a series of steps is also a struggle for students with learning disabilities. They tend to skip steps by accident or take shortcuts to save time and end up with a series of incorrect answers. • Limit the number of steps or the number or problems involved in one problem solving task, at least until students have gained a higher level of accuracy and independence. • Encourage students to use a place marker to cover the other problems and only focus on one step at a time. **Accommodation** • Provide a step-by-step checklist for students to follow when solving multi-step problems. • Provide blank problem-solving boxes for students to use. Some students may need help choosing which formula to use for each step in the problem until they gain more understanding and confidence.

Strategies for Students with Limited English-Language Proficiency

The following is a list of terms that have appeared in *Thrills and Chills*.

acceleration
centripetal acceleration
centripetal force
displacement
gravitational field
gravitational potential energy
gravity
Hooke's law
inverse-square relationship
joule
kinetic energy
mechanical energy
Newton's law of universal gravitation
normal force
power
scalar
speed
vector
velocity
watt
weight
work

Have students take on the role of an experienced roller-coaster designer helping a less-experienced colleague understand the fundamentals of roller-coaster design. Ask them to create a two-page tutorial explaining some basic design considerations and safety issues to their colleagues. They should discuss types of energy, forces, and one or two other factors, such as acceleration, speed, direction, height, loops, curves, and so on. Have them include one mathematical problem and/or one labeled diagram in their tutorials.

Instruct students to write in complete sentences. Encourage them to use as many vocabulary terms related to the content as possible. (Note: There are no new vocabulary terms introduced in this section, but students should use as many as possible from previous sections.) You may wish to have students work in pairs. Review and correct students' work.

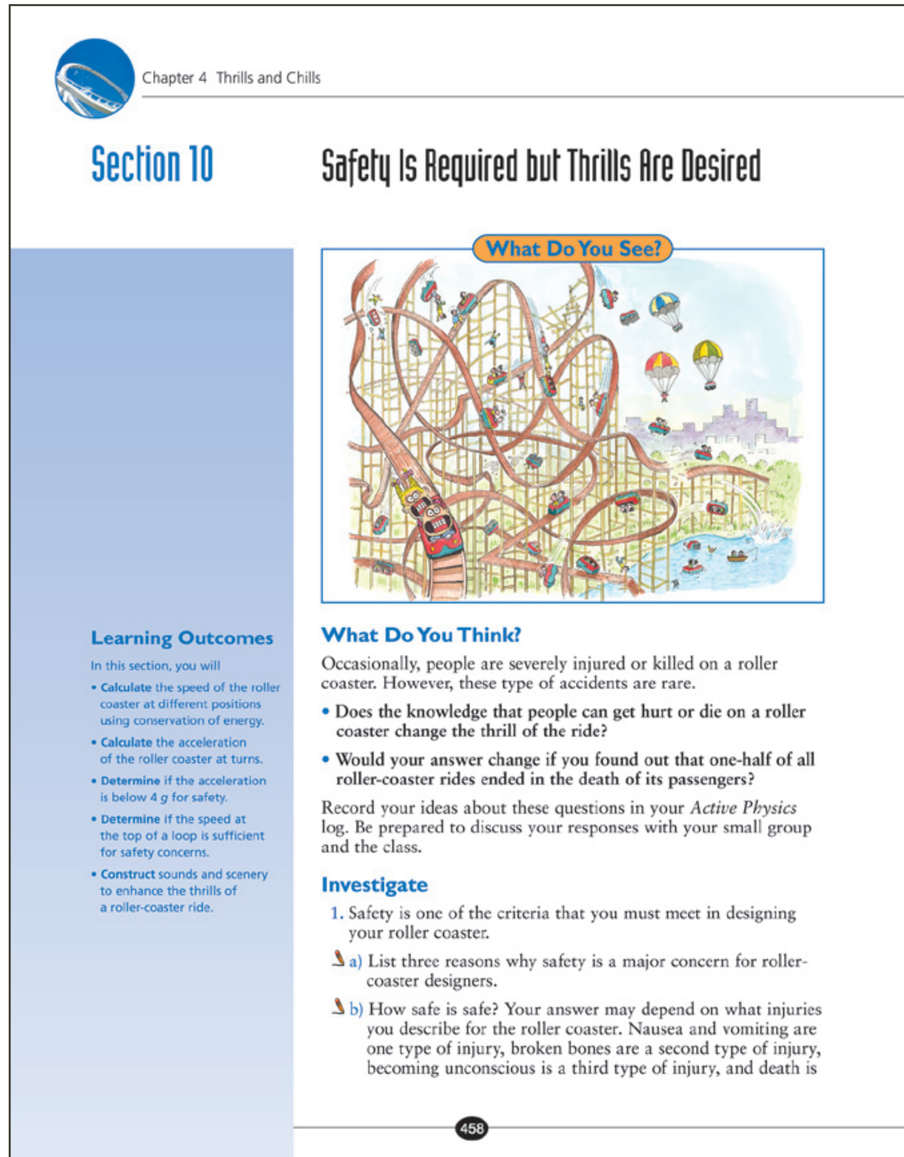
Rapid feedback about students' sentences is essential, because the sentences and errors will be fresh in their minds. A quick and powerful method for providing this feedback is to prepare a list of examples of sentences containing errors from the students' work. Divide examples into the following categories: incorrect science, incorrect usage of vocabulary, incorrect sentence structure, and incorrect grammar. Choose several examples from the collected work to use in each category and edit the sentences until they contain only one or two obvious errors. At the beginning of class the next day, provide each student with a page containing a double-spaced, typed list of the incorrect sentences, with headings for the four categories. Allow students 10 minutes to silently make corrections to the sentences. Then place a copy of the list on the overhead projector and ask students to suggest ideas for how to correct the sentences. During the exercise, guide students toward correct science and correct English usage.

SECTION 10

Teaching Suggestions and Sample Answers

What Do You See?

This illustration conveys a feeling of thrill and tension. The facial expressions of the riders also suggest a sense of anxiety as well as the parachutes of the riders thrown from the tracks and the falling coaster cars. Students will most likely be drawn to different aspects of this visual. Initiate a discussion on the shape of the roller coaster and other images that the artist has included. Ask the students if they think the roller coaster illustrated is safe, and whether they would be willing to ride it. Point out that each visual is significant in this section. A close observation should help in connecting with the main purpose of this section. Draw students' attention to



Chapter 4 Thrills and Chills

Section 10 Safety Is Required but Thrills Are Desired

What Do You See?

Learning Outcomes

In this section, you will

- **Calculate** the speed of the roller coaster at different positions using conservation of energy.
- **Calculate** the acceleration of the roller coaster at turns.
- **Determine** if the acceleration is below 4 *g* for safety.
- **Determine** if the speed at the top of a loop is sufficient for safety concerns.
- **Construct** sounds and scenery to enhance the thrills of a roller-coaster ride.

What Do You Think?

Occasionally, people are severely injured or killed on a roller coaster. However, these type of accidents are rare.

- **Does the knowledge that people can get hurt or die on a roller coaster change the thrill of the ride?**
- **Would your answer change if you found out that one-half of all roller-coaster rides ended in the death of its passengers?**

Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your responses with your small group and the class.

Investigate

1. Safety is one of the criteria that you must meet in designing your roller coaster.

a) List three reasons why safety is a major concern for roller-coaster designers.

b) How safe is safe? Your answer may depend on what injuries you describe for the roller coaster. Nausea and vomiting are one type of injury, broken bones are a second type of injury, becoming unconscious is a third type of injury, and death is

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Students' Prior Conceptions

This section allows you to review the concepts developed in the previous sections and to assess student understanding of balanced and unbalanced forces while traveling in coasters that have both vertical and horizontal loops. Gravity is always present but is not always balanced by opposing forces, and friction is important in providing the vital component of centripetal forces. Therefore, it is important to examine the following prior conceptions:

1. **Students believe objects are pushed down rather than being pulled down by gravity.** The force of gravity always acts down toward the center of Earth on everything on the surface of the planet or in the air above it. Listen to the students while they are explaining the physics involved in the safety of their coasters and ascertain that they recognize the ever-present force of gravity pulling all objects down toward the center of Earth. There is an upward force due to the seat of the coaster that balances this force while the coaster is in the horizontal position. This force is the normal force; however, this normal force changes as coasters travel around both horizontal and vertical circles. This is critical physics for the challenge presented in this chapter. You need to guide students to understand what happens to the speed of the roller coaster at different positions and why safety is a factor. Using the conservation of kinetic and gravitational potential energy is an excellent avenue of instruction.

2. **Students must not continue to hold these two beliefs: A force pushes you outward as you ride around a curve and circular motion is accelerated motion.** As students

the broken track of the roller coaster, riders falling off, people parachuting down, and large and small loops. This *What Do You See?* illustration skillfully captures the topic of this section and renders it on many levels of interpretation. Encourage students to return to this illustration, once they have investigated the concepts in this section.

What Do You Think?

These questions introduce students to ideas that they will be probing in the following *Investigate*. The potential risks involved in a roller-coaster ride will stimulate students' curiosity. Prompt them to recall their personal experiences on a roller coaster and if they ever felt scared. A quick search of the Internet may reveal local roller-coaster accidents that have led to injury. Highlight prior concepts such as conservation of energy, centripetal acceleration, normal force, and gravitational force to guide students toward the situations and answers. At the same time, emphasize that you are not evaluating their responses for "wrong" or "right" answers. Ask students to record their ideas in their logs. Generate a whole-class discussion with safety of roller coasters as the main focus. This section provides a formative assessment of student engagement. Post key points of the class discussion on the board so that students can also see the relevance of their ideas when they are updating their responses.

Investigate

1.a)

- Attendance—people will only go on rides they know are safe
- Lawsuits—people will sue if they are hurt on a ride
- Injury—nobody wants to be injured

1.b)

Students' answers will vary.

- Nausea—1 person per day and the ride should be closed
- Broken bones—1 injury every 3 years
- Unconsciousness—1 injury every 3 years
- Death—1 injury every 20 years if due to the ride and not negligence on the part of the rider.

What Do You Think?

A Physicist's Response

Students should distinguish between the thrill of the fear of being in danger and actually being in danger. The damage that could occur to a human body by the high velocity of the roller coaster, as well as the rapid changes in direction where the riders fear they may fall out are all part of the psychological thrill of the coaster. The idea behind a roller-coaster ride is make you fear you are out of control and may be in danger. If one-half of all the riders on a roller coaster were killed, it would change the thrill of a roller-coaster ride to dread at the thought of riding it. A survivor might feel exhilarated at the rides conclusion, but would not likely volunteer for a second ride.

CHAPTER 4

determine if the acceleration is below 4 *g* for safety and determine if the speed at the top of a loop is sufficient for safety concerns, the teacher should listen to explanations. Also examine challenge designs to ascertain that students grasp the concept that centripetal acceleration is a radial acceleration acting in toward the center of the circle or loop at all times and that the associated centripetal force interacts with the force of gravity and the normal force at all times. It is critical for students to understand that the normal force goes to zero instantaneously at the top of vertical loops. This means that the force of gravity is unbalanced and that it relates directly to the centripetal force at the top of the loop.

NOTES

the greatest injury. For the four types of injuries listed, make an estimate of how many people could get injured on a roller coaster ride before it would be closed to the public. Be sure to include whether these are injuries in a day, a month, or a year.

2. Astronauts going into space have to withstand very large accelerations during rocket launch. After many tests of test pilots and race-car drivers, it was determined that people will become unconscious if the acceleration is greater than about 9 *g* (or 9 times the acceleration due to gravity, that is, 9 × 9.8 m/s^2). Some people black out at 5 *g* or 6 *g*. This unconsciousness results from the blood leaving the brain during the high acceleration.

The roller-coaster manufacturer has indicated that the maximum acceleration at any place on the roller coaster should not exceed 4 *g* (or 4 times the acceleration due to gravity, that is, 4 × 9.8 m/s^2).

a) At what locations on the roller coaster are there accelerations?

b) If the roller coaster were to fall straight down, what would be the acceleration?

c) Is this a safe acceleration?

d) If you find that your roller coaster accelerations are greater than 4 *g* at the position shown, what changes in speed or shape of the curve can be made to decrease the acceleration?

3. The roller-coaster designer can play all sorts of tricks to produce extra thrills on the same roller coaster.

a) Describe how you can add suspense to the trip up to the top of the first hill.

b) Describe how you can use sounds during the roller-coaster ride to add to the thrills.

4. The choice of scenery surrounding the roller coaster can also add to the thrill. The roller coaster can look like it will dive into water when it is descending. The roller coaster can look like it will hit a building when it rounds a curve.

a) Describe three visual effects through the use of scenery that will add to the thrill of your roller coaster.

5. Thrills can come from high speeds. Thrills more often come from acceleration (changing velocity with respect to time). Change in velocity can be a change in speed or direction.

a) Describe three ways in which you can add thrills to a roller-coaster design by having changes in velocity.

b) Describe three ways in which you can add thrills to a roller-coaster design by having changes in acceleration.

6. The photo shows a roller coaster at an amusement park.

a) Using what you have learned in this chapter, describe the different parts of the roller-coaster ride shown in the picture. Where are you likely to experience the most thrills? Why?

2.a)

Accelerations occur on the turns and loops, as well as when the roller coaster goes up or down a hill.

2.b)

The acceleration in free fall is 9.8 m/s^2, or 1 *g*.

2.c)

It is not a safety concern with regard to consciousness. You will pick up speed quickly and will need a safe way to slow down at the bottom of the ride.

2.d)

Modifications to reduce the acceleration might include decreasing the speed of the coaster at this point, or increasing the radius of the curve to reduce the centripetal acceleration.

Note: For the following questions, students' answers will vary. Encourage their creativity. Some sample answers are provided.

3.a)

You can make the roller coaster move very slowly; you can make it shake a bit. You could also increase the height of the hill while keeping the size of the first drop the same.

3.b)

You can make the roller coaster have some creaks and other noises to sound like it may be broken.

4.a)

The roller coaster can come close to walls. The roller coaster can feel like it is higher than it is. The roller coaster can make it seem like the track will end, or that it will plunge into water.

5.a)

The roller coaster can change directions. The roller coaster can come close to free fall. The roller coaster can make a sharp turn.

5.b)

The roller coaster can change acceleration by having it zigzag back and forth with turns of different sizes. The loops provide changes in the acceleration. The roller coaster can turn and tilt at the same time.

6.a)

For the ride shown, the greatest thrills occur where the acceleration and change in acceleration is the greatest. This would include the loop and points with sharp turns. Simply being upside down in the loops may also add thrills for some people.

6.b)

The loop is more sharply curved at the top than at the bottom (a clothoid loop) to keep the acceleration of the coaster cars constant. Since the speed decreases as the car goes up the loop, if the radius of the loop is decreased accordingly, the centripetal acceleration will remain the same. The centripetal acceleration is by the equation $a_c = v^2/r$. If v^2 and r go down by the same amount, the acceleration will remain the same.

Physics Talk

The safety of roller-coaster rides serves as a framework for calculating accelerations and forces acting on roller coasters in the *Physics Talk*. Students analyze the design of a roller coaster that provides thrills but does not make the ride unsafe for its passengers. It is important for students to understand why the acceleration should be less than 4 *g*. You might want to point out that the letter *g* is not a unit but a constant representing the gravitational field strength, and that is why it is italicized in the text. Ask students how accelerations and forces can be increased or decreased when a roller-coaster car navigates a loop and how they can find the value of acceleration at any given point. Discuss sample problems by writing the important steps of the strategy on the board and ask students what they are learning from the sample problem that will help them in modifying the design of the roller coaster.

Review the concept of apparent weight. Explain why the normal force required at the bottom of a vertical loop is greater than the normal force that acts on the roller coaster when the coaster is at rest. Consider asking students why people at the top of a roller coaster feel lighter while at the bottom they feel heavier. Discuss the diagram in Sample Problem 1 of the *Physics Talk* and highlight the difference in magnitude of the normal force in the different positions. Emphasize how the design of the roller-coaster track must take into account the mass and speed of the car, and the radius of the loops. Point out that at the bottom of the loop the normal force required to hold up the roller coaster will be more than the normal force that supports the roller coaster at the bottom of an incline when the coaster cars are at rest. Ask students why the net force is

b) In the vertical loops you will notice that the track is more sharply curved at the top than in the curved sections between the loops? Why is the track designed that way? (Hint: Think about speed and the *g*'s you would pull on various parts of the loop.)

Physics Talk

ROLLER-COASTER SAFETY

The roller coaster has to be safe in order to be fun. Analysis of the safety requirements of the roller coaster is a valuable way of reviewing some of the physics in earlier sections. Your class and teacher can decide the level of mathematics that will be required for your challenge. In each of the examples below, there is both a qualitative and quantitative discussion.

You know from research with test pilots that people will not be safe if their acceleration is greater than 4 *g*. A free fall provides an acceleration of 1 *g*. Roller coasters may have steep inclines but they are generally not in free fall and therefore have an acceleration less than 1 *g* on straight inclines.

When the roller coaster rips around a corner or moves through the bottom of a vertical loop, the acceleration can be much more than 1 *g*. Analyze the acceleration at the bottom of a loop. The acceleration can be computed by recognizing that the roller coaster at this location is moving in an arc of a circle. The centripetal acceleration must be toward the center of the circle and can be calculated by using the equation

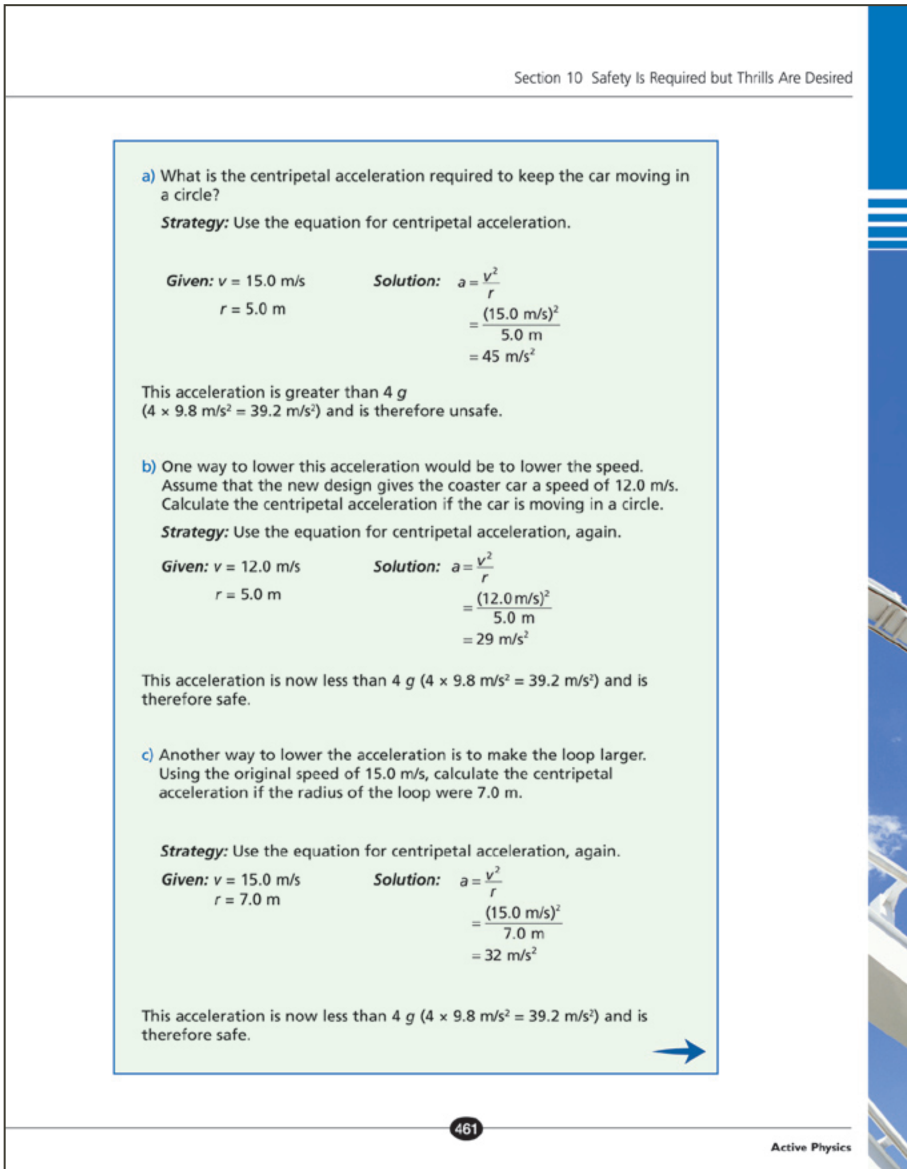
$$a = \frac{v^2}{r}$$

By varying the speed or the radius of the circle in the roller-coaster design, you can limit the acceleration to less than 4 *g*.

Qualitative (no numbers): Decreasing the speed at that point will lower the acceleration. This can be accomplished by changing the height from which the coaster descends. Less gravitational potential energy, *GPE*, will result in less kinetic energy, *KE*, and therefore a lower speed. Alternatively, you can make the curve gentler. This increases the radius of the curve and decreases the acceleration as well.

Sample Problem 1 (Quantitative – with numbers)

A roller-coaster car with a mass of 800 kg is traveling at 15.0 m/s at the bottom of a loop. The loop has a radius of 5.0 m.



a) What is the centripetal acceleration required to keep the car moving in a circle?

Strategy: Use the equation for centripetal acceleration.

Given: $v = 15.0$ m/s, $r = 5.0$ m

Solution:

$$a = \frac{v^2}{r} = \frac{(15.0 \text{ m/s})^2}{5.0 \text{ m}} = 45 \text{ m/s}^2$$

This acceleration is greater than 4 *g* ($4 \times 9.8 \text{ m/s}^2 = 39.2 \text{ m/s}^2$) and is therefore unsafe.

b) One way to lower this acceleration would be to lower the speed. Assume that the new design gives the coaster car a speed of 12.0 m/s. Calculate the centripetal acceleration if the car is moving in a circle.

Strategy: Use the equation for centripetal acceleration, again.

Given: $v = 12.0$ m/s, $r = 5.0$ m

Solution:

$$a = \frac{v^2}{r} = \frac{(12.0 \text{ m/s})^2}{5.0 \text{ m}} = 29 \text{ m/s}^2$$

This acceleration is now less than 4 *g* ($4 \times 9.8 \text{ m/s}^2 = 39.2 \text{ m/s}^2$) and is therefore safe.

c) Another way to lower the acceleration is to make the loop larger. Using the original speed of 15.0 m/s, calculate the centripetal acceleration if the radius of the loop were 7.0 m.

Strategy: Use the equation for centripetal acceleration, again.

Given: $v = 15.0$ m/s, $r = 7.0$ m

Solution:

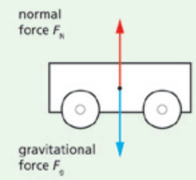
$$a = \frac{v^2}{r} = \frac{(15.0 \text{ m/s})^2}{7.0 \text{ m}} = 32 \text{ m/s}^2$$

This acceleration is now less than 4 *g* ($4 \times 9.8 \text{ m/s}^2 = 39.2 \text{ m/s}^2$) and is therefore safe.

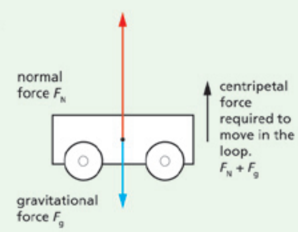
greater in the second diagram and how it is calculated.

Students should realize that in a free fall a person feels weightless because of the absence of a support force. Riders feel light or heavy based on the amount of normal force exerted during different points of the ride. This normal force is equal to 1 *g* when the cart it is at the bottom of the incline traveling horizontally or at rest. Have students explain in their *Active Physics* logs why the speed at the top of a loop must be high enough for the roller coaster to complete the loop without losing contact with the track.

The largest centripetal acceleration (at the bottom of the loop) also requires the largest centripetal force. This maximum force will inform you as roller-coaster designer of the strength of materials required to build this part of the roller coaster. The force acting on the coaster car is a combination of its weight and the normal force from the track. The normal force required when the coaster car is moving in a circle at the bottom of the loop is much greater than the normal force that would support the car at rest at the bottom of the incline. The at-rest cart requires no net force. The normal force up (provided by the track) must equal the gravitational force (weight) down. This is shown with the first vector diagram to the right.



When the car is moving in a vertical circle, a centripetal force is required. The sum of the normal force from the track and the gravitational force must equal the centripetal force required. Since the gravitational force is down, the normal force, must be greater at the bottom of the loop to provide the additional upward force needed. This is shown in the second vector diagram to the right.



The centripetal force required can be calculated using Newton's second law:

$$F_{net} = ma.$$

In this case $F_{net} = \frac{mv^2}{r}$.

In this section, you equated an acceleration with the concept of pulling "*g*'s." Pulling 4 *g*'s is actually different that experiencing an acceleration of 4 x 9.8 m/s². For example, when you stand still on Earth your acceleration is 0 *g*'s, but you are experiencing 1 *g*. Here, 1 *g* means you feel normal. The support force under your feet is equal to your weight. Experiencing 2 *g*'s means that you feel twice as heavy as normal, because the support force (often normal force) is twice your weight. The heaviness that you experience is strictly based on the support force. When you are in free fall, your acceleration is 1 *g*, but you feel weightless because there is no support force.

In the chapter, it indicates that an acceleration of greater than 4 *g*'s (as produced by $a = v^2/r$) is the maximum acceptable. Actually, if you are at the bottom of the loop, an acceleration of 4 *g*'s will require a support of 5 *g*'s (five times the rider's weight) and you will feel five times as heavy as normal. At the top of the loop, an acceleration of 4 *g*'s only requires 3 *g*'s of support force, because gravity is providing a force toward the center as well.

Sample Problem 2

A roller-coaster car with a mass of 800.0 kg is traveling at 15.0 m/s at the bottom of a loop. The loop has a radius of 5.0 m.

a) What is the centripetal force required to keep the car moving in the circle?

Strategy: Use Newton's second law to relate net force and acceleration.

Given:

$$v = 15.0 \text{ m/s}$$
$$r = 5.0 \text{ m}$$
$$m = 800.0 \text{ kg}$$

Solution:

$$F_{net} = ma = \frac{mv^2}{r}$$
$$= \frac{(800.0 \text{ kg})(15.0 \text{m/s})^2}{5.0 \text{ m}}$$
$$= 36{,}000 \text{ N}$$

This net force of 36,000 N up will allow the car to move in the vertical circle.

b) What is the normal force that the track exerts on the car?

Strategy: The normal force must be 36,000 N greater than the gravitational force to provide a net force of 36,000 N as required.

Solution: The gravitational force (weight) is:

$$F_w = mg = (800.0 \text{ kg})(9.8 \text{ m/s}^2)$$
$$= 7840 \text{ N}$$

Therefore, the normal force must equal 36,000 N + 7840 N = 43,840 N or about 44,000 N.

This indicates that the track and support structure of the roller coaster must be able to exert a force of at least 44,000 N or the track will break.

Similar calculations can be completed with the force required to make a turn on a horizontal part of the roller coaster and a turn where the roller coaster banks on its side as it whips around a turn.

Checking Up

1.

The maximum safe acceleration for a roller coaster is 4 *g*.

2.

Two methods to maintain the acceleration of the coaster cars within safe boundaries during turns and loops would be to control the speed of the car as it travels around the loop. Also to ensure that the radius of the loop is sufficiently large so that a car does not accelerate more than 4 *g* when traveling at a maximum speed in that part of the loop.

3.

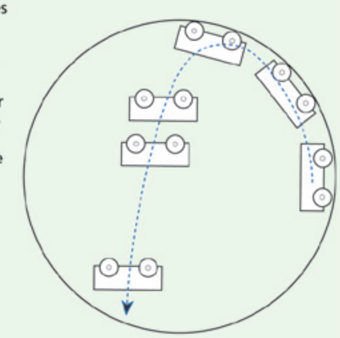
The centripetal acceleration will be greatest at the bottom of the loop because that is where the car's velocity is the greatest.

4.

The normal force will be greatest at the bottom of the loop.

Chapter 4 Thrills and Chills

Another safety feature requires that the speed at the top of the loop is great enough to complete the loop. A cart that has too little speed will not make it to the top of the roller coaster and will not be able to move in the circle. It will fall to the ground, as shown in the diagram at the right.



If gravity were the only force acting at the top of the roller coaster, then the car must require a centripetal acceleration equal to that of free fall.

Sample Problem 3

What is the minimum speed required at the top of the loop to ensure that the coaster car does not leave the track? The car has a mass of 800.0 kg. The loop has a radius of 5.0 m.

Strategy: The minimum speed pertains to the centripetal acceleration of $9.8\ m/s^2$. Using the equation for centripetal acceleration, you can find the required speed.

Given: $a = 9.8\ m/s^2$, $r = 5.0\ m$

Solution:

$$a = \frac{v^2}{r}$$

$$v = \sqrt{ar}$$

$$= \sqrt{(9.8\ m/s^2)(5.0\ m)}$$

$$= 7.0\ m/s$$

A coaster car traveling with a speed of 7.0 m/s will be able to complete the upper part of the loop. A speed greater than 7.0 m/s will also be able to make the loop. The greater speed will require a larger centripetal force. The additional force will be provided by the track pushing down on the car. In real roller coasters, the cars have special attachments under the wheels to keep the cars on the track if, for some reason, the speed gets too low near the top of the loop.

Checking Up

1. What is the maximum safe acceleration for a roller coaster?
2. List two ways to keep the acceleration of coaster cart low enough to be safe.
3. At what part of the loop is the acceleration the greatest on a roller-coaster cart?
4. At what part of the loop is the normal force the greatest?

Active Physics Plus

Students apply quantitative analysis of rollers-coaster safety to solve problems. They calculate the speed by using energy considerations and the acceleration at different points of a loop as it is determined by the speed of the coaster car and the loop's radius. Students identify the source of the forces that keep the coaster car moving in a circle, and calculate the force required for a roller coaster with a specific mass at various points around a loop. Encourage students to share their answers with other students in their class.

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
••			

More Quantitative (with numbers) Analysis

1. A roller-coaster car is traveling at 30.0 m/s at the bottom of a circular loop. The radius of the loop is 9.0 m.

   a) Using the conservation of mechanical energy,
   $KE + GPE = \text{constant}$
   $mgh + \frac{1}{2}mv^2 = \text{constant}$
   calculate the initial height of a roller-coaster car to give it a speed of 30.0 m/s at the bottom of a loop. At the highest point of the roller-coaster ride, the velocity is 0 m/s. At the bottom of the loop of the roller-coaster ride, the height h equals 0 m.

   b) Using the equation $a = \frac{v^2}{r}$, calculate the acceleration at the bottom of the loop.

   c) Is this a safe acceleration?

   d) At what speed would this loop with a radius of 9.0 m begin to be a safety concern?

   e) At what speed would a loop with a smaller radius of 7.0 m begin to be a safety concern?

   f) How fast would the roller-coaster car be traveling at the top of the loop? (Because the loop's radius is 9.0 m and the top of the loop is 18.0 m above the ground, the diameter is 18.0 m.) You must use the initial height of the roller coaster that you calculated above to solve this problem.

   g) Using the equation $a = \frac{v^2}{r}$, calculate the acceleration at the top of the loop.

   h) Is this a safe acceleration?

   i) There are two safety concerns regarding accelerations in a loop. The acceleration cannot exceed 4 g. This excessive acceleration would occur at the bottom of the loop, if at all. The acceleration at the top must be greater than 1 g (9.8 m/s²). If the acceleration required for circular motion at the top of the loop is less than 1 g, the roller-coaster car will leave the track and plummet to the ground. The speed at the top of the roller coaster must be large enough to require acceleration at least as great at 9.8 m/s².

   j) Describe if the results you found in *b) – g)* fit these safety limits.

2. The track must be strong enough to hold the roller-coaster car without breaking. You can calculate the minimum strength of a track by assuming that the roller-coaster car is filled with big football players or Sumo wrestlers.

   What force would a roller-coaster track on a horizontal section of track have to supply to hold up a car filled with passengers if the total mass were 1000.0 kg? (Remember the equation for weight is $F_w = mg$ where g = 9.8 N/kg.)

3. When the roller-coaster car makes turns, there must be a centripetal force pushing the roller-coaster car toward the center of the circle.

1.a)

Given:
$v_{\text{bottom}} = 30\text{ m/s};\ r = 9\text{ m}$

$KE_{\text{top}} + GPE_{\text{top}} =$
$KE_{\text{bottom}} + GPE_{\text{bottom}}$
$0 + mgh_{\text{top}} = \frac{1}{2}mv^2_{\text{bottom}} + 0$
$2gh_{\text{top}} = v^2_{\text{bottom}}$
$2\left(9.8\text{ m/s}^2\right)(h) = \left(30\text{ m/s}\right)^2$
$h_{\text{top}} = 46\text{ m}$

1.b)

$a_c = v^2/r$
$a_c = (30\text{ m/s})^2/9\text{ m}$
$a_c = 100\text{ m/s}^2$

1.c)

No, this acceleration is much larger than 4 g's or 40 m/s².

1.d)

$a_c = v^2/r$
$40\text{ m/s}^2 = v^2/9\text{ m}$
$19\text{ m/s} = v$

1.e)

Given:
$r = 7\text{ m}$

$a_c = v^2/r$
$40\text{ m/s}^2 = v^2/7\text{ m}$
$17\text{ m/s} = v$

1.f)

$KE_{\text{top}} + GPE_{\text{top}} =$
$KE_{\text{bottom}} + GPE_{\text{bottom}}$
$0 + mgh_{\text{top}} = \frac{1}{2}mv^2_{\text{loop top}} + mgh_{\text{loop top}}$
$2g\left(h_{\text{top}} - h_{\text{loop top}}\right) = v^2_{\text{loop top}}$
$2\left(9.8\text{ m/s}^2\right)\left(40\text{ m} - 18\text{ m}\right) = v^2_{\text{loop top}}$
$23\text{ m/s} = v_{\text{loop top}}$

1.g)

$a_c = v^2/r$
$a_c = (23\text{ m/s})^2/9\text{ m}$
$a_c = 59\text{ m/s}^2$

1.h)

No, this acceleration is not safe, since it is more than 4 g.

1.j)

The results are the same as described in *Step 1.i)* of the *Student Edition*.

2.

Given:
$m = 1000\text{ kg};\ g = 9.8\text{ m/s}^2$

$F_w = mg$
$F_w = \left(1000\text{ kg}\right)\left(9.8\text{ m/s}^2\right) =$
9800 N

3.a)

A sharp horizontal left turn on a flat track would need a centripetal force to the left pointing toward the center of the circle. A loop always needs centripetal force acting toward the center of the circle.

CHAPTER 4

3.b)

For the horizontal, flat track friction supplies the centripetal force. For the horizontal, vertical track, the normal force supplies the force. For the loop, the normal force provides the force on the bottom, and a combination of the normal force and the weight provide the centripetal force at the top.

4.

Given:
$m = 1000$ kg; $r = 12$ m; $v = 15$ m/s

$F_c = mv^2/r$
$F_c = (1000 \text{ kg})(15 \text{ m/s})^2/12 \text{ m} = 18,800 \text{ N}$

5.

The same force as in *Question 4* is required. For the vertical track the force is supplied by the normal. For a flat track it must be supplied by friction.

6.a)

The same centripetal force as *Step 4* is required.

6.b)

Students copy the diagram for the question into their logs

6.c)

Given:
$m = 1000$ kg; $r = 12$ m; $v = 15$ m/s

$F_w = mg$
$F_w = (1000 \text{ kg})(9.8 \text{ m/s}^2) = 9800 \text{ N}$

6.d)

$F_c = F_{normal} - F_{weight}$
$18,800 \text{ N} = F_{normal} - 9800 \text{ N}$
$28,600 \text{ N} = F_{normal}$

The following are three types of turns that will be analyzed:

- a sharp left turn on a flat track,
- a sharp left turn where the car is turned on its side and the track is vertical, and
- a loop where the car is moving in a vertical circle.

a) For each of the three turns, identify the direction of the force keeping the car moving in a circle.

b) For each of the three turns, identify the source of this force. (The force could be the force of friction, the gravitational force, the normal force, the force of tension on a rope, the force of the wheels in the track or any combination of these forces.)

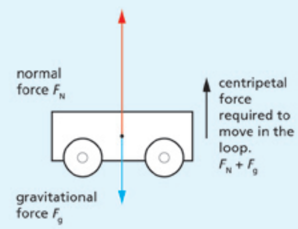
4. A roller-coaster car has a mass of 1000.0 kg and takes a sharp left turn where the car is turned on its side and the track is vertical with a radius of 12.0 m and the car is moving at 15.0 m/s. Calculate the centripetal force required to keep the car moving in the curve.

5. A roller-coaster car has a mass of 1000.0 kg and takes a sharp left turn on a flat track with a radius of 12.0 m and the car is moving at 15.0 m/s. Calculate the centripetal force required to keep the car moving in the curve.

6. A roller-coaster car has a mass of 1000.0 kg and is about to enter a vertical loop that has a radius of 12.0 m. The car is moving at 15.0 m/s.

a) Calculate the centripetal force required to keep the car moving in the curve.

b) This centripetal force that you calculated is the sum of the normal force of the track up toward the center of the roller-coaster car and the force of gravity down to the ground. Copy the following diagram into your log:



c) Calculate the force of gravity on the car.

d) Calculate the normal force on the car. This is the magnitude of the force that the tracks and frame of the roller-coaster structure must be able to exert on the car to keep it moving in a circle.

What Do You Think Now?

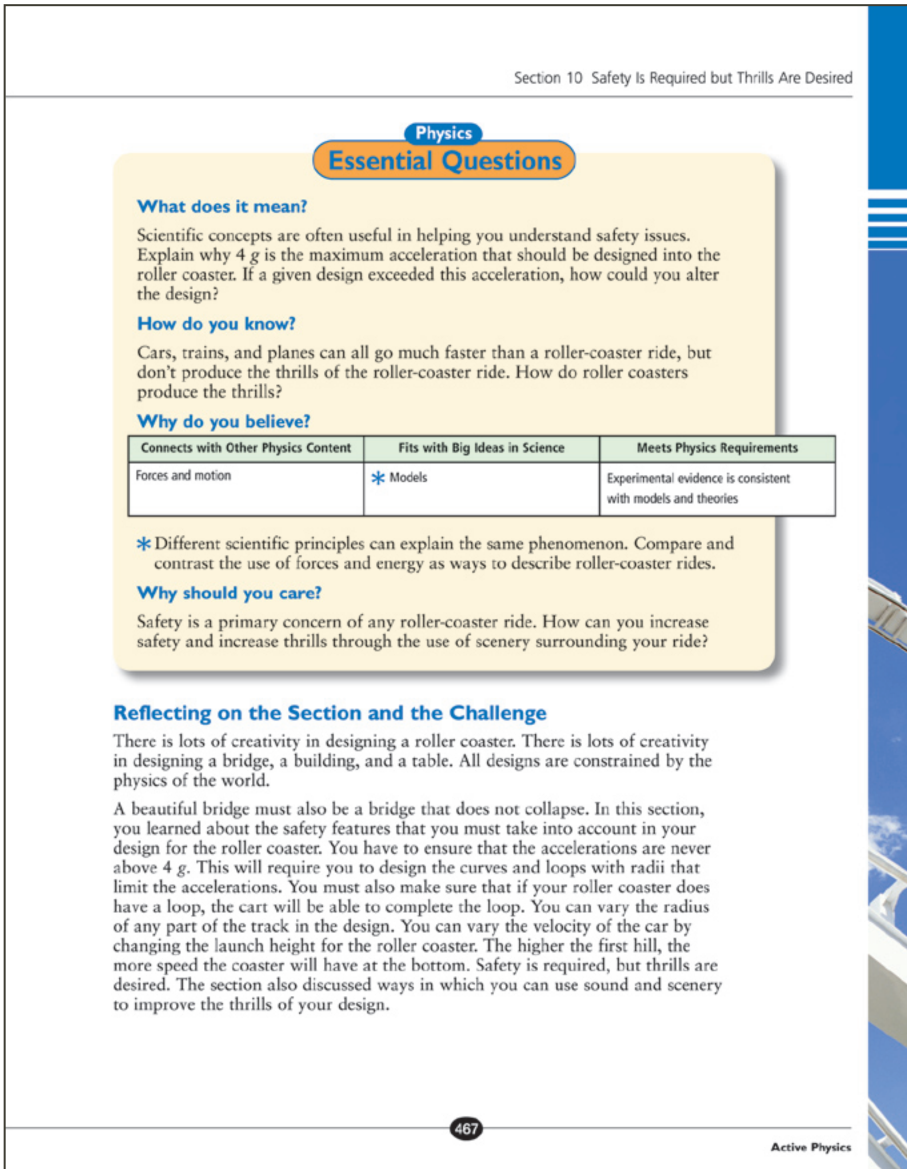
At the beginning of this section, you were asked the following:

- **Does the knowledge that people can get hurt or die on a roller coaster change the thrill of the ride?**
- **Would your answer change if you found out that one-half of all roller-coaster rides ended in the death of its passengers?**

Record you ideas about these questions now. What aspects of the roller-coaster ride are most likely to be dangerous if the ride is not properly designed? Describe how you would use what you have learned about the physics of roller coasters to make sure your roller-coaster ride is safe.

What Do You Think Now?

Ask students to refer to the responses you had previously posted on the board during a discussion of the *What Do You Think?* questions. Students now have an opportunity to update and revise their original answers. Emphasize that there is still time for them to clear earlier misconceptions by sharing their answers with other members of their group. Encourage students to return to the *What Do You See?* illustration and review their responses. Discuss the answer in *A Physicist's Response*. Ask students to incorporate how forces acting on the tracks and the coaster cars would be significant to the design of their roller coaster. Also, highlight the importance of keeping acceleration below 4 *g* and the tracks strong enough to provide a normal force equal to the sum of the net force and the gravitational force.



Physics
Essential Questions

What does it mean?

Scientific concepts are often useful in helping you understand safety issues. Explain why 4 *g* is the maximum acceleration that should be designed into the roller coaster. If a given design exceeded this acceleration, how could you alter the design?

How do you know?

Cars, trains, and planes can all go much faster than a roller-coaster ride, but don't produce the thrills of the roller-coaster ride. How do roller coasters produce the thrills?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	* Models	Experimental evidence is consistent with models and theories

* Different scientific principles can explain the same phenomenon. Compare and contrast the use of forces and energy as ways to describe roller-coaster rides.

Why should you care?

Safety is a primary concern of any roller-coaster ride. How can you increase safety and increase thrills through the use of scenery surrounding your ride?

Reflecting on the Section and the Challenge

There is lots of creativity in designing a roller coaster. There is lots of creativity in designing a bridge, a building, and a table. All designs are constrained by the physics of the world.

A beautiful bridge must also be a bridge that does not collapse. In this section, you learned about the safety features that you must take into account in your design for the roller coaster. You have to ensure that the accelerations are never above 4 *g*. This will require you to design the curves and loops with radii that limit the accelerations. You must also make sure that if your roller coaster does have a loop, the cart will be able to complete the loop. You can vary the radius of any part of the track in the design. You can vary the velocity of the car by changing the launch height for the roller coaster. The higher the first hill, the more speed the coaster will have at the bottom. Safety is required, but thrills are desired. The section also discussed ways in which you can use sound and scenery to improve the thrills of your design.

Reflecting on the Section and the Challenge

Allow students the time to reflect on a safe roller-coaster design for their *Chapter Challenge*. Discuss the factors that affect the acceleration of a roller coaster. Ask students to consider the radii of the loops and the highest speed that the roller coaster will reach. Emphasize that students can vary the velocity of the ride by changing the launch height. Ask them to write down a list of equations in their logs that will help them calculate the roller-coaster's velocity at different locations of the ride. Students should refer to their *Active Physics* log responses and bear in mind that velocity must not exceed 4 *g* at any instance of the ride. Point out that they will be responsible for calculations of the acceleration of the roller-coaster car at various points along the track to ensure that the acceleration limit of 4 *g*'s is not exceeded.

Physics Essential Questions

What does it mean?

With too much acceleration, humans can become unconscious as the blood rushes from parts of the brain. If there are too many *g*'s for a design, the curve can be changed to lower the *g* forces.

How do you know?

The thrill comes from the accelerations, not the speeds.

Why do you believe?

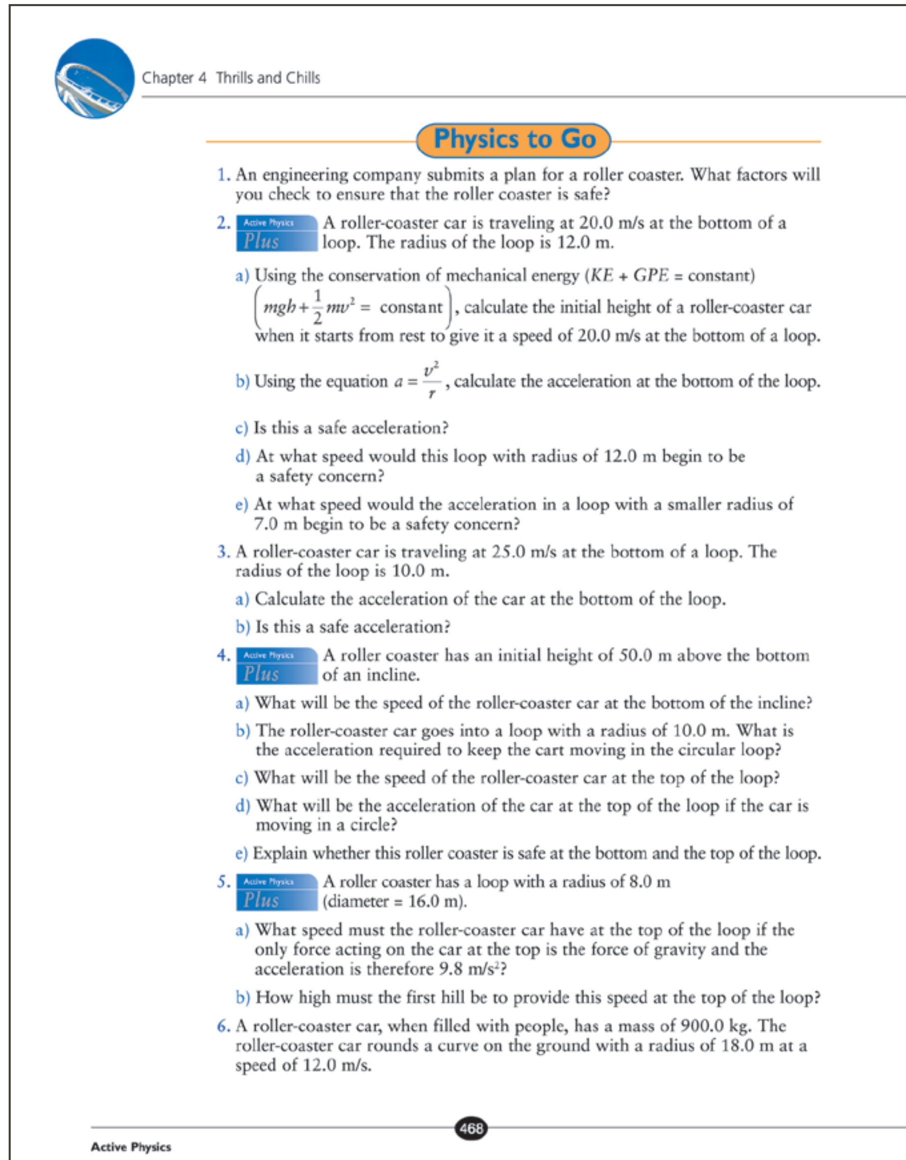
Energy is a scalar and can help you calculate the speed at each point of the roller coaster.

Force is a vector and can help you calculate your apparent weight on you at different locations of the roller coaster.

Why should you care?

It can appear that the roller coaster is going to hit something right before it turns.

Physics to Go



Chapter 4 Thrills and Chills

Physics to Go

1. An engineering company submits a plan for a roller coaster. What factors will you check to ensure that the roller coaster is safe?

2. Active Physics Plus A roller-coaster car is traveling at 20.0 m/s at the bottom of a loop. The radius of the loop is 12.0 m.

a) Using the conservation of mechanical energy ($KE + GPE$ = constant) $\left(mgh + \frac{1}{2}mv^2 = \text{constant}\right)$, calculate the initial height of a roller-coaster car when it starts from rest to give it a speed of 20.0 m/s at the bottom of a loop.

b) Using the equation $a = \frac{v^2}{r}$, calculate the acceleration at the bottom of the loop.

c) Is this a safe acceleration?

d) At what speed would this loop with radius of 12.0 m begin to be a safety concern?

e) At what speed would the acceleration in a loop with a smaller radius of 7.0 m begin to be a safety concern?

3. A roller-coaster car is traveling at 25.0 m/s at the bottom of a loop. The radius of the loop is 10.0 m.

a) Calculate the acceleration of the car at the bottom of the loop.

b) Is this a safe acceleration?

4. Active Physics Plus A roller coaster has an initial height of 50.0 m above the bottom of an incline.

a) What will be the speed of the roller-coaster car at the bottom of the incline?

b) The roller-coaster car goes into a loop with a radius of 10.0 m. What is the acceleration required to keep the cart moving in the circular loop?

c) What will be the speed of the roller-coaster car at the top of the loop?

d) What will be the acceleration of the car at the top of the loop if the car is moving in a circle?

e) Explain whether this roller coaster is safe at the bottom and the top of the loop.

5. Active Physics Plus A roller coaster has a loop with a radius of 8.0 m (diameter = 16.0 m).

a) What speed must the roller-coaster car have at the top of the loop if the only force acting on the car at the top is the force of gravity and the acceleration is therefore 9.8 m/s²?

b) How high must the first hill be to provide this speed at the top of the loop?

6. A roller-coaster car, when filled with people, has a mass of 900.0 kg. The roller-coaster car rounds a curve on the ground with a radius of 18.0 m at a speed of 12.0 m/s.

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Active Physics

1.

To ensure that the roller coaster is safe, you should check to see if

- no acceleration is greater than 4 *g*,
- the acceleration at the top of a vertical loop is at least 1 *g*, and
- the tracks are strong enough to supply the forces required.

2.a) Active Physics Plus

Given:

$v_{bottom} = 20$ m/s; $r = 12$ m

$$GPE_{top} + KE_{top} = GPE_{bottom} + KE_{bottom}$$
$$mgh_{top} + 0 = 0 + \tfrac{1}{2}mv_{bottom}^2$$
$$h_{top} = v_{bottom}^2 / 2g$$
$$h_{top} = (20\text{ m/s})^2 / 2(9.8\text{ m/s}^2)$$
$$h_{top} = 20\text{ m}$$

2.b)

$$a = v^2/r$$
$$a = (20\text{ m/s})^2/(12\text{ m})$$
$$a = 33\text{ m/s}^2$$

2.c)

There is no safety concern because the acceleration is less than 4 *g* (40 m/s²).

2.d)

If the acceleration were limited to 40 m/s², you can find the corresponding velocity.

$$a = v^2/r$$
$$v = \sqrt{ar} = \sqrt{(40\text{ m/s})^2(12\text{ m})} = 22\text{ m/s}$$

2.e)

Given:

$r = 7$ m

$$a = v^2/r$$
$$v = \sqrt{ar} = \sqrt{(40\text{ m/s})^2(7\text{ m})} = 17\text{ m/s}$$

3.a)

Given:

$v = 25$ m/s; $r = 10$ m

$$a = v^2/r$$
$$a = (25\text{ m/s})^2/(10\text{ m})$$
$$a = 63\text{ m/s}^2$$

3.b)

There is a safety concern since the acceleration is more than 4 *g* (40 m/s²).

4.a) Active Physics Plus

Given:
$h = 50$ m

$GPE_{top} + KE_{top} =$
$GPE_{bottom} + KE_{bottom}$
$mgh + 0 = 0 + \frac{1}{2}mv^2_{bottom}$
$v^2_{bottom} = 2gh$
$v_{bottom} = \sqrt{2gh} =$
$\sqrt{2(9.8 \text{ m/s}^2)(50 \text{ m})} = 31 \text{ m/s}$

4.b)

Given:
$r = 10$ m

$a = v^2/r$
$a = (31 \text{ m/s})^2/(10 \text{ m})$
$a = 96 \text{ m/s}^2$

4.c)

Top of ride to top of loop

$GPE_{top} + KE_{top} =$
$GPE_{top\ of\ loop} + KE_{top\ of\ loop}$
$mgh_{top} + 0 = mgh_{top\ of\ loop} + \frac{1}{2}mv^2$
$gh_{top} = gh_{top\ of\ loop} + \frac{1}{2}v^2$
$v^2 = 2g(h_{top} - h_{top\ of\ loop})$
$v^2 = 2(9.8 \text{ m/s}^2)(50 \text{ m} - 20 \text{ m})$
$v = 24$ m/s

4.d)

$a = v^2/r$
$a = (24 \text{ m/s})^2/(10 \text{ m})$
$a = 58 \text{ m/s}^2$

4.e)

Yes, there is a safety concern at both the top and bottom of the loop. The acceleration is greater than 4 *g*.

5.a) Active Physics Plus

Given:
$r = 8$ m

$a = v^2/r$
$v = \sqrt{gr}$

(The acceleration must equal the free-fall acceleration due to gravity, $g = 9.8 \text{ m/s}^2$.)

$v = \sqrt{(9.8 \text{ m/s}^2)(8 \text{ m})} = 8.9 \text{ m/s}$

5.b)

Top of hill to top of loop

$GPE_{top} + KE_{top} =$
$GPE_{top\ of\ loop} + KE_{top\ of\ loop}$
$mgh_{top} + 0 = mgh_{top\ of\ loop} + \frac{1}{2}mv^2$
$gh_{top} = gh_{top\ of\ loop} + \frac{1}{2}v^2$
$h_{top} = h_{top\ of\ loop} + \frac{1}{2}v^2/g$
$h_{top} =$
$16 \text{ m} + \frac{1}{2}(8.9 \text{ m/s})^2/(9.8 \text{ m/s}^2)$
$h_{top} = 16 \text{ m} + 4 \text{ m}$
$h_{top} = 20$ m

NOTES

CHAPTER 4

NOTES

6.a)

Given:
$m = 900$ kg; $r = 18$ m;
$v_{bottom} = 12$ m/s

$a = v^2/r$
$a = (12 \text{ m/s})^2/(18 \text{ m})$
$a = 8.0 \text{ m/s}^2$

6.b)

$F_c = ma = mv^2/r$
$F_c = (900 \text{ kg})(8.0 \text{ m/s}^2)$
$F_c = 7200$ N

6.c)

The centripetal force will be provided by the track on the wheels.

7.a)

Given:
$m = 900$ kg; $r = 15$ m; $v = 20$ m/s

$a = v^2/r$
$a = (20 \text{ m/s})^2/(15 \text{ m})$
$a = 26.7 \text{ m/s}^2$

7.b)

$F_c = ma = mv^2/r$
$F_c = (900 \text{ kg})(26.7 \text{ m/s}^2)$
$F_c = 24{,}000$ N

7.c)

Yes, the coaster is safe because the wheels only have to supply a force of 24,000 N.

8.a)

No, the acceleration is not dependent on the mass.

Section 10 Safety Is Required but Thrills Are Desired

a) What is the centripetal acceleration of the car?

b) What is the centripetal force on the car?

c) What will provide this centripetal force?

7. A roller-coaster car, when filled with people, has a mass of 900.0 kg. The roller-coaster cart rounds a curve on the ground with a radius of 15.0 m at a speed of 20.0 m/s.

a) What is the centripetal acceleration of the car?

b) What is the centripetal force on the car?

c) The wheels in the tracks can provide a force of 25,000 N. Is the roller coaster safe?

8. A roller coaster is able to complete a loop when the car has two passengers. The car is loaded with six people.

a) Will the centripetal acceleration change as a result of the change in mass?

b) Will the roller coaster be going faster, slower, or the same speed at the bottom of the loop with the extra passengers?

c) Will the roller-coaster track require a stronger material because of the increased number of riders?

Inquiring Further

Amusement-park physics

Visit an amusement-park physics site as directed by your teacher and explore some of the safety factors in designing roller coasters. Report your findings back to your team.

CHAPTER 4

8.b)

The roller coaster will be traveling at the same speed.

8.c)

Yes, the roller-coaster track will require a stronger material. The force is proportional to the mass $(F_{net} = ma)$.

Inquiring Further

An excellent opportunity to investigate roller coasters is often available during "physics days" at many amusement parks. Generally, these days occur during the spring, and the parks are closed to the general public and available to students from various local schools to ride and analyze the various rides at the park. The students will have a chance to question the park operators about the various safety features on the rides. Some of these features may include the following:

- Interlocks that do not allow the ride to operate unless all the seat bars that keep students in the car are locked down
- Sensors along the path to indicate if the cars have the proper speed to negotiate the loops and turns
- Fail-safe mechanisms to stop the cart if there is a failure in any of the systems, such as the brakes, and simple mechanical safe guards, such as opposing wheels below the wheels of the coaster cars to prevent the cars from leaving the track.

SECTION 10 QUIZ

4-10a **Blackline Master**

(Assume the acceleration of gravity is 10 m/s^2 for all questions)

1. A roller-coaster designer wants to add a horizontal loop to the roller coaster, but wants to keep the maximum acceleration equal to 4 *g*'s or 40 m/s^2. If the loop has a radius of 10 m, approximately what is the maximum speed the cart can reach during the loop and not exceed the 4-*g* maximum?

   a) 10 m/s b) 20 m/s
   c) 30 m/s d) 40 m/s

2. A roller coaster has a curve of radius 12 m at the top of a hill. If the car has a speed of 9.0 m/s at the top of the hill, what is the cars centripetal acceleration?

   a) 9.8 m/s^2 b) 8.5 m/s^2
   c) 6.8 m/s^2 d) 22 m/s^2

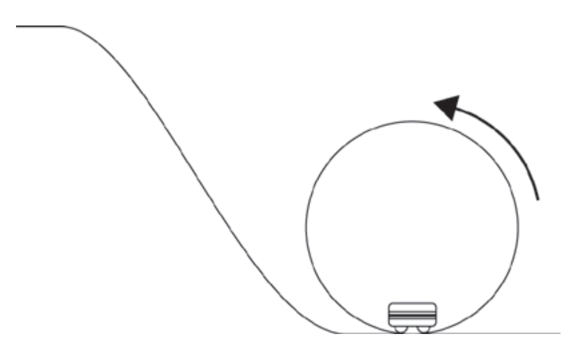
3. In *Question 2*, what would be the apparent weight of a 50-kg student riding the roller coaster at the top of the hill?

   a) 160 N b) 400 N
   c) 650 N d) 840 N

4. The vertical loop in a roller coaster has a radius of 15 m. At the very peak, if all the centripetal force is provided by the force of gravity (the normal force of the track is zero), approximately what is the minimum speed that a roller-coaster car must have at the peak of the loop to maintain contact with the track?

   a) 9.8 m/s b) 12 m/s
   c) 16 m/s d) 19 m/s

5. A roller-coaster car's loop is located at the bottom of the first hill as shown in the diagram to the right. Approximately what should be the height of the hill if the coaster car starts at rest from the top of the hill and is to have a speed of 25 m/s as the car enters the loop?



   a) 18 m
   b) 21 m
   c) 26 m
   d) 32 m

SECTION 10 QUIZ ANSWERS

1. b) Using the formula for centripetal acceleration $a_c = v^2/r$ and then solving for v gives $40\ \text{m/s}^2 = v^2/(10\ \text{m})$ or $v = 20\ \text{m/s}$.

2. c) Using the formula for centripetal acceleration $a_c = v^2/r$ and then solving for the acceleration gives $a_c = (90\ \text{m/s})^2/(12\ \text{m}) = 6.8\ \text{m/s}^2$.

3. a) Using Newton's second law for the forces acting on the car at the top of the loop gives $F_c = F_w - F_N$. The centripetal force is found by multiplying the centripetal acceleration times the mass, giving $ma_c = mg - F_N$. Solving for F_N gives $(50\ \text{kg})(6.8\ \text{m/s}^2) = (50\ \text{kg})(10\ \text{m/s}^2) - F_N$ or $F_N = 160\ \text{N}$.

4. b) At the peak, if all the centripetal force is provided by gravity, then $F_c = F_w$ or $mv^2/r = mg$. Canceling the mass and solving for v gives $v^2/(15\ \text{m}) = 10\ \text{m/s}^2$ or $v \approx 12\ \text{m/s}$.

5. d) Using conservation of energy, the *GPE* at the top of the hill equals the *KE* at the bottom of the hill, or $GPE = KE$. Putting the values into the formulas and solving for the height of the hill gives $mgh = \frac{1}{2}mv^2$ or after canceling the mass, $(10\ \text{m/s}^2)(h) = \frac{1}{2}(25\ \text{m/s})^2$. This gives a hill height of approximately 32 m.

CHAPTER 4