SECTION 2

Gravitational Potential Energy and Kinetic Energy: What Goes Up and What Comes Down

Section Overview

Students find out if the final speed of a steel ball is affected by the angle of incline or the height of a sloped track. They vary the height of a track and roll a steel ball down the track to determine the final speed of the ball from different distances. If the values of the final speed obtained are close to each other, students calculate the average speed for the ball at the bottom of the track. They record four different initial heights and their corresponding distances and find that the speed of the ball increases with an increase if the height and/or the distance increase. Next, students keep one of the two variables of distance or height constant and vary the angle of incline to find the speed of the ball at the bottom of the track.

In the inquiry investigations that follow, students review the data in the second step of the *Investigate* to determine if there is a pattern between the starting heights and the speed or between the starting distances along the track and speed. Students then repeat the previous investigations. They change the angle of the track and predict the speed at the bottom of the track.

Students conduct previous investigations using a curved track and compare the predicted speed with the measured speeds. In the *Investigate* with a pendulum, students determine if a pattern exists between the initial height of the mass of a string and its speed at the bottom of the swing and plot the data on a graph for height versus speed squared.

Background Information

The conservation of energy is arguably the most important principle in science. The roller-coaster ride is a wonderful application of the conservation of mechanical energy. If you ignore loss of mechanical energy due to friction producing sound and heat, you can state that the changes in the kinetic energy (and speed) of the roller coaster are due to changes in the gravitational potential energy of the roller coaster.

Mechanical energy is the sum of the kinetic energy (*KE*) and potential energy (*PE*). The *PE* is the energy a system has by virtue of its position or spatial configuration. Gravitational potential energy (*GPE*) near Earth is expressed in terms of the position of an object relative to Earth. In this chapter, the students also meet up with spring potential energy associated with the compression or extension of a spring.

If mechanical energy is converted into other forms, such as heat energy (thermal energy), sound energy, light energy and so on, you say that some of the mechanical energy is "dissipated." You can also use other forms of energy to change the kinetic energy or potential energy of an object. In that case, you say that the source of the other form of energy is "doing work" on the object. *Section 7* explores the concept of work in terms of getting a roller-coaster car to the top of the first hill of the ride.

The gravitational potential energy of the roller coaster can be expressed as mgh, where h is taken to be the height of the roller-coaster car above the lowest point of the ride. Strictly speaking, all we need to consider are changes in the height of the roller-coaster car. What one call's zero height doesn't matter. Most students, however, are more comfortable if there is a definite "zero point." Using the lowest point of the ride as the zero point ($h = 0$ there) simplifies the analysis of *GPE* and avoids the possible confusion of thinking about different zero points.

In this section, g is introduced as the "strength of the gravitational field" rather than as the "acceleration due to gravity." Near the surface of Earth, $g = 9.8$ N/kg is the strength of the gravitational field. In the first applications of g, in computing the gravitational potential energy, the object is not always accelerating in free-fall. Later on, when weight is introduced, g is again referred to as the strength of the gravitational field, so that the weight of force $F_w = mg$. If an object is moving under the influence only of gravity (the so-called free-fall case) then its acceleration is numerically equal to $g = 9.8\ \text{m/s}^2$. Referring to g as the strength of the gravitational field should make it easier for students to understand why the weight of an object is less on the Moon than it is on Earth: The Moon has a weaker gravitational field.

Energy is measured in joules (J) (pronounced "jewels"). You can find the equivalent units for joule by looking at any equation for energy. For example, for gravitational potential energy (*GPE*):

$$GPE = mgh = (\text{kg})(\text{N/kg})(\text{m}) = (\text{N})(\text{m}) = (\text{kg})(\text{m/s}^2)(\text{m}) = (\text{kg})(\text{m}^2/\text{s}^2)$$

$$1\ \text{J} = 1\ \text{kg}\left(\text{m}^2/\text{s}^2\right)$$

$$KE = \tfrac{1}{2}mv^2 = (\text{kg})(\text{m/s})^2 = (\text{kg})\left(\text{m}^2/\text{s}^2\right)$$

$$1\ \text{J} = 1\ \text{kg}\left(\text{m}^2/\text{s}^2\right)$$

If the mechanical energy of the roller coaster is conserved, you can compare the mechanical energy at any two points on the roller coaster without concern for what happened at other points on the roller coaster. As a consequence, the speed of the roller coaster at any one point depends only on the height of the roller coaster at that point and not on whether the roller coaster is moving up or down, or is level at that point. If you know the total mechanical energy at any point on the roller coaster, you know that every other point has an identical mechanical energy (if frictional forces are insignificant). If you know the height at that point, you can easily calculate the gravitational potential energy. From there, you can then calculate the kinetic energy because the remainder of the mechanical energy is kinetic energy. Knowing the kinetic energy and the mass of the cart allows us to find the speed. Equating the total mechanical energy at any point to the total mechanical energy at another point, you see that the mass in the equation "drops out."

To help the students think about the conservation of mechanical energy, the text introduces energy bar charts, which show *GPE* and *KE* in terms of the heights of the bars. If the roller-coaster car starts at the top of the first hill almost at rest, its *KE* will be very small (just about zero) and all its mechanical energy will be in the form of *GPE*. Halfway down the hill, the mechanical energy will be equally divided between *GPE* and *KE*. At the bottom of the hill ($h = 0$), the mechanical energy will be all *KE*. At any location along the hill the sum of the heights of the two energy bars should be the same.

Mechanical Energy at Point A =
Mechanical Energy at Point B

$$GPE_A + KE_A = GPE_B + KE_B$$

$$mgh_A + \tfrac{1}{2}mv_A^2 = mgh_B + \tfrac{1}{2}mv_B^2$$

$$gh_A + \tfrac{1}{2}v_A^2 = gh_B + \tfrac{1}{2}v_B^2$$

The physical significance of the mass "dropping out" of the equation is that the speed of the roller-coaster car at any point is not dependent on the mass of the roller-coaster car.

The conservation of mechanical energy can also be applied to a ball falling or (as in the *Inquiring Further*) a skateboarder on the vert. It can also be used to explain the changes in speed of a pendulum or a playground swing.

Crucial Physics

- The speed increase of an object that has descended a vertical distance under the action of gravity depends only on the height h and not on the details of the path as long as friction and air resistance are not factors. In fact, the square of the speed gained is proportional to h.
- Gravitational potential energy (GPE) is the energy an object has due to its position in a gravitational field, such as Earth's. For an object near the surface of Earth, $GPE = mgh$, where m is the object's mass, h is the height above the ground or the object's lowest possible position, and g is the strength of the gravitational field, which is sometimes called the acceleration due to gravity. For Earth, $g = 9.8$ N/kg or $9.8\ \text{m/s}^2$. GPE is a scalar and is measured in joules.
- Kinetic energy (KE) is the energy an object has due to its speed and is related to its mass m, and its speed v.
  $KE = \frac{1}{2}mv^2$.
  KE is a scalar and is measured in joules.
- For a rising or descending mass with no outside forces, mechanical energy is the sum of KE and GPE. If no dissipative forces (such as friction or air resistance) act on the system, then the mechanical energy is conserved and remains constant. In this situation
  mechanical energy $= KE + GPE =$ constant.
- If mechanical energy is conserved, then the speed of an object after falling a vertical distance depends only on the vertical displacement and not on the path traveled.

Learning Outcomes	Location in the Section	Evidence of Understanding
Detect the speed of an object at the bottom of a ramp.	***Investigate*** Step 1	Students find the speed of a ball at the bottom of a ramp by placing the ball at several positions.
Identify the relationship between the speed at the bottom of a ramp and both the height and angle of the ramp.	***Investigate*** Steps 2-5	Students find the speed of a ball at the bottom of a ramp by varying the angle of the ramp and keeping the height constant.
Complete a graph of speed versus height of the ramp.	***Investigate*** Step 2	Students find the speed of a ball as it relates to the initial height it was placed.
Define and calculate gravitational potential energy and kinetic energy.	***Physics Talk***	Students learn the definition of gravitational potential energy and kinetic energy and learn how to calculate the *KE* and *GPE*.
State the conservation of energy.	***Physics Talk***	Students use the principle that the total mechanical energy at the top and bottom of the roller-coaster ride remains constant.
Relate the conservation of energy to a roller-coaster ride.	***Physics Talk***	Students read how the total energy at all stages of the ride remains constant. They also solve a practice problem to see how energy at the top and bottom of a roller-coaster ride remains the same.

Section 2 Materials, Preparation, and Safety

Materials and Equipment

PLAN A		
Materials and Equipment	**Group (4 students)**	**Class**
Beespi	1 per group	
Ball, steel sphere, 3/4 in.	1 per group	
Track (plastic piece for roller coaster)	1 per group	
Track, wood, base pack (to include hardware)	1 per group	
Holder, right-angle, cast iron	1 per group	
Rod, aluminum, 3/8 in. x 12 in. (to act as crossarm)	1 per group	
Ring stand, large	1 per group	
Meter stick	1 per group	
C-clamp, 3 in.	1 per group	
Ruler, metric, in./cm	1 per group	
Mass, hanging, 100 g	1 per group	
Scissors		1 per class
Battery, alkaline, AAA	2 per group	
Paper, graph, pkg. of 50		1 per class
String, ball		1 per class

*Additional items needed not supplied

Time Requirements

- Allow one and a half class periods or 65 minutes for the *Investigate*.

Teacher Preparation

- If you are using the velocimeters, familiarize yourself and the students with how to convert the readings to meters/second. If you are using photogates, show the students how to place the photogate so the center of the ball passes through the gate and how to calculate the ball's speed.
- If tracks are not available for the ball, inclined planes with meter sticks taped on may be used to provide a path.
- For *Part B*, cylindrical hooked masses work best, since the diameter that passes through the photogate is uniform.
- Photogates and velocimeters often will not work correctly in direct sunlight.

Safety

- Warn the students that the string that holds the swinging mass in *Part B* may break. Be sure that no feet or other body parts are below any area where the mass might fall if the string breaks.
- Instruct the students on how to release the swinging mass in *Part B* so that it does not strike and damage the velocimeter.
- The students should keep careful control over the steel balls used in *Part A*. They should not land on the floor where someone may slip on them.

Materials and Equipment

PLAN B		
Materials and Equipment	**Group (4 students)**	**Class**
Beespi		1 per class
Ball, steel sphere, 3/4 in.		1 per class
Track (plastic piece for roller coaster)		1 per class
Track, wood, base pack (to include hardware)		1 per class
Holder, right-angle, cast iron		1 per class
Rod, aluminum, 3/8 in. x 12 in. (to act as crossarm)		1 per class
Ring stand, large		1 per class
Meter stick		1 per class
C-clamp, 3 in.		1 per class
Ruler, metric, in./cm		1 per class
Mass, hanging, 100 g		1 per class
Scissors		1 per class
Battery, alkaline, AAA		2 per class
Paper, graph, pkg. of 50		1 per class
String, ball		1 per class

*Additional items needed not supplied

Time Requirements

- Allow one class period or 45 minutes to do the *Investigate* and *Physics Talk* discussion.

Teacher Preparation

- Most of the planning for *Plan A* still applies, but only one material setup is needed. Prepare one track and velocimeter or photogate setup for the front of the classroom. Several students should be recruited to assist you in taking the measurements and recording them on the board for the rest of the class to record in their journals. If a camera and projection system is available it should be used so the students can see more clearly what is occurring.
- Practice the trials prior to the class to be certain consistent results are obtained. Mark your starting positions for the ball on the track, and make the conversions from the km/h recorded by the velocimeter to the m/s needed for calculation prior to the start of class.
- Have a ring stand with a horizontal bar extending out with a 60-cm piece of string dangling down ready for *Part B*. A 100-gram cylindrical mass should be tied to the end of the string to allow it to freely swing through the velocimeter. A meter stick or similar measuring device should be at hand to measure the height and the diameter of the mass before it starts swinging.
- Make a Blackline Master of the diagram for *Step 2* of the *Investigate* and the Energy Graphs for the *Physics Talk* in the *Student Edition*.
- Summarize the *Investigate* to familiarize the students with what they will be observing.

Safety

- Keep careful control over the steel balls used in *Part A*. They should not land on the floor where someone may slip on them.

NOTES

Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

Learning Issue	Reference	Augmentation and Accommodations
Constructing a graph	***Investigate*** Part B, Steps 2 and 3	**Augmentation** • Students often struggle to compare decimal values, especially when required to set up the scale on the axis of a graph. If possible, ask students to think of the decimal values in terms of money. For example, 0.03 is three cents. • Review place value. Ask students to identify their largest and smallest values from their data set and decide on a scale for their graph axes. **Accommodation** • Provide a graph that is already set up with labels and scales, and then ask students to plot their data points.
Reading comprehension	***Physics Talk***	**Augmentation** • Students who struggle with reading comprehension have an even more difficult time reading explanations of mathematical concepts with new vocabulary and symbols. Use the pictures and graphs in this section to aid students' comprehension. • Provide direct instruction to teach students the value and use of pictures, diagrams, and graphs in science textbooks as a resource for gathering information. • Ask students to explain the relationship between KE and PE_{grav} in their own words. For whole-to-part learners, understanding the big concept of conservation of energy will help them understand the calculations.
Understanding derivation of formulas	***Physics Talk***	**Augmentation** • Understanding the derivation of a formula is an extremely difficult concept for students. They often do not attach concrete meaning to abstract variables. Provide direct instruction to model the derivation. • In pairs, ask students to assign values to the variables and test the derivation. If each pair of students tests a different set of values, students will have more examples to view. These examples make the abstract concept more concrete. **Accommodation** • Some students may not have the pre-requisite skills or be developmentally ready to understand this concept. Teach these students to use the derived formula to solve for velocity rather than focusing on the derivation.
Completing calculations in charts	***Physics to Go*** Questions 4, 6, and 11	**Augmentation** • For students who have a difficult time processing visual information, tables are not an ideal format for doing calculations and comparing numbers because this format is visually over-stimulating. Encourage students to show their work step-by-step and then fill answers into the table. • Students could also recopy the table in a larger format. • Provide time for students to complete *Physics to Go*, *Question 4* in class, and then assign *Physics to Go*, *Questions 6 and 11* for homework. This allows students to ask questions and get immediate feedback. **Accommodation** • Writing tasks are often laborious for students who have difficulty with visual processing. Provide these students with a large-print format of the table to complete. • Reduce the assignment by asking student to complete one of the tables and focus on taking their time and self-monitoring their accuracy.

Strategies for Students with Limited English-Language Proficiency

Learning Issue	Reference	Augmentation
Comprehension	***Investigate*** Part A, Steps 2.a), b); 4.a), b); 5.a); and Part B, Step 1.a)	Check students' data tables to make sure the rows and columns are labeled correctly and that students are recording the correct information in the correct table cells.
Identifying relationships	***Investigate*** Part A, Step 3.a)	Make sure students correctly identify the relationship between starting height and ball speed.
Graphing skills Answering higher order questions	***Investigate*** Part B, Steps 3.a) and c)	Presenting scientific data in graph form is an important science skill. Some students may be struggling with the vocabulary of graphing, while others may be learning the content. • Check students' graphs for missing information, such as axis labels that specify units. Make sure students give titles to their graphs. • It may be helpful to identify common graphing errors for students and have them check their own graphs or a partner's graph. A typical student error is to mark a scale with uneven intervals. • Challenge students to explain why squaring the speed values and plotting these new data points on a graph does not compromise the integrity of the results.
Comprehension Graph-reading skills	***Physics Talk***	Check students' interpretations of the energy bar graphs.

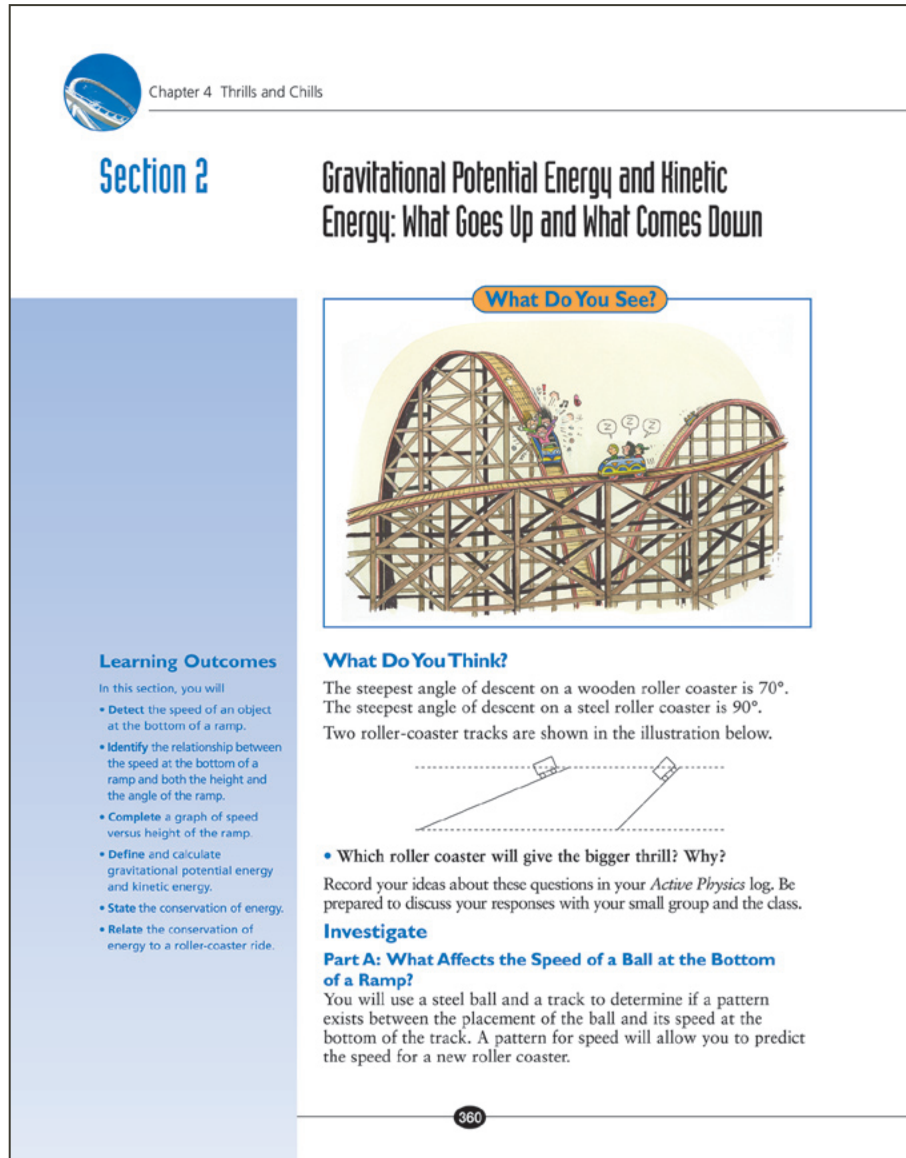
Two important aspects of learning a new language are speaking and writing in that language. Some ELL students will be self-conscious and shy about speaking in front of their peers, while others will be less reluctant to try. Be sure to encourage all ELL students to speak in class, and give them opportunities to write on the board from time to time. Experience will broaden their comfort level. Over time, the shy students will become increasingly less self-conscious about speaking in front of their classmates.

With that in mind, hold a class discussion to review *Section 2*. Call on ELL students to respond to the bulleted items below.

- What determines the speed of a roller-coaster car at the bottom of a hill, assuming the brakes were not applied? [the height of the slope]
- What is mechanical energy? [the sum of kinetic energy and potential energy]
- What can be said about the mechanical energy of a roller-coaster car at any two points on a roller coaster? [It is the same, assuming there are no energy losses due to friction.]
- What are two ways to give the value of the strength of the gravitational field near the surface of Earth? [$g = 9.8$ N/kg and $g = 9.8$ m/s^2]
- Write these two equations on the board and ask volunteers to state each of them in words.
  $KE = \frac{1}{2}mv^2$ [Kinetic energy equals one half the object's mass times the square of the object's velocity.]
  $PE_{grav} = mgh$ [Gravitational potential energy equals the object's mass times the strength of the gravitational field times height.]

SECTION 2

Teaching Suggestions and Sample Answers



Section 2

Gravitational Potential Energy and Kinetic Energy: What Goes Up and What Comes Down

What Do You See?

Learning Outcomes

In this section, you will

- **Detect** the speed of an object at the bottom of a ramp.
- **Identify** the relationship between the speed at the bottom of a ramp and both the height and the angle of the ramp.
- **Complete** a graph of speed versus height of the ramp.
- **Define** and calculate gravitational potential energy and kinetic energy.
- **State** the conservation of energy.
- **Relate** the conservation of energy to a roller-coaster ride.

What Do You Think?

The steepest angle of descent on a wooden roller coaster is 70°. The steepest angle of descent on a steel roller coaster is 90°.

Two roller-coaster tracks are shown in the illustration below.

- **Which roller coaster will give the bigger thrill? Why?**

Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your responses with your small group and the class.

Investigate

Part A: What Affects the Speed of a Ball at the Bottom of a Ramp?

You will use a steel ball and a track to determine if a pattern exists between the placement of the ball and its speed at the bottom of the track. A pattern for speed will allow you to predict the speed for a new roller coaster.

What Do You See?

This illustration will most likely prompt students to discuss the different effects the roller coasters are having on each group of riders. Encourage them to look at different aspects of each visual. Steer the class discussion towards the title of this section. Ask students if the title provides them with clues to the *What Do You See?* question. Remind them that they will have an opportunity to update their responses as they continue to investigate and learn new physics concepts. The main purpose of this section is to spark an interest in the topic and prepare the students for an inquiry-based approach to learning.

Students' Prior Conceptions

1. **Students believe potential energy is a thing that objects hold like fuel stored in the gas tank of a car, and that stored energy is something that causes a reaction later. It is not energy until it has been released.** Rooting out this preconception at the start of *Section 2* will be difficult for the student to do. It is only when students compare the results of calculations for gravitational potential energy and kinetic energy that they begin to relate the change in one form of energy with the corresponding change in another. This pattern may lead students to recognize that the height of the cart correlates with its potential energy. This energy is not contained within the cart itself but only becomes evident with a change in position of the cart on the plane or due to the angle of the inclined plane. The evidence for the potential energy is manifested in the change in speed of the cart as it travels down the plane.
2. **Students believe gravitational potential energy depends only upon the height of an object.** This prior conception merits classroom discussion to encourage students to compare the gravitational potential energy of carts with different masses so that students recognize that mass is a component of gravitational potential energy. It is in transforming gravitational potential energy into ideal kinetic energy (no friction is involved) that students recognize that mass is a component of both types of energies. Mass can be

As students describe what they see in the illustration, consider asking them why the artist has shown roller coasters of different shapes and sizes. Why one roller coaster is higher than the other? Why the slopes of the roller coasters are not the same? Point out to students that the more questions they will raise, the more they will unravel the logic of science.

What Do You Think?

This section further engages the students with the design of roller coasters. As they present answers, ask them to record their ideas in their *Active Physics* logs. Engage students to discuss what they already know and ask them to share their ideas with other students in the class. A good discussion should lead to the transfer of prior learning and enable you to uncover misconceptions students might have. The main idea behind a *What Do You Think?* question is to plant the seed of inquiry. The question leads students into the *Investigate* after they have spent a few minutes thinking about a problem and considering possible answers. Students should know that their responses must be supported by a logical explanation.

What Do You Think?

A Physicist's Response

Most students will correctly recognize that the steeper roller coaster will produce the bigger thrill. They might incorrectly attribute this thrill to a faster speed. In fact, as students will discover in this section, the speed at the bottom of the inclines is identical if the initial heights are the same. The steeper slope allows the cart to reach the identical top speed in less time. That will result in a larger acceleration. It is the acceleration, the rate of change of velocity, which produces the thrill.

cancelled out of the two equations for energy to predict the velocity of the cart at the bottom of a ramp. Velocity is proportional to height.

3. **Students believe that doubling the velocity of a moving object doubles its kinetic energy.** Mathematical applications will ensure that students relate kinetic energy with the square of the velocity; doubling the velocity quadruples the kinetic energy.
4. **When thinking about the conservation of energy, students believe that energy can be changed completely from one form to another with no loss of useful energy; energy must be conserved.** It is vital that the teacher encourage students to observe and to record everything as the cart moves down the ramp. For example, is there sound or is heat generated? This will enable students to make connections with the next misconception.
5. **Students believe energy is literally lost in many energy transformations; heat is a substance and not energy.** It takes energy to create heat, which is itself another form of energy. Encourage students to see the transfer of mechanical energy into heat energy, as well as other forms of energy, such as sound and light. Relating the conservation of energy to what students may hear on a roller-coaster ride and how they presume roller coasters stop also promotes understanding.

NOTES

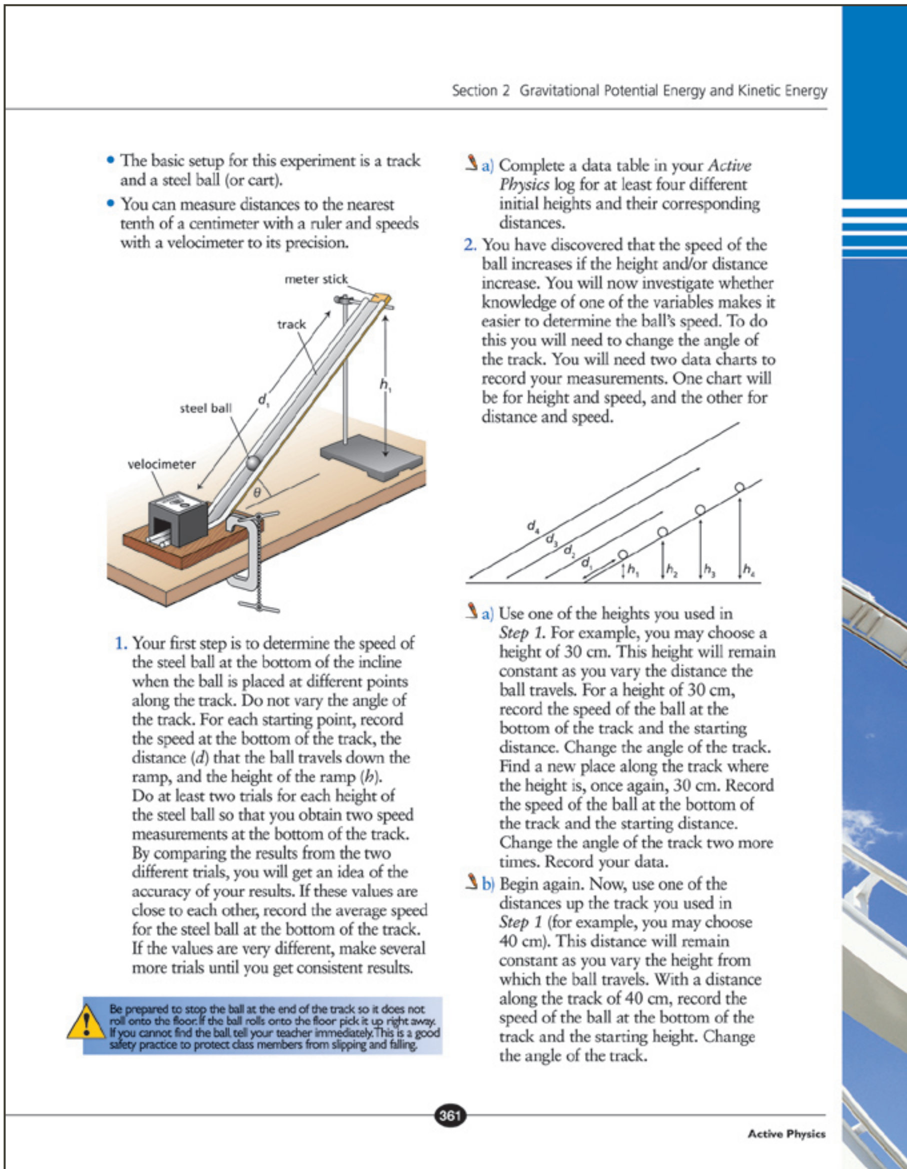
Investigate

1.

Students should acquaint themselves with the apparatus and how they will measure the distances along the incline, the initial height of the ball, and the speed at the bottom of the incline.

1.a)

The data table can be copied into students' logs and the students can record the height, distance, and speed. This table is provided as a Blackline Master at the end of this section. Be sure to remind students to write down the angle of the track at the top of the chart as you move about the classroom. Some students will know how to use a protractor and can give the correct angle for the track. If you do not wish to have the students use protractors, they can record the angle as small, medium, or large.



- The basic setup for this experiment is a track and a steel ball (or cart).
- You can measure distances to the nearest tenth of a centimeter with a ruler and speeds with a velocimeter to its precision.

1. Your first step is to determine the speed of the steel ball at the bottom of the incline when the ball is placed at different points along the track. Do not vary the angle of the track. For each starting point, record the speed at the bottom of the track, the distance (*d*) that the ball travels down the ramp, and the height of the ramp (*h*). Do at least two trials for each height of the steel ball so that you obtain two speed measurements at the bottom of the track. By comparing the results from the two different trials, you will get an idea of the accuracy of your results. If these values are close to each other, record the average speed for the steel ball at the bottom of the track. If the values are very different, make several more trials until you get consistent results.

Be prepared to stop the ball at the end of the track so it does not roll onto the floor. If the ball rolls onto the floor pick it up right away. If you cannot find the ball tell your teacher immediately. This is a good safety practice to protect class members from slipping and falling.

a) Complete a data table in your *Active Physics* log for at least four different initial heights and their corresponding distances.

2. You have discovered that the speed of the ball increases if the height and/or distance increase. You will now investigate whether knowledge of one of the variables makes it easier to determine the ball's speed. To do this you will need to change the angle of the track. You will need two data charts to record your measurements. One chart will be for height and speed, and the other for distance and speed.

a) Use one of the heights you used in *Step 1*. For example, you may choose a height of 30 cm. This height will remain constant as you vary the distance the ball travels. For a height of 30 cm, record the speed of the ball at the bottom of the track and the starting distance. Change the angle of the track. Find a new place along the track where the height is, once again, 30 cm. Record the speed of the ball at the bottom of the track and the starting distance. Change the angle of the track two more times. Record your data.

b) Begin again. Now, use one of the distances up the track you used in *Step 1* (for example, you may choose 40 cm). This distance will remain constant as you vary the height from which the ball travels. With a distance along the track of 40 cm, record the speed of the ball at the bottom of the track and the starting height. Change the angle of the track.

Students should be encouraged to measure the speed at least twice for each height. The repeated measurements will give some sense of the reproducibility of the results. The spread in results for a given setup will help the students decide whether the same height with different angles gives the same speed or not. Students may need a lot of coaching to be able to think about how the spread in results for a single setup is related to the agreement or disagreement of results when they change the incline angle but use the same height.

Technical note: If students use too steep an angle for the incline, the ball may slide without rolling during the initial part of the motion. In that case, the speed at the bottom may be a bit different from the speed found when the ball starts rolling immediately.

2.

This will require two additional data tables like the one in *Step 1*. In the first data table, the values for height will be identical as the table in *Step 1*. In the second data table, the values for distance will be identical to the table in *Step 1*.

4-2a Blackline Master

2.a)

Students record the varying distances along the plane and the speed at the bottom of the track for varying angles, while always starting the ball from the same height above the ground. Students observe that the speed of the ball is essentially constant, although the distance along the track increases.

2.b)

Students record the speed of the ball at the bottom of the track when the varying angles cause the starting height of the ball to change because they always roll the ball the same distance along the track. Students should find that the speed of the ball increases with increasing height.

3.

Students should recognize that the same height produces the same speed at the bottom, irrespective of the angle of the track, taking into account the spread of results of a series of measurements for a single height or distance. In other words, the speeds won't be exactly the same, but they should be the same within the spread indicated for that height or distance. They will also recognize that the same distance along the steeper track results in a greater speed.

4.a)

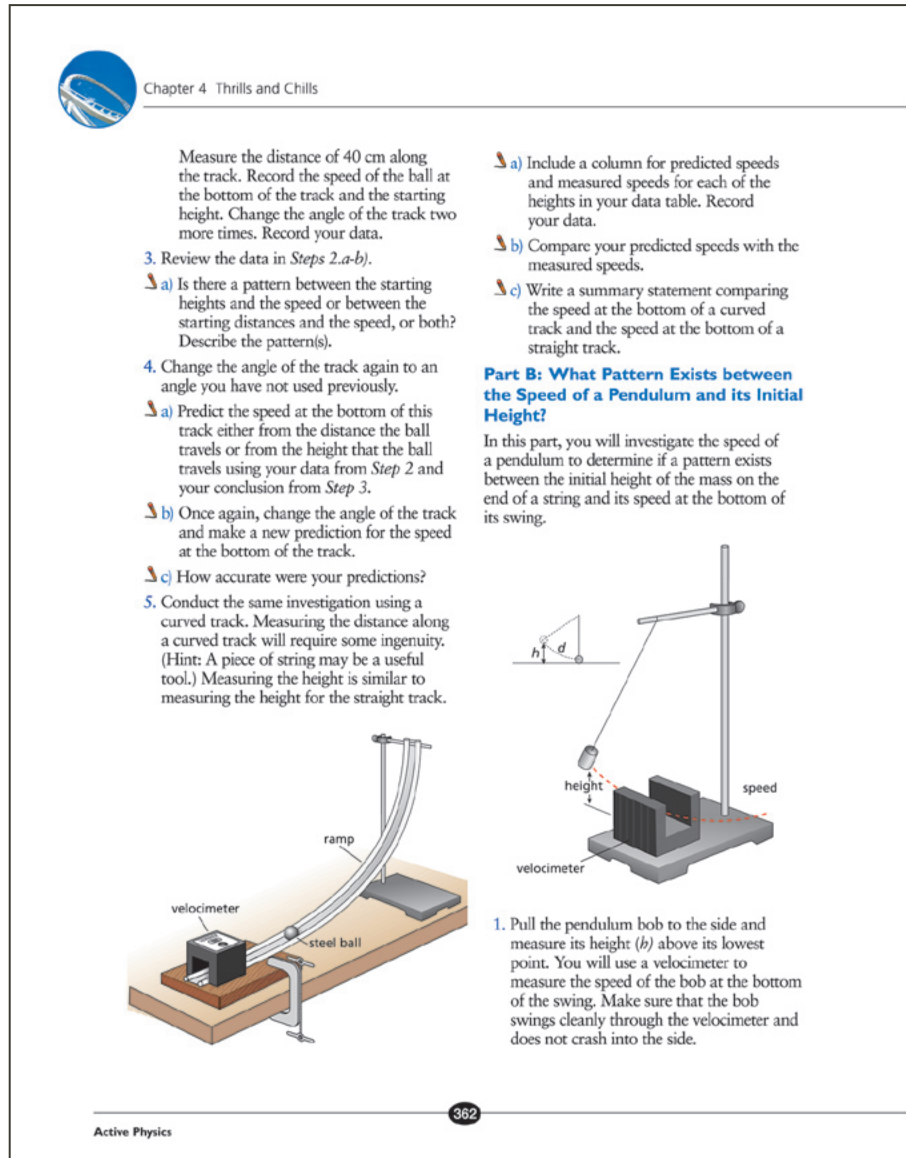
As a test for their understanding, students should create a chart that has the same heights and predict the same speeds at the bottom.

4.b)

Students change the angle of the track and make a new prediction of speed.

4.c)

Although the predicted and measured speeds may not be exactly the same, they are very close (within the spread of values for a single height).



Chapter 4 Thrills and Chills

Measure the distance of 40 cm along the track. Record the speed of the ball at the bottom of the track and the starting height. Change the angle of the track two more times. Record your data.

3. Review the data in *Steps 2.a-b)*.

a) Is there a pattern between the starting heights and the speed or between the starting distances and the speed, or both? Describe the pattern(s).

4. Change the angle of the track again to an angle you have not used previously.

a) Predict the speed at the bottom of this track either from the distance the ball travels or from the height that the ball travels using your data from *Step 2* and your conclusion from *Step 3*.

b) Once again, change the angle of the track and make a new prediction for the speed at the bottom of the track.

c) How accurate were your predictions?

5. Conduct the same investigation using a curved track. Measuring the distance along a curved track will require some ingenuity. (Hint: A piece of string may be a useful tool.) Measuring the height is similar to measuring the height for the straight track.

a) Include a column for predicted speeds and measured speeds for each of the heights in your data table. Record your data.

b) Compare your predicted speeds with the measured speeds.

c) Write a summary statement comparing the speed at the bottom of a curved track and the speed at the bottom of a straight track.

Part B: What Pattern Exists between the Speed of a Pendulum and its Initial Height?

In this part, you will investigate the speed of a pendulum to determine if a pattern exists between the initial height of the mass on the end of a string and its speed at the bottom of its swing.

1. Pull the pendulum bob to the side and measure its height (*h*) above its lowest point. You will use a velocimeter to measure the speed of the bob at the bottom of the swing. Make sure that the bob swings cleanly through the velocimeter and does not crash into the side.

Active Physics 362

5.

As an additional part of the inquiry, students have to invent a way to measure the distance traveled along a curved track.

5.a)

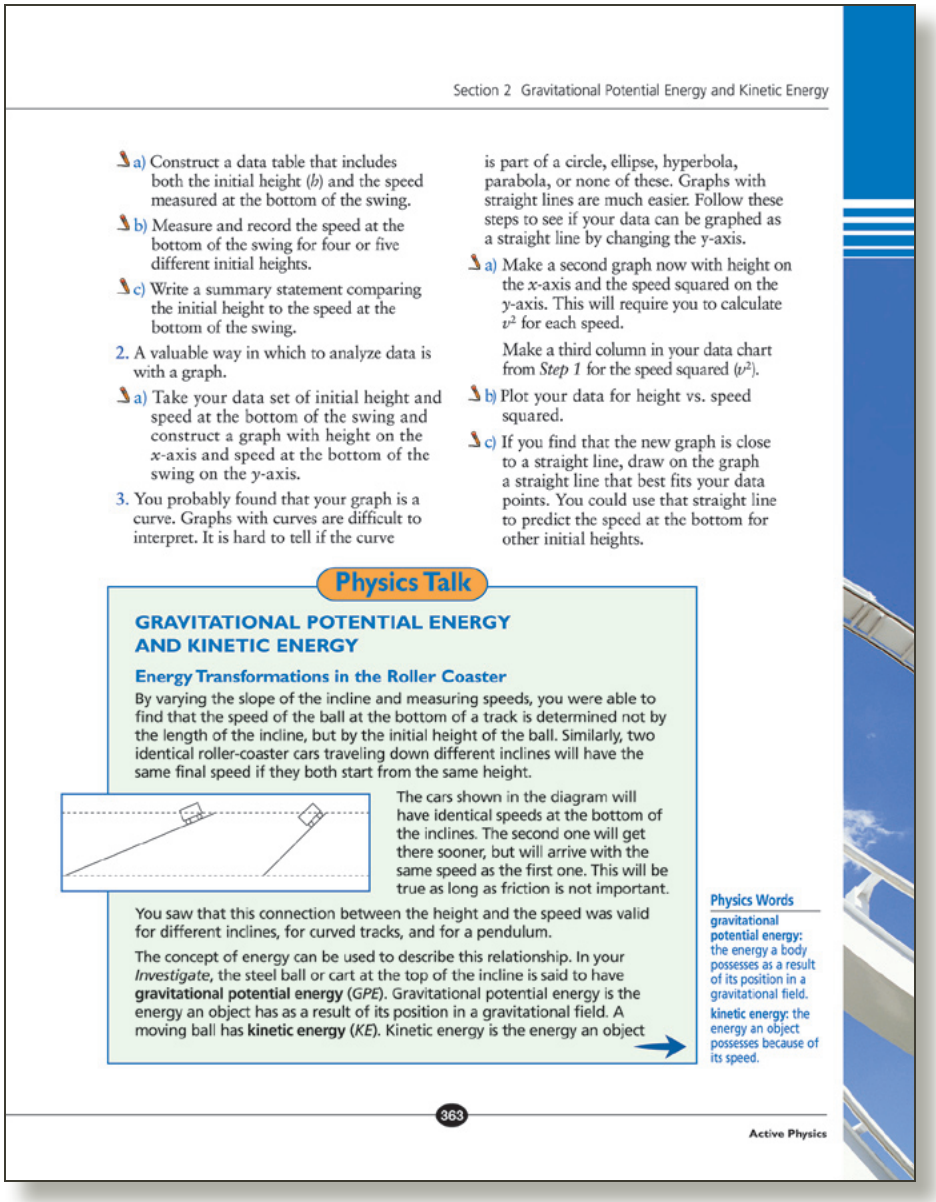
Students construct a data table.

5.b)

Students should find that the initial height of the ball on the ramp determines the final speed as it did for the straight tracks.

5.c)

Students discover that the speed at the bottom of the curved track and the speed at the bottom of the straight track is the same if the balls start from the same height.



Section 2 Gravitational Potential Energy and Kinetic Energy

a) Construct a data table that includes both the initial height (h) and the speed measured at the bottom of the swing.

b) Measure and record the speed at the bottom of the swing for four or five different initial heights.

c) Write a summary statement comparing the initial height to the speed at the bottom of the swing.

2. A valuable way in which to analyze data is with a graph.

a) Take your data set of initial height and speed at the bottom of the swing and construct a graph with height on the x-axis and speed at the bottom of the swing on the y-axis.

3. You probably found that your graph is a curve. Graphs with curves are difficult to interpret. It is hard to tell if the curve is part of a circle, ellipse, hyperbola, parabola, or none of these. Graphs with straight lines are much easier. Follow these steps to see if your data can be graphed as a straight line by changing the y-axis.

a) Make a second graph now with height on the x-axis and the speed squared on the y-axis. This will require you to calculate v^2 for each speed.

Make a third column in your data chart from *Step 1* for the speed squared (v^2).

b) Plot your data for height vs. speed squared.

c) If you find that the new graph is close to a straight line, draw on the graph a straight line that best fits your data points. You could use that straight line to predict the speed at the bottom for other initial heights.

Physics Talk

GRAVITATIONAL POTENTIAL ENERGY AND KINETIC ENERGY

Energy Transformations in the Roller Coaster

By varying the slope of the incline and measuring speeds, you were able to find that the speed of the ball at the bottom of a track is determined not by the length of the incline, but by the initial height of the ball. Similarly, two identical roller-coaster cars traveling down different inclines will have the same final speed if they both start from the same height.

The cars shown in the diagram will have identical speeds at the bottom of the inclines. The second one will get there sooner, but will arrive with the same speed as the first one. This will be true as long as friction is not important.

You saw that this connection between the height and the speed was valid for different inclines, for curved tracks, and for a pendulum.

The concept of energy can be used to describe this relationship. In your *Investigate*, the steel ball or cart at the top of the incline is said to have **gravitational potential energy** (*GPE*). Gravitational potential energy is the energy an object has as a result of its position in a gravitational field. A moving ball has **kinetic energy** (*KE*). Kinetic energy is the energy an object

Physics Words

gravitational potential energy: the energy a body possesses as a result of its position in a gravitational field.

kinetic energy: the energy an object possesses because of its speed.

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Active Physics

Part B: What Pattern Exists between the Speed of a Pendulum and its Initial Height?

1.a)

Students construct a data table.

1.b)

Students take measurements and record the data for height of release vs. speed.

1.c)

A pendulum's speed at the bottom of the pendulum's path is also determined by the height from which the pendulum is released, just as for the ball on the track.

2.a)

Students' graphs should have a title, axes labeled with value and units, and the axes should be drawn with a ruler. This graph is similar to many graphs students have created in the past. The graph should be a curve with v increasing as the square root of h.

3.a)

Students should add another column onto their data table for v^2 and fill in the column by squaring the values they found for the velocity of the different heights.

3.b)

By plotting v^2 versus h, the data should fall close to a straight line. The second graph has v^2 on the y-axis. This has units of m^2/s^2. You should encourage the students to use the graph to predict the speed for a height that they did not measure.

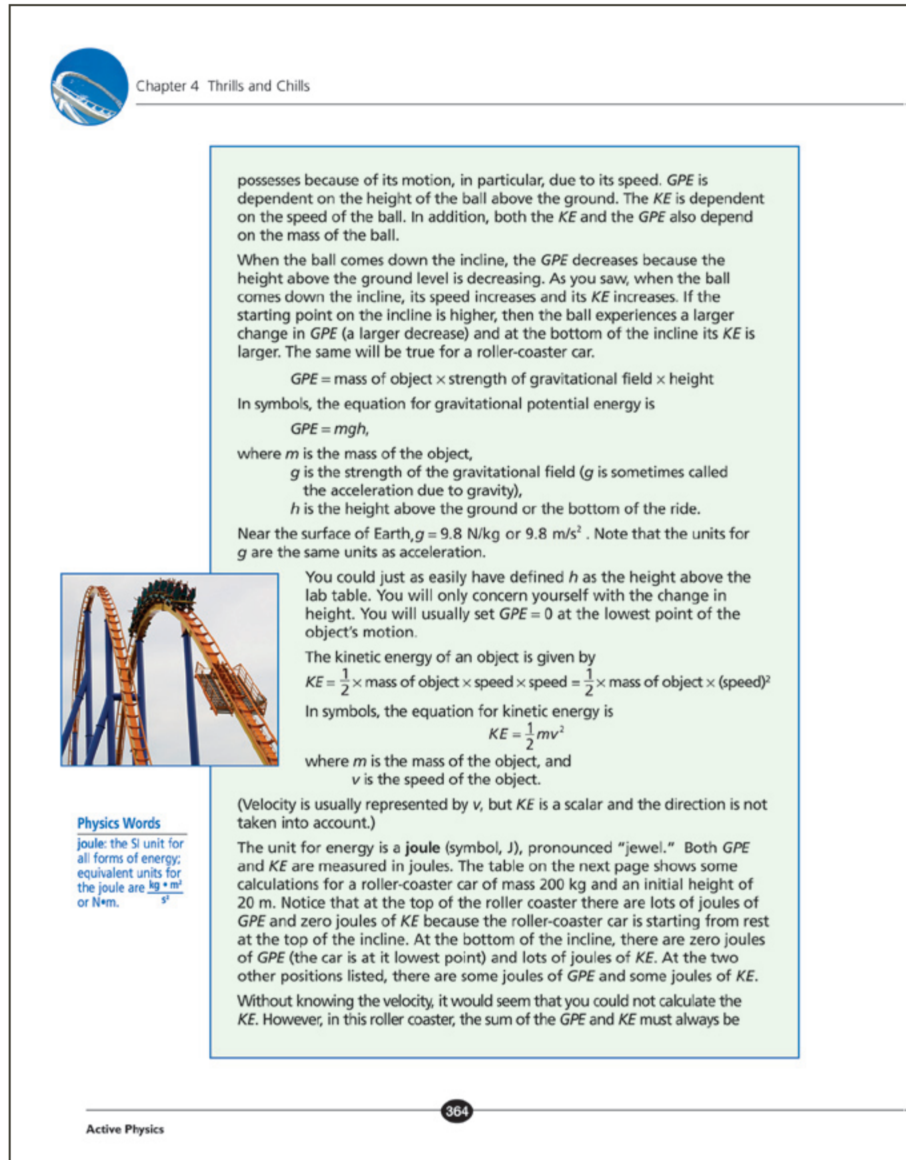
3.c)

Students make the graph. You may choose to have them predict a speed from the graph when you give them a different height than they measured, and then test their prediction.

Physics Talk

The *Physics Talk* explains how the concepts of kinetic energy and gravitational potential energy can be used to explore the transformation of gravitational potential energy into kinetic energy as an object rolls freely down a ramp. Students read how their findings in the *Investigate* relate to the concept of energy. A comparison between the ball rolling down an incline and roller-coaster cars also traveling on a similar track is continuously made for students to see how their *Investigate* is significant to the design of a roller coaster.

The *Physics Talk* also analyzes the definitions of gravitational potential energy and kinetic energy and discusses the formulas for calculating these quantities. Students are then introduced to the unit of energy, the joule. The physics behind the conservation of energy is illustrated by examples, using a bar graph.


Chapter 4 Thrills and Chills

possesses because of its motion, in particular, due to its speed. *GPE* is dependent on the height of the ball above the ground. The *KE* is dependent on the speed of the ball. In addition, both the *KE* and the *GPE* also depend on the mass of the ball.

When the ball comes down the incline, the *GPE* decreases because the height above the ground level is decreasing. As you saw, when the ball comes down the incline, its speed increases and its *KE* increases. If the starting point on the incline is higher, then the ball experiences a larger change in *GPE* (a larger decrease) and at the bottom of the incline its *KE* is larger. The same will be true for a roller-coaster car.

$$GPE = \text{mass of object} \times \text{strength of gravitational field} \times \text{height}$$

In symbols, the equation for gravitational potential energy is

$$GPE = mgh,$$

where m is the mass of the object,
g is the strength of the gravitational field (g is sometimes called the acceleration due to gravity),
h is the height above the ground or the bottom of the ride.

Near the surface of Earth, $g = 9.8$ N/kg or 9.8 m/s^2. Note that the units for g are the same units as acceleration.

You could just as easily have defined h as the height above the lab table. You will only concern yourself with the change in height. You will usually set $GPE = 0$ at the lowest point of the object's motion.

The kinetic energy of an object is given by

$$KE = \frac{1}{2} \times \text{mass of object} \times \text{speed} \times \text{speed} = \frac{1}{2} \times \text{mass of object} \times (\text{speed})^2$$

In symbols, the equation for kinetic energy is

$$KE = \frac{1}{2}mv^2$$

where m is the mass of the object, and
v is the speed of the object.

(Velocity is usually represented by v, but *KE* is a scalar and the direction is not taken into account.)

The unit for energy is a **joule** (symbol, J), pronounced "jewel." Both *GPE* and *KE* are measured in joules. The table on the next page shows some calculations for a roller-coaster car of mass 200 kg and an initial height of 20 m. Notice that at the top of the roller coaster there are lots of joules of *GPE* and zero joules of *KE* because the roller-coaster car is starting from rest at the top of the incline. At the bottom of the incline, there are zero joules of *GPE* (the car is at it lowest point) and lots of joules of *KE*. At the two other positions listed, there are some joules of *GPE* and some joules of *KE*.

Without knowing the velocity, it would seem that you could not calculate the *KE*. However, in this roller coaster, the sum of the *GPE* and *KE* must always be

Physics Words
joule: the SI unit for all forms of energy; equivalent units for the joule are $\frac{kg \cdot m^2}{s^2}$ or N•m.

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Active Physics

To reinforce the concepts articulated in the *Physics Talk*, begin a discussion on the factors that affect the speed of a ball at the bottom of a track. Ask students how the diagram in the beginning of the *Physics Talk* illustrates the connection between the speed of a ball at the bottom of a slope and the height from which the ball was released. Prompt students to answer why the speed of the ball at the bottom of the incline remains the same when the angle of the slope is varied but the height is the same.

Discuss the energy transformations of a roller coaster as it comes down the incline. Consider asking the students to draw a graph that shows that the energy the change from gravitational potential energy to kinetic energy as the roller-coaster car descends. Students will benefit by writing a description of how the change in energy takes place. Ask them to explain how the total mechanical energy of a system remains the same, and what factors in the real world cause losses of mechanical energy. Discuss the problem provided in the *Physics Talk* and determine if the students understand that the speed of the roller coaster is independent of its mass. Also, check to see if students understand the significance of height in determining the gravitational potential energy and how that affects the speed of a roller coaster.

4-2b Blackline Master

40,000 J. This is because that was the total **mechanical energy** at the beginning. Mechanical energy in this case is the sum of *GPE* and *KE*. When the roller coaster was at a height of 20 m, there was no movement and 0 J of KE. All of the energy at this point was the 40,000 J of *GPE*.

This 40,000 J becomes very important for this roller coaster. The sum of *GPE* and *KE* must always be 40,000 J at any point on the roller coaster. (There is, of course, in real life, some loss of energy to the environment due to friction that must be taken into consideration. However, you may neglect that for now.)

Physics Words

mechanical energy: the sum of kinetic energy and potential energy.

At the bottom of the roller coaster (see row 2 on the table), there are 0 J of *GPE*. To total 40,000 J, there must be 40,000 J of *KE* at the bottom.

Halfway down (see row 3 on the table), the *KE* must equal 20,000 J, so that the sum of the *GPE* (20,000 J) and the *KE* (20,000 J) once again equals 40,000 J.

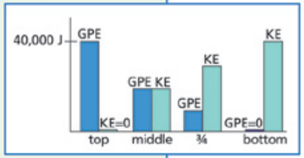
Mass of car = 200 kg and g = 10 N/kg or 10 m/s^2 (approximate value)			
Position of car (height) (m)	*GPE* (J) = mgh	*KE* (J) = $\frac{1}{2}mv^2$	*GPE* + *KE* (J)
top (20 m)	40,000	0	40,000
bottom (0 m)	0	40,000	40,000
halfway down (10 m)	20,000	20,000	40,000
three-quarters way down (5 m)	10,000	30,000	40,000

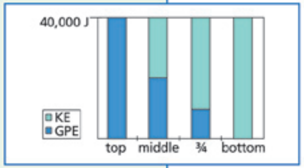
Three-quarters of the way down (see row 4 on the table), the *KE* must equal 30,000 J. The sum of the *GPE* (10,000 J) and the *KE* (30,000 J) once again equals 40,000 J.

Given any height, you can determine the *GPE* and then determine the *KE*. In this roller coaster, the *GPE* plus the *KE* must equal 40,000 J.

In a higher roller coaster, the *GPE* plus *KE* might equal 60,000 J. You can still calculate the *GPE* at any height and then find the corresponding *KE*.

To the right is an energy bar chart that shows *GPE* and *KE* at different parts of the car's trip. Notice that when *GPE* decreases, *KE* increases and vice versa. The second energy stacked bar chart below shows that the sum of the heights of the two types of energy bars remains the same. That indicates that the sum of *KE* and *GPE* remains the same.





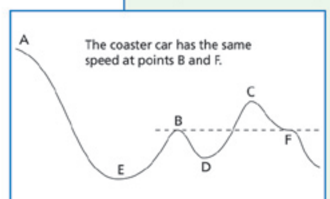
When discussing the conservation of energy, you must recognize that the sum of the *GPE* and *KE* only remains the same if there are no losses of energy due to friction, sound, or other outside sources and no additions of energy from motors. At the end of the roller-coaster ride, brakes are applied to bring the roller-coaster cars to a stop. In that process, the KE of the roller-coaster cars is converted into thermal energy, sound, and some light. The brakes certainly get hot! If the track is level at the end of the ride, there is no change in *GPE* during stopping.

CHAPTER 4

Chapter 4 Thrills and Chills

Calculating Kinetic Energy from Gravitational Potential Energy

In a system like your roller coaster, the sum of the *GPE* and *KE* is constant (as long as friction is not important). *GPE* + *KE* = constant.



In the roller coaster to the left, suppose the *GPE* at point A is 30,000 J and the *KE* at point A is 0 J. Then the total energy at point A and each and every other point on the roller coaster is 30,000 J. The total energy is 30,000 J at points B, C, D, E, and F and every point in between. Since points B and F have the same height, the roller-coaster cart must also have the same *GPE*. That implies that the roller-coaster car also has the same *KE* and is therefore going at the same speed at points B and F.

The height determines the *GPE*. The total energy at every point is the same. If you know the *GPE*, you can easily find the *KE*. The *KE* informs you about the speed.

Calculating Speed from Kinetic and Gravitational Potential Energy

The conservation of energy provides a way to find the kinetic energy if you know the change in height of the roller coaster. From the *KE*, you can find the speed of the roller coaster. Using algebra, you can calculate the speed.

First, you recognize that the total energy, the mechanical energy, of the roller coaster stays the same.

$$\text{Mechanical energy (bottom)} = \text{Mechanical energy (top)}$$

Since you consider only *KE* and *GPE*, the sum of those two must be the same at the top as it is at the bottom.

$$KE\text{ (bottom)} + GPE\text{ (bottom)} = KE\text{ (top)} + GPE\text{ (top)}$$

Since $GPE = 0$ at the bottom and $KE = 0$ at the top, you have

$$KE\text{ (bottom)} + 0 = 0 + GPE\text{ (top)}$$

$$KE\text{ (bottom)} = GPE\text{ (top)}$$

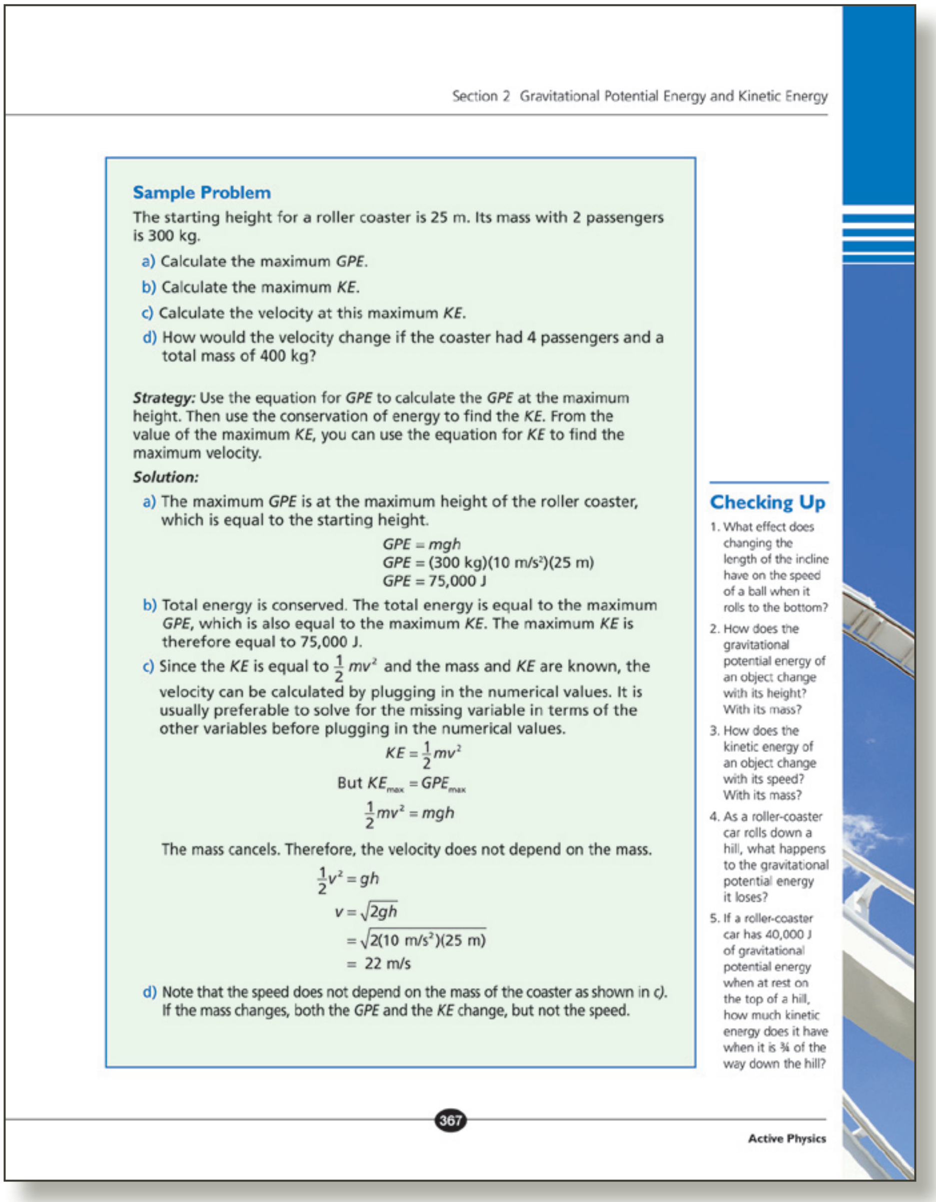
You now use $KE = \frac{1}{2}mv^2$ and $GPE = mgh$

$$\frac{1}{2}mv^2\text{ (bottom)} = mgh\text{ (top)}$$

Solving for v^2 gives

$$v^2\text{ (bottom)} = 2gh\text{ (top)}$$

In the preceding equation, the mass (*m*) cancelled out. This means that the speed is independent of the mass of the car. (It doesn't matter whether the roller-coaster car has two or four passengers.)



Sample Problem

The starting height for a roller coaster is 25 m. Its mass with 2 passengers is 300 kg.

a) Calculate the maximum *GPE*.

b) Calculate the maximum *KE*.

c) Calculate the velocity at this maximum *KE*.

d) How would the velocity change if the coaster had 4 passengers and a total mass of 400 kg?

Strategy: Use the equation for *GPE* to calculate the *GPE* at the maximum height. Then use the conservation of energy to find the *KE*. From the value of the maximum *KE*, you can use the equation for *KE* to find the maximum velocity.

Solution:

a) The maximum *GPE* is at the maximum height of the roller coaster, which is equal to the starting height.

$$GPE = mgh$$
$$GPE = (300\ \text{kg})(10\ \text{m/s}^2)(25\ \text{m})$$
$$GPE = 75{,}000\ \text{J}$$

b) Total energy is conserved. The total energy is equal to the maximum *GPE*, which is also equal to the maximum *KE*. The maximum *KE* is therefore equal to 75,000 J.

c) Since the *KE* is equal to $\frac{1}{2}mv^2$ and the mass and *KE* are known, the velocity can be calculated by plugging in the numerical values. It is usually preferable to solve for the missing variable in terms of the other variables before plugging in the numerical values.

$$KE = \frac{1}{2}mv^2$$
$$\text{But } KE_{max} = GPE_{max}$$
$$\frac{1}{2}mv^2 = mgh$$

The mass cancels. Therefore, the velocity does not depend on the mass.

$$\frac{1}{2}v^2 = gh$$
$$v = \sqrt{2gh}$$
$$= \sqrt{2(10\ \text{m/s}^2)(25\ \text{m})}$$
$$= 22\ \text{m/s}$$

d) Note that the speed does not depend on the mass of the coaster as shown in *c*). If the mass changes, both the *GPE* and the *KE* change, but not the speed.

Checking Up

1. What effect does changing the length of the incline have on the speed of a ball when it rolls to the bottom?
2. How does the gravitational potential energy of an object change with its height? With its mass?
3. How does the kinetic energy of an object change with its speed? With its mass?
4. As a roller-coaster car rolls down a hill, what happens to the gravitational potential energy it loses?
5. If a roller-coaster car has 40,000 J of gravitational potential energy when at rest on the top of a hill, how much kinetic energy does it have when it is ¾ of the way down the hill?

Checking Up

1.

Changing the length of the incline does not have an effect on the speed of a ball when it rolls to the bottom, if the ball always starts from the same height. However, if the height changes with the length of the ramp, the speed of the ball also changes.

2.

The gravitational potential energy of an object increases with an object's height and its mass.

3.

The kinetic energy of an object increases linearly with an increase in the object's velocity squared. As the mass of the object increases, the kinetic energy of the object increases proportionately.

4.

The roller coaster's gravitational potential energy changes to kinetic energy.

5.

The roller coaster has 30,000 J of energy of kinetic energy when it reaches $\frac{3}{4}$ of the way down the hill.

Active Physics Plus

This exercise leads the students through a "derivation" of the expression for *KE*. In particular, the exercise shows where the factor of $\frac{1}{2}$ comes from in the expression for *KE*. The calculation involves several algebraic steps. All students will not be ready to carry out these steps successfully, and many, even if they can carry out the steps, may not see the overall logic of the calculation. Also, the calculation leads to the expression of force multiplied by distance for work. Some students may not have encountered this concept before.

1.

The important aspect of this exercise is to note a straight-line relationship between v^2 and h, not the value of the slope. The students calculate the slope of the line from their data. For a mass sliding down an incline without friction, the slope would be twice the acceleration due to gravity. For a ball rolling down a track without slipping, the slope should be 10/7 times the acceleration due to gravity. Since the ball will probably slide in the beginning before starting to roll, a value somewhere between these is to be expected, depending upon the loss of friction during the sliding phase.

2.

Students compare slopes with other groups. The slopes should be fairly consistent with each other.

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
••	•		

Finding an Equation for the Height vs. Speed Squared Graph

In the *Investigate*, you plotted a graph with height on the x-axis and speed squared on the y-axis for your pendulum. You can find an equation that describes the straight line you drew. Recall that the equation for any straight line on an x-y graph is

$$y = (\text{slope} \times x) + b$$

where b is the y-intercept.

Note that in most math books the equation is written as $y = mx + b$ where m is the slope. To avoid confusing m for mass with m for slope, the word slope was used in the equation.

In your straight-line graph, $(\text{speed})^2$ is the y-variable and starting height is the x-variable.

If the car starts out at $h = 0$, (the bottom of the incline), its final speed will be zero.

The graph intersects the origin, and the value of the y-intercept, b, is 0.

The equation for the graph becomes

$$y = (\text{slope} \times x) + 0 \text{ or}$$

$$y = (\text{slope} \times x)$$

Substituting for the variables in your graph:

$$(\text{speed})^2 = \text{slope} \times \text{height}$$

$$v^2 = \text{slope} \times h$$

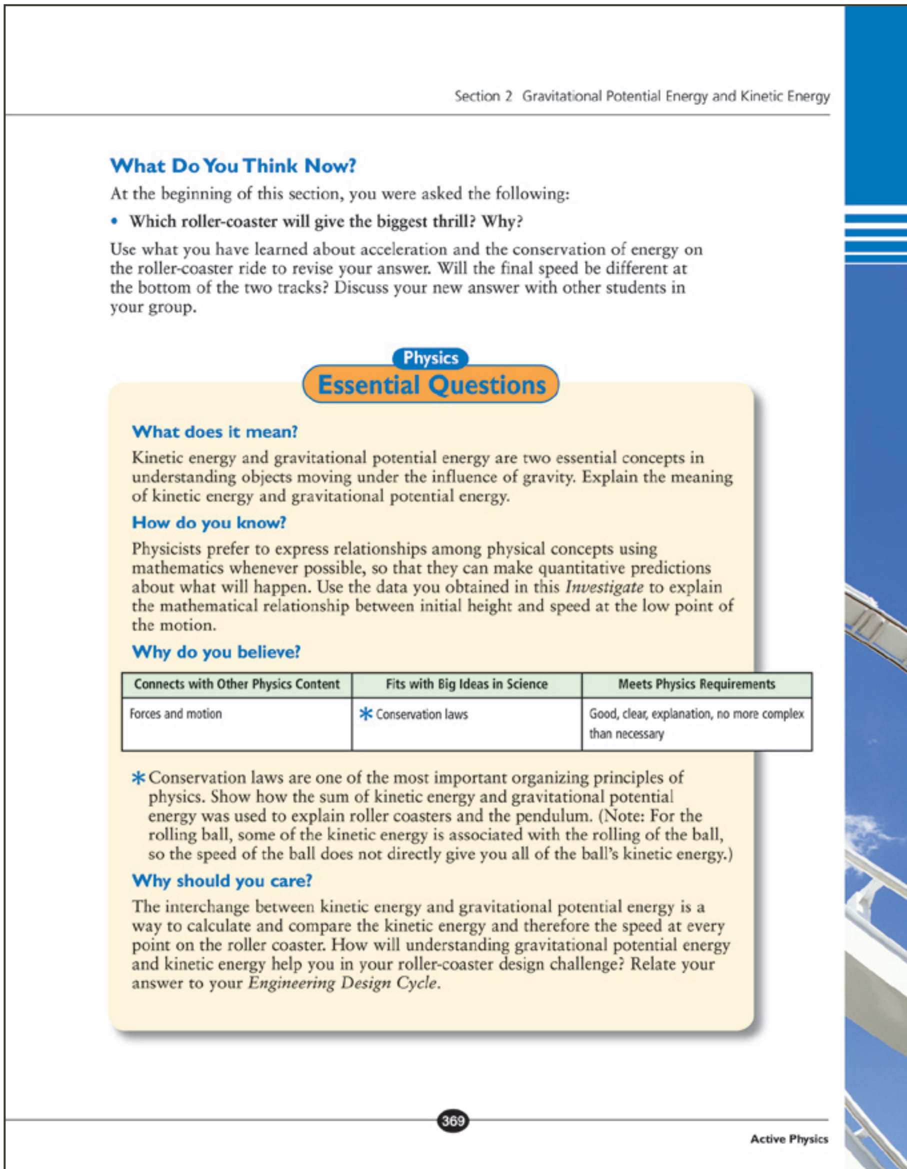
1. Calculate the slope (slope = "rise" over "run") of your graph from the pendulum data, and record its value.
2. Compare the value of your slope with those of other groups in the class.
3. Create a similar v^2 versus h graph with the data from the ball rolling down the incline from different heights (*Investigate Part A, 2.b*). Calculate the slope of the graph. If your data plotted as v^2 as a function of h fall along a straight line (at least fairly close to a straight line), then you can conclude that there is a direct relationship between the square of the speed at the bottom of the ramp and the height at which the ball starts its trip.

In the *Physics Talk*, you saw that $v^2 = 2gh$. Now, you can see why your graph of v^2 versus h was a straight line.

If the car were sliding (no rotation of the wheels), the slope of the line representing the car's speed squared versus h should also be (approximately): $2 \times 9.8 \text{ N/kg} = 2 \times 9.8 \text{ m/s}^2 = 19.6 \text{ m/s}^2$. The slope of your graph is equal to $2g$. Measurement of the slope is a measurement of "g" — the strength of the gravitational field and the acceleration due to gravity. If your slope is 19.6 m/s^2, then $g = 9.8 \text{ m/s}^2$. When the wheels are rotating or when your steel ball was rolling down the incline, there is some *KE* energy of rotation as well as the *KE* energy of the linear motion of the car or ball. For most roller-coaster cars, the rotational energy is rather small and you can use the equations above to determine the speed of the car.

3.

The students repeat the exercise with different heights to verify the straight-line relationship between v^2 and h.


Section 2 Gravitational Potential Energy and Kinetic Energy

What Do You Think Now?

At the beginning of this section, you were asked the following:

- **Which roller-coaster will give the biggest thrill? Why?**

Use what you have learned about acceleration and the conservation of energy on the roller-coaster ride to revise your answer. Will the final speed be different at the bottom of the two tracks? Discuss your new answer with other students in your group.

Physics **Essential Questions**

What does it mean?

Kinetic energy and gravitational potential energy are two essential concepts in understanding objects moving under the influence of gravity. Explain the meaning of kinetic energy and gravitational potential energy.

How do you know?

Physicists prefer to express relationships among physical concepts using mathematics whenever possible, so that they can make quantitative predictions about what will happen. Use the data you obtained in this *Investigate* to explain the mathematical relationship between initial height and speed at the low point of the motion.

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	* Conservation laws	Good, clear, explanation, no more complex than necessary

*Conservation laws are one of the most important organizing principles of physics. Show how the sum of kinetic energy and gravitational potential energy was used to explain roller coasters and the pendulum. (Note: For the rolling ball, some of the kinetic energy is associated with the rolling of the ball, so the speed of the ball does not directly give you all of the ball's kinetic energy.)

Why should you care?

The interchange between kinetic energy and gravitational potential energy is a way to calculate and compare the kinetic energy and therefore the speed at every point on the roller coaster. How will understanding gravitational potential energy and kinetic energy help you in your roller-coaster design challenge? Relate your answer to your *Engineering Design Cycle.*

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Active Physics

What Do You Think Now?

Students should now revise their previous responses to the *What Do You Think?* question and be able recognize how their understanding of energy has grown. This is an opportunity for them to ponder previous learning. From the answers that students give, find out if they still need to clear up misconceptions. *A Physicist's Response* is provided, so that you can share it with your class. Ask students to discuss their answer with each other and determine how their experiments in the *Investigate* helped them in deciding which roller coaster will give them the bigger thrill. You might want to point out that the same diagrams are in the *Physics Talk*.

Physics Essential Questions

What does it mean?

Kinetic energy is the energy of a moving object. It can be calculated using the equation: $KE = \frac{1}{2}mv^2$ Potential energy is the energy an object has due to its position with respect to Earth. A roller coaster has more potential energy at the top of the hill than it has at the bottom. It can be calculated using the equation: $GPE = mgh$

How do you know?

As the initial height increased, the speed at the bottom of the incline increased. Doubling the height did not double the speed. The relationship is: $v_f = \sqrt{2gh}$

Why do you believe?

In both the roller-coaster cars and the pendulum, the sum of the gravitational potential energy (*GPE*) at the top and the kinetic energy (*KE*) at the top was equal to the sum at the bottom. At the top, all the energy was *GPE* while at the bottom all the energy was *KE.*

Why should you care?

The roller coaster is dragged to a starting height. This height determines the maximum speed that the roller coaster will have.

Reflecting on the Section and the Challenge

Have a student volunteer read this section aloud. Knowing the speed at each point of the ride is crucial to ensuring safety; so is knowing the sum of *GPE* and *KE*. Students should reflect on how the speed of the roller coaster will vary. They should focus on the height of the hill from which their roller coaster is designed to descend. Emphasize that the height will determine the speed of the roller coaster at each and every point. Ask students to think of how *GPE* during the ride will vary and how the *KE* will also change. Remind them that this also time for them to reflect on how their knowledge of energy transformation has progressed from the *What Do You See?* section to now.

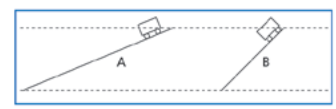
Chapter 4 Thrills and Chills

Reflecting on the Section and the Challenge

In designing a roller coaster, it is necessary to know how the speed of the roller coaster will vary. Knowing that the sum of the *GPE* and *KE* is constant is crucial in finding the speed at each point on the ride. How to calculate the *GPE* and *KE* will also be important when you want to ensure safety. You cannot let the roller coaster fall off the tracks, nor can you build a roller coaster that injures people. If your roller coaster begins on top of a tall hill, the height of that hill will determine the speed for each and every point along the roller-coaster ride. As you design your roller coaster, you should think about where the *GPE* will be largest and where it will be smallest. You should also think about how the *KE* changes in different parts of the ride.

Physics to Go

1. For which track is the speed of the car the greatest at the bottom? (Assume no friction.)



2. State the conservation of energy as it applies to roller coasters. Include in your statement *GPE*, *KE*, *mgh*, and $\frac{1}{2}mv^2$.
3. Complete the table below for a roller coaster starting from rest at the top.

Mass of car = 200 kg and g = 10 N/kg or 10 m/s² (approximate value)			
Position of car → height (m)	*GPE* (J) = *mgh*	*KE* (J) = $\frac{1}{2}mv^2$	*GPE* + *KE* (J)
top (30 m)	60,000		
bottom (0 m)			
halfway down (15 m)			
three-quarters way down (7.5 m)			

4. Draw a *GPE* and *KE* energy bar chart for the situation given in *Question 3*.
5. Complete the table below for a roller coaster starting from rest at the top.

Mass of car = 300 kg and g = 10 N/kg or 10 m/s² (approximate value)			
Position of car → height (m)	*GPE* (J) = *mgh*	*KE* (J) = $\frac{1}{2}mv^2$	*GPE* + *KE* (J)
top (25 m)			
bottom (0 m)			
halfway down (12.5 m)			
further down (5 m)			

6. Draw a *GPE* and *KE* energy bar chart for the situation given in *Question 5*.

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Active Physics

Physics to Go

1.

Both carts will have the same speed at the bottom because they both start from the same height.

2.

As the roller coaster is lifted to its highest point, it gains gravitational potential energy ($GPE = mgh$). Once it starts its descent, the roller coaster loses gravitational potential energy and gains kinetic energy $\left(KE = \frac{1}{2}mv^2\right)$. A gain in kinetic energy is a gain in speed (since the mass stays the same). The total mechanical energy of the roller coaster at every point is identical (as long as friction can be ignored). This conservation of mechanical energy equation can be stated mathematically as the sum of the kinetic and gravitational potential energy at one point on the roller coaster and is equal to the sum of the kinetic and gravitational potential energy at any other point on the roller coaster.

3.

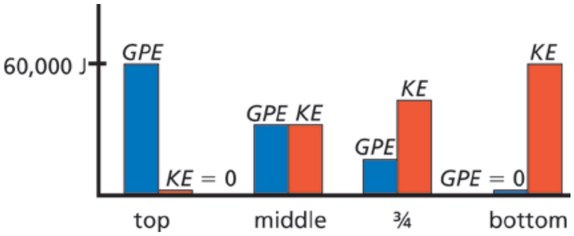
GPE at top = (200 kg)(10 N/kg)(30 m) = 60,000 J

Refer to table below:

Mass of car= 200 kg and *g* = 10 N/kg or 10 m/s² (approximate value)			
Position of car = height (m)	*GPE* (J) = *mgh*	*KE* (J) = ½mv^2	*GPE* + *KE* (J)
top (30 m)	60,000	0	60,000
bottom (0 m)	0	60,000	60,000
halfway down (15 m)	30,000	30,000	60,000
three-quarters way down (7.5 m)	15,000	45,000	60,000

4.

The energy bar chart should look like the bar chart below:



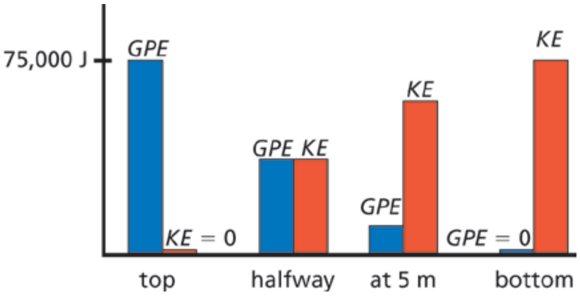
5.

See table below:

Mass of car = 300 kg and *g* = 10 N/kg or 10 m/s² (approximate value)			
Position of car = height (m)	*GPE* (J) = *mgh*	*KE* (J) = ½mv^2	*GPE* + *KE* (J)
top (25 m)	75,000	0	75,000
bottom (0 m)	0	75,000	75,000
halfway down (12.5 m)	37,500	37,500	75,000
further down (5 m)	15,000	60,000	75,000

6.

The chart should look like the one below:



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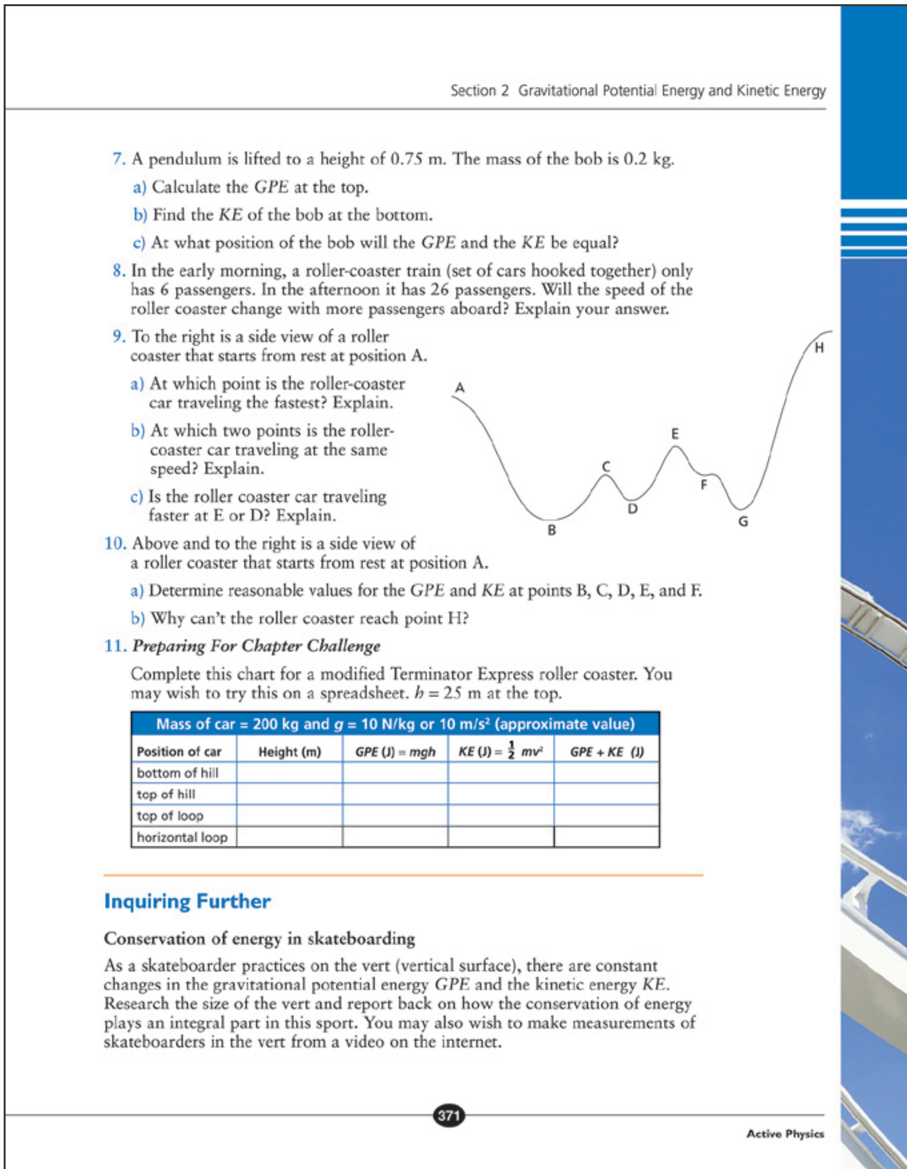


7. A pendulum is lifted to a height of 0.75 m. The mass of the bob is 0.2 kg.
   a) Calculate the *GPE* at the top.
   b) Find the *KE* of the bob at the bottom.
   c) At what position of the bob will the *GPE* and the *KE* be equal?
8. In the early morning, a roller-coaster train (set of cars hooked together) only has 6 passengers. In the afternoon it has 26 passengers. Will the speed of the roller coaster change with more passengers aboard? Explain your answer.
9. To the right is a side view of a roller coaster that starts from rest at position A.
   a) At which point is the roller-coaster car traveling the fastest? Explain.
   b) At which two points is the roller-coaster car traveling at the same speed? Explain.
   c) Is the roller coaster car traveling faster at E or D? Explain.

10. Above and to the right is a side view of a roller coaster that starts from rest at position A.
    a) Determine reasonable values for the *GPE* and *KE* at points B, C, D, E, and F.
    b) Why can't the roller coaster reach point H?
11. ***Preparing For Chapter Challenge***

    Complete this chart for a modified Terminator Express roller coaster. You may wish to try this on a spreadsheet. $h = 25$ m at the top.

Mass of car = 200 kg and g = 10 N/kg or 10 m/s² (approximate value)				
Position of car	Height (m)	*GPE* (J) = *mgh*	*KE* (J) = $\frac{1}{2}mv^2$	*GPE* + *KE* (J)
bottom of hill				
top of hill				
top of loop				
horizontal loop				

Inquiring Further

Conservation of energy in skateboarding

As a skateboarder practices on the vert (vertical surface), there are constant changes in the gravitational potential energy *GPE* and the kinetic energy *KE*. Research the size of the vert and report back on how the conservation of energy plays an integral part in this sport. You may also wish to make measurements of skateboarders in the vert from a video on the internet.

7.a)

Given:
$h = 0.75$ m; $m = 0.2$ kg

$GPE = mgh =$
$(0.2 \text{ kg})(10 \text{ m/s}^2)(0.75 \text{ m}) = 1.5 \text{ J}$

7.b)

KE at bottom is equal to *GPE* at top = 1.5 J

7.c)

The *GPE* and *KE* will be equal when the *GPE* has half its original value. This occurs at half its original height or 0.375 m (or 0.37 m) above the ground.

8.

The roller coasters will have identical speeds at the bottom. The speed at the bottom will be independent of the mass since the *KE* at the bottom equals the *GPE* at the top. Mathematically, $mgh = \frac{1}{2}mv^2$, so the mass cancels.

9.a)

The roller coaster is traveling fastest at B. That is where it has the least *GPE* and must have the largest *KE*.

9.b)

C and F. The roller coaster has the same speed because it is at the same height. The same height has the same *GPE* and therefore has the same *KE*.

9.c)

The roller coaster is faster at D than E because at D it has less *GPE* and therefore more *KE*.

CHAPTER 4

10.a)

See table below.

Mass of car = 300 kg and *g* = 10 N/kg or 10 m/s² (approximate value)			
Position of car = height (m)	*GPE* (J) = mgh	*KE* (J) = ½mv^2	*GPE* + *KE* (J)
top (25 m)	75,000	0	75,000
bottom (0 m)	0	75,000	75,000
halfway down (12.5 m)	37,500	37,500	75,000
further down (5 m)	15,000	60,000	75,000

You might have the students produce a *KE* + *GPE* bar chart for this situation.

10.b)

The roller coaster cannot reach point H because it is higher than point A and this would require a potential energy greater than the 50,000 joules that the roller coaster started with.

Preparing for the Chapter Challenge

11.

GPE at top = (200 kg)(10 N/kg)(25 m) = 50,000 J

See table below.

Mass of car = 200 kg and *g* = 10 N/kg or 10 m/s² (approximate value)				
Position of car	Height	*GPE* (J) = *mgh*	*KE* (J) = ½mv^2	*GPE* + *KE* (J)
bottom of hill	0 m	0	50,000	50,000
top of hill	25 m	50,000	0	50,000
top of loop	15 m	30,000	20,000	50,000
horizontal loop	0 m	0	50,000	50,000

Inquiring Further

You might have the students produce a *KE* + *GPE* bar chart for this situation. The typical height of a “vert” ramp is 10-14 ft, with anywhere from 6 in. to 3 ft of vertical wall on the top. The students should recognize the trade-off between *GPE* and *KE* as the skateboarder goes up and down a vertical surface (the “vert”). As the skateboarder goes up, the speed decreases as the *KE* decreases and the *GPE* increases. The reverse happens on the way down: *GPE* decreases and the *KE* increases, leading to an increase in speed on the way down.

CHAPTER 4

NOTES

SECTION 2 QUIZ

4-2c **Blackline Master**

1. A ramp 1.2 m long rests on a level table with one end elevated 0.40 m above the table surface. A steel ball starts at the elevated end of the ramp and rolls down the ramp through a velocimeter. If the ball has a mass of 0.045 kg, what is the ball's potential energy at the top of the ramp?

   a) 0.18 J b) 0.45 J

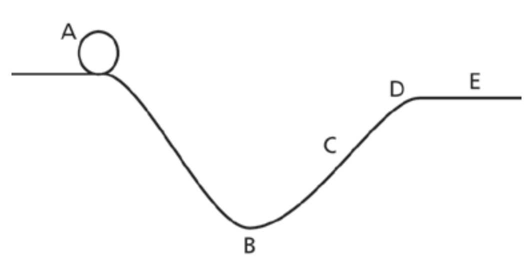
   c) 0.54 J d) 12 J

2. What is the ball's kinetic energy as it rolls through the velocimeter at the bottom of the ramp?

   a) 0.18 J b) 0.45 J

   c) 0.54 J d) 0.27 J

3. In the diagram below, the object's kinetic energy at point C is less than at point



   a) A. b) B.

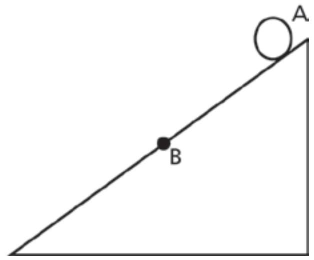
   c) E. d) D.

4. If the kinetic energy of an object is 32 J when its speed is 4 m/s, then the object's mass must be

   a) 1 kg. b) 2 kg.

   c) 3 kg. d) 4 kg.

5. In the diagram below, a 1.0 kg sphere at point A has a potential energy of 5.0 J. What is the potential energy of the sphere at point B, halfway down the plane?



a) 0.0 J

b) 3.0 J

c) 2.5 J

d) 5.0 J

SECTION 2 QUIZ ANSWERS

1 a) Gravitational potential energy is defined as $GPE = mg\Delta h = (0.045 \text{ kg})(10 \text{ m/s}^2)(0.40 \text{ m}) = 0.18 \text{ J}$

2 a) If no energy is lost the *KE* at the bottom of the ramp should equal the *GPE* at the top of the ramp or 0.18 J.

3 b) The ball gains *KE* as it rolls down the ramp, with maximum *KE* at point B, and then loses *KE* as it rolls up the ramp. Point B is where the *KE* is maximum, so at point C the *KE* must be less than at point B. Since the ball continues rolling up the ramp it loses additional *KE* as it gains *GPE*, so the *KE* is greater at B than at points A, C, and D.

4 d) Using the formula $KE = \frac{1}{2}mv^2$ and solving for the mass gives $32 \text{ J} = \frac{1}{2}m(4 \text{ m/s})^2$ and $m = 4$ kg.

5 c) As the mass rolls down the incline, it loses *KE* and gains *GPE*. When halfway down the ramp, the object would have lost half of its *GPE* since it height was half the previous value at the top of the ramp, which would have been converted into *KE*. If the *GPE* at the top of the ramp was 5 J, the *GPE* halfway down the ramp would now be $\frac{1}{2}(5 \text{ J}) = 2.5 \text{ J}$.