SECTION 8

Modeling Human Motion: Bounding on the Moon

Section Overview

Students observe the *Active Physics* video of astronauts and compare how astronauts "walk" on the Moon to how they walk on Earth. Using the model of a pendulum, students analyze how the length of the pendulum affects the time it takes for the pendulum to swing back and forth. They pull the longer pendulum halfway up and record the period of a swing by averaging the time it takes to complete 10 swings. Students then observe the motion of a leg while they walk and notice how the leg moves during a stiff-legged walk and how it bends during a relaxed walk. Then they measure the length of the lower leg and record how long it takes to swing forward during a stride. Doubling the time measurement gives them the period of a person's lower leg (the time required to swing back and forth). Using the measurements from students of different length legs, the students construct a graph of period versus length to find out if a person's lower leg behaves like a pendulum while walking.

Using an equation that incorporates the length of pendulum and the acceleration due to gravity, students calculate the period for each cylindrical pendulum and compare the predicted to the observed values. Students then use the same equation to calculate the period of their lower leg and compare their predicted results to the measured values for their lower leg. They compare the predicted period of their lower leg's swing on the Moon to the lower leg's swing on Earth by replacing "g" in the equation with "$(1/6)g$." On further analysis, they learn that it takes longer for the lower leg to swing forward on the Moon than it does on Earth due to reduced gravity, explaining why the astronauts use the bounding motion rather than a normal walk.

Background Information

This section introduces the pendulum as a model of human legs during walking, showing that the pendulum action of legs during walking is a problem in the reduced gravitational field of the Moon. This effect, in addition to the reduced frictional force and loose soil on the Moon, renders walking (or running) virtually impossible on the Moon. That is why astronauts on the Moon developed the "bounding" technique for locomotion on the Moon's surface.

The general equation for the period of a pendulum (the time for a pendulum to complete one complete cycle of motion, such as to swing over-and-back when displaced from the equilibrium position and released) is $T = 2\pi\sqrt{I/(mgd)}$, where T is the period in seconds, I is the moment of inertia in (kg)m^2 of the object acting as a pendulum (to be described immediately below), m is the mass of the object in kilograms, g is the acceleration due to gravity in m/s^2, and d is the distance in meters from the point of suspension of the pendulum to the object's center of mass.

Just as an object's mass is a measure of its tendency to resist translational acceleration (acceleration along a straight-line path), an object's moment of inertia is a measure of its tendency to resist angular acceleration (changes in angular speed during rotation of the object about an axis). When an object rotates on an axis, its resistance to angular acceleration is determined not only by the object's mass, but also by how the object's mass is distributed in respect to the object's axis of rotation.

If m_1, m_2, m_3, etc., represent the masses of infinitely small particles of a rotating object, and if r_1, r_2, r_3, etc., represent their respective distances from the axis of rotation, the moment of inertia of the object in respect to the axis of rotation is: $I = \left(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\right)$, or $I = \sum_i m_ir_i^2$.

For the simple case of an object being twirled on the end of a string (when the object's size is negligible compared to the length of the string and the string has negligible mass) the moment of inertia is $I = mr^2$; the same equation would apply to a thin cylindrical shell, hoop, or ring (such as a bicycle wheel) because, even though the mass is spread along the circumference, all of the mass is at the same distance from the spin axis. For more complex shapes where the mass is distributed at various distances from the axis of rotation, calculus methods are used to determine the moment of inertia. When an object acts as a pendulum, the moment of inertia is involved because a pendulum is an object which has its rotation arrested by gravity acting somewhat as a spring; if there were no gravity, a pendulum would continue in circular motion at constant angular speed. For this activity wherein the human leg is modeled as a solid cylinder rotated about one end of the rod, application of calculus results in the moment of inertia: $I = (1/3)\left(mL^2\right)$ where m is the mass of the cylinder and L is the length of the cylinder (or leg). Substituting the above as the moment of inertia and substituting $L/2$ for d in the general equation for the period of a pendulum:

$$T = 2\pi\sqrt{I/mgd} =$$
$$2\pi\sqrt{\left[(1/3)mL^2\right]/\left[mg(L/2)\right]} =$$
$$2\pi\sqrt{2L/3g} = 2\pi\sqrt{2/3}\sqrt{L/g} =$$
$$5.1\sqrt{L/g}.$$

The above equation dictates the period of a human leg swinging freely as a pendulum; this is what a human leg does during the forward swing of the leg during each stride. Notice that the above equation does not include either the mass of the leg or the amplitude of the leg's swing as variables; it is true that neither affects the leg's period (swing time). If ever needed, the moments of inertia of objects of many shapes in respect to various axes of rotation are listed in most college physics textbooks.

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Crucial Physics

- The natural swinging motion of the human leg can be modeled by that of a cylindrical pendulum.
- The period of a cylindrical pendulum depends upon the square root of the ratio of the length of the pendulum and the acceleration due to gravity, and can be found using the formula $T = 2\pi\sqrt{(2L/3g)}$.
- Due to the lowered gravity on the Moon, the period of a freely swinging leg on the Moon will be $\sqrt{6}$ times longer than on Earth. The slow swing rate is responsible for the astronauts changing their locomotion style to "bounding" to conserve energy.

Learning Outcomes	Location in the Section	Evidence of Understanding
Apply a cylinder as a model of a human leg acting as a pendulum during walking.	***Investigate*** Steps 2, 3, 6, and 7	Students record the lengths of cylinders with different sizes hung from a horizontal bar, allow them to swing by pulling them up, and then record how long it takes each cylinder to move back and forth to complete one swing. They compare the movement of the pendulum to the model of a human leg while walking.
Measure the amount of time for a human leg to move forward and back as a human walks on Earth.	***Investigate*** Step 7	Students measure the amount of time required for their lower leg to swing forward and double that amount of time to record it as the period of their lower leg's swing while walking on Earth.
Predict the amount of time for a human leg to swing forward as a human walks on the Moon.	***Investigate*** Step 9	Students multiply the forward swing of their lower leg by the square root of six, or 2.5, to find how much time it would take their lower leg to swing forward on the Moon.
Explain why it is not possible to walk unaffected on the Moon.	***Investigate*** Step 9 ***Physics Talk***	Students try to walk with the swing time that their lower leg would have when powered by gravity on the Moon and explain why it is not possible to walk unaffected when analyzing the affects of reduced gravity on the Moon.
Discover how the period of a pendulum depends on the length, mass, and angle of a swing.	***Investigate*** Steps 2–7	Students record the periods of a series of cylindrical pendulums of different lengths. They vary the angle of swing and observe how it affects the period by pulling the pendulum halfway up and then one-quarter of the way up.

NOTES

Section 8 Materials, Preparation, and Safety

Materials and Equipment

PLAN A		
Materials and Equipment	**Group (4 students)**	**Class**
Multimedia DVD/CD Set		1 per class
Meter sticks	1 per group	
Ring stand, large	1 per group	
Holder, right angle, cast iron	1 per group	
Rod, aluminum, 1/8 in. x 12 in. (to act as crossarm)	1 per group	
Cylinder, hangable, set	1 per group	
Stopwatch	1 per group	
Paper, graph, pkg. of 50		1 per class

Time Requirements

- Allow two class periods or 90 minutes for the students to do the *Investigate* and other parts of the section.

Teacher Preparation

- Reserve a TV monitor and VCR for viewing segments of astronauts bounding on the Moon in the *Active Physics* video, or search the Internet for a video of astronauts walking on the Moon.
- A series of cylinders of different lengths should be prepared. PVC pipe with a hole drilled through the end so that it can swing on a cross-bar will work well (see *Step 2* of the *Investigate*). Lengths should vary from 20 cm to more than a meter. Three-quarter inch PVC is available from home products stores. Members of your school technology department can assist with the cutting and drilling necessary.

Safety Requirements

- Students should wear goggles for this *Investigate*.
- Caution the students to ensure that the area where the cylinders are swinging is free from obstructions.
- If the cylinders are to be hung from a ring stand as shown in the student text, a counter balance on the bottom plate of the ring stand will be necessary to prevent the ring stand from tipping over.

Materials and Equipment

PLAN B		
Materials and Equipment	**Group (4 students)**	**Class**
Multimedia DVD/CD Set		1 per class
Meter sticks		1 per class
Ring stand, large		1 per class
Holder, right angle, cast iron		1 per class
Rod, aluminum, 1/8 in. x 12 in. (to act as crossarm)		1 per class
Cylinder, hangable, set		1 per class
Stopwatch		1 per class
Paper, graph, pkg. of 50		1 per class

Time Requirements

- Allow one class period or 45 minutes for the teacher to do the *Investigate* portion of the section as a class demonstration, the *Physics Talk*, plus all associated sections in the *Pacing Guide*.

Teacher Preparation

- Reserve a TV monitor and VCR for viewing segments of astronauts bounding on the Moon in the *Active Physics* video, or search the Internet for a video of astronauts walking on the Moon.
- A series of cylinders of different lengths should be prepared. PVC pipe with a hole drilled through the end so that it can swing on a cross-bar will work well (see *Step 2* of the *Investigate*). Lengths should vary from 20 cm to more than a meter. Three-quarter inch PVC is available from home products stores. Members of your school technology department can assist with the cutting and drilling that is required.
- Set up the hanging cylinder apparatus in an area clearly visible to all the students.
- Have several student volunteers assist you with the "stiff-legged walk" portion of the activity.
- Prepare transparencies to be used as graphs for the data from the hanging cylinder period vs. cylinder length, and leg length period vs. leg length graphs.

Safety Requirements

- Caution the students to ensure the area where the cylinders are swinging is free from obstructions.
- If the cylinders are to be hung from a ring stand as shown in the student text, a counter balance on the bottom plate of the ring stand will be necessary to prevent the ring stand from tipping over.

Meeting the Needs of All Students

Differentiated Instruction: Augmentation and Accommodations

Learning Issue	Reference	Augmentation and Accommodations
Creating a graph	***Investigate*** Steps 5.b) and 7.c)	**Augmentation** • Ask students to set up and label their axes. Remind students that their scale must be evenly spaced with the same number of blocks between each number. Students may need help deciding what scale to use, especially when using decimal numbers. • After students have completed their graphs, take a break from the *Investigate* and check in with students. Graphing is very frustrating for some students and a productive break may help them relax. Ask students questions about what they have completed. Do they see any patterns developing? Do they have any questions that they need answered before they continue? **Accommodation** • If a particular student is still struggling to set up a graph, provide a graph with the scale marked but not labeled with numbers. Help students decide what scale could be used to represent the data, and ask them to mark the scale. Check their scales before they begin plotting points.
Sharing personal space	***Investigate*** Step 7	**Augmentation** • Some students are not good at sharing personal space and may offend someone or be incited to extreme anger by someone touching them. One way to avoid this problem is to ask one group member to hold a meter stick next to their own leg, touching the floor with the zero end of the meter stick, while a classmate measures the length of their lower leg. Also, providing students with a sewing measuring tape may make it easier for them to measure their own leg.
Adding a column to a data table	***Investigate*** Steps 8.a) and 8.b)	**Augmentation** • Students are being asked to add a column to the data table they created in *Step 2* first, however, they have created another data table in *Step 7*. It will be difficult for students to compare the measured and predicted periods if they add the new column and calculations to the incorrect table. Before students get to this step, remind them that the new column for *Step 8.a)* should be added to the table from *Step 2*.
Solving pendulum equations	***Investigate*** Step 8 ***Physics to Go*** Question 4	**Augmentation** • Students may need a quick review of the order of operations before they begin solving problems with the pendulum equation. Students may try to do the square root before dividing, which will result in a lot of incorrect answers to compare. Remind students of the mnemonic device used for order of operations in their school. For example, PEMDAS is one common example of a mnemonic device for order of operations. • It will also be helpful to assist students in solving the equation for length. The square root confuses many students when solving for *L*. Arriving at $L = T^2 g/(5.1)^2$ would be very difficult for students who struggle to manipulate formulas.

Strategies for Students with Limited English-Language Proficiency

Learning Issue	Reference	Augmentation
Following complex procedures	***Investigate*** Steps 2–7	Break down *Steps 2–7* into smaller chunks that allow students to comprehend each portion of the activity before moving on to the next one. This will allow them to become comfortable following the procedures outlined within each step. The steps here are not complicated, but to be executed properly they do require patience and diligence. To collect data that is as accurate as possible, students should perform each step multiple times and average the results. As you walk around the classroom, glance at the tables in students' *Active Physics* logs to be sure they are on target with their data. Do the same to check students' graphs.
Understanding concepts	***Investigate*** Steps 8 and 9	Check students' calculations for accuracy. After students have completed their written statements in their logs for *Steps 8.a)* and *b),* hold a brief class discussion so students can share results and interpret them. Encourage ELL students to participate as much as possible. Make sure students are able to verbalize *why* the leg swing takes 2.5 times longer on the Moon than it does on Earth.
Identifying plurals	***Physics Talk***	ELL students likely are familiar with plural nouns formed by adding "-s" or "-es" to the singular form, but of course English is full of exceptions to this rule. ELL students may not be aware that in English some words have more than one acceptable option for forming a plural. For example, a word ending in "-um" can be made plural by adding an "-s," as with "pendulums," and the plural may also be formed by changing "–um" to "-a," as in "pendula." A more familiar word with the same two plural options is curriculum ("curriculums" and "curricula").
Understanding concepts	***Active Physics Plus*** Question 2	The concepts here may seem complicated, especially for students who struggle with visual concepts. To help students understand simple harmonic motion, it may help to use an alternate visual model. On the board, draw a horizontal line and its midpoint. Model simple harmonic motion by showing the horizontal line sliding back and forth through the point that was originally its midpoint, always remaining horizontal. If time allows, have pairs of students explore on their own the shadow of a rotating pendulum. Make sure students maintain constant speed, which is much easier to do with the pendulum moving in a horizontal circle. Students should easily observe that the speed of the pendulum bob is zero at the extremes of motion, and at the central point the speed is at maximum.
Higher-order thinking	***Physics Essential Questions***	Elicit students' ideas as to why the period of a cylindrical pendulum differs from the period of a mass on a string (assuming pendulums of equal length). Encourage verbal participation by ELL students.

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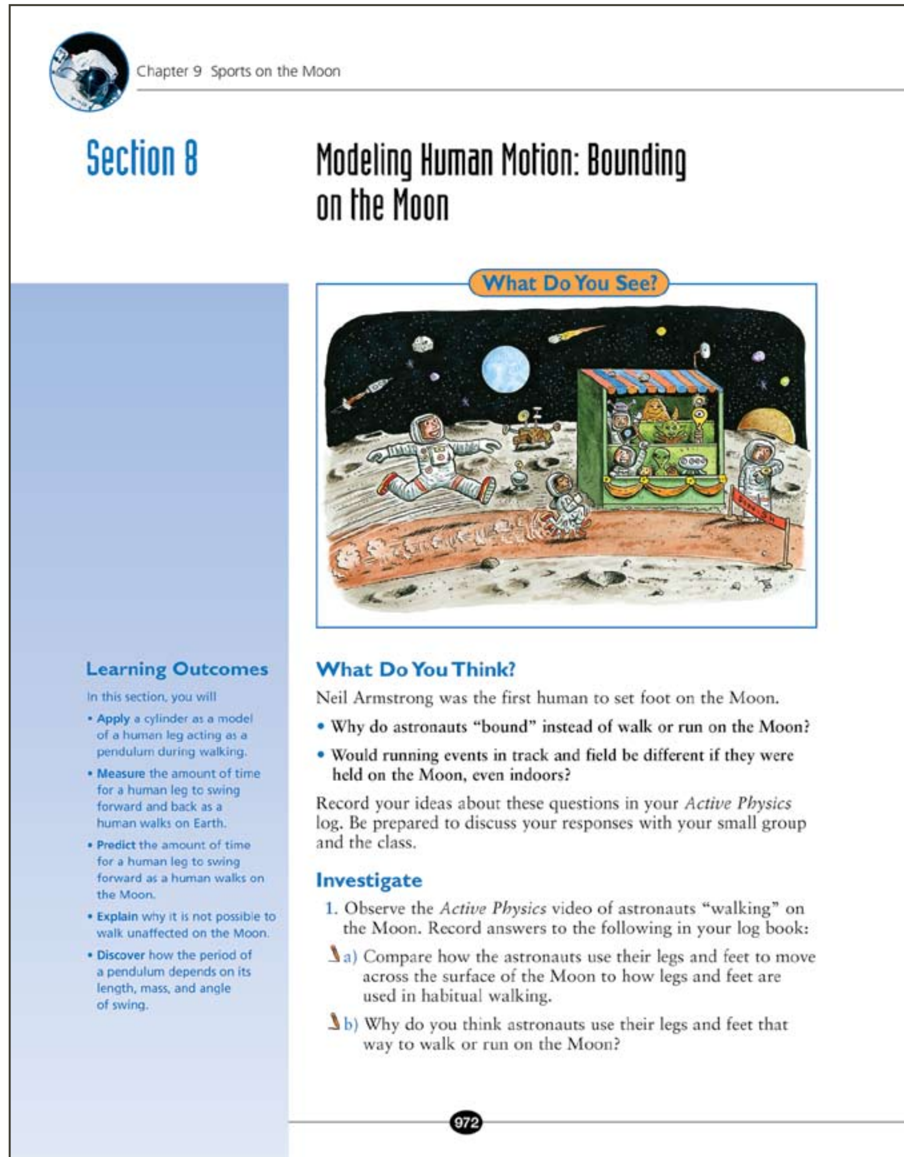
SECTION 8
Teaching Suggestions and Sample Answers

What Do You See?

Three astronauts involved in the game arouse much curiosity. This engaging visual will most likely elicit numerous responses from students. Encourage them to observe each image closely and determine what the illustration is trying to convey. You might want to ask students, "Why is the taller astronaut making the longer strides behind the shorter astronaut making quicker strides? Or, "Why is the audience in an enclosed area?" These questions will guide student impressions. When they return to *What Do You See?* at a later point during the section, they will be able to understand why the artist chose to convey physics concepts using images of astronauts playing a game on the Moon.


Chapter 9 Sports on the Moon

Section 8 Modeling Human Motion: Bounding on the Moon

What Do You See?

Learning Outcomes

In this section, you will
- **Apply** a cylinder as a model of a human leg acting as a pendulum during walking.
- **Measure** the amount of time for a human leg to swing forward and back as a human walks on Earth.
- **Predict** the amount of time for a human leg to swing forward as a human walks on the Moon.
- **Explain** why it is not possible to walk unaffected on the Moon.
- **Discover** how the period of a pendulum depends on its length, mass, and angle of swing.

What Do You Think?

Neil Armstrong was the first human to set foot on the Moon.
- Why do astronauts "bound" instead of walk or run on the Moon?
- Would running events in track and field be different if they were held on the Moon, even indoors?

Record your ideas about these questions in your *Active Physics* log. Be prepared to discuss your responses with your small group and the class.

Investigate

1. Observe the *Active Physics* video of astronauts "walking" on the Moon. Record answers to the following in your log book:
   a) Compare how the astronauts use their legs and feet to move across the surface of the Moon to how legs and feet are used in habitual walking.
   b) Why do you think astronauts use their legs and feet that way to walk or run on the Moon?

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Students' Prior Conceptions

This section presents the opportunity for the teacher to uncover two specific prior conceptions that students believe about actions on the Moon as compared to actions on Earth. You need to emphasize the effect of reduced gravity on the Moon to the nature of the physiological forces needed to walk or to jump on the Moon.

1. **Students may believe that their physiological effort will result in the same effect on the Moon as on Earth.** Take the opportunity to have students look into how astronauts train on Earth in preparation for travel to the Moon or to the Space Station. Astronauts train in large pools while wearing packs and suits weighing several hundred pounds so that they become experienced with moving around in lower gravity while still having the inertia of the space suit. The buoyant force of the water counteracts the force of gravity on Earth to simulate the gravity of the Moon or apparent lack of gravity on the Space Station. Students will discover that even while wearing massive space suits and environmental and safety packs, a small jump on Earth becomes a large bound on the Moon.
2. **Students may misconstrue the transformations of kinetic and potential energy on the Moon as compared with that on Earth.** Because a small leap on Earth becomes a large bound on the Moon, students may not consider the conservation of energy to be a universal principle. The change in the mechanical energy on the Moon results in the same amount of change in the gravitational potential energy on the Moon, but the relationship between $\frac{1}{2}mv^2$ and mgh differ because g on the Moon is approximately 1⁄6 g on Earth. Thus, a given amount of mechanical energy on Earth transforms to the same amount of mechanical energy on the Moon, but the change in height would be much greater on the Moon.

What Do You Think?

The introduction of Neil Armstrong being the first human to set foot on the Moon evokes a feeling of awe and is meant to grab students' attention. Encourage them to record their initial impressions and recall previous physics concepts in answering these questions. Students may think that the bounding motion of the astronauts is because of the lack of air on the Moon. Students are likely to say that if astronauts bound and do not walk on the Moon, as mentioned in the previous question, the running events in track and field will be affected by how astronauts move from one location to another. Point out that answers in the *Active Physics* logs will not be evaluated but you will be checking to see if the answers are earnestly written and students are thinking constructively using logic and reasoning. Emphasize the link between the title of this section and the questions asked. After a thought-provoking discussion, allow time for students to frame their responses.

What Do You Think?

A Physicist's Response

Astronauts bound on the Moon because they can't walk due to low friction, loose soil, and the effect of the Moon's reduced gravity on human legs acting as pendulums during walking. While running involves voluntarily bringing the trailing leg forward with each stride, not allowing the leg to swing as a pendulum, the great dependence on frictional force for running would seem to make running on the Moon impossible. There would also be great need to adjust the timing of running strides on the Moon because the body would fly for a much longer time after each push.

Investigate

1.a)

Students will observe that astronauts bound on the Moon.

1.b)

Astronauts bound on the Moon because they can't walk due to low friction, loose soil, and the effect of the Moon's reduced gravity on human legs acting as pendulums during walking.

NOTES

Teaching Tip

Three-quarters inch diameter PVC pipes may be used for the cylinders. PVC pipe may be obtained from any hardware or home supply store inexpensive in 10-ft lengths, which is sufficient to make the cylinders for two groups. If possible, drill a hole through the PVC pipe close to one end that is slightly larger than the diameter of the support rod on which they will hang. Hang the cylinders from the support rod by passing the rod through the hole in the pipe, rather than hanging from a string as shown in the student edition. For added discussion, you may choose to have one or more groups use PVC pipe of different diameters (i.e., 1 in. or 1/2 in. vs. 3/4 in.). This would be analogous to people with different diameter legs (e.g., football players vs. marathon runners).

2.a)

Students' results will vary depending upon the length of the pendulums chosen. Students record this data in their logs.

3.a)

Students record the period for the cylinder they choose.

4.a)

Students may note a small difference in the period when the cylinder is pulled a quarter of the way up to when it is pulled halfway up. To prevent introducing a new variable, students should always pull up the cylinders at the same angle for each trial.

5.a)

The table below shows some possible sample data.

Cylindrical Pendulum

Pendulum length (m)	Period (s)
0.10	0.50
0.20	0.80
0.40	1.00
0.80	1.50
1.20	1.90

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2. Use a set of cylinders of various lengths as pendulums. Each cylinder has a hole at one end to allow it to be hung as shown in the diagram. The length of the string should be as short as possible while still allowing the cylinder to swing freely. If string is unavailable, you may choose to slip the drilled end of the cylinder directly on to the cross arm. This will allow the cylinder to swing freely. Measure the length of each cylinder in centimeters (to the nearest 0.1 cm) from the pivot point where the string is tied to the horizontal bar to the bottom edge of the cylinder.

a) Record the lengths of the cylinders (in centimeters) in a column in your *Active Physics* log. Leave room for several more columns of data to the right of this first one.

3. Choose one of the longer cylinders, pull it aside about ½ way up, and allow it to swing as a pendulum. Use a stopwatch to measure the *period* of this pendulum. (The period of a pendulum is the time, in seconds, for the pendulum to complete one full swing over and back. The symbol for period is "*T*" and it is capitalized to show that it is not the usual time, but the specific time for one full swing to take place.) A good way to measure the period accurately is to measure the time to complete 10 swings over and back and then divide the measurement by 10.

a) Record the measurement of this period in your *Active Physics* log in a separate column next to the length of the cylinder.

4. Using the same cylinder, pull it aside about ¼ of the way up, and allow it to swing. Measure the period of the pendulum.

a) Record it in your *Active Physics* log. Does it differ from the period measured in *Step 3*? If so, keep this in mind as you make measurements.

5. Measure the period of each of the cylinders using the method of *Step 3*.

a) Record the period of each cylinder, in seconds, in the column next to the lengths of the cylinders.

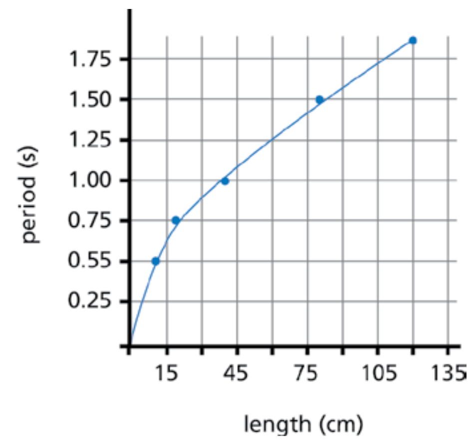
b) Plot a graph of period versus length for the cylindrical pendulums. Plot period, T, (in seconds) on the vertical axis, and length (in centimeters) on the horizontal axis. Mark the data points on your plot and sketch a smooth line that has an even distribution of data points around it. Observe carefully to decide whether the line should be straight or curved.

6. Now you will examine two types of walking, stiff-legged and the more usual style.

Observe a member of your group as he or she walks stiff-legged (without bending the knees). This can look pretty funny. Notice how after one foot hits the ground, that the opposite leg, trailing behind, swings forward as a pendulum before it is used for the next step. Also notice that a human leg is similar in shape to a cylinder. It is suspended at the top from the hip joint. The person does not use much muscular force to swing the leg forward because the force of gravity helps swing the leg forward. Therefore, the forward swing of a stiff human leg can be modeled by the cylinders used above.

5.b)

The graph of period vs. length should have a shape similar to the one shown below.



6.

Students observe a member of their group walk stiff-legged and then normally.

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7.a)

Student leg lengths will vary and should be recorded their *Active Physics* logs.

7.b)

Students should note that the time required for the lower leg to move from back to front is about 1 s. A student with a leg length of 1 m would act as a cylindrical pendulum with a period of approximately 1.6 s, and half that time would be 0.8 s, or roughly 1 s.

Name	Leg Length (m)	Period (s)
Tonya	0.90	1.02
Eric	1.10	1.18
Jose	1.20	1.10
Keshawn	1.40	1.27

7.c)

Students' graphs should appear similar to the one in *Step 5.b)*. Students should find that the cylindrical pendulum serves as a good model for the swinging of the lower leg, since its motion is primarily driven by gravity, rather than the use of muscles of the body.

8.a)

Students' calculations will vary with the lengths used for the pendulums. The completed table using the data given above is shown below.

Cylindrical Pendulum		
Pendulum length (m)	**Measured period (s)**	**Calculated period (s)**
0.10	0.50	0.52
0.20	0.80	0.74
0.40	1.00	1.03
0.80	1.50	1.47
1.20	1.90	1.80

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Observe a member of the group walking more normally. Notice that the leg bends at the knee, making the motion more complicated. Now observe a group member walking again, focusing on the lower leg, from the center of the knee down. Maybe this part of the leg can be modeled by a swinging cylinder.

7. Measure the length of the lower leg (in centimeters), from the knee to the bottom of the foot, for each member of your group.

 a) Create a table in your *Active Physics* log to record the names and lengths.

 b) As each member of your group walks in a normal way, other members should use stopwatches to measure the time for the person's lower leg to swing forward during one stride. For accuracy, take the average of several measurements. Since the forward swing of the lower leg is only ½ of its period, double the measurement and record that as the period of each person's lower leg in the table in your *Active Physics* log.

 c) Create a graph of period versus length to find out how well a cylindrical pendulum models the forward swing of your lower leg. Explain whether a cylindrical pendulum is a reasonably good model of a person's lower leg while walking.

8. Use the following equation to calculate the predicted periods of each cylindrical pendulum for which you made measurements (see *Physics Talk* for the source of this equation):

$$T = 5.1\sqrt{\frac{L}{g}}$$

Here L is the length of the cylinder (in centimeters) and g is the acceleration due to gravity ($9.8\ m/s^2 = 980\ cm/s^2$). Include units of measurement when you do the calculations to be sure that the answer is in seconds. Divide the work among members of your group and then share the results.

 a) Add a column to the data table in your *Active Physics* log, and record the values predicted by the equation. Write a comment comparing the measured and predicted periods.

 b) Use the same equation to calculate the period of your lower leg. Share results within your group, enter the data in your log, and compare the predicted results to the measured values. Write a comment in your log comparing them.

9. Since acceleration due to gravity on the Moon is ⅙ the acceleration due to the gravity on Earth, the pendulum and your lower leg will require more time to swing. You might expect that the leg would take 6 times longer to swing since gravity is 6 times smaller on the Moon. Notice that the "g" in the equation for the period is within the square root sign. This informs you that the leg will not take 6 times as long to swing but $\sqrt{6}$ or 2.5 times as long.

 a) Multiply the time for the forward swing of your lower leg (half of the period) by 2.5 to find how much time it would take your lower leg to swing forward on the Moon. Try to walk with the "swing time" that your lower leg would have when powered by the Moon's gravity. You could ask a group member to give you time signals to help you get it right.

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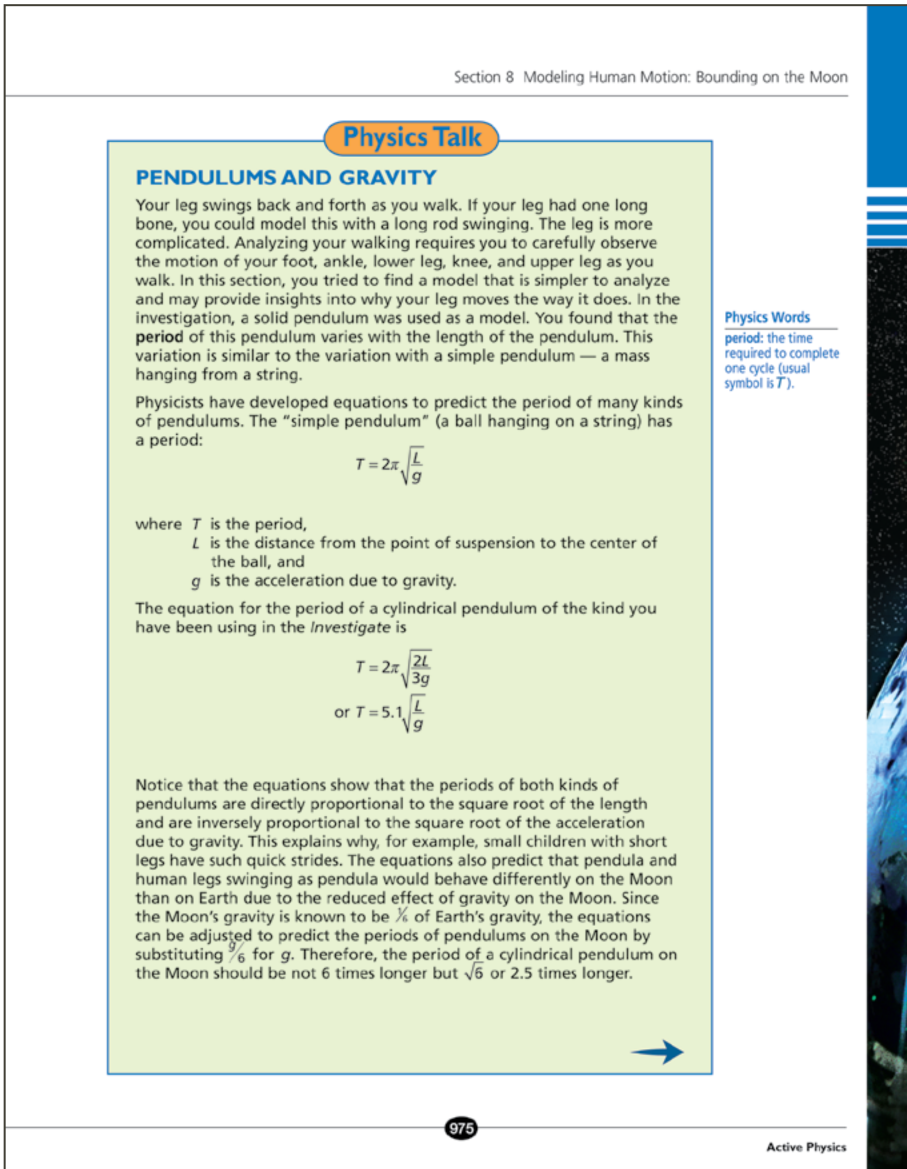
8.b)

Students' calculations will vary with the leg lengths. The completed table using the data given above is shown below.

Name	Leg length (m)	Measured period (s)	Calculated period (s)
Tonya	0.38	0.90	1.00
Eric	0.52	1.1	1.18
Jose	0.49	1.20	1.14
Keshawn	0.57	1.40	1.23

9.a)

Students try to walk with the period their leg would have on the Moon.



Physics Talk

PENDULUMS AND GRAVITY

Your leg swings back and forth as you walk. If your leg had one long bone, you could model this with a long rod swinging. The leg is more complicated. Analyzing your walking requires you to carefully observe the motion of your foot, ankle, lower leg, knee, and upper leg as you walk. In this section, you tried to find a model that is simpler to analyze and may provide insights into why your leg moves the way it does. In the investigation, a solid pendulum was used as a model. You found that the **period** of this pendulum varies with the length of the pendulum. This variation is similar to the variation with a simple pendulum — a mass hanging from a string.

Physics Words
period: the time required to complete one cycle (usual symbol is T).

Physicists have developed equations to predict the period of many kinds of pendulums. The "simple pendulum" (a ball hanging on a string) has a period:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where T is the period,
L is the distance from the point of suspension to the center of the ball, and
g is the acceleration due to gravity.

The equation for the period of a cylindrical pendulum of the kind you have been using in the *Investigate* is

$$T = 2\pi\sqrt{\frac{2L}{3g}}$$

$$\text{or } T = 5.1\sqrt{\frac{L}{g}}$$

Notice that the equations show that the periods of both kinds of pendulums are directly proportional to the square root of the length and are inversely proportional to the square root of the acceleration due to gravity. This explains why, for example, small children with short legs have such quick strides. The equations also predict that pendula and human legs swinging as pendula would behave differently on the Moon than on Earth due to the reduced effect of gravity on the Moon. Since the Moon's gravity is known to be $\frac{1}{6}$ of Earth's gravity, the equations can be adjusted to predict the periods of pendulums on the Moon by substituting $\frac{g}{6}$ for g. Therefore, the period of a cylindrical pendulum on the Moon should be not 6 times longer but $\sqrt{6}$ or 2.5 times longer.

Physics Talk

This *Physics Talk* discusses how the swinging motion of a leg is similar to the motion of a pendulum. Students read why the period of the pendulum varies with the length of the pendulum. To reinforce the purpose of the *Investigate* and to analyze it further, draw students' attention to the equation that physicists have developed that shows that the time period of a pendulum is inversely proportional to the square root of gravity and directly proportional to the square root of the length of the pendulum. The pendulum's length is the distance from the point of suspension to the center of mass of the cylinder for a cylindrical pendulum. For a simple pendulum it is the distance from the suspension point to the center of the mass on the string. Ask students why children with short legs have quick strides as compared to an adult. Because gravity on the Moon is less, students should be able to explain how it affects the rhythm of walking on the Moon. Ask them to explain in their *Active Physics* logs why astronauts jump on the Moon from one location to another, instead of walking. Have them include the equation that shows how the period of a stride on the Moon is 2.5 times greater than an ordinary stride on Earth.

Checking Up

1.

The period of a cylindrical pendulum is determined by the square root of the pendulum's length divided by the acceleration due to gravity at the location the pendulum is swinging.

2.

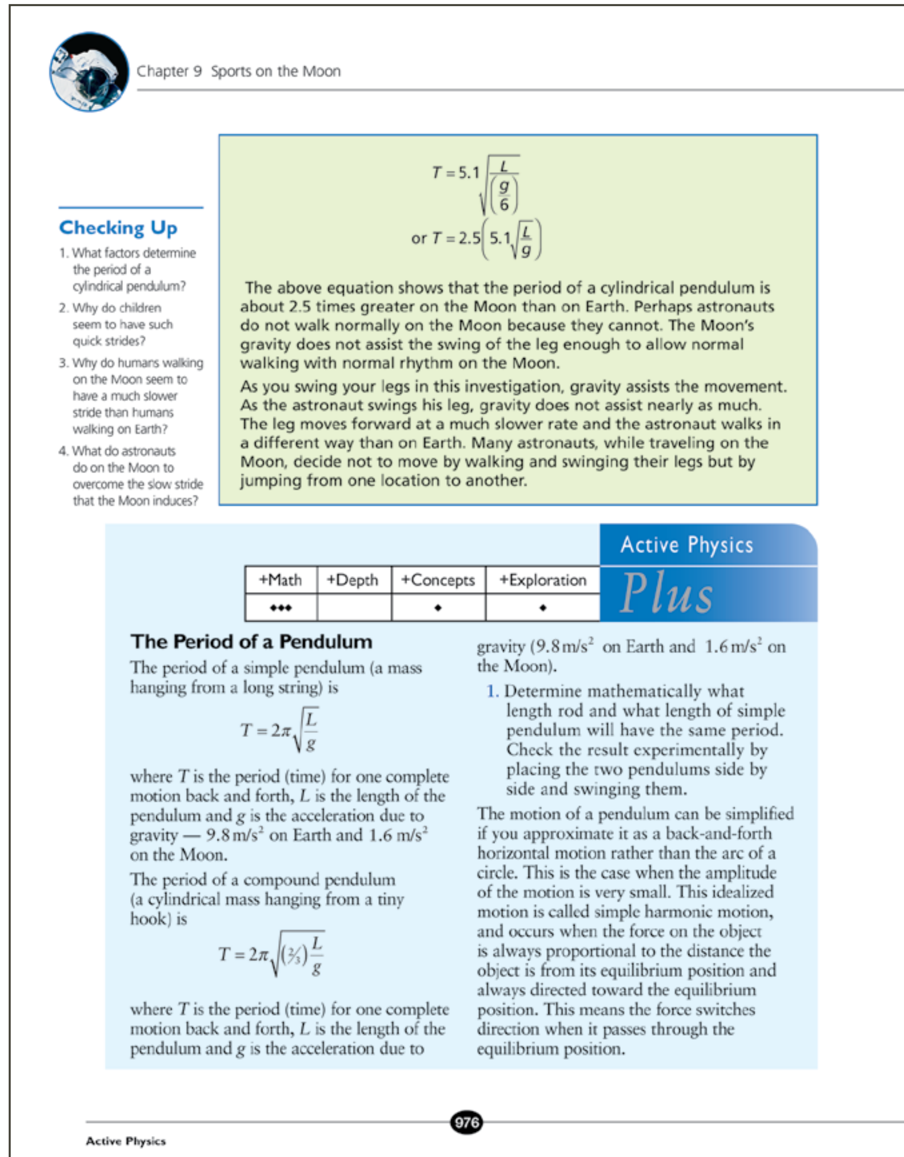
Children seem to have very quick strides relative to adults because their legs are shorter than adults and thus have a faster natural pendulum motion.

3.

Humans walking on the Moon have a much slower stride than similar people walking on Earth because the gravity on the Moon is weaker than Earth's gravity. Because the natural pendulum motion of the leg depends inversely on the acceleration due to gravity, a smaller acceleration due to gravity implies a longer period (perceived as a slower stride).

4.

To overcome the slow stride on the Moon, astronauts would have to use their leg muscles much more to accelerate the leg forward for a faster stride. This would soon become very tiring, so the astronauts use the kangaroo-like bounding technique, which allows the legs more time to swing naturally.



Chapter 9 Sports on the Moon

$$T = 5.1\sqrt{\frac{L}{\left(\frac{g}{6}\right)}}$$

$$\text{or } T = 2.5\left(5.1\sqrt{\frac{L}{g}}\right)$$

The above equation shows that the period of a cylindrical pendulum is about 2.5 times greater on the Moon than on Earth. Perhaps astronauts do not walk normally on the Moon because they cannot. The Moon's gravity does not assist the swing of the leg enough to allow normal walking with normal rhythm on the Moon.

As you swing your legs in this investigation, gravity assists the movement. As the astronaut swings his leg, gravity does not assist nearly as much. The leg moves forward at a much slower rate and the astronaut walks in a different way than on Earth. Many astronauts, while traveling on the Moon, decide not to move by walking and swinging their legs but by jumping from one location to another.

Checking Up

1. What factors determine the period of a cylindrical pendulum?
2. Why do children seem to have such quick strides?
3. Why do humans walking on the Moon seem to have a much slower stride than humans walking on Earth?
4. What do astronauts do on the Moon to overcome the slow stride that the Moon induces?

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
•••		•	•

The Period of a Pendulum

The period of a simple pendulum (a mass hanging from a long string) is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where T is the period (time) for one complete motion back and forth, L is the length of the pendulum and g is the acceleration due to gravity — $9.8\ \text{m/s}^2$ on Earth and $1.6\ \text{m/s}^2$ on the Moon.

The period of a compound pendulum (a cylindrical mass hanging from a tiny hook) is

$$T = 2\pi\sqrt{\left(\tfrac{2}{3}\right)\frac{L}{g}}$$

where T is the period (time) for one complete motion back and forth, L is the length of the pendulum and g is the acceleration due to gravity ($9.8\ \text{m/s}^2$ on Earth and $1.6\ \text{m/s}^2$ on the Moon).

1. Determine mathematically what length rod and what length of simple pendulum will have the same period. Check the result experimentally by placing the two pendulums side by side and swinging them.

The motion of a pendulum can be simplified if you approximate it as a back-and-forth horizontal motion rather than the arc of a circle. This is the case when the amplitude of the motion is very small. This idealized motion is called simple harmonic motion, and occurs when the force on the object is always proportional to the distance the object is from its equilibrium position and always directed toward the equilibrium position. This means the force switches direction when it passes through the equilibrium position.

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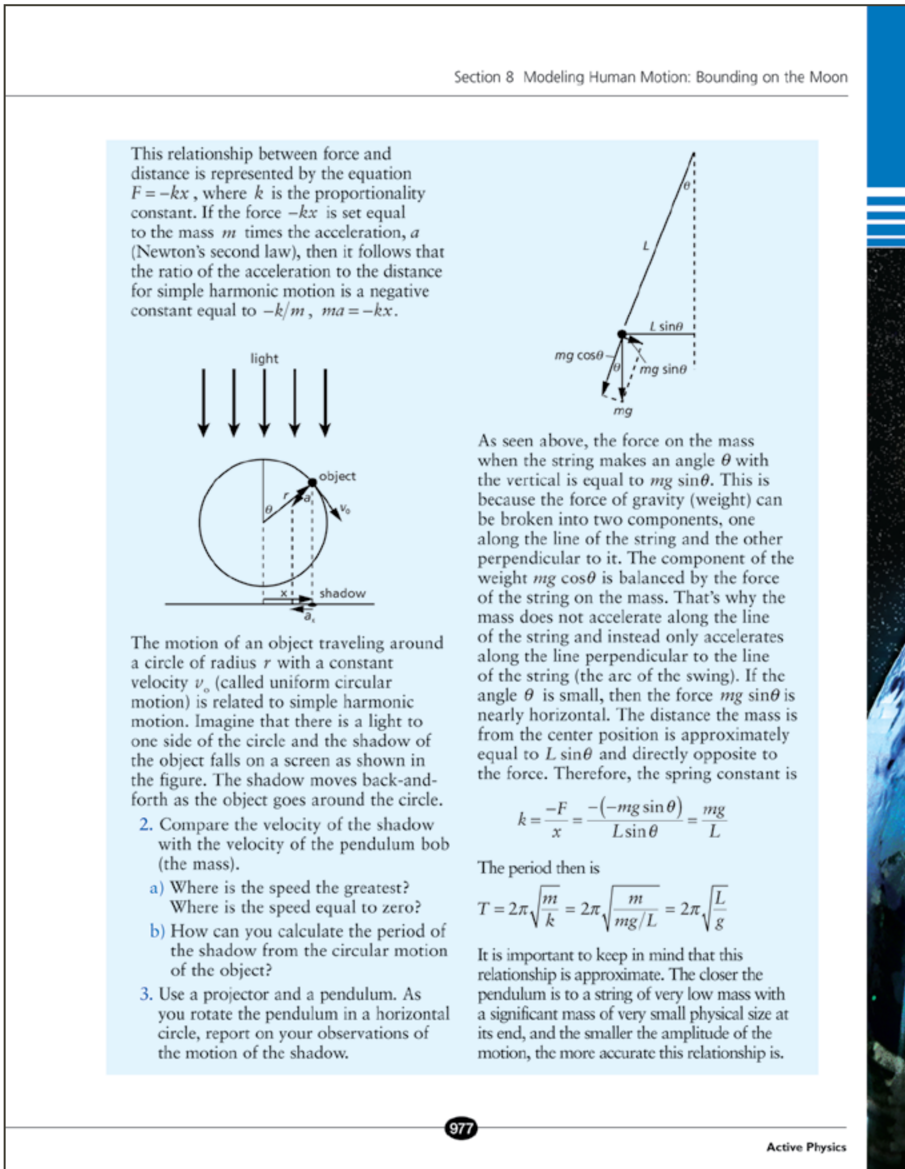
Active Physics

Active Physics Plus

This section explores the relationship between simple harmonic motion and circular motion. The first exercise compares the period of a cylindrical pendulum with a simple pendulum. It illustrates the connection between an object traveling in uniform circular motion and the one-dimensional projection of its motion to simple harmonic motion. In a problem, students calculate the period of the circular motion and compare the speed of the shadow of the motion to that of a pendulum. The equation for the period of a simple pendulum is derived using the relationship between these two motions.

1.

Arbitrarily choosing a pendulum with a period of one second, the length the pendulum needed to have a period of 1 s can be found by solving $T = 2\pi\sqrt{L/g}$ for L which gives $L = T^2 g/4\pi^2$ and substituting in 1 second for T. This gives $L = 0.25$ m. Using the equation for a cylindrical pendulum $T = 2\pi\sqrt{2L/3g}$ and solving for L with a period of 1 s, gives $T = 3T^2 g/8\pi^2$ which yields $L = 0.37$ m. The students then set up the experiment to verify that the periods are equal.



This relationship between force and distance is represented by the equation $F = -kx$, where k is the proportionality constant. If the force $-kx$ is set equal to the mass m times the acceleration, a (Newton's second law), then it follows that the ratio of the acceleration to the distance for simple harmonic motion is a negative constant equal to $-k/m$, $ma = -kx$.

The motion of an object traveling around a circle of radius r with a constant velocity v_o (called uniform circular motion) is related to simple harmonic motion. Imagine that there is a light to one side of the circle and the shadow of the object falls on a screen as shown in the figure. The shadow moves back-and-forth as the object goes around the circle.

2. Compare the velocity of the shadow with the velocity of the pendulum bob (the mass).
   a) Where is the speed the greatest? Where is the speed equal to zero?
   b) How can you calculate the period of the shadow from the circular motion of the object?
3. Use a projector and a pendulum. As you rotate the pendulum in a horizontal circle, report on your observations of the motion of the shadow.

As seen above, the force on the mass when the string makes an angle θ with the vertical is equal to $mg\sin\theta$. This is because the force of gravity (weight) can be broken into two components, one along the line of the string and the other perpendicular to it. The component of the weight $mg\cos\theta$ is balanced by the force of the string on the mass. That's why the mass does not accelerate along the line of the string and instead only accelerates along the line perpendicular to the line of the string (the arc of the swing). If the angle θ is small, then the force $mg\sin\theta$ is nearly horizontal. The distance the mass is from the center position is approximately equal to $L\sin\theta$ and directly opposite to the force. Therefore, the spring constant is

$$k = \frac{-F}{x} = \frac{-(-mg\sin\theta)}{L\sin\theta} = \frac{mg}{L}$$

The period then is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}} = 2\pi\sqrt{\frac{L}{g}}$$

It is important to keep in mind that this relationship is approximate. The closer the pendulum is to a string of very low mass with a significant mass of very small physical size at its end, and the smaller the amplitude of the motion, the more accurate this relationship is.

2.

The velocity of the shadow equals the exact velocity of a pendulum bob of the same length as the radius.

2.a)

The speed of the shadow is greatest in the middle and least at either end.

2.b)

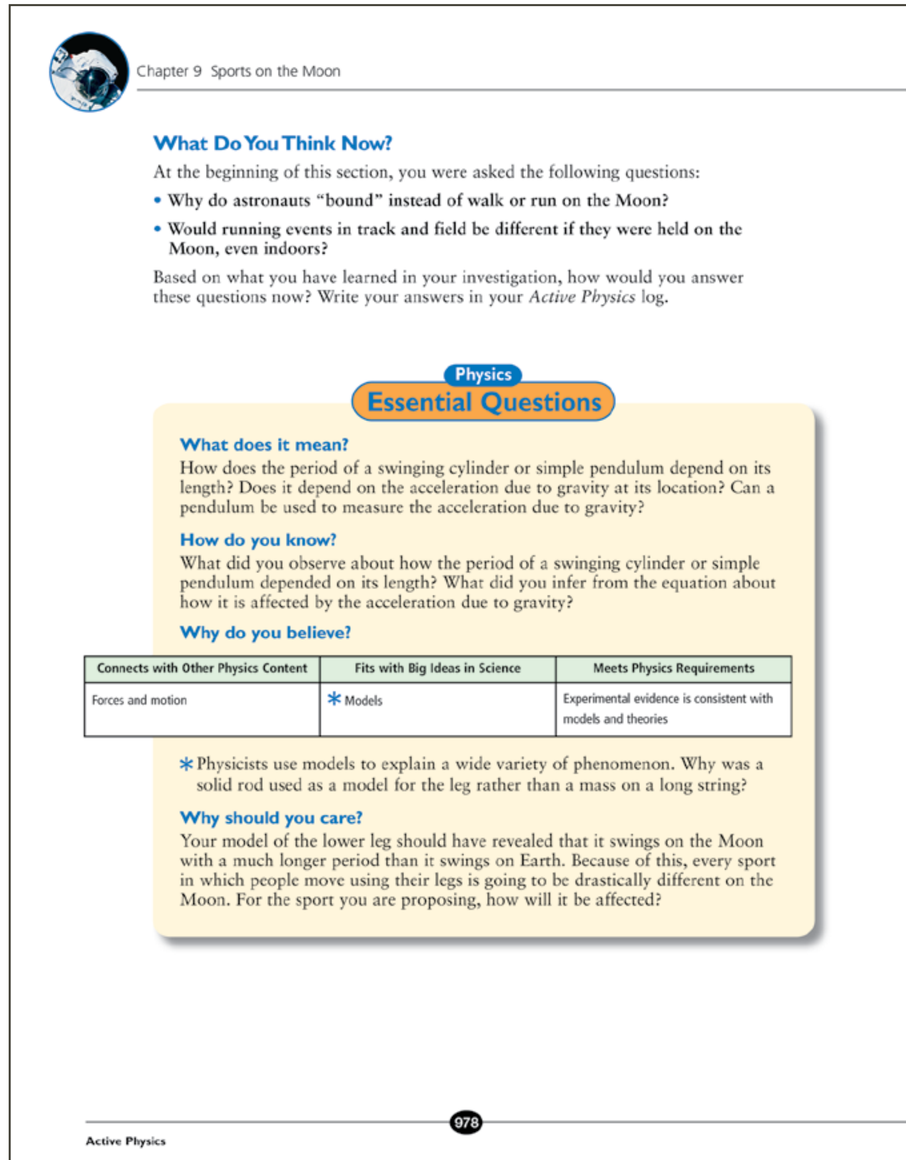
The period of the shadow will be exactly the same as the time it takes the object going around the circle to complete its cycle. Using $v_0 = d/t = 2\pi r/t$ and solving for t gives $t = 2\pi r/v_0$.

3.

Students set up a pendulum rotating in a circle and light to test the principles discussed.

What Do You Think Now?

As students begin to revise and edit their answers to *What Do You Think?*, share *A Physicist's Response* with them and discuss the importance of friction and the pendulum motion of the leg in relation to walking and running. This is the time to uncover recurring misconceptions by discussing the factors which affect gravity on the Moon. Ask students to explain how running on the Moon would be nearly impossible and how pendulum motion can be used as a model to show how the lower leg behaves while walking or running. Point out to students that they should show a connection between what they have learned in the *Investigate* to the questions that are asked in this section. Remind students to update their answers and make sure that they have a firmer grasp of physics concepts that were investigated and discussed in this section.



What Do You Think Now?

At the beginning of this section, you were asked the following questions:

- Why do astronauts "bound" instead of walk or run on the Moon?
- Would running events in track and field be different if they were held on the Moon, even indoors?

Based on what you have learned in your investigation, how would you answer these questions now? Write your answers in your *Active Physics* log.

Physics **Essential Questions**

What does it mean?
How does the period of a swinging cylinder or simple pendulum depend on its length? Does it depend on the acceleration due to gravity at its location? Can a pendulum be used to measure the acceleration due to gravity?

How do you know?
What did you observe about how the period of a swinging cylinder or simple pendulum depended on its length? What did you infer from the equation about how it is affected by the acceleration due to gravity?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Forces and motion	* Models	Experimental evidence is consistent with models and theories

* Physicists use models to explain a wide variety of phenomenon. Why was a solid rod used as a model for the leg rather than a mass on a long string?

Why should you care?
Your model of the lower leg should have revealed that it swings on the Moon with a much longer period than it swings on Earth. Because of this, every sport in which people move using their legs is going to be drastically different on the Moon. For the sport you are proposing, how will it be affected?

Physics Essential Questions

What does it mean?

The period of a pendulum is dependent on its length and on the acceleration due to gravity. The relevant equation is $T = 2\pi\sqrt{L/g}$. The acceleration due to gravity can be determined by measuring the period and length of a pendulum.

How do you know?

The observations showed that as the length of the pendulum increased, the period increased. A quadrupling of the length will provide a doubling of the period. On the Moon, the acceleration due to gravity is less than that on Earth, and the pendulum will have a greater period.

Why do you believe?

The leg does not have all its mass at one end as a pendulum with a bob does. The solid rod better represents the distribution of mass in the leg.

Why should you care?

Running will be very different on the Moon. Athletes may decide to jump from one location to another instead of running.

Section 8 Modeling Human Motion: Bounding on the Moon

Reflecting on the Section and the Challenge

There is a problem with walking on the Moon, and perhaps the same problem would extend to running on the Moon. Your legs will swing more slowly on the Moon. The period of the natural swing will be 2.5 times longer on the Moon. However, when running, does the runner simply allow the leg to swing forward? This swing delay, if it happens to runners also, could have serious implications for many sports on the Moon, unless "bounding" like astronauts is an acceptable substitute for walking or running. It probably can't even be said that a good runner on Earth would necessarily be a good "bounder" on the Moon because different muscles and skills are used. Maybe an Olympic champion who finished first in the 100-m dash would finish last in the "100-m bound" on the Moon! The time is nearing to write your proposal, so it's time to sort out the possibilities for sports on the Moon.

Physics to Go

1. The period of a "simple pendulum," a small massive object hanging from a string, is given by the equation $T = 2\pi\sqrt{(L/g)}$, where T is the time for the pendulum to swing once over and back, L is the distance from the point of suspension of the string to the center of mass of the object, and g is the acceleration due to gravity. Make a simple pendulum, let it swing, and see if the equation works. In using the equation, make sure the distance units of L and g are both measured in meters and m/s^2.
2. Describe how difficulty with walking or running on the Moon would affect at least one sport of your choice.
3. How would walking and running be affected on a planet that has an acceleration due to gravity greater than g on Earth?
4. How long would a simple pendulum need to be to have a period of 1.0 s? Make one and see if it works. (Hint: Solve for L in the equation $T = 2\pi\sqrt{(L/g)}$ or try different values of L and calculate T.)
5. Pendulums were used as the mechanical basis for making the first accurate clocks. What is it about the period of pendulums, even as they swing less and less, that makes them good for clocks?
6. You also use your arms as pendulums when you walk. Do you think you use your arms for a reason? Why or why not?
7. Why do you "shorten" the length of your arms by bending at the elbows when you are running?
8. Obtain data for a small child's leg swing. Does it fit the data on your graph?

Reflecting on the Section and the Challenge

As result of their investigation, students should now be able to make a direct connection between the *Chapter Challenge* and the effect of gravity on the Moon. The analysis of how running on the Moon would affect a sport will prompt students to consider their options in choosing and designing a sport. Ask students to read *Reflecting on the Section and the Challenge*, and reflect on the model of a pendulum that was used to investigate how humans use their legs. Refer back to earlier misconceptions and discuss how students were able to address them. Emphasize how learning happens with addressing prior beliefs and working like an investigator to arrive at the truth. Remind students that a review of the *What Do You See?* illustration should show them how much further they have progressed in their understanding of physics concepts.

Physics to Go

1.

The students will find that the equation works very well at predicting the period.

2.

Student answers will vary but should include the slow swing of the legs.

3.

You weigh more so your legs may not be able to support your body. In contrast the legs would swing very fast due to increased gravity.

4.

$T = 2\pi\sqrt{L/g}$

$L = T^2\, g/4\pi^2 =$

$L = (1.00\text{ s})^2\,(9.8\text{ m/s}^2)/4\pi^2$

$L = 0.249\text{ m}$

The standard value of g expressed to three significant figures was used in the above calculation; Thomas Jefferson once proposed such a pendulum as a standard for one second of time.

5.

Pendulums served as the basis for clocks because even as a pendulum "winds down" (decreases in amplitude, or swing distance) the period remains constant. Also, pendulum clocks that run too fast or too slow can have their period changed by slightly adjusting the position of the swinging mass (much like the position of the swinging mass on a grandfather's clock).

6.

We swing our arms for two reasons while walking. The first, and less important reason, is that they act as a counterbalance for the legs as we walk, since usually the right arm swings forward as the left arm swings back. The second, and probably more important reason, is that scientists have found that swinging the arms actually requires less energy than not swinging them. Since the arms and the legs are similar in length, they both have approximately the same period, so swinging both in sync is easy.

7.

Runners shorten the length of their arms and legs when they run to shorten the natural period of swing. A shorter natural period requires less effort to move them when running, since the object is to decrease the period by moving the legs quickly.

8.

The students will find that the data does fit on the graph line.

9.a)

The period depends upon the square root of the length. The length must be increased by a factor of four to double the period.

9.b)

The period of a pendulum is independent of the mass, as long as the length is kept constant.

9.c)

The period of a pendulum varies inversely with the square root of the acceleration due to gravity. If the acceleration of gravity is one-fourth as strong as on Earth, the period of the pendulum will be twice as long as an earthly pendulum.

Chapter 9 Sports on the Moon

9. Using the equation for a pendulum, answer the following questions:

a) How does the period of a swinging cylinder or simple pendulum depend on its length?

b) How does the period of a swinging cylinder or simple pendulum depend on its mass?

c) How does the period of a swinging cylinder or simple pendulum depend on the acceleration due to gravity?

d) Can a pendulum be used to measure the acceleration due to gravity?

10. *Preparing for the Chapter Challenge*

On Earth, many competitive sports consist of people running from one place to another while something else is occurring. A tennis player chases an opponent's shot, a baseball player runs the bases when the ball is hit, and so on. How could the game of tennis be modified so that an athlete playing on the Moon with a much slower stride could still return an opponent's shot in time?

Inquiring Further

Factors affecting the period of a pendulum

The equations for both simple and cylindrical pendulums presented in *Physics Talk* make no mention of mass or amplitude (swing distance) as variables that may affect the period. Do you think it is true that such characteristics do not affect the period? Design experiments to test the effects of these and other properties of pendulums on the period and report your procedures and results. If you measure data and graph it, be certain the scales for both axes start at the zero point.

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9.d)

Yes, a pendulum can be used to measure the acceleration due to gravity. By measuring the period of a pendulum of known length, and then substituting these values into the formula $T = 2\pi\sqrt{L/g}$, the acceleration due to gravity at that location may be determined by solving the equation for g.

10.

Preparing for the Chapter Challenge

Because it takes much longer to walk or run on the Moon (where the leg is acting as a slow-moving pendulum), tennis players would have an easier time jumping in the direction of the opponent's shot. Student answers should accommodate this need.

Inquiring Further

This is a student-designed inquiry activity. Students should vary the amplitude (angle of swing) that a pendulum goes through to see how it affects the period. Although generally ignored for low amplitude swings, the period of a pendulum is dependent upon the amplitude, and the calculations can become quite complex. Students should only verify that the period does make a difference when the amplitude starts to exceed a value of approximately 20°.

NOTES

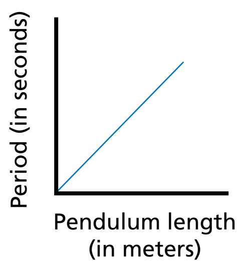
SECTION 8 QUIZ

9-8a **Blackline Master**

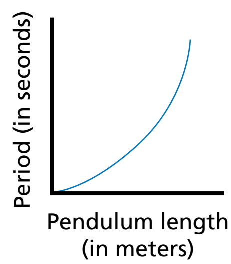
For the purpose of answering the following questions, the acceleration due to gravity on the Moon is 1.6 m/s^2 and on Earth 10 m/s^2.

1. Which graph below represents the relationship between the period of a cylindrical pendulum and the pendulum's length?

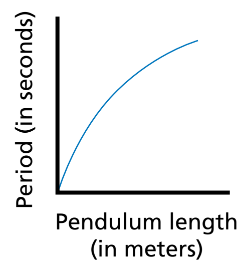
a)



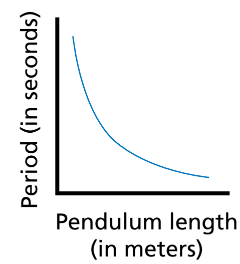
b)



c)



d)



2. Compared to the period of pendulum on the Moon, the period of a pendulum on Earth is

a) 6 times longer.
b) $\sqrt{6}$ times longer.
c) 6 times shorter.
d) $\sqrt{6}$ times shorter.

3. The reason humans walking on the Moon seem to have a much slower stride is because the Moon's weaker gravity

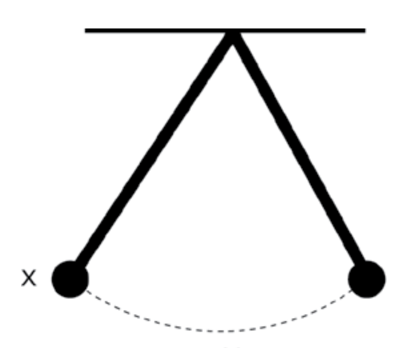
a) causes the human's legs to swing more slowly.
b) makes the human's legs longer.
c) makes it easier for humans to swing their legs.
d) makes bounding easier.

4. To allow runners to move their legs faster when running, which strategy would work best?

a) add weights to the ankles and run stiff-legged
b) bend the legs at the knees to shorten the length they swing
c) run in an area with lower gravity
d) increase the length of their legs to have a longer stride

5. A cylindrical pendulum swings as shown in the diagram at right. The period of the pendulum is the time required for the pendulum to swing from X to

a) Y.
b) Z.
c) Z and back to X.
d) Z and divided by 10.



SECTION 8 QUIZ ANSWERS

1. b) The period squared varies with the length, forming a graph like b).

2. d) The period depends upon one over the square root of the acceleration due to gravity of the planet. Since Earth's gravity is six times stronger than that of the Moon, the period of a pendulum on Earth is $\sqrt{6}$ times shorter.

3. a) Because gravity on the Moon is so much weaker, the human legs naturally swing more slowly when acted upon by gravity.

4. b) Shortening a cylindrical pendulum decreases the period, allowing the legs to swing faster. Adding weights would not change the period, since gravity accelerates all masses equally.

5. c) By definition, the period of a pendulum is the time required for the pendulum to swing from X to Z and back to X.

NOTES

CHAPTER 9