SECTION 4

Newton's Law of Universal Gravitation: The Ups and Downs of a Roller Coaster

Section Overview

Students explore the force of gravity by investigating the direction in which an object falls. They drop a penny and observe that the penny falls vertically down with an increasing speed. Students learn that this increase in speed or acceleration is identical from any location on Earth. They draw vertical lines in a map to represent the direction of Earth's gravitational field. They determine what a map indicating the direction of the falling pennies would look like if the penny experiment was performed in a classroom there. By crumpling a piece of paper and treating it as a globe, they mark Australia's location on the map and compare the direction of gravitational field arrows in Australia to the direction of those arrows in their classroom.

Students analyze the force of gravity exerted by Earth is weak or strong by determining that a mass as large as a planet is needed to accelerate a small mass like a penny. From the graph they determine the value of acceleration due to gravity at different locations from the center of Earth. By calculating the ratios of accelerations at different distances, students discover the inverse-square relationship between the magnitude of the gravitational pull and the distance from the mass exerting the force of gravity.

Background Information

Gravity is one of the four fundamental forces of nature. Each bit of matter produces a gravitational field (a region of space where the force of gravity acts). An object placed in this field will experience an attractive force. The force of gravity is always attractive and acts on all objects that have mass or the equivalent of mass (energy).

The gravitational force is the weakest of all the fundamental forces. Consequently, the gravitational force between ordinary objects such as baseballs and people is extremely small. Gravity becomes an important force only when at least one of the objects is extremely massive. You feel the force of Earth's gravity on you because of Earth's gigantic mass. The force of gravity also provides the "cosmic glue" that holds solar systems and galaxies together.

Sir Isaac Newton proposed that the gravitational force was universal: it acts on all masses everywhere in the universe. This was significant because before Newton, it was thought that "terrestrial gravity" (or Earthly gravity) pulls a falling apple to Earth, whereas a different "celestial gravitation" held the Moon and the planets in their orbits. Newton showed that it was the same force of gravity that makes the apple fall and holds the Moon in its orbit. He showed that the magnitude of the gravitational force is directly related to the product of the two masses involved in the interaction. He also demonstrated that the force of gravity depends on the distance separating two bodies in a unique way. Gravity obeys the "inverse square law"— the attractive force decreases inversely with the square of the distance separating two masses. When the distance between two masses doubles, the force of attraction is reduced by one-fourth. When the distance between the masses is tripled, the force of attraction is reduced by one-ninth, and so on. As with any force obeying the inverse square law, gravity's range is infinite. It is thought that the force of gravity works by these same principles throughout the universe.

Newton verified his law of universal gravitation by comparing the acceleration of a terrestrial object to the acceleration of the Moon. Newton knew that the distance from the center of Earth to the center of the Moon was 60 times larger than the radius of Earth, a ratio of 60:1. Hence the ratio of the acceleration should be $1/60^2$, or 1/3600, which it is.

In 1798, Henry Cavendish confirmed Newton's law of universal gravitation and measured the gravity constant, *G*, which puts the force of gravity in the metric unit of newtons. His apparatus consisted of two spheres of known mass at either end of a light rod suspended by a thin fiber from the center of the rod.

He had earlier found the small force that was needed to twist the fiber. By bringing a third sphere close to one of the suspended spheres, he was able to measure the force of gravity between the spheres, and hence *G*.

NOTES

Crucial Physics

- Earth's gravitational field always points toward Earth's center. The gravitational field is in the direction of the force exerted by Earth on a mass that is located on or near Earth. Field lines are used to depict Earth's gravitational field. The gravitational field is stronger where the lines are closer together and weaker where the lines are further apart. The gravitational field is everywhere and extends out to infinity.

- As the distance between a mass and Earth increases, the force exerted by Earth on the mass decreases in a manner proportional to the inverse square of the distance.

- Newton's law of universal gravitation describes the force between two masses, which is an inverse square law force, as was proven by Cavendish's experiment. Mathematically it is written as
  $$F_G = \frac{Gm_1m_2}{r^2},$$
  where F_G is the force between two bodies, r is the distance between their centers, m_1 and m_2 are the masses of the two bodies, and G is a universal constant equal to $6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$.

- The constant G relates the strength of the force of gravity to the system of units we use.

Learning Outcomes	Location in the Section	Evidence of Understanding
Analyze and map Earth's gravitational field.	***Investigate*** Step 1	Students make a field map with vertical lines and at five points along the line draw vertical arrows to show the direction of the penny's fall. The map represents Earth's gravitational field.
State Newton's law of universal gravitation.	***Physics Talk***	Students read about Newton's law of universal gravitation and record it in their *Active Physics* logs
Describe the inverse-square pattern.	***Investigate*** Step 2 ***Physics Talk***	Students find the ratios of accelerations at given distances, note the inverse- square pattern between two objects and describe it later while examining this pattern in the *Physics Talk*.
Express Newton's law of universal gravitation as a mathematical formula.	***Physics Talk***	Students express Newton's law of universal gravitation in an equation.

Section 4 Materials, Preparation, and Safety

Materials and Equipment

PLAN A		
Materials and Equipment	**Group (4 students)**	**Class**
Paper, graph, pkg. of 50		1 per class
Paper, sheet*	1 per group	
Pennies*		100 per class

*Additional items needed not supplied

Time Requirements

- Allow one class period or 45 minutes for the *Investigate* of this section.

Teacher Preparation

- Cut 8½ by 11 sheets of paper into three strips to allow each student to make a paper spring.
- Determine how the students will attach pointers to the masses suspended from the springs to take their readings.
- If there are an insufficient number of springs for each group to have a set, the students may swap springs to get a different one.
- Determine good examples of "unknown" masses that the students can weigh. The mass of these samples should be in the weight range measured by the students.
- Ensure that the given weights are able to stretch the spring, but not beyond its point of elasticity. You may want to consider how the students can mount the ruler so that it does not shift as they add weights to the spring.

Safety

- Students should be aware that masses might fall off the spring or mass hanger. Make certain that no feet are located below the spring where the masses may fall. Placing a soft surface underneath the area where the masses might fall will also protect the floor.
- When the masses are removed from the springs quickly while in the stretched position, the springs might be launched off their holders. Eye protection is again absolutely necessary.

Materials and Equipment

PLAN B		
Materials and Equipment	**Group (4 students)**	**Class**
Paper, graph, pkg. of 50		1 per class
Paper, sheet*		1 per class
Pennies*		100 per class

*Additional items needed not supplied

Time Requirements

- Allow one class period or 45 minutes for this investigation.

Teacher Preparation

- Make a Blackline Master of the table the students will be making to compare their results with the accepted graph.
- Have a globe available in the classroom so that the students will have a chance to locate the position of Australia and compare it to their location in the United States.

Safety

There are no specific safety issues for this *Investigate*.

CHAPTER 4

Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

Learning Issue	Reference	Augmentation and Accommodations
Creating graphs	***Investigate*** Step 2.a)	**Augmentation** • Creating graphs is a difficult task for many students, but it is especially difficult when a large range of numbers must be included on the scale, such as 6400 m to 391,000 m. Make sure students have the correct variable on each axis, assist students in deciding a reasonable scale for both axes, and monitor their point plotting as the graph will be used for further analysis. • Another method of creating graphs that requires some direct instruction but can be very helpful is teaching students to create graphs on spreadsheet software. Show students a step-by-step approach to entering data and creating graphs. Pair this modeled instruction with a written instruction sheet for students who struggle with auditory comprehension and attention to tasks. With some practice, students will become more independent. **Accommodation** • Provide a graph that has already been set up and ask students to plot the points. • Provide a copy of an accurate graph of this data if the class is focusing more on the inverse square law pattern than the skill of graphing.
Understanding the inverse square law by interpreting graphs and comparing values	***Investigate*** Steps 2.b) and 2.c)	**Augmentation** • The main reason students struggle with this skill is because they lack prior experiences with interpreting more complex functions on a graph. *Steps 2.c), 2.d)*, and *2.e)* require students to be proficient in some basic number concepts which are difficult for many students including accurately identifying points on a graph, finding ratios, and comparing fractions or decimals. • Remind students that a ratio is another way to represent the relationship between two numbers and can be represented 1:4 or ¼. Ratios can also be reduced like a fraction, and students may need to review this skill. • Provide direct instruction and model a thought process for interpreting a graph and comparing values using a think aloud. Students can also participate in this step by explaining their thought process to the class. **Accommodation** • Some students may not be developmentally ready to conceptually understand the inverse square law. They may be able to repeat the law or plug in numbers using the pattern, but if asked to apply the law to a novel situation, they would really struggle.
Imagining a model	***Investigate*** Step 3	**Augmentation** • Providing visual models are a great strategy for helping students with learning disabilities understand and remember a new concept. However, many of these students are not skilled at imagining a model that has been described to them because this is a complex task that requires a lot of practice. • Create a visual model to show the class. • Ask pairs of students to create their own models to display with their own explanation of how the effects of gravity are represented in the model.
Conceptually understanding formulas	***Physics to Go*** Questions 1, 2, and 7-10	**Augmentation** • If students are struggling to conceptually understand the formula for Newton's law of universal gravitation to answer the *Physics to Go* questions, encourage students to plug in numbers to test what would happen if the values for each variable were manipulated. **Accommodation** • Provide direction instruction and model the effects of changing values in the law of universal gravitation formula as a whole group.

Strategies for Students with Limited English-Language Proficiency

Learning Issue	Reference	Augmentation
Understanding concepts	***Investigate*** Step 1.g)	Make sure students recognize that all arrows point toward the center of Earth.
Vocabulary comprehension	***Investigate*** Step 2 and data table	Students will likely need help determining the meaning of "geosynchronous." Encourage students to think of familiar words with "geo" or "synch" as a root or prefix. As a prompt, ask them to define "geography" and "geology." Students may be familiar with the term "sync" as it is used to describe updating the music on an MP3 player. They also may have heard of synchromesh transmissions or seen spy thrillers in which people synchronize their watches.
Understanding concepts	***Investigate*** Step 3	Make sure students understand that the toothpicks represent Earth's gravitational field, but that they do not represent the limits of the field; the model of Earth's gravitational field extends beyond the ends of the toothpicks.

There is much information about the following terms in this section:

gravitational field

inverse-square relationship

Newton's law of universal gravitation

Kepler's first law

Kepler's second law

Kepler's third law

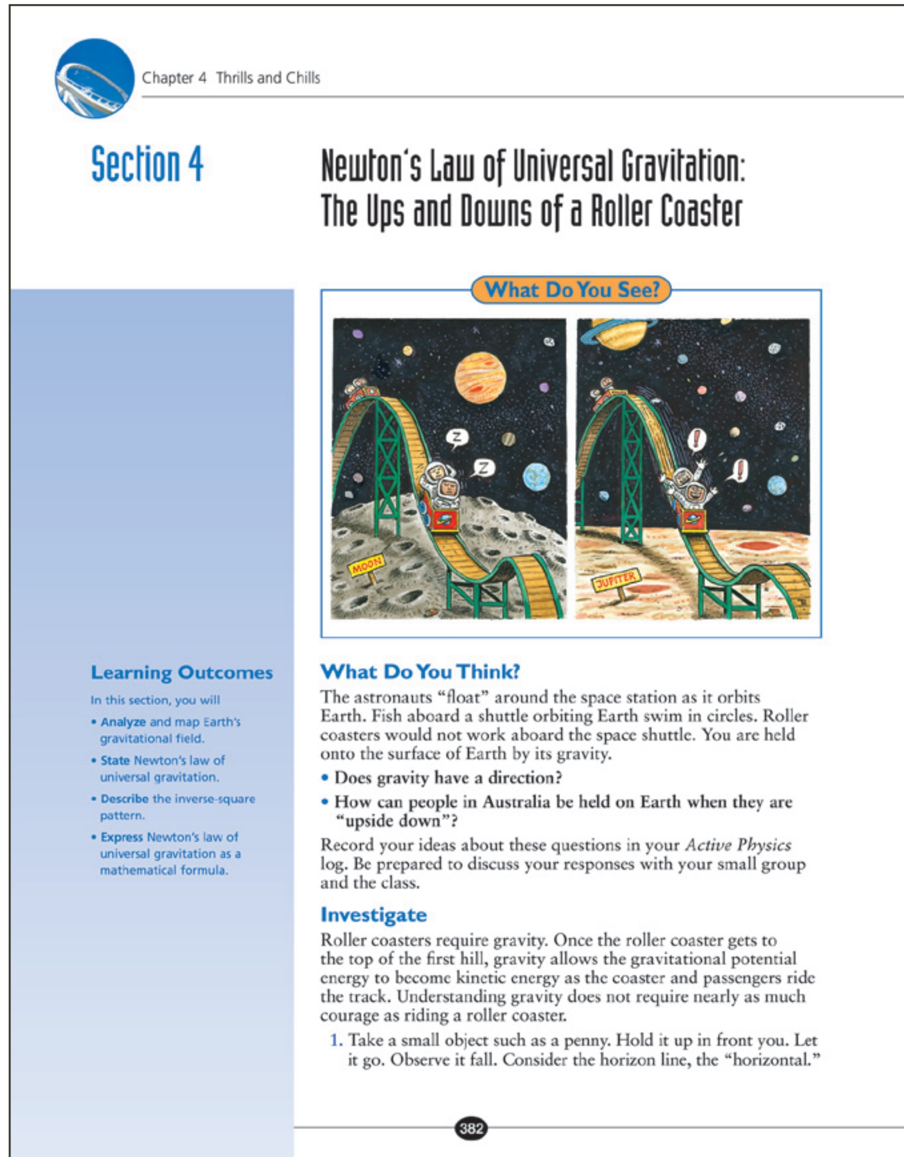
As you work through the section, have students write these terms in their *Active Physics* log and add definitions in their own words. Encourage students to add as much information as possible about each term, including labeled diagrams, mathematical equations, relationships, and calculations.

One way to practice using these terms is for students to work in teams to write meaningful sentences about the content in this section. Encourage students to write in their own words and to use simple sentence structures. The goals are to correctly use as many terms as possible, and to include as much information about the terms as possible. The sentences should be in proper English. The rubric for grading these sentences should include four elements: correct science, correct usage of vocabulary, correct sentence structure and grammar, and quantity of work.

Rapid feedback about students' sentences is essential, because the sentences and errors will be fresh in the students' minds. A quick and powerful method for providing this feedback is to prepare a list of examples of incorrect sentences from the work collected. Divide examples into the following categories: incorrect science, incorrect usage of terms, incorrect sentence structure, and incorrect grammar. Choose several examples from the collected work to use in each category and edit the sentences until they contain only one or two obvious errors, or limit the choices to these kinds of sentences. At the beginning of class on the day following the sentence-writing activity, provide each student with a page containing a double-spaced, typed list of the incorrect sentences, with headings for the categories. Allow students 10 minutes to silently make corrections to the sentences. Then, place a copy of the list on the overhead projector and collect students' ideas on how to repair the sentences, guiding them toward correct science and English usage.

SECTION 4

Teaching Suggestions and Sample Answers


Chapter 4 Thrills and Chills

Section 4

Newton's Law of Universal Gravitation: The Ups and Downs of a Roller Coaster

What Do You See?

Learning Outcomes

In this section, you will

- **Analyze** and map Earth's gravitational field.
- **State** Newton's law of universal gravitation.
- **Describe** the inverse-square pattern.
- **Express** Newton's law of universal gravitation as a mathematical formula.

What Do You Think?

The astronauts "float" around the space station as it orbits Earth. Fish aboard a shuttle orbiting Earth swim in circles. Roller coasters would not work aboard the space shuttle. You are held onto the surface of Earth by its gravity.

- Does gravity have a direction?
- How can people in Australia be held on Earth when they are "upside down"?

Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your responses with your small group and the class.

Investigate

Roller coasters require gravity. Once the roller coaster gets to the top of the first hill, gravity allows the gravitational potential energy to become kinetic energy as the coaster and passengers ride the track. Understanding gravity does not require nearly as much courage as riding a roller coaster.

1. Take a small object such as a penny. Hold it up in front you. Let it go. Observe it fall. Consider the horizon line, the "horizontal."

382

What Do You See?

This eye-catching illustration is bound to spark interest among students. Project this illustration on an overhead and highlight the bored expression of the astronauts when they are on the Moon and then contrast it with how thrilled the astronauts seem when they are on Jupiter. Lead students to question what is different between the Moon and Jupiter that would cause such a change in expression to take place. Point out to them that a closer look at the visuals will provide a clue to understanding the main purpose of this section. Prompt them to recall what they have learned in previous sections, and encourage them to use prior knowledge to interpret the images in the illustration.

Students' Prior Conceptions

The concepts of gravitational fields, inverse-square relationships between the distance from center to center separating the objects, field strength, and how universal gravity acts among heavenly bodies to maintain orbits in space may seem non-relevant to the student studying how and why roller coaster rides are thrilling. You may emphasize that the gravitational force is the weakest fundamental force, and in almost all cases the forces of gravity experienced every day by a student are so incredibly small that they are not noticed; certainly they are not easily measured with the tools available to most students. One of the important ideas to convey in this section is that the weight of the student is directly associated with the magnitude and the direction of the gravitational forces between a student's body and Earth. Weight is mass times acceleration of gravity, and the gravitational force close to the surface of Earth is assumed to be g or 9.8 m/s^2. The gravitational force decreases as the object moves further away from Earth; again, this distance is not measurable with everyday classroom tools.

1. **Students commonly believe that there is no gravity in space.** When teachers highlight the content of the *Physics Talk* for *Section 4* and review what the gravitational field lines indicate in the toothpick model, they should encourage students to accommodate new ideas about the gravity field in space by extending the patterns evident in the toothpick model to an understanding of why you have a lab in space. By extending the patterns evident in the toothpick model to the orbital radius of the space shuttle above Earth (about

Students should be reminded that returning to the *What Do You See?* section is significant in realizing how their initial impressions connect with what they learn subsequently in each section, as they progress through different steps of an inquiry-based approach to learning.

What Do You Think?

Read or have a student read the introductory paragraph of the *What Do You Think?* section. These questions are meant to make students curious about gravity. They also prepare them for the *Investigate* and provide a frame of reference on which to base their inquiry. Ask students to recall concepts they learned in previous sections. How does energy relate to gravity? What do students observe about things when they fall in their everyday experience? Is the force of gravity the same all over the globe? These are some of the questions that you could choose to have a discussion on. Encourage students to record all their answers and remind them that they will have an opportunity later to review their responses. Reiterating that you only expect a well-thought out response will boost student confidence and you will also get an opportunity to gauge what misconceptions need to be resolved, once students engage in an interactive discussion.

What Do You Think?

A Physicist's Response

Gravity, like all fields, is a vector quantity that has both magnitude and direction. The source of a gravitational field is the mass of the object that is producing the field. For Earth, a large spherical mass, the field is directed symmetrically toward Earth's center. Since Earth is spherical, the direction toward the center will be different in different places. The term "upside down" is not relevant, since the "downward direction" is determined by direction of gravity. Although the people in Australia may have a different direction for "down," their "down" is also toward Earth's center, allowing them to have their feet firmly on the ground.

300 miles above Earth's surface) indicates that Earth's gravitational field is still quite strong at this distance. Explain to the students that the reason the astronauts appear "weightless" is not due to the lack of gravity, but due to the fact that they are constantly falling toward (but missing) Earth. Someone who jumps into the air experiences the same weightlessness momentarily.

2. **Students believe that "*g*" is gravity.** The model developed in this section should prod students to recognize that *g* is acceleration due to gravity, a measure of the effect of gravity that is related to the strength of the attracting object, creating the gravitational force field and the location of the attracted object within that field.

3. **Students believe in the concept of "weightlessness"—that there is no pull of gravity on astronauts or on objects that "float" in space.** If there were no gravity in space, the space shuttle could not orbit Earth. The content matter and the sequential development of patterns in *Section 4* are useful tools to adjust student thinking. Teachers may persuade student beliefs to accommodate the concept of apparent weight and how it relates to what students experience on a roller coaster. When discussing apparent weight, emphasize how weight changes with a change in the net acceleration exerted on an object or on a rider in a roller coaster. Discussions of a non-visible force holding floating objects in space in orbits will relate to *Circular Motion: Riding on the Curves*.

NOTES

Investigate

1.a)

The penny falls straight down. This is done to set up the answer to *Step 4*.

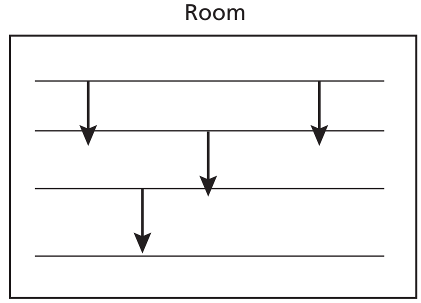
1.b)

The students should recognize that the speed increases, since the penny started from rest in their hands.

1.c)

Students' diagram should be similar to the one to the right:

Room



1.d)

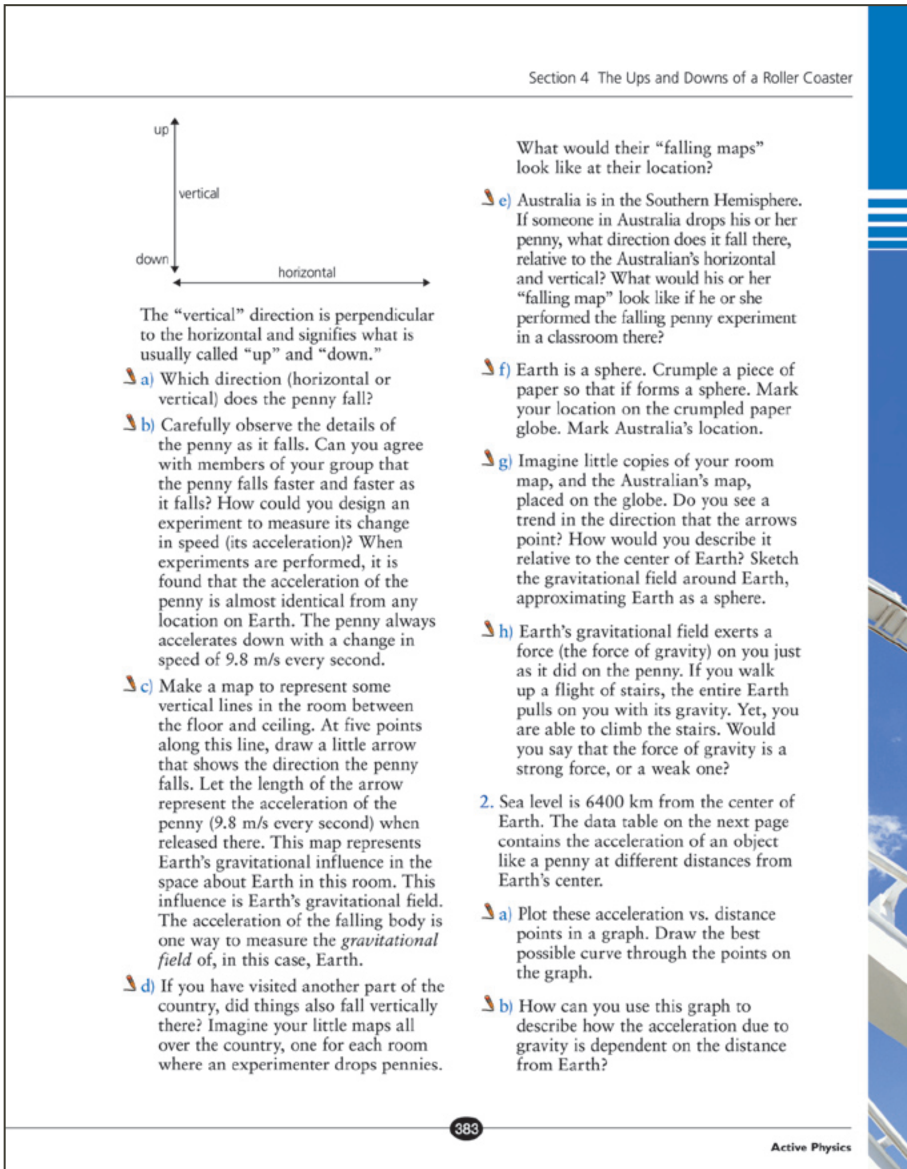
All the pennies would fall down toward the center of Earth, but that would be in different directions for each location. If a globe is available, tape some action figures onto the globe at different points to indicate the different "down" orientations.

1.e)

For the Australians, the penny would also fall "down" toward Earth's center, which would be a different direction from the classroom's "down."

1.f)

Students crumple paper and mark their location and Australia's on the piece of crumpled paper. It might be interesting to see where various students place Australia on their crumpled paper relative to their location.



Section 4 The Ups and Downs of a Roller Coaster

The "vertical" direction is perpendicular to the horizontal and signifies what is usually called "up" and "down."

a) Which direction (horizontal or vertical) does the penny fall?

b) Carefully observe the details of the penny as it falls. Can you agree with members of your group that the penny falls faster and faster as it falls? How could you design an experiment to measure its change in speed (its acceleration)? When experiments are performed, it is found that the acceleration of the penny is almost identical from any location on Earth. The penny always accelerates down with a change in speed of 9.8 m/s every second.

c) Make a map to represent some vertical lines in the room between the floor and ceiling. At five points along this line, draw a little arrow that shows the direction the penny falls. Let the length of the arrow represent the acceleration of the penny (9.8 m/s every second) when released there. This map represents Earth's gravitational influence in the space about Earth in this room. This influence is Earth's gravitational field. The acceleration of the falling body is one way to measure the *gravitational field* of, in this case, Earth.

d) If you have visited another part of the country, did things also fall vertically there? Imagine your little maps all over the country, one for each room where an experimenter drops pennies. What would their "falling maps" look like at their location?

e) Australia is in the Southern Hemisphere. If someone in Australia drops his or her penny, what direction does it fall there, relative to the Australian's horizontal and vertical? What would his or her "falling map" look like if he or she performed the falling penny experiment in a classroom there?

f) Earth is a sphere. Crumple a piece of paper so that if forms a sphere. Mark your location on the crumpled paper globe. Mark Australia's location.

g) Imagine little copies of your room map, and the Australian's map, placed on the globe. Do you see a trend in the direction that the arrows point? How would you describe it relative to the center of Earth? Sketch the gravitational field around Earth, approximating Earth as a sphere.

h) Earth's gravitational field exerts a force (the force of gravity) on you just as it did on the penny. If you walk up a flight of stairs, the entire Earth pulls on you with its gravity. Yet, you are able to climb the stairs. Would you say that the force of gravity is a strong force, or a weak one?

2. Sea level is 6400 km from the center of Earth. The data table on the next page contains the acceleration of an object like a penny at different distances from Earth's center.

a) Plot these acceleration vs. distance points in a graph. Draw the best possible curve through the points on the graph.

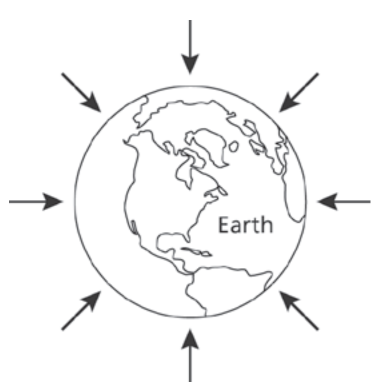
b) How can you use this graph to describe how the acceleration due to gravity is dependent on the distance from Earth?

383

Active Physics

1.g)

The students should see that "down" refers toward the center of Earth. Their diagram should look like the one to the right.

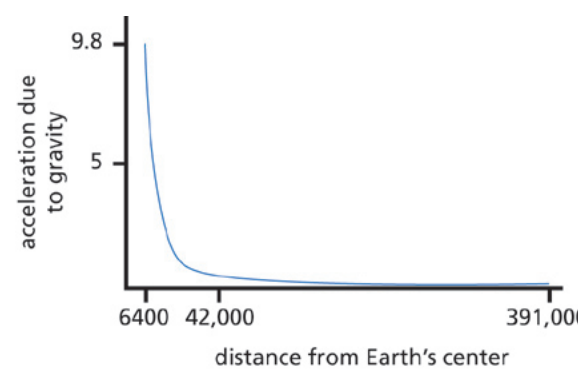


1.h)

The students may claim the force of gravity is strong, but point out that the whole Earth is pulling down on them, and even a baby is able to oppose all that pull with its legs. Gravity is really a very weak force that requires huge amounts of mass to exert a force on another small mass.

2.a)

Prompt students to set up their axes scales prior to plotting the points. After the students set up an x-axis scale to include the point for 391,000 km, indicate that the y-axis does not have to have the same scale. Only then should students start to plot the points. They will quickly conclude that the first few points all seem to overlap. Students' graphs should appear similar to the one below. (Note: the graph is not drawn to scale!)



2.b)

Students should see how quickly the acceleration due to gravity decreases with increasing distance from Earth.

CHAPTER 4

4-4a Blackline Master

2.c)

The students should have values of approximately.

i) 4 m/s² ii) 1 m/s²

iii) 0.44 m/s² iv) 0.25 m/s²

2.d)

i)

$20{,}000{:}10{,}000 =$

$\left(1\ \text{m/s}^2\right)/\left(4\ \text{m/s}^2\right) = 0.25 = 1/4$

ii)

$30{,}000{:}10{,}000 =$

$\left(0.44\ \text{m/s}^2\right)/\left(4\ \text{m/s}^2\right) = 0.11 = 1/9$

iii)

$40{,}000{:}10{,}000 =$

$\left(0.25\ \text{m/s}^2\right)/\left(4\ \text{m/s}^2\right) =$

$0.0625 = 1/16$

2.e)

1/25 of the acceleration due to gravity at 10,000 km, $1/25\left(4\ \text{m/s}^2\right) = 0.16\ \text{m/s}^2$. Depending upon how well the students graphed the data, they should have a close answer.

3.a)

The graphical and mathematical models are similar as each represents how the field varies with distance. The toothpicks move apart in two dimensions as they get further away from the center of the sphere. That means the area between the toothpicks increases as the length times the width, which is a square. This increasing area is a model for the decreasing field strength as the gravitational field is spread over larger areas.

The models are different because the toothpick model also provides a direction for the field, whereas the mathematical model does not.

Distance from the center of Earth (km)	Acceleration due to gravity (m/s²)
6400	9.81 (Earth's surface)
6403	9.80
6411	9.77 (Mount Everest - highest point on Earth)
6560	9.33
6800	8.68 (shuttle orbit)
8000	6.27
14,400	1.93
22,400	0.80
42,000	0.22 (geosynchronous orbit for communications satellite)
391,000	0.003 (orbit of the Moon)

c) From this graph, determine the value of the acceleration due to gravity at the following distances from the center of Earth:

i) 10,000 km ii) 20,000 km

iii) 30,000 km iv) 40,000 km

d) Recall that the acceleration due to gravity represents the change in velocity with respect to time for a falling object at that location. For example, 9.8 m/s² represents a change in velocity of 9.8 m/s every second. Find the ratios of the acceleration at the following distances:

i) 20,000 km:10,000 km

ii) 30,000 km:10,000 km

iii) 40,000 km:10,000 km

e) The change in acceleration is an *inverse-square rule*. Notice that the acceleration at 20,000 km is ¼ (0.25) the acceleration of the acceleration at 10,000 km. When the distance doubled, the acceleration became ¼ of the original value. When you tripled the distance (30,000 km from 10,000 km), the force became ⅑ (0.11) of the original value.

When you increased the distance by a factor of 4, the force became 1/16 (0.06) of the original value.

Predict the value of the acceleration when the distance is 50,000 km by assuming that the acceleration should be 1/25 of the value at 10,000 km. Check this prediction against the graph.

3. Imagine a sphere with a radius of 10 cm. This will represent Earth. Inserted into this sphere are toothpicks at equal distances around the sphere so that it looks something like a blowfish shown in the photograph. These toothpicks represent the gravitational field lines of Earth as you move out from the center. Back on Earth's surface, the influence of the gravitational field had a certain magnitude (such as 9.81 m/s² on Earth's surface). At a distance above the surface, this gravitational field and lines now have to be "spread out" over a larger area. The source of the field is Earth's mass, *m*. Since the surface area of a sphere is proportional to r^2, the gravitational field is getting weaker by the square of the distance from Earth.

a) Describe how the mathematical model (values of the acceleration due to gravity), the graphical model, and the toothpick model are all similar and how they are different.


Section 4 The Ups and Downs of a Roller Coaster

Physics Talk

NEWTON'S LAW OF UNIVERSAL GRAVITATION

Earth's Gravitational Field

You have made a map of Earth's **gravitational field** that exists in your room, and about the entire Earth. A "field" is an influence that one object (in this case, Earth) sets up in the space around it. The first object is called the source of the field. Earth is the source of its gravitational field.

A second object (your penny, your body, or the Moon) interacts with this field. The second object is called the response object or the test object. It responds to the field, and you can use it to test for the existence of the field and map it.

The lines of the gravitational field in the model you imagined were created using toothpicks.

These gravitational field lines show you

- The direction of the gravitational field is the direction of the force on a mass. (All objects will accelerate along these field lines toward the center of Earth.)
- The gravitational field is stronger where the lines are close together and weaker where the lines are further apart. (The acceleration due to gravity is largest near Earth and gets weaker as you move further from Earth.)
- The gravitational field is present everywhere. The lines just show the field at some points.
- The gravitational field extends out to infinity.

The gravitational field lines from a large mass such as Earth resemble the points on a blowfish like the one shown on the previous page.

In the case of gravitational fields, the field is mapped with objects that have mass; in the case of magnetic fields, you have to use magnets; in the case of the electric field, you have to use electric charges. In mapping the gravitational field about Earth, your map gives information about the direction and size of the force that Earth's gravity would exert on an object with mass that is placed there.

The Inverse-Square Relationship

You saw a pattern in the *Investigate*. You were told that acceleration due to gravity becomes less as an object moves further from the surface of Earth. In simple terms for gravity, the **inverse-square relationship** says that the force of gravity between two objects decreases by the square of the distance between them.

Physics Words

gravitational field: the gravitational influence in the space around a massive object.

inverse-square relationship: the relationship between the magnitude of a gravitational force and the distance from the mass. This also describes how electrostatic forces depend on the distance from an electrical charge.

385

Active Physics

Physics Talk

Students read how the toothpick model in the *Investigate* explains Earth's gravitational field. The main features of the gravitational field are highlighted in this section and Newton's law of gravitation is then mathematically expressed. The *Physics Talk* builds up an explanation of how Earth exerts its gravitational field on other objects. This introduces students to a comprehensive picture of Newton's law of universal gravitation.

To facilitate students' understanding, ask them to explain how objects respond to the force of gravity on Earth in their *Active Physics* logs. Encourage students to connect their experiments in the *Investigate* to what they learn in the *Physics Talk*. Students should include how the force of gravity varies with distance from the center of Earth, where it is stronger, and where it is weaker on Earth's surface. Review the inverse-square relationship of how the force of gravity between two objects changes with distance. You could invite a student volunteer to explain this relationship on the board using numerical values. Emphasize Newton's third law as it relates to the force of gravity, pointing out that the masses attract one another with equal forces. Ask the students to use Newton's second law to explain why the small mass is rapidly accelerated toward Earth and not vice versa.

As the concept of a gravitational field extends into Newton's law of universal gravitation, discuss how Newton unified the "terrestrial" and "celestial" conceptions of gravity into one principle of universal gravitation. Ask students to state Newton's law of universal gravitation in their logs and explain it using an example. Emphasize the importance of Henry Cavendish's experiment to measure the force between two masses as a method to experimentally verify Newton's conjectures as well as establish the magnitude of "*G*." Show students how the mathematical expression of Newton's law of universal gravitation means exactly the same when stated in words. Ask them why the planets do not move in perfectly elliptical orbits. This question will further tie in all the significant information presented in this *Physics Talk*.

Physics Words

Newton's law of universal gravitation: all bodies with mass attract all other bodies with mass; the force is proportional to the product of the two masses and gets stronger as either mass gets larger; the force decreases as the square of the distances between the two bodies increases.

gravity: the force of attraction between two bodies due to their masses.

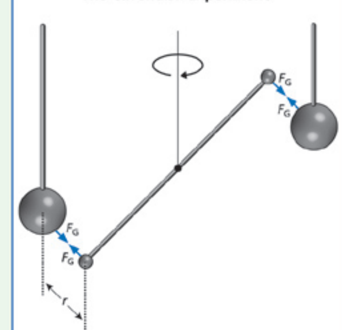
If you triple (3×) the distance, the force is $\frac{1}{3^2}$ or $\frac{1}{9}$ the original force. If you quadruple (4×) the distance, the force is $\frac{1}{4^2}$ or $\frac{1}{16}$ the original force. If you increase the distance by 10 times (10×), the force is $\frac{1}{10^2}$ or $\frac{1}{100}$ the original force.

Newton's law of universal gravitation describes the gravitational attraction of objects for one another. Isaac Newton first recognized that all objects with mass attract all other objects with mass. The strength of gravity is quite small. (Recall how you can climb stairs despite the entire Earth pulling down on you.) It takes a very large concentration of mass, such as a planet, to exert large gravitational forces.

All objects have mass and Earth attracts all objects. Newton reasoned that the Moon must have mass, and that Earth must also attract the Moon. This he called universal gravitation. The same field of **gravity** that pulls an apple to the ground goes on reaching out across space, all the way to the Moon. Before Newton, people thought there was "terrestrial gravity" that described the falling of apples and projectiles near Earth's surface, but some other kind of "celestial force" held the Moon in its orbit. Newton unified these "terrestrial" and "celestial" conceptions into one principle: universal gravitation. He calculated the acceleration of the Moon in its orbit and measured the acceleration of an apple falling in Earth's gravity and saw that both were related by the inverse-square relationship. It is a tribute to Newton's genius that he then guessed that not only Earth, but all bodies with mass attract each other. Yes, you and the table attract each other with a gravitational force.

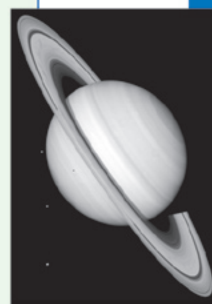
Newton initially had astronomical quantitative evidence for his inverse-square force concept. Almost 100 years passed before Newton's idea that all bodies with mass attract all other bodies with mass was supported by table-top laboratory experiments. To do so, the very small gravitational force that small bodies exert on one another had to be measured. Because this force is very small compared to the force of the massive Earth, the experiments were very difficult. But in 1798, Henry Cavendish, a British physicist, finally measured the gravitational force between two masses (lead spheres) of a few kilograms each. He used the tiny twist of a quartz fiber caused by the force between two suspended lead spheres to detect and measure the force between them. Gravitation was, indeed, universal.

The Cavendish Experiment



Newton's law of universal gravitation states

- All bodies with mass attract all other bodies with mass.
- The force is proportional to the product of the two masses and gets stronger as either mass gets larger.
- The force decreases as the square of the distance between the two bodies increases.



Is There an Equation?

Complex laws often look easier in mathematical form. You can express Newton's law of universal gravitation mathematically as

$$F_G = \frac{Gm_1m_2}{r^2}$$

where F_G is the force between the bodies,
r is the distance between their centers,
m_1 and m_2 are the masses of the bodies, and
G is a universal constant equal to $6.67 \times 10^{-11} \text{N} \bullet \text{m}^2/\text{kg}^2$.

You can see that the equation says exactly the same thing as the words in a much smaller package.

With Newton's law of gravitation, you now have a concept in terms of which you can reason about how things behave in a gravitational field. When this force law is put into Newton's laws of motion, you can show, by calculation, that according to this inverse-square relationship, the Moon would orbit Earth and the planets would orbit the Sun in elliptical paths. This calculation, based on the concept of universal gravitation with its inverse-square dependence, can be checked against the real orbits of the planets. The planets do move in ellipses (to the first approximation), hence to that degree of precision, these "laws" are confirmed.

The planets do not move exactly in ellipses because the planets also tug on one another besides each being "tethered" by or "tied" to the Sun. But these small changes from the ellipse can also be calculated and checked against data, and that program has shown Newton's laws of motion, with universal gravitation as a force, to be very reliable. This back-and-forth of guessing a principle within the pattern of facts, then working out the consequences of that principle, testing it against reality, and using the results of those tests to refine the principles, is how you do science.

Newton's law of gravitation can describe and predict accurately the forces. It does not describe a mechanism for the force. For example, it does not explain how Earth "communicates" to the falling penny which way is "down." What is the mechanism for this "force at a distance?" Newton was puzzled by this and chose not to answer it. Scientists today are still exploring how this force at a distance is transmitted.

Checking Up

1. What is the direction of the gravitational field in your classroom?
2. Using the idea of field lines, where is the gravitational field the strongest?
3. If you triple the distance between two masses, what happens to the force of gravity between the two masses?
4. What is the force that holds the Moon in its orbit around Earth?
5. Approximately what is the shape of the orbit of the planets around the Sun?

Checking Up

1.

The direction of the gravitational field in the classroom will be the direction in which the force acts on a mass, and will be directed down toward Earth's center.

2.

The gravitational field is the strongest where the lines are close together.

3.

If you triple the distance between masses, the force becomes $1/9^{th}$ the strength of the original force.

4.

The force of gravity holds the Moon in orbit around Earth.

5.

The shape of the orbits of the planets around the Sun is approximately an ellipse.

CHAPTER 4

Active Physics Plus

Students use the concept of centripetal acceleration (which will be studied further in *Section* 7) to calculate the acceleration due to gravity at the Moon's distance. They read about Kepler's laws and how the laws are related to the force of gravity. Using Kepler's third law they find the value of the mathematical relationship that defines the time it takes for Earth to orbit the Sun relative to its radius of orbit. Using the same data for the other planets of the solar system they also calculate the value of the same relationship for different planets to demonstrate the third law's universal nature.

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
••	•	••	

Acceleration of the Moon

The Moon is 60 times further away from the center of Earth than objects on the surface of Earth, and moves about Earth in an approximately circular path.

1. If the acceleration due to gravity on the surface of Earth is 9.8 m/s², use the inverse square relationship and your calculator to determine the acceleration due to gravity at the Moon's distance.

   Since the Moon orbits Earth, you can calculate its acceleration. The equation for acceleration of any object moving in a circle at constant speed is

   $$a = \frac{v^2}{r}$$

   where v is the speed
   r is the distance (radius) of the object from the center

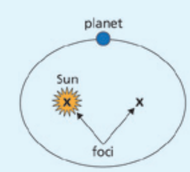
The Moon's distance is 60 times the radius of Earth ($60 \times 6.4 \times 10^6$ m = 3.84×10^8 m). The Moon travels once about Earth in 28.25 days.

2. Find the speed of the Moon using the equation $v = d/t$ where d is the circumference of the Moon's orbit ($2\pi r$).
3. Calculate the acceleration of the Moon.
4. Compare the acceleration of the Moon using these two different approaches. (When Newton made this comparison, he was confident that his inverse square law of gravity was correct.)

Kepler's Laws

Newton once stated, "If I have seen further than others, it is because I have stood on the shoulders of giants." Tycho Brahe and Johannes Kepler were two of those giants. Tycho Brahe observed and recorded the positions of the planets every night over many years. Kepler viewed the data and tried for years to form mathematical relationships that could explain all of Brahe's data. At first, it appeared that the planet's data could be adequately described by stating that the planets moved in circles about the Sun. This description was only approximately true. Kepler had to decide if Brahe's data was a bit imprecise or whether the circular path was incorrect. Kepler's confidence in Brahe's observations forced him to keep trying other descriptions of the paths. Kepler eventually did find three mathematical relationships that fit the data very well. These are now known as Kepler's laws.

Kepler's first law: The planets travel in ellipses about the Sun. An ellipse is a mathematical shape that is defined by two foci. The Sun is at one focus. There is nothing at the other focus.



Acceleration of the Moon

1.

$g_{\text{earth}}/g_{\text{moon}} = (1/d^2_{\text{earth}})/(1/d^2_{\text{moon}})$

$(9.8 \text{ m/s}^2)/g_{\text{moon}} = (1/r^2_{\text{earth}})/(1/60r^2_{\text{earth}})$

$g_{\text{moon}} = (9.8 \text{ m/s}^2)/(3600 \text{ s}) = 0.0027 \text{ m/s}^2$

2.

$$v = 2\pi r/T = \frac{2(3.14)(3.84 \times 10^8 \text{ m})}{(28.25 \text{ days})(24 \text{ h/day})(3600 \text{ s/h})}$$

$v = 990$ m/s

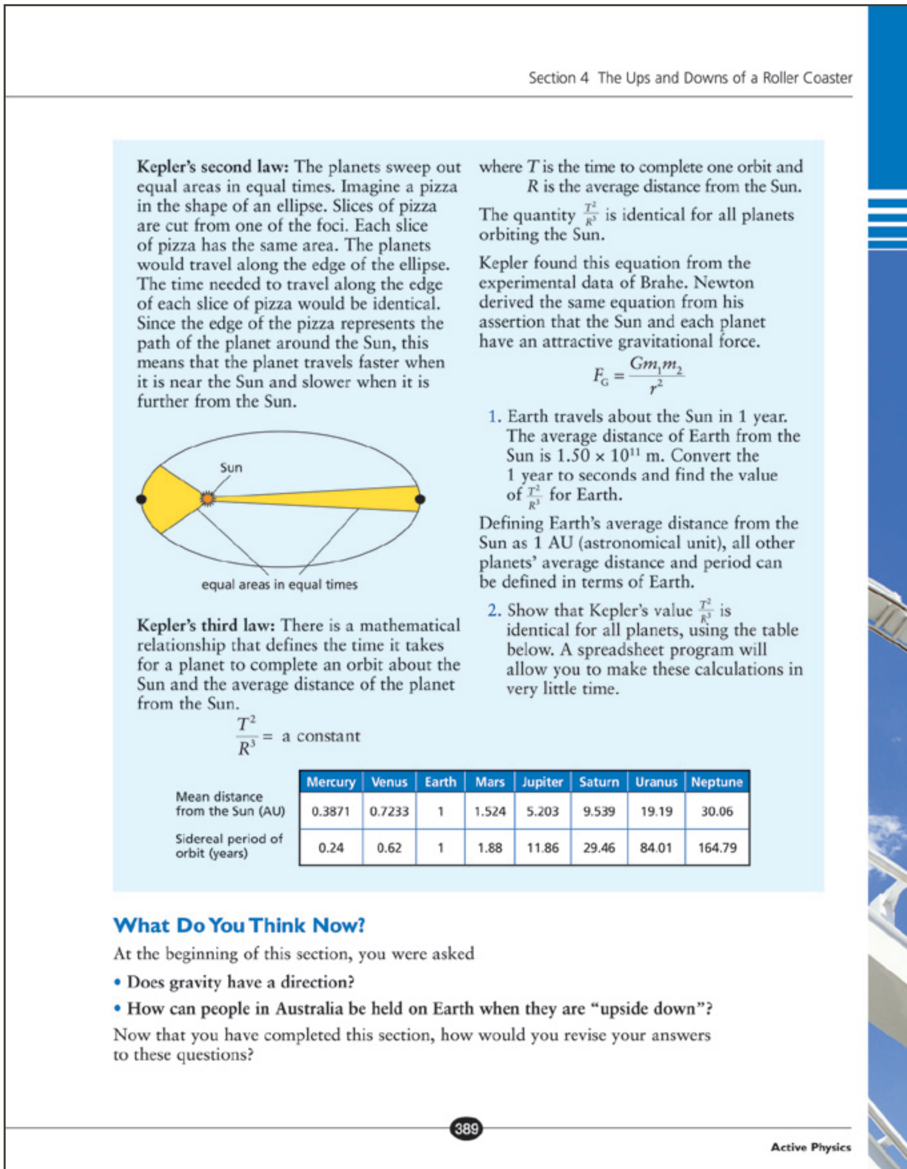
3.

$a = v^2/r = (990 \text{ m/s})^2/(3.84 \times 10^8 \text{ m})$

$a = 0.0026 \text{ m/s}^2$

4.

The result using $a = v^2/r$ is quite close to the value given by the inverse square relationship of Newton's law of gravity.



Section 4 The Ups and Downs of a Roller Coaster

Kepler's second law: The planets sweep out equal areas in equal times. Imagine a pizza in the shape of an ellipse. Slices of pizza are cut from one of the foci. Each slice of pizza has the same area. The planets would travel along the edge of the ellipse. The time needed to travel along the edge of each slice of pizza would be identical. Since the edge of the pizza represents the path of the planet around the Sun, this means that the planet travels faster when it is near the Sun and slower when it is further from the Sun.

Kepler's third law: There is a mathematical relationship that defines the time it takes for a planet to complete an orbit about the Sun and the average distance of the planet from the Sun.

$$\frac{T^2}{R^3} = \text{a constant}$$

where T is the time to complete one orbit and R is the average distance from the Sun.

The quantity $\frac{T^2}{R^3}$ is identical for all planets orbiting the Sun.

Kepler found this equation from the experimental data of Brahe. Newton derived the same equation from his assertion that the Sun and each planet have an attractive gravitational force.

$$F_G = \frac{Gm_1m_2}{r^2}$$

1. Earth travels about the Sun in 1 year. The average distance of Earth from the Sun is 1.50×10^{11} m. Convert the 1 year to seconds and find the value of $\frac{T^2}{R^3}$ for Earth.

Defining Earth's average distance from the Sun as 1 AU (astronomical unit), all other planets' average distance and period can be defined in terms of Earth.

2. Show that Kepler's value $\frac{T^2}{R^3}$ is identical for all planets, using the table below. A spreadsheet program will allow you to make these calculations in very little time.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Mean distance from the Sun (AU)	0.3871	0.7233	1	1.524	5.203	9.539	19.19	30.06
Sidereal period of orbit (years)	0.24	0.62	1	1.88	11.86	29.46	84.01	164.79

What Do You Think Now?

At the beginning of this section, you were asked

- **Does gravity have a direction?**
- **How can people in Australia be held on Earth when they are "upside down"?**

Now that you have completed this section, how would you revise your answers to these questions?

389 Active Physics

What Do You Think Now?

Students should now revise and update previous answers to the *What Do You Think?* section. You might want to ask them how their initial impressions of gravity may have been altered. This is the time to review ideas that students brought up during the *What Do You See?* Share *A Physicist's Response* with your students and ask them to describe how the *Investigate* and the *Physics Talk* helped them in learning more about gravity. Divide students into small groups and facilitate a discussion on how roller coasters depend on gravity to come down, once they are on top of a hill. Highlight important ideas raised during discussions to clear misconceptions that students may still have. Remind them that they seeing how their knowledge evolved is key to developing an in-depth understanding of science.

CHAPTER 4

Kepler's Laws

1.

One year = (365.25 days)(24 h/day) × (3600 s/h) = 3.15×10^7 s

(This is quite close to $\pi \times 10^7$ seconds)

$T^2/R^3 =$

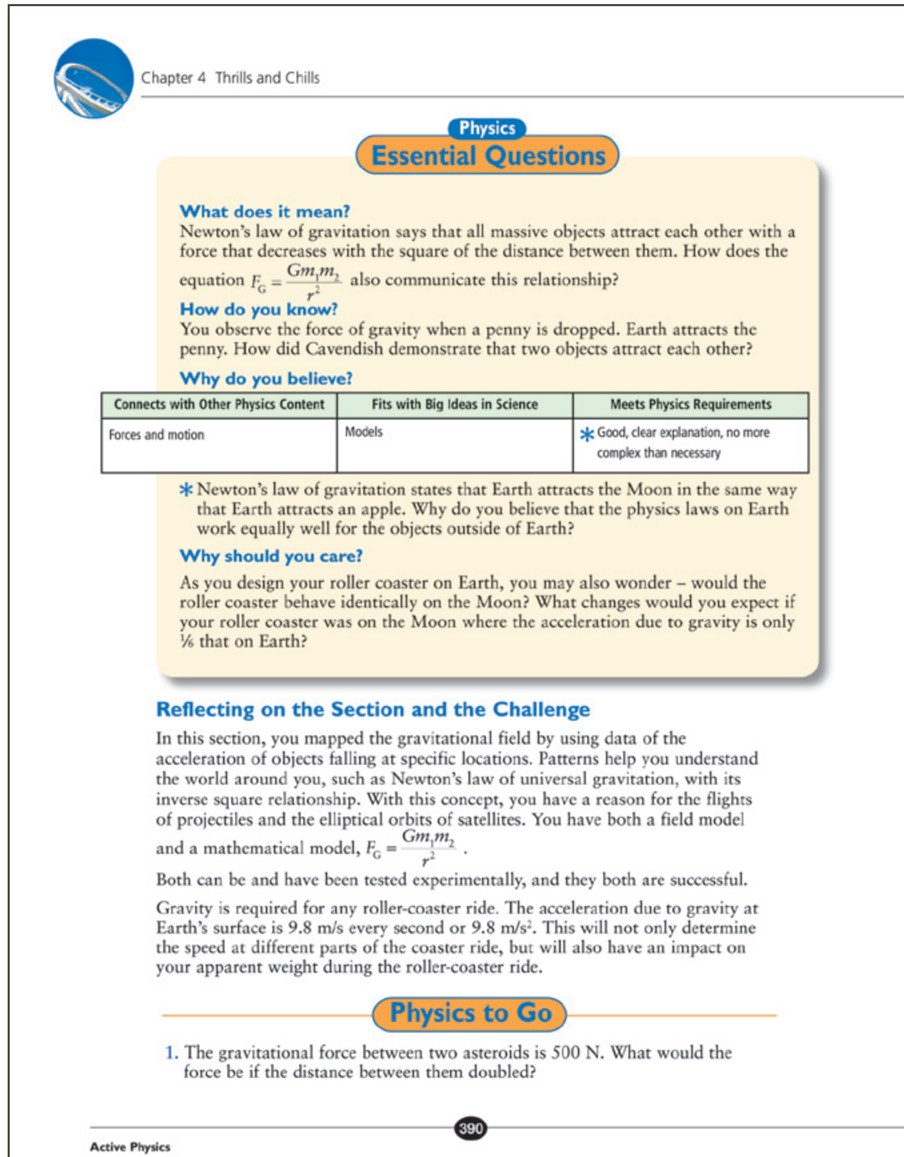
$(3.15 \times 10^7 \text{ s})^2 / (1.5 \times 10^{11} \text{ m})^3 = 2.9 \times 10^{-19} \text{ s}^2/\text{m}^3$

2.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Distance	0.3871	0.7233	1	1.524	5.203	9.539	19.19	30.06
Period	0.24	0.62	1	1.88	11.86	29.46	84.01	164.79
T^2/R^3	0.993	1.02	1	0.998	0.998	0.999	0.999	1.000

Reflecting on the Section and the Challenge

Students have an opportunity to reflect on Newton's law of universal gravitation. Ask students to pay particular attention to the inverse square nature of the force, and point out this is their first exposure to a law of this nature. Consider asking students why planets move in elliptical orbits. Prompt them to reflect on how gravity determines a roller coaster's speed at different stages of the ride. Begin a class discussion that allows students to reflect on the mathematical and field models of Newton's law of universal gravitation. Invite a student volunteer to describe a model on the board, so that others in the class can ask questions and clear any doubts they might have. Point out that each physics concept they have learned in this section will help them to meet the goal of their *Chapter Challenge*.



Physics

Essential Questions

What does it mean?

Newton's law of gravitation says that all massive objects attract each other with a force that decreases with the square of the distance between them. How does the equation $F_G = \frac{Gm_1m_2}{r^2}$ also communicate this relationship?

How do you know?

You observe the force of gravity when a penny is dropped. Earth attracts the penny. How did Cavendish demonstrate that two objects attract each other?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	Models	* Good, clear explanation, no more complex than necessary

* Newton's law of gravitation states that Earth attracts the Moon in the same way that Earth attracts an apple. Why do you believe that the physics laws on Earth work equally well for the objects outside of Earth?

Why should you care?

As you design your roller coaster on Earth, you may also wonder – would the roller coaster behave identically on the Moon? What changes would you expect if your roller coaster was on the Moon where the acceleration due to gravity is only ⅙ that on Earth?

Reflecting on the Section and the Challenge

In this section, you mapped the gravitational field by using data of the acceleration of objects falling at specific locations. Patterns help you understand the world around you, such as Newton's law of universal gravitation, with its inverse square relationship. With this concept, you have a reason for the flights of projectiles and the elliptical orbits of satellites. You have both a field model and a mathematical model, $F_G = \frac{Gm_1m_2}{r^2}$.

Both can be and have been tested experimentally, and they both are successful.

Gravity is required for any roller-coaster ride. The acceleration due to gravity at Earth's surface is 9.8 m/s every second or 9.8 m/s^2. This will not only determine the speed at different parts of the coaster ride, but will also have an impact on your apparent weight during the roller-coaster ride.

Physics to Go

1. The gravitational force between two asteroids is 500 N. What would the force be if the distance between them doubled?

Physics Essential Questions

What does it mean?

The force is on the left side of the equation. An increase in either mass is an increase in the numerator of the fraction on the right which increases the value of the fraction and therefore increases the force. An increase in distance provides a decrease in the force. Since the distance is squared, a tripling of the distance will result in a decrease of the force by a factor of $3 \times 3 = 9$.

How do you know?

Cavendish set up spheres on the end of a rod. He then placed fixed masses near these spheres. He observed the spheres attracted to the masses. By measuring the twist of the wire supporting the rod and spheres, he calculated the force.

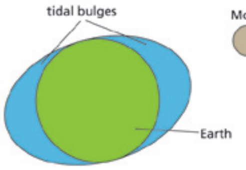
Why do you believe?

The world is a simpler place if the laws of physics are the same at different locations on Earth and in space. The motion of the planets can be explained with the same physics principles as the motion of objects on Earth.

Why should you care?

The decrease in the acceleration due to gravity would result in a slower ride for a roller coaster of the same dimensions. We would still expect that the energy conservation would describe the relation between height and speed.

2. A satellite sitting on the launch pad is one Earth radius away from the center of Earth (6.4×10^6 m).
   a) How would the gravitational force between them be changed after launch when the satellite was two Earth radii (1.28×10^7 m) from the center of Earth?
   b) What would the gravitational force be if the satellite was three Earth radii (1.92×10^7 m) from the center of Earth?
   c) What would the gravitational force be if the satellite was four Earth radii (2.56×10^7 m) from the center of Earth?
3. Why does everyone trust in gravity?
4. Compare the acceleration due to gravity at the top and bottom of a roller-coaster ride.
5. a) Which is closer to the Moon—the middle of Earth or the water on the side of Earth facing the Moon?
   b) Suggest an explanation for high tides on the side of Earth facing the Moon. *Please note these diagrams are not to scale.*
   c) Use your answer to *a)* to propose an explanation for the uneven distribution of water on Earth's surface, as shown in the diagram.



6. Astronauts on many Shuttle flights study the effects of "free fall." Fish taken aboard the Shuttle react to "free fall" by swimming in circles. "Free fall" mimics what you feel if gravity were not present.
   a) How would a fish's life be different without gravity?
   b) Does gravity hold a fish "down" on Earth?
7. Two objects have a tiny, but measurable gravitational force of attraction between them. How will that force change if the distance between the objects is
   a) doubled b) tripled c) quadrupled d) halved
8. Two objects have a tiny, but measurable gravitational force of attraction between them. How will that force change if the mass of one of the objects is
   a) doubled b) tripled c) quadrupled d) halved
9. Two objects have a tiny, but measurable gravitational force of attraction between them. How will that force change if the mass of both of the objects is
   a) doubled b) tripled c) quadrupled d) halved
10. Two objects have a tiny, but measurable gravitational force of attraction between them. How will that force change if the following changes in mass occur?

Mass 1	Mass 2
same	doubles
triples	triples
doubles	triples

Physics to Go

1.

Because the force of gravity falls off as the inverse square of the distance between the object's centers, if the distance between the asteroids is doubled, the force will be reduced to 1/4 its previous value, or 125 N.

2.a)

Doubling the distance would reduce the force by a factor of $1/2^2$, so the force would be 1/4 times larger.

2.b)

Tripling the distance would reduce the force by a factor of $1/3^2$, so the force would be 1/9 times larger.

2.c)

Increasing the distance by a factor of 4 would reduce the force by a factor of $1/4^2$, so the force would be 1/16 times larger.

3.

Everyone trusts gravity because the force always acts between any two masses. As long as Earth has mass and you have mass, the force of gravity will exist between both of you. If the mass of either you or Earth disappears, the loss of gravity will only be a part of the problem you will face!

4.

The force of gravity at the top and bottom of a roller-coaster ride is essentially the same, since it depends upon the distance from the center of Earth.

5.a)

The water on the side of Earth near the Moon is closer than the center of Earth.

5.b)

Because the water is closer to the Moon than Earth's center, the water has a slightly stronger force of gravity acting on it than Earth's center. As a result the water is slightly pulled away from the surface of Earth. Parts of Earth, water included, that are nearer to the Moon feel a stronger gravitational force from the Moon than those parts further away.

5.c)

The high tide on the side of Earth nearest the Moon is due to the stronger gravitational force discussed in the answer to *Question 5.b)*. The high tide on the side of Earth away from the Moon is due to the center of Earth being closer to the Moon than the water on the far side. The center of Earth then is pulled slightly away from the bottom of the ocean on the far side of Earth from the Moon, leading to the simultaneous high tide on that side (but not quite as high as on the near side). Newton was the first to recognize the reason for this high tide on the far side.

6.a)

Without gravity the only force holding water together would be surface tension. Unless the water is confined by an external system, the force of fish swimming in the water would probably be strong enough to break the water apart, leaving them as "fish out of water."

6.b)

Yes, gravity holds a fish "down" on Earth.

7.

Since the force of gravity depends upon one over the distance between the object squared, the force of gravity between the objects would be

a) one-fourth as strong.
b) one-ninth as strong.
c) one-sixteenth as strong.
d) four times as strong.

8.

Since the force of gravity depends upon the product of the objects masses, the force of gravity between them would be

a) twice as strong.
b) three times as strong.
c) four times as strong.
d) half as strong.

9.

Since the force of gravity depends upon the product of the objects' masses, the force of gravity between them would be

a) four times as strong.
b) nine times as strong.
c) sixteen times as strong.
d) one-fourth as strong.

10.

Mass 1	Mass 2	Force
same	doubles	doubled
triples	triples	nine times stronger
doubles	triples	six times stronger

NOTES

NOTES

SECTION 4 QUIZ

4-4b **Blackline Master**

1. Two bodies attract one another with a gravitational force of 10.0 N. What will be the force of attraction if the mass of each body is doubled?

   a) 2.5 N b) 5.0 N

   c) 20 N d) 40 N

2. Which of the following statements about the force of gravity between two masses is incorrect?

   a) The force depends upon the product of the masses.

   b) The force decreases with the inverse square of the distance between the masses.

   c) The force of gravity requires a medium such as air to travel through.

   d) The two masses may also attract other masses.

3. If the distance between two masses is tripled, the gravitational force between them would be

   a) 1/9 as great. b) 1/3 as great.

   c) 3 times as great. d) 9 times as great.

4. Which graph below best represents the relationship between the force of gravity between two masses, and the distance between those masses?

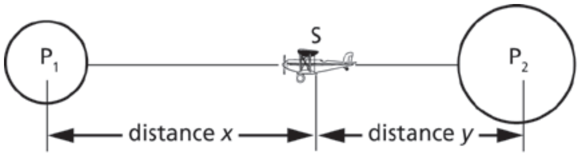
a) 

b) 

c) 

d) 

5. Spacecraft S is traveling between two planets P_1 and P_2 as shown in the diagram below.



At the position shown, the gravitational force on the spacecraft due to the planets is equal. If distance x is greater than distance y, then compared to the mass of planet P_2 the mass of planet P_1

a) must be greater than planet P_2.

b) must be less than planet P_2.

c) must be equal to the mass of planet P_2.

d) cannot be determined without knowing the mass of the spacecraft.

SECTION 4 QUIZ ANSWERS

1. d) The force of gravitational attraction between two masses is given by Newton's law of gravitation, $F_G = Gm_1m_2/d^2$. To see what happens when both masses are doubled, first put a value of 1 in for all the variables. This will give $F_G = G(1)(1)/(1)^2 = G$. When both masses are doubled, the equation becomes $F_G = G(2)(2)/(1^2) = 4G$, or four times greater. Thus, if the original force was 10 N, and both masses are doubled, the new force will be four times larger, or 40 N.

2. c) The gravitational field that serves to exert the force of gravity requires no medium to pass through as is easily seen by the force of gravity acting between Earth and the Moon

3. a) The force of gravitational attraction between two masses is given by Newton's law of gravitation $F_G = Gm_1m_2/d^2$. To see what happens when both masses are doubled, first put a value of 1 in for all the variables. This will give $F_G = G(1)(1)/(1)^2 = G$. When the distance between the masses is tripled, the equation becomes $F_G = G(1)(1)/(3^2) = G/9$, or one-ninth as large.

4. b) The force of gravity is an inverse square law force which would be represented by graph b. Although graph c shows the force decreasing as the distance increases, it will pass through zero, and become negative or a repulsive force. Gravity is always attractive. In addition, the graph is the wrong shape for an inverse graph of any type.

5. a) Because the forces are equal, the two equations for force can be set equal to each other, giving $F_{net} = mv$ or $Gm_1m_r/x^2 = Gm_2m_r/y^2$ where m_1 and m_2 are the mass of the planets, and m_r is the mass of the rocket. After canceling, the equation becomes $m_1/x^2 = m_2/y^2$. Solving for m_2 gives $m_2 = m_1\left(y^2/x^2\right)$. Since y is smaller than x, y^2/x^2 is a number less than 1, meaning m_2 equals m_1 multiplied by a number less than one, or is less than m_1. Thus, Planet one is more massive than planet two.