SECTION 4

Projectile Motion on the Moon

Section Overview

Students trace the path of a projectile launched on Earth at a 30-degree angle by setting up a scale drawing. They mark off time and distance on an inclined line and distance on the horizontal line, which meet each other at an angle of 30˚. Using the equation $d = \frac{1}{2}gt^2$, students calculate the total distance an object would fall from the inclined line on Earth. They divide each fall distance by 10 to fit the 1/10 scale of their drawing. Students use this distance scale to measure the real-world maximum height above ground level before the projectile strikes the ground, as well as the horizontal distance traveled by the projectile, until it strikes the ground. Next, students calculate the total distance traveled and the maximum height reached by a projectile launched at the same speed and angle on the Moon and plot its trajectory on a scale drawing. The calculations reveal that the maximum height, range, and time of flight of a projectile launched at the same speed and angle on the Moon as that on Earth is six times greater than the comparable values on Earth. Later, students learn how the horizontal and vertical velocities of a projectile can be calculated by solving a sample problem in the *Physics Talk*.

Background Information

In this section, the mathematical and physical models of projectile motion developed by students in *Physics in Action, Section 5* are adapted to the $1/6\,g$ environment of the Moon. You may wish to review the student text and the *Background Information* for *Physics in Action*, *Section 5* before proceeding in this section. All of the equations for projectile motion presented in that section may be applied to comparing quantities on the Moon to corresponding quantities on Earth by substituting $g/6$ for g in the equations shown in the next column.

In each equation, g represents the acceleration due to gravity on Earth. For a projectile launched horizontally at speed v from height h on the Moon, the time of flight is

$$t = \sqrt{2h/(g/6)} = \sqrt{(6)\,2h/g} = \sqrt{6}\sqrt{2h/g} = (2.45)\sqrt{2h/g}.$$

The range (horizontal distance) is $R = vt = v\sqrt{6}\sqrt{2h/g} = v(2.45)\sqrt{2h/g}$ Compared to Earth, both the time of flight and the range of a projectile launched horizontally are increased by a factor of $\sqrt{6} = 2.45$ on the Moon. For the general case of a projectile launched from ground level at speed v at an angle θ above the horizontal, and traveling over flat ground on the Moon (again, in each equation, g represents the acceleration due to gravity on Earth), the speed in the horizontal direction is $v_x = v\cos\theta$ (which remains constant).

The initial speed in the vertical direction is $v_{y0} = v\sin\theta$. The speed in the vertical direction at time t is $v_y = (v\sin\theta) - (g/6)t$. The horizontal position at time, t is $v_x t = (v\cos\theta)t$. The vertical position at time t is found using $y = (v\sin\theta)t - \frac{1}{2}(g/6)t^2$. The time to reach maximum height is $t_{max} = (v\sin\theta)/(g/6) = 6(v\sin\theta)/g$. The total time of flight is twice the time required to reach the maximum height, or $t = 2t_{max}$ (see t_{max}, above). The range (horizontal distance) is $R = \left(v^2\sin 2\theta\right)/(g/6) = 6\left(v^2\sin 2\theta\right)/g$. Compared to Earth, both the range and time of flight of a projectile launched at an angle θ are increased by a factor of 6 on the Moon. Since the time for which the projectile rises is increased by a factor of 6, it follows that the maximum height of the projectile is also increased by a factor of 6.

Crucial Physics

- The rules governing projectile motion are the same on the Moon as on Earth.
- The accelerated vertical motion of a projectile and the constant velocity horizontal motion of a projectile are independent of each other.
- The horizontal distance covered by a projectile is found by multiplying the horizontal velocity times the time of flight, $d_x = v_x \cdot t$. The vertical position of a projectile is found by multiplying the vertical velocity by the flight time, and then subtracting the distance the projectile would fall under the influence of gravity in that time, or $d_y = v_y t - \frac{1}{2} g t^2$.
- The range and time of flight of a projectile launched at an angle on the Moon would be six times greater that the range of an identical projectile on Earth.

Learning Outcomes	Location in the Section	Evidence of Understanding
Apply the acceleration due to gravity on Earth to projectile motion on Earth.	***Investigate*** Step 1	Students apply the acceleration due to gravity on Earth to plot a projectile's path by combining its horizontal and vertical components of motion on a scale drawing.
Apply this understanding to describe the acceleration due to gravity on the Moon to projectile motion on the Moon.	***Investigate*** Steps 2 and 3	Students apply the acceleration due to gravity on the Moon to plot a projectile's path by combining its horizontal and vertical components of motion on a scale drawing.
Design a mathematical model and a physical model of a trajectory of a projectile on the Moon.	***Investigate*** Steps 2 and 3	Students sketch the trajectory of a projectile on the Moon to the same scale as a projectile launched on Earth. They calculate the ratio of the projectile height, range and time of flight on the Moon as compared to Earth.

Section 4 Materials, Preparation, and Safety

Materials and Equipment

PLAN A		
Materials and Equipment	**Group (4 students)**	**Class**
Ruler, metric, in./cm	1 per group	
Protractor	1 per group	
Piece of paper*	4 per group	
Pencil*	8 per group	

*Additional items needed not supplied

Time Requirement

- Allow two class periods or 90 minutes for students to complete the *Investigate* portion of the section.

Teacher Preparation

- Students will need a blank sheet of 8½ by 11 copy paper or graph paper for *Step 1* of the *Investigate*, and at least 6 sheets of the same size paper to be taped together for *Step 2* of the *Investigate*.
- You may wish to obtain larger sheets of paper, such as freezer paper used in commercial establishments, butcher paper, or paper from a small publishing firm in place of taping together individual sheets of paper for *Step 2*.
- Each student should have a ruler and protractor to complete the *Investigate*.

Safety Requirements

- There are no particular safety requirements for this section.

Materials and Equipment

PLAN B		
Materials and Equipment	**Group (4 students)**	**Class**
Ruler, metric, in./cm		1 per class
Protractor		1 per class
Piece of paper*	4 per group	
Pencil*	8 per group	

*Additional items needed not supplied

Time Requirement

- Allow one class period or 45 minutes for the teacher demonstration of the section.

Teacher Preparation

- You may choose to have the students plot several points on their own copies as you construct the transparency.

Safety Requirements

- There are no particular safety requirements for this section.

Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

Learning Issue	Reference	Augmentation and Accommodations
Making a scaled drawing	***Investigate*** Steps 1-2 ***Physics to Go*** Question 6	**Augmentation** • Following a list of directions is difficult for students with reading and/or executive function struggles. These difficulties are compounded when the directions are intricate and require students to accurately measure and do calculations. • Guide students through the *Investigate* step-by-step. Ask students to read each step individually and complete that step on their papers. Then model that step on a teacher drawing to allow students to check their progress. Students who feel confident to follow directions independently could move on and work at their own pace while students who are struggling could keep pace with you. • Remind students to use the metric sides of their rulers. If the scales are confusing students, cover the inch scale with tape. • If the fractions and scaled conversions are confusing students, develop a class drawing that is full-scale instead of $1/10$ scale. • Tell students to use a different color to represent their Moon data on the scaled drawing. **Accommodation** • Provide a copy of the *Investigate* that has each step highlighted with alternating colors. For example, in *Step 1.a)*, there are three different directions for students to follow, so the first direction could be highlighted yellow, the second could be highlighted pink, and the third highlighted blue. The highlighting helps students focus on one step at a time. Then they could check off the steps as they complete them. • Provide a scaled drawing that has been completed for *Steps 1.a), 1.b),* and *1.c)*. Ask students to complete the chart in *Step 1.d)* and finish *Steps 1.f)-1.h)*.
Comparing and summarizing results	***Investigate*** Steps 3 and 4	**Augmentation** • Accomplishing *Steps 1* and *2* accurately may have been very difficult for some students. To help students write an accurate summary for *Step 4*, have a group discussion to compare results for *Step 3*. Ask students to share their maximum heights, ranges, and times of flight. Were the results similar? If not, what might be the reasons for the differences? Once the class has agreed on some common values, students can individually complete the calculations in *Step 3*. Then they can write and share their summaries in *Step 4*.

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Learning Issue	Reference	Augmentation and Accommodations
Solving projectile motion problems	***Physics Talk*** ***Active Physics Plus*** ***Physics to Go*** Questions 2-4	**Augmentation** • Projectile motion problems require students to complete a series of calculations to solve one problem. Students who struggle with math computation and problem-solving or struggle to organize their work usually have a difficult time with multi-step problems. Make sure students understand that the vertical component and horizontal component are different and should be calculated separately. • Ask students to read through the sample problem individually, model a similar problem that is teacher-directed, and then provide an opportunity for guided practice that is student-directed. • Ask students to generate a two-column chart with the math calculations in the left-hand column and the corresponding explanations in the right-hand column. This will help you analyze student misunderstandings and errors in problem-solving. • Make sure students understand the more basic computation in the *Physics Talk* before moving on to use the equations in *Active Physics Plus*. The subscripts in these equations are especially confusing for students. Creating a key to clarify variables may help students. **Accommodation** • Provide a two-column chart with some sample problems completed and explained. Then students can use this chart to work on solving similar problems.

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Strategies for Students with Limited English-Language Proficiency

Learning Issue	Reference	Augmentation
Following complex procedures	***Investigate***	Break down the *Investigate* into smaller chunks to allow students to comprehend each portion of the *Investigate* before moving on to the next one. This approach will allow students to become comfortable following the procedures outlined within each step, and also to internalize new concepts introduced within a step. Make sure students take care with their measurements and markings, and take time to understand what the markings represent. Because the markings are extensive and intricate, you may wish to work through the steps ahead of time, making a set of drawings for your own quick reference when walking among students to monitor their progress. When students make and fill in their tables in their *Active Physics* logs, remind them to record their data carefully and be diligent when calculating the scaled values. When students work on *Step 3*, encourage them again to take care with their calculations, and to include a detailed explanation of their work in their logs. For *Step 4*, remind students to organize their thoughts before they begin writing and to write in complete sentences and use content vocabulary when appropriate. When you review the summaries, comment on students' grammar, sentence structure, and punctuation, as well as the science content.

Two important aspects of learning a new language are speaking and writing in that language. Some ELL students will be self-conscious and shy about speaking in front of their peers, while others will be less reluctant. Be sure to encourage all ELL students to speak in class, and give them opportunities to write on the board from time to time. Experience will broaden their comfort level. Over time, the shy students will become increasingly less self-conscious about speaking in front of their classmates.

With that in mind, hold a class discussion to review this section. Call on ELL students to answer or address the bulleted items below.

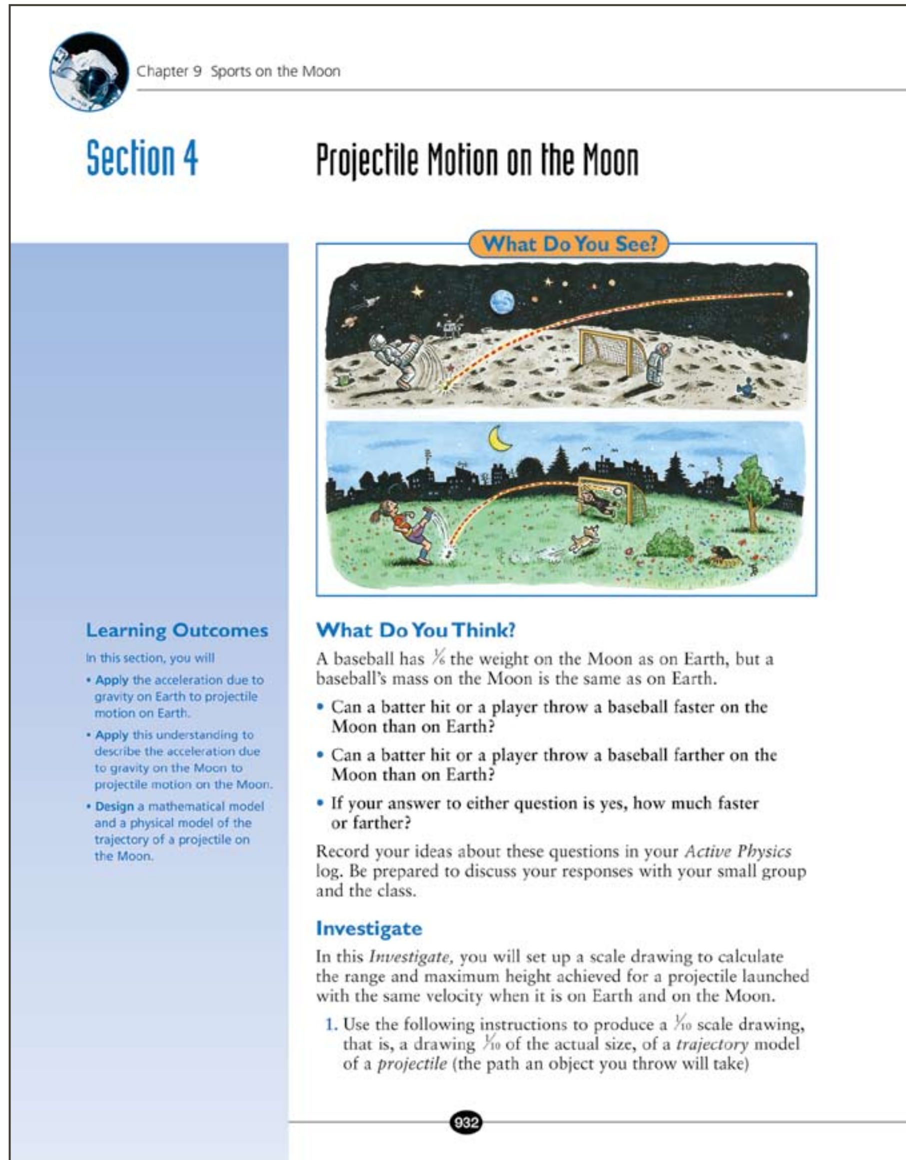
- The motion of a projectile is composed of two independent motions. What are they? [Constant horizontal velocity and accelerated vertical motion]
- What is a trajectory? [The path of a projectile] For motion that starts and ends at the surface of a planet, what shape will a trajectory take? [A parabola]
- Think of the time a projectile takes to reach its maximum height, and then of the time the projectile takes to come down from its maximum height. What is true of these two times? [They are equal.]
- In the context of this section, what does "six times the distance and six times the time" mean? [A projectile launched on the Moon will travel a distance six times greater than the distance the same projectile will travel on Earth when launched at the same angle and the same speed. The projectile will be in motion six times longer on the Moon than on Earth.]

SECTION 4

Teaching Suggestions and Sample Answers

What Do You See?

The two visuals that have contrasting images evoke an immediate curiosity about the soccer ball being kicked at the goal. You might want to initiate a question on why it goes flying past its target on the Moon. Record students' responses as they come forth. Consider asking students if they believe it is the lack of air on the Moon that is the cause of the different trajectories, or if there could be another reason. Ask students, "What physics concepts is the artist trying to capture?" Encourage students to look closely at the *What Do You See?* visual in order to link it to the title of this section.


Chapter 9 Sports on the Moon

Section 4 Projectile Motion on the Moon

What Do You See?

Learning Outcomes

In this section, you will

- **Apply** the acceleration due to gravity on Earth to projectile motion on Earth.
- **Apply** this understanding to describe the acceleration due to gravity on the Moon to projectile motion on the Moon.
- **Design** a mathematical model and a physical model of the trajectory of a projectile on the Moon.

What Do You Think?

A baseball has $\frac{1}{6}$ the weight on the Moon as on Earth, but a baseball's mass on the Moon is the same as on Earth.

- Can a batter hit or a player throw a baseball faster on the Moon than on Earth?
- Can a batter hit or a player throw a baseball farther on the Moon than on Earth?
- If your answer to either question is yes, how much faster or farther?

Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your responses with your small group and the class.

Investigate

In this *Investigate*, you will set up a scale drawing to calculate the range and maximum height achieved for a projectile launched with the same velocity when it is on Earth and on the Moon.

1. Use the following instructions to produce a $\frac{1}{10}$ scale drawing, that is, a drawing $\frac{1}{10}$ of the actual size, of a *trajectory* model of a *projectile* (the path an object you throw will take)

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Students' Prior Conceptions

This section allows students to put into practice scale modeling, ratio and proportion. As they construct scale models to determine the range and maximum height of a projectile on the Moon as compared to that on Earth, review the following misconceptions.

1. **An object traveling fast enough in the horizontal direction can defy gravity**. You might ask, "How fast does a projectile travel in the horizontal direction and what is acting upon it at every instant along this horizontal path?" Regardless of the horizontal speed, gravity acts on the projectile and constantly pulls it down toward the center of Earth or the Moon. This pull may be less on the Moon but gravity does not turn off with speed. You might review *Section 4* from *Physics in Action* and have students project two pennies or two heavy marbles from the tabletop at the same time with two different forces. Ask, which will hit the floor first? Extend this demonstration to two identical objects, one of which has an extremely high speed and one of which drops from rest from the same location. Which hits the ground first? Revisiting these conversations may alert you to a student who continues to hold the misconception mentioned in bold text.
2. **The faster something travels horizontally, the slower it falls.** The conversations and the interventions conducted by you for the first misconception will apply to this one, too.
3. **Objects fall with a constant velocity.** A great way to intervene in this case is to emphasize the distances the hammer and the feather fell on the Moon when they were

What Do You Think?

The questions in this section are designed for students to analyze what they know about the difference between mass and weight and apply that understanding to how a baseball's motion would be affected on the Moon in terms of velocity and distance. Ask students what factors might affect how fast they could throw a ball and if these would be different on the Moon, and what factors might affect how far they could throw a ball on Earth and the Moon. Have them recall what they learned in the previous section while they think of possible answers for these questions. This will be an opportunity for you to note the misconception students might have and address them at a later stage. You may want to discuss students' personal experiences of playing baseball and how they were or were not able to affect the motion of a baseball while batting. Encourage students to record their answers in their *Active Physics* logs.

What Do You Think?

A Physicist's Response

Since the inertial properties of a bat and ball are the same on the Moon as on Earth, a player cannot throw or hit a ball faster on the Moon than on Earth. Because *g* on the Moon is 1/6 *g* on Earth, a ball can be thrown six times farther on the Moon than on Earth.

dropped by Commander Scott in the video that students watched in *Section 2* of this chapter. Reinforce that the hammer and the feather fell at increasing distances in successive time intervals.

4. **Gravity only operates when an object falls.** You can root out this prior conception in an extension of the discussion on falling objects and their speeds in the horizontal and the vertical directions. "Would you be sitting in your chair if gravity only operated when an object falls?" is a question you might want to ask.

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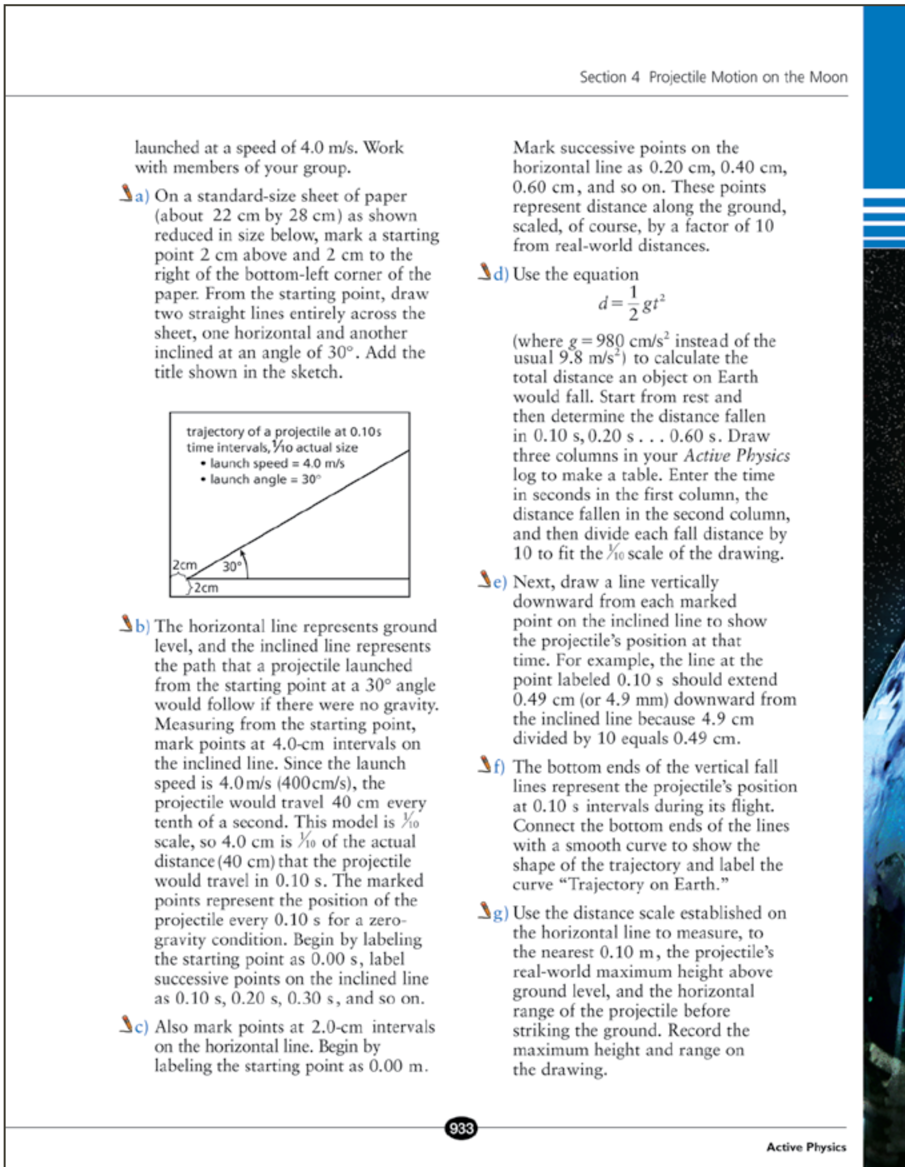

launched at a speed of 4.0 m/s. Work with members of your group.

a) On a standard-size sheet of paper (about 22 cm by 28 cm) as shown reduced in size below, mark a starting point 2 cm above and 2 cm to the right of the bottom-left corner of the paper. From the starting point, draw two straight lines entirely across the sheet, one horizontal and another inclined at an angle of 30°. Add the title shown in the sketch.

b) The horizontal line represents ground level, and the inclined line represents the path that a projectile launched from the starting point at a 30° angle would follow if there were no gravity. Measuring from the starting point, mark points at 4.0-cm intervals on the inclined line. Since the launch speed is 4.0 m/s (400 cm/s), the projectile would travel 40 cm every tenth of a second. This model is 1/10 scale, so 4.0 cm is 1/10 of the actual distance (40 cm) that the projectile would travel in 0.10 s. The marked points represent the position of the projectile every 0.10 s for a zero-gravity condition. Begin by labeling the starting point as 0.00 s, label successive points on the inclined line as 0.10 s, 0.20 s, 0.30 s, and so on.

c) Also mark points at 2.0-cm intervals on the horizontal line. Begin by labeling the starting point as 0.00 m. Mark successive points on the horizontal line as 0.20 cm, 0.40 cm, 0.60 cm, and so on. These points represent distance along the ground, scaled, of course, by a factor of 10 from real-world distances.

d) Use the equation

$$d = \frac{1}{2} g t^2$$

(where $g = 980\ \text{cm/s}^2$ instead of the usual $9.8\ \text{m/s}^2$) to calculate the total distance an object on Earth would fall. Start from rest and then determine the distance fallen in 0.10 s, 0.20 s . . . 0.60 s. Draw three columns in your *Active Physics* log to make a table. Enter the time in seconds in the first column, the distance fallen in the second column, and then divide each fall distance by 10 to fit the 1/10 scale of the drawing.

e) Next, draw a line vertically downward from each marked point on the inclined line to show the projectile's position at that time. For example, the line at the point labeled 0.10 s should extend 0.49 cm (or 4.9 mm) downward from the inclined line because 4.9 cm divided by 10 equals 0.49 cm.

f) The bottom ends of the vertical fall lines represent the projectile's position at 0.10 s intervals during its flight. Connect the bottom ends of the lines with a smooth curve to show the shape of the trajectory and label the curve "Trajectory on Earth."

g) Use the distance scale established on the horizontal line to measure, to the nearest 0.10 m, the projectile's real-world maximum height above ground level, and the horizontal range of the projectile before striking the ground. Record the maximum height and range on the drawing.

Investigate

1.a)

Students prepare the sheet of paper to plot the trajectories.

1.b)

Students mark off the position of the projectile at 4-cm intervals along the inclined line representing the distance traveled every 1/10 of a second and label the successive points on the inclined line.

1.c)

Students mark off the successive distance points on the *x*-axis.

1.d)

Students use the equation $d = \frac{1}{2} g t^2$ to calculate the distance the projectile falls from the inclined line during the time from launch and then divide each distance by 10 to fit the scale drawing.

Time (in seconds)	Earth Fall distance (in cm)	Earth Fall distance / 10 (in cm)
0.10	4.9	0.49
0.20	19.6	1.96
0.30	44.1	4.41
0.40	78.4	7.84
0.50	122.5	12.3
0.60	176.4	17.6

1.e)

Students plot the scaled-down fall positions of the projectile by measuring downward from the inclined line at the time intervals associated with that fall distance.

1.f)

Students draw a smooth curve through the points and label the curve "Trajectory on Earth."

1.g)

Students should measure a height of 2 cm or about 20 cm real-world height. The measured range should be approximately about 14 cm or 1.4 m real-world distance.

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1.h)

The time of flight will be very close to 0.4 s.

2.a)

Values for the first second are shown in the table below.

Time (in seconds)	Moon Fall distance (in cm)	Moon Fall distance / 10 (in cm)
0.10	0.8	0.08
0.20	3.2	0.32
0.30	7.2	0.72
0.40	12.8	1.28
0.50	20	2.0
0.60	28.8	2.88
0.70	39.2	3.92
0.80	51.2	5.12
0.90	64.8	6.48
1.0	80	8.0

2.b)

Students plot scaled-down fall positions of the projectile by measuring points on the inclined line at time intervals associated with the fall distance, then measuring down from the inclined line the associated fall distance to plot each point.

2.c)

Students draw a smooth curve through the points and label the curve "Trajectory on the Moon."

2.d)

The measured height should be 12.5 cm or 1.25 m in the real world. The measured range should be 86.6 cm or 8.66 m in the real world, and the time of flight should be 2.5 s.

3.a)

The ratio of the maximum height on Earth to the maximum height on the Moon is 20 cm/125 cm ≈ 1/6.

h) Use the time scale established on the inclined line to measure, to the nearest 0.010 s, the projectile's time of flight. Record the time of flight on the drawing.

2. You will now draw the trajectory that would result if the projectile were launched at the same speed and in the same direction on the Moon.

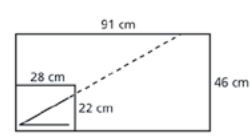
a) Use the same equation

$$d = \frac{1}{2}gt^2$$

to calculate the total distance an object on the Moon falls, starting from rest, in 0.10 s, 0.20 s, 0.30 s, and so on. The value of acceleration to use in the equation is the acceleration due to gravity on the Moon, 1.6 m/s^2, or 160 cm/s^2.

Prepare a table in your *Active Physics* log, making it similar to the table for Earth distances fallen, to show the calculated value of the total distance of fall at the end of each 0.10 s of flight on the Moon. Enter the time in seconds in the first column, the distance fallen in the second column, and then divide each fall distance by 10 to fit the 1/10 scale of the drawing. Draw the trajectory for the projectile on the Moon in a similar manner as the trajectory you drew on Earth.

b) Draw a vertical line downward from each marked point on the inclined line to show the projectile's position at that time on the Moon. For example, the line at the point labeled 0.30 s should extend 0.72 cm, or 7.2 mm, downward from the inclined line. This line, and others, will need to be drawn on top of or immediately next to the lines drawn earlier for fall distances on Earth.



Extend the size of the paper to be able to show the entire trajectory on the Moon as shown in the sketch. Tape the sheet of paper containing your drawing to the lower left-hand corner of a sheet of wrapping paper approximately 46 cm high and 91 cm wide.

c) The bottom ends of the vertical fall lines represent the projectile's position at 0.10 s intervals during its flight on the Moon. Connect the bottom ends of the lines with a smooth curve to show the shape of the trajectory and label the curve "Trajectory on the Moon."

d) Use the distance and time scales on the drawing to measure the projectile's maximum height, range, and time of flight on the Moon. Record the values on the drawing. Fold and save your drawing.

3. Create a table to show the above measurements of the maximum heights, ranges, and times of flight of a projectile launched with equal initial velocities on Earth and the Moon to complete the calculations below.

a) $\frac{\text{Max height of projectile on Earth}}{\text{Max height of projectile on the Moon}}$

b) $\frac{\text{Range of projectile on Earth}}{\text{Range of projectile on the Moon}}$

c) $\frac{\text{Time of flight on Earth}}{\text{Time of flight on the Moon}}$

d) Show your work and discuss it in your *Active Physics* log.

4. Write a summary of the effects of the Moon's "$\frac{1}{6}g$" on the maximum height, range, and time of flight of a projectile launched on the Moon as compared to the same projectile launched at the same speed and angle of elevation on Earth.

a) Record your summary in your *Active Physics* log.

3.b)

The ratio of the range on Earth to the range on the Moon is 1.4 m/8.66 m ≈ 1/6.

3.c)

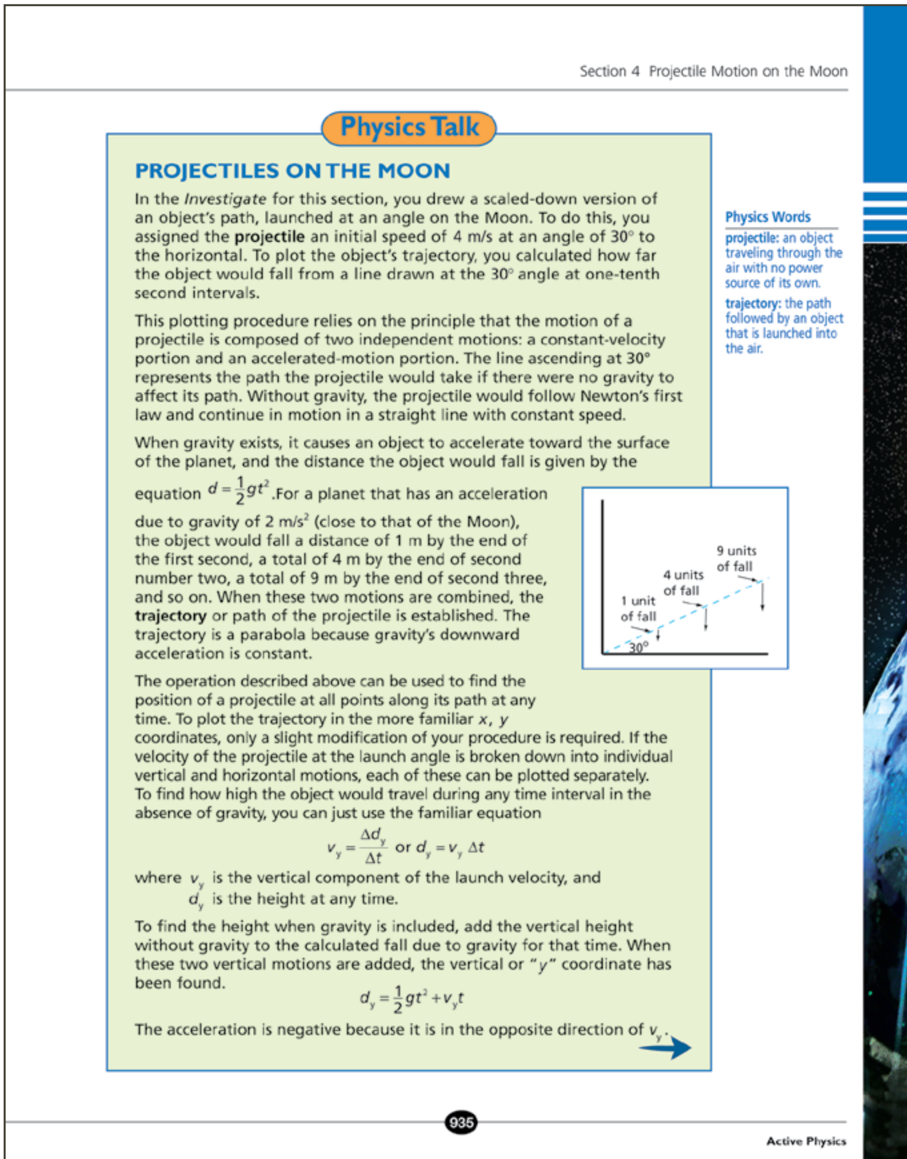
The ratio of the time of flight on Earth to the time of flight on the Moon is 0.4 s/2.5 s ≈ 1/6.

3.d)

Students should record their calculations in their logs.

4.a)

Summarizing, the flight times, maximum height and range for a projectile launched at an upward angle to the horizontal on the Moon increases by a factor of six over those values for Earth.



Section 4 Projectile Motion on the Moon

Physics Talk

PROJECTILES ON THE MOON

In the *Investigate* for this section, you drew a scaled-down version of an object's path, launched at an angle on the Moon. To do this, you assigned the **projectile** an initial speed of 4 m/s at an angle of 30° to the horizontal. To plot the object's trajectory, you calculated how far the object would fall from a line drawn at the 30° angle at one-tenth second intervals.

This plotting procedure relies on the principle that the motion of a projectile is composed of two independent motions: a constant-velocity portion and an accelerated-motion portion. The line ascending at 30° represents the path the projectile would take if there were no gravity to affect its path. Without gravity, the projectile would follow Newton's first law and continue in motion in a straight line with constant speed.

When gravity exists, it causes an object to accelerate toward the surface of the planet, and the distance the object would fall is given by the equation $d = \frac{1}{2}gt^2$. For a planet that has an acceleration due to gravity of 2 m/s^2 (close to that of the Moon), the object would fall a distance of 1 m by the end of the first second, a total of 4 m by the end of second number two, a total of 9 m by the end of second three, and so on. When these two motions are combined, the **trajectory** or path of the projectile is established. The trajectory is a parabola because gravity's downward acceleration is constant.

The operation described above can be used to find the position of a projectile at all points along its path at any time. To plot the trajectory in the more familiar x, y coordinates, only a slight modification of your procedure is required. If the velocity of the projectile at the launch angle is broken down into individual vertical and horizontal motions, each of these can be plotted separately. To find how high the object would travel during any time interval in the absence of gravity, you can just use the familiar equation

$$v_y = \frac{\Delta d_y}{\Delta t} \text{ or } d_y = v_y \Delta t$$

where v_y is the vertical component of the launch velocity, and
d_y is the height at any time.

To find the height when gravity is included, add the vertical height without gravity to the calculated fall due to gravity for that time. When these two vertical motions are added, the vertical or "y" coordinate has been found.

$$d_y = \frac{1}{2}gt^2 + v_y t$$

The acceleration is negative because it is in the opposite direction of v_y.

Physics Words

projectile: an object traveling through the air with no power source of its own.

trajectory: the path followed by an object that is launched into the air.

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Active Physics

Physics Talk

Students recall the *Investigate* to analyze the motion of a projectile. The *Physics Talk* establishes the purpose of plotting an inclined line and highlights the two independent motions of a projectile under the influence of gravity. They learn that in the absence of gravity, the projectile would continue in motion in a straight line with constant speed. In the presence of gravity, the object accelerates downward.

Discuss the combination of horizontal and vertical motions that are used to plot the path of a projectile. Point out that the path of a trajectory is a parabola because the downward acceleration due to gravity is constant. Ask students how they would calculate the fall distance of an object thrown upward at an angle. Have them describe how the trajectory of a projectile can be plotted from x, y coordinates and on the graph to determine how high the object would travel using the distance-time equation. Ask students why a scaled-down version of velocities helps in plotting the path of a projectile. Discuss the sample problem and relate it to how the scale drawing helps in the calculation of fall distance.

9-4a **Blackline Master**

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The projectile's horizontal position at any time can be found from the equation

$$v_x = \frac{\Delta d_x}{\Delta t} \text{ or } d_x = v_x \Delta t$$

where v_x is the horizontal component of velocity and
d_x is the horizontal distance at any time.

Nothing needs to be added to the horizontal position, since the force of gravity works only in the vertical direction. The path of a projectile affected only by gravity (no air resistance) has a specific curved shape. This curve is referred to as a parabola. You drew a parabolic path on Earth and the Moon in the *Investigate*.

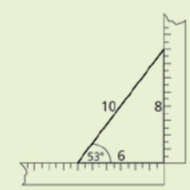
The vertical and horizontal components of the launch velocity can be found either by graphical methods (measuring) or by calculation. To find the velocities graphically, draw a right triangle with an angle to the horizontal the same as the launch velocity. Make the length of the hypotenuse equal to a scaled down version of the launch velocity. When you use a ruler to measure the size of the vertical and horizontal sides of the triangle, you have found the scaled down size of the vertical and horizontal launch velocities. You then scale the velocities up to the correct values.

To calculate the vertical and horizontal launch velocities, trigonometry can be used.

Knowing these velocities and using this method allows you to solve numerous problems in trajectories.

Sample Problem

A field-goal kicker in a football game on the Moon kicks the ball at an angle of 53° to the horizontal with a speed of 10.0 m/s. Will the football clear the crossbar 3.0 m above the ground if the goal post is 54 m away?



Strategy: First, measure the vertical and horizontal components of the velocity by setting up a scale diagram similar to the one on the right. The measured vertical and horizontal velocities then will be $v_y = 8$ m/s and $v_x = 6$ m/s, respectively.

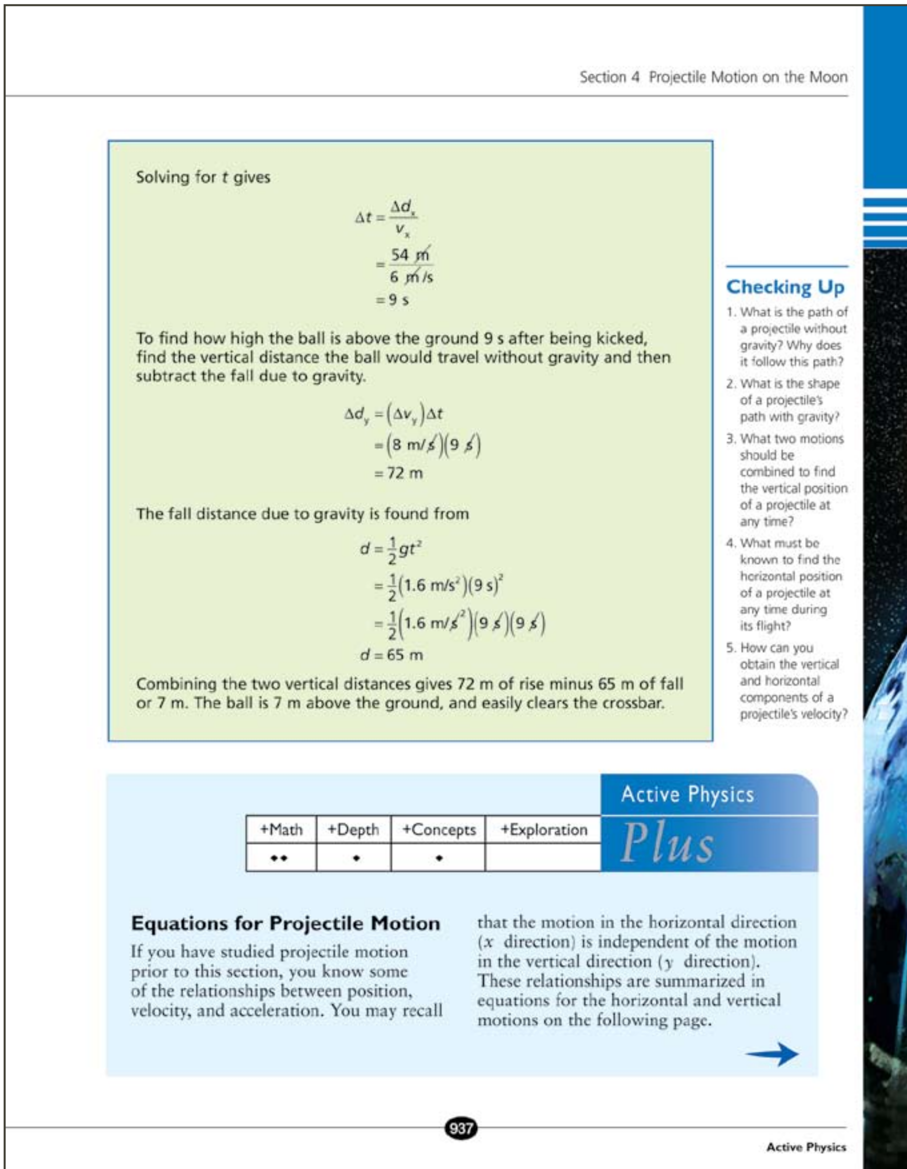
Given: $a_g = 1.6 \text{ m/s}^2$

$v_y = 8$ m/s
$v_x = 6$ m/s

Solution:

The time it takes the ball to travel the 54 m horizontally to the goal post is found using the horizontal velocity equation

$$v_x = \frac{\Delta d_x}{\Delta t}$$



Solving for t gives

$$\Delta t = \frac{\Delta d_x}{v_x} = \frac{54\ \text{m}}{6\ \text{m/s}} = 9\ \text{s}$$

To find how high the ball is above the ground 9 s after being kicked, find the vertical distance the ball would travel without gravity and then subtract the fall due to gravity.

$$\Delta d_y = (\Delta v_y)\Delta t = (8\ \text{m/s})(9\ \text{s}) = 72\ \text{m}$$

The fall distance due to gravity is found from

$$d = \frac{1}{2}gt^2 = \frac{1}{2}(1.6\ \text{m/s}^2)(9\ \text{s})^2 = \frac{1}{2}(1.6\ \text{m/s}^2)(9\ \text{s})(9\ \text{s})$$
$$d = 65\ \text{m}$$

Combining the two vertical distances gives 72 m of rise minus 65 m of fall or 7 m. The ball is 7 m above the ground, and easily clears the crossbar.

Checking Up

1. What is the path of a projectile without gravity? Why does it follow this path?
2. What is the shape of a projectile's path with gravity?
3. What two motions should be combined to find the vertical position of a projectile at any time?
4. What must be known to find the horizontal position of a projectile at any time during its flight?
5. How can you obtain the vertical and horizontal components of a projectile's velocity?

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
••	•	•	

Equations for Projectile Motion

If you have studied projectile motion prior to this section, you know some of the relationships between position, velocity, and acceleration. You may recall that the motion in the horizontal direction (x direction) is independent of the motion in the vertical direction (y direction). These relationships are summarized in equations for the horizontal and vertical motions on the following page.

Checking Up

1.

The path of projectile without gravity would be a straight line. The projectile follows this path due to its inertia, in agreement with Newton's first law, whereby an object in motion continues in motion in a straight line unless acted upon by a net force.

2.

Under the influence of gravity, the path of a projectile is a parabola.

3.

The two motions that must be combined to find the vertical position at any time are the initial vertical velocity and the vertical acceleration. The initial vertical velocity multiplied by the time of flight minus the distance the projectile would fall under the effect of the vertical acceleration for that time will give the vertical position of the projectile.

4.

To find the horizontal position of a projectile at any time, the horizontal component of the projectile's velocity must be known, as well as how long the projectile has been traveling with that speed (the time since the launch).

5.

The vertical and horizontal velocities of a projectile may be found either by making a scale drawing of the velocity and finding the right angle components or by using trigonometry.

Active Physics Plus

This *Active Physics Plus* first analyzes the conditions and equations that govern the flight of a projectile and then derives an equation that is useful in predicting a projectile's time of flight. Students use these equations in problems to develop equations that predict the maximum height and range of a projectile launched at an angle to the horizontal.

CHAPTER 9

1.

Students should agree that the result of the *Investigate* and what is shown by the equation match. To get $y_{max} = -v_{y0}^2/2g$ substitute $t_{max} = -v_{y0}/g$ into $y = v_{y0}t + \frac{1}{2}gt^2$. This yields

$$y_{max} = v_{y0}\left(-v_{y0}/g\right) + \tfrac{1}{2}g(-v_{y0}/g)^2 = -v_{y0}^2/2g.$$

2.

Using the values from the *Investigate* of $v_0 = 4$ m/s and $v_{y0} = 2$ m/s, and inserting into the equation yields

$$y_{max} = \frac{(2 \text{ m/s})^2}{(-2)\times\left(1.6 \text{ m/s}^2\right)} = 1.25 \text{ m}$$

which agrees with *Step 2.b)* of the *Investigate*.

All quantities are considered positive if they are directed upward. The object is assumed to have started at the position (0,0).

$$v_x = v_{x0} \qquad x = v_{x0}t$$

$$v_y = v_{y0} + gt \qquad y = v_{y0}t + \frac{1}{2}gt^2$$

In these equations, x and y are the horizontal and vertical positions, v_{x0} and v_{y0} are the horizontal and vertical components of the velocity at time $= 0$. Here, g is the acceleration due to gravity, and t is the time.

Notice that the horizontal component of the velocity is constant and equal to its value at time $= 0$. Finally, g is a negative number since the acceleration due to gravity is down ($-9.8\,\text{m/s}^2$ on Earth and $-1.6\,\text{m/s}^2$ on the Moon).

In analyzing projectile motion where the object starts at some height and returns to the same height, there is an easy way to use these relationships. Note that the motion from the start up to the point of maximum height is the same as the motion from the maximum height down to the finish except that one is the reverse of the other. The important point is that the time it takes for the projectile to reach its maximum height is the same as the time it takes for it to descend from its maximum height to the finish. Find the time it takes to reach its maximum height first, and use this result to find other quantities of interest.

As the projectile goes up, the vertical component of its velocity v_y decreases. At the same time the horizontal component of its velocity v_x remains constant. At some point in time v_y decreases to zero, then through zero, and finally becoming negative. When v_y is positive, the projectile is rising, and when v_y is negative, the projectile is falling. The point at which v_y is zero is the point of maximum height of the projectile. Setting $v_y = 0$ yields

$$0 = v_{y0} + gt_{max} \qquad \text{or} \qquad t_{max} = \frac{v_{y0}}{-g}.$$

The total time the projectile is in the air t_{total} is twice the time it takes to reach maximum height, so

$$t_{total} = 2t_{max} = \frac{2v_{y0}}{-g}.$$

From this result, it is easy to see that if two projectiles on Earth and on the Moon start off with the same value of v_{y0}, the time in flight is six times longer on the Moon because the value for the acceleration due to gravity g is one sixth as large on the Moon.

1. Did your results for the two trajectories you constructed in the *Investigate* agree with this?

Finding the maximum height is not difficult now that you know the time it takes the projectile to reach its maximum height. Simply substitute t_{max} into the equation for y (let $y_0 = 0$) and see if you can get the equation

$$y_{max} = \frac{v_{y0}^2}{-2g}$$

2. How should the maximum heights of a projectile launched with the same value of v_{y0} on Earth and the Moon compare? Compare this with your constructed trajectories.

The range of the projectile (denoted R or x_{max}) can be determined by finding the horizontal position x at time equal to t_{total}. Try this for yourself. The result is

$$R = x_{max} = \frac{2v_{x0}v_{y0}}{-g}$$

3. How should the ranges of projectiles on Earth and on the Moon compare if they start with the same values for both of the two components of the velocity? Is this what you found in the *Investigate*?

4. *Question 3* in *Physics to Go* states that if a projectile is launched at 45°, the range is the velocity squared divided by the acceleration due to gravity (considered positive). See if you can derive this result starting with the more general equation for range given above.

What Do You Think Now?

At the beginning of this section you were asked the following questions:

- **Can a batter hit or a player throw a baseball faster on the Moon than on Earth?**
- **Can a batter hit or a player throw a baseball farther on the Moon than on Earth?**
- **If your answer to either question is yes, how much faster or farther?**

Based on what you have learned about projectile motion, how would you answer these questions now? Record your answers in your *Active Physics* log.

What Do You Think Now?

As students are allowed time to revise their *What Do You Think?* answers, ask them to consider the difference between inertia and gravity. Share *A Physicist's Response* and discuss what should happen when a batter hits a baseball on the Moon and how the motion of the baseball compares to its motion on Earth. Encourage students to recall the explanation of projectile motion in the *Physics Talk* and make sure that their answers are accurate. It is important for students to discuss their responses with their classmates and feel confident about how well they know the physics of projectile motion. Remind students that they can revisit the *What Do You See?* section to better appreciate the artists' rendition of projectile motion.

3.

To get the range, use the range equation $R = x_{max} = v_{x0}(t)$ and substitute in the total time $t_{total} = -2v_{y0}/g$ which gives $R = x_{max} = v_{x0}(-2v_{y0}/g) = -2v_{x0}v_{y0}/g$. The range of a projectile on the Moon will be six times greater than on Earth since g_{Moon} is one-sixth the size of g_{Earth}.

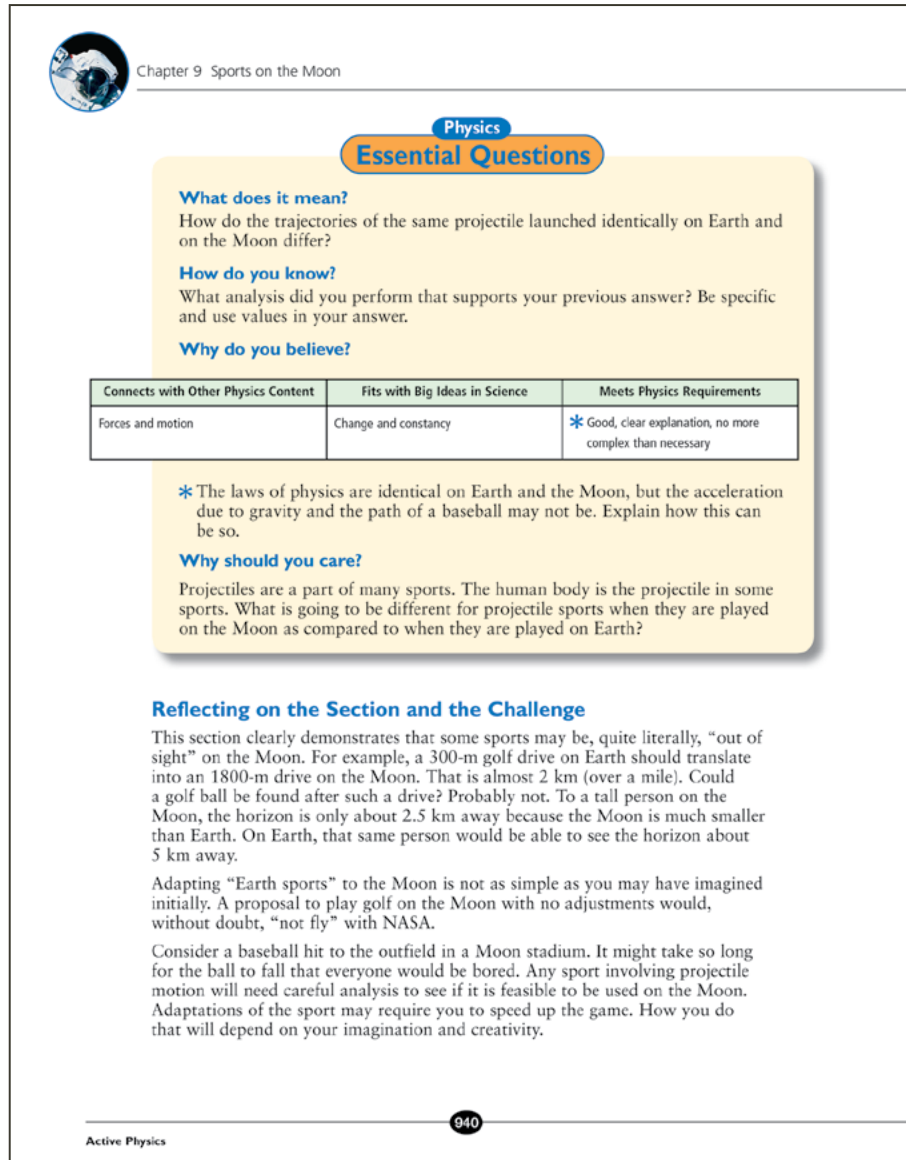
4.

Using $R = x_{max} = -2v_{x0}v_{y0}/g$, substitute in for $v_{x0} = v_0(\cos\theta)$ and for $v_{y0} = v_0(\sin\theta)$. At 45°, $\sin\theta = \cos\theta = 0.7071$, so $v_{x0} = (0.7071)v_0$ and $v_{y_0} = (0.7071)v_0$. Inserting these values into the equation gives $R = -2(0.7071)v_0(0.7071)v_0/g = -v_0^2/g$.

CHAPTER 9

Reflecting on the Section and the Challenge

Students now have the chance to read and apply the text of *Reflecting on the Section and the Challenge*. Have them highlight what obstacles would have to be overcome if certain sports were played on the Moon. Ask them to reflect on how a sport should be adapted to the Moon and encourage them to share their reflections in class. Allow time for students to prepare a careful analysis of their *Chapter Challenge*. Encourage them to recall what they have learned so far and apply it in designing a sport that is both creative and adaptable to conditions on the Moon. Emphasize that the sport they choose should be convincing enough for NASA to consider in a proposal.



Physics

Essential Questions

What does it mean?

How do the trajectories of the same projectile launched identically on Earth and on the Moon differ?

How do you know?

What analysis did you perform that supports your previous answer? Be specific and use values in your answer.

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	Change and constancy	* Good, clear explanation, no more complex than necessary

* The laws of physics are identical on Earth and the Moon, but the acceleration due to gravity and the path of a baseball may not be. Explain how this can be so.

Why should you care?

Projectiles are a part of many sports. The human body is the projectile in some sports. What is going to be different for projectile sports when they are played on the Moon as compared to when they are played on Earth?

Reflecting on the Section and the Challenge

This section clearly demonstrates that some sports may be, quite literally, "out of sight" on the Moon. For example, a 300-m golf drive on Earth should translate into an 1800-m drive on the Moon. That is almost 2 km (over a mile). Could a golf ball be found after such a drive? Probably not. To a tall person on the Moon, the horizon is only about 2.5 km away because the Moon is much smaller than Earth. On Earth, that same person would be able to see the horizon about 5 km away.

Adapting "Earth sports" to the Moon is not as simple as you may have imagined initially. A proposal to play golf on the Moon with no adjustments would, without doubt, "not fly" with NASA.

Consider a baseball hit to the outfield in a Moon stadium. It might take so long for the ball to fall that everyone would be bored. Any sport involving projectile motion will need careful analysis to see if it is feasible to be used on the Moon. Adaptations of the sport may require you to speed up the game. How you do that will depend on your imagination and creativity.

Physics Essential Questions

What does it mean?

Projectiles on the Moon with the same initial velocity and angle will travel six times farther than they would on Earth and require six times more time.

How do you know?

The amount an object drops during a specific time was computed on Earth and the Moon using the equation $d = \frac{1}{2}gt^2$ with g being either 9.8 m/s² for Earth and 1/6 m/s² for the Moon. This "drop" was plotted from the straight-line path that the object would have without the existence of gravity. When this was performed for many different time intervals, it showed that the trajectory on the Moon was six times farther and six times higher than it would be on Earth.

Why do you believe?

The laws of physics are summarized in equations. However, the values of some of the variables (for example, the mass of the planet) could be different. The acceleration due to gravity on a planet or the Moon are dependent on the same variables, but provide different values due to the difference in mass and size of Earth and the Moon.

Why should you care?

The projectile (a human body or a football) will remain in the air six times as long on the Moon and travel six times farther on the Moon.

Physics to Go

1. Due to the increased time and distance of travel of a projectile, discuss potential adjustments you need to make to play each of the following sports on the Moon:
   a) football
   b) gymnastics
   c) trapeze
   d) baseball
2. A typical sports arena on Earth has a playing field 120-m long and 100-m wide surrounded by tiered seats for spectators. World-class shot-put athletes throw the steel shot 23 m on Earth. Explain whether the spectators would be safe if a shot-put event were held in a stadium of similar size on the Moon.
3. The maximum range of a projectile launched at ground level occurs when the launch angle is 45°. Physicists have shown that the range of a projectile launched at 45° is given by the equation $R = v^2/g$, where R is the range, v is the launch speed, and g is the acceleration due to gravity on the planet or moon where the projectile is launched. How would this equation be useful for estimating the size of facilities needed for sports on the Moon?
4. If a golf ball were hit at a speed of 40 m/s at a launch angle of 45° on the Moon, what would be its range? (Hint: Use the equation $R = v^2/g$, where R is the range, v is the launch speed, and g is the acceleration due to gravity on the Moon.)
5. Since the Moon's gravity is weaker than that on Earth, and since projectiles near the surface of the Moon do not experience air resistance, is it possible for an object thrown straight upward from the surface of the Moon to "escape" the Moon's gravity, never to fall back down to the surface of the Moon? Write a brief statement about your thoughts on this possibility.
6. You have found that the path of a trajectory from the ground to the ground requires six times the distance and six times the time. In basketball, the ball does not start at the ground and the hoop is not on the ground. You can, however, predict the trajectory in the following way:
   a) Draw a person 1.8 m tall and a hoop some distance away that is 3.5 m high.
   b) Draw a parabola that shows the ball moving from the player's head up and down through the hoop.
   c) On the same diagram, draw a horizontal line at the original location of the ball. Extend the basketball's path through the basket until it crosses this line.

Physics to Go

1.a)

For football, the size of the playing area need not be increased unless it is desired to allow the football to be thrown farther. The kicking part of the game would have to be eliminated however, since a kicked football would travel six times farther and higher.

1.b)

The ceiling height, size of the mats for floor exercise and distance between some of the apparatuses in gymnastics would need to be increased since the gymnasts would be able to spend a longer time in the air and jump higher on the Moon.

1.c)

The distance between trapezes and the height of the area where the stunts are performed would both have to increase.

1.d)

Unless there are some modifications in the equipment, a baseball field would need to be six times larger in almost all dimensions, including the height of the dome.

2.

A shot put throw could travel six times 23 m or 138 m on the Moon. This is a large enough distance for it to land in the spectator seats of a typical sports arena on Earth. This would be a very dangerous situation for the spectators.

3.

Since g is one-sixth its value on Earth, this equation tells us that all maximum ranges are six times larger on the Moon. Thus, all sports facilities that might be governed by this equation should be six times larger in all dimensions.

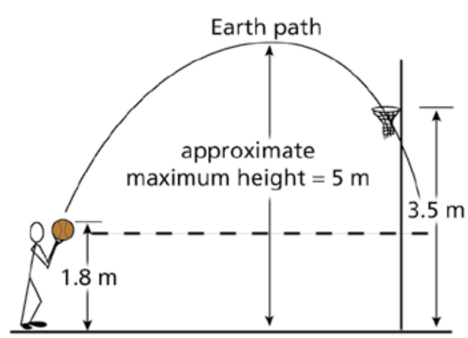
4.

Using $R = v^2/g$ and substituting in the velocity of 40 m/s and $g = 1.6\ \text{m/s}^2$ on the Moon gives $R = (40\ \text{m/s})^2/(1.6\ \text{m/s}^2) = 1000\ \text{m}$.

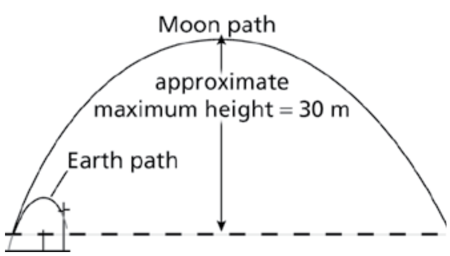
5.

It is possible to give an object on either Earth or the Moon a large enough velocity that it will "escape" the planet's gravitational force and never fall back. Because of the lower value of g and the lack of air resistance on the Moon, the required velocity is much less on the Moon than on Earth. The escape velocity for the Moon is about 1.7 km/s, so the projectile must have a speed significantly higher than most objects we are familiar with.

6.a-c)



6.d)



6.e)

Basketball on the Moon would have to use a more massive ball so that the players could not launch it with a speed as high as on Earth, to reduce the distance and height the ball travels.

7.

Preparing for the Chapter Challenge

Since gravity makes all objects fall at the same rate regardless of the objects mass, just increasing the mass of a projectile while not decreasing its launch velocity will not affect the projectile's range. Making an object more massive will help to reduce the launch velocity if the same force is applied to the massive projectile as to its less massive counterpart. Increasing the mass means that the object's acceleration $a = F/m$ will be less, and if accelerated for approximately the same distance ($v^2 = 2ad$) will yield a proportionately smaller velocity.

d) The basketball on the Moon will travel six times higher from this line and six times further along this line. Approximate the high point of the ball and where it will hit this line. Draw the complete path on the Moon.

e) How will you adapt the game of basketball given this new insight into how the ball moves on the Moon?

7. *Preparing for the Chapter Challenge*

You have seen in this section that the range of a projectile on the Moon is greatly increased. Some of your classmates may suggest that the way to reduce the range of the projectiles is to increase their mass. Using what you know about how objects of different masses are affected by gravity, explain why this will not solve the problem once the projectiles are in the air with equal speeds. But use your knowledge of the principle of inertia to explain why increasing the mass might lead to projectiles with reduced velocity and thus, reduced range.

Inquiring Further

Orbital velocity

You have seen that as celestial bodies get smaller, their gravity becomes weaker, and this weaker gravity leads to increased range. During this investigation, it was assumed that gravity was acting straight down off a flat surface. Real planets are curved. When the curvature is taken into account, is it possible for the range of the projectile to increase so much that the object never comes down? Look up Newton's canon to investigate what happens when the speed of a projectile is increased on a spherical body. Find a reference for the term "orbital velocity" and relate that to the speed required for a projectile never to return to the surface once launched.

Inquiring Further

Students will be able to find numerous descriptions and several excellent Java script simulations of Newton's cannon on the Web. Some simulations are interactive and will allow the student to find the orbital velocity for Earth. Calculation for the orbital velocity of other celestial bodies can also be simply done using the expression $v = \sqrt{G M_p / r_p}$.

NOTES

SECTION 4 QUIZ

9-4b Blackline Master

The acceleration due to gravity on the Moon is 1.6 m/s^2 and is 10 m/s^2 on Earth.

1. Two identical projectiles are launched with the same speed at an angle of 60° to the horizontal. Projectile A is located on Earth, and projectile B is located on the Moon. Compared to the horizontal distance traveled by the projectile on Earth, the horizontal distance traveled by the projectile on the Moon would be

   a) the same.
   b) six times as far.
   c) $\sqrt{6}$ times further.
   d) one-sixth as far.

2. A projectile is launched at an angle of 37° to the horizontal on the Moon. The projectile has a vertical speed of 9 m/s and a horizontal speed of 12 m/s. After 3 s of flight, how far has the projectile traveled horizontally?

   a) 27 m
   b) 36 m
   c) 45 m
   d) $45/6$ m

3. For the projectile in *Question 2*, how far will the projectile fall in three seconds?

   a) One-sixth of a meter
   b) Three-sixths of a meter
   c) 4.8 m
   d) 7.2 m

4. If air resistance is ignored, the path of a projectile on Earth is a parabola. The path of a projectile launched on the Moon would be closest to

   a) a parabola.
   b) half a circle.
   c) a straight line.
   d) an ellipse.

5. A projectile is launched on the surface of the Moon at an angle of 45° with a vertical velocity of 10 m/s. After 5 seconds, how high above the surface would this projectile be?

   a) 50 m
   b) $50/6$ m
   c) 42 m
   d) 30 m

SECTION 4 QUIZ ANSWERS

1. b) Since a projectile spends six times as long in flight on the Moon as compared to Earth, it will travel six times as far horizontally.

2. b) Horizontally, a projectile travels with constant speed. The horizontal distance traveled is given by $d_x = v_x t$ or $d_x = (12\ \text{m/s})(3\ \text{s}) = 36\ \text{m}$. The acceleration due to gravity on the Moon being one-sixth of that on Earth is not a factor in the horizontal direction.

3. d) The distance a projectile falls on the Moon is given by $d_y = \frac{1}{2}gt^2$ or $d_y = \frac{1}{2}\left(1.6\ \text{m/s}^2\right)(3\ \text{s})^2 = 7.2\ \text{m}$.

4. a) The path of a projectile on the Moon is also a parabola, but the parabola would be spread out more on the Moon.

5. d) The height of the projectile above the surface is given by the distance it would travel without the effect of gravity minus the distance it would fall under the influence of gravity. The height above the surface without gravity would be given by $d_y = v_y t$ or $d_y = (10\ \text{m/s})(5\ \text{s}) = 50\ \text{m}$. The distance the projectile falls is given by $d_y = \frac{1}{2}gt^2$ or $d_y = \frac{1}{2}\left(1.6\ \text{m/s}^2\right)\left(5\ \text{s}\right)^2 = 20\ \text{m}$. The difference between these values is the height above the ground, or 30 m.