SECTION 8

Energy Stored within the Nucleus

Section Overview

Students begin their investigation of the energy equivalence of Einstein's equation, $E = mc^2$, by first comparing the time it takes light to travel short and long distances. They then calculate the energy equivalence of the mass of a pea and equate it to the kinetic energy of a sprinter. Using this energy, they determine how fast the sprinter would have to run in order to have the same kinetic energy. They also compare the energy equivalence of the mass of a pea to the height a bowling ball would have to reach to acquire the same gravitational potential energy. Students are then introduced to the terms atomic mass units, nucleons, and mass defect. To find the binding energy holding the nucleus together, they calculate the mass defect and average binding energy per nucleon of nitrogen-15. By comparing the binding energies of the nuclei of a nitrogen isotope, students calculate the energy required to remove a neutron from the nucleus. Subsequently, students examine Einstein's equation to learn that when a particle (electron) collides with an antiparticle (positron) their mass is destroyed and light energy is created.

Background Information

In the year 1905, Albert Einstein recognized that mass and energy are related, and described that relationship in his now famous equation, $E = mc^2$. In introducing $E = mc^2$, it should be noted that this expression gives the conversion factor of mass into energy for a mass at rest. The general expression for the energy of a noninteracting mass, moving with velocity v, is

$$E = mc^2 \Big/ \left(1 - v^2/c^2\right)^{\frac{1}{2}}$$

Some perceptive student, recalling $KE = \frac{1}{2}mv^2$ and noting there's no v in $E = mc^2$, may ask about it. A particle's kinetic energy, according to Einstein, is

$$mc^2\left[\left(1 - v^2/c^2\right)^{-\frac{1}{2}} - 1\right],$$

which reduces to $\frac{1}{2}mv^2$ when $v << c$. (To see this requires the series expansion of the terms in the equation.)

Each bit of mass has a tremendous amount of energy. You have daily reminders of this as many cities receive their entire electrical energy from the conversion of mass to energy in nuclear power plants. You also live at a time where you have nuclear weapons capable of destroying entire cities. When protons and neutrons come together to form a nucleus, some of the mass is released as energy. This binding energy holds the nucleus together. It is necessary to add energy in order to remove a proton or a neutron from the nucleus. By adding up the masses of the constituent parts of a nucleus and comparing this total mass with the mass of the formed nucleus, you can calculate the mass difference or mass defect and then compute the binding energy of the nucleus. You will find that the binding energy of a proton to a nucleus is equivalent to millions of electron volts. This is millions of times larger than the binding energy of an electron to an atom, which was found earlier to be of the order of a few electron volts. Creating nuclear energy will be the topic for the next section.

Crucial Physics

- Einstein was the first to indicate that mass and energy are two forms of the same quantity related by a constant, the speed of light squared. Einstein's equation $E = mc^2$ determines the quantity of energy that can be obtained from the destruction of an amount of mass and vice-versa.
- When a nucleus is assembled from its constituent particles, part of the mass of the particles is converted into energy to provide the binding energy of the nucleus.
- To break a nucleus up into its individual parts requires an addition of energy equal to the binding energy of the nucleus.
- Removing a particle from the nucleus is similar to removing an electron from the atom; however, a much larger amount of energy is required to remove a nuclear particle.

Learning Outcomes	Location in the Section	Evidence of Understanding
Explain the meaning of Einstein's equation.	***Investigate*** Steps 2–5	Students investigate the speed of light by calculating how long it takes light to travel short and long distances. They then investigate Einstein's equation and compare the required *KE* of a sprinter and the *GPE* of a bowling ball needed to equal the energy equivalent of the mass of a pea. They then explain Einstein's equation by reading the information presented in the *Physics Talk*.
Calculate the energy equivalent of different masses.	***Investigate*** Steps 3 ***Physics to Go*** Question 1	Students calculate the energy equivalent of a pea, an electron, and a 50-kg student using Einstein's equation
Discover the relative stability of the nuclei by calculating how much energy would be required to take the nucleus apart into individual protons and neutrons (the "binding energy").	***Investigate*** Step 6	Students calculate the total mass of separate protons and neutrons and compare it to the mass of the nucleus to determine the mass difference or the binding energy of the nucleus.
Compare the binding energy of the nucleus with the binding energy of electrons in the atoms.	***Investigate*** Step 6	Students compare the binding energy of a nucleon to the binding energy of the electrons in the atoms by calculating the average binding energy per nucleon.

Section 8 Materials, Preparation, and Safety

Materials and Equipment

- There are no materials or equipment needed for this section.

Time Requirement

- Allow one class period or 45 minutes for the students to complete the *Investigate* portion of this section.

Teacher Preparation

- Each student should have a basic calculator so that the members of the groups can check each other's calculations.

Safety Requirements

- There are no particular safety requirements for this activity.

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Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

Learning Issue	Reference	Augmentation and Accommodations
Calculating binding energy of a nucleon	***Investigate*** Step 6 ***Physics to Go*** Question 4	**Augmentation** • Calculating binding energy is a multi-step problem solving process that is complicated by the decimal numbers used for mass. • Review the rules for rounding decimals before students begin solving these problems. • Students who struggle with problem solving need a step-by-step process to solve these problems. 1. Calculate total mass of protons by multiplying the number of protons (atomic number) by 1.007825 u. 2. Find the number of neutrons by subtracting the number of protons from the mass number. 3. Calculate total mass of the neutrons by multiplying the number of neutrons by 1.008665 u. 4. Add up the mass of the protons and neutrons to find the total mass of the separate nucleons. 5. Find the *mass defect* by subtracting the mass of the combined nucleus from the total mass of the separate nucleons. 6. Find the total binding energy (TBE) by multiplying the mass defect by the conversion factor of 931.5 MeV per 1 u. 7. Find the average binding energy by dividing the total binding energy by the number of nucleons. • This checklist should be paired with an example to be most effective. The steps should be on the left side of the paper and the corresponding step for the example should be lined up on the right side of the paper. • Some students may be able to create their own checklist using the *Investigate*, *Physics Talk*, and *Active Physics Plus*. **Accommodation** • Provide a graphic organizer that has the steps for solving the problems on the left side of the page and blank space on the right side for students to show their work.

Strategies for Students with Limited English-Language Proficiency

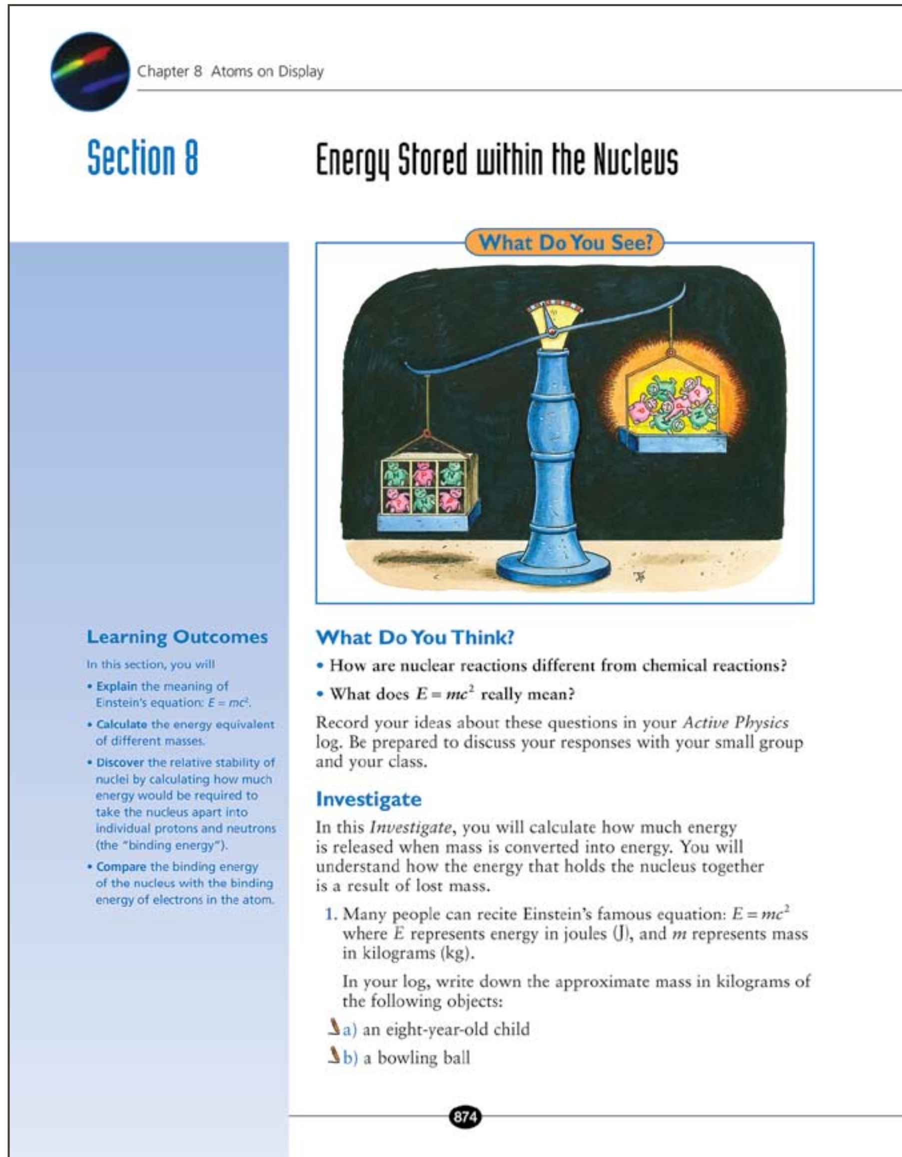
Learning Issue	Reference	Augmentation
Understanding concepts	***Investigate*** Step 2	One way to help students remember that *c* in the equation $E = mc^2$ stands for the speed of light is to remember that the speed of light in a vacuum is a constant. In fact, some historians have suggested that the notation *c* was used by physicists because in early works, such as Newton's writings, it was common to use Latin, and the Latin term used for speed is *celeritas.* Although Newton did not write equations using variable abbreviations, *c* was later used as a variable for speed by scientists and mathematicians in the 1700s.
Vocabulary comprehension	***Investigate*** Example	Remind students that they learned about analogies in *Section 3.* Call their attention to the word "analogous" here and have them figure out its meaning. Then challenge them to use the "rock in a well" analogy to explain the term "mass defect" in their own words.
Vocabulary comprehension Understanding concepts	***Physics Talk***	Good readers often use context clues to determine the meaning of an unfamiliar word. Have ELL students read the text in the last bullet in the *Physics Talk* and then suggest a meaning for the word "annihilate." Discuss antiparticles with the class. Ask: If a positron, like a proton, has a positive charge, what is the difference between the two particles? Be sure students can state that a positron has significantly less mass than a proton; its mass is identical to the mass of an electron.
Understanding concepts	***Checking Up*** Question 2	If students have trouble answering *Checking Up, Question 2,* they likely will have trouble understanding why nuclear processes are so much more energetic than chemical processes. Remind them that the term "nuclear" simply means "related to the nucleus." Chemical reactions involve electrons that move from one atom to another. Because electrons are outside the nucleus, they are held in the atom by relatively little energy. Therefore, only a relatively small amount of energy is required to release them from the atom. Nuclear reactions, by contrast, involve neutrons and protons, particles that exist within the nucleus of the atom. They are held in the atom by much greater forces, so a great amount of energy is required to release them from the nucleus. Another point of confusion for students may be the term "binding energy" itself. The term makes sense when used to describe the energy required to keep a particle bound to an atom. But it may seem counterintuitive in the context of chemical and nuclear reactions, given that the reactions require enough energy to "unbind" the particles from the atom.
Vocabulary comprehension	***Active Physics Plus***	Challenge students to use context clues to decipher the term "abundance." They should recognize that almost all of the naturally occurring uranium on Earth is the U-238 isotope.

SECTION 8

Teaching Suggestions and Sample Answers

What Do You See?

The balance tipping on one side of the scale is a good starting point for discussion of *What Do You See?* Consider asking students why the box on one side of the scale has puppets arranged in different squares while in the other box they are all jumbled up without any boundaries. Why is there an explosion of light around one side of the scale while the other doesn't have any light around it? How could the title be a link to the illustration? These questions, among others, raised through discussion, will consolidate the ideas that students have and will prepare them for the further inquiry in the next section. Remind students that they will be given other opportunities


Chapter 8 Atoms on Display

Section 8 Energy Stored within the Nucleus

What Do You See?

Learning Outcomes

In this section, you will

- **Explain** the meaning of Einstein's equation: $E = mc^2$.
- **Calculate** the energy equivalent of different masses.
- **Discover** the relative stability of nuclei by calculating how much energy would be required to take the nucleus apart into individual protons and neutrons (the "binding energy").
- **Compare** the binding energy of the nucleus with the binding energy of electrons in the atom.

What Do You Think?

- How are nuclear reactions different from chemical reactions?
- What does $E = mc^2$ really mean?

Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your responses with your small group and your class.

Investigate

In this *Investigate*, you will calculate how much energy is released when mass is converted into energy. You will understand how the energy that holds the nucleus together is a result of lost mass.

1. Many people can recite Einstein's famous equation: $E = mc^2$ where E represents energy in joules (J), and m represents mass in kilograms (kg).

   In your log, write down the approximate mass in kilograms of the following objects:

   a) an eight-year-old child

   b) a bowling ball

874

Students' Prior Conceptions

Students are inclined to equate energy with mass, and they may find it difficult to grasp the meaning of a conversion factor. The equivalence of energy in terms of mass may be confusing to them.

From the equation $E = mc^2$, students might think that the conversion of mass to energy is a practical proposition. In theory, mass can be converted to pure energy, but in reality, even if a small amount of mass were to be converted to pure energy, the destruction would be immense. If you take one kilogram of a substance then the equivalent energy would be the amount of mass multiplied by the speed of light, which is $1\ \text{kg} \times (3 \times 10^8\ \text{m/s})^2 = 9 \times 10^{16}$ J. To put this number in perspective, this is approximately the same amount of mass you could haul in a pickup truck that would provide more energy than is required to power our entire planet for a whole year. In nuclear reactors, however, the conversion of mass releases energy, but it is only a partial conversion, and even that is extremely hard to control. Therefore, equivalence in $E = mc^2$ is not the same thing as conversion.

Chemical reactions and nuclear reactions use the same amount of energy. This misconception can be addressed when you show that the binding energy of an electron is much smaller than the binding energy of a neutron, which is in millions of electron volts. Chemical reactions involve the exchange or sharing of electrons, while nuclear reactions involve the exchange or transformation of nucleons.

during their investigations in this section to analyze what this illustration seeks to convey.

What Do You Think?

Based on what students have learned about binding energy and radioactive decay, they should come up with a wide range of responses. Accept all answers and ask them to enter their responses in their *Active Physics* logs. Encourage them to share their answers with their classmates, pointing out that prior conceptions they have now will subsequently be addressed as they investigate nuclear reactions. Consider asking students to think about the structure of an atom and whether the location of the electrons or the neutrons and protons affect how nuclear reactions take place. Emphasize to students that this section is mainly meant to brainstorm ideas so that they can progress with an increased focus of physics concepts that they will be examining as they examine $E = mc^2$.

What Do You Think?

A Physicist's Response

Nuclear reactions are interactions among and within nuclei, while chemical reactions are interactions among and within atoms. In chemical reactions, electrons jump from one atom to another, or electrons are shared between atoms, but the nuclei of the atoms remain unchanged. Chemical reactions are much weaker than nuclear reactions. A chemical reaction may have the energy of a few electron volts. Nuclear reactions, where the nuclei of the atoms are changed, have a typical energy of millions of electron volts. A violent chemical reaction is a stick of dynamite that can blast a hole in a rock face. A violent nuclear reaction is an atomic bomb that can level an entire city.

Investigate

There will be some variations in the students' answers.

1.a)

Child = 20 kg

1.b)

Bowling ball = 6 kg

NOTES

NOTES

c) a box of spaghetti

d) a compact car

2. The c in the equation $E = mc^2$ represents the speed of light, which is equal to 3.0×10^8 m/s. Use the equation:

$$d = vt$$

to calculate how long it takes light to travel:

a) from one side of a football field to the other if the distance is approximately 100 m.

b) from Atlanta to Miami (approximately 1000 km or 10^6 m)

c) from the Moon to Earth (approximately 380,000 km or 3.8×10^8 m)

d) from the Sun to Earth (approximately 150,000,000 km or 1.5×10^{11} m)

3. The mass of a pea is 1 g or 1×10^{-3} kg. Consider the energy equivalent of such a small mass.

The energy equivalent of an object at rest can be determined by $E = mc^2$.

a) Calculate the energy equivalent in joules of a pea using $E = mc^2$. (Assume 100% conversion of mass into energy.)

4. There are many forms of energy. Kinetic energy, associated with moving cars, people, planets, and subatomic particles, can be calculated using the equation:

$$KE = \frac{1}{2}mv^2$$

where KE is kinetic energy in joules (J),

m is the mass in kilograms (kg), and

v is the velocity in meters per second (m/s).

a) If a sprinter with a mass of 50 kg is running at 8 m/s, calculate the kinetic energy in joules.

b) How fast would a sprinter have to be running to have the energy equivalent of converting the mass of one pea entirely into energy?

c) What conclusion can be drawn about the amount of energy in an object even as small as a pea?

5. Gravitational potential energy is energy associated with position. It can be calculated using the equation:

$$GPE = mgh$$

where GPE is gravitational potential energy in joules (J),

m is the mass in kilograms (kg),

g is the acceleration due to gravity equal to about $9.8\ \text{m/s}^2$, and

h is the elevation in meters (m).

a) How high would a bowling ball (mass = 6 kg) have to be elevated to have the same gravitational potential energy as the mass-energy equivalent of a pea?

6. In *Section 3*, you studied electrons in the atom. The *binding energy* of an electron in hydrogen is the energy that holds the electron to the nucleus. In other words, the binding energy is the amount of energy you would have to "spend" to remove the electron from the hydrogen atom. In the $n = 1$ (ground) state of hydrogen, the binding energy is –13.6 eV. The negative sign indicates to you that the electron is bound to the nucleus. That electron requires +13.6 eV in order for it to be pulled free of the nucleus with no kinetic energy left over.

1.c)

A box of spaghetti = 0.4 kg

1.d)

A compact car = 500 kg

2.a)

$t = d/v =$

$100\ \text{m}/(3 \times 10^8\ \text{m/s}) = 3.3 \times 10^{-7}\ \text{s}$

2.b)

$t = d/v =$

$10^6\ \text{m}/(3 \times 10^8\ \text{m/s}) = 3.3 \times 10^{-3}\ \text{s}$

2.c)

$t = d/v =$

$3.8 \times 10^8\ \text{m}/(3 \times 10^8\ \text{m/s}) = 1.3\ \text{s}$

2.d)

$t = d/v =$

$1.5 \times 10^{11}\ \text{m}/(3 \times 10^8\ \text{m/s}) = 500\ \text{s}$

3.

$E = mc^2 =$

$1 \times 10^{-3}\ \text{kg}(3 \times 10^8\ \text{m/s})^2 = 9 \times 10^{13}\ \text{J}$

4.a)

$KE = \frac{1}{2}mv^2 =$

$\frac{1}{2}(50\ \text{kg})(8\ \text{m/s})^2 = 1600\ \text{J}$

4.b)

The energy equivalent of a pea = 9×10^{13} J, so $9 \times 10^{13}\ \text{J} = \frac{1}{2}(50\ \text{kg})v^2$; therefore, $v = 1.9 \times 10^6\ \text{m/s}$.

4.c)

Even very small objects have the potential to liberate tremendous amounts of energy when their mass is converted to energy.

5.a)

$GPE = mgh =$

$9 \times 10^{13}\ \text{J} = (6\ \text{kg})(9.8\ \text{m/s}^2)(h)$

$h = 1.5 \times 10^{12}\ \text{m}$

or 10 times the distance to the Sun!

CHAPTER 8

6.a)

chlorine-37

mass of 17 protons =
17(1.007825 u) = 17.133025 u

mass of 20 neutrons =
20(1.008665 u) = 20.1733 u

total nucleon mass = 37.306325 u

mass of chlorine-37 = 36.965898 u

mass difference =
37.306325 u − 36.965898 u =
0.340427 u

total binding energy =
(931.5 MeV/u)(0.340427 u) =
317.1 MeV

average binding energy per nucleon = 317.1 MeV/37 nucleons
= 8.6 MeV/nucleon

Neutrons and protons are also bound to the nucleus, held there by the strong nuclear force. A nitrogen-15 ($^{15}_{7}N$) nucleus has 7 protons and 8 neutrons. To free a baryon (proton or neutron) requires an addition of energy equal to the binding energy of the baryon. You can calculate that binding energy.

The *atomic mass unit* is used to compare the masses of atoms. It is defined in terms of the mass of carbon-12. The carbon-12 nucleus has a mass of exactly 12 atomic mass units, by definition of the atomic mass unit! The atomic mass unit is denoted by "u" so that the mass of carbon-12 is 12 u. Each atomic mass unit (u) is approximately equal to 1.7×10^{-27} kg. (You can see from this value why scientists prefer to use atomic mass units instead of kilograms.)

Example:

Calculate the mass of all the protons and neutrons in a nitrogen-15 nucleus.

In atomic mass units, the proton's mass is 1.007825 u, and the neutron's mass is 1.008665 u. The mass of 7 separate protons, and 8 separate neutrons can, therefore, be found as follows:

Mass of 7 protons = 7 (1.007825 u)
= 7.054775 u

Mass of 8 neutrons = 8 (1.008665 u)
= 8.069320 u

Total mass of the separate (protons + neutrons) = 15.124095 u

- Compare this to the mass of the assembled nitrogen-15 nucleus. The mass of the nitrogen-15 nucleus (15.000108 u) can be found from a chart of the nuclides in a reference book. The nitrogen-15 mass is less than the total mass of the same number of separate protons and neutrons, which are also called *nucleons*.

Energy must be supplied to remove, say, a neutron from the assembled nucleus. This situation is analogous to pulling a rock out of a well—you have to use energy to raise the rock out of the well.

Total nucleon mass	=	15.124095 u
Mass of nucleus of ^{15}N	=	15.000108 u
Mass difference	=	0.123987 u

This difference in mass, known as the *mass defect*, is the binding energy of the nucleus.

Conversion factor between atomic mass units and energy (this comes from $E = mc^2$):

$$1 \text{ u} = 931.5 \text{ MeV}$$

$$1 \text{ MeV} = 1{,}000{,}000 \text{ eV}$$

(The derivation of 931.5 MeV is given in *Active Physics Plus*.)

- To calculate the total binding energy (*TBE*) of the nucleus,

$$TBE = 0.123987 \text{ u} \times 931.5 \frac{\text{MeV}}{\text{u}}$$
$$= 115.5 \text{ MeV}$$

- To calculate the average binding energy per nucleon,

Total number of nucleons = 15 (7 protons and 8 neutrons)

Average binding energy/nucleon = 115.5 MeV/15 = 7.7 MeV

On average, it requires 7.7 MeV or 7.7 million eV to remove a proton or neutron from the nucleus of nitrogen-15 (The removal of the first baryon actually requires more energy than this—the 7.7 MeV is an average.) Recall that removing an electron from hydrogen required only 13.6 eV.

6.a) Calculate the average binding energy per baryon for chlorine-37, $^{37}_{17}Cl$. The mass of chlorine-37 is equal to 36.965898 u. Use the masses of protons and neutrons given in the sample problem.

b) A neutron can be removed to create $^{14}_{7}\text{N}$ from $^{15}_{7}\text{N}$. Compare the mass of $^{15}_{7}\text{N}$ to the masses of $^{14}_{7}\text{N}$ plus one separate neutron to determine the energy required to free the neutron. The mass of nitrogen-14 equals 14.003074 u. The other values are given in previous text.

c) Compare this binding energy to remove the neutron with the average binding energy that you calculated in the sample problem.

Physics Talk

ENERGY AND MATTER

The speed of light is the "speed limit" of the universe. No material object can travel faster than this speed. In the *Investigate*, you examined Einstein's equation, $E = mc^2$, which gives the "exchange rate" between energy and mass and can be interpreted in several ways.

$$E = mc^2$$

- provides the energy equivalent of a piece of mass. (In the section, you found the equivalent energy of a pea.)
- tells you that energy and mass are equivalent entities, but one is given in joules and the other is given in kilograms.
- tells you that the conversion factor between energy and mass is the square of the speed of light (c^2). To change kilograms to joules, you have to multiply by c^2, equal to $9 \times 10^{16}\ \text{m}^2/\text{s}^2$.
- explains how much energy is produced when an electron (particle) and a **positron (antiparticle)** can annihilate each other and create light energy. A positron is identical to the electron but has a positive charge. When a particle and an antiparticle collide their mass is destroyed and the process is called **particle-antiparticle annihilation**.

In *Section 3*, you investigated the binding energy of electrons in an atom. In the $n = 1$ (ground) state of hydrogen, the binding energy was -13.6 eV. To pull the electron from the $n = 1$ state and take it completely out of the atom, it takes 13.6 eV of energy. The binding energy of the electron in the $n = 2$ state is -3.4 eV. You would have to give that electron $+3.4$ eV to free it from the $n = 2$ state. Similarly, if the nucleus of hydrogen captures an electron to make a hydrogen atom, and the electron drops into the $n = 1$ state, 13.6 eV is given off as a photon of light.

The conservation of energy requires that the energy of a system before an event must equal the energy after the event. In our examples of light and electrons in atoms, the system consists of the atom, its electron, and light. An electron absorbs a photon of light of $+13.6$ eV, bringing its total energy to 0 and it is free. Or, a free electron of 0 energy becomes bound to the proton nucleus with an energy of -13.6 eV and gives off a light photon with energy equal to $+13.6$ eV.

Physics Words

positron: a nuclear particle identical to the electron but with a positive charge.

particle-antiparticle annihilation: the process in which a particle and an antiparticle collide and their mass becomes energy.

Physics Talk

This section interprets Einstein's equation based on the problems students solved in the *Investigate*. By relating the energy equivalent of a pea to $E = mc^2$, the *Physics Talk* states that energy and mass are equivalent quantities, but energy is given in joules while mass is given in kilograms. The conversion factor between energy and mass is the speed of light, and energy is produced when an electron and a positron annihilate each other, creating light energy. The binding energy of electrons examined in *Section 3* are compared to the binding energy of nucleons to give students a sense of why nuclear reactions have so much more energy than chemical reactions.

Discuss Einstein's equation with students and ask them to describe how they would interpret this equation. Review the investigations from *Section 3* and the significance of conservation of energy of a system during a nuclear or chemical reaction. Emphasize the binding energy of electrons at different ground levels, and that this energy decreases as the distance of the electron from the nucleus increases. Ask students why the binding energy of a nucleon would be more. Determine if they understand the meaning of mass defect and how it is calculated and measured. Consider asking students to make diagrams to explain the new terms they have learned in this section.

6.b)

$\left(^{14}_{7}\text{N} + ^{1}_{0}\text{n}\right) - ^{15}_{7}\text{N} = 14.003074\ \text{u} +$
$1.008665\ \text{u} - 15.000108\ \text{u} =$
$1.011631\ \text{u}$;
$1.011631\ \text{u} \times 931.5\ \text{MeV/u} =$
$10.8\ \text{MeV}$

6.c)

The 10.8 MeV to remove the first nucleon required much more energy than the average binding energy per nucleon (derived in the text as 7.7 MeV/nucleon).

CHAPTER 8

Checking Up

1.

The energy of 6 kg in joules can be found by using $E = mc^2$. Substituting in the values for the mass (6 kg) and the speed of light $(3\times10^8\ \text{m/s})$ gives

$$E = (6\ \text{kg})(3\times10^8\ \text{m/s})^2$$
$$= 5.4\times10^{17}\ \text{J}.$$

2.

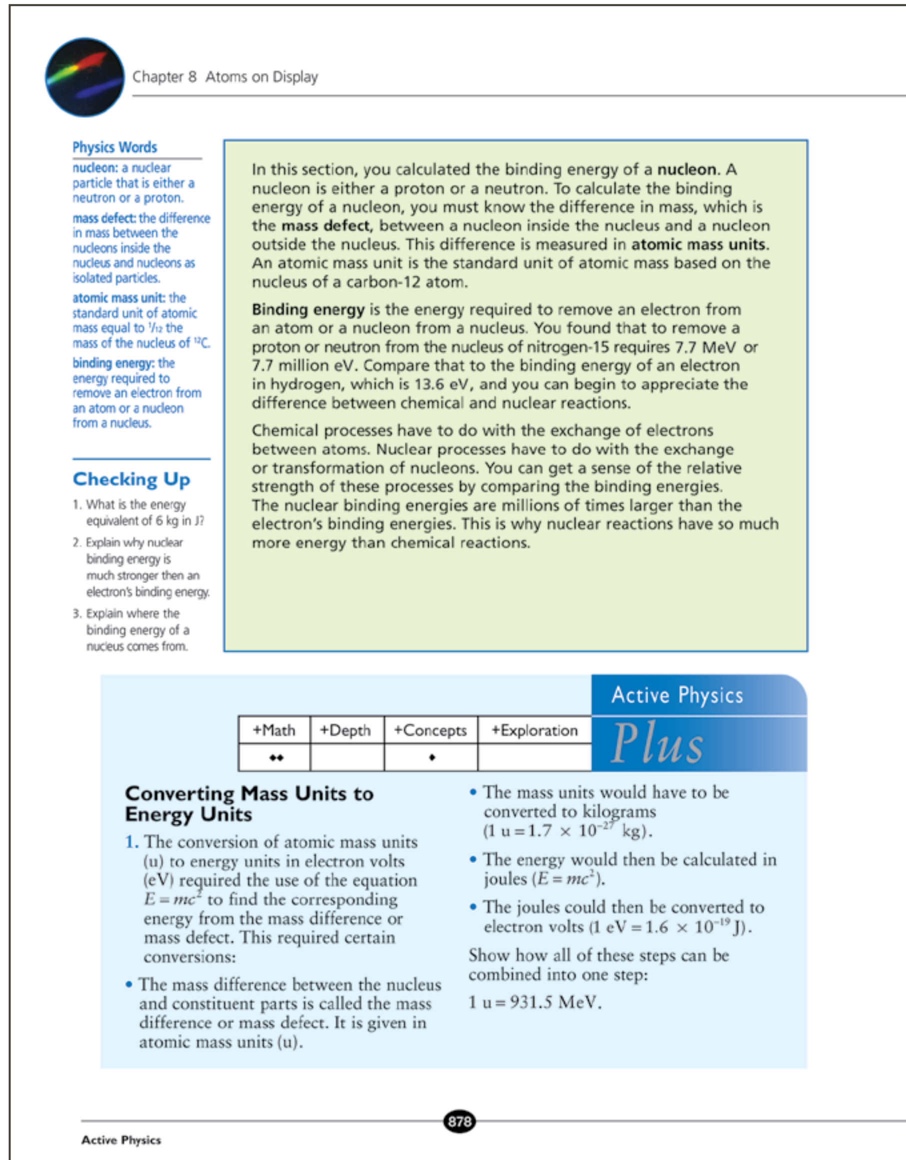
Nuclear binding energy is much stronger than electron binding energy because nuclear binding energy is governed by a force much stronger than the Coulomb force, which determines the electron binding energy. In addition, electrons are much further away from the nucleus, which diminishes the force holding them to the nucleus.

3.

The binding energy of a nucleus comes from the mass that is converted into energy when the protons and neutrons that form the nucleus are brought together.

Active Physics Plus

This section shows how the mass defect in atomic mass units is converted into its corresponding energy. Students calculate the average mass of carbon and iron nuclei by using their atomic masses and abundance, which are listed in a table. They then compare the calculated mass average of iron and carbon nuclei to the masses shown for carbon and iron on the periodic table.



Chapter 8 Atoms on Display

Physics Words

nucleon: a nuclear particle that is either a neutron or a proton.

mass defect: the difference in mass between the nucleons inside the nucleus and nucleons as isolated particles.

atomic mass unit: the standard unit of atomic mass equal to 1/12 the mass of the nucleus of ^{12}C.

binding energy: the energy required to remove an electron from an atom or a nucleon from a nucleus.

Checking Up

1. What is the energy equivalent of 6 kg in J?
2. Explain why nuclear binding energy is much stronger then an electron's binding energy.
3. Explain where the binding energy of a nucleus comes from.

In this section, you calculated the binding energy of a **nucleon**. A nucleon is either a proton or a neutron. To calculate the binding energy of a nucleon, you must know the difference in mass, which is the **mass defect**, between a nucleon inside the nucleus and a nucleon outside the nucleus. This difference is measured in **atomic mass units**. An atomic mass unit is the standard unit of atomic mass based on the nucleus of a carbon-12 atom.

Binding energy is the energy required to remove an electron from an atom or a nucleon from a nucleus. You found that to remove a proton or neutron from the nucleus of nitrogen-15 requires 7.7 MeV or 7.7 million eV. Compare that to the binding energy of an electron in hydrogen, which is 13.6 eV, and you can begin to appreciate the difference between chemical and nuclear reactions.

Chemical processes have to do with the exchange of electrons between atoms. Nuclear processes have to do with the exchange or transformation of nucleons. You can get a sense of the relative strength of these processes by comparing the binding energies. The nuclear binding energies are millions of times larger than the electron's binding energies. This is why nuclear reactions have so much more energy than chemical reactions.

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
♦♦		♦	

Converting Mass Units to Energy Units

1. The conversion of atomic mass units (u) to energy units in electron volts (eV) required the use of the equation $E = mc^2$ to find the corresponding energy from the mass difference or mass defect. This required certain conversions:
   - The mass difference between the nucleus and constituent parts is called the mass difference or mass defect. It is given in atomic mass units (u).
   - The mass units would have to be converted to kilograms ($1\ \text{u} = 1.7 \times 10^{-27}$ kg).
   - The energy would then be calculated in joules ($E = mc^2$).
   - The joules could then be converted to electron volts ($1\ \text{eV} = 1.6 \times 10^{-19}$ J).

   Show how all of these steps can be combined into one step:

   $1\ \text{u} = 931.5\ \text{MeV}$.

878

Active Physics

1.

Using $E = mc^2$ to find the energy in one atomic mass unit gives

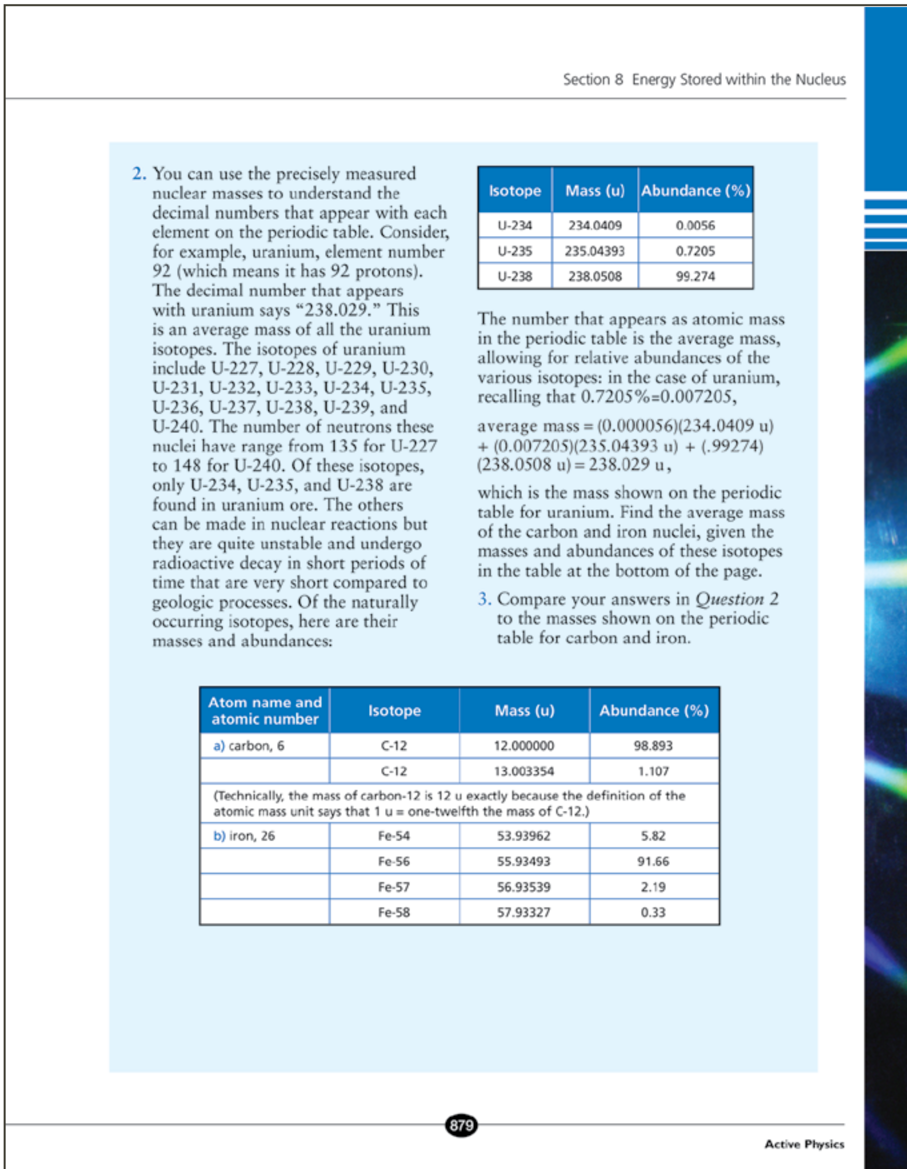
$$E = (1.7\times10^{-27}\ \text{kg})(3\times10^8\ \text{m/s})^2$$
$$= 1.53\times10^{-10}\ \text{J/u}$$

Converting joules to electron volts gives

$$\left(\frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}}\right)(1.53\times10^{-10}\ \text{J/u}) =$$

956 MeV

If the more accurate 1.67×10^{-27} kg/u value is used, then 1 u = 931.5 MeV.



Section 8 Energy Stored within the Nucleus

2. You can use the precisely measured nuclear masses to understand the decimal numbers that appear with each element on the periodic table. Consider, for example, uranium, element number 92 (which means it has 92 protons). The decimal number that appears with uranium says "238.029." This is an average mass of all the uranium isotopes. The isotopes of uranium include U-227, U-228, U-229, U-230, U-231, U-232, U-233, U-234, U-235, U-236, U-237, U-238, U-239, and U-240. The number of neutrons these nuclei have range from 135 for U-227 to 148 for U-240. Of these isotopes, only U-234, U-235, and U-238 are found in uranium ore. The others can be made in nuclear reactions but they are quite unstable and undergo radioactive decay in short periods of time that are very short compared to geologic processes. Of the naturally occurring isotopes, here are their masses and abundances:

Isotope	Mass (u)	Abundance (%)
U-234	234.0409	0.0056
U-235	235.04393	0.7205
U-238	238.0508	99.274

The number that appears as atomic mass in the periodic table is the average mass, allowing for relative abundances of the various isotopes: in the case of uranium, recalling that 0.7205%=0.007205,

$$\text{average mass} = (0.000056)(234.0409\ \text{u}) + (0.007205)(235.04393\ \text{u}) + (.99274)(238.0508\ \text{u}) = 238.029\ \text{u},$$

which is the mass shown on the periodic table for uranium. Find the average mass of the carbon and iron nuclei, given the masses and abundances of these isotopes in the table at the bottom of the page.

3. Compare your answers in *Question 2* to the masses shown on the periodic table for carbon and iron.

Atom name and atomic number	Isotope	Mass (u)	Abundance (%)
a) carbon, 6	C-12	12.000000	98.893
	C-12	13.003354	1.107
(Technically, the mass of carbon-12 is 12 u exactly because the definition of the atomic mass unit says that 1 u = one-twelfth the mass of C-12.)			
b) iron, 26	Fe-54	53.93962	5.82
	Fe-56	55.93493	91.66
	Fe-57	56.93539	2.19
	Fe-58	57.93327	0.33

879

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2.

For carbon
$(0.98893)(12.0000\ \text{u}) + (0.01107) \times (13.003354\ \text{u}) = 12.0111\ \text{u}$.

For iron
$(0.0582)(53.93962\ \text{u}) + (0.9166)(55.93493\ \text{u}) + (0.0219)(56.93539\ \text{u}) + (0.0033)(57.93327\ \text{u}) = 55.8473\ \text{u}$

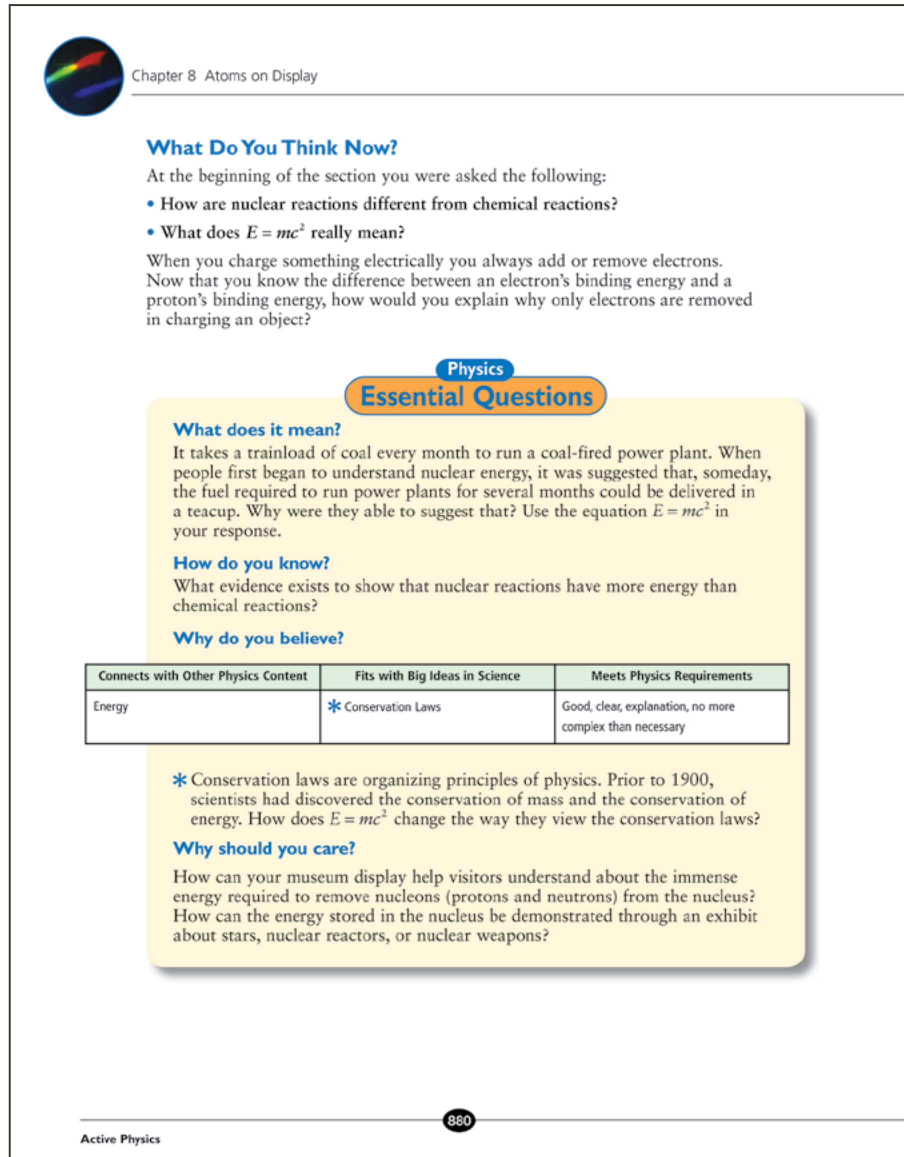
3.

The periodic table lists the atomic mass of iron as 55.847 u and the atomic mass of carbon as 12.011 u.

CHAPTER 8

What Do You Think Now?

Students will most likely be able to describe the difference between chemical and nuclear reactions. Discuss these differences in detail, sharing the answers provided in *A Physicist's Response*. Encourage students to clear the doubts they might still have and ask them to revise and update their answers. Ask them why a proton's binding energy is more than an electron's binding energy, and include the explanations in their final responses. Re-emphasize that the c^2 term in $E = mc^2$ is the conversion factor between mass and energy.



Chapter 8 Atoms on Display

What Do You Think Now?

At the beginning of the section you were asked the following:

- **How are nuclear reactions different from chemical reactions?**
- **What does $E = mc^2$ really mean?**

When you charge something electrically you always add or remove electrons. Now that you know the difference between an electron's binding energy and a proton's binding energy, how would you explain why only electrons are removed in charging an object?

Physics

Essential Questions

What does it mean?

It takes a trainload of coal every month to run a coal-fired power plant. When people first began to understand nuclear energy, it was suggested that, someday, the fuel required to run power plants for several months could be delivered in a teacup. Why were they able to suggest that? Use the equation $E = mc^2$ in your response.

How do you know?

What evidence exists to show that nuclear reactions have more energy than chemical reactions?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Energy	* Conservation Laws	Good, clear, explanation, no more complex than necessary

* Conservation laws are organizing principles of physics. Prior to 1900, scientists had discovered the conservation of mass and the conservation of energy. How does $E = mc^2$ change the way they view the conservation laws?

Why should you care?

How can your museum display help visitors understand about the immense energy required to remove nucleons (protons and neutrons) from the nucleus? How can the energy stored in the nucleus be demonstrated through an exhibit about stars, nuclear reactors, or nuclear weapons?

880

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Physics Essential Questions

What does it mean?

The amount of energy in a teacup of mass is enormous. A teacup of water has a mass of approximately 100 grams or 0.1 kg. Using the relation $E = mc^2$, you calculate the equivalent energy to be $E = (0.1\text{ kg})(3 \times 10^8\text{ m/s})^2 = 9 \times 10^{15}\text{ J}$.

How do you know?

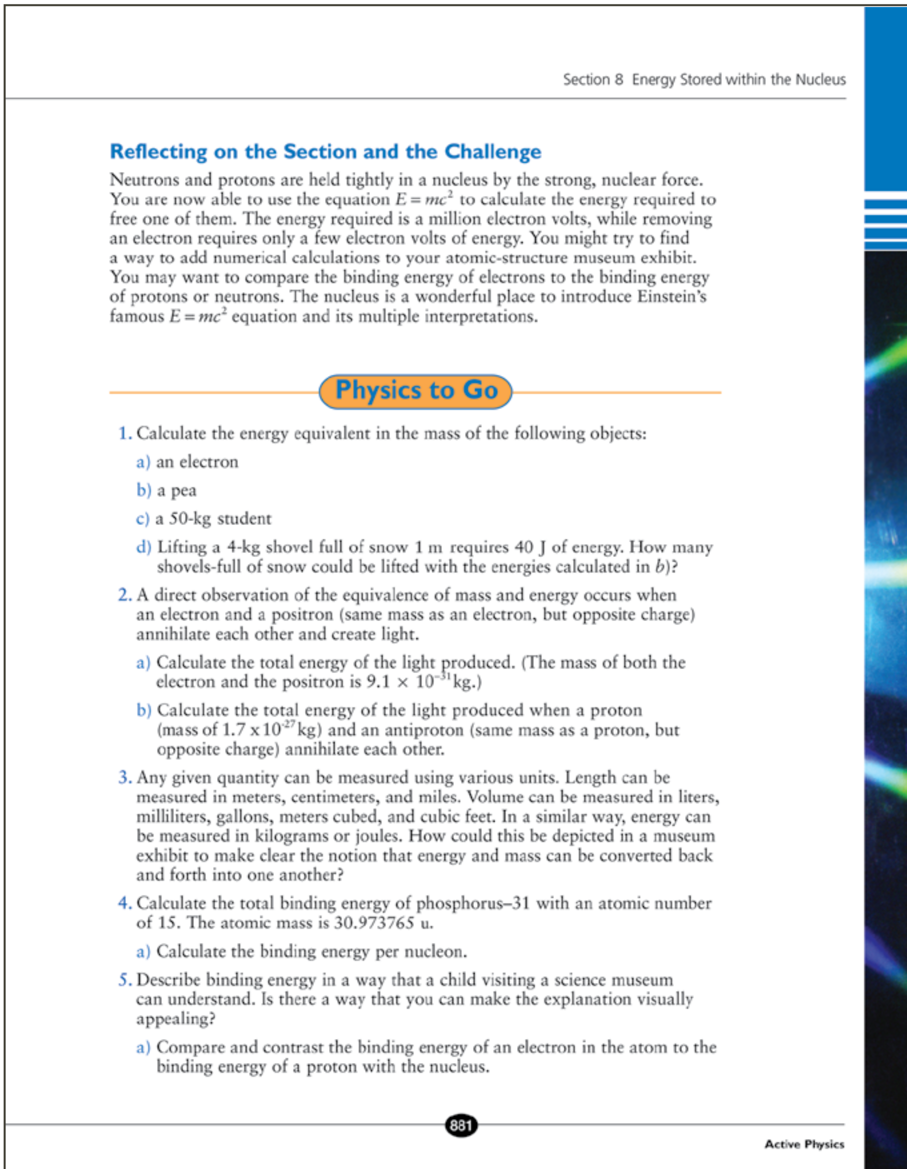
Dynamite explosions rely on chemical reactions. Nuclear bombs rely on nuclear reactions. Our body uses chemical reactions to keep providing us with heat.

Why do you believe?

The equation $E = mc^2$ shows that what you thought were two conservation laws (conservation of mass and conservation of energy) can be stated as one conservation law (conservation of mass/energy). The equation also shows that energy and mass are equivalent—one can become the other.

Why should you care?

Visitors can be asked to move two things—one of which requires one million times the energy as the other. For example, lifting one gram above your head verses lifting 1,000,000 g.



Section 8 Energy Stored within the Nucleus

Reflecting on the Section and the Challenge

Neutrons and protons are held tightly in a nucleus by the strong, nuclear force. You are now able to use the equation $E = mc^2$ to calculate the energy required to free one of them. The energy required is a million electron volts, while removing an electron requires only a few electron volts of energy. You might try to find a way to add numerical calculations to your atomic-structure museum exhibit. You may want to compare the binding energy of electrons to the binding energy of protons or neutrons. The nucleus is a wonderful place to introduce Einstein's famous $E = mc^2$ equation and its multiple interpretations.

Physics to Go

1. Calculate the energy equivalent in the mass of the following objects:
   a) an electron
   b) a pea
   c) a 50-kg student
   d) Lifting a 4-kg shovel full of snow 1 m requires 40 J of energy. How many shovels-full of snow could be lifted with the energies calculated in *b*)?
2. A direct observation of the equivalence of mass and energy occurs when an electron and a positron (same mass as an electron, but opposite charge) annihilate each other and create light.
   a) Calculate the total energy of the light produced. (The mass of both the electron and the positron is 9.1×10^{-31} kg.)
   b) Calculate the total energy of the light produced when a proton (mass of 1.7×10^{-27} kg) and an antiproton (same mass as a proton, but opposite charge) annihilate each other.
3. Any given quantity can be measured using various units. Length can be measured in meters, centimeters, and miles. Volume can be measured in liters, milliliters, gallons, meters cubed, and cubic feet. In a similar way, energy can be measured in kilograms or joules. How could this be depicted in a museum exhibit to make clear the notion that energy and mass can be converted back and forth into one another?
4. Calculate the total binding energy of phosphorus–31 with an atomic number of 15. The atomic mass is 30.973765 u.
   a) Calculate the binding energy per nucleon.
5. Describe binding energy in a way that a child visiting a science museum can understand. Is there a way that you can make the explanation visually appealing?
   a) Compare and contrast the binding energy of an electron in the atom to the binding energy of a proton with the nucleus.

881

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Reflecting on the Section and the Challenge

Consider asking students why the nucleus is a wonderful place to introduce Einstein's equation, $E = mc^2$. Have students read this section and reflect on what they have learned about the structure of an atom that will relate to their museum display. Ask them to incorporate the difference in binding energy of an electron and a proton and how they can add the numerical calculations to demonstrate it. Have students highlight a few steps from the *Investigate* and write a brief summary of what they have learned so far, including why a few electron volts are needed to remove an electron from an atom while a million electron volts are required to remove a proton.

Physics to Go

1.a)

electron: $E = mc^2 =$

$(9.1 \times 10^{-31}\ \text{kg})(3 \times 10^{8}\ \text{m/s})^2 =$

8.2×10^{-14} J

1.b)

pea: $E = mc^2 =$

$(10^{-3}\ \text{kg})(3 \times 10^{8}\ \text{m/s})^2 = 9 \times 10^{13}$ J

1.c)

50-kg student: $E = mc^2 =$

$(50\ \text{kg})(3 \times 10^{8}\ \text{m/s})^2 = 4.5 \times 10^{18}$ J

1.d)

$(9 \times 10^{13}\ \text{J})/(40\ \text{J/shovels-full}) =$

2.25×10^{12} shovels-full

2.a)

$E = mc^2 =$

$(2 \times 9.1 \times 10^{-31}\ \text{kg})(3 \times 10^{8}\ \text{m/s})^2 =$

1.6×10^{-13} J

2.b)

$E = mc^2 =$

$(2 \times 1.67 \times 10^{-27}\ \text{kg})(3 \times 10^{8}\ \text{m/s})^2 =$

3×10^{-10} J

3.

Answers will vary.

4.a)

phosphorus-31

mass of 15 protons =

15(1.007825 u) = 15.117375 u

mass of 16 neutrons =

16(1.008665 u) = 16.13864 u

total nucleon mass =

15.117375 u + 16.13864 u =

31.256015 u

CHAPTER 8

mass of phosphorus-31 = 30.973768 u

mass difference = 31.256015 u − 30.973768 u = 0.2822475 u

total binding energy = (931.5 MeV/u)(0.2822475 u) = 262.9 MeV

average binding energy per nucleon = 262.9 MeV/31 nucleons = 8.5 MeV/nucleon

5.a)

Answers will vary. The binding energy of a proton to the nucleus is millions of times greater than the binding energy of an electron to an atom.

6.

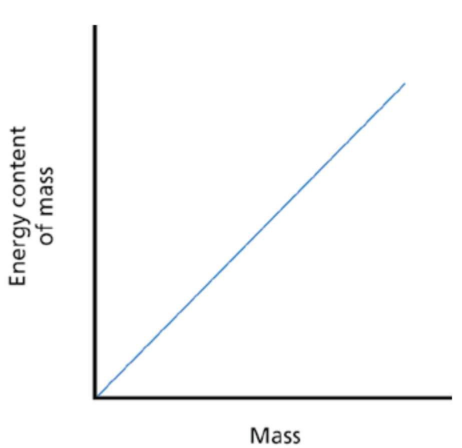
The matter and antimatter will annihilate each other with the mass becoming energy, and that energy is used to supply the ship.

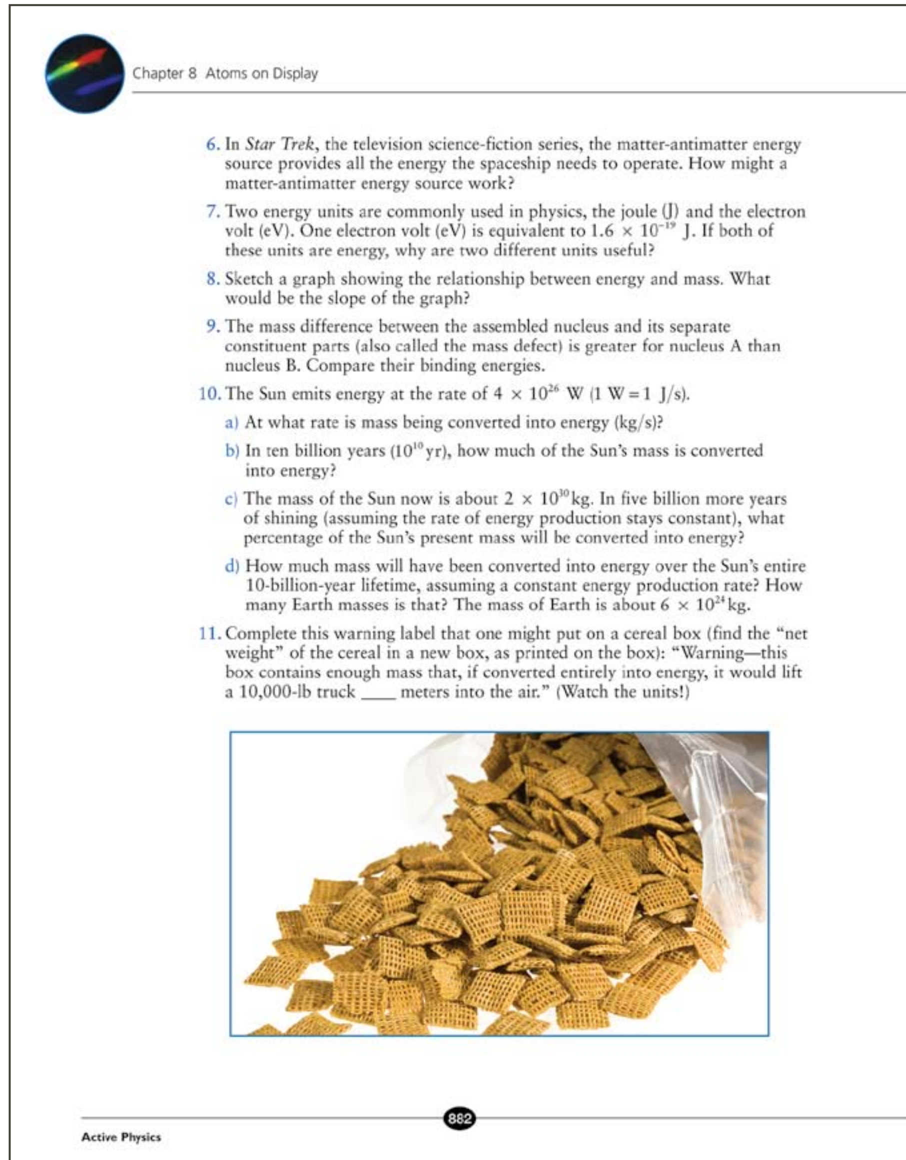
7.

When dealing with small energies, it is often easier to discuss the energy in eV rather than joules. This is similar to discussing room dimensions in meters rather than miles.

8.

The graph is shown below.





Chapter 8 Atoms on Display

6. In *Star Trek*, the television science-fiction series, the matter-antimatter energy source provides all the energy the spaceship needs to operate. How might a matter-antimatter energy source work?

7. Two energy units are commonly used in physics, the joule (J) and the electron volt (eV). One electron volt (eV) is equivalent to 1.6×10^{-19} J. If both of these units are energy, why are two different units useful?

8. Sketch a graph showing the relationship between energy and mass. What would be the slope of the graph?

9. The mass difference between the assembled nucleus and its separate constituent parts (also called the mass defect) is greater for nucleus A than nucleus B. Compare their binding energies.

10. The Sun emits energy at the rate of 4×10^{26} W (1 W = 1 J/s).

a) At what rate is mass being converted into energy (kg/s)?

b) In ten billion years (10^{10} yr), how much of the Sun's mass is converted into energy?

c) The mass of the Sun now is about 2×10^{30} kg. In five billion more years of shining (assuming the rate of energy production stays constant), what percentage of the Sun's present mass will be converted into energy?

d) How much mass will have been converted into energy over the Sun's entire 10-billion-year lifetime, assuming a constant energy production rate? How many Earth masses is that? The mass of Earth is about 6×10^{24} kg.

11. Complete this warning label that one might put on a cereal box (find the "net weight" of the cereal in a new box, as printed on the box): "Warning—this box contains enough mass that, if converted entirely into energy, it would lift a 10,000-lb truck ____ meters into the air." (Watch the units!)

882

Active Physics

The slope will be the velocity of light squared (from the equation $E = mc^2$).

9.

Larger mass defects correspond to larger binding energies, so nucleus A would have a larger total binding energy than nucleus B.

10.a)

Using $E = mc^2$ and solving for m gives

mass loss rate = (energy production rate)$/c^2$ = $(4 \times 10^{26}\ \text{J/s})/(3 \times 10^8\ \text{m/s})^2$ = 4.4×10^9 kg/s.

10.b)

10 billion years =
$(10^{10} \text{ years})(365 \text{ days/year}) \times$
$(24 \text{ h/day}) \times (3600 \text{ s/h}) =$
3.15×10^{17} s.

The total mass lost by the Sun in this time would be
$(3.15 \times 10^{17} \text{ s})(4.4 \times 10^{9} \text{ kg/s}) \approx$
1.4×10^{27} kg.

10.c)

In five billion years, one-half of the mass in *Step 10.b)* would have been converted to mass or 7×10^{26} kg, this is

$$\frac{7 \times 10^{26} \text{ kg}}{2 \times 10^{30} \text{ kg}} = 0.00035 \text{ or } 0.035\%$$

of the mass of the Sun.

10.d)

Over the total life of the Sun the mass converted to energy would be the same as *Step 10.b)*, or 1.4×10^{27} kg. This is

$$\frac{1.4 \times 10^{27} \text{ kg}}{6 \times 10^{24} \text{ kg/Earth}} = 233 \text{ Earths.}$$

11.

Using a net weight of approximately 500 grams (0.50 kg) and using $E = mc^2$ gives
$E = (0.50 \text{ kg})(3 \times 10^{8} \text{ m/s})^2 =$
4.5×10^{16} joules.

A 10,000 lb truck has a mass of approximately 4500 kg. Using $PE_{grav} = mgh$ and substituting in the energy and mass gives
$4.5 \times 10^{16} \text{ J} = 4500 \text{ kg}(9.8 \text{ m/s}^2)h$
or $h = 1 \times 10^{12}$ meters, or about 630 million miles!

NOTES

SECTION 8 QUIZ

8-8a **Blackline Master**

1. A mass of 4 grams (0.004 kg) is to be totally converted into energy. The energy equivalent of the mass would be

   a) 4×10^{8} J.
   b) 1.2×10^{9} J.
   c) 3.6×10^{14} J.
   d) 9×10^{16} J.

2. The atomic mass unit (u) is based on the mass of which of the following atomic isotopes?

   a) ${}^{12}_{6}C$
   b) ${}^{4}_{2}He$
   c) ${}^{1}_{1}H$
   d) ${}^{235}_{92}U$

3. If the mass defect for nucleus X is larger than the mass defect for nucleus Y, then nucleus X must have

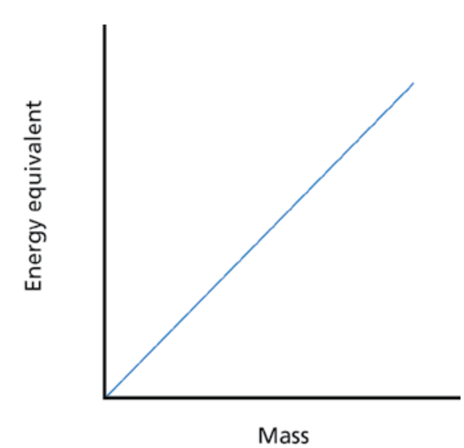
   a) a smaller binding energy than nucleus Y.
   b) a larger binding energy than nucleus Y.
   c) more protons than nucleus Y.
   d) fewer protons than nucleus Y.

4. The mass of a nucleus of lithium ${}^{7}_{3}Li$ is 7.0160 u. The mass of the proton is 1.007825 u and the neutron mass is 1.008665 u. What is the binding energy of a lithium nucleus?

   a) 0.0421 u
   b) 0.0639 u
   c) 0.0716 u
   d) 0.0160 u

5. The graph at right represents the relationship between mass and its energy equivalent. The slope of the graph represents

   a) the electrostatic constant (k).
   b) the gravitational constant (G).
   c) the speed of light squared (c^2).
   d) Planck's constant (h).



SECTION 8 QUIZ ANSWERS

1 c) 3.6×10^{14} J. The conversion of mass into energy is given by Einstein's $E = mc^2$ equation. Using the equation $E = mc^2$ yields
$E = (0.004\ \text{kg})(3 \times 10^8\ \text{m})^2 = 3.6 \times 10^{14}$ J.

2 a) ${}^{12}_{6}C$. The atomic mass unit (u) is defined as being exactly one twelfth of the mass of the Carbon 12 atom, which by convention has a mass of exactly 12 atomic mass units.

3 b) The binding energy of a nucleus is directly proportional to the amount of mass converted into energy as the nucleus was formed. The larger the mass defect, the larger the binding energy, so if nucleus X has a larger mass defect than nucleus Y, it must also have a larger binding energy.

4 a) 0.0421 u. The binding energy is the difference between the mass of the atom and the mass of its constituent particles. For lithium, which is composed of 3 protons and 4 neutrons, this is $3(1.007825) + 4(1.008665) - 7.0160 = 0.0421$ u.

5 c) the speed of light squared (c^2). The slope of a straight-line graph is the constant that relates the two variables on the axes. For mass and energy, this is the speed of light squared, as shown in the equation $E = mc^2$.