SECTION 3

Spring Potential Energy: More Energy

Section Overview

Students explore a pop-up toy by compressing its spring and then investigating how high the toy rises after the spring is released. This extends the concept of the conservation of mechanical energy to include the potential energy of a spring. Students observe how the pop-up toy's gravitational potential energy and kinetic energy remain the same when coins are taped to the toy although the toy does not fly as fast or as high. They learn how spring potential energy changes to kinetic energy, and the kinetic energy turns to gravitational potential energy.

Later in the section, students read that in the roller-coaster ride, the energy is not supplied by a spring but rather by a motor. The electrical motor lifts the roller coaster to its highest point.

Background Information

In this section, the principle of the conservation of energy is expanded to include spring potential energy. When a spring is compressed or extended, it has spring potential energy. If the spring is attached to an object (the pop-up toy) and is released, the potential energy of the spring becomes kinetic energy of the object. The expanded equation for conservation of mechanical energy would now be:

$SPE + KE + GPE$ at point A =
$SPE + KE + GPE$ at point B.

The spring potential energy can be calculated using the following equation:

$SPE = \frac{1}{2}kx^2$ where k is the spring constant (a measure of the stiffness of the spring) and x is the compression (or stretch) of the spring. The details of finding the spring constant k from Hooke's law is not a part of this section. That will be taken up in *Section 5*. Once again, all energies are measured in joules.

As before, the conservation of mechanical energy holds as long as no mechanical energy is lost via dissipative forces such as friction, air resistance, and so on.

This expansion of the law of conservation of mechanical energy to include spring potential energy can also explain jumping on a trampoline where the spring potential energy would be a combination of the stretch of the springs supporting the trampoline as well as the stretch of the trampoline fabric itself. The equations can also be used to explain the conservation of energy in a pole vault. In the pole vault, the spring potential energy is stored in the bend of the pole.

Crucial Physics

- A compressed or stretched spring has spring potential energy (*SPE*) associated with its configuration (compressed or stretched relative to its relaxed position). If the relaxed position is taken as zero, x is the compressed or stretched distance, and k is the strength of the spring known as the spring constant, then the spring potential energy is given by $SPE = \frac{1}{2}kx^2$.
- If both a spring and gravity act on a system, then the potential energy is due to both the spring and gravity and the mechanical energy is the sum of the kinetic energy, the gravitational potential energy and the spring potential energy.
- If dissipative forces can be neglected then the mechanical energy is conserved.

Learning Outcomes	Location in the Section	Evidence of Understanding
Calculate the kinetic energy of a pop-up toy.	***Investigate*** Step 7	Students calculate the kinetic energy of the pop-up toy at different heights and record the values in a table.
Calculate the spring potential energy from the conservation of energy.	***Investigate*** Step 7	Students calculate the kinetic energy of the pop-up toy when it is at rest and record the values in a table.
Calculate the spring potential energy by using an equation.	***Physics Talk***	Students calculate the spring potential energy of the pop-up toy when it is at rest.
Relate spring potential energy with conservation of mechanical energy using an equation.	***Investigate***	Students examine the relationship between spring potential energy, gravitational potential energy, and kinetic energy under the conservation of mechanical energy.
Recognize the general nature of the conservation of energy as it involves heat, sound, chemical, and other forms of energy.	***Physics Talk***	Students recognize conservation of energy in various forms with graphs and equations.

Section 3 Materials, Preparation, and Safety

Materials and Equipment

PLAN A		
Materials and Equipment	**Group (4 students)**	**Class**
Beespi	1 per group	
Meter stick	1 per group	
Toy, pop-up, type A	2 per group	
Safety glasses, impact	4 per group	
Scale, electronic, 0.1-g readability, 0-1500 g		1 per class
Battery, alkaline, AAA	2 per group	
Tape, masking		6 per class
Nickel*	4 per group	

*Additional items needed not supplied

Time Requirements

- Allow one class period or 45 minutes for the *Investigate*.

Teacher Preparation

- Ensure that all photogate or velocimeter timers are working properly.
- Check to make sure the pop-up toys work well.
- It is important to try this *Investigate* with the equipment you have on hand first to make certain it will work correctly.
- If the pop-up toys that you are using do not fit through the velocimeter or photogate, attach a paper "flag" to the side of the toy that will pass through the gate to use for timing.
- Ensure you have enough nickels available for the day's investigations.

Safety

- Eye protection is particularly important in this *Investigate*.

Materials and Equipment

PLAN B		
Materials and Equipment	**Group (4 students)**	**Class**
Beespi	1 per group	
Meter stick		1 per class
Toy, pop-up, type A	2 per group	
Safety glasses, impact	4 per group	
Scale, electronic, 0.1-g readability, 0-1500 g		1 per class
Battery, alkaline, AAA	2 per group	
Tape, masking		6 per class
Nickel*	4 per group	

*Additional items needed not supplied

Time Requirements

- Allow one class period or 45 minutes for this *Investigate*.

Teacher Preparation

- Ensure that all photogate or velocimeter timers are working properly.
- Check to make sure the pop-up toys work well.
- It is important to try this *Investigate* with the equipment you have on hand first to make certain it will work correctly.
- If the pop-up toys that you are using do not fit through the velocimeter or photogate, attach a paper "flag" to the side of the toy that will pass through the gate to use for timing.
- Ensure you have enough nickels available for the day's investigations.

Safety

- Eye protection is particularly important in this *Investigate*.

NOTES

Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

Learning Issue	Reference	Augmentation and Accommodations
Designing an experiment to determine *KE*	***Investigate*** *Questions 2 and 3*	**Augmentation** • Determining what type of data needs to be collected is often the most difficult step for students. Once they determine what type of data needs to be collected, they are often very creative in designing an experiment. • Ask students how they can calculate *KE*. Someone will eventually offer the formula as a way to calculate *KE*. Then ask students how they can determine mass and velocity. At this point, students can work with their groups to figure out how to calculate velocity using the supplies available. • Allow students to use the lab equipment while they are designing their experiment. This hands-on approach will help kinesthetic learners. **Accommodation** • Provide a step-by-step experimental design for students to follow if the students are unable to design their own process.
Reading comprehension	***Physics Talk***	**Augmentation** • Sometimes the concept being described is lost in the meanings of the words for students who struggle with reading comprehension. However, these same students benefit from the type of analogy described in this section. Model the exchange of money described in the opening paragraphs to help students create a visual model for energy transfer. Create colored cards that represent each form of energy in this activity and transfer the energy cards in place of the coins. • The energy cards could also be taped to a large piece of chart paper to create a color-coded energy bar graph similar to graphs used in this chapter.

CHAPTER 4

Strategies for Students with Limited English-Language Proficiency

Learning Issue	Reference	Augmentation
Comprehension	***Investigate*** Questions 6.c) and 7	• It may help to review the law of conservation of energy from the preceding section.
Vocabulary comprehension	***Investigate*** Question 7	• ELL students may not be familiar with the term "ascend." It may help to ask students if there are cognates for this term in their native language. You might also introduce the term "descend" to help students remember the meaning. • The paragraph uses several verbs to describe going up. Have students find these verbs and make a list, including the words that follows each verb (e.g., "rose above," "popping up," and "leaped off"). It may be helpful to model the grammar of a term like "pop up," which can be confusing for ELL students. For example, we say "the toy pops up" or "while the toy is popping up," but when converting the term to an adjective we say "pop-up toys." • The term "peak" may not be familiar to students. Explain that, as when referring to a mountain peak, the word in the paragraph means the maximum height.
Comprehension	***Physics Talk*** Conservation of Energy	Students may not have seen triple bar charts before, so take the time to make sure they are able to correctly interpret both versions of the energy bar chart.
Graphing skills	***Physics to Go*** Question 2	Have students create an energy bar chart with three bars, one for each type of energy. Make it clear that they must not include a bar for the sum of the types of energy. If they have difficulty understanding why, it may help to have them represent the same information with a stacked energy bar chart.
Comprehension	***Physics to Go*** Questions 5 and 6	A fundamental rule of roller-coaster design is that the first hill has to be the tallest hill. Make sure students are able to explain why. Their understanding of this concept will be crucial to their roller-coaster design.

Consider finishing this section with a cloze activity. Cloze activities are useful tools for summarizing material and for giving English-language learners an opportunity to practice using their science vocabulary words in context. Write the following paragraphs on the board, or type and photocopy them, replacing the underlined words with a write-on line. Encourage volunteers to fill in the blanks, or have all students do the activity on paper.

4-3a **Blackline Master**

When compressed, a spring has *SPE*, or spring potential energy. A spring toy has the greatest kinetic energy just after it leaps off the table, and the greatest gravitational potential energy at the very top of its path. The law of conservation of energy states that the sum of the energies at one point during the spring toy's path equals the sum of the energies at any other point along the path. The total energy at any point can be represented with an energy bar chart, which shows a separate bar for each type of energy, or a stacked bar chart, which shows one bar at each point in time.

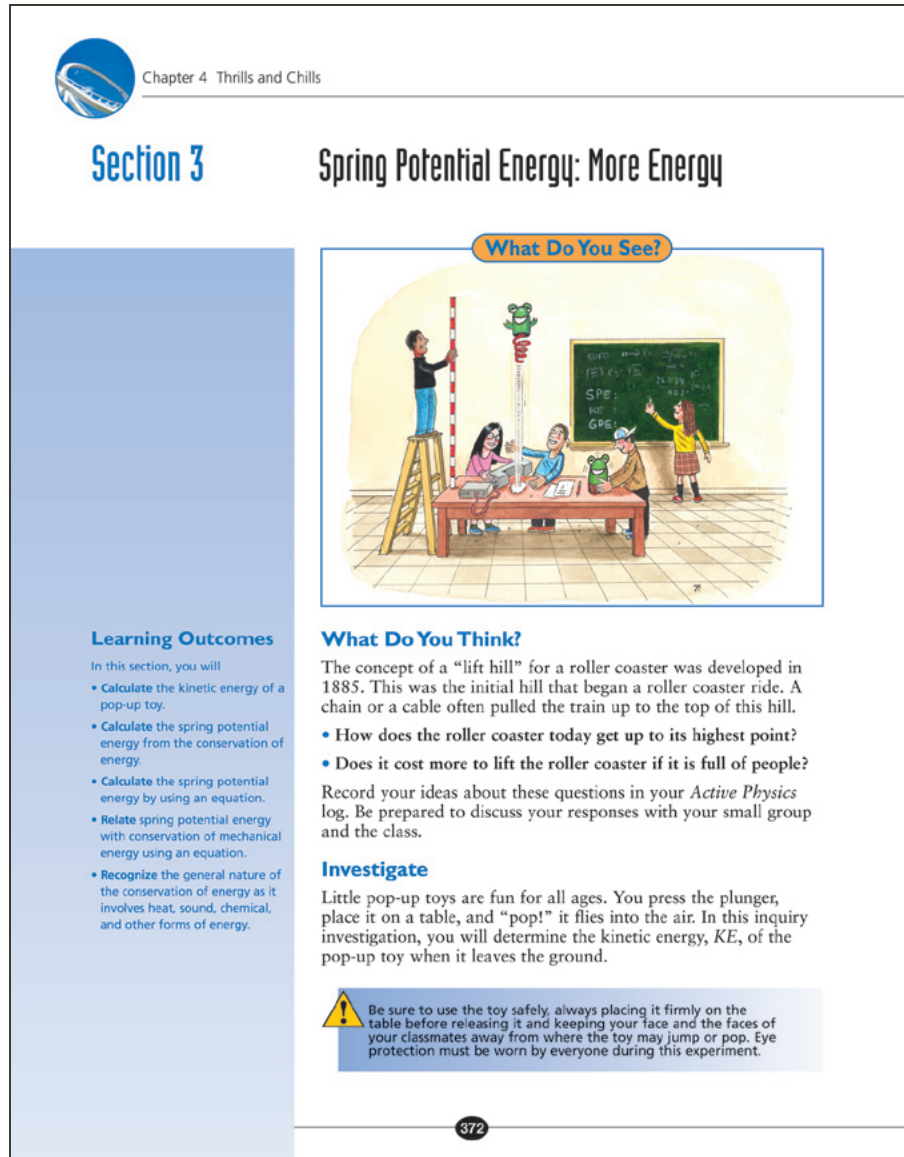
The equation for spring potential energy is $SPE = \frac{1}{2}kx^2$, where k is the spring constant and x is the amount of compression or stretch in the spring. A spring that is easy to compress has a smaller spring constant than a spring that is difficult to compress. $GPE + KE + SPE =$ constant. So, $mgh + \frac{1}{2}mv^2 + \frac{1}{2}kx^2 =$ constant for a given spring. If you increase the mass of a spring toy, it will reach a lower maximum height.

SECTION 3

Teaching Suggestions and Sample Answers

What Do You See?

The meter stick, the pop-up toy, and the girl writing on the board are visuals that immediately strike the reader's eye. Students will most likely comment on these visuals and relate them to the title of *Section 3*. You will elicit quite a few responses if you guide their attention toward the events happening in the illustration. Ask students to keep their original responses in mind and revisit these responses when they investigate and read more about the physics concepts in this section. You might wish to record highlights of your discussion in a corner on the board, so that students can revisit prior responses at a glance.



Chapter 4 Thrills and Chills

Section 3 Spring Potential Energy: More Energy

What Do You See?

Learning Outcomes

In this section, you will

- **Calculate** the kinetic energy of a pop-up toy.
- **Calculate** the spring potential energy from the conservation of energy.
- **Calculate** the spring potential energy by using an equation.
- **Relate** spring potential energy with conservation of mechanical energy using an equation.
- **Recognize** the general nature of the conservation of energy as it involves heat, sound, chemical, and other forms of energy.

What Do You Think?

The concept of a "lift hill" for a roller coaster was developed in 1885. This was the initial hill that began a roller coaster ride. A chain or a cable often pulled the train up to the top of this hill.

- How does the roller coaster today get up to its highest point?
- Does it cost more to lift the roller coaster if it is full of people?

Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your responses with your small group and the class.

Investigate

Little pop-up toys are fun for all ages. You press the plunger, place it on a table, and "pop!" it flies into the air. In this inquiry investigation, you will determine the kinetic energy, *KE*, of the pop-up toy when it leaves the ground.

Be sure to use the toy safely, always placing it firmly on the table before releasing it and keeping your face and the faces of your classmates away from where the toy may jump or pop. Eye protection must be worn by everyone during this experiment.

372

Students' Prior Conceptions

You have the opportunity to facilitate student recognition of other types of potential energy in this section. Use mathematical analysis to assist students to avoid some of the conceptual pitfalls inherent in prior conceptions about mechanical energy and how conservation of energy involves heat, sound, and other forms of energy. Teachers should assess conceptual frameworks for student misconceptions by building on what students know or believe they know from *Section 2*.

1. **Students do not believe objects have any energy if they are not moving.** Discussing what happens to the energy of pushing the pop-up toy into its "down" position and what happens to the toy's energy as it jumps up encourages students to postulate that energy is stored in the compressed spring of the toy and that this energy is a form of potential energy. There is spring potential energy in relation to gravitational potential energy and this potential energy exists when the toy is not moving. Drawing a force diagram to show the weight of the toy at rest, on the table or floor, and the normal force helps students relate the transformation of energy to the change in the forces acting on the toy as it pops up. A force, that is not the normal force, must oppose the gravitational force or the weight of the toy. There must be a net force in the upward direction to propel the toy upwards. This may lead students to recognize a spring force.
2. **Students believe that the only type of potential energy is gravitational.** Teachers may address this prior conception at the same time that students are measuring the kinetic

What Do You Think?

The questions in this section are bound to make the students curious about roller coasters full of people being lifted up a hill. Will it affect the speed of the roller coaster as it goes up? Does the *GPE* change with more people in the roller coaster? How does the *KE* get affected if the roller coaster becomes heavier? Does the law of conservation of energy still apply? These are a few questions that should be helpful in guiding students' discussion. Ask students to recall prior learning and relate the question to the energy transformations that took place while the roller coaster was descending down the hill.

Have them record their responses in their *Active Physics* logs.

What Do You Think?

A Physicist's Response

Any roller coaster needs the input of energy to raise the car to the top of the first hill. This is typically done with an electric motor, but some may use other methods, such as compressed air to provide sufficient kinetic energy to the car to reach the top. Many students think that a motor is used for the entire roller-coaster ride, rather than at just the beginning of the ride to bring the roller coaster to its highest point. Although it may seem obvious that a roller coaster filled with people will be more difficult to lift, (and thus cost more in terms of energy and money) some students may think that the mass of the coaster does not matter because it did not make a difference to the roller coaster's speed as it descended.

energy of the pop-up toy. Remind students that the kinetic energy is a transformation of the potential energy.

3. **Students believe that energy is literally lost in many energy transformations.** This prior conception is a fundamental student construct within the chapter. Students have the opportunity to observe the transformation of spring potential energy into both kinetic energy and sound energy as the pop-up toy springs up with a "pop." Energy is not lost; it is transformed, and although other energies may not be measurable with the tools available to the student, "hearing" and "seeing" other energy transformations may stamp out this preconceived notion.

Investigate

1.a)

The heights will vary depending upon the size and type of pop-up toy used. A hop height of 0.5 m would be considered ideal.

1.b)

The range of heights achieved would be a way of describing consistency.

2.a)

Determining the kinetic energy requires a determination of the mass of the toy and the speed with which it leaves the table. $\left(KE = \frac{1}{2}mv^2\right)$. The mass can be measured with a balance. The speed could be measured with a velocimeter. If the toy does not fit through the velocimeter, a "flag" (a stiff strip of paper) may be fitted onto the side of the pop-up toy so that it fits through the velocimeter. The speed of the flag would be the same as the speed of the toy. A second way in which to calculate the *KE* would be to equate the *KE* with the gravitational potential energy the toy has at the top of its jump. The *GPE* at the top would be identical to the *KE* at the bottom. You use $GPE = mgh$ and a balance to measure the mass and a ruler to measure the height.

3.a)

Students record their results using both methods, also recording any adaptations they make to their methods to increase the accuracy.

3.b)

The two *KE* values should be approximately the same.

4.

The students will record the mass of the pop-up toy with the nickels attached.

5.a)

The heavier pop-up toy has more mass. It may have the same *KE* when it jumps, and thus the same *GPE* when at the peak, but with more mass it will have less speed. With less speed it will not go as high. The students may have more difficulty measuring the lower height, since it is only in the air for a very short time.

6.a)

The *KE* as it sits on the table is 0 J. The *GPE* as it sits on the table is 0 J, since you will take $h = 0$ at the surface of the table.

6.b)

As the top of the toy rises from the table, the *KE* and *GPE* both increase, since the spring is expanding to its "normal" length, converting the spring potential energy into *KE* and *GPE* of the system. *GPE* increases a bit because the center of mass of the pop-up toy is rising during this initial phase. After the pop-up toy leaves the table, the spring no longer plays a role and the *KE* decreases as the *GPE* increases.

6.c)

The energy comes from the compressed spring.

6.d)

The *KE* is greatest when the pop-up toy just leaves the table. The *GPE* is greatest at the peak of the jump.

7.a)

See table below.

Position height above table	*SPE* (J)	*KE* (J)	*GPE* (J)	*SPE* + *KE* + *GPE*
$h = 0$	20	0	0	20
Just after popping: height = 0 m	0	20	~0	20
At peak: height = 0.30 m	0	0	20	20
½ the way up: $h = 0.15$ m	0	10	10	20
With the spring partially opened: height = 0 m	10	10	~0	20
Some other position: height = ? m	?	?	?	20

At "some other position," answers will vary. But the sum of the energies should be 20.

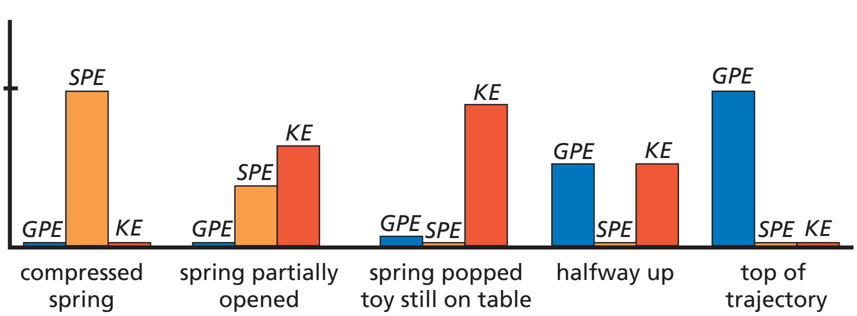
1. Play with the toy to get a sense of how high it jumps.
   a) What is the approximate height of a jump?
   b) How consistent is the pop-up toy from one jump to the next?

2. Discuss ideas with your group to identify two distinct methods you can use to determine the *KE* when the pop-up toy leaves the table. One method will use the velocimeter. The second method will use a meter stick to measure the height of the jump.
   a) Record your two methods in your *Active Physics* log. Since another team may want to understand what you have done, be quite careful to list all the steps. Indicate how all measurements are completed, and what is recorded or calculated.
3. Conduct the investigation into *KE* using both of your methods.
   a) Record your results. If you changed your procedure during the experiment, you should also record any changes here. These modifications are similar to the *Process* step of your *Engineering Design Cycle*.
   b) Compare the *KE* determinations from the two methods.
4. Measure the mass of the pop-up toy using a balance. Tape some coins to the top of the pop-up toy in order to approximately double its mass. The mass of a nickel is approximately 5 g. You can probably come close to doubling the mass of the pop-up toy by adding nickels.
5. Repeat the investigation and find the *KE* of the pop-up toy as it leaves the ground. Be sure the coins are taped securely on the toy. Retape after every two or three trials.
   a) Why do you think that the heavier pop-up toy behaved differently? Use the terms *GPE* and *KE* in your explanation.

Remember to wear eye protection during this *Investigate* and to have team members step back before the toy is released.

6. Answer the following questions in your *Active Physics* log:
   a) What is the *KE* and *GPE* of the toy when it sits on the table?
   b) What happens to the *GPE* and *KE* of the toy as it rises from the table?
   c) If the total energy of the toy is conserved, where does the *KE* and *GPE* come from as it rises?
   d) Where is the toy when its *KE* is greatest and where is it when its *GPE* is greatest?
7. The pop-up toy had both *KE* and *GPE* as it rose above the table. While the toy was sitting on the table, it also had *spring potential energy*, *SPE*. This *SPE* was converted to *KE* when the toy leaped off the table. The *KE* then became increasing *GPE* and decreasing *KE* as the pop-up toy ascended and slowed down. Using the concept of conservation of energy from the last section, you notice that before popping up, the energy of the toy was all *SPE*. Just after popping up, it was all *KE*. When reaching the highest point, the energy was all *GPE*.

7.b)

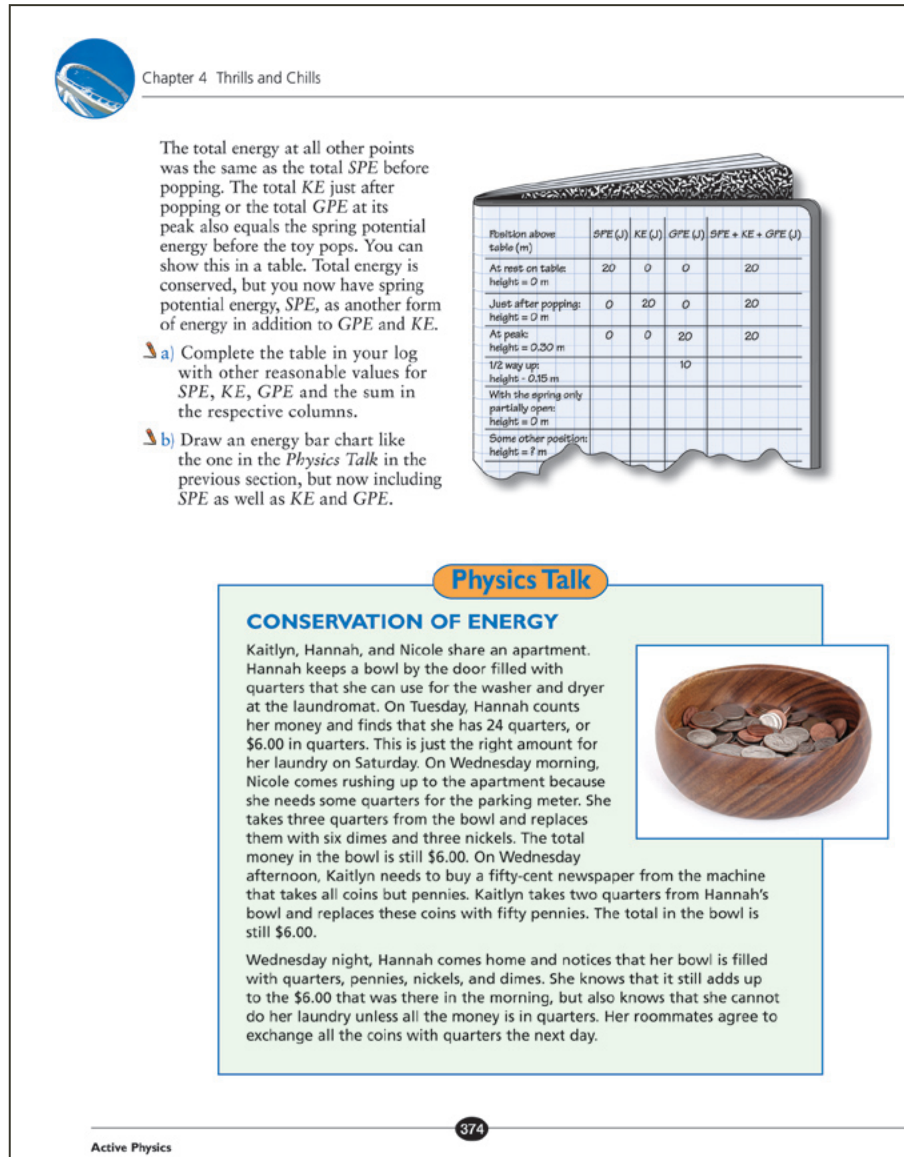


Physics Talk

Using an analogy of a bowl full of quarters, students read how replacing some quarters with nickels, dimes, and pennies does not affect the total amount of money in the bowl; in the same way as the total amount of energy in a closed system is conserved regardless of the intermediary changes in the energy in a system.

The bar graphs in the *Physics Talk* extend those developed in the previous section to illustrate the conservation of energy when spring potential energy is included. Point out that the total number of joules (like the total amount of money in the bowl) before and after the energy changes in a system remains the same. As students analyze the bar graphs, discuss the *Investigate* and how these graphs interpret the types of energy of a pop-up toy at different locations. Ask the students why the heavier pop-up toy did not go as high as the lower mass pop-up toy if both toys had identical spring potential energies. Students should also be able to tell where the energy at the top of the ride came from. Consider having the students do a second sample problem using different numbers to reinforce the concepts.

Have the students write down the equation for spring potential energy in their *Active Physics* logs. Find out if they know which type of spring would have a large spring constant. Write the sample problem on the board and have student volunteers come up and write a step for each strategy, so



Chapter 4 Thrills and Chills

The total energy at all other points was the same as the total *SPE* before popping. The total *KE* just after popping or the total *GPE* at its peak also equals the spring potential energy before the toy pops. You can show this in a table. Total energy is conserved, but you now have spring potential energy, *SPE*, as another form of energy in addition to *GPE* and *KE*.

a) Complete the table in your log with other reasonable values for *SPE*, *KE*, *GPE* and the sum in the respective columns.

b) Draw an energy bar chart like the one in the *Physics Talk* in the previous section, but now including *SPE* as well as *KE* and *GPE*.

Position above table (m)	SPE (J)	KE (J)	GPE (J)	SPE + KE + GPE (J)
At rest on table: height = 0 m	20	0	0	20
Just after popping: height = 0 m	0	20	0	20
At peak: height = 0.30 m	0	0	20	20
1/2 way up: height - 0.15 m			10	
With the spring only partially open: height = 0 m				
Some other position: height = ? m				

Physics Talk

CONSERVATION OF ENERGY

Kaitlyn, Hannah, and Nicole share an apartment. Hannah keeps a bowl by the door filled with quarters that she can use for the washer and dryer at the laundromat. On Tuesday, Hannah counts her money and finds that she has 24 quarters, or $6.00 in quarters. This is just the right amount for her laundry on Saturday. On Wednesday morning, Nicole comes rushing up to the apartment because she needs some quarters for the parking meter. She takes three quarters from the bowl and replaces them with six dimes and three nickels. The total money in the bowl is still $6.00. On Wednesday afternoon, Kaitlyn needs to buy a fifty-cent newspaper from the machine that takes all coins but pennies. Kaitlyn takes two quarters from Hannah's bowl and replaces these coins with fifty pennies. The total in the bowl is still $6.00.

Wednesday night, Hannah comes home and notices that her bowl is filled with quarters, pennies, nickels, and dimes. She knows that it still adds up to the $6.00 that was there in the morning, but also knows that she cannot do her laundry unless all the money is in quarters. Her roommates agree to exchange all the coins with quarters the next day.

374

Active Physics

that you can emphasize how the *GPE*, *SPE*, and *KE* are calculated.

Section 3 Spring Potential Energy: More Energy

The money in the bowl could represent the energy in a system. The total amount of energy may have been 600 J. As the coins in the bowl change from quarters to dimes and nickels to pennies and back to quarters, the energy in the system can vary from kinetic energy to gravitational potential energy to **spring potential energy** in any combination. (*Note*: In Chapter 2, the term *EPE* (elastic potential energy) was used. Bungee cords, trampolines, and bent poles in pole vaulting all have elastic potential energy (*EPE*). The best approximation for *EPE* is that these behave like springs. In this chapter, we refer to *SPE*, or spring potential energy.)

If Kaitlyn had taken the two quarters and not replaced them with pennies, then the total money would be less. The loss in money due to Kaitlyn would have resulted in that money being somewhere else. In some systems, energy is also lost. A bouncing ball does not get to the same height in each successive bounce. Some of the energy of the ball becomes sound energy and heat energy. These can be measured and will indicate that some energy left the system but did not disappear. In the pop-up toy and the roller coaster, the total energy can be *GPE*, *KE*, or *SPE*, but the sum of the energies must always be the same.

As you followed the changes in Hannah's bowl of money, you knew that there were ways to measure the total amount of money. Fifty pennies is identical in value to two quarters. Scientists look for all the energies in a system. There is electrical energy, light energy, nuclear energy, sound energy, heat (thermal) energy, chemical energy, and others. Each one is able to be calculated using measurements. All the energies are measured in joules. If you take into account all forms of energy, the total number of joules must always remain the same. The total energy is conserved.

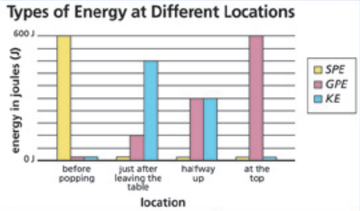
In this section, you were able to observe how the spring potential energy of the pop-up toy became the kinetic energy of the pop-up toy, which then became the gravitational potential energy of the pop-up toy. A graph of the three types of energy at different locations in the top diagram to the right shows that each type of energy changes in value, but that the total energy remains the same. The total energy can best be shown in a stacked bar chart of the same data in the bottom diagram to the right.

In this *Investigate*, you were also able to observe how the pop-up toy's behavior changed when you doubled its mass.

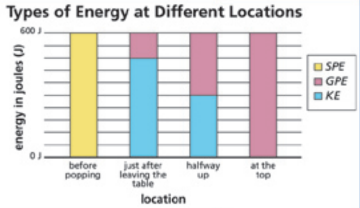
Physics Words

spring potential energy: the energy stored in a spring due to its compression or stretch.

Types of Energy at Different Locations



Types of Energy at Different Locations



The larger mass pop-up toy did not go as high as the original, lower-mass pop-up toy. Both pop-up toys had identical spring potential energies. Since "before popping" the *SPE* represents the total energy, both pop-up toys had identical total energies as well. The graphs shown on the previous page could describe the heavier pop-up toy as well. The less massive and more massive pop-up toys can have the same *GPE* if and only if the more massive pop-up toys do not go as high. Since $GPE = mgh$, the larger mass has a smaller height and the smaller mass has a larger height.

In a real roller coaster, the roller coaster has all its energy as *GPE* (gravitational potential energy) as it sits on the highest hill. Most of this energy becomes *KE* (kinetic energy) as the roller coaster is released. Some small amount of the energy is converted to thermal energy and a smaller part to sound energy.

Where does the roller coaster get all of that *GPE* that drives the rest of the ride? Something has to pull the roller coaster up to the top of the hill. The energy to pull the roller coaster is usually electric. The electrical energy comes from a power plant (that burns oil, gas, coal, or uses nuclear energy or water's potential energy) or from a local generator that may use gasoline.

After the cars are pulled to the top of the hill, the total *GPE* and *KE* of the roller coaster remains the same except for losses due to thermal and sound energy. At the end of the ride, the *KE* is converted to thermal energy as the brakes bring the cars to a halt. If the brakes fail, there may be a large spring that will stop the car as the car compresses it.

Calculating Spring Potential Energy

In this section, you extended the conservation of energy principle to include the spring potential energy. It is possible to calculate the spring potential energy.

The equation for spring potential energy is

$$SPE = \frac{1}{2}kx^2,$$

where k is the spring constant and

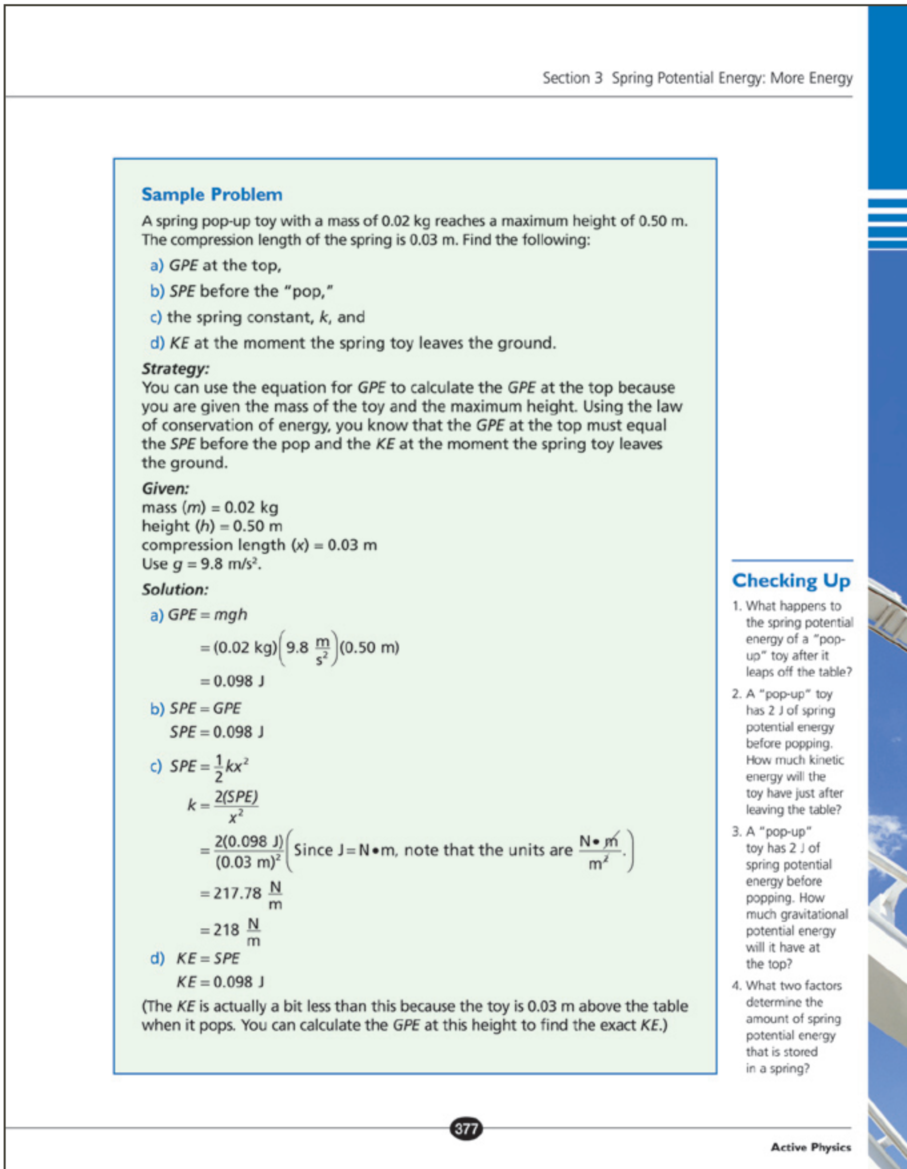
x is the amount of stretch or compression of the spring.

A spring that is difficult to compress or stretch will have a large spring constant (k). That spring will "pack" more *SPE* for an identical compression than a spring that is easy to compress.

The total energy of a spring toy that can jump into the air is the sum of the *SPE*, the *GPE*, and the *KE*. Once the spring is compressed, the sum of these three energies, *GPE*, *KE*, and *SPE* must remain constant.

$$GPE + KE + SPE = \text{constant}$$

$$mgh + \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$



Sample Problem

A spring pop-up toy with a mass of 0.02 kg reaches a maximum height of 0.50 m. The compression length of the spring is 0.03 m. Find the following:

a) *GPE* at the top,

b) *SPE* before the "pop,"

c) the spring constant, k, and

d) *KE* at the moment the spring toy leaves the ground.

Strategy:
You can use the equation for *GPE* to calculate the *GPE* at the top because you are given the mass of the toy and the maximum height. Using the law of conservation of energy, you know that the *GPE* at the top must equal the *SPE* before the pop and the *KE* at the moment the spring toy leaves the ground.

Given:
mass $(m) = 0.02$ kg
height $(h) = 0.50$ m
compression length $(x) = 0.03$ m
Use $g = 9.8\ \text{m/s}^2$.

Solution:

a) $GPE = mgh$

$= (0.02\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.50\ \text{m})$

$= 0.098\ \text{J}$

b) $SPE = GPE$

$SPE = 0.098\ \text{J}$

c) $SPE = \frac{1}{2}kx^2$

$k = \frac{2(SPE)}{x^2}$

$= \frac{2(0.098\ \text{J})}{(0.03\ \text{m})^2}$ (Since $\text{J} = \text{N}\bullet\text{m}$, note that the units are $\frac{\text{N}\bullet\text{m}}{\text{m}^2}$.)

$= 217.78\ \frac{\text{N}}{\text{m}}$

$= 218\ \frac{\text{N}}{\text{m}}$

d) $KE = SPE$

$KE = 0.098\ \text{J}$

(The *KE* is actually a bit less than this because the toy is 0.03 m above the table when it pops. You can calculate the *GPE* at this height to find the exact *KE*.)

Checking Up

1. What happens to the spring potential energy of a "pop-up" toy after it leaps off the table?
2. A "pop-up" toy has 2 J of spring potential energy before popping. How much kinetic energy will the toy have just after leaving the table?
3. A "pop-up" toy has 2 J of spring potential energy before popping. How much gravitational potential energy will it have at the top?
4. What two factors determine the amount of spring potential energy that is stored in a spring?

Checking Up

1.

The spring potential energy of a pop-up toy gets converted to kinetic energy, after it initially leaps off the table, and then changes to gravitational potential energy.

2.

The pop-up toy will have 2 J of kinetic energy just after leaving the table.

3.

The toy will have 2 J of gravitational potential energy at the peak of its rise.

4.

The two factors responsible for the amount of spring potential energy stored in a spring are the spring constant and the amount of stretch or compression of the spring.

Active Physics Plus

1.

Before the toy "pops," it is at rest, so $KE = 0$. The spring is compressed, so $SPE = \frac{1}{2}kx^2$, where x is the amount of compression of the spring.

2.

$$KE + GPE + SPE =$$
$$\tfrac{1}{2}mv^2 + mgh + \tfrac{1}{2}kx^2 = 0 + 0 + \tfrac{1}{2}kx^2$$

3.

When it reaches its highest point, its velocity is zero, so $KE = 0$. Since the spring has now relaxed, $x = 0$ and $SPE = 0$.

4.

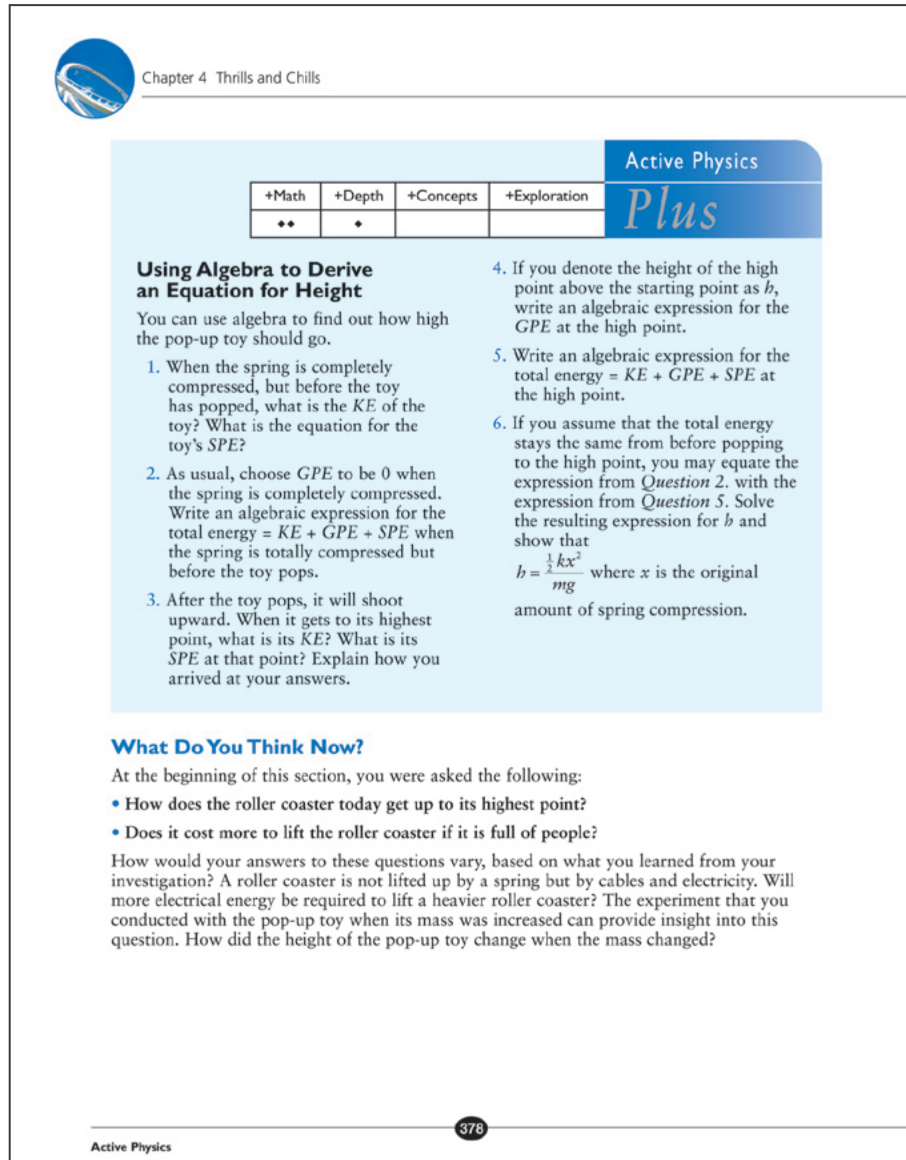
At the height $h, GPE = mgh$.

5.

$$KE + GPE + SPE =$$
$$\tfrac{1}{2}mv^2 + mgh + \tfrac{1}{2}kx^2 = 0 + mgh + 0$$

6.

Initial energy equals final energy or $\frac{1}{2}kx^2 = mgh$. Solving for h, $h = \frac{1}{2}kx^2/mg$.



Chapter 4 Thrills and Chills

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
••	•		

Using Algebra to Derive an Equation for Height

You can use algebra to find out how high the pop-up toy should go.

1. When the spring is completely compressed, but before the toy has popped, what is the *KE* of the toy? What is the equation for the toy's *SPE*?
2. As usual, choose *GPE* to be 0 when the spring is completely compressed. Write an algebraic expression for the total energy = *KE* + *GPE* + *SPE* when the spring is totally compressed but before the toy pops.
3. After the toy pops, it will shoot upward. When it gets to its highest point, what is its *KE*? What is its *SPE* at that point? Explain how you arrived at your answers.
4. If you denote the height of the high point above the starting point as h, write an algebraic expression for the *GPE* at the high point.
5. Write an algebraic expression for the total energy = *KE* + *GPE* + *SPE* at the high point.
6. If you assume that the total energy stays the same from before popping to the high point, you may equate the expression from *Question 2.* with the expression from *Question 5.* Solve the resulting expression for h and show that $h = \frac{\frac{1}{2}kx^2}{mg}$ where x is the original amount of spring compression.

What Do You Think Now?

At the beginning of this section, you were asked the following:

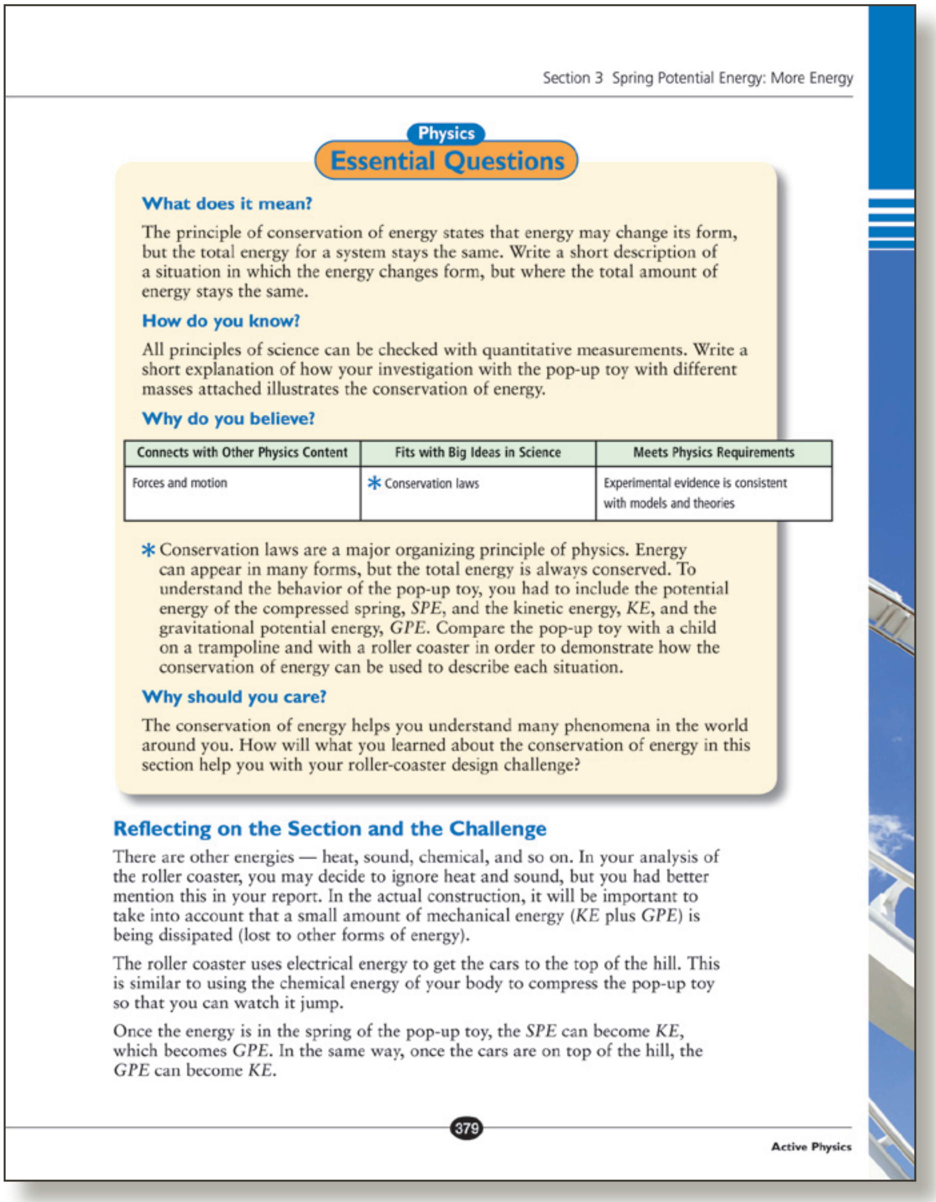
- **How does the roller coaster today get up to its highest point?**
- **Does it cost more to lift the roller coaster if it is full of people?**

How would your answers to these questions vary, based on what you learned from your investigation? A roller coaster is not lifted up by a spring but by cables and electricity. Will more electrical energy be required to lift a heavier roller coaster? The experiment that you conducted with the pop-up toy when its mass was increased can provide insight into this question. How did the height of the pop-up toy change when the mass changed?

Active Physics 378

What Do You Think Now?

This is a good time for students to go back to the *What Do You See?* illustration and determine if they can now see the artist's purpose in showing a pop-up toy being released from a table. Most students should be able to answer the *What Do You Think Now?* questions based on their experiments in the *Investigate* and reading of the *Physics Talk*. Consider sharing *A Physicist's Response* and encourage students to update their original responses to the *What Do You Think?* questions. These questions cover different aspects of energy and an insightful discussion in class should promote an enriched understanding of how roller coasters operate.



Physics
Essential Questions

What does it mean?

The principle of conservation of energy states that energy may change its form, but the total energy for a system stays the same. Write a short description of a situation in which the energy changes form, but where the total amount of energy stays the same.

How do you know?

All principles of science can be checked with quantitative measurements. Write a short explanation of how your investigation with the pop-up toy with different masses attached illustrates the conservation of energy.

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	* Conservation laws	Experimental evidence is consistent with models and theories

* Conservation laws are a major organizing principle of physics. Energy can appear in many forms, but the total energy is always conserved. To understand the behavior of the pop-up toy, you had to include the potential energy of the compressed spring, *SPE*, and the kinetic energy, *KE*, and the gravitational potential energy, *GPE*. Compare the pop-up toy with a child on a trampoline and with a roller coaster in order to demonstrate how the conservation of energy can be used to describe each situation.

Why should you care?

The conservation of energy helps you understand many phenomena in the world around you. How will what you learned about the conservation of energy in this section help you with your roller-coaster design challenge?

Reflecting on the Section and the Challenge

There are other energies — heat, sound, chemical, and so on. In your analysis of the roller coaster, you may decide to ignore heat and sound, but you had better mention this in your report. In the actual construction, it will be important to take into account that a small amount of mechanical energy (*KE* plus *GPE*) is being dissipated (lost to other forms of energy).

The roller coaster uses electrical energy to get the cars to the top of the hill. This is similar to using the chemical energy of your body to compress the pop-up toy so that you can watch it jump.

Once the energy is in the spring of the pop-up toy, the *SPE* can become *KE*, which becomes *GPE*. In the same way, once the cars are on top of the hill, the *GPE* can become *KE*.

Reflecting on the Section and the Challenge

Students should reflect on energies other than gravitational, spring, and kinetic energy. They should know that for a roller coaster some energy is always lost to the surroundings generally in the form of heat, or sound, or rarely chemical energy. Emphasize that these small losses of energy explain why the *GPE* measured in the beginning may not always equal the *KE* at a later point. It is important to mention the loss of a small amount of energy when writing a report for the *Chapter Challenge*. This section requires students to focus on energy transformations. Learning about the phenomenon of energy conversion should help them to modify the design of a roller coaster, keeping the safety of passengers in mind. Students should read *Reflecting on the Section and the Challenge* to develop specific connections between energy transformation and the design of a roller coaster.

Physics Essential Questions

What does it mean?

In both the roller-coaster cars and the pendulum, the sum of the gravitational potential energy (*GPE*) at the top and the kinetic energy (*KE*) at the top was equal to the sum at the bottom. At the top, all the energy was *GPE* while at the bottom all the energy was *KE*.

How do you know?

The pop-up toy investigation demonstrated that the sum of the spring potential energy (*SPE*) and the *GPE* and the kinetic energy *KE* was constant. When mass was added to the toy, the *SPE* was identical. That *SPE* sets the maximum *GPE*. Since $GPE = mgh$, an increase in the mass must have a corresponding decrease in the height for the same *GPE*.

Why do you believe?

A child on a trampoline has maximum *GPE* at her height. She has maximum *KE* when she has descended a bit on the trampoline. She has maximum *SPE* when the trampoline is at its maximum stretch. At that maximum stretch, the *GPE* is zero and the *KE* is zero.

Why should you care?

The *GPE* at the maximum height determines the maximum *KE* of the roller coaster as well as the *SPE* that may be necessary to stop the roller coaster if brakes should fail.

Physics to Go

1.

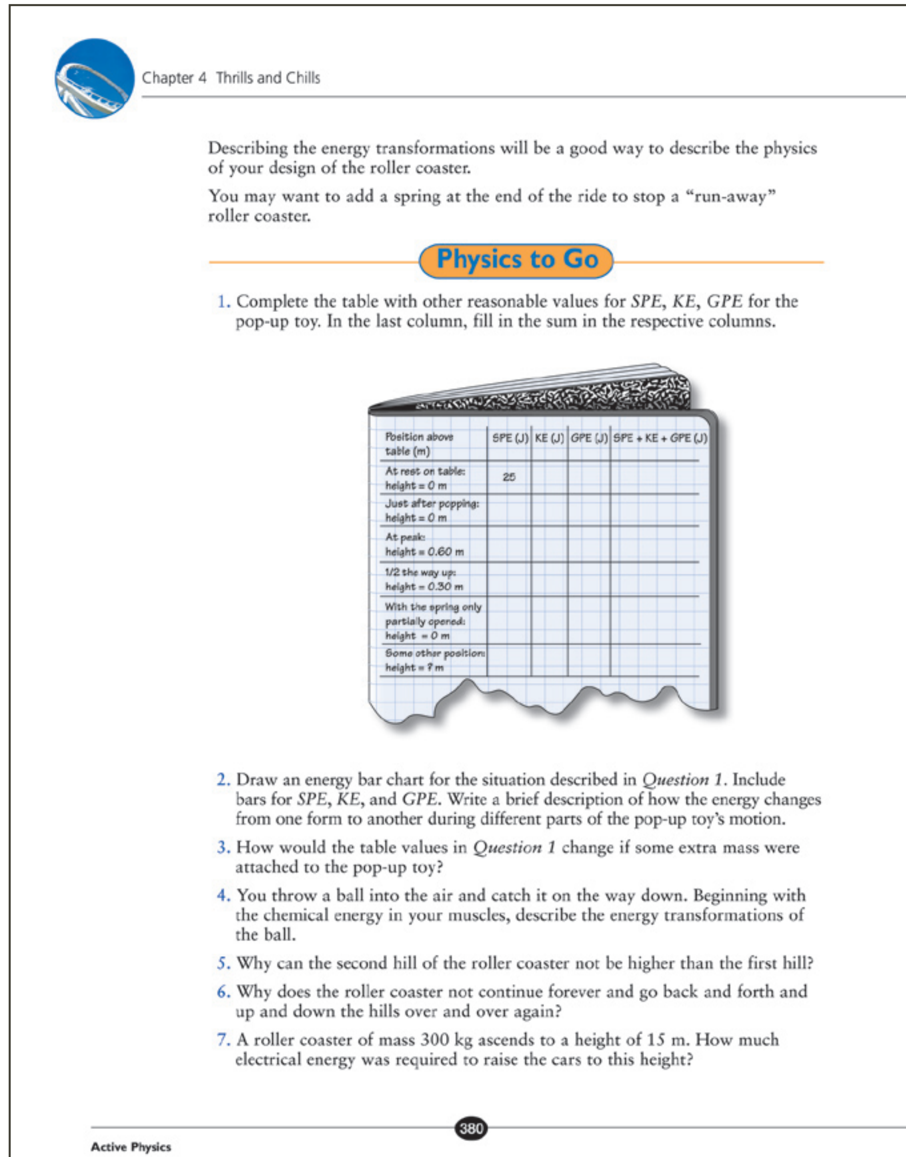
See table at the bottom of this page. Answers may vary, but the total in the final column should always read 25 J.

2.

Description: At $h = 0$ m, the mechanical energy is all spring potential energy. When the spring pops, the mechanical energy turns into kinetic energy, but the toy has not yet left the table so the *GPE* is still close to zero. Halfway up, the *KE* has decreased and the *GPE* has increased. The *SPE* is zero because the spring remains uncompressed. At the top of the trajectory, the $KE = 0$ and the *GPE* is now 25 J.

3.

If the mass were increased, the *SPE* would remain the same and therefore the total energy would remain the same. The *GPE* would remain the same because it is based on the height only, and the *KE* would remain the same as well. However, the spring toy with the same *GPE* would not reach the same height as before nor would it reach the same speed.



Describing the energy transformations will be a good way to describe the physics of your design of the roller coaster.

You may want to add a spring at the end of the ride to stop a "run-away" roller coaster.

Physics to Go

1. Complete the table with other reasonable values for *SPE*, *KE*, *GPE* for the pop-up toy. In the last column, fill in the sum in the respective columns.

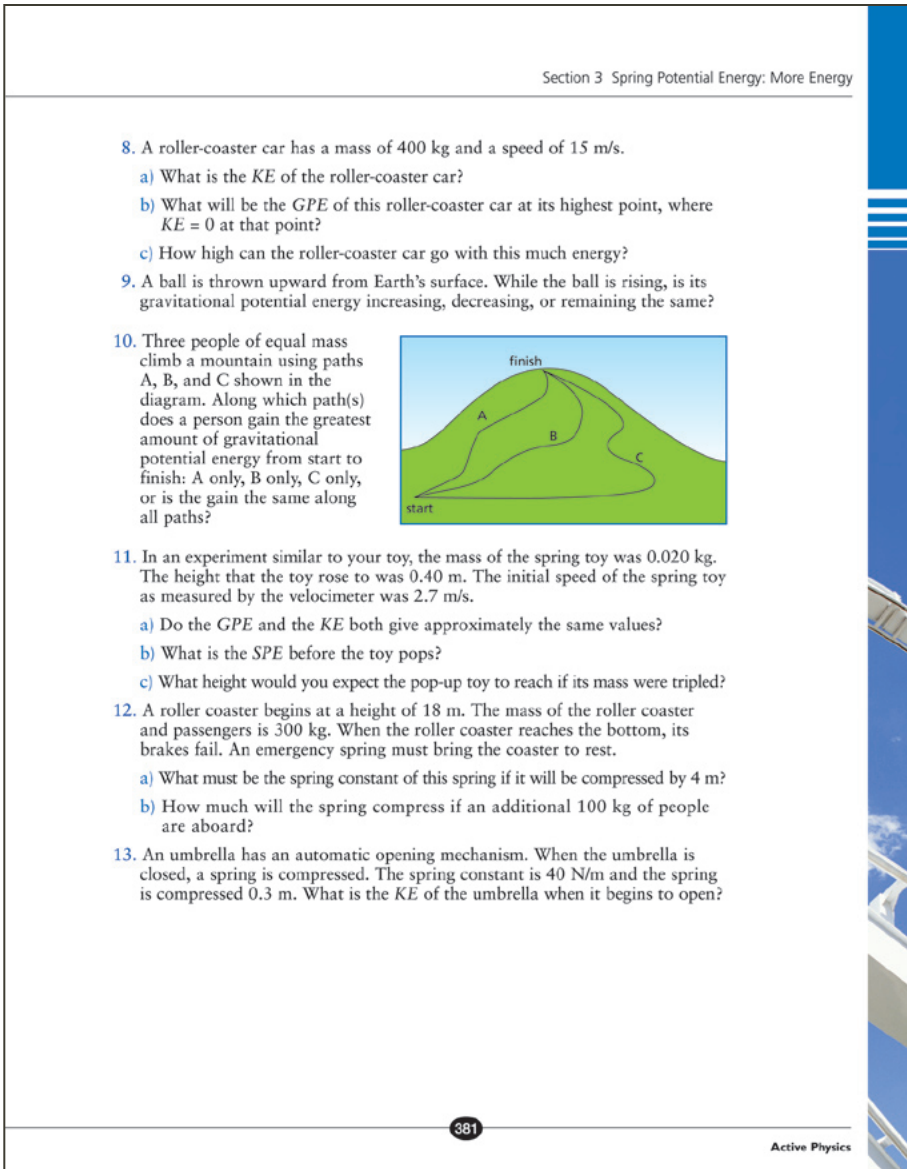
Position above table (m)	SPE (J)	KE (J)	GPE (J)	SPE + KE + GPE (J)
At rest on table: height = 0 m	25			
Just after popping: height = 0 m				
At peak: height = 0.60 m				
1/2 the way up: height = 0.30 m				
With the spring only partially opened: height = 0 m				
Some other position: height = ? m				

2. Draw an energy bar chart for the situation described in *Question 1*. Include bars for *SPE*, *KE*, and *GPE*. Write a brief description of how the energy changes from one form to another during different parts of the pop-up toy's motion.
3. How would the table values in *Question 1* change if some extra mass were attached to the pop-up toy?
4. You throw a ball into the air and catch it on the way down. Beginning with the chemical energy in your muscles, describe the energy transformations of the ball.
5. Why can the second hill of the roller coaster not be higher than the first hill?
6. Why does the roller coaster not continue forever and go back and forth and up and down the hills over and over again?
7. A roller coaster of mass 300 kg ascends to a height of 15 m. How much electrical energy was required to raise the cars to this height?

Position above table (m)	SPE (J)	KE (J)	GPE (J)	SPE + KE + GPE (J)
At rest	25	0	0	25
Just after popping	0	~25	~0	25
Peak height	0	0	25	25
Halfway up	0	12.5	12.5	25
Spring partially open	~12.5	~12.5	0	25
Some other position	?	?	?	25

4.

Electrical signals are sent to the muscles ordering them to contract. Chemical energy allows the muscles to contract. The contracted muscles push the ball into the air. The ball begins its motion with a large kinetic energy. As it rises, the kinetic energy is transformed into gravitational potential energy. When it reaches its highest point, the ball has maximum



8. A roller-coaster car has a mass of 400 kg and a speed of 15 m/s.
   a) What is the *KE* of the roller-coaster car?
   b) What will be the *GPE* of this roller-coaster car at its highest point, where *KE* = 0 at that point?
   c) How high can the roller-coaster car go with this much energy?
9. A ball is thrown upward from Earth's surface. While the ball is rising, is its gravitational potential energy increasing, decreasing, or remaining the same?
10. Three people of equal mass climb a mountain using paths A, B, and C shown in the diagram. Along which path(s) does a person gain the greatest amount of gravitational potential energy from start to finish: A only, B only, C only, or is the gain the same along all paths?

11. In an experiment similar to your toy, the mass of the spring toy was 0.020 kg. The height that the toy rose to was 0.40 m. The initial speed of the spring toy as measured by the velocimeter was 2.7 m/s.
    a) Do the *GPE* and the *KE* both give approximately the same values?
    b) What is the *SPE* before the toy pops?
    c) What height would you expect the pop-up toy to reach if its mass were tripled?
12. A roller coaster begins at a height of 18 m. The mass of the roller coaster and passengers is 300 kg. When the roller coaster reaches the bottom, its brakes fail. An emergency spring must bring the coaster to rest.
    a) What must be the spring constant of this spring if it will be compressed by 4 m?
    b) How much will the spring compress if an additional 100 kg of people are aboard?
13. An umbrella has an automatic opening mechanism. When the umbrella is closed, a spring is compressed. The spring constant is 40 N/m and the spring is compressed 0.3 m. What is the *KE* of the umbrella when it begins to open?

gravitational potential energy and is momentarily at rest with zero kinetic energy. As it descends it picks up speed and kinetic energy, gradually losing its gravitational potential energy. When you catch the ball, your muscles dissipate the ball's *KE*, bringing it to rest.

5.

For the roller coaster to reach the top of a second hill that is higher than the first, it would require additional gravitational potential energy. The total mechanical energy of the ride cannot increase once it is rolling on its own.

6.

In real life, the roller coaster is constantly losing mechanical energy due to friction with the track and with the air. The energy is lost as heat or heard as a sound.

7.

Given:
$m = 300$ kg; $h = 15$ m;
The electrical energy required will be equal to the change in gravitational potential energy. $GPE = mgh =$
$(300 \text{ kg})(10 \text{ N/kg})(15 \text{ m}) =$
$45{,}000$ J.

8.a)

Given:
$m = 400$ kg; $v = 15$ m/s

$KE = \frac{1}{2}mv^2 =$
$\frac{1}{2}(400 \text{ kg})(15 \text{ m/s})^2 = 45{,}000 \text{ J}$

8.b)

$GPE = 45{,}000$ J

8.c)

$GPE = mgh$
$45{,}000 \text{ J} = (400 \text{ kg})(10 \text{ N/kg})(h)$
$h = 11$ m

9.

As it rises, the *GPE* is increasing.

10.

The gain is the same along all paths.

11.a)

Given:
$m = 0.02$ kg; $h = 0.4$ m; $v = 2.7$ m/s

$GPE = mgh =$
$(0.020 \text{ kg})(9.8 \text{ m/s}^2)(0.40 \text{ m}) =$
0.078 J
$KE = \frac{1}{2}mv^2 =$
$\frac{1}{2}(0.020 \text{ kg})(2.7 \text{ m/s})^2 = 0.073 \text{ J}$
Yes, the answers are approximately equal.

CHAPTER 4

11.b)

The *SPE* would be approximately equal to the *KE* or *GPE* or around 0.075 J

11.c)

$GPE = mgh$

$0.078\text{ J} = (0.060\text{ J})(9.8\text{ m/s}^2)(h)$

$0.13\text{ m} = h$

Since the mass is tripled, the height would be about 1/3 as high.

12.a)

The gravitational potential energy of the roller coaster must be stored in the spring potential energy when the coaster is stopped (hopefully with a latching mechanism at that point). Solving for the spring constant:

Given:

$h = 18\text{ m};\ m = 300\text{ kg}$

$GPE = SPE$

$mgh = \frac{1}{2}kx^2$

$(300\text{ kg})(9.8\text{ m/s}^2)(18\text{ m}) =$

$\frac{1}{2}k(4\text{ m})^2$

$k \approx 6600\text{ N/m}$

12.b)

With an additional 100 kg of passengers, the equation becomes

$mgh = \frac{1}{2}kx^2$

$x^2 = \dfrac{2mgh}{k}$

$x = \sqrt{\dfrac{2mgh}{k}} =$

$\sqrt{\dfrac{2(400\text{ kg})(9.8\text{ m/s}^2)(18\text{ m})}{6600\text{ N/m}}} =$

4.62 m

13.

Given:

$k = 40\text{ N/m}$

$x = 0.3\text{ m}$

The *SPE* stored in the spring is converted into *KE* when the spring is released, so the

$KE = SPE$

$SPE = \frac{1}{2}kx^2 =$

$\frac{1}{2}(40\text{ N/m})(0.3\text{ m})^2 = 1.8\text{ J}$

Just about all of the *SPE* converted to *KE* would be lost to friction in the air as the umbrella opens.

NOTES

NOTES

SECTION 3 QUIZ

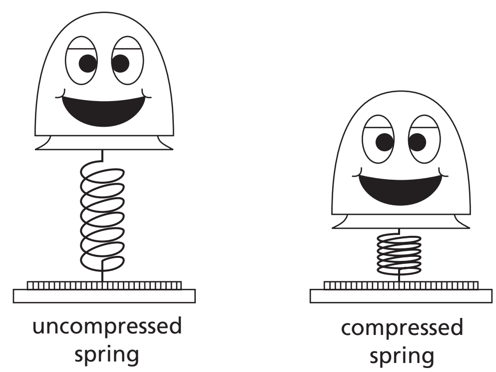
4-3c **Blackline Master**

1. A spring has a spring constant of 120 N/m. How much potential energy is stored in the spring as it is stretched 0.20 m?

   a) 2.4 J
   b) 12 J
   c) 4.8 J
   d) 24 J

2. In the diagram below, a student compresses the spring in a pop-up toy a distance of 0.020 m. If 0.068 J of energy are stored in the toy, what is the toy's spring constant?

   a) 120 N/m
   b) 170 N/m
   c) 225 N/m
   d) 340 N/m



3. In *Question 2*, how much kinetic energy would the spring toy have immediately after "popping"?

   a) 0.068 J
   b) 0.340 J
   c) 0.124 J
   d) 1.24 J

4. In *Question 2*, if the mass of the pop-up toy is 0.008 kg, what will be the pop-up toy's gravitational potential energy at the peak of its rise?

   a) 0.085 J
   b) 0.054 J
   c) 0.068 J
   d) 0.15 J

5. For *Question 4*, how high is the pop-up toy at the peak of its rise?

   a) 0.85 m
   b) 0.54 m
   c) 0.68 m
   d) 0.15 m

SECTION 3 QUIZ ANSWERS

1. a) Spring potential energy *(SPE)* is given by the formula $SPE = \frac{1}{2}kx^2$. Calculating the *SPE* with the values given yields $SPE = \frac{1}{2}(120\ \text{N/m})(0.20\ \text{m})^2 = 2.4\ \text{J}$

2. d) Using $SPE = \frac{1}{2}kx^2$ and solving for the spring constant k gives $SPE = \frac{1}{2}kx^2$ or $0.068\ \text{J} = \frac{1}{2}(k)(0.020\ \text{m})^2$ becomes $0.068\ \text{J} = (k)0.0002\ \text{m}^2$. Solving for k yields $k = 340\ \text{N/m}$.

3. a) When the spring pops, the *SPE* is converted into *KE* as the pop-up toy starts to rise. By conservation of energy the $SPE = KE = 0.068\ \text{J}$.

4. c) After the spring pops, the *SPE* is converted into *KE*, which then goes into *GPE* as the toy rises to its peak. Assuming there is no loss of energy as the toy rises, the *GPE* at the peak equals the *KE* immediately after the spring pops, which is equal to the *SPE* stored in the spring. Once again the answer is 0.068 J. The mass of the pop-up toy has no effect on the energy it possesses, but will affect the height the toy reaches.

5. a) Using the formula for $GPE = mgh$ gives $0.068\ \text{J} = (0.008\ \text{kg})(10\ \text{m/s}^2)(h)$. Solving for h gives 0.85 m.