## SECTION 6

## Momentum and Gravity: Golf on the Moon

## Section Overview

Students investigate how balls made of different material bounce when dropped on the floor. They apply their findings to the bounce height of a golf ball and explore different strategies through which the range of a golf ball can be reduced to play golf on the Moon. In one strategy, they consider altering the bounciness of the ball or changing its mass. In another strategy, they consider changing the mass of the head of the golf club. Students also consider a combination of masses for the golf ball and the head of the golf club to reduce the range of the golf ball on the Moon. Later, students investigate the physics of controlling the range of a ball using energy equations. They learn that a ball that bounces $1 / 6$ as high as a golf ball dropped from the same height has its range decreased by a factor of $1 / 6$.

## Background Information

Two new phenomena are introduced in this section:

- The "bounciness" of various balls compared to a golf ball. That is, how well they rebound as compared to a golf ball.
- Relative motion in a head-on collision between a golf ball and a golf club.
When testing the possibility of "taming" the ball to adapt golf to the Moon, a golf ball and a variety of "less bouncy" balls are tested in a simulation of a golf club hitting a ball. Balls of different sizes and different bounce characteristics are dropped from a standard height to bounce back upward from a hard floor surface. In this simulation, the floor represents the face of the golf club hitting the ball. The assumption is that the forces acting on the ball during the collision and the resulting velocity of the ball after impact are equal. This is independent of
whether it is the face of the golf club (simulated by the floor surface) or the ball which is moving before the collision. The effect of a collision between a golf club and ball depends on the relative motion of one object with respect to the other before the collision. It is true that the change in velocity of a stationary ball after being hit by a club head moving at a certain speed would be equal to the change in velocity that would result if the same ball, moving in the opposite direction at the same speed, collided with a stationary golf club head. Therefore, the initial speeds of a variety of balls all of the same mass hit by the moving head of a golf club would compare in the same way as the initial rebound speeds of a variety of balls dropped from equal heights to strike a hard-floor surface.

Conservation of energy is used to calculate the rebound speeds of balls dropped from a standard height:

$$
\begin{aligned}
& (P E \text { of ball at peak of rebound })= \\
& \text { (Initial rebound } K E \text { ) } \\
& m g h=\frac{1}{2} m v^{2} ; v=\sqrt{2 g h} .
\end{aligned}
$$

It is desirable to find a kind of ball which would have a rebound speed equal to $\sqrt{6}$ times the rebound speed of a golf ball. A durable ball about the same size as a golf ball which would have such a speed when hit by a golf club would be difficult, if not impossible, to find. Therefore, taming the golf ball does not seem to be a good possibility. The second possibility considered for limiting the range of golf balls on the Moon considered in the Investigate is to tame the club. As explored in the Investigate, to find a mass ratio for a golf ball and club head would result in reducing the speed of a golf ball hit by a club from the typical value of $60 \mathrm{~m} / \mathrm{s}$ by a factor of $\sqrt{6}$ or to $60 \div 2.45=24 \mathrm{~m} / \mathrm{s}$. This value is about half of the speed of the head of a golf club for a normal
golfer's swing. To meet this condition, conservation of momentum demands that the mass ratio of the head of the golf club to the golf ball would be about 1:3 (a very lightweight club head compared to standard golf club heads). More problematic, the club head would need to bounce back from the collision at a speed equal to about half of its speed before the collision-follow through of the golfer's swing would not be possible, and the bounce-back of the club would not be pleasant for the golfer. For detailed analysis of the conservation of momentum of a head-on collision between objects having a 1:3 mass ratio when the less massive object is moving before the collision, refer to Safety, Section 6, Question 8 in the Physics to Go. Only the velocities need to be changed to apply to this situation.

Therefore, taming the golf club also does not seem a good possibility for adapting the game of golf to the Moon. Only for your information, the "bounciness" of balls, which of course depends on the materials from which balls are made, usually is quantified by the "coefficient of restitution," which is defined as the ratio of the rebound height to drop height for a ball striking a hard surface. The coefficient of restitution ranges from zero (no bounce at all) to, ideally, one. Balls made of highly "bouncy" materials such as golf balls and "super balls" have coefficients of restitution in the range 0.8 to 0.9 . Ball bearings made of hardened steel have high coefficients of restitution compared to balls made of most other materials.

## NOTES

## Crucial Physics

- A golf ball dropped on the floor converts its GPE to $K E$ as it falls. When the ball hits the floor, much of its $K E$ is stored as spring potential energy, which is then released to cause the ball to rebound up to a new height.
- During the collision with the floor, the golf ball loses some energy, and thus does not rebound to its original height.
- When a golf club collides with a golf ball, only part of the energy of the club is imparted to the ball. The amount of energy the ball gains depends upon the mass of the ball, the mass of the club, and the amount of energy lost in the collision.
- The range of a projectile such as a golf ball is the maximum when the launch angle is $45^{\circ}$. The range is given by the equation $R=v^{2} / g$.

| Learning Outcomes | Location in <br> the Section | Evidence of Understanding |
| :--- | :--- | :--- |
| Compare the bouncing <br> qualities of balls made <br> from a variety of <br> materials. | Investigate <br> Step 2 | Students drop balls of different bounce ability made from different materials <br> and compare their rebound height and speed relative to the floor. |
| Relate bounce height <br> to velocity immediately <br> after impact. | Physics Talk | Students use the concepts of conservation of energy to see how the rebound <br> height of a golf ball is proportional to the square of the rebound velocity <br> immediately after impact. |
| Analyze the required <br> characteristics of a <br> replacement for a <br> standard golf ball that <br> would limit the range of <br> a ball hit on the Moon <br> to the typical range of a <br> golf ball hit on Earth. | Investigate <br> Step 4 | Students try various replacements for golf balls by trying balls of various <br> bounce characteristics to limit the range of a ball hit on the Moon. |
| Analyze how a golf <br> club would need to be <br> modified to limit the <br> range of standard golf <br> balls hit on the Moon <br> to the typical range of a <br> golf ball hit on Earth. | Investigate <br> Steps 5 and 6 | Students find various combinations of masses representing the golf ball <br> and the head of the golf club for which the ball moves away just after the <br> collision at 0.6 times the speed of the head of the golf club just before |
| Discover collision and <br> rebound difference when <br> masses of the objects are <br> not the same. | Investigate <br> Steps 6 and 7 | Students try various combinations of golf club head mass and golf ball mass <br> to find a combination that reduces the speed of the ball after collision from <br> 1.5 to 0.6 relative speeds, and observe the motion of the "club head" ball <br> compared to the golf ball. |

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## Section 6 Materials, Preparation, and Safety

Materials and Equipment

| Materials and Equipment |  | Group <br> (4 students) <br> M per group |
| :--- | :---: | :--- |
| Meter stick | Class |  |
| Ball, golf (w/ hole) | 2 per group |  |
| Ball, super | 1 per group |  |
| Ball, table tennis | 1 per group |  |
| Ring stand, large | 2 per group |  |
| Rod, aluminum, 3/8 in. $\times 12$ in. <br> (to act as crossarm) | 2 per group |  |
| Holder, right angle, cast iron | 2 per group |  |
| Ball, wood, drilled | 1 per group |  |
| Ball, aluminum, drilled | 1 per group |  |
| Ball, steel, drilled | 1 per group |  |
| Scissors | 1 per group |  |
| Ball, golf | 1 per group |  |
| String, ball |  | 2 per class |
| Tape, masking |  | 6 per class |
| Access to clear area with a wall* | 1 per group |  |

*Additional items needed not supplied

## Time Requirement

- Allow two class periods or 90 minutes for students to complete the Investigate and other parts of the section.


## Teacher Preparation

- A set of balls containing one golf ball and a variety of balls made from other materials but having about the same diameter as a golf ball will be needed for each group. Possibilities for balls similar in size to golf balls include plastic, practice golf balls, small super balls, table-tennis balls, rubber balls used for playing jacks, and rubber balls used on paddle ball sets. If you can secure other kinds, use them too.
- Each group will also need a collision apparatus (see the diagram in the student text). The apparatus can be arranged using a ring stand having a horizontal rod clamped to the upright rod. A variety of spheres need to be able to be mounted two-at-a-time from V-shaped suspensions to assure good alignment for head-on collisions. It is not critical in this part of the Investigate for the balls to be of nearly equal diameters; instead, it is important to have several combinations of mass ratios available for students to test. Try to include two spheres having a mass ratio of about 1:3.
- Epoxy cement will work when threads cannot be attached to the spheres by simpler methods such as tying. Also, consider that students will need to be able to exchange the spheres easily, so use a simple method of attaching the threads to the horizontal rod.


## Safety Requirements

- Students must wear safety glasses for this Investigate.
- Students must pick up any balls lying on the floor immediately to prevent anyone from slipping on them.
- A clear area with a smooth floor will be required to drop the balls in Steps 3 and 4.
- Caution students that golf balls do not always bounce straight due to the "dimpled" shape of the surface.
- When students are finished with Step 5, caution them to only release the "club head" ball from an angle less than 90 degrees, and not to give it any extra speed for the collision.


## Materials and Equipment

| Materials and Equipment |  | Group <br> (4 students) |
| :--- | :--- | :--- |
| Meter stick |  | Class |
| Ball, golf (w/ hole) |  | 2 per class |
| Ball, super |  | 1 per class |
| Ball, table tennis |  | 1 per class |
| Ring stand, large |  | 2 per class |
| Rod, aluminum, 3/8 in. $\times 12$ in. <br> (to act as crossarm) |  | 2 per class |
| Holder, right angle, cast iron |  | 2 per class |
| Ball, wood, drilled |  | 1 per class |
| Ball, aluminum, drilled |  | 1 per class |
| Ball, steel, drilled |  | 1 per class |
| Scissors |  | 1 per class |
| Ball, golf |  | 2 per class |
| String, ball |  | 2 per class |
| Tape, masking |  | 6 per class |
| Access to clear area with a wall* |  | 1 per class |

*Additional items needed not supplied

## Time Requirements

- Allow one class period or 45 minutes to complete the Investigate portion of the section as a wholeclass demonstration, discuss the Physics Talk, plus all associated material in the Pacing Guide.


## Teacher Preparation

- A set of balls containing one golf ball and a variety of balls made from other materials but having about the same diameter as a golf ball will be needed for each group. Possibilities for balls similar in size to golf balls include plastic, practice golf balls, small super balls, table-tennis balls, rubber balls used for playing jacks, and rubber balls used on paddle ball sets. If you can secure other kinds, use them too.
- Set up the demonstration collision apparatus (see the diagram in the student text) in an area where it can be clearly seen by all students. The apparatus can be arranged using a ring stand having a horizontal rod clamped to the upright rod. A variety of spheres need to be able to be mounted two-at-a-time from V-shaped suspensions to assure good alignment for headon collisions. It is not critical in this part of the Investigate for the balls to be of nearly equal diameters; instead, it is important to have several combinations of mass ratios available for students to test. Try to include two spheres having a mass ratio of about 1:3.
- Epoxy cement will work when threads cannot be attached to the spheres by simpler methods such as tying. Also, consider that you will need to be able to exchange the spheres easily, so prepare a series of rods with strings pre-attached of the appropriate length. This will allow you to just remove and replace the rods after each use with the next in the series to speed up the investigation.


## Safety Requirements

- Students must wear safety glasses for this Investigate.
- Pick up any balls lying on the floor immediately to prevent anyone from slipping on them.
- A clear area with a smooth floor will be required to drop the balls in Steps 3 and 4.
- Be aware that golf balls do not always bounce straight due to the "dimpled" shape of the surface.
- When performing Step 5, only release the "club head" ball from an angle less than 90 degrees.


## Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

| Learning Issue | Reference | Augmentation and Accommodations |
| :---: | :---: | :---: |
| Reading comprehension | Investigate | Augmentation <br> - This Investigate has longer paragraphs explaining the procedures and making conceptual connections more so than other Investigates. For students who struggle with reading, it would be helpful to preview the main procedures and concepts as a class before students break into smaller groups. <br> - Assign group member tasks including such jobs as director (to read directions), measurer, recorder, and so on. Assigning tasks helps with classroom management and allows you to strategically request that students perform tasks that play into their individual strengths. <br> - Allow students 5-10 minutes to read through the Investigate with their groups before they are allowed to use any equipment. Then give students five minutes to ask clarifying questions before they begin. |
| Comparing with fractions | Investigate Steps 4.d) and 6 | Augmentation <br> - Comparing numbers is a difficult task for students who struggle with number sense, estimation, and number comparison. This task is infinitely more difficult when students are asked to compare fractions. <br> Ask students if " $1 / 6$ as high" means that the ball will bounce more or less than the golf ball. This will help them focus on the concept more than the computation. Then ask students how to solve for a height that is $1 / 6$ as high. At least one student in the class should remember that "as" means multiply in this case. <br> Accommodation <br> - Some students may need a visual example of a height that is $1 / 6$ or $1 / 3$ as high to be able to compare to measured heights. |
| Understanding mathematical relationships | Physics Essential Questions What does it mean? | Augmentation <br> - Students who struggle with reading and/or math skills have a difficult time learning math concepts from text. The Physics Talk explains the concepts needed to answer this question, but without direct instruction, some students may still not understand the concept. Provide direct instruction to explain the relationships represented by $\frac{1}{2} m v^{2}=m g h$ and $R=v^{2} / g$. <br> - If you would like to scaffold student independence, ask students to work with a strategically chosen partner to develop an understanding of these two equations by focusing on the Physics Talk explanation and substituting numbers to verify the relationships as described in the Physics Talk. Students could make a poster to explain their understanding of the relationship of rebound height and maximum range to take-off speed and launch speed. <br> - If students are still struggling, provide sample values for each of the variables. Ask students to solve the equations using the given values. Lead a group discussion to make sense of the equation solutions. |

## Strategies for Students with Limited English-Language Proficiency

$\left.$| Learning Issue | Reference |  |
| :--- | :--- | :--- |
| Understanding <br> concepts | Investigate <br> Step 2.b) | Have students give some thought to each physicist statement (i, ii, and iii). <br> Make sure they feel comfortable with their understanding of each one. <br> Encourage students to discuss the statements within their groups and ask for <br> clarification if necessary. They should keep the statements in mind as they work <br> through the Investigate. |
| Higher-order <br> thinking | Investigate <br> Step 4.c) <br> Physics Talk | Have students discuss the question within their groups and write down the <br> thoughts of all group members. They can look back to these ideas when they <br> read through Physics Talk and learn the two equations that prove the answer. |
| concepts |  |  |$\quad$| Investigate |
| :--- |
| Step 6.a) |$\quad$| Check students' Active Physics logs for what they determined to be the ratio of |
| :--- |
| the masses (golf ball: head of golf club). Make sure they correctly discovered that |
| the conditions they are searching for are best met when the mass of the head of |
| the golf club is smaller than the mass of the golf ball. This fact will be important |
| when students address the Why should you care? question later, and may factor |
| prominently in any modifications that they suggest for their chosen sport in their |
| proposal to NASA. | \right\rvert\,

## SECTION 6 <br> Teaching Suggestions and Sample Answers

## What Do You See?

The images of two astronauts playing golf on the Moon will elicit a stream of responses. Ask students to focus on what this illustration is trying to convey. Point out that each image, including the labels, are significant to the purpose of their future investigations. As students discuss their responses, record a few on the board. Remind them that they will have other opportunities to return to the illustration as they investigate new physics concepts. Allow students time to consider different aspects of this visual and relate it to their previous knowledge, as well as the concepts mentioned in the title of this section.


## Students' Prior Conceptions

1. A dropped ball can bounce higher than the distance from which it is dropped. This misconception comes from students throwing balls downward and getting a bounce higher than the start height. Repeatedly, having them drop various balls from a given height with no initial velocity and then repeating the test with the balls having an initial downward speed generally clears up this prior conception. In addition, the simple name "super ball" implies that the ball can perform feats such as bouncing higher than the drop height, because they are "super."
2. A lighter golf club will hit the ball further than a heavy one. This misconception may come from baseball, where a lighter bat allows the swinger to respond more quickly to a pitch, and swing the bat faster. Having the students repeat Step 5 of the Investigate with a series of progressively heavier "club head" balls should convince students that the more massive the club head for the same speed, the greater the distance.

## What Do You Think?

Students might be amazed to learn that an astronaut has actually hit a golf ball on the Moon. Some spontaneous questions that might come up could be, "How far did the ball travel?" "Did the ball bounce? How much force did the astronaut have to use to launch the ball in the air?" Encourage students to relate these and other questions they might have to the physics of motion they are already familiar with and record them in their Active Physics logs. During this stage, students will most
likely reveal the misconceptions they have about modifications that could be introduced if golf were to be played on the Moon. Ask them to share their responses with their group members and prepare questions for future discussions. Students must be assured that their immediate responses to these questions will not be evaluated. However, these should be thoughtfully written out so that they can be revised and updated as they learn new physics concepts.

## What Do You Think?

A Physicist's Response
Show the video segment of astronaut Alan Shepard hitting a golf ball on the Moon. Although no one recorded the distance the golf ball traveled after Shepard hit it, the golf ball flew much further than on Earth, even though the astronaut was wearing a spacesuit, which restricted the swing. As seen on the video, taming the game of golf to be played on Earth-sized golf courses on the Moon is beset with serious problems. Due to the Moon's weaker gravity, projectiles have greatly enhanced ranges, which would make locating the shot very difficult. Many sports which involve objects moving as projectiles (for example, baseball) would need to be tamed to be adapted to the Moon.

NOTES

## NOTES

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## Investigate

## 1.a)

One way to limit the distance a golf ball travels on the Moon might be to make the ball out of a high air resistance material, so that the force due to air resistance is much greater (rough surface). A second method would be make the golf club lighter, use a material with poor rebound characteristics for the head of the golf club or
the ball, or change the rules on how the golf club is swung.

## 2.a)

Students' descriptions will depend on the type of ball selected.
2.b) i)-iii)

Students should agree with each statement.

## 2.c)

In ease of understanding, the concepts are probably ii), i) and iii).

## 3.

Students position a $2-\mathrm{m}$ stick, vertically, with the zero-end resting on the floor and secure it to a wall or the edge of a table to observe the height of a falling ball bouncing in front.

## 4.a)

Below is sample data for balls dropped from a 2-m height to a concrete floor.

Golf ball:
Bounce height $=1.60 \mathrm{~m}$.
Ball used for playing jacks:
Bounce height $=0.94 \mathrm{~m}$.
Plastic golf ball used for practice:
Bounce height $=0.74 \mathrm{~m}$.

## 4.b)

From the data above:
Ball used for playing jacks golf ball $0.94 \mathrm{~m} / 1.60 \mathrm{~m}=0.587$.

Plastic golf ball for used practice golf ball
$0.74 \mathrm{~m} / 1.60 \mathrm{~m}=0.463$.

## 4.c)

Students would typically respond by saying "yes" to this question.

## 4.d)

The ball that comes closest to the 0.167 factor will depend upon the ball chosen for the Investigate.
Using the data above, it would be the plastic golf ball.

## 5.

Student practice the collision a few times.

## 9-6a Blackline Master

## 6.a)

Students should find that the ratio of the masses for the golf ball to the club head should be about 3:1, or the club head should have one third the mass of the golf ball.

## 6.b)

Students will find that when the golf club's head is reduced to one-third the mass of the golf ball, the golf club head will reverse direction after impact when the speed of the golf ball after impact is reduced to 0.6 times the speed of the club. Golfers almost certainly would not find this to be an acceptable modification of the game. In addition, a secondary problem would be that the golf ball would be more difficult for the golfer to control when the club bounces back after striking the ball.

## 7.a)

Student answers will vary, but it appears that unless a golfer accepts some unpleasant side effects, such as the golf club's head bouncing back or long walks after hitting a golf ball, golf on the Moon does not seem feasible.

## 7.b)

Moon golfers would find the game very unappealing for the reasons stated above.


## 7.c)

7.c)
Other methods suggested by
students might consist of altering
the swing, using rubber-shafted
clubs, or hitting the ball through
a screen to slow its speed after
the swing
7.c)
Other methods suggested by
students might consist of altering
the swing, using rubber-shafted
clubs, or hitting the ball through
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the swing the swing.

Section 6 Momentum and Gravity. Golf on the Moon
8. Golf may not be your game. Lots of other sports involve hitting a ball with a bat, a foot, a hand, or a racquet. For each of these sports on the Moon, you may be bothered by how far the ball travels and you may want to "tame" the sport by altering the ball or the
object that hits the ball. Choose two sports in which a ball gets hit and describe how you might alter the ball or the object that hits the ball to decrease the distance the ball travels.
دa) Record your descriptions in your log.

## Physics Talk

## THE PHYSICS OF TAMING THE GOLF BALL

The conservation of energy informs you that the total energy of an object must remain constant unless it does work or work is done on it. A bouncing ball has gravitational potential energy $G P E$, then it gains kinetic energy $K E$ as it falls, which then leads to a gain once again of GPE as it travels up followed by KE followed by GPE, and so on. It is as if the energy is "bouncing" between GPE and KE as the ball is bouncing up and down. Since each successive bounce is not as high as the previous bounce, there is a loss of energy. In this section, you measured the change in height of a golf ball and could calculate the loss in energy of each successive bounce. Where does the energy go? Since energy is conserved and you noted a loss in the gravitational potential energy after each bounce, you must look or listen for where this energy went. Some of the energy went into sound (each bounce made some sound) and into heat (the temperature of the ball probably increased a tiny bit, and into vibration of the floor (yes, the floor vibrates - if it were a bowling ball, you would notice it). If you were able to measure all these energies with precision, their sum would equal the energy loss in the height of each bounce.

By introducing a different ball, you can have a poorer bounce and a greater loss of energy during each bounce. This could limit the range of the golf ball so that the game of golf on the Moon does not send the ball so far away.
Physicists like to view the same phenomenon from different perspectives in order to better understand all that is happening. When a ball bounces, one perspective is that the ball and floor collided. The very massive floor appears to remain at rest after the collision, but could be moving a bit, and it would take some ingenuity to measure the vibration of the floor. By changing the floor, you could change the bounce. The floor's bounce properties could change with padding or by using a different floor material. It is difficult to change the mass of the floor
A golf club and a golf ball also undergo a collision. You can change the mass of the golf club in order to change this collision with the golf ball. You investigated this change and the affect on the golf ball in this Investigate.

## 8.a)

Students' answers will vary depending upon the sport chosen.
Most answers should follow similar suggestions made for golf.

## Physics Talk

Students discover that each time a golf ball bounces, it loses height and therefore, energy. However, the energy that is lost by the ball is gained by the surrounding environment in the form of sound, heat, or vibration. If the energy that goes into sound, heat, or vibration can be measured with precision, their sum would equal the energy lost by the golf ball as it loses its height. Students also note that the range of a golf ball driven by a golf club is proportional to the square of the velocity. In the Investigate, students noted that to reduce the range of a golf ball by a factor of six, the bounce height must be reduced by the same factor. The range of the golf ball is therefore directly proportional to the bounce height.

Ask students how the range of the ball can be limited. Have them describe their reasoning in their Active Physics logs. Ask them to include how the ball's bounce can be changed using a different floor material. Discuss how the kinetic energy of a ball changes as its height drops with each bounce. Draw students' attention to the energy equation that equates the gravitational potential energy to kinetic energy. Ask them how they can find out the "take-off" speed of the golf ball. Then draw students' attention to the second equation that provides the range or the horizontal distance that a golf ball travels. Discuss the bounce heights of the ball using the data students have gathered during the Investigate.

Chapter 9 Sports on the Moon


It is true that a ball that bounces only $1 / 6$ as high as a golf ball when dropped from the same height would have $1 / 6$ of the range of a golf ball when hit by a golf club. Two equations help to show this.
The first equation is found by equating the kinetic energy of the bouncing ball at the instant it leaves the floor to the gravitational potential energy the ball has at the peak of its bounce:

$$
\frac{1}{2} m v^{2}=m g h
$$

For a specific ball, the mass remains constant as does the acceleration due o gravity. If the height decreases by a factor of 6 (on the right side of the equation), then the $v^{2}$ must also decrease by a factor of 6 (on the left side of the equation).
This means that if the "take-off" speed squared of an object doubles, then the height it reaches doubles. Or if the "take-off" speed squared of an object decreases to half its value, then the height the object reaches decreases to half its value.

Active Physics


## Checking Up

## 1.

The energy lost by a bouncing golf ball as it goes through successive bounces is lost to heat, sound, and the vibration of the floor.

## 2.

When the energy lost to sound, heat, and floor vibration is added to the energy retained by the golf ball as kinetic energy, the total energy is conserved before and after the ball strikes the floor.

## Active Physics Plus

This Active Physics Plus revisits the concepts and equations for projectile motion, and explores them in greater depth. Students read how a generalized equation for the range of a projectile launched at an angle to the horizontal is derived. Using this equation, they plot a graph of $2 \sin \theta \cos \theta$ vs. the angle $\theta$ to find the angle that yields the maximum range. From the graph, students discover the symmetry of range for angles equally above and below the range when launched at $45^{\circ}$. Students are then shown how using a trigonometric identity yields the identical result.

## 3.

The range of a projectile depends upon the square of its launch velocity.

## 4.

If the height of a golf ball decreases by a factor of six, the range of the golf ball will also
decrease by a factor of six. This decrease by a factor of six. This decrease by a factor of six. This of the golf ball, which depends
upon velocity squared, determines of the golf ball, which depends
upon velocity squared, determines the golf ball's height.
constructing the right triangle with sides $v_{x}$ and $v_{\text {, }}$ and hypotenuse $v_{0}$. From this triangle, you can see that

$$
\cos \theta=\frac{v_{0 \mathrm{x}}}{v_{0}} \text { and } \sin \theta=\frac{v_{0 \mathrm{ox}}}{v_{0}}
$$

Using algebra, then
$v_{\text {ox }}=v_{0} \cos \theta$ and $v_{0 \mathrm{y}}=v_{0} \sin \theta$
These two relationships will be useful later.

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## 1.

Students graph the equation $y=2 \sin \theta \cos \theta$ and find $\theta_{\text {max }}$ occurs at $45^{\circ}$
2.

Students should see that both $40^{\circ}$ and $50^{\circ}$ gives a range of 98 percent of the maximum range. Symmetric angles above and below $45^{\circ}$ yields the same range. Thus, $40^{\circ}$ and $50^{\circ}, 30^{\circ}$ and $60^{\circ}$, and so on, will have the same range.
3.

Student reading.


## What Do You Think Now?

Encourage students to review their original responses to the What Do You Think? questions. Students' responses should include an explanation of why the range of a golf ball has to be reduced. Ask them why physicists have to view the same phenomenon from different perspectives. How would it help
in modifying a game played on the Moon? Students should be more confident about their responses now. Check to see if their misconceptions have been addressed. Consider sharing A Pbysicist's Response and invite them to discuss doubts they might still have, emphasizing at the same time that their ideas should reflect a thorough understanding of physics concepts discussed in this section.


## Reflecting on the Section and the Challenge

This is a time for students to reflect on what they have learned so far, and to apply how the range of a golf ball can be changed to make it possible for the sport to be played on the Moon. Ask students to review the equations associated with the transformation and conservation of energy when a golf ball undergoes a collision. Students should understand that for a specific ball, if the mass and acceleration due to gravity remain constant, the height decreases or increases by a factor of six, $v^{2}$ also increases or decreases by a factor of six. Ask students to reflect on how what they learned in this section can apply to other sports as well. Have them prepare a list of problems that help in adapting a sport to the Moon. Point out that considering prospective solutions will allow students to present a convincing proposal to NASA for their Chapter Challenge.

## Physics Essential Questions

## What does it mean?

Both the rebound height and the range of the projectile are proportional to the square of the speed of the object. If the square of the velocity is twice as large, the object will rebound to twice the height.

## How do you know?

If an object of a small mass collides with a more massive object, the small mass will bounce back.

## Why do you believe?

The conservation of energy accurately predicts what will happen when a ball is dropped or when two objects collide. The rebound height is less than predicted by the conservation of energy, implying that energy is lost in the collision, not that the law of conservation of energy is incorrect.

Why should you care?
The masses of the bat and ball can be modified to limit the distance the ball will travel. This can be done in baseball, tennis and golf.

## Physics to Go

## 1.

Because a ball on the Moon falls much more slowly than a ball on Earth, the range of a tennis ball on the Moon is much longer. Getting the tennis ball to fall onto the court inbounds means that the ball would require a much lower velocity; either the racquet or the ball would have to be modified to slow the ball's velocity to shorten the range.

## 2.

Hitting a golf ball resting on lunar soil would be similar to hitting a golf ball out of a sand trap on a golf course on Earth. Particles ranging in size from sand to dust would be ejected from the surface as projectiles. On Earth, air resistance causes such particles to slow down quickly, so they do not travel a great horizontal distance from the point of impact before falling back to the surface; also on Earth, tiny dust-like particles fall back to the surface slowly because air resistance causes them to reach terminal velocity soon after they begin to fall. With no air resistance on the Moon, particles launched in a divot all would fly along parabolic trajectories, some traveling great distances, and all particles would accelerate downward at the Moon's $g$ of $1.6 \mathrm{~m} / \mathrm{s} / \mathrm{s}$.

## 3.

There is no air to transmit sound waves on the Moon, so communication is accomplished via radio. Communication is possible only with those who are "tuned" to the same radio frequency. To warn another golfer about an incoming

golf ball, the golfers might need an override channel that would alert anyone with their radio tuned to only specific frequency of the warning. Unfortunately, this would go to all the golfers on the course, not just those in danger!

## 4.a)

Some reasons in favor of golf on the Moon:

- Many people enjoy golf and it provides exercise.
- There is plenty of available
space on the Moon's surface for golf courses.
- The differences between trajectories of golf balls on the Moon and on Earth would be interesting.


## 4.b)

Some reasons against golf on the Moon:

- It may not be possible to find a way to limit the range of golf
工,
balls to reasonable distances on the Moon, while at the same time preserving the nature of the game.
- Need for space suits and survival equipment would make it time-consuming and expensive to play a round of golf on the Moon.
- It is not possible to walk in a normal way on the Moon, so the nature of the exercise provided would be different from playing golf on Earth.


## 5.

Sports that use bats, clubs or racquets might include cricket, baseball, softball, tennis and squash. To make all these sports playable on the Moon might require using a more massive ball. Although a heavy ball will
not fall any faster than a light ball, the added mass of the ball would reduce the initial velocity of the ball when struck by the bat or racquet, reducing the range. Alternatively, the bat or racquet could be modified to reduce the velocity of the ball after impact.

## 6.

Preparing for the Chapter Challenge

The sport of soccer might be quite difficult to adapt to the Moon. Having a more massive soccer ball would certainly slow the ball's velocity, but would quite likely prove very painful for the players to kick, and a slowmoving ball would make it much easier to defend, leading to many scoreless ties!

## Inquiring Further

The coefficient of restitution (COR) of an object is defined as the ratio of the kinetic energy of an object before a collision to its kinetic energy after the collision. A simple way to measure the COR of a golf ball is by measuring the velocity before and after the ball hits a hard surface, such as a concrete floor. Alternatively, the COR is found by the ratio of the square root of the height divided by the bounce height, or COR $=\sqrt{H_{\text {initial }} / H_{\text {final }}}$. According to the USGA, the COR of a golf ball is limited to 0.83 for sanctioned events.

## SECTION 6 QUIZ

## 9-6b Blackline Master

For the purposes of these question, the acceleration due to gravity on the Moon is $1.6 \mathrm{~m} / \mathrm{s}^{2}$ and is $10 \mathrm{~m} / \mathrm{s}^{2}$ on Earth.

1. A golf ball is dropped from a height of 2 m on Earth onto a hard surface and bounces back up to a height of 1 m . What was the speed of the golf ball as it was hitting the ground on the way down?
a) $2 \mathrm{~m} / \mathrm{s}$.
b) $4 \mathrm{~m} / \mathrm{s}$.
c) $\sqrt{20} \mathrm{~m} / \mathrm{s}$.
d) $\sqrt{40} \mathrm{~m} / \mathrm{s}$.
2. In Question 1, what was the velocity of the ball as it was leaving the surface on the bounce upward?
a) $2 \mathrm{~m} / \mathrm{s}$.
b) $4 \mathrm{~m} / \mathrm{s}$.
c) $\sqrt{20} \mathrm{~m} / \mathrm{s}$.
d) $\sqrt{40} \mathrm{~m} / \mathrm{s}$.
3. Two golf balls are to be hit at the same angle outdoors on the Moon. Golf ball A is to go farther than golf ball B. Which of the following changes will make this happen?
a) The two balls should have the same launch speed, but golf ball B has a greater mass than golf ball A.
b) The two golf balls have the same launch speed, but gravity should be stronger for golf ball B than for golf ball A.
c) The two golf balls have the same mass, but golf ball A has a higher launch velocity than golf ball B.
d) Any of the three methods would cause golf ball A to go further.
4. Two balls fall onto a steel plate from the same height. Ball A rebounds off the plate with twice the speed of ball B. Compared to the bounce height of ball $B$, the bounce height of ball A will be
a) $\sqrt{2}$ times as great.
b) 2 times as great.
c) 4 times as great.
d) 8 times as great.
5. A golf ball is dropped onto a floor from a height of 2 meters and rebounds to a height of 1.4 m . Which of the following would not be a source of lost energy for the golf ball?
a) Gravitational potential energy.
b) Sound energy.
c) Heat energy.
d) Vibration energy.

## SECTION 6 QUIZ ANSWERS

(1) d) Using conservation of energy, the gravitational potential energy of the ball as it is dropped is converted to kinetic energy as it is just striking the floor. Thus $\frac{1}{2} m v^{2}=m g h$ or $v^{2}=2 g h$. Inserting the correct values gives $v^{2}=2\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)(2 \mathrm{~m})$ or $v^{2}=40 \mathrm{~m}^{2} / \mathrm{s}^{2}$ or $v=\sqrt{40} \mathrm{~m} / \mathrm{s}$.
(2) c) Using conservation of energy, the kinetic energy of the ball as it rebounds from the floor is converted to gravitational potential energy as the ball rises to the peak. Thus $\frac{1}{2} m v^{2}=m g h$ or $v^{2}=2 g h$. Inserting the correct values gives $v^{2}=2\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)(1 \mathrm{~m})$ or $v^{2}=20 \mathrm{~m}^{2} / \mathrm{s}^{2}$ or $v=\sqrt{20} \mathrm{~m} / \mathrm{s}$.
(3) Giving ball A a higher launch velocity will increase its range. Changing the ball's mass but keeping the launch velocity the same will not affect the range, since all objects fall at the same rate if there is no air resistance. Changing gravity is not really an option.
(4) c) The range and bounce height of an object depends upon the initial velocity squared. If the rebound velocity of ball $A$ is twice that of ball $B$, it will rise to a height of $(2)^{2}$ or 4 times as great.
(5) The gravitational potential energy is stored energy and would be recovered by the ball in the form of kinetic energy as the ball starts to fall. The sound, heat, and vibration energy would not be recoverable from the ball.

