## SECTION 5

## Hooke's Law: Finding Your "At Rest" Weight

## Section Overview

Students investigate the difference between mass and weight and how springs are used to determine an object's weight using Hooke's law. They begin by calculating and comparing the weights of different objects and learn that in the metric system the mass of an object is measured in kilograms and weight in newtons. They find what the weight in newtons is of a bowling ball weighing $11-\mathrm{lb}$ and a $1 / 4-\mathrm{lb}$ burger. Students then find the metric equivalent weight of a roller-coaster car that weighs 1500 lbs .

To investigate how a spring is used to measure the weight of an object, students record the changing stretch of a spring by hanging different masses from a spring that is attached to a ring stand. They plot a graph of the stretch of the spring on the $x$-axis versus the weight hung on the spring on the $y$-axis, and determine the slope of the line and the spring's "spring constant."

Students repeat their investigations for a second spring that looks different, make a prediction about the stretch of the spring, and then test the accuracy of their prediction. They compare the slope of the line obtained from this graph to the slope from the previous graph. They also plot a graph for an "invented" third spring and write the description of the spring that would have that data. Finally, using the spring constant for each spring the students create a scale to measure the weight of different masses, three known and two whose weight is unknown.

## Background Information

Students often have a great deal of difficulty in distinguishing mass and weight. That is probably because in everyday experience a type of conversion
between mass and weight has been created.
When you want to know how far you can throw something, you first lift it up and down to determine its weight. Knowing the weight, you can deduce its corresponding mass and know how hard it will be to push or throw. You also tend to confuse mass and weight because in common English usage, you use the same terms like ounces or pounds for both mass and weight. In countries that use the SI system, groceries such as meat or fruits, are sold by the kg, but the "mass" is determined by weighing them on a spring scale.

In physics, the distinction between mass and weight is crucial to our understanding. Mass is inertia-a resistance to motion. It is also the amount of matter. Weight is a force. Our weight on Earth is the force that Earth exerts on us. The unit for mass is the kilogram. The unit for weight is the newton. From Newton's second law, you can see that $1 \mathrm{~N}=1 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}$. It is sometimes useful to use alternate units for $g$ of $\mathrm{N} / \mathrm{kg}$. Since an object resting on the table has mass and weight, it might clear up some misunderstandings if students are calculating the weight by multiplying the mass by $g$ in units of $\mathrm{N} / \mathrm{kg}$, rather than the equivalent $\mathrm{m} / \mathrm{s}^{2}$. In symbols, you write $F_{\mathrm{w}}=m g . F_{\mathrm{w}}$ is the weight (or weight force).

If gravity is the only force acting on an object, then Newton's second law tells you that $F_{\mathrm{w}}=m a=m g$. Then the object's acceleration equals $g,\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right.$ near the surface of Earth). In that case, the object is said to be in "free-fall." Hooke's law states the relationship between the force that a spring can exert on an object and the stretch of the spring needed to exert that much force. The larger the stretch (or compression) of the spring, the larger the exerted force. This is written mathematically as $F=-k x$. The negative sign indicates that when you
stretch a spring to the right, the restoring force from the spring is to the left. The force exerted by the spring and the displacement (stretch) are in opposite directions.

Hooke's law is sufficiently accurate for measuring weights of food, gold, and many other materials throughout the ages. You either hang objects from the scale and stretch the spring or place objects on the scale and compress the spring. Hooke's law is also used as a first approximation when other
materials are stretched or compressed. If a piece of rubber was to be stretched, the first guess for the force would be Hooke's law-the force is directly proportional to the stretch. Springs can stretch beyond their ability to restore themselves to their original shape. As they stretch to this point, they no longer follow Hooke's law. Graphing the data, one would notice that the graph of force versus displacement (stretch) would begin to diverge from the straight line of the earlier data.

## Crucial Physics

- Weight is the force of gravity acting on an object. If the object is on or near Earth's surface then the strength of the gravitational field is nearly constant and is given by $9.8 \mathrm{~N} / \mathrm{kg}$. This is also called the acceleration due to gravity. The object's weight is then given by $F_{\mathrm{w}}=m g$. Weight, like all forces is a vector and acts toward Earth's center. The units for force are newtons.
- The force exerted on an object by a spring is a restoring force that is directly proportional to the distance the spring is stretched or compressed from its natural length. The spring force is described by Hooke's law: $F_{\mathrm{s}}=-k x$, where $F_{\mathrm{s}}$ is the force exerted by the spring on an object, $x$ is the distance the spring is stretched or compressed from its natural length, and $k$ is the spring constant. The spring constant of a spring describes how easy or difficult it is to stretch or compress a spring and is measured in newtons per meter.

| Learning Outcomes | Location in the Section | Evidence of Understanding |
| :--- | :--- | :--- |
| Distinguish between mass and <br> weight. | Investigate <br> Part A, Step 1 | Students learn the definition of weight in physics and find <br> that the metric unit of mass is kilograms and weight is <br> newtons. |
| Calculate weight in newtons for <br> a given mass. | Investigate <br> Part A, Step 2 <br> Physics Talk | Students calculate and compare the weights of a gymnast <br> and football player with different masses. |
| Measure the effect of weight on <br> the vertical stretch of a spring. | Investigate <br> Part B, Step 7.a) <br> Physics Talk | Students record measurements of the stretch of a spring <br> from it zero position length using weights of different <br> masses. |
| Illustrate the relationship <br> between weight and stretch of a <br> spring in a graph. | Investigate <br> Part B, Steps 7-11 <br> Physics Talk | Students plot a graph with the stretch of the spring on <br> x-axis and weight on y-axis. Students learn that slope <br> of the graph is a straight line. Students draw graphs for <br> different springs. |
| Use a spring to create a scale <br> and explain how Newton's <br> second law is used in the <br> creation of the scale. | Investigate <br> Part C, Steps 1-3 <br> Physics Talk | Students create a scale and choose five objects, three of <br> known masses and two of unknown masses and explain <br> how Newton's second law is used to determine the weight <br> on an object. |
| Calculate spring forces using <br> Hooke's law. | Physics to Go <br> Questions 3, 4, 6, and 7 | Students solve problems to calculate the spring constant of <br> a spring. |

## Section 5 Materials, Preparation, and Safety

## Materials and Equipment

| P A N A |  |  |
| :---: | :---: | :---: |
| Materials and Equipment | Group (4 students) | Class |
| Holder, right-angle, cast iron | 1 per group |  |
| Rod, aluminum, $3 / 8$ in. $x 12$ in. (to act as crossarm) | 1 per group |  |
| Ring stand, large | 1 per group |  |
| Meter stick | 1 per group |  |
| Ruler, metric, in./cm | 1 per group |  |
| Springs, set, Hooke's law | 1 per group |  |
| Hanger, mass | 1 per group |  |
| Weight, slotted, 12-piece set, 1 g-500 g | 1 per group |  |
| Scale, electronic, 0.1-g readability, 0-1500 g |  | 1 per class |
| Paper, graph, pkg. of 50 |  | 1 per class |
| Tape, masking |  | 6 per class |
| Card, index, unlined, 3 in. x 3 in. |  | 1 per class |
| Paper, sheet* | 1 per group |  |
| Pad, easel** |  | 1 per class |
| Markers** |  | 6 per class |

*Additional items needed not supplied
**Item used in Mini-Challenge.

## Time Requirement

Allow one class period or 45 minutes for the Investigate of this section.

## Teacher Preparation

- Cut $8^{112}$ by 11 sheets of paper into three strips to allow each student to make a paper spring.
- Determine how the students will attach pointers to the masses suspended from the springs to take their readings.
- If there are an insufficient number of springs for each group to have a set, the students may swap springs to get a different one.
- Determine good examples of "unknown" masses that the students can weigh. The mass of these samples should be in the weight range measured by the students.
- Ensure that the given weights are able to stretch the spring, but not beyond its point of elasticity. You may want to consider how the students can mount the ruler so that it does not shift as they add weights to the spring.


## Safety

- Students should be aware that masses might fall off the spring or mass hanger. Make certain that no feet are located below the spring where the masses may fall. Placing a soft surface underneath the area where the masses might fall will also protect the floor.
- When the masses are removed from the springs quickly while in the stretched position, the springs might be launched off their holders. Eye protection is again absolutely necessary.


## Materials and Equipment

| Materials and Equipment | Group <br> (4 students) | Class |
| :--- | :--- | :--- |
| Holder, right angle, cast iron |  | 1 per class |
| Rod, aluminum, 3/8 in. x 12 in. (to <br> act as crossarm) |  | 1 per class |
| Ring stand, large |  | 1 per class |
| Meter stick |  | 1 per class |
| Ruler, metric, in./cm |  | 1 per class |
| Springs, set, Hooke's law | 1 per class |  |
| Hanger, mass | 1 per class |  |
| Weight, slotted, 12-piece set, <br> 1 g-500 g |  | 1 per class |
| Scale, electronic, $0.1-\mathrm{g}$ readability, <br> $0-1500 \mathrm{~g}$ |  | 1 per class |
| Paper, graph, pkg. of 50 |  | 1 per class |
| Tape, masking | 6 per class |  |
| Card, index, unlined, 3 in. x 3 in. |  | 1 per class |
| Paper, sheet* |  | 1 per class |
| Pad, easel** | 1 per class |  |
| Markers** |  |  |
| per class |  |  |

*Additional items needed not supplied
**Item used in Mini-Challenge.

## Time Requirements

- Allow one class period or 45 minutes for the Investigate (all parts), the Physics Talk, as well as other parts of the section from the Pacing Guide.


## Teacher Preparation

- Note the preparations for Plan A as well as these below.
- Several students should be recruited to assist you in taking the measurements and recording them on the board for the rest of the class to record in their journals. If a camera and projection system are available they should be used so the students can see more clearly what is occurring.
- Have a ring stand with a horizontal bar attached set up in the front of the classroom. Have different springs, a mass hanger, and a series of slotted masses available at the front of the room, along with a meter stick.
- Test the various springs that will be used prior to class to ensure they provide repeatable results when the masses are hung from them.
- Cut $81 / 2$ by 11 sheets of paper into three strips, enough for one per group plus one for you to allow the students to make paper springs. Have students follow your example as the paper springs are tested.
- Have several "unknown masses" available to hang on the springs and have the students determine the mass from the recorded stretch.
- Summarize the Investigate to familiarize the students with what they will be observing.


## Safety

- When the masses are removed from the springs quickly while in the stretched position the springs might be launched off their holders. Eye protection is again absolutely necessary.


## Meeting the Needs of All Students

## Differentiated Instruction: Augmentation and Accommodations

| Learning Issue | Reference | Augmentation and Accommodations |
| :---: | :---: | :---: |
| Doing multi-step unit conversions | Investigate <br> Part A, Step 4 | Augmentation <br> - Learning how to complete unit conversions is an important skill in science classes. Using direct instruction, teach students a strategy for unit conversions that can be applied to all unit conversions. $\frac{11 \mathrm{~Kb}}{1} \times \frac{1 \mathrm{~kg}}{2.2 \mathrm{~Kb}} \times \frac{9.8 \mathrm{~N}}{1 \mathrm{~kg}}=49 \mathrm{~N}$ <br> - Learning a problem-solving process takes repetition and practice, but once the process is mastered, most students with learning disabilities have a lot of success applying procedures. |
| Making graphs | Investigate <br> Part B, Steps 7 and 9 <br> Physics to Go Question 3 | Augmentation <br> - Students have created a graph in the three sections preceding this one and have probably done so with some support. At this point, students should be gaining some independence with graphing, however, sometimes students do not generalize skills to new situations. Explicitly show students how graphing the stretch of the spring vs. weight and then interpolating data is the same set of skills they practiced in Section 4. Then ask students to work in pairs to create the graph and make predictions. |
| Applying new concepts | Investigate <br> Part B, 10.a) <br> Physics to Go Question 3 | Augmentation <br> - The two previous graphs in this Investigate were comparing amount of stretch vs. weight and now students are being asked to graph spring force vs. amount of stretch. While these concepts are related, students may have a difficult time applying what they have learned to create a graph for an "invented" spring that does not have any known data points. <br> - One way to differentiate this activity would be to allow students to draw a graph for the "invented" spring and then write a statement to defend their graphs. Other students could choose between two graphs that have been created by someone else, and write a statement to defend that choice. <br> Accommodation <br> - Make sure students can view all three of their graphs side-by-side at one time. This will make comparisons and differences much more obvious. Many students do not notice similarities and differences unless they are right in front of them. |
| Calculating slope | Investigate <br> Part B, Step 9.c) <br> Physics Talk <br> Sample Problem 2 <br> Active Physics Plus <br> Physics to Go Questions 3.c) and 7 | Augmentation <br> - Most students can define slope as rise over run. When asked to apply this definition to find the slope of a graph, students often have difficulty determining the values for rise and run. Using the formula slope $=\Delta y / \Delta x$ may help students. If they are able to identify how much the $y$-values changed and how much the $x$-values changed using the line on the graph, they can divide to find the slope. <br> - Provide direct instruction to review the method for calculating slope from a graph or a chart. <br> Accommodation <br> - Highlight the $y$-axis with one color and the $x$-axis with another color. Then use the corresponding colors to highlight rise and run in the slope formula. This is a visual cue to remind students which scale they should be using to determine values for rise and run. |

## Strategies for Students with Limited English-Language Proficiency

| Learning Issue | Reference | Augmentation |
| :---: | :---: | :---: |
| Understanding concepts | Investigate <br> Part A, Step 3.a) | After students have answered the question, check their understanding of the difference between weight and mass by asking: What would be the mass of the gymnast and the football player on the Moon? |
| Research skills | Investigate <br> Part A, Step 4 | If time and curiosity allow, have students see whether they can find out why we use pounds instead of newtons as the nonscientific unit of weight. Students may find it interesting that the abbreviation " lb " comes from the Latin libra, which means scales or balance. |
| Vocabulary comprehension <br> Higher-order thinking | Investigate <br> Part B, Step 7.b) | - Help students use context clues to infer the meaning of the term "unconventional," if necessary. It may help to ask what convention they usually use in graphing. <br> - Before students draw their graphs, hold a class discussion to get students' opinions on why the independent variable (the weight) is graphed on the $y$-axis instead of on the x-axis. Have the same discussion after they have drawn their graphs, to see whether their thinking has changed. |
| Vocabulary comprehension | Investigate <br> Part B, Step 7.c) | "Interpolation" is an important term, and an important skill, in science. To make sure students have a solid grasp of what it means, it may help to discuss how interpolation differs from extrapolation. Discuss why an interpolation might be more reliable than an extrapolation. |
| Comprehension | Investigate <br> Part B, Step 9.c) | Check to make sure that students understand "best-fit line." When drawing this line, they should not connect the data points. |
| Understanding concepts <br> Vocabulary comprehension | Physics Words | Encourage students to write a summary of Hooke's law in their own words, which will allow you to check their understanding. ELL students may need assistance to properly comprehend "restoring force." |
| Understanding concepts | Physics Talk <br> Hooke's Law <br> Describes the <br> Restoring Force a <br> Spring Exerts | To make sure students understand the meaning of the negative sign in the equation for Hooke's law, have them explain it in their own words. |
| Comprehension | Physics Talk <br> Sample Problem 1 | Make sure students understand what omitting the negative sign from their calculations really means. The negative sign is not necessary in any mention of the magnitude of the force. It is important only when indicating the direction of the force. |
| Vocabulary comprehension | Physics Talk <br> Sample Problem 2 | ELL students may need help interpreting "rise over run" as a definition of slope. One way to help students visualize the relationship is to use the example of a set of stairs. The solid vertical piece of wood in each step is called a riser, so it represents the rise. The depth of each step is the run. |
| Vocabulary comprehension | Active Physics Plus | Check students' interpretations of the word "reciprocal," and correct if necessary. |
| Understanding concepts | Physics Essential <br> Questions <br> What does it mean? | To fully grasp the content of this section, students need to be clear on the differences in meaning of "spring force," "spring stretch," and "spring constant." Point out that the term "spring" can be a noun or a verb, but in these three terms it is an adjective. |

## SECTION 5

## Teaching Suggestions and Sample Answers

## What Do You See?

This is an illustration that communicates its message through images that clearly indicate that a woman buying something to eat at a deli is having her purchase weighed again on a different scale. Project the illustration on a transparency using an overhead projector. Consider asking students why the weight is written in newtons and not in kilograms and why the woman is using a second scale to measure the weight. In a discussion prompt students to observe the setting of the two visuals in the What Do You See? section. Remind students that their answers are all valuable at this point because they will be shaping up their original


## Students' Prior Conceptions

Student understanding of the previous sections ladders their comprehension of the differences between mass and weight. This section enables students to discover what they believe and to compare it with what they actually know through direct experimentation. This section also furthers cognitive development of more than one type of potential energy.

1. Students confuse the ideas of mass and weight. The teachers' attention is directed to the extensive discussion of prior conceptions associated with mass and weight in Physics in Action. The opportunity to assess the coherence of student thinking on "at rest" weight as measured with the spring scale provides measured data around which mass can be discerned from weight.
2. Everyday language leads students to think that mass is something you get by weighing an object. Using the spring scale to measure the weight of various known masses will help students to build templates for correlating mass and weight. Calculation of the restoring force of the spring or the "spring force" and relating force to acceleration will also establish another connection between weight and apparent weight. Additionally, the opening exists for the teacher to emphasize differences between weight and apparent weight. The acceleration component of the net force or the change in the acceleration due to certain types of motion indicate why students may "feel" weightless at certain points on roller-coaster rides or when riding in an elevator. Mass remains constant, weight may change.
responses as they progress further in the section. Your main emphasis should be in engaging students' curiosity and getting them involved in thinking about the topic they will be investigating.

## What Do You Think?

The What Do You Think? questions encourage students to think about weight. The enormous differences in the weight of a canary and elephant will prompt student to the think of the scale used for measurement. Ask students to record their answers in their Active Pbysics logs, and assure them at the same time that you are not evaluating their answers at this point. Students should feel confident in discussing their ideas. You might want to ask students to record the questions that come up in their minds after they have written down their responses to the What Do You Think? section. Note the misconceptions that students might have and post
them on a wall in your classroom so that you can address them at a later stage when students have carried out their experiments in the Investigate. There is no reason to expect that the students would know how a bathroom scale works, particularly if the scale they have at home is electronic.

## Investigate

## Part A: Mass and Weight

 1.a)A pound is a unit of weight, the force that Earth exerts on an object.

## 1.b)

The pound is related to the mass of an object. The greater the mass, the greater the weight. A pound is approximately 0.45 kg .

## 1.c)

Strictly speaking, weight is the force that Earth's gravity exerts on an object. If you are on the Moon, it is the force that the Moon's gravity exerts on you.

## What Do You Think?

## A Physicist's Response

The same scale could not be used for an elephant and a canary. The canary scale must be able to differentiate between grams, while the elephant scale needs to be accurate only to the nearest kilogram. Although you could build an elephant scale to the precision needed for a canary scale, it would be prohibitively expensive and serve no obvious purpose. The bathroom scale is similar to the spring that the students used in the pop-up toy. The greater the force applied, the more the spring is compressed. If a pointer is attached to the spring, and a properly calibrated scale is near the pointer, the spring can be used for determining the force of the weight for a given stretch of the spring.

NOTES

## NOTES

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## 2.a)

For the gymnast,
$F_{\mathrm{w}}=m g=(40 \mathrm{~kg})(9.8 \mathrm{~N} / \mathrm{kg})=$ 392 N (or about 400 N ).

For the football player, $F_{\mathrm{w}}=m g=(110 \mathrm{~kg})(9.8 \mathrm{~N} / \mathrm{kg})=$ 1078 N (or about 1100 N ).

## 3.a)

For the gymnast
$F_{\mathrm{w}}=m g_{\text {moon }}=$ $40 \mathrm{~kg} \times 1.6 \mathrm{~N} / \mathrm{kg}=64 \mathrm{~N}$.

For the football player
$F_{\mathrm{w}}=m g_{\text {moon }}=$
$110 \mathrm{~kg} \times 1.6 \mathrm{~N} / \mathrm{kg}=176 \mathrm{~N}$.
The weights are considerably smaller on the Moon, but of course the masses are the same.

## 4.a)

$(11 \mathrm{lb}) \times(1 \mathrm{~kg} / 2.2 \mathrm{lb}) \times$
$(9.8 \mathrm{~N} / \mathrm{kg})=49 \mathrm{~N}$
4.b)
$(0.25 \mathrm{lb}) \times(1 \mathrm{~kg} / 2.2 \mathrm{lb}) \times$
$(9.8 \mathrm{~N} / \mathrm{kg})=1.1 \mathrm{~N}$

## 5.a)

1500 pounds is approximately 6000 1/4-lb burgers.
$1500 \mathrm{lb}=6000 \mathrm{~N}$

## Part B: The Properties of Springs

## 1.

Students fold an $8 \frac{1}{2} \times 11$ piece of paper in the shape of an accordion. The creases in the paper will let the piece of paper act like a spring.

## 2.a)

When you release the paper, it springs back to its original shape.
3.a)

When you release the paper, it springs back to its original shape.

## 4.a)

If you stretch it too far, it won't return to its shape.

## 5.

You could use a metal paper clip as a spring for heavier weights than the paper. But if you stretch the paper clip too far, like the paper its shape is distorted and it does not return to its original shape.

## 6.a)

Students create a data table in their log. See table below:

| Mass | Weight | Stretch |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## 7.a)

Data will vary depending upon the springs used. Note that it is important that the amount of stretch be measured from the relaxed position of the spring.


## 9.a)

Students describe the difference in the springs. If you are using different springs for different groups, each group of students can exchange the springs.

## 9.b)

Students' graphs should differ in the stretch values recorded. The students will probably use the same value for the weights
attached to the springs. The result of graphing force versus distance will again be a straight line. The slope of the line will be different from the previous graph.

## 9.c)

The slope of the line is a measure of the difficulty in stretching the spring. The greater the slope, the "stiffer" the spring, and the less it will stretch for a given force.


## 10.a)

The graph should be a straight line with a third slope.

## 10.b)

Students' description should include whether the spring was harder to stretch than the previous springs, based on the slope of the graph they have drawn.

## 11.a)

Students complete the fourth column in their data table.

## Physics Talk

The Physics Talk describes the experiments done in the Investigate to discover the property of springs. Students are introduced to Hooke's law that is expressed both in words and as a mathematical equation. The sample problems show how the spring constant is calculated from the Hooke's law equation and also from a graph of force versus stretch of the spring. Students read about the difference between mass and weight and how bathroom scales measure weight. They are shown how the force exerted by the spring in a bathroom scale is equal to the force of gravity that acts downward on a person who steps on the scale.

While discussing the linear relationship between the force applied and the stretch of a spring, ask them to explain Hooke's law. Determine if students understand how a restoring force of a spring is related to the applied force. Point out that the spring constant for different springs is different and determines the stiffness of a spring. Discuss the equation of Hooke's law and ask students about the absolute value of the force exerted by a spring, and why there is a negative sign in the equation. Consider writing the sample problems on the board to detail the strategy used to solve each problem. Discuss how the spring constant can be calculated from a graph of force versus stretch of a spring.


To understand how bathroom scales read weight, the difference between mass and weight should be emphasized during a discussion of this Physics Talk. Ask students to write the definitions of mass and weight in their Active Physics logs. Students should be able to connect Newton's second law to the net force and know that this force is zero when the spring in the scale stops moving.

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where F}\mathrm{ , is the force exerted by the spring,
    x}\mathrm{ is the stretch (or compression) of the spring, and
    k}\mathrm{ is the spring constant.
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The negative sign in the equation indicates that the pull by the spring is opposite to the direction it is stretched or compressed. Stretch a spring down and it pulls up. Stretch a spring to the right and it pulls to the left. Compress a spring to the left and it pushes to the right.
The spring constant $(k)$ is an indication of how easy or difficult it is to stretch or compress a spring. You can determine the value of the spring constant $(k)$ by measuring the force exerted by the spring and the stretch of the spring. This is what you did in the Investigate. You varied the force by using different weights and measured the stretch. The measure of the stiffness of the spring is represented by $k$. A stiff spring will have a large value for $k$; a "soft" spring will have a small value for $k$. The spring constant, $k$, depends on the material from which the spring is made and the shape and size of the coils.
Sample Problem I
A 3.0-N weight is suspended from a spring. The spring stretches 2.0 cm $(0.020 \mathrm{~m})$. Calculate the spring constant.
Strategy: If a $3.0-\mathrm{N}$ weight is suspended at rest from the spring, the spring must be applying a force of 3.0 N . If the spring were applying a force of less than 3.0 N , the weight would accelerate down. If the spring were applying a force of more than 3.0 N , the weight would accelerate up. When the force of gravity on the mass is 3.0 N down and the spring exerts a force of 3.0 N up, then the mass has no net force on it and it remains at rest once the friction brings it to rest. In the Investigate, you always took your measurements when the mass had stopped moving.

| Given: | Solution: |  |
| :---: | :---: | :---: |
| $F=3.0 \mathrm{~N}$ | The magnitude of the force is |  |
| $x=0.020 \mathrm{~m}$ |  | $F_{\text {s }}=k x$ |
|  | Solving for $k$ | $k=\frac{F_{5}}{x}$ |
|  |  | $=\frac{3.0 \mathrm{~N}}{0.02 \mathrm{~m}}$ |
|  |  | $=150 \mathrm{~N} / \mathrm{m}$ |

Note that the stretch of the spring, $x$, was converted from centimeters to meters. The result tells you that it would take a force of 150 N to stretch the spring 1 m . You also ignored the negative sign in calculating the magnitude (size) of the force. From here on, the negative sign in the equation will be (size) of the force. From here on, the negative sign in the equation will be
omitted. In mathematics, you would say that you are calculating the absolute omitted. In mathematics, you would say that you are calculating the absolute
value of $F$ using the absolute value of the stretch, $x$. You must always keep in value of $F$ using the absolute value of the stretch, $x$. You must always keep in
mind that the direction of the force is opposite to the direction of the stretch.

## 4-5a

With a set of data points for the weight and the stretch, you would find that all values of $k$ are the same or constant (at leas approximately) for a given spring. The spring constant is $k$. As you determined in the Investigate, you can also record the data on a graph. The graph can also be used to determine the spring constant ( $k$ ). If the data are graphed so that the force is on the $y$-axis and the stretch is on the $x$-axis, then the spring constant will be the slope of the graph. You can see this if you compare the equations for Hooke's law and the equation for a straight line (when the $y$-intercept is zero).
Sample Problem 2
Weights are hung from a spring and the stretch is measured. The data collected is shown in the graph to the right. Calculate the graph to the right. Calculate the spring constant from the graph
Strategy: Since the force is on the $y$-axis and the stretch is on the $x$-axis, you can compare the equations for a straight line and
 Hooke's law.
Hooke's law: $F_{s}=k x \quad$ Straight line: $y=$ slope $\times x$
The slope of the graph will be equal to the spring constant, $k$.
Given: Solution:


Mass and Weight
In this section, you used specific masses as weights. A 1 -kg mass on Earth has a weight of 9.8 N .

$$
\text { weight }=m g
$$

where $m$ is mass in kilograms and
$g$ is the gravitational field strength of the Earth ( $9.8 \mathrm{~N} / \mathrm{kg}=9.8 \mathrm{~m} / \mathrm{s}^{2}$ ). weight $=(1 \mathrm{~kg})(9.8 \mathrm{~N} / \mathrm{kg})$

$$
=9.8 \mathrm{~N}
$$

A $1-\mathrm{kg}$ mass has a specific amount of matter and a specific volume. Of these three properties, only the mass is related to the weight. When the $1-\mathrm{kg}$ mass is on Earth, there is a force attracting the $1-\mathrm{kg}$ mass to Earth. This force is also


## Active Physics Plus

Students plot the stretch of the spring on the vertical axis and the weight of the suspended object on the horizontal axis of a graph. Some people prefer to plot the data this way because they view the weight of the suspended object as the independent variable (the "cause") and like to put independent variables on the $x$ (horizontal) axis. In this case, the slope of the straight line is equal to $1 / k$, the reciprocal of the spring constant. A stiffer spring will be represented by a line with a smaller slope.

The experimental design for testing a rubber band will be the same as for determining Hooke's law for a spring. Generally, a rubber band is not as linear as a spring, but the stretch does increase with the force.

## What Do You Think Now?

The What Do You Think Now? section focus students' attention on what they have learned so far in the section. Encourage them to apply what they have learned to review and update their answers. Have them return to the What Do You See? section and discuss how their original impressions of the illustration may have been altered. Students should reassess their understanding of physics concepts and share their answers with their classmates in small groups. Sharing $A$ Physicist's Response with students will provide students with a comprehensive answer to the

Chapter 4 Thrills and Chills


Graphing Force on the $\boldsymbol{X}$-axis
If a graph for a stretched spring is constructed so that the force is on the $x$ axis and the stretch is on the $y$-axis, then the slope will be the reciprocal of the spring constant or $1 / k$. You can see this by comparing the equations for Hooke's law and for a straight line.
In the following equation, capital $X$ and $Y$ are used to label the axes to avoid confusion with $x$, for stretch. The equation for a straight line passing through the origin is
$Y=$ slope $\times X$
Hooke's law for the magnitude of the force (omit the minus sign) is
$F_{\mathrm{s}}=k x$
$x=\left(\frac{1}{k}\right) F_{\text {s }}$

Since the force is on the X -axis and the stretch $x$ is on the Y -axis, the slope of the straight-line graph is $1 / k$.

$$
\begin{aligned}
& x=\frac{1}{k} F_{\mathrm{s}} \\
& Y=\text { slope } \times X
\end{aligned}
$$

Graph the data in Physics to Go, Question 7 by putting the amount of stretch $(x)$ on the $y$-axis and the force on the $x$-axis. From the slope of the graph, find the spring constant, $k$.
Hooke's Law and the Stretch of a Rubber Band
Design an experiment to see if the amount of stretch of a rubber band is described by Hooke's law. With your teacher's permission, carry out the experiment.

What Do You Think Now?
At the beginning of this section, you were asked the following:

- Can you use the same scale to weigh a canary and an elephant?
- How does a bathroom scale work?

Now that you have completed this section, how would you answer these questions? Include the concepts of weight and Hooke's law and springs in your answer.

What Do You Think? questions. Point out that their revision should incorporate the key information they now have on bathroom scales.


## Reflecting on the Section and the Challenge

Students now get a chance to reflect on what they have learned in this section and ponder how their knowledge can be applied to the Chapter Challenge. Consider asking students why they experience different sensations on a roller-coaster ride, how does the weight change during the ride, and how is Newton's second law related to Hooke's law. You might want to have a student read this section aloud so that you can highlight important aspects of this section and its broader relation to the Goal for this chapter. Encourage them to record their reflections in their Active Physics logs.

## Physics Essential Questions

## What does it mean?

As you pull on a spring, the spring pulls back. When you compress a spring, the spring pushes back. The force of the spring varies as the stretch (or compression). The spring constant tells you how difficult it is to stretch a spring and the spring's force for a given stretch.

## How do you know?

Hooke's law states that $F=-k x$. When the force of the spring was measured for each stretch, it was
found that the ratio of the force to the stretch was a constant for a given spring.

## Why do you believe?

You can design an experiment that measures the force of the rubber band and its stretch. If the plot of your data of force versus distance is a straight line, you know that the rubber band is obeying Hooke's law.

Why should you care?
If you carry a spring scale on the roller coaster, you will be able to observe changes in the scale reading during these motions.

## Physics to Go

## 1.a)

Given:
$m=100 \mathrm{~kg}$
$F_{\mathrm{w}}=m g=(100 \mathrm{~kg})(9.8 \mathrm{~N} / \mathrm{kg})=$ $980 \mathrm{~N} \approx 1000 \mathrm{~N}$
1.b)

Given:
$m=10 \mathrm{~kg}$
$F_{\mathrm{w}}=m g=(10 \mathrm{~kg})(9.8 \mathrm{~N} / \mathrm{kg})=$ $980 \mathrm{~N} \approx 100 \mathrm{~N}$

## 1.c)

Given:
$m=60 \mathrm{~kg}$
$F_{\mathrm{w}}=m g=(60 \mathrm{~kg})(9.8 \mathrm{~N} / \mathrm{kg})=$ $590 \mathrm{~N} \approx 600 \mathrm{~N}$

## 2.a)

Given:
$\frac{1}{4} \mathrm{lb}=1 \mathrm{~N}$
$130 \mathrm{lb}=520$ quarter-pounds $=$ 520 N
2.b)
$1000 \mathrm{lb}=4000 \mathrm{~N}$
2.c)
$50 \mathrm{lb}=200 \mathrm{~N}$


Chapter 4 Thrills and Chills

## Physics to Go

1. Calculate the weight of the following objects:
a) a football player with a mass of 100 kg
b) a toddler with a mass of 10 kg
c) an adult with a mass of 60 kg
2. Use the approximation that the weight of a $1 / 4-\mathrm{lb}$ burger is one newton. (Likewise, one stick of butter $=1 / 4-\mathrm{lb}$ butter has a weight of one newton.) Write down the approximate weights (in newtons) of the following objects:
a) a $130-\mathrm{lb}$ student
b) a $1000-\mathrm{lb}$ roller-coaster car $\quad$ c) a $50-\mathrm{lb}$ child
3. Weights were hung from a spring and the stretch of the spring was measured. The data is given in the table below.

| Weight (N) | Stretch (cm) |
| :---: | :---: |
| 0.0 | 0.0 |
| 0.3 | 2.0 |
| 0.7 | 4.6 |
| 1.2 | 8.0 |
| 2.0 | 13.0 |
| 2.4 | 16.2 |
| 3.1 | 21.0 |

a) Graph the data with the stretch of the spring on the $x$-axis and the weight on the $y$-axis.
b) If the data points do not fall exactly on a straight line, draw the best-fit line through the data points.
c) Find the slope of the graph.
d) What is the meaning of the slope?
e) Devise a graph for a second spring. Sketch the devised spring's graph Write a description of a spring that would have such data. The description should include the ease or difficulty of stretching the spring.
4. A weight of 12 N causes a spring to stretch 3.0 cm . What is the spring constant ( $k$ ) of the spring?

## 3.a)

Students' graph should have the weight on the vertical axis and the stretch on the horizontal axis. The points should be graphed and should fall on a straight line rising to the right.

## 3.b)

The students should draw a "best fit" line that is close to as many points as possible.
3.c)
slope $=$ rise $/$ run $=3.1 \mathrm{~N} / 21.0 \mathrm{~cm}=$ $3.1 \mathrm{~N} / 0.210 \mathrm{~m}=1.47 \mathrm{~N} / \mathrm{m}$

## 3.d)

The slope gives us the spring constant, a measure of the stiffness of the spring.

## 3.e)

Students' answers will vary. Graphs that double or triple the values in the data table would be

acceptable. If the spring drawn has a steeper slope, it will be harder to stretch.

## 4.

Given:
$F=12 \mathrm{~N} ; x=3 \mathrm{~cm}$
$F=k x$
$k=F / x=12 \mathrm{~N} / 3.0 \mathrm{~cm}=$
$4.0 \mathrm{~N} / \mathrm{cm}=400 \mathrm{~N} / \mathrm{m}$

## 5.

As the weight pulling on the spring increases, the stretch of the spring increases in proportion. (Double the weight, you double the stretch.)

## 6.

The $15.0-\mathrm{N} / \mathrm{cm}$ spring requires 15 N to stretch it one centimeter, while the $10.0 \mathrm{~N} / \mathrm{cm}$ requires only 10 N to stretch it one centimeter.

## 7.

The spring constant will be the change in $F$ (rise) divided by the change in stretch (run).
$k=F / x=3.0 \mathrm{~N} / 2.0 \mathrm{~cm}=$ $1.5 \mathrm{~N} / \mathrm{cm}=150 \mathrm{~N} / \mathrm{m}$
8.

Preparing for the Chapter Challenge

A bathroom scale has a spring attached to the top and bottom. When you stand on the scale, the spring is compressed. The compression is proportional to your weight. The weight corresponding to the compression is displayed as the number on the scale.

## Inquiring Further

For this Inquiring Further, it is assumed that the students will be disassembling a spring-based bathroom scale rather than a digital scale. The spring-based scale relies upon a spring that undergoes compression with a pointer attached to the spring. The scale will often have a calibration dial that allows the user to set the zero mark when there is no weight on the platform. The digital scale relies upon a strain gauge, where the electrical resistance of the sensitive element varies with the applied force. This value is then converted to a reading on the display. For the spring-based scale, the students will list parts such as the case, the spring, the dial, the mounting mechanism, etc.

## SECTION 5 QUIZ

## 4-5b Blackline Master

1. The graph below shows the relationship between the force applied to a spring and the spring's stretch. What is the spring constant of the spring?

a) $20 \mathrm{~N} / \mathrm{m}$
b) $9.8 \mathrm{~N} / \mathrm{m}$
c) $0.60 \mathrm{~N} / \mathrm{m}$
d) $0.50 \mathrm{~N} / \mathrm{m}$
2. An unstretched spring with a spring constant of $100 \mathrm{~N} / \mathrm{m}$ stretches a distance of 0.40 m when a mass is attached to the spring. The size of the mass must be of
a) 9.8 kg .
b) 0.004 kg .
c) 25 kg .
d) 4.0 kg .
3. A $5.0-\mathrm{kg}$ stone is dropped from a bridge $40-\mathrm{m}$ high. What is the weight of the stone?
a) 8 N
b) 50 N
c) 200 N
d) 0.20 N
4. The ratio of an object's weight to its mass is equal to the
a) object's inertia.
b) acceleration due to gravity.
c) object's kinetic energy.
d) object's gravitational potential energy.
5. A $2.0-\mathrm{kg}$ mass attached to a spring scale is being lifted upward with constant velocity. The reading on the spring scale must be
a) equal to 20 N .
b) greater than 20 N .
c) equal to 2.0 N .
d) greater than 2.0 N .

## SECTION 5 QUIZ ANSWERS

(1) a) The spring constant may either be found from the slope of the graph, or by using the equation $F_{\mathrm{s}}=k x$. Choosing values from the graph of $F_{\mathrm{s}}=12 \mathrm{~N}$ and a stretch of 0.60 m and solving for $k$ gives $k=F_{\mathrm{s}} / x=12 \mathrm{~N} / 0.60 \mathrm{~m}=20 \mathrm{~N} / \mathrm{m}$.
(2) d) Using the equation $F_{\mathrm{s}}=k x$, and solving for the force gives $F_{\mathrm{s}}=(100 \mathrm{~N} / \mathrm{m})(0.40 \mathrm{~m})=40 \mathrm{~N}$ for the force. This force is the weight of the mass. To find the mass use the equation $F_{\mathrm{w}}=m g$. Solving the equation for the mass gives $m=F_{\mathrm{w}} / g$ or $m=(40 \mathrm{~N}) /\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)=4.0 \mathrm{~kg}$.
(3) The weight can be found from the mass using the equation $F_{\mathrm{w}}=m g$. Solving for $F_{\mathrm{w}}=(5 \mathrm{~kg})\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)=50 \mathrm{~N}$.

4 b) The ratio of an objects weight $(m g)$ divided by its inertia (measured in terms of mass, $m$ ) yields $m g / m=g$, the acceleration due to gravity.
(5) a) When an object is traveling with constant velocity, the net force on the object must be zero. The upward force supplied by the spring scale must exactly balance the downward force provided by the weight of the mass. To find the weight of the mass, use the equation $F_{\mathrm{w}}=m g . F_{\mathrm{w}}=2.0 \mathrm{~kg}\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)=20 \mathrm{~N}$, so the spring scale must read 20 N . Students may think it has to read more than 20 N to keep moving upward, but once the object is traveling with constant upward speed only 20 N is needed to maintain the motion.

