SECTION 7

Circular Motion: Riding on the Curves

Section Overview

Students investigate circular motion and apply their analysis of centripetal forces and accelerations to a roller-coaster design. They tie a battery-operated car to a string and hold the other end in place so that it goes around in a circle. They describe the forces acting on the car when it is attached to the string and the motion of the car when the string is released. The diagrams they sketch show the centripetal forces acting on a roller coaster moving on vertical and horizontal curves. Students then investigate the force required to twirl a string attached to a stopper and what happens when the either the length of the string is increased or the mass on the end of the string is increased by adding stoppers. This experiment helps them evaluate the forces required to keep a roller coaster traveling in a vertical loop at the top and bottom of the ride.

Background Information

Any object moving in a circle requires a centripetal force—a force acting toward the center of the circle. This force may be a force due to friction, a force due to tension in a string, a force due to gravity, a normal force (a contact force perpendicular to a surface), a force due to a spring, a magnetic force, and so on. Regardless of what force (or combination of forces) produces the circular motion, the net force directed toward the center of the circle is referred to as the centripetal force.

The centripetal force, according to Newton's second law, has a centripetal acceleration associated with it. This acceleration is always directed to the center of the circle if the object is moving with constant speed. Even if the speed of the object moving in a circle does not change, the direction of the velocity does change. The change in direction of the velocity is an acceleration. The centripetal acceleration can be calculated from the equation (derived in the student's text) $a = v^2/r$ where v is the speed and r is the radius of the circle around which the object is traveling. Applying Newton's second law, you arrive at the equation for centripetal force $F = ma = m v^2/r$ in terms of the centripetal acceleration.

When an object travels in a vertical circle, several forces combine to give the centripetal force. The weight of the object is always down. At the bottom of the vertical circle, a net centripetal force acting vertically upward (toward the center of the circle) is required if the object is to move in a circle. The downward weight cannot supply this force. If the object is in contact with a surface at the bottom of the vertical circle, the normal force of the surface that is in contact with the object is able to supply the required upward force. This upward force must be larger than the weight by the amount required for centripetal motion. The sum of the normal force and the weight must equal mv^2/r.

CRUCIAL PHYSICS

- An object moving along a curved path must have a force pointing toward the center of the curve acting on it, otherwise it would move in a straight line. If the force is removed, the object travels in a straight-line tangent to the path at that point.
- The normal force is a contact force exerted on an object by the surface it is in contact with, and it points perpendicular to the surface, toward the object.
- Centripetal force is the force pointing toward the center of a curved path that causes an object to follow a curved path. When the path is circular and the object moves at a constant speed, then the centripetal force is given by:

  $$F_c = \frac{mv^2}{r},$$

  where F_c is the centripetal force, m is the mass of the object, v is the speed, and r is the radius of the circle.
- Centripetal acceleration is the acceleration directed toward the center of a curved path an object is traveling on and is given by:

  $$a_c = \frac{v^2}{r}.$$
- For an object travelling in a circular path at constant speed, the centripetal acceleration and force are always perpendicular to the object's velocity.

Learning Outcomes	Location in the Section	Evidence of Understanding
Recall the idea that an object in motion remains in motion with a constant velocity unless acted upon by a force—Newton's first law.	***Investigate*** Part A, Steps 1 and 2; Part B, Steps 1-3 ***Physics Talk***	Students run a battery-operated car on a string, release it, and notice that the car goes straight at a tangent to the circle at the point of release when centripetal force ceases.
Explore how a force directed toward a center (a centripetal force) will allow a roller-coaster car to travel circular motion.	***Investigate*** Part A, Steps 1-5	Students compare the force that keeps the car attached to a string traveling in a circular motion to that of a roller coaster going around a circular track and sketch diagrams to show how that force acts on a roller coaster.
Describe how the centripetal force is dependent on the speed and the radius of the curve and the mass of the car.	***Investigate*** Part B, Steps 1-5	Students compare the forces acting on a roller coaster to that of a string attached to a stopper that is twirled in a circle. They change the length of the string to observe how the amount of force applied changes with the radius of the car. Then they add more stoppers to the string to see how the mass changes the degree of centripetal force.
Solve problems using the equation for centripetal force.	***Physics to Go*** Questions 5-7	Students use the equation for centripetal force by solving problems.
Recognize that safety considerations limit the acceleration of a roller-coaster to below 4 *g*.	***Physics Talk***	Students learn that safety on a roller coaster requires them to stay below an acceleration of 4 *g*.

Section 7 Materials, Preparation, and Safety

Materials and Equipment

PLAN A		
Materials and Equipment	**Group (4 students)**	**Class**
Safety glasses, impact	4 per group	
Stopper, rubber, #6 (w/hole)	1 per group	
Toy, car, battery operated	1 per group	
String, ball		1 per class
Scissors		1 per class

*Additional items needed not supplied

Time Requirements

Allow one class period or 45 minutes for this section's *Investigate*.

Teacher Preparation

- Check the toy car to make sure that the batteries will operate the car. You may wish to do this part of the *Investigate* with the entire class if supplies are limited. Do not use toy cars with wheels in front that would steer the cars around a curve. The car should go straight when released from the string.
- You will need to provide each group with a string that has a rubber stopper attached. Additional stoppers with holes in them may be added by sliding along the string until they are in contact with the first stopper.
- If possible, have the students use the hallways or other open areas to provide extra space.

Safety

- Eye protection is extra important in this *Investigate*.
- Ensure that the stoppers are attached to the string in such a way that they will not fly off when twirled.
- Caution the students to be very careful while moving around the room when the stoppers are being twirled to avoid being hit.
- The student who is twirling the stopper should have the string wrapped around the finger that is doing the twirling to ensure the string does not fly loose.

Materials and Equipment

PLAN B		
Materials and Equipment	**Group (4 students)**	**Class**
Safety glasses, impact	4 per group	
Stopper, rubber, #6 (w/hole)	1 per group	
Toy, car, battery operated	1 per group	
String, ball		1 per class
Scissors		1 per class

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NOTES

Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

Learning Issue	Reference	Augmentation and Accommodations
Being impulsive	***Investigate*** Part B, Steps 1-4	**Augmentation** • When everyone if the classroom starts twirling their stoppers, it may be very difficult for students who struggle with hyperactivity and impulsivity to maintain control. These students might try to have a competition to see who could swing it the fastest, or they might let go of the string to see what happens. • Structure this activity with very specific directions and time limits. Allow each student to experience the force of twirling the stopper by giving each student 20 seconds to twirl, tell them to stop, and then tell them to switch with another member of the group. While the next student is twirling, the previous student can record their observations. This strategy limits the opportunity to be impulsive because the activity is very structured.
Imagining a scenario	***Investigate*** Part B	**Augmentation** • Students will have an easier time labeling forces and making predictions about speed if they can see simulations of a roller coaster under different constraints. Find Web sites that offer free roller-coaster simulations and allow the user to change different features of the coaster. **Accommodation** • Guide students through the simulations step-by-step. Change different features, one at a time, and ask students to record their observations.
Understanding essential concepts Solving centripetal force and acceleration problems	***Physics Talk*** ***Physics Essential Questions*** ***Physics to Go*** Question 9	**Augmentation** • Students need to understand that "centripetal force is not an additional force." This concept is embedded in a paragraph in the *Physics Talk* section. Students with reading comprehension and focus issues may skim over this important concept. Provide a two-column graphic organizer with situations that involve an object traveling in a circle in the left-hand column and a blank right-hand column. As students read the section, tell them to record the centripetal force that keeps the object traveling in a circle. Include some new situations that are not described in the reading to check for understanding. • Include a third column in the chart and ask students to draw a picture of the described situation and label the forces acting on the object. • The following Web site provides simulations to illustrate centripetal force and centripetal acceleration as described in *Physics Talk* (*www.physicsclassroom.com*). **Accommodation** • Provide choices for the second column that requires students to identify the centripetal force. • In the graphic organizer, include the drawings in the third column and ask students to label the forces.
Knowing when to add and subtract vectors	***Physics to Go*** Question 10	**Augmentation** • Refer students to the table in *Physics Talk* to use as a model when completing this problem. If students can figure out the pattern in the example, they will be able to complete the other table independently. Also, to check for misconceptions, ask students to explain how they arrived at their answers.

Strategies for Students with Limited English-Language Proficiency

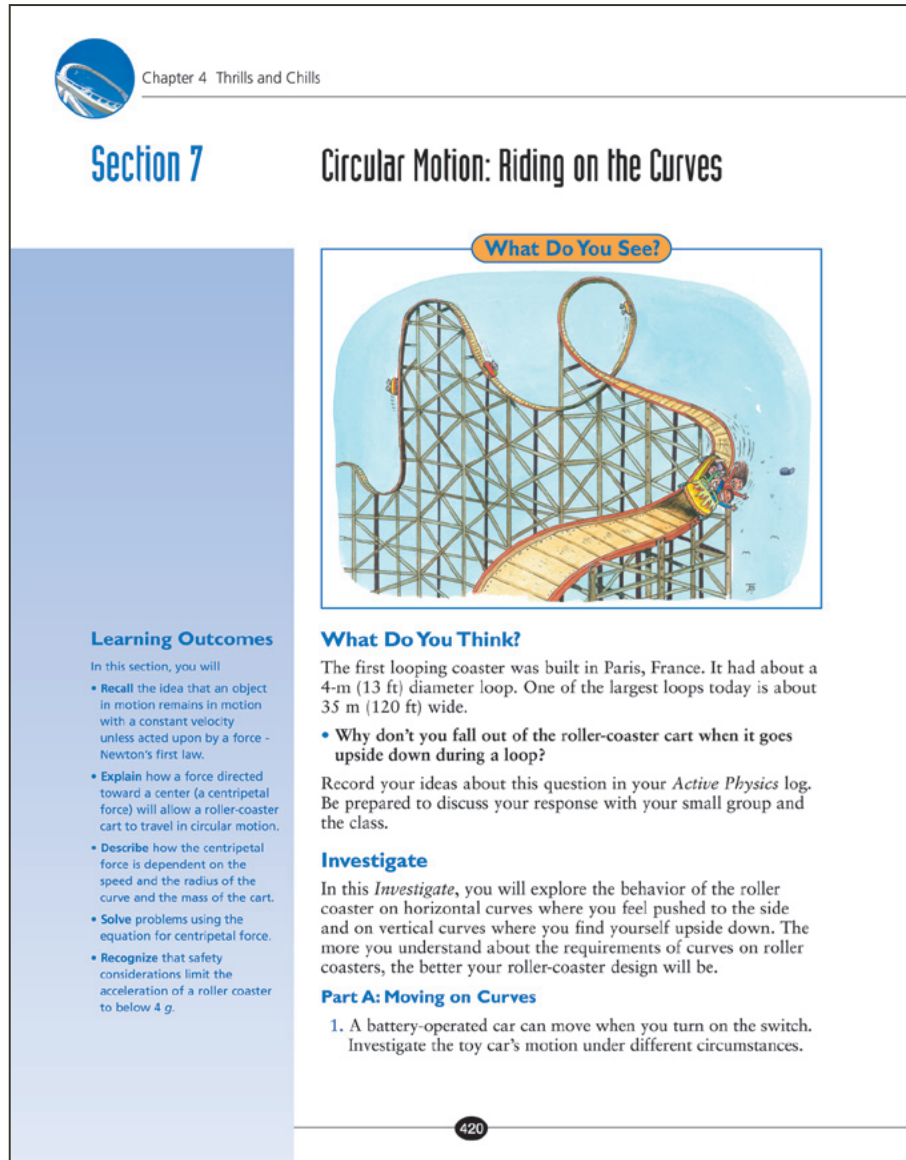
Learning Issue	Reference	Augmentation
Comprehension	***Physics Words***	To make sure students understand the difference between centripetal force and centripetal acceleration, ask them which one causes the other.
Comprehension	***Physics Talk*** Centripetal Force and Acceleration	Regarding swinging rubber stoppers on a string, make sure students are able to identify how the magnitude of the centripetal force changes with a change in mass, change in length of the string, and change in speed of the mass.
Understanding concepts Comprehension	***Physics Talk***	Hold a class discussion to make sure students understand the main point in this paragraph. The centripetal force is always toward the center, and the vector arrow shows this. But because the roller-coaster car is moving, it is constantly changing direction, which means the direction of the centripetal force is constantly changing direction as well. The vector will be pointing toward the center, but the center may be left of the car, or above the car, etc., as the car moves around the track. ELL students benefit from speaking in English. Write the equation for centripetal force on the board and have a student explain it in words.
Understanding concepts	***Physics Talk***	To assess students' understanding, ask: In a vertical loop, when does the centripetal force acting on a roller-coaster car point toward the center? [at all times]. What is the direction of the centripetal force at the top of the loop? [straight down]. If the net force is straight down, then why doesn't a roller-coaster car fall down at that point? [The centripetal force is necessary to keep the roller-coaster car moving in a circle. If the centripetal force were not present, the roller-coaster car would fly off horizontally.]
Comprehension	***Physics Talk*** Apparent Weight and the Roller-Coaster Ride	To check comprehension, ask an ELL volunteer to explain to the class why a clothoid loop is safer than a circular loop. Correct any misunderstandings.
Comprehension	***Active Physics Plus***	Write the equation for centripetal force on the board and have a volunteer explain it in words. Ask another volunteer to elaborate how changing one of the variables on the right side of the equation would change the force.
Higher-order thinking	***Reflecting on the Section and the Challenge***	Return to the discussion of the Space Shuttle's orbit that you held at the end of the preceding section. Challenge students to explain how the Shuttle can be in free fall, using what they now know about centripetal force.
Comprehension	***Physics to Go*** Question 10	It is a good idea to review students' work with tables now and again. Make sure they can correctly record data in a table and are able to interpret table data correctly. Make sure they include a title and clearly label rows and columns.

SECTION 7

Teaching Suggestions and Sample Answers

What Do You See?

This roller coaster is a prominent image in this illustration and students will most likely comment on its shape. Direct their attention to the horizontal and vertical shapes and ask them why the artist has chosen to show a roller coaster with a vertical loop. Prompt students to observe other features of this illustration. For instance, why do the riders appear to be so thrilled or why they appear to be almost falling off the track of the roller coaster? Encourage students to comment on different images that they see and point out that the artist may have had a purpose in showing these images. The *What Do You See?* illustration is designed to make students curious about the section they are about to study.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Section 7 Circular Motion: Riding on the Curves

What Do You See?

Learning Outcomes

In this section, you will

- **Recall** the idea that an object in motion remains in motion with a constant velocity unless acted upon by a force - Newton's first law.
- **Explain** how a force directed toward a center (a centripetal force) will allow a roller-coaster cart to travel in circular motion.
- **Describe** how the centripetal force is dependent on the speed and the radius of the curve and the mass of the cart.
- **Solve** problems using the equation for centripetal force.
- **Recognize** that safety considerations limit the acceleration of a roller coaster to below 4 *g*.

What Do You Think?

The first looping coaster was built in Paris, France. It had about a 4-m (13 ft) diameter loop. One of the largest loops today is about 35 m (120 ft) wide.

- **Why don't you fall out of the roller-coaster cart when it goes upside down during a loop?**

Record your ideas about this question in your *Active Physics* log. Be prepared to discuss your response with your small group and the class.

Investigate

In this *Investigate*, you will explore the behavior of the roller coaster on horizontal curves where you feel pushed to the side and on vertical curves where you find yourself upside down. The more you understand about the requirements of curves on roller coasters, the better your roller-coaster design will be.

Part A: Moving on Curves

1. A battery-operated car can move when you turn on the switch. Investigate the toy car's motion under different circumstances.

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Students' Prior Conceptions

There are six ideas rooted in naïve student conceptions that deal with motion on curves or circular motion; one misconception is a component of what students think about inertia.

1. **If an object moves with the same speed around a curve then the motion is steady; there is no acceleration.** Observing what happens to the motion of the toy car attached to the string encourages students to theorize that there must be a force along the string (tension in the string) maintaining the circular motion. You need to guide students to make the conceptual leap that any object moving in a circle or along a curve must have a force pulling it in toward the center of the circle, whether this force is visible, like in the case of the string, or invisible.

2. **If an object speeds up as it moves around a curve or in a circle then the accelerating force acts parallel to the direction of the motion along the curve; there is no centripetal force (the force directed toward the center of the curve).** Students may wonder what happens when a car speeds up as it rounds a curve and conjecture that the acceleration of the vehicle is tangential to the path of the vehicle. Students may not recognize the existence of the centripetal force nor understand that two forces may act independently upon the vehicle to influence its motion.

3. **Acceleration and velocity vectors act parallel to each other.** Students do not comprehend that motion along a circle must have a force acting along a radius directed toward the center of the circle. Their experience with linear

This section is meant to evoke an inquiry and response-based approach to science where students continuously learn to accommodate new ideas into existing ones or revise their previous conceptions through investigations. Remind students that they will have an opportunity to return to the *What Do You See?* illustration when they have advanced further in their understanding of the topic they will be exploring.

What Do You Think?

The question in the *What Do You Think?* section embodies the physics concepts discussed later in this section. Students will know that riders don't fall out of the roller coaster when it is upside down, but few will recognize that this is due to the circular motion. Ask them to recall their own experiences on roller coasters if they've been on one. Some students might say that the seat belts kept them from falling off and that is why they had to hold tight. Acknowledge all answers and point out that these answers will be revisited later after experiments and discussions, when students have gained more knowledge of the physics involved in a roller-coaster ride. Students should record their responses in their *Active Physics* logs. The *What Do You Think?* question will invariably lead to further questions and reveal students' prior conceptions. A useful strategy to redirect meaningful responses is to highlight a few answers by writing them on the board.

What Do You Think?

A Physicist's Response

The force of gravity pulls you down and would accelerate you downward at the acceleration due to gravity if you were released from rest. In a roller-coaster car going around a circular loop, the car is actually pulling you downward faster than gravity at the top of the loop. This might seem disastrous, but this is what keeps you firmly pinned to the seat at the top of the ride. The changing direction of the force on you then brings you safely around the loop, rather than the abrupt upward force exerted on an object hitting the ground.

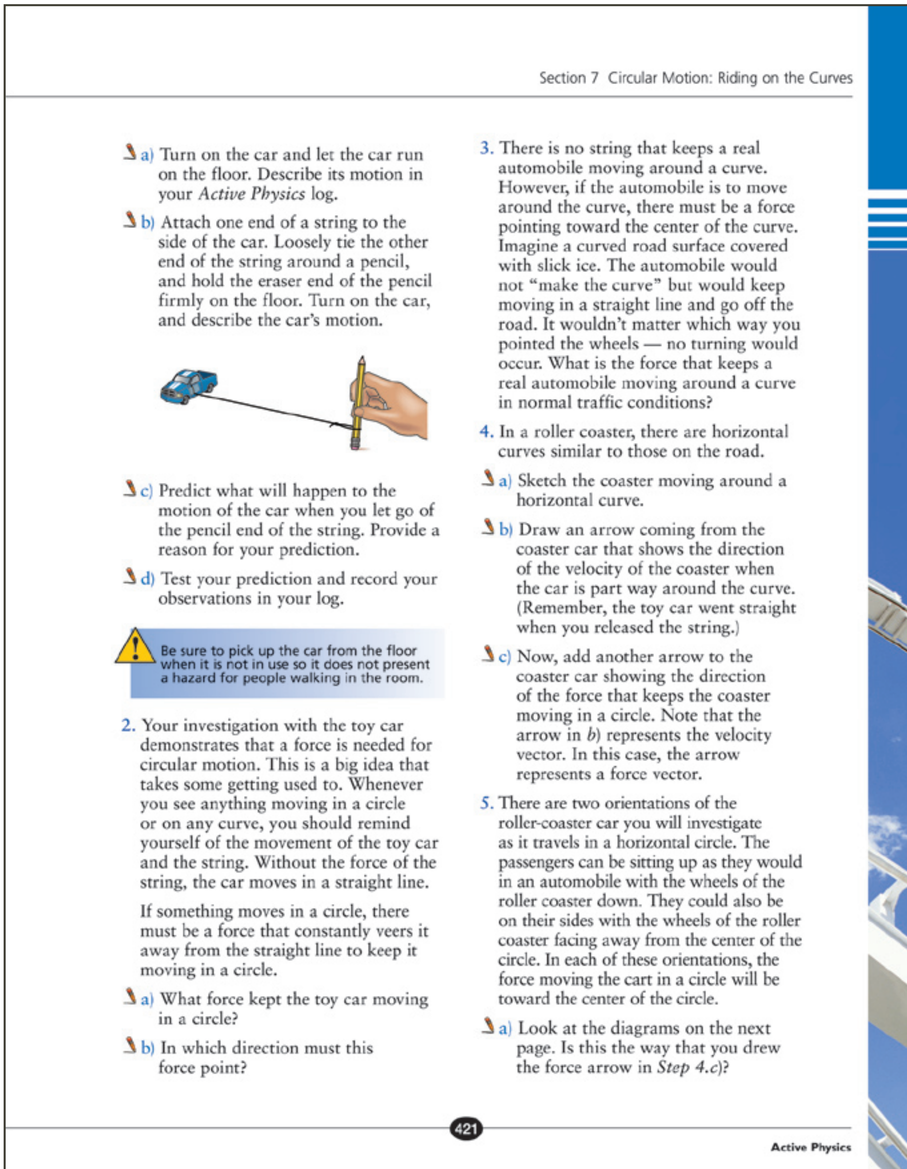
motion indicates that acceleration and velocity vectors usually parallel each other. Students will model motion along a curve with similar constructs. When discussing the *Investigate*, teachers should highlight the direction of the acceleration vector, parallel to the radius, and contrast it to the velocity vector, which is tangential to the path of the vehicle. On a circular curve, the acceleration and the velocity vectors are perpendicular to each other.

4. **If the force causing acceleration ceases to act, the speed of the object decreases as it continues to move along the curve.** Experience with the toy car traveling in a straight path when the string is released enables students to revamp their thinking about what happens to the path of a vehicle when the centripetal force stops. You may elect to relate what happens to the motion of a car when the tires hit a slippery spot as everyday reasoning, such as "Why should you care?"

5. **There is a force that pushes you to the side or out and away from the center of the curve.** Teachers may wish to revisit the concepts of inertia and have students recall that inertia wants you to keep traveling along a straight path when riding in a vehicle executing a turn, so you believe or "feel" that a force pushes you out, away from the center of the curve.

6. **Friction only impedes motion.** In this section, students learn that friction provides the centripetal force to maintain motion along a curve.

NOTES



a) Turn on the car and let the car run on the floor. Describe its motion in your *Active Physics* log.

b) Attach one end of a string to the side of the car. Loosely tie the other end of the string around a pencil, and hold the eraser end of the pencil firmly on the floor. Turn on the car, and describe the car's motion.

c) Predict what will happen to the motion of the car when you let go of the pencil end of the string. Provide a reason for your prediction.

d) Test your prediction and record your observations in your log.

Be sure to pick up the car from the floor when it is not in use so it does not present a hazard for people walking in the room.

2. Your investigation with the toy car demonstrates that a force is needed for circular motion. This is a big idea that takes some getting used to. Whenever you see anything moving in a circle or on any curve, you should remind yourself of the movement of the toy car and the string. Without the force of the string, the car moves in a straight line.

If something moves in a circle, there must be a force that constantly veers it away from the straight line to keep it moving in a circle.

a) What force kept the toy car moving in a circle?

b) In which direction must this force point?

3. There is no string that keeps a real automobile moving around a curve. However, if the automobile is to move around the curve, there must be a force pointing toward the center of the curve. Imagine a curved road surface covered with slick ice. The automobile would not "make the curve" but would keep moving in a straight line and go off the road. It wouldn't matter which way you pointed the wheels — no turning would occur. What is the force that keeps a real automobile moving around a curve in normal traffic conditions?

4. In a roller coaster, there are horizontal curves similar to those on the road.

a) Sketch the coaster moving around a horizontal curve.

b) Draw an arrow coming from the coaster car that shows the direction of the velocity of the coaster when the car is part way around the curve. (Remember, the toy car went straight when you released the string.)

c) Now, add another arrow to the coaster car showing the direction of the force that keeps the coaster moving in a circle. Note that the arrow in *b)* represents the velocity vector. In this case, the arrow represents a force vector.

5. There are two orientations of the roller-coaster car you will investigate as it travels in a horizontal circle. The passengers can be sitting up as they would in an automobile with the wheels of the roller coaster down. They could also be on their sides with the wheels of the roller coaster facing away from the center of the circle. In each of these orientations, the force moving the cart in a circle will be toward the center of the circle.

a) Look at the diagrams on the next page. Is this the way that you drew the force arrow in *Step 4.c)*?

Investigate

Part A: Moving on Curves

Teaching Tip

To make this investigation work best, tape the string to the side of the car at the center. The car's front wheels will not be aimed around the circle, but the car will travel in a circle.

1.a)

The car runs in a straight line at constant speed.

1.b)

The car moves in a circle. The radius of the circle is the length of the string.

1.c)

The students might predict the car will fly away from the center, or go in a straight line, or some other choice.

1.d)

When the car is released from the string, it moves in a straight line. The line is tangent to the circle at the point where the string was released.

2.a)

The tension in the string supplies the force.

2.b)

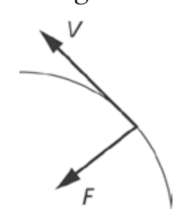
The force is toward the center of the circle.

3.

The force of friction between the tires and the road keeps a real car moving in a circle.

4.a)–c)

Refer to the diagram below:



5.a)

Students should modify their diagrams in *Step 4.c)*, if necessary.

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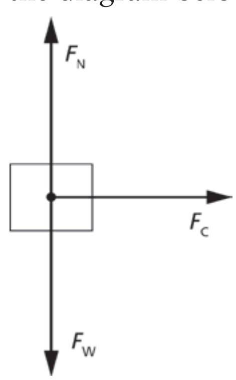
5.b)

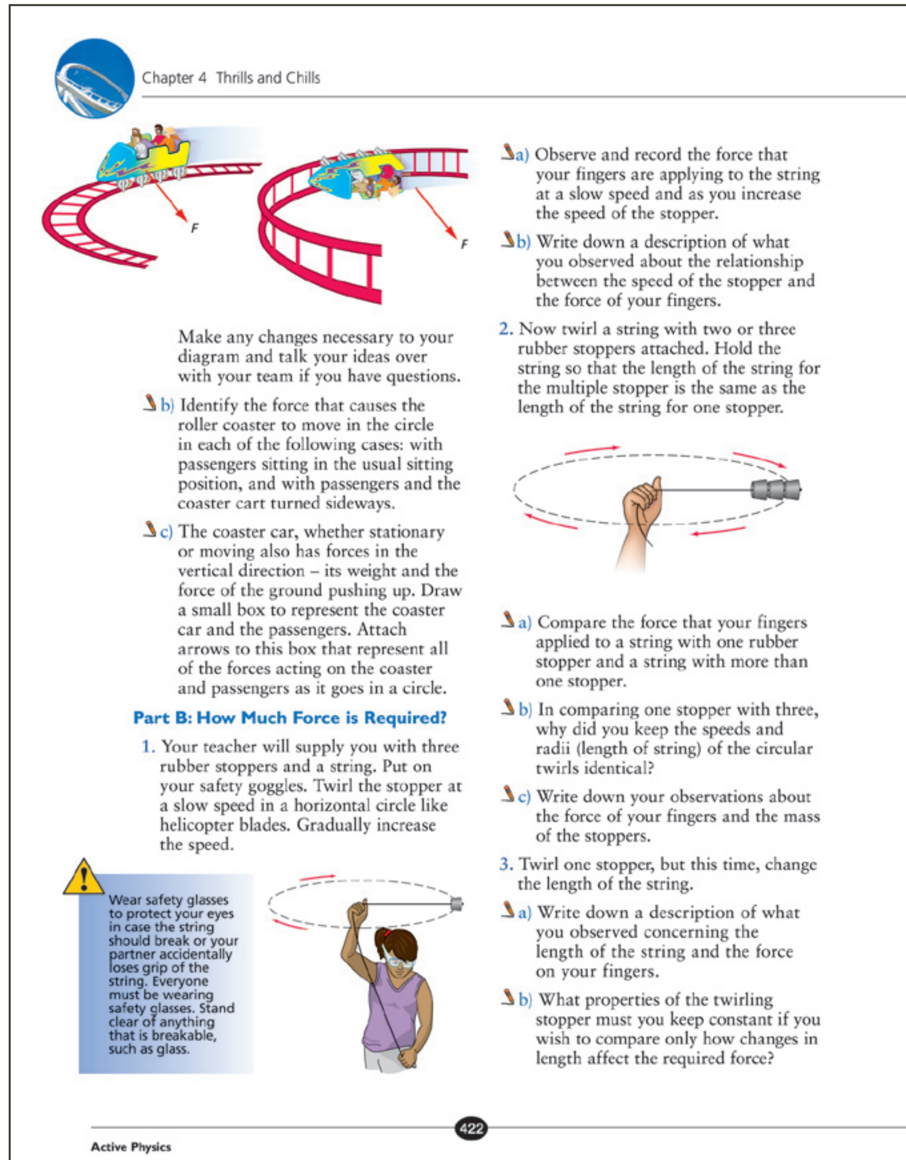
When the car is flat on the ground, the force between the wheels and the side of the track holding the wheels provides the force. When the roller coaster is on its side, the force of the face of the track (the vertical track) on the wheels provides the force.

5.c)

Refer to the diagram below:



Strictly speaking, a roller-coaster car does not go in a completely horizontal direction, since the passenger would have too great a tendency to slide down toward the lower side. The track will be tilted to allow the passengers to stay in their seats better. The actual path is in the shape of a helix.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Make any changes necessary to your diagram and talk your ideas over with your team if you have questions.

b) Identify the force that causes the roller coaster to move in the circle in each of the following cases: with passengers sitting in the usual sitting position, and with passengers and the coaster cart turned sideways.

c) The coaster car, whether stationary or moving also has forces in the vertical direction – its weight and the force of the ground pushing up. Draw a small box to represent the coaster car and the passengers. Attach arrows to this box that represent all of the forces acting on the coaster and passengers as it goes in a circle.

Part B: How Much Force is Required?

1. Your teacher will supply you with three rubber stoppers and a string. Put on your safety goggles. Twirl the stopper at a slow speed in a horizontal circle like helicopter blades. Gradually increase the speed.

Wear safety glasses to protect your eyes in case the string should break or your partner accidentally loses grip of the string. Everyone must be wearing safety glasses. Stand clear of anything that is breakable, such as glass.

a) Observe and record the force that your fingers are applying to the string at a slow speed and as you increase the speed of the stopper.

b) Write down a description of what you observed about the relationship between the speed of the stopper and the force of your fingers.

2. Now twirl a string with two or three rubber stoppers attached. Hold the string so that the length of the string for the multiple stopper is the same as the length of the string for one stopper.

a) Compare the force that your fingers applied to a string with one rubber stopper and a string with more than one stopper.

b) In comparing one stopper with three, why did you keep the speeds and radii (length of string) of the circular twirls identical?

c) Write down your observations about the force of your fingers and the mass of the stoppers.

3. Twirl one stopper, but this time, change the length of the string.

a) Write down a description of what you observed concerning the length of the string and the force on your fingers.

b) What properties of the twirling stopper must you keep constant if you wish to compare only how changes in length affect the required force?

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Part B: How Much Force is Required?

1.a)

The force is acting toward the center of the circle.

1.b)

As you twirl the stopper at larger speeds, your fingers tighten up to supply a bigger force.

2.a)

As the mass of the stoppers on the end of the spinning string increases, the fingers tighten to supply a bigger force.

2.b)

If you change more than one variable, you will not know what causes the change. Therefore, you should change only one variable, the mass.

2.c)

As the mass increases, the force required also increases.

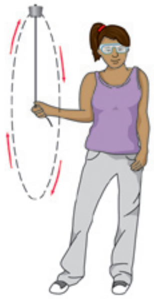
3.a)

As you twirl the stopper in a smaller circle, you tighten your fingers to supply a larger force.

3.b)

You must hold the mass and the speed of the stopper constant.

4. Now twirl the stopper in a vertical circle.



a) Observe how the force your fingers exert on the string is different when the stopper is near the top of the circle and when it is at the bottom of the circle.

b) Write down a description of how this force changes. Is it larger when the stopper is at the top of the loop or when it is at the bottom?

c) Twirl the stopper in a vertical circle and gradually reduce the speed of the stopper. At some point, the speed will be so low that the string goes slack and the stopper no longer moves in a circular path. When the string goes slack, it is no longer exerting a force on the stopper. Record your observation of where the string goes slack.

5. To keep a roller coaster moving in a circle, a force is required toward the center of the circle. The track pushing on the edges of the wheels, the surface of the track pushing on the rims of the wheels, the force of gravity on the coaster car, or some combination of these forces can supply this force.

a) How does the required force change when the speed of the roller coaster changes?

b) How does the required force change when the mass of the roller coaster changes?

c) How does the required force change when the radius of the curve changes?

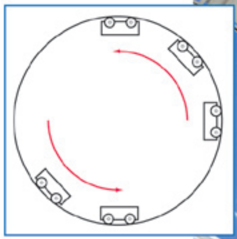
d) If the speed of a roller coaster were increased, how might you strengthen the track to provide the additional force required?

6. In physics, scientists often look at "limiting cases" (or "extreme cases") to help understand a concept better. A limiting case is the most extreme case that you may imagine. For instance, analyze the limiting cases for a roller coaster going around a horizontal curve that is not banked. If the coaster car's speed got much larger, an extreme case would be a very high speed.

a) If the coaster car were going at a very high speed, would the force required from the track to keep the car moving in the curve be very large or very small? Write your response down in your *Active Physics* log.

b) The other extreme case is a coaster car with zero speed. Would the force that the track would have to provide if the car were moving very slowly around a curve be very large or very small? Write your response down in your *Active Physics* log.

7. The roller coaster may also do a loop as it travels in a vertical circle. If the loop were a perfect circle, as illustrated to the right, there would always have to be a force toward the center of circle.



a) Make a sketch of the loop in your *Active Physics* log.

b) Draw the velocity vector for the coaster at each of the positions shown in the diagram.

c) Draw the centripetal force vectors at each position. This is the force toward the center of the circle that keeps the roller-coaster car moving in a circle at each position in a circle.

CHAPTER 4

5.a)

As the speed of the roller coaster increases, a larger force is required to keep it moving in a circle at constant speed.

5.b)

As the mass of the roller coaster increases, a larger force is required to keep it moving in a circle at constant speed.

5.c)

As the radius of the curve of the roller coaster decreases, a larger force is required. If the radius increases, the required force is smaller.

5.d)

You could strengthen the track by building it with stronger steel or reinforcing the steel around the curve.

6.a)

A very high speed would require a very large force.

6.b)

If there were zero speed, there would be no force required. If you were going very slowly, a very small force would be required.

4.a)

Students observe the force that their fingers exert on the string when the stopper is near the top of the circle and when the stopper is near the bottom of the circle.

4.b)

The force that your fingers provide is less when the stopper is at the top of the vertical circle and larger when it is at the bottom.

4.c)

As the speed of the stopper decreases, the string will eventually go slack, indicating that the string is no longer exerting a force on the stopper. Then the stopper will no longer travel on a circular path and your fingers will not be providing any force to the string.

7.a)

The students copy the diagram into their logs.

7.b)

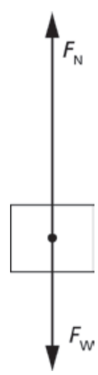
The velocity vectors should be tangent to the circle at each position.

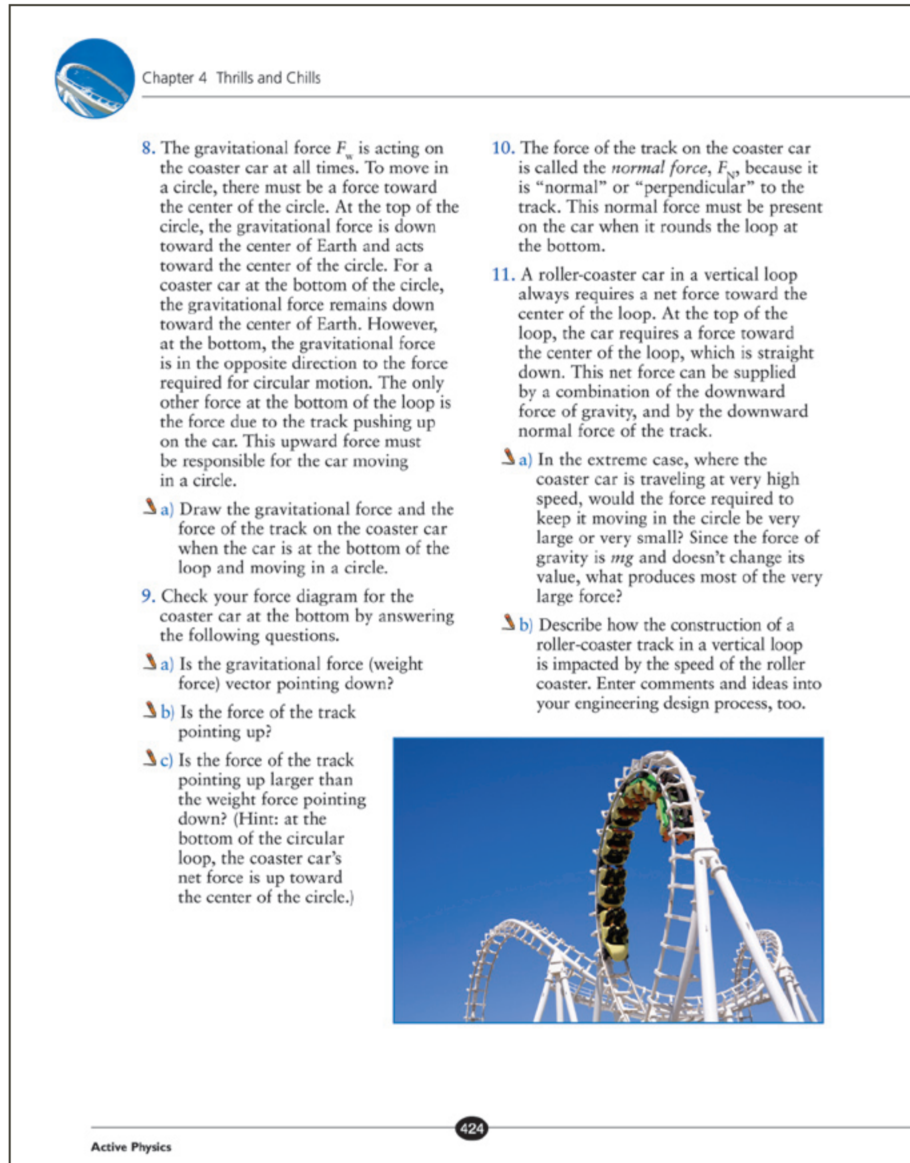
7.c)

The centripetal force should always point toward the center of the circle.

8.a)

The gravitational force is always downward. At the bottom of the loop, the normal force due to the track acting on the car will be upward. The student's diagram should look like the one below:




Chapter 4 Thrills and Chills

8. The gravitational force F_w is acting on the coaster car at all times. To move in a circle, there must be a force toward the center of the circle. At the top of the circle, the gravitational force is down toward the center of Earth and acts toward the center of the circle. For a coaster car at the bottom of the circle, the gravitational force remains down toward the center of Earth. However, at the bottom, the gravitational force is in the opposite direction to the force required for circular motion. The only other force at the bottom of the loop is the force due to the track pushing up on the car. This upward force must be responsible for the car moving in a circle.
   a) Draw the gravitational force and the force of the track on the coaster car when the car is at the bottom of the loop and moving in a circle.
9. Check your force diagram for the coaster car at the bottom by answering the following questions.
   a) Is the gravitational force (weight force) vector pointing down?
   b) Is the force of the track pointing up?
   c) Is the force of the track pointing up larger than the weight force pointing down? (Hint: at the bottom of the circular loop, the coaster car's net force is up toward the center of the circle.)
10. The force of the track on the coaster car is called the *normal force*, F_N, because it is "normal" or "perpendicular" to the track. This normal force must be present on the car when it rounds the loop at the bottom.
11. A roller-coaster car in a vertical loop always requires a net force toward the center of the loop. At the top of the loop, the car requires a force toward the center of the loop, which is straight down. This net force can be supplied by a combination of the downward force of gravity, and by the downward normal force of the track.
   a) In the extreme case, where the coaster car is traveling at very high speed, would the force required to keep it moving in the circle be very large or very small? Since the force of gravity is mg and doesn't change its value, what produces most of the very large force?
   b) Describe how the construction of a roller-coaster track in a vertical loop is impacted by the speed of the roller coaster. Enter comments and ideas into your engineering design process, too.

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Active Physics

9.a)

The gravitational force vector should be pointing down.

9.b)

The force of the track should be pointing up, opposite the gravitational force vector.

9.c)

The force of the track pointing up should be larger than the weight force acting down.

10.

The normal force at the bottom must be larger than the weight to provide a net force upward toward the center of the circle.

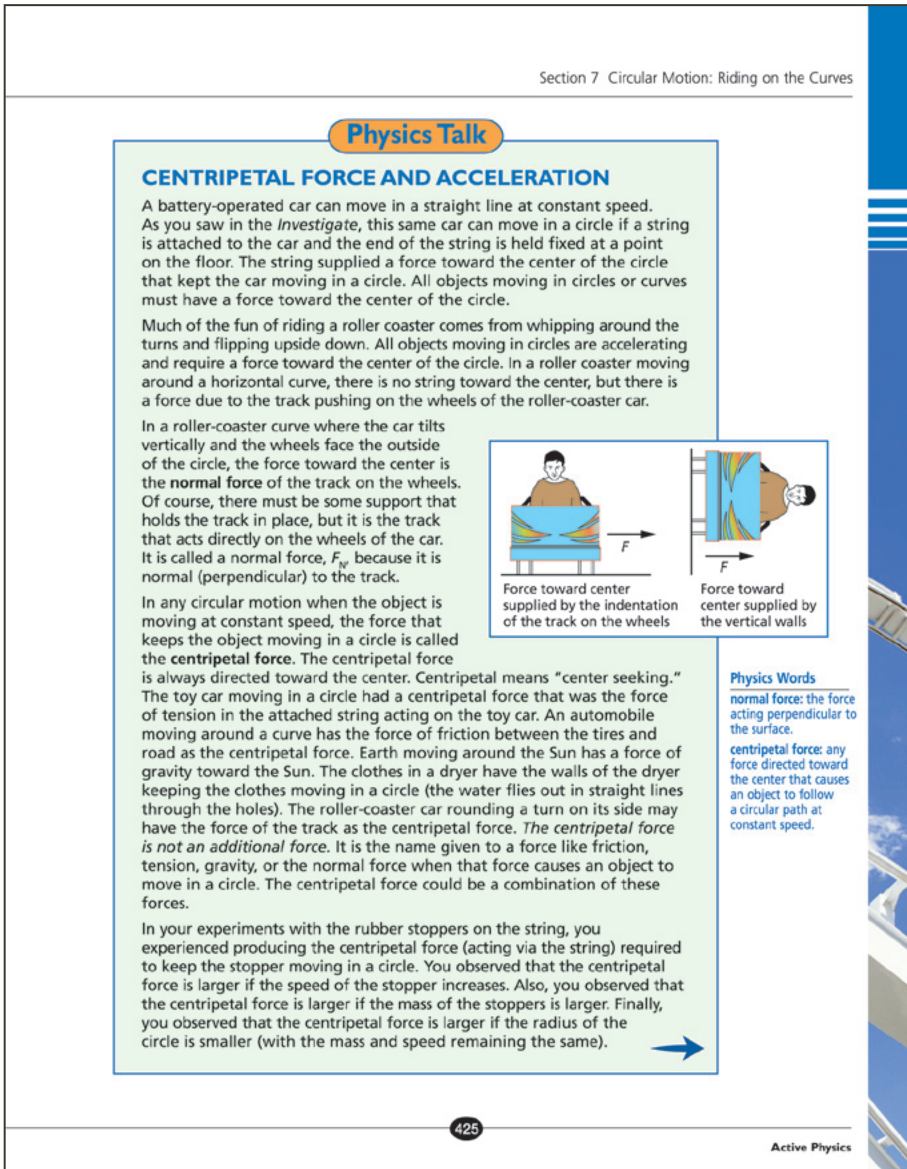
11.a)

The force required to keep the roller coaster moving in a circle would be very large. At the top of the loop for a roller coaster going very fast, most of the downward force is provided by the track.

For a person sitting in the roller-coaster car, the downward force would be provided by the seat. Depending upon the speed of the roller coaster, at the top of the loop, the riders may either feel that they weigh more than normal or less. In either case, the seat is pressing down on the rider to accelerate the rider toward the center of the circle.

11.b)

The roller coaster goes slower at the top of the loop than at the bottom, and at the top of the loop the weight contributes to the force toward the center. This means that the loop does not have to supply as much force at the top as it would at the bottom. Therefore, the track would not have to be constructed to withstand as much force at the top as at the bottom.


Section 7 Circular Motion: Riding on the Curves

Physics Talk

CENTRIPETAL FORCE AND ACCELERATION

A battery-operated car can move in a straight line at constant speed. As you saw in the *Investigate*, this same car can move in a circle if a string is attached to the car and the end of the string is held fixed at a point on the floor. The string supplied a force toward the center of the circle that kept the car moving in a circle. All objects moving in circles or curves must have a force toward the center of the circle.

Much of the fun of riding a roller coaster comes from whipping around the turns and flipping upside down. All objects moving in circles are accelerating and require a force toward the center of the circle. In a roller coaster moving around a horizontal curve, there is no string toward the center, but there is a force due to the track pushing on the wheels of the roller-coaster car.

In a roller-coaster curve where the car tilts vertically and the wheels face the outside of the circle, the force toward the center is the **normal force** of the track on the wheels. Of course, there must be some support that holds the track in place, but it is the track that acts directly on the wheels of the car. It is called a normal force, F_N, because it is normal (perpendicular) to the track.

Force toward center supplied by the indentation of the track on the wheels

Force toward center supplied by the vertical walls

In any circular motion when the object is moving at constant speed, the force that keeps the object moving in a circle is called the **centripetal force**. The centripetal force is always directed toward the center. Centripetal means "center seeking." The toy car moving in a circle had a centripetal force that was the force of tension in the attached string acting on the toy car. An automobile moving around a curve has the force of friction between the tires and road as the centripetal force. Earth moving around the Sun has a force of gravity toward the Sun. The clothes in a dryer have the walls of the dryer keeping the clothes moving in a circle (the water flies out in straight lines through the holes). The roller-coaster car rounding a turn on its side may have the force of the track as the centripetal force. *The centripetal force is not an additional force.* It is the name given to a force like friction, tension, gravity, or the normal force when that force causes an object to move in a circle. The centripetal force could be a combination of these forces.

In your experiments with the rubber stoppers on the string, you experienced producing the centripetal force (acting via the string) required to keep the stopper moving in a circle. You observed that the centripetal force is larger if the speed of the stopper increases. Also, you observed that the centripetal force is larger if the mass of the stoppers is larger. Finally, you observed that the centripetal force is larger if the radius of the circle is smaller (with the mass and speed remaining the same).

Physics Words

normal force: the force acting perpendicular to the surface.

centripetal force: any force directed toward the center that causes an object to follow a circular path at constant speed.

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Active Physics

Physics Talk

The *Physics Talk* explains how a centripetal force acts on objects traveling in a circular motion. Students learn how a car tied to a string (traveling in a circle) continues to go around a curve and the force that causes the circular motion is the same force that keeps the roller coaster on a curved path. When students are trying to understand the forces that keep an object moving in a circular path, the forces that are responsible for causing the centripetal force should be clearly emphasized. Ask students to refer to the diagrams they drew in the *Investigate*. Reiterate that the centripetal force is not an additional force. Friction, gravity, or the normal force could all be acting together to produce a centripetal force. When giving an example of centripetal force, start with a horizontal case, such as a roller-coaster car going around a horizontal curve because the centripetal force would be easier for the students to identify.

Ask students to write the definition of normal force in their *Active Physics* logs and explain why the normal force contributes to a centripetal force when the roller coaster is going round a curve. Point out that the direction of the centripetal force on the roller coaster keeps changing, even though it is always acting toward the center. Have students show the direction of centripetal force acting on roller coaster by drawing a sketch. Discuss the direction of different forces shown in the *Physics Talk*. Ask students why the net force at the top of the loop is smaller than the net force at the bottom of the loop. Point out that as the coaster car goes around a vertical loop, both the normal force and the centripetal force change in magnitude, while the weight force is constant. Consider highlighting the black force vector and drawing students' attention to how its length changes with the change in the position of the roller coaster. Discuss the changes in speed in a roller-coaster ride and how the diagrams demonstrate that a person's weight at the top and bottom of a ride remains the same, but the apparent weight would change because of the net upward or downward force acting on the roller coaster. Determine if students understand why the roller-coaster car cannot have an acceleration more than 4 *g*'s at the bottom of the loop in a roller coaster, and why roller coasters use a clothoid loop and not a circular loop.

CHAPTER 4

4-7a Blackline Master

All of these observations are summarized in the equation for centripetal force.

$$F_c = \frac{mv^2}{r}$$

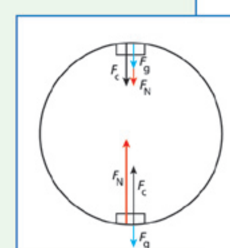
where F_c is the centripetal force,
m is the mass of the object,
v is the speed, and
r is the radius of the circle.

When you are riding in the roller-coaster car and the car goes around a curve, you are also accelerating. You know from Newton's second law that if there is a net force, an object must be accelerating. In the case of circular motion, this force is toward the center of the circle. Therefore, the acceleration must be toward the center of the circle. This is called **centripetal acceleration**. You can feel the contact forces between you and your seat and between you and the side of the coaster car that cause you to accelerate. It is the acceleration and the related contact forces that give you the thrill of riding on the roller coaster.

Physics Words

centripetal acceleration: the acceleration directed toward the center of a circle experienced by an object traveling in a circular path at constant speed.

When the roller coaster is in a vertical loop, the direction of the centripetal force is always changing to ensure that the centripetal force vector always points toward the center of the circular track. Pay particular attention to how this is phrased. Although the centripetal force is always toward the center, the direction is always changing since in the circle, the centripetal force may be toward the left or the right or up, but still point toward the center.



In the vertical loop, this centripetal force can be either the gravitational force, the normal force of the track on the coaster car or a combination of the two. When it is a combination of the two, you must add the forces as vectors. At the bottom of the circle, the normal force (red vector) points toward the center of the circle (upward) while the gravitational force (blue vector) points downward. The vector sum of these two forces must be toward the center of the circle. You can therefore conclude that the normal force is larger than the gravitational force. The normal force corresponds to your apparent weight, as it did in the investigation in the previous section. This is why you feel as if you weigh more at the bottom of the loop of the roller coaster.

At the top of the loop-the-loop, the gravitational force (blue vector) and the normal force (red vector) both act downward, toward the center of the loop. The sum of these two vectors provides the required centripetal force.

Section 7 Circular Motion: Riding on the Curves

How much of the normal force required to keep the car moving in a circle will depend on the mass and speed of the car.

The black force vectors show the net centripetal force required to keep the car moving in a circular path at the top and bottom of the loop. Notice that the length of the black vectors is different since the coaster car has a larger speed at the bottom of the loop. The directions are different because the centripetal force must always point toward the center of the circle.

The blue force vector represents the force of gravity or weight of the coaster cars. Both weight vectors are identical because the weights of the roller-coaster car are identical at the top and bottom.

The red vector represents the normal force of the track on the car. The sum of the normal force plus the weight must be equal to the required net (centripetal) force. At the top, the normal force is smaller since the weight contributes to the centripetal force. If the speed decreases, the required net centripetal force would be less and less. There comes a point where the gravitational force (weight) would be all that is required to keep the coaster car moving in a circle. In that case, the normal force is zero. In this special situation, where no normal force is required, you could actually have a small gap at the top between the track and the car. The car would continue to move in a circle. If the car were to slow down more, the car would leave the track completely and no longer travel in a circle. That is something you don't want to have happen with your roller coaster!

At the bottom of the roller-coaster loop, the car would need a normal force of the track on the car greater than the weight since the weight is downward and the car needs a net upward force (toward the center of the circle).

This is summarized in the following tables.

Fast-moving Roller Coaster

	Required centripetal force	Force of gravity (weight)	Normal force (the force of the track on the car)
at the top of the loop	5000 N	1000 N	4000 N
at the bottom of the loop	9000 N	1000 N	10,000 N

Slow-moving Roller Coaster

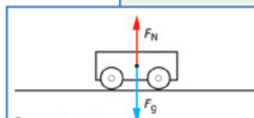
	Required centripetal force	Force of gravity (weight)	Normal force (the force of the track on the car)
at the top of the loop	2100 N	1000 N	1100 N
at the bottom of the loop	6100 N	1000 N	7100 N

4-7b Blackline Master

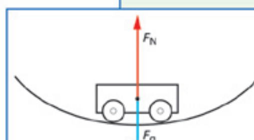
Apparent Weight and the Roller-Coaster Ride

You discovered earlier that an elevator ride could give you a sense of weight changes during accelerations. In the roller coaster loop-the-loop, the passenger will also experience changes in apparent weight. The normal force on the passenger due to the seat is an indication of the apparent weight, as it was in the elevator. A passenger on the roller coaster feels lighter at the top of the loop because the contact force between the passenger and the seat is smaller. This is similar to the elevator because in both cases you feel lighter because acceleration is directed downward. A passenger on the roller coaster feels heavier at the bottom of the loop. Once again, this is similar to the elevator because in both cases you feel heavier because acceleration is upward.

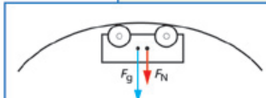
In the slow-moving roller coaster in the chart, the apparent weight (normal force) at the top of the loop may only be 1100 N, while the apparent weight (normal force) at the bottom of the loop may be 7100 N.



Forces on you:
The force of the seat on you = 500 N (apparent weight).
The force of gravity on you is 500 N (weight).



Forces on you:
The force of the seat on you = 1000 N.
The force of gravity on you is 500 N (weight).
You feel as if you weigh 1000 N (apparent weight).



Forces on you:
The force of the seat on you = 100 N.
The force of gravity on you is 500 N (weight).
You feel as if you weigh 100 N (apparent weight).

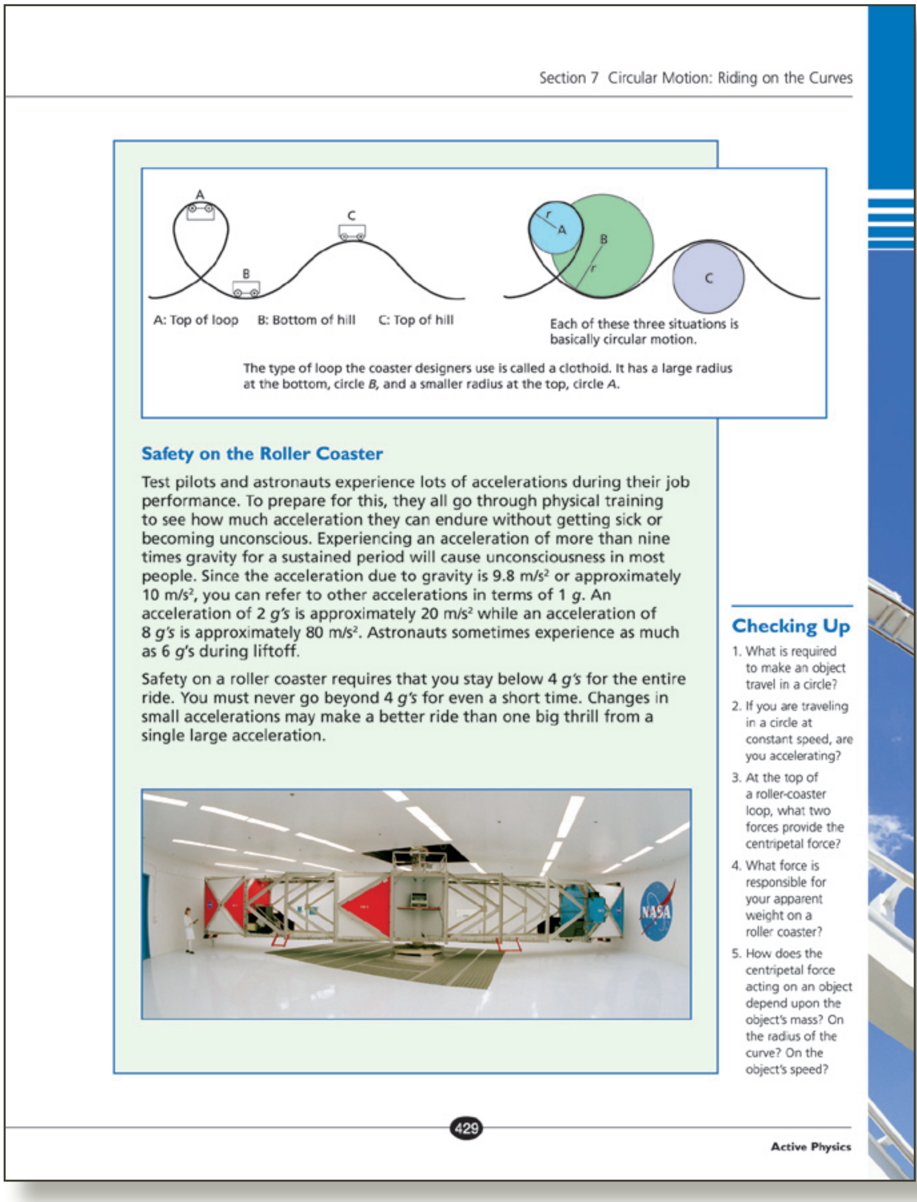
Three locations can be used to summarize the discussion on forces and weight. On a level track with the coaster car moving at constant speed, the sum of the forces must be zero.

At the bottom of the loop, there must be a net force up toward the center of the circle to keep you moving in a circular path.

At the top of the loop, there must be a net force down toward the center of the circle to keep you moving in a circular path.

Roller coasters do not use loops that are circular. They use a clothoid loop (it has a big radius at the bottom and a small radius at the top). In this way, at the top of the loop the roller coaster is moving in a small circle (smaller radius), while at the bottom it is moving in a larger circle (larger radius). This kind of loop is used to ensure that the roller-coaster car can make the turn at the top but not have an acceleration at the bottom of the loop that exceeds about 4 *g's*. The speed at the bottom is determined by the height of the loop as you saw in *Section 2*. If *r* is larger at the bottom of the loop, then the acceleration experienced by the riders at the bottom of the loop will be smaller, keeping it within a thrilling but safe range.

CHAPTER 4


Section 7 Circular Motion: Riding on the Curves

The type of loop the coaster designers use is called a clothoid. It has a large radius at the bottom, circle *B*, and a smaller radius at the top, circle *A*.

Safety on the Roller Coaster

Test pilots and astronauts experience lots of accelerations during their job performance. To prepare for this, they all go through physical training to see how much acceleration they can endure without getting sick or becoming unconscious. Experiencing an acceleration of more than nine times gravity for a sustained period will cause unconsciousness in most people. Since the acceleration due to gravity is 9.8 m/s^2 or approximately 10 m/s^2, you can refer to other accelerations in terms of 1 *g*. An acceleration of 2 *g's* is approximately 20 m/s^2 while an acceleration of 8 *g's* is approximately 80 m/s^2. Astronauts sometimes experience as much as 6 *g's* during liftoff.

Safety on a roller coaster requires that you stay below 4 *g's* for the entire ride. You must never go beyond 4 *g's* for even a short time. Changes in small accelerations may make a better ride than one big thrill from a single large acceleration.

Checking Up

1. What is required to make an object travel in a circle?
2. If you are traveling in a circle at constant speed, are you accelerating?
3. At the top of a roller-coaster loop, what two forces provide the centripetal force?
4. What force is responsible for your apparent weight on a roller coaster?
5. How does the centripetal force acting on an object depend upon the object's mass? On the radius of the curve? On the object's speed?

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Active Physics

Checking Up

1.

A net centripetal force is required to make the object travel in a circle.

2.

Yes, you are accelerating because the direction of your motion is changing.

3.

The normal force and the force of gravity provide the centripetal force acting on an object at the top of a roller-coaster loop.

4.

The normal force is responsible for your apparent weight on a roller coaster.

5.

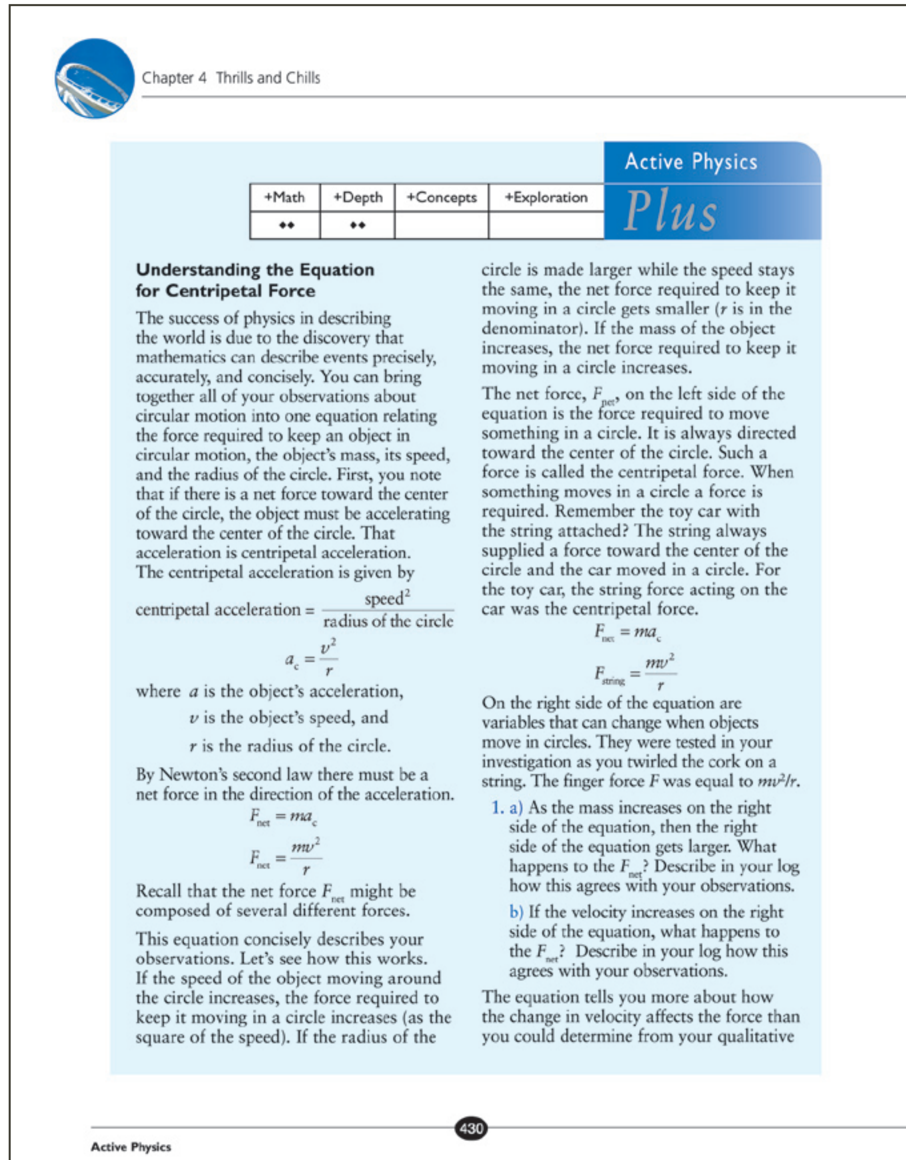
The centripetal force acting on an object depends directly on the object's mass. The larger the mass, the greater the centripetal force required to keep the object moving in a circle. The centripetal force varies inversely with the radius of the curve being traveled. The larger the curve, the less centripetal force is required. The centripetal force increases with the square of the object's velocity. For a fixed curve radius and mass, doubling the velocity would require four times the centripetal force.

Active Physics Plus

This section emphasizes how all observations about circular motion relating the centripetal force to the mass, speed, and radius of the curve can fit into one equation. Students learn that centripetal acceleration is given by the following equation:

$a_c = v^2/r$; (a is the object's acceleration, v the object's speed, and r the radius of the circle).

This equation is further extended to accommodate Newton's second law. Therefore, $F_{net} = mv^2/r$, explains why increasing the radius of the curve decreases the centripetal force and increasing the speed and/or the mass increases the force required to keep the object moving in a circle.



Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
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Understanding the Equation for Centripetal Force

The success of physics in describing the world is due to the discovery that mathematics can describe events precisely, accurately, and concisely. You can bring together all of your observations about circular motion into one equation relating the force required to keep an object in circular motion, the object's mass, its speed, and the radius of the circle. First, you note that if there is a net force toward the center of the circle, the object must be accelerating toward the center of the circle. That acceleration is centripetal acceleration. The centripetal acceleration is given by

$$\text{centripetal acceleration} = \frac{\text{speed}^2}{\text{radius of the circle}}$$

$$a_c = \frac{v^2}{r}$$

where a is the object's acceleration,
v is the object's speed, and
r is the radius of the circle.

By Newton's second law there must be a net force in the direction of the acceleration.

$$F_{net} = ma_c$$

$$F_{net} = \frac{mv^2}{r}$$

Recall that the net force F_{net} might be composed of several different forces.

This equation concisely describes your observations. Let's see how this works. If the speed of the object moving around the circle increases, the force required to keep it moving in a circle increases (as the square of the speed). If the radius of the circle is made larger while the speed stays the same, the net force required to keep it moving in a circle gets smaller (r is in the denominator). If the mass of the object increases, the net force required to keep it moving in a circle increases.

The net force, F_{net}, on the left side of the equation is the force required to move something in a circle. It is always directed toward the center of the circle. Such a force is called the centripetal force. When something moves in a circle a force is required. Remember the toy car with the string attached? The string always supplied a force toward the center of the circle and the car moved in a circle. For the toy car, the string force acting on the car was the centripetal force.

$$F_{net} = ma_c$$

$$F_{string} = \frac{mv^2}{r}$$

On the right side of the equation are variables that can change when objects move in circles. They were tested in your investigation as you twirled the cork on a string. The finger force F was equal to mv^2/r.

1. a) As the mass increases on the right side of the equation, then the right side of the equation gets larger. What happens to the F_{net}? Describe in your log how this agrees with your observations.

   b) If the velocity increases on the right side of the equation, what happens to the F_{net}? Describe in your log how this agrees with your observations.

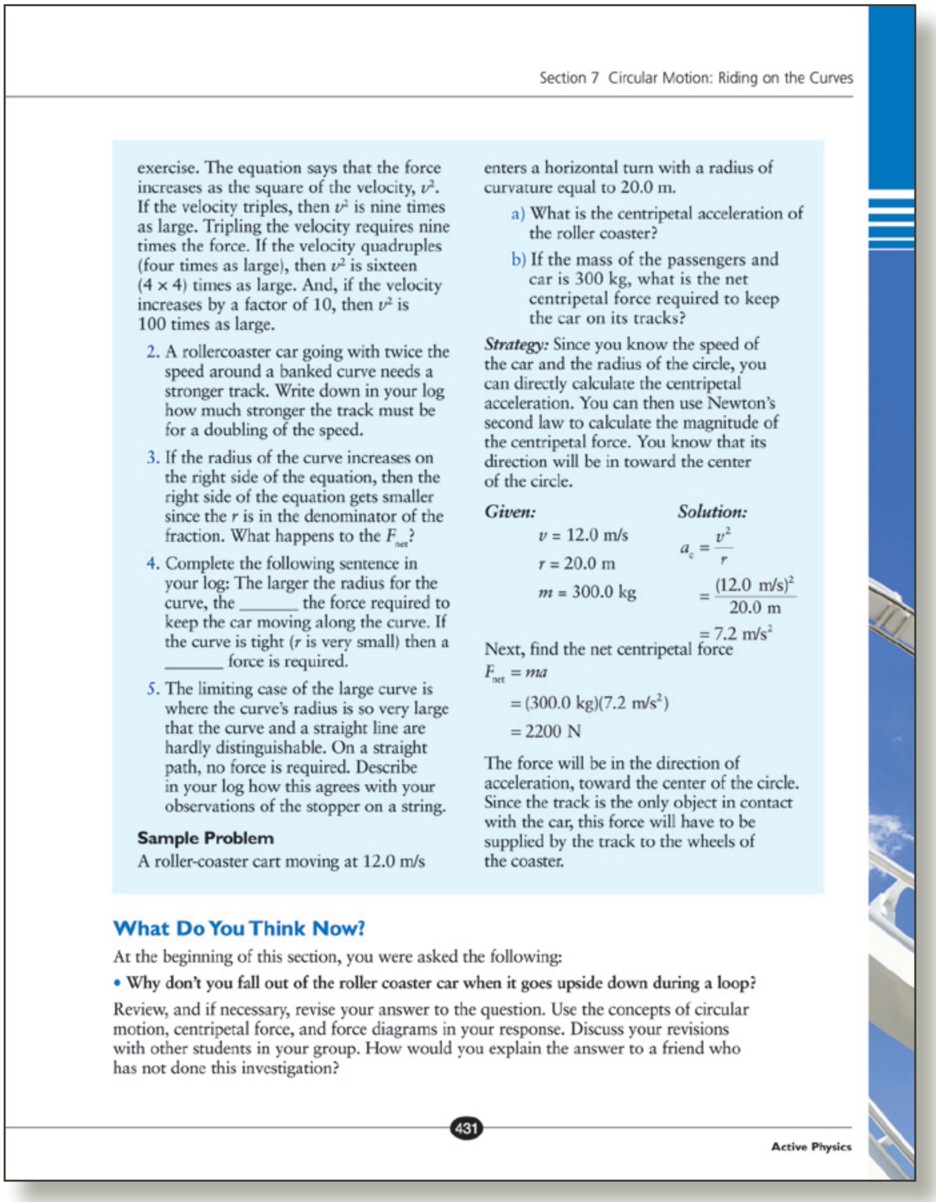
The equation tells you more about how the change in velocity affects the force than you could determine from your qualitative

1.a)

As the mass increases on the right side of the equation, the net force must also increase on the left side to keep the two sides equal. When more stoppers were added to the string that was being twirled in the horizontal circle, the force needed to keep the stopper going in the circle at constant speed increased.

1.b)

As the velocity on the right side of the equation increases, the net force on the left-hand side has to increase. As the stoppers were twirled faster in a horizontal circle of constant radius, the person twirling them had to increase the force exerted to keep them going in a circle at the increased speed.



exercise. The equation says that the force increases as the square of the velocity, v^2. If the velocity triples, then v^2 is nine times as large. Tripling the velocity requires nine times the force. If the velocity quadruples (four times as large), then v^2 is sixteen (4×4) times as large. And, if the velocity increases by a factor of 10, then v^2 is 100 times as large.

2. A rollercoaster car going with twice the speed around a banked curve needs a stronger track. Write down in your log how much stronger the track must be for a doubling of the speed.
3. If the radius of the curve increases on the right side of the equation, then the right side of the equation gets smaller since the r is in the denominator of the fraction. What happens to the F_{net}?
4. Complete the following sentence in your log: The larger the radius for the curve, the _______ the force required to keep the car moving along the curve. If the curve is tight (r is very small) then a _______ force is required.
5. The limiting case of the large curve is where the curve's radius is so very large that the curve and a straight line are hardly distinguishable. On a straight path, no force is required. Describe in your log how this agrees with your observations of the stopper on a string.

Sample Problem

A roller-coaster cart moving at 12.0 m/s enters a horizontal turn with a radius of curvature equal to 20.0 m.

a) What is the centripetal acceleration of the roller coaster?

b) If the mass of the passengers and car is 300 kg, what is the net centripetal force required to keep the car on its tracks?

Strategy: Since you know the speed of the car and the radius of the circle, you can directly calculate the centripetal acceleration. You can then use Newton's second law to calculate the magnitude of the centripetal force. You know that its direction will be in toward the center of the circle.

Given:

$v = 12.0$ m/s

$r = 20.0$ m

$m = 300.0$ kg

Solution:

$$a_c = \frac{v^2}{r} = \frac{(12.0 \text{ m/s})^2}{20.0 \text{ m}} = 7.2 \text{ m/s}^2$$

Next, find the net centripetal force

$$F_{net} = ma = (300.0 \text{ kg})(7.2 \text{ m/s}^2) = 2200 \text{ N}$$

The force will be in the direction of acceleration, toward the center of the circle. Since the track is the only object in contact with the car, this force will have to be supplied by the track to the wheels of the coaster.

What Do You Think Now?

At the beginning of this section, you were asked the following:

- **Why don't you fall out of the roller coaster car when it goes upside down during a loop?**

Review, and if necessary, revise your answer to the question. Use the concepts of circular motion, centripetal force, and force diagrams in your response. Discuss your revisions with other students in your group. How would you explain the answer to a friend who has not done this investigation?

2.

Doubling the speed requires the track to be four times stronger, since the centripetal force varies with the velocity squared.

3.

Increasing the radius, while the mass and the speed are kept constant, decreases the force needed to keep the roller-coaster car going in a circle.

4.

"The larger the radius for the curve, the smaller the force required to keep the car moving along the curve. If the curve is tight (r is very small), then a larger force is required."

5.

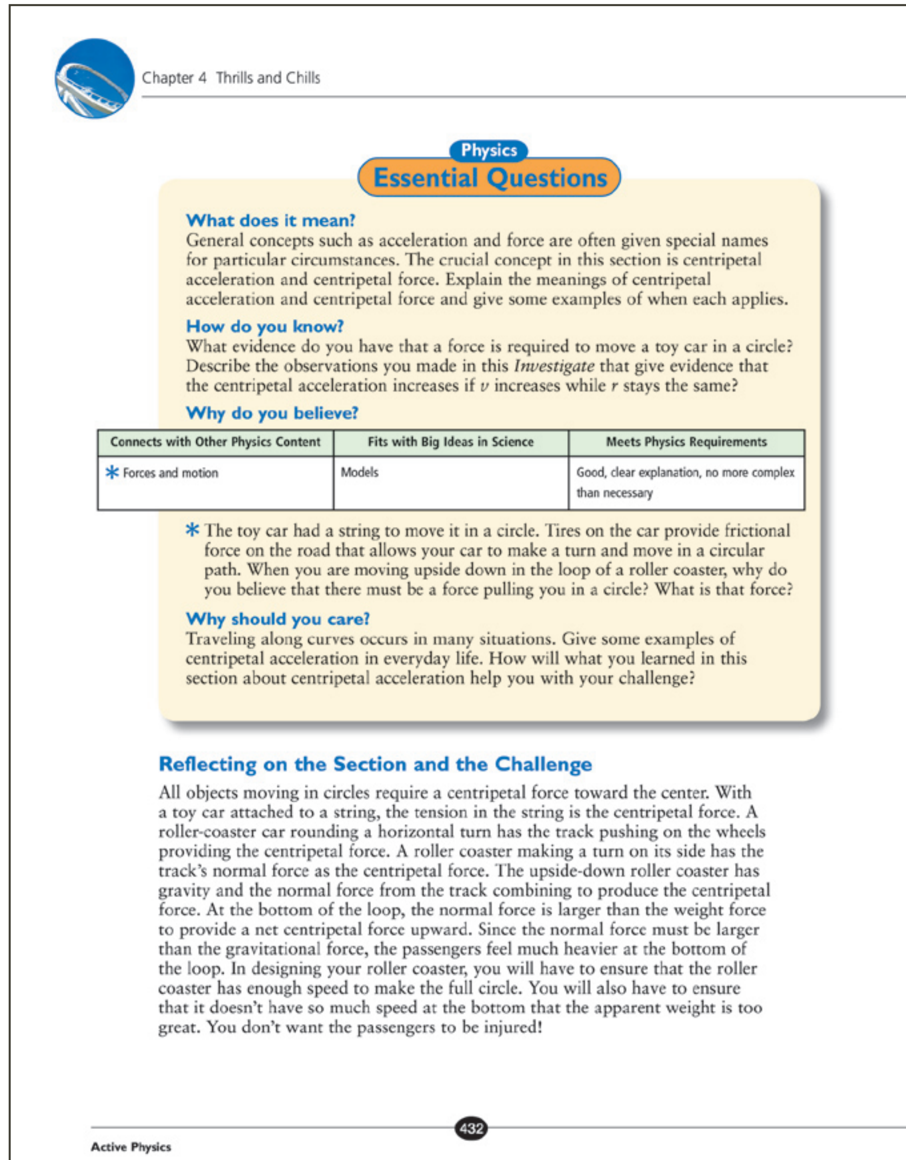
A circle with an extremely large radius would be almost like a straight line and require a very small (or zero) force to keep the car going along the path. This agrees with what happened to the battery-powered car when the string was released. The car went straight (or in a circle with an infinite radius).

What Do You Think Now?

Students should be able to correctly answer this question based on their experiments in *Investigate* and the *Physics Talk* that explained the physics of centripetal force. However, they might still have doubts that need further clarification. Share *A Physicist's Response* with your students and ask students to review their answers. At this stage, you must emphasize that their answers must incorporate the concepts of centripetal force and how it relates to radius of the curve, mass of the roller-coaster car, and speed of the ride. Students must share their answers with other members in their group and reevaluate their own responses. This is also a good time to review earlier responses to the *What Do You Think?* section and ask students to reflect on how their understanding of circular motion has evolved.

Reflecting on the Section and the Challenge

Students should now reflect on how centripetal force keeps a roller coaster moving around horizontal and vertical loops around a circular path. Ask students to explain the forces that contribute to a centripetal force during a roller coaster's ascent and descent in circular motion. Determine if students understand why normal force may be smaller than the force of gravity when the roller-coaster car is upside-down, and why the weight force is smaller than the normal force when coaster car is at the bottom of a loop. Point out that while modifying the design of their roller coaster, students should remember that the roller coaster must not exceed forces of 4 *g*'s during its turns. Ask them how they will be able to determine if the 4-*g* limit on acceleration is being exceeded. Consider asking students to sketch a preliminary design for a roller coaster that incorporates the essential aspects of centripetal force required to keep the roller coaster safe and at the same time thrilling for its passengers.



Chapter 4 Thrills and Chills

Physics
Essential Questions

What does it mean?
General concepts such as acceleration and force are often given special names for particular circumstances. The crucial concept in this section is centripetal acceleration and centripetal force. Explain the meanings of centripetal acceleration and centripetal force and give some examples of when each applies.

How do you know?
What evidence do you have that a force is required to move a toy car in a circle? Describe the observations you made in this *Investigate* that give evidence that the centripetal acceleration increases if v increases while r stays the same?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
* Forces and motion	Models	Good, clear explanation, no more complex than necessary

* The toy car had a string to move it in a circle. Tires on the car provide frictional force on the road that allows your car to make a turn and move in a circular path. When you are moving upside down in the loop of a roller coaster, why do you believe that there must be a force pulling you in a circle? What is that force?

Why should you care?
Traveling along curves occurs in many situations. Give some examples of centripetal acceleration in everyday life. How will what you learned in this section about centripetal acceleration help you with your challenge?

Reflecting on the Section and the Challenge

All objects moving in circles require a centripetal force toward the center. With a toy car attached to a string, the tension in the string is the centripetal force. A roller-coaster car rounding a horizontal turn has the track pushing on the wheels providing the centripetal force. A roller coaster making a turn on its side has the track's normal force as the centripetal force. The upside-down roller coaster has gravity and the normal force from the track combining to produce the centripetal force. At the bottom of the loop, the normal force is larger than the weight force to provide a net centripetal force upward. Since the normal force must be larger than the gravitational force, the passengers feel much heavier at the bottom of the loop. In designing your roller coaster, you will have to ensure that the roller coaster has enough speed to make the full circle. You will also have to ensure that it doesn't have so much speed at the bottom that the apparent weight is too great. You don't want the passengers to be injured!

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Active Physics

Physics Essential Questions

What does it mean?

Objects moving in circles are accelerating and therefore must have a net force. This net force for circular motion is toward the center of the circle. Regardless of what caused this net force, it is referred to as a centripetal force. The acceleration toward the center is referred to as a centripetal acceleration.

How do you know?

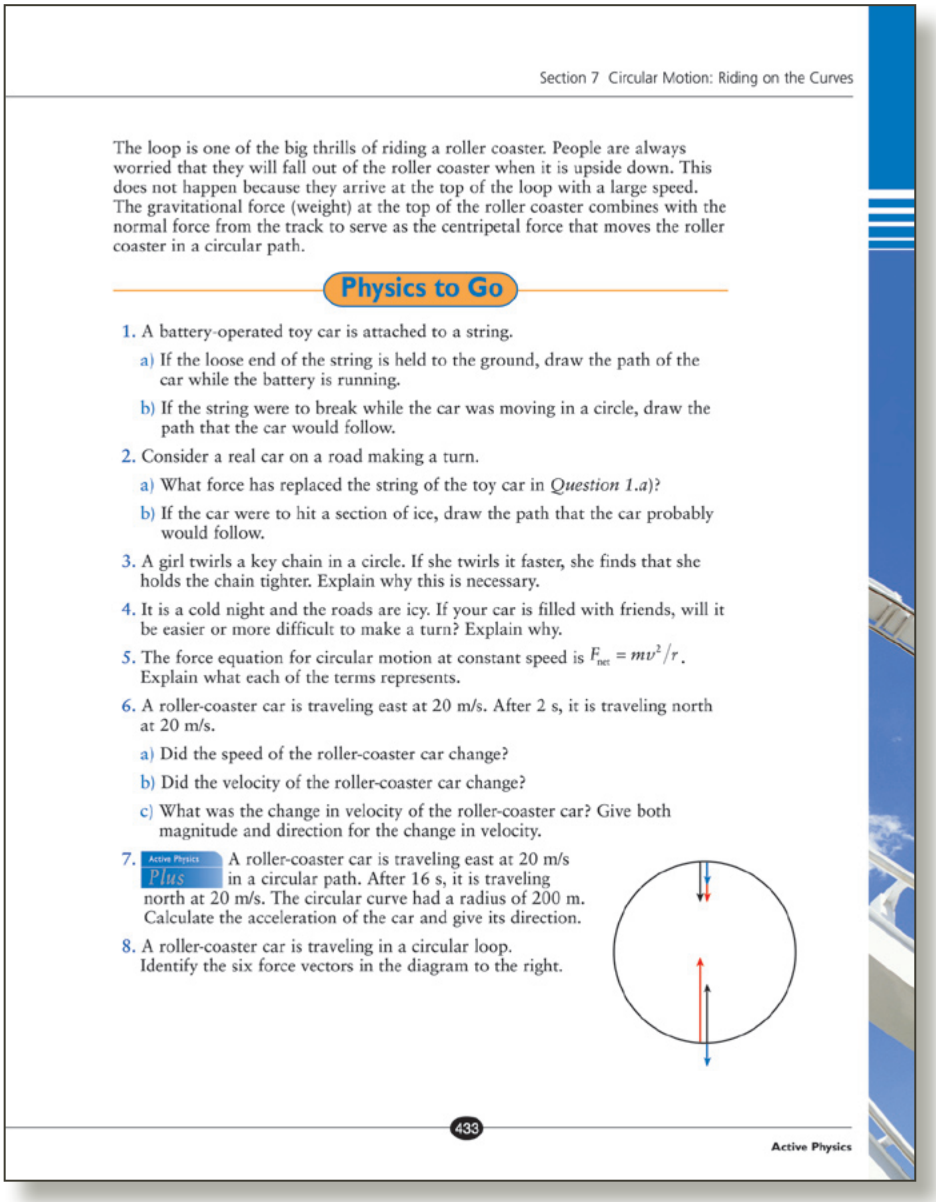
You observed that a car attached to a string would move in a circle if the end of the string were held fixed. If the string broke, the car traveled in a straight line.

Why do you believe?

Since your direction is changing, there is an acceleration, the result of a net force. At the top of the roller coaster, this net force must be vertically down and is the sum of the force of gravity and the normal force of the seat pushing down.

Why should you care?

When you make a sharp turn in your car, there is a centripetal acceleration. When you place food in a mixture, there is a centripetal force. When clothes spin in a dryer, there is a centripetal force. Recognizing that the net force is a centripetal force toward the center of the circle will help you in determining the forces at all points on the roller coaster.



The loop is one of the big thrills of riding a roller coaster. People are always worried that they will fall out of the roller coaster when it is upside down. This does not happen because they arrive at the top of the loop with a large speed. The gravitational force (weight) at the top of the roller coaster combines with the normal force from the track to serve as the centripetal force that moves the roller coaster in a circular path.

Physics to Go

1. A battery-operated toy car is attached to a string.
   a) If the loose end of the string is held to the ground, draw the path of the car while the battery is running.
   b) If the string were to break while the car was moving in a circle, draw the path that the car would follow.
2. Consider a real car on a road making a turn.
   a) What force has replaced the string of the toy car in *Question 1.a)*?
   b) If the car were to hit a section of ice, draw the path that the car probably would follow.
3. A girl twirls a key chain in a circle. If she twirls it faster, she finds that she holds the chain tighter. Explain why this is necessary.
4. It is a cold night and the roads are icy. If your car is filled with friends, will it be easier or more difficult to make a turn? Explain why.
5. The force equation for circular motion at constant speed is $F_{net} = mv^2/r$. Explain what each of the terms represents.
6. A roller-coaster car is traveling east at 20 m/s. After 2 s, it is traveling north at 20 m/s.
   a) Did the speed of the roller-coaster car change?
   b) Did the velocity of the roller-coaster car change?
   c) What was the change in velocity of the roller-coaster car? Give both magnitude and direction for the change in velocity.
7. Active Physics Plus A roller-coaster car is traveling east at 20 m/s in a circular path. After 16 s, it is traveling north at 20 m/s. The circular curve had a radius of 200 m. Calculate the acceleration of the car and give its direction.
8. A roller-coaster car is traveling in a circular loop. Identify the six force vectors in the diagram to the right.

Physics to Go

1.a)

The car will travel in a circular path.

1.b)

If the string were to break, the cart will travel in a straight line tangent to the circle at the location of the car when the string broke. Refer to the diagram to the right:

2.a)

Friction between the road and the car has replaced the string of the toy car.

2.b)

The car would travel in a straight line, tangent to the circle at the point where the car hit the ice. Refer to diagram for *Question 1.b)*.

3.

An object moving in a circle at a greater speed requires a larger centripetal force. That force is supplied by the grip of the fingers. To increase the grip, the girl squeezes her fingers more tightly.

4.

It will be more difficult to make the turn because the mass has increased. A larger mass requires a larger centripetal force. (A complication: In many situations, the friction force increases as the weight of the car increases. If that is true, then adding more weight doesn't make a significant difference in making the turn more difficult.)

5.

F_{net} is the centripetal force required for circular motion, m is the mass of the object moving in a circle, v is the speed of the object, r is the radius of the circular path in which the object is moving.

6.a)

No, the speed is still 20 m/s.

6.b)

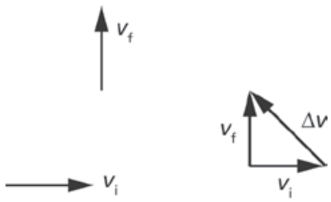
Yes, the velocity changed because the direction changed.

6.c)

By constructing a vector diagram, you can see that the change in velocity is equal to 28 m/s northwest. (It increased its speed

CHAPTER 4

toward north by 20 m/s and decreased its speed toward east by 20 m/s, which when added using the Pythagorean theorem produces a change of 28 m/s NW.)



Given:

$v_i = 20$ m/s; $v_f = 20$ m/s; $\Delta t = 2$ s

$$|\Delta v| = \sqrt{v_i^2 + v_f^2} =$$
$$\sqrt{(20\text{ m/s})^2 + (20\text{ m/s})^2} = 28\text{ m/s}$$

7. Active Physics *Plus*

Use the expression for centripetal acceleration, $a = v^2/r = (20\text{ m/s})^2/(200\text{ m}) = 2.0\text{ m/s}^2$ with the direction of the acceleration being toward the center of the circle. Note that this is a constantly changing direction, so the acceleration of the roller-coaster car is constant in magnitude but not direction.

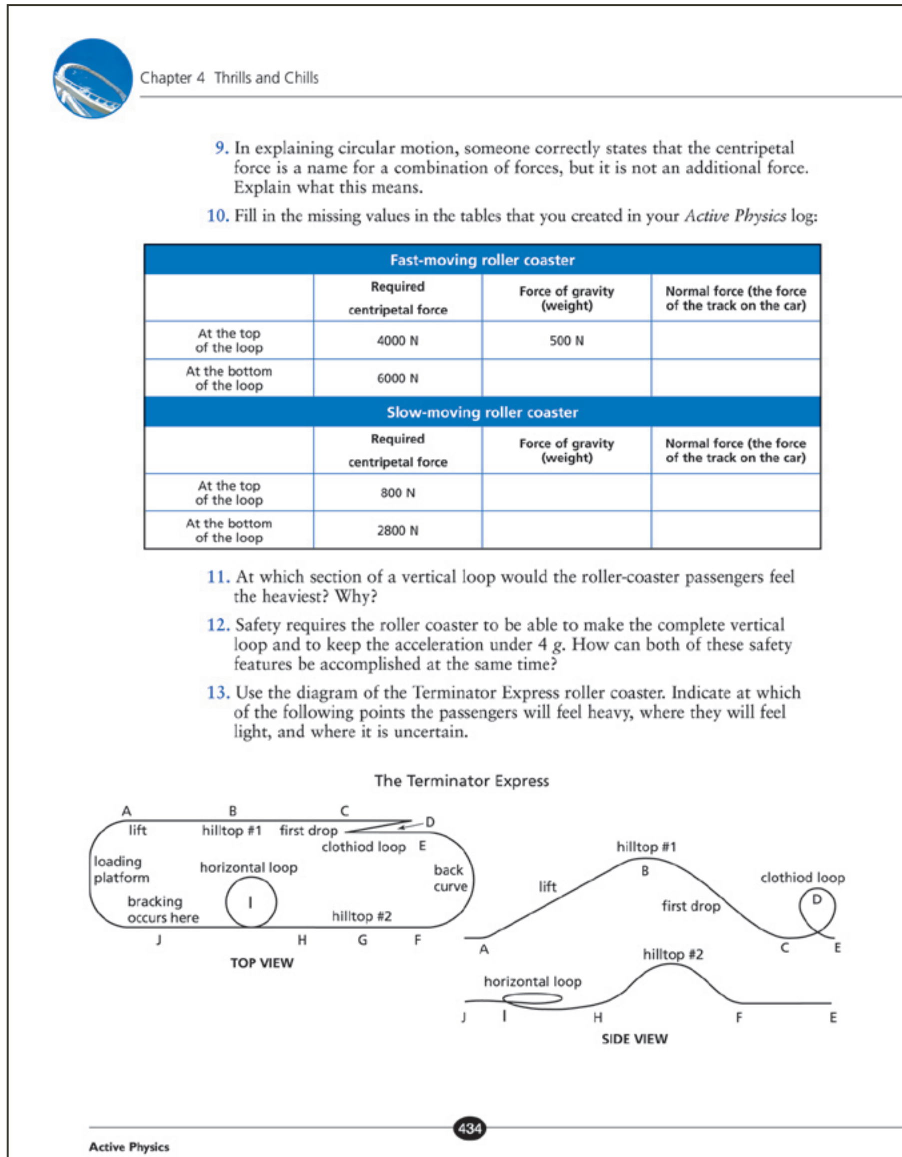
8.

At the top:

The first downward force vector (blue) is the force of gravity (the weight of the car); the other downward force vector (red) is the normal force of the track pushing down; the largest downward force vector (black) is the vector sum of the weight and the normal force. This vector sum of the two forces is the net force, the centripetal force.

At the bottom:

The downward force vector (blue) is the force of gravity (the weight of the car); the large upward force vector (red) is the normal force of the track pushing up; the other upward force vector (black) is the vector sum of the weight and the normal force. This vector sum of the two forces is the net force, which is the centripetal force.



9. In explaining circular motion, someone correctly states that the centripetal force is a name for a combination of forces, but it is not an additional force. Explain what this means.

10. Fill in the missing values in the tables that you created in your *Active Physics* log:

Fast-moving roller coaster			
	Required centripetal force	Force of gravity (weight)	Normal force (the force of the track on the car)
At the top of the loop	4000 N	500 N	
At the bottom of the loop	6000 N		
Slow-moving roller coaster			
	Required centripetal force	Force of gravity (weight)	Normal force (the force of the track on the car)
At the top of the loop	800 N		
At the bottom of the loop	2800 N		

11. At which section of a vertical loop would the roller-coaster passengers feel the heaviest? Why?

12. Safety requires the roller coaster to be able to make the complete vertical loop and to keep the acceleration under 4 *g*. How can both of these safety features be accomplished at the same time?

13. Use the diagram of the Terminator Express roller coaster. Indicate at which of the following points the passengers will feel heavy, where they will feel light, and where it is uncertain.

The Terminator Express

9.

The force that moves something in a circle may be the tension in a string, weight, friction, a normal force or other kinds of forces. Whichever force (or combination of forces) keeps the object moving in a circle is called the centripetal force.

10.

Fast–moving roller coaster			
	Required centripetal force	Force of gravity (Weight)	Normal force (the force of the car on the track)
At the top of the loop	4000 N	500 N	3500 N
At the bottom of the loop	6000 N	500 N	6500 N

Slow–moving roller coaster			
	Required centripetal force	Force of gravity (Weight)	Normal force (the force of the car on the track)
At the top of the loop	800 N	500 N	300 N
At the bottom of the loop	2800 N	500 N	3300 N

11.

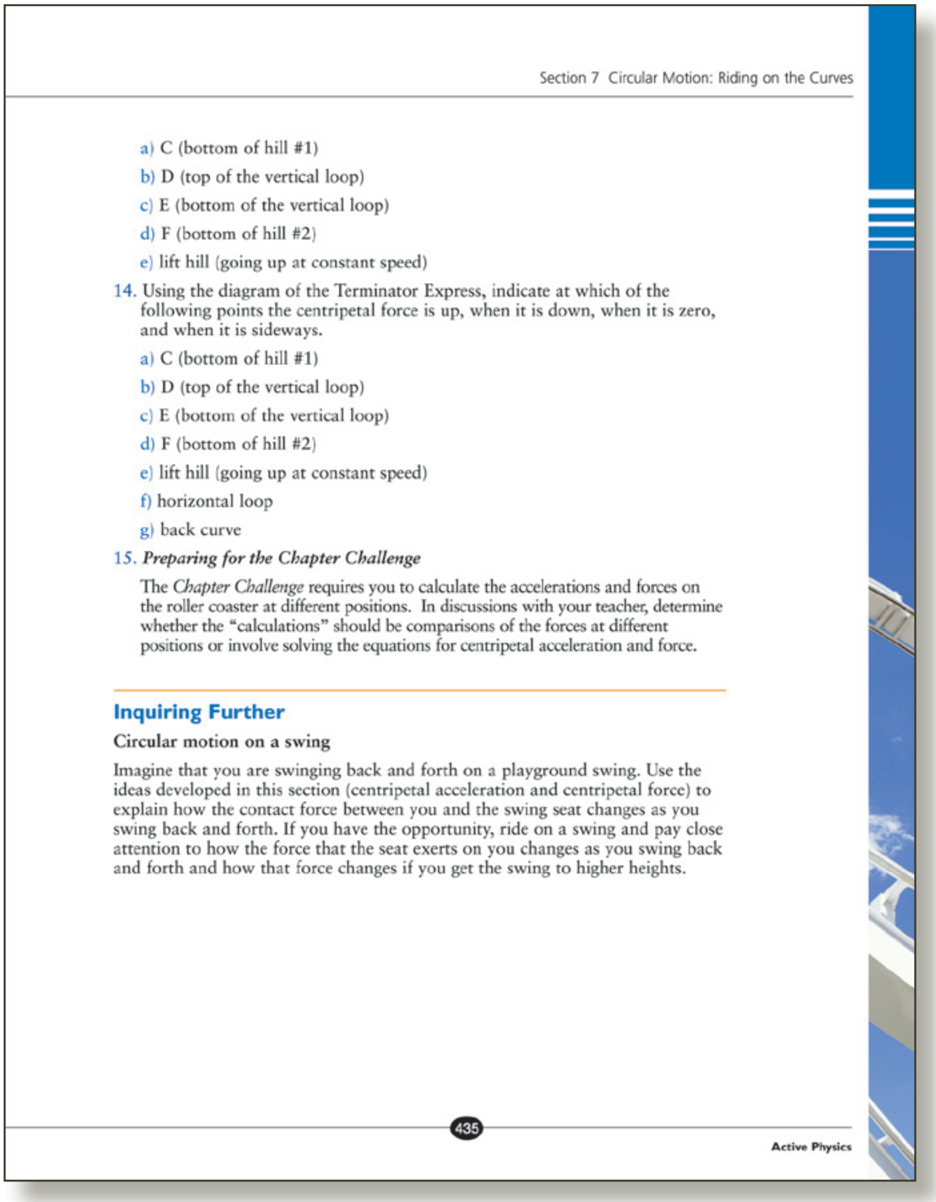
At the bottom of the roller coaster loop, the normal force (supplied by the surface you are seated on) is larger and you would feel heaviest because you interpret that contact force between you and the seat as your "weight."

12.

The speed and radius of the curve must be adjusted. Using a curve with a smaller radius at the top allows the roller coaster to navigate the loop with slower speed at the top.

NOTES

NOTES



a) C (bottom of hill #1)

b) D (top of the vertical loop)

c) E (bottom of the vertical loop)

d) F (bottom of hill #2)

e) lift hill (going up at constant speed)

14. Using the diagram of the Terminator Express, indicate at which of the following points the centripetal force is up, when it is down, when it is zero, and when it is sideways.

a) C (bottom of hill #1)

b) D (top of the vertical loop)

c) E (bottom of the vertical loop)

d) F (bottom of hill #2)

e) lift hill (going up at constant speed)

f) horizontal loop

g) back curve

15. ***Preparing for the Chapter Challenge***

The *Chapter Challenge* requires you to calculate the accelerations and forces on the roller coaster at different positions. In discussions with your teacher, determine whether the "calculations" should be comparisons of the forces at different positions or involve solving the equations for centripetal acceleration and force.

Inquiring Further

Circular motion on a swing

Imagine that you are swinging back and forth on a playground swing. Use the ideas developed in this section (centripetal acceleration and centripetal force) to explain how the contact force between you and the swing seat changes as you swing back and forth. If you have the opportunity, ride on a swing and pay close attention to how the force that the seat exerts on you changes as you swing back and forth and how that force changes if you get the swing to higher heights.

13.a)

C (bottom of hill #1): heavy

13.b)

D (top of the vertical loop): light

13.c)

E (bottom of the vertical loop): heavy

13.d)

F (bottom of hill #2): heavy

13.e)

lift hill (going up at constant speed): normal

14.a)

C (bottom of hill #1): up

14.b)

D (top of the vertical loop): down

14.c)

E (bottom of the vertical loop): up

14.d)

F (bottom of hill #2): up

14.e)

lift hill (going up at constant speed): zero

14.f)

horizontal loop: sideways

14.g)

back curve: sideways

15.

Preparing for the Chapter Challenge

Students discuss with the teacher where the calculations will be made. The level of mathematical sophistication of the class and the desired goals of the teacher should determine this.

Inquiring Further

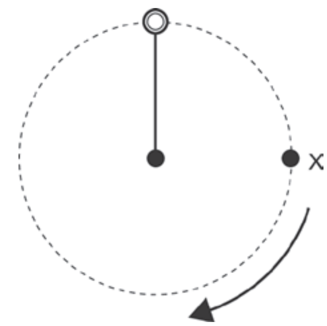
On a playground swing, you travel in a circular path (with its center at the pivot point of the swing's chains), but your speed changes significantly during the motion. At the top of the swing's path, the speed is instantaneously zero. At that point you have no centripetal acceleration. At the bottom of the swing's path, you have a velocity, which, as usual, is tangent to the swing's path at that point. Now you have a centripetal acceleration directed upward toward the chains' pivot point. Because your weight force is downward, the seat of the swing (and the chains) must provide an upward force larger in magnitude than your weight. Thus, you feel heavier at the bottom of the swing's path.

CHAPTER 4

SECTION 7 QUIZ

4-7c **Blackline Master**

1. A mass is rotating clockwise in a horizontal circle as shown in the diagram to the right:



   If the string breaks when the object is at point X, which arrow below best represents the path of the object after the string has broken?

a) 

b) 

c) 

d) 

2. In *Question 1*, when the mass is at position X before the string breaks, which arrow below best represents the direction of the mass's acceleration?

a) 

b) 

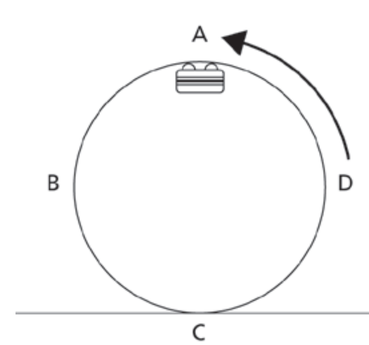
c) 

d) 

3. A mass of 10 kg is revolving in a circle with a speed of 5 m/s. If the circle has a radius of 10 m, what centripetal force must be acting on the mass?

   a) 5 N
   b) 10 N
   c) 20 N
   d) 25 N

4. A roller-coaster car is going around a vertical circular loop with constant speed as shown in the diagram. At which point on the loop would a student riding in the car experience the greatest normal force?



   a) A
   b) B
   c) C
   d) D

5. In *Question 4*, a 50-kg student riding in the coaster car is sitting on a bathroom scale when the car is in position A. The circle has a radius of 8 m, and the car's velocity at point A is 10 m/s. What would be the reading on the bathroom scale when the student and car are at point A?

a) 0 N   b) 125 N

c) 500 N   d) 625 N

SECTION 7 QUIZ ANSWERS

1 a) If the spring breaks at position X when the object is moving in the direction shown, the mass will have no net force acting on it. According to Newton's first law, the object will then move in a straight line with constant speed. At the instant the string broke, the object was moving directly toward the bottom of the page, so it will continue moving in that direction.

2 d) When the mass is at position X, the only force acting on it is the string, which is exerting a centripetal force toward the center of the circle to make it move in a circular motion.

3 d) The magnitude of the centripetal force is found using the equation $F_c = mv^2/r$ Solving gives $F_c = (10\ \text{kg})(5\ \text{m/s})^2/(10\ \text{m}) = 25\ \text{N}$

4 c) The centripetal force acting on the rider is the vector sum of the normal force (F_N) and the rider's weight (F_w) or $F_c = F_N - F_w$. Because the car has a constant speed as it goes around the loop, the centripetal force will be constant. The greatest normal force then would occur when the normal force provides an upward force to balance the weight acting downward, and also supply an additional upward force to provide for the force needed for the centripetal acceleration. This occurs at the bottom of the loop.

5 b) The centripetal force acting on the rider is the vector sum of the normal force (F_N) and the rider's weight (F_w) or $F_c = F_N + F_w$. Because the car has a constant speed as it goes around the loop, the centripetal force will be constant. At point A, the normal force and the weight act in the same direction (down) to provide the centripetal force. The weight is found using $F_w = mg = (50\ \text{kg})(10\ \text{m/s}^2) = 500\ \text{N}$. The centripetal force is found using the equation $F_c = mv^2/r = (50\ \text{kg})(10\ \text{m/s})^2/(8\ \text{m}) = 625\ \text{N}$. Since $F_c = F_N + F_w$, solving for F_N gives $625\ \text{N} = F_N - 500\ \text{N}$ or $F_N = 125\ \text{N}$.