

# Electric Charge and Electric Field

## Static Electricity; Electric Charge and Its Conservation

Early experiments with the effects of **static electricity** on different substances placed in contact or proximity provided many of the conventions for electricity used today.

- The **electric charge** of an object is a discrete quantity that can be acquired or transferred. The charges of objects affect their interaction.
- There are two types of charge, **positive** and **negative**, such that two objects of like charge repel, whereas two objects of opposite charge attract. Objects with no charge are described as **neutral**.
- Charge can be transferred or induced onto objects, but the net quantity of charge always remains constant. This is called the **law of conservation of electric charge**.

## Electric Charge in the Atom

Electric charges are responsible for most forces at the microscopic level.

- The atom itself is composed partly of individual negative charges, called **electrons**, orbiting around a much larger nucleus. Positively charged protons are part of the nucleus such that the net charge of the atom is zero. Atoms that gain or lose electrons during interactions with other atoms are called **ions** and have a net nonzero charge.
- Some molecules distribute their electrons such that there is a charge difference between parts of the molecule, even though the overall charge is zero. Examples include water molecules, which are described as **polar** because of the nonuniform distribution of positive and negative charges within each molecule.

## Insulators and Conductors

The ability of a material to be influenced by external charge depends on the mobility of its electrons. Materials that readily allow charge to cross them are called **conductors**. The electrons in conductors are relatively fluid in their movement within the materials. Materials whose electrons are less fluid and consequently do not allow charge to cross them are called **insulators**.

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## Insulators and Conductors

The ability of a material to be influenced by external charge depends on the mobility of its electrons. Materials that readily allow charge to cross them are called **conductors**. The electrons in conductors are relatively fluid in their movement within the materials. Materials whose electrons are less fluid and consequently do not allow charge to cross them are called **insulators**.

## Induced Charge; The Electroscope

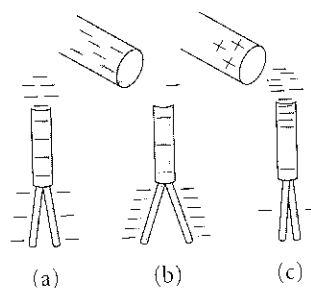
Charge can be imparted to neutral objects by contact or induction.

When a charged object touches a neutral conductive object, electrons are transferred such that two objects have the same type of charge. The direction of electron transfer in such a case depends on the initial type of charge.

When a charged object approaches a neutral conductive object without touching it, the electrons within the neutral object shift such that the near end has the opposite charge and the far end has the same charge.

If this process were repeated, but with the far end attached to a **ground**, a charge reservoir such as the earth, the entire object would receive an overall charge opposite that of the nearby object.

These properties are utilized by **electroscopes**, devices consisting of two metal "leaves" attached to a central conducting node for measuring charge. Charge can be imparted to the electroscope leaves, and the behavior of the leaves indicates the relative charge of other objects brought into its proximity. While the electroscope indicates whether charges are of the same or opposite type, it cannot independently identify them as positive or negative. However, the strength of charge is reflected in the angle of separation of the leaves.



## Coulomb's Law and Its Applications

The force resulting from the interaction of charged objects is directly proportional to their charges, and it is inversely proportional to the square of their separation distance.

Since Coulomb noted these relations, the equation (developed later) is known as Coulomb's law.  $F = kQ_1Q_2/d^2$ , where  $k$  represents the proportionality constant  $9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$ , and the charges  $Q_1$  and  $Q_2$  are in coulombs, C. Functional units of charges are often expressed in microcoulombs, where  $1 \mu\text{C} = 10^{-6} \text{ C}$ . Coulomb's law can be expressed also as  $F = Q_1Q_2/4\pi\epsilon_0d^2$ , where  $\epsilon_0 = 1/4\pi k = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$ , and this is called the **permittivity of free space**.

The smallest unit of charge is called an **elementary charge**, and it has the value  $e = 1.602 \times 10^{-19} \text{ C}$ . A single electron has a charge of  $-e$  and a single proton has a charge of  $+e$ . Because electrons and protons are functionally indivisible, all charges must be integer multiples of this elementary charge, and charge is thus described as **quantized**.

- 2 While Coulomb's law gives the magnitude of force, its direction is along the line joining the charges—mutually repelling for the same charges and mutually attracting for different charges. The law simplifies charges as immobile, one-dimensional positions in space called **point charges**. It assumes that their size is small relative to their separation distance.
- 3 Calculating the forces involving more than two particles at rest requires vectors. The force components are added in each of two perpendicular directions. If the point charges are not collinear, the magnitude of the resultant force can be determined by the Pythagorean formula. The direction of the resultant force can be found using trigonometry. Note that the signs on the components of this resultant force correspond to directions along each axis (not to the signs of the point charges).

## The Electric Field

Electrical forces do not require contact for objects to influence each other's motion. As with gravitational force, this circumstance is described as "action at a distance."

- 1 The force radiating from an electrical charge creates a **field** around the charge. The interaction between point charges is explained by the behavior of each charge in the field of the other charge.
- 2 The strength of a field at a point in space can be measured by its force imposed on a **test charge**, which is a relatively insignificant positive charge placed at that point. The strength of the **electric field** at a point in space is the ratio of the force on a test charge at that point to the magnitude of the test charge, so  $E = F/q$ .
- 3 Combining this with Coulomb's law, we find that the electric field at a distance  $r$  from a single point charge  $Q$  has a magnitude of  $E = kQ/r^2$ .
- 4 For several point charges, the total strength of their electric fields at a point can be determined by summing their vector components,  $E = E_1 + E_2 + E_3 + \dots$ , which is called the **superposition principle**.

## Field Lines

**Electric fields** are represented graphically with arrows extending away from positive charges or toward negative charges.

- 1 Electric field lines signify the direction of force caused by the electric field. The number of field lines in a given region is proportional to the magnitude of the force there.
- 2 The field's direction at any point is tangent to the field line at that point.
- 3 The strength of forces is indicated by the proximity of field lines to each other, such that they are closer together nearer the charge source.
- 4 For oppositely charged parallel plates, the magnitude of the electric field is constant everywhere between the plates, so the field lines are evenly spaced.

## Electric Fields and Conductors

When a conductor is placed inside the electric field produced by stationary charges, the strength of the electric field inside the conductor is zero. The electrons within the conductor arrange themselves by induction so that  $E = 0$  everywhere inside the conductor. However, the external field caused by the charge continues from the exterior surface of the conductor as though the conductor were not present. The direction of the field is perpendicular to the external surface of a conductor.

### For Additional Review

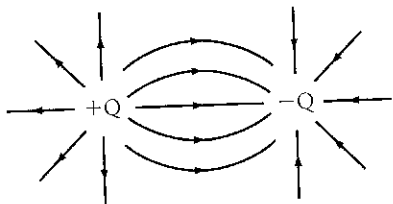
Calculate the field strength due to several charges where no more than two charges are collinear.

### Multiple-Choice Questions

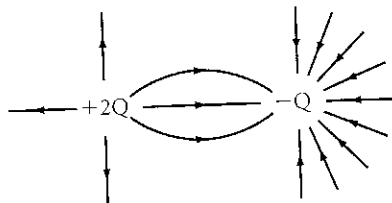
1. What is the magnitude of force between two negative point charges, one of  $-2 \times 10^{-5}$  C, the other of  $-4 \times 10^{-5}$  C, that are  $4.3 \times 10^{-4}$  m apart?
  - (A)  $4.8 \times 10^6$  N
  - (B)  $7.2 \times 10^6$  N
  - (C)  $3.9 \times 10^7$  N
  - (D)  $8.9 \times 10^7$  N
  - (E)  $4.3 \times 10^8$  N

2. Which of the following is NOT a consistent representation of charges and their field lines?

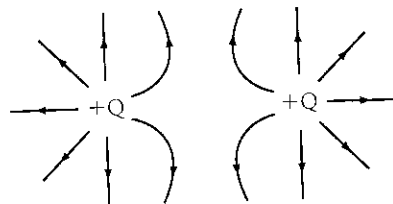
I.



II.



III.



- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) II and III only

3. A negative charge of  $-5.0 \mu\text{C}$  is equidistant from two positive charges as shown below. One positive charge is  $+3.0 \mu\text{C}$ , the other is  $+4.7 \mu\text{C}$ , and each is 0.01 m away from the negative charge. What is the net electrostatic force on the central charge if all three point charges are collinear?

$+4.7 \mu\text{C}$  ———  $-5.0 \mu\text{C}$  ———  $+3.0 \mu\text{C}$

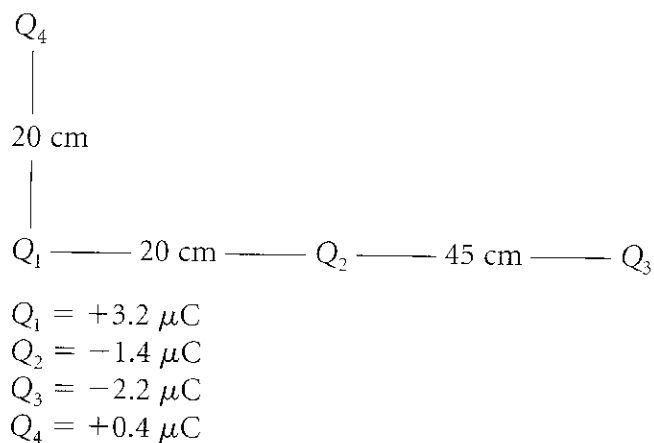
- (A) 3500 N to the left
- (B) 700 N to the left
- (C) 700 N to the right
- (D) 3500 N to the right
- (E) 0 N

4. Find the magnitude of an electric field 1 mm from  $+6.0 \times 10^{-3}$  C point charge.
  - (A)  $5.4 \times 10^2$  N/C
  - (B)  $5.4 \times 10^4$  N/C
  - (C)  $5.4 \times 10^7$  N/C
  - (D)  $5.4 \times 10^{11}$  N/C
  - (E)  $5.4 \times 10^{13}$  N/C

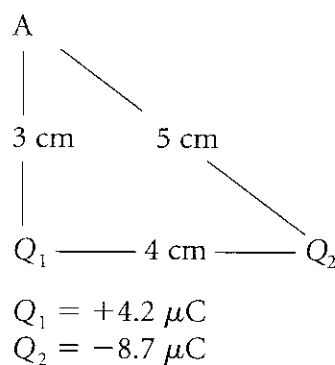
5. What is the electric field halfway between two collinear negative charges separated by 6 cm. One charge is  $-2.1 \mu\text{C}$ , whereas the other is  $-1.3 \mu\text{C}$ .
- (A)  $2.6 \times 10^5 \text{ N/C}$   
 (B)  $6.3 \times 10^5 \text{ N/C}$   
 (C)  $3.2 \times 10^6 \text{ N/C}$   
 (D)  $8.0 \times 10^6 \text{ N/C}$   
 (E)  $1.1 \times 10^7 \text{ N/C}$
6. A metal rod of unknown charge charges an electroscope by induction. As a second metal rod, also of unknown charge, approaches the charged electroscope, the electroscope leaves separate more. This means that
- (A) the charge of both rods is negative  
 (B) the charge of both rods is positive  
 (C) the charge of the first rod is positive, and the charge of the second is negative  
 (D) the charge of the first rod is negative, and the charge of the second is positive  
 (E) Not enough information is given to determine the answer.
7. The electric field halfway between two positive point charges of  $1.7 \times 10^{-5} \text{ C}$  is
- (A)  $34 \text{ N/C}$             (D)  $-17 \text{ N/C}$   
 (B)  $17 \text{ N/C}$             (E)  $-34 \text{ N/C}$   
 (C)  $0 \text{ N/C}$
8. As the charge doubles for two oppositely charged particles, initially  $+Q$  and  $-Q$ , the strength of the electric field halfway between them
- (A) is quartered  
 (B) is halved  
 (C) remains the same  
 (D) is doubled  
 (E) is quadrupled
9. As the distance between two identical charges is halved, the magnitude of the force between them
- (A) is quartered  
 (B) is halved  
 (C) remains the same  
 (D) is doubled  
 (E) is quadrupled
10. What is the electric force of two electrons  $10^{-10} \text{ m}$  apart?
- (A)  $2.3 \times 10^{-8} \text{ N}$   
 (B)  $4.6 \times 10^{-8} \text{ N}$   
 (C)  $9.2 \times 10^{-8} \text{ N}$   
 (D)  $2.3 \times 10^{-9} \text{ N}$   
 (E)  $4.6 \times 10^{-10} \text{ N}$

### Free-Response Questions

1. Calculate the net forces on particle  $Q_2$  as shown below.



2. Calculate the electric field at point A as shown below.



## ANSWERS AND EXPLANATIONS

### Multiple-Choice Questions

1. **(C) is correct.** Force between two point charges is given by  $F = kQ_1Q_2/d^2$   
 $= (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(2 \times 10^{-5} \text{ C})(4 \times 10^{-5} \text{ C})/(4.3 \times 10^{-4} \text{ m})^2$   
 $= 3.9 \times 10^7 \text{ N}.$
2. **(B) is correct.** The number of field lines should reflect the relative strength of a charge. In Figure II, the field lines are reversed, with twice as many heading to the  $-Q$  charge as the  $+2Q$  charge. Figures I and III are accurate.
3. **(B) is correct.** Consider the force between  $+4.7 \mu\text{C}$  and  $-5.0 \mu\text{C}$  as  $F_{21}$  and the force between  $+3.0 \mu\text{C}$  and  $-5.0 \mu\text{C}$  as  $F_{23}$ .  
 $F_{21} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(5.0 \times 10^{-6} \text{ C})(4.7 \times 10^{-6} \text{ C})/(10^{-2} \text{ m})^2$   
 $= 2.1 \times 10^3 \text{ N},$   
 $F_{23} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(5.0 \times 10^{-6} \text{ C})(3.0 \times 10^{-6} \text{ C})/(10^{-2} \text{ m})^2$   
 $= 1.4 \times 10^3 \text{ N}.$   
 Force is additive, but the direction of the force must be accounted for. Each positive charge exerts an attractive force on the negative charge in opposite directions, with the stronger positive charge exerting a stronger force.  
 Thus  $F = F_{21} + F_{23} = 2.1 \times 10^3 \text{ N} - 1.4 \times 10^3 \text{ N} = 700 \text{ N}$  in the direction of the  $+4.7 \mu\text{C}$  charge.
4. **(E) is correct.** The electric field is given by  $E = kQ/r^2$   
 $= (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(6.0 \times 10^{-3} \text{ C})/(1.0 \times 10^{-3} \text{ m})^2 = 5.4 \times 10^{13} \text{ N/C}.$
5. **(D) is correct.** The field from each charge is pulling in its own direction. As such,  $E = kQ_1/r_1^2 - kQ_2/r_2^2$  in the direction of the stronger charge.

$$E = \left( 9.0 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \right) \left[ \frac{(2.1 \times 10^{-6} \text{ C})}{(3.0 \times 10^{-2} \text{ m})^2} - \frac{(1.3 \times 10^{-6} \text{ C})}{(3.0 \times 10^{-2} \text{ m})^2} \right]$$

$$E = \left( 9.0 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \right) [2.3 \times 10^{-3} \text{ C/m}^2 - 1.4 \times 10^{-3} \text{ C/m}^2]$$

$$E = 8.0 \times 10^6 \text{ N/C}.$$

6. (E) is correct. The behavior of this electroscope indicates that the two rods have opposite charges, but the specific charge on each rod cannot be determined.
7. (C) is correct. Without any calculation, the field strength is 0 N/C because the fields from the two charges exactly cancel there. Note that the fields do *not* cancel at other points that are equidistant from the two charges, because the vectors are not colinear.
8. (D) is the correct answer. As the superposition principle states, the electrical field is equal to the sum of electrical fields. Halfway between the charges, the electrical field is in the same direction for each. Initially, the electric field is  $E = kQ/r^2 + kQ/r^2 = 2kQ/r^2$ . If the charge doubles, the strength of the electric field is  $E = 2kQ/r^2 + 2kQ/r^2 = 4kQ/r^2$ . As such, the electric field has doubled.
9. (E) is correct. By Coulomb's law,  $F = kQ_1Q_2/d^2$ , as the distance between the charges is halved,  $F = kQ_1Q_2/(d/2)^2 = 4kQ_1Q_2/d^2$ . As such the force quadruples.
10. (A) is correct. Force between two point charges is given by  $F = kQ_1Q_2/d^2 = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})(1.6 \times 10^{-19} \text{ C})/(10^{-10} \text{ m})^2 = 2.3 \times 10^{-8} \text{ N}$ .

### Free-Response Questions

1. First, from trigonometry, the angle between  $Q_2$  and  $Q_4$  must be  $45^\circ$ . Then, reference axes must be established for each, here assuming up and right to be positive and down and left to be negative. Next, the forces for each must be calculated.

$$F_{23} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.4 \times 10^{-6} \text{ C})(2.2 \times 10^{-6} \text{ C})/(0.45 \text{ m})^2 = 0.14 \text{ N (repulsion to the left, so } -0.14 \text{ N)}$$

$$F_{21} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.4 \times 10^{-6} \text{ C})(3.2 \times 10^{-6} \text{ C})/(0.2 \text{ m})^2 = 1.0 \text{ N (attraction to the left, so } -1.0 \text{ N)}$$

$$F_{24} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.4 \times 10^{-6} \text{ C})(4.0 \times 10^{-7} \text{ C})/(0.28 \text{ m})^2 = 0.064 \text{ N (attraction to the left, so } -0.064 \text{ N)}$$

Then combine these facts, breaking the forces into vectors.

$$F_x = F_{23} + F_{21} + F_{24} \cos \theta = -0.14 \text{ N} + -1.0 \text{ N} + -0.045 \text{ N} = -1.2 \text{ N},$$

$$\text{and } F_y = F_{24} \sin \theta = 0.045 \text{ N}.$$

The magnitude and direction can be determined using

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-1.2)^2 + (0.045 \text{ N})^2} = 1.2 \text{ N}$$

$$\text{and } \tan \theta = F_y/F_x = 0.045 \text{ N}/-1.2 \text{ N} = 178^\circ.$$

This is understandable, since the preponderance of force comes from the  $-x$  direction.

*This response would receive full credit because it correctly computes the forces between the charges, separates and sums each force into its correct vector components, and converts this into the correct magnitude and direction of force.*

2. The field from each point charge must be computed before the vector components can be summed:

$$E_{A1} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4.2 \times 10^{-6} \text{ C})/(3.0 \times 10^{-2} \text{ m})^2 = 4.2 \times 10^7 \text{ N/C}$$

$$E_{A2} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(8.7 \times 10^{-6} \text{ C})/(5.0 \times 10^{-2} \text{ m})^2 = 3.1 \times 10^7 \text{ N/C}$$



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$E_{Ax} = E_{A2} \cos \theta$ , where  $\theta$  is the angle at  $Q_2$ . By trigonometric definitions,  $\cos \theta = \text{adjacent/hypotenuse} = 4/5$ , even if  $\theta$  is not explicitly presented (although it could be determined if necessary) and  $E_{Ax} = (3.1 \times 10^7 \text{ N/C})(0.8) = 2.5 \times 10^7 \text{ N/C}$ .  $E_{Ay} = E_{A1} + -E_{A2} \sin \theta$  where  $\theta$  is the angle at  $Q_2$ . Note that  $E_{A1Y}$  is upward (positive) and  $E_{A2Y}$  is downward (negative). Again, by trigonometric definitions,  $\sin \theta = \text{opposite/hypotenuse} = 3/5 = 0.6$  and  $E_{Ay} = 4.2 \times 10^7 \text{ N/C} + -(3.1 \times 10^7 \text{ N/C})(0.6) = 2.3 \times 10^7 \text{ N/C}$ . For magnitude and direction,

$$E = \sqrt{E_{Ax}^2 + E_{Ay}^2} = \sqrt{(2.5 \times 10^7 \text{ N/C})^2 + (2.3 \times 10^7 \text{ N/C})^2} = 3.4 \times 10^7 \text{ N/C}$$

where  $\tan \theta = E_{Ay}/E_{Ax} = 2.3 \times 10^7 \text{ N/C}/2.5 \times 10^7 \text{ N/C}$ .  $\theta = 43^\circ$ .

*This response would receive full credit because it correctly computes the field at the point due to each charge, separates and sums each field into its correct vector components, and converts this into the correct magnitude and direction for the electric field.*

# Electric Potential and Electric Energy; Capacitance

Electric energy provides further quantitative descriptions of electrical phenomena beyond force and fields.

## Electric Potential and Potential Difference

Since potential energy is measured as a difference of potential energies rather than as an absolute value, **electric potential energy** refers to the change in potential energy as a charge is moved between two points.

Electric potential energy is the negative of the work done by the electric field to move the charge. Because only a difference in potential energy is measured, a zero potential energy can be assigned to either point.

**Electric potential**,  $V$ , is defined as the quotient of potential energy to charge, such that  $V_a = PE_a/q$ . Further,  $V_{ab} = V_a - V_b = -W_{ba}/q$ . The units for electric potential difference are **volts**, equivalent to J/C, and the terms **potential difference** and **voltage** are interchangeable.

The potential energy difference is defined as  $\Delta PE = PE_b - PE_a = qV_{ba}$ .

## Relation Between Electric Potential and Electric Field

For a uniform electric field, the relation to electric potential is given  $E = V_{ba}/d$  for a positive charge  $q$  moved from a point  $b$  to a point  $a$  separated by  $d$  meters.

The **electric field** is a vector that can have the equivalent units V/m or N/C.

**Electric potential** is a scalar with equivalent units of V or J/C.

## Equipotential Lines

As field lines graphically represent electric fields, **equipotential lines** represent electric potential.

The points that comprise an equipotential line have the same electric potential. Equipotential lines run perpendicular to field lines at any point.

Equipotential lines are parallel to charged parallel plates, and they are in concentric circles around single charges.

The electric potential of these lines depends on their distance from the sources of potential.

## The Electron Volt, a Unit of Energy

On a microscopic scale, the **electron volt** is often used for energy.  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ , representing the energy gained by an electron or other particle with a charge of  $1.6 \times 10^{-19} \text{ C}$  traveling across a one-volt potential difference.

## Electric Potential Due to Point Charges

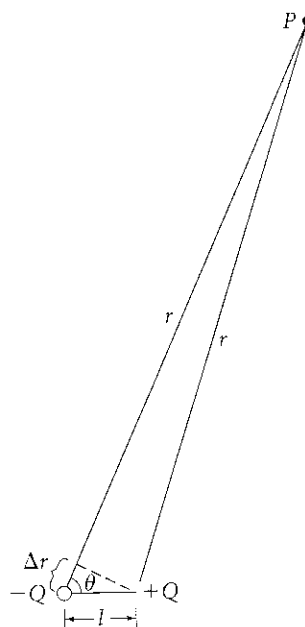
Assuming the electric potential is zero at a distance of infinity, the electric potential at a distance  $r$  from a point charge  $Q$  is given by the equation  $V = kQ/r$ .

The combined electric potential of several point charges is given by the scalar sum of their individual potentials. Note that the sign for each potential must match the sign of the charge.

## Electric Dipoles

An **electric dipole** consists of two equal but oppositely charged point charges in space.

The electric potential at a point that is  $r$  meters away from the positive charge of an electric dipole, for which the charges are a distance apart,  $l$ , is given by  $V = kQl \cos \theta / r^2$ , where  $\theta$  is the interior angle between a line from the negative charge to the positive charge and a second line from the negative charge to the point of potential. This is valid when the distance  $r$  is much greater than the distance of charge separation,  $l$  ( $r \gg l$ ).



The product of charge and the distance of separation is called a **dipole moment**,  $p$ , so  $V = kp \cos \theta / r^2$ . This relation explains the behavior of electrically neutral polar molecules, which have a nonsymmetrical distribution of the molecular charge.

## Capacitance

A **capacitor** is a charge-storing device often made from two conducting parallel plates separated by a small layer of air or a thin film.

The plates become equally and oppositely charged when the capacitor is in a circuit pathway that has a potential difference.

The charge quantity on each plate,  $Q$ , is proportional to the voltage applied, such that  $Q = VC$ .  $C$  is the **capacitance**, whose value of  $C$  depends on the capacitor's design.

Capacitance is a function of the distance between plates,  $d$ , and the area of each plate,  $A$ , such that  $C = \epsilon_0 A/d$ , where  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$ .  $\epsilon_0$  is called the permittivity of space.

The unit of **capacitance** is  $C/V$ , also known as a **farad**.

### For Additional Review

Consider why the electric dipole has 0 voltage at point  $P$  when  $\theta = 90^\circ$ , seemingly contradicting the results of Example 17-4 (in Giancoli's *Physics*, page 510), in which point  $A$  is at a  $90^\circ$  angle and has a nonzero voltage.

### Multiple-Choice Questions

1. What is the change in potential energy when a  $+2.5 \text{ C}$  point charge is moved from a negative plate to a positive plate across a potential difference of  $8 \text{ V}$ ?  
(A)  $24 \text{ J}$  gained  
(B)  $20 \text{ J}$  gained  
(C)  $3 \text{ J}$  gained  
(D)  $0 \text{ J}$  net change  
(E)  $20 \text{ J}$  lost
2. What is the speed of a  $-3.3 \text{ C}$  charge of mass  $5.4 \times 10^{-10} \text{ kg}$  when it is moved from a negative plate to a positive plate across a potential difference of  $6.4 \text{ V}$ ?  
(A)  $2.1 \times 10^1 \text{ m/s}$   
(B)  $9.0 \times 10^2 \text{ m/s}$   
(C)  $6.5 \times 10^3 \text{ m/s}$   
(D)  $2.8 \times 10^5 \text{ m/s}$   
(E)  $1.3 \times 10^{10} \text{ m/s}$
3. What is the magnitude of the electric field between two parallel plates that have a potential difference of  $10 \text{ V}$  and are  $25 \text{ cm}$  apart?  
(A)  $0.025 \text{ V/m}$   
(B)  $0.25 \text{ V/m}$   
(C)  $2.5 \text{ V/m}$   
(D)  $4 \text{ V/m}$   
(E)  $40 \text{ V/m}$
4. A  $-3.5 \text{ C}$  charge that is moved from a positive plate to a negative plate gains  $70 \text{ J}$  of potential energy. What is the electric field for these parallel plates if they are  $8 \text{ mm}$  apart?  
(A)  $2500 \text{ V/m}$   
(B)  $3500 \text{ V/m}$   
(C)  $4500 \text{ V/m}$   
(D)  $5500 \text{ V/m}$   
(E)  $8500 \text{ V/m}$
5. How much work is needed to decrease the distance between a  $+15 \mu\text{C}$  charge and a  $-20 \mu\text{C}$  charge from  $1 \text{ m}$  to  $0.25 \text{ m}$  by moving the positive charge?  
(A)  $-8.1 \text{ J}$   
(B)  $-2.7 \text{ J}$   
(C)  $0 \text{ J}$   
(D)  $2.7 \text{ J}$   
(E)  $8.1 \text{ J}$

6. Determine the electric potential at a point  $A$  where  $Q_1 = -20 \mu\text{C}$  and  $Q_2 = +20 \mu\text{C}$   
 $Q_1$  — 10 cm —  $Q_2$  — 5 cm —  $A$   
 (A)  $8.0 \times 10^1 \text{ V}$   
 (B)  $1.9 \times 10^2 \text{ V}$   
 (C)  $2.4 \times 10^6 \text{ V}$   
 (D)  $5.5 \times 10^7 \text{ V}$   
 (E)  $6.7 \times 10^{10} \text{ V}$
8. Find the potential at point  $B$  where the distance between  $\text{H}_3$  complex and  $A$  is  $1.5 \times 10^{-6} \text{ m}$  and the interior angle to point  $N$  is  $90^\circ$ ?  
 (A)  $9.1 \times 10^{-4} \text{ V}$   
 (B)  $2.5 \times 10^{-3} \text{ V}$   
 (C)  $8.2 \times 10^{-3} \text{ V}$   
 (D)  $7.5 \times 10^{-2} \text{ V}$   
 (E) None of the above

Questions 7 and 8 refer to the following diagram, which is not to scale.

This is an  $\text{NH}_3$  molecule, which has a dipole moment of  $5.0 \times 10^{-30} \text{ C}\cdot\text{m}$ , arranged as shown below.



•  
B

•  
A

7. Find the potential at point  $A$  from where the distance between  $\text{H}_3$  complex and  $A$  is  $3.0 \times 10^{-9} \text{ m}$  and the interior angle at  $N$  is  $60^\circ$ .  
 (A)  $9.1 \times 10^{-4} \text{ V}$   
 (B)  $2.5 \times 10^{-3} \text{ V}$   
 (C)  $8.2 \times 10^{-3} \text{ V}$   
 (D)  $7.5 \times 10^{-2} \text{ V}$   
 (E) None of the above
9. What is the charge on each plate of a capacitor whose plates are  $3 \text{ cm} \times 3 \text{ cm}$  and  $2 \text{ mm}$  apart when connected to a  $9 \text{ V}$  battery?  
 (A)  $4.0 \times 10^{-12} \text{ C}$   
 (B)  $3.6 \times 10^{-11} \text{ C}$   
 (C)  $7.2 \times 10^{-10} \text{ C}$   
 (D)  $2.1 \times 10^{-9} \text{ C}$   
 (E)  $1.0 \times 10^{-8} \text{ C}$
10. Which of the following are valid units for an electric field?  
 I.  $\text{N/C}$   
 II.  $\text{J/C}\cdot\text{m}$   
 III.  $\text{V/m}$   
 (A) I and II only  
 (B) II and III only  
 (C) I and III only  
 (D) I, II, and III  
 (E) None of the above

### Free-Response Questions

- Assume a zero potential at infinity for this question.
  - Calculate the dipole moment for a polar molecule which has a potential of  $0.15 \text{ V}$  at a distance  $1.24 \times 10^{-10} \text{ m}$  away at an interior angle of  $41^\circ$  from the negative charge.
  - Assuming the charges are separated by a distance of  $6.8 \times 10^{-11} \text{ m}$ , calculate each of the charges.
  - Along what line would the potential be zero?
- A  $-35 \mu\text{C}$  charge and a  $+40 \mu\text{C}$  charge are separated by  $15 \text{ cm}$ . The potential is tested at a point in space  $75 \text{ cm}$  from the negative charge. Is there a distance from the positive charge at which this point can be placed such that it has the potential is equal to  $40,000 \text{ V}$ ?

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## ANSWERS AND EXPLANATIONS

### Multiple-Choice Questions

- 1. (B) is correct.** The change in potential energy is given by  $\Delta PE = PE_b - PE_a = qV_{ba} = (2.5 \text{ C})(8 \text{ V}) = 20 \text{ J}$ , so potential energy will increase by 20 J.
- 2. (D) is correct.** The change in potential energy is given by  $\Delta PE = PE_b - PE_a = qV_{ba} = (-3.3 \text{ C})(6.4 \text{ V}) = -21 \text{ J}$ , so it will lose 21 J of electric potential energy. Because energy is conserved, the change in kinetic energy is equal to the change in potential energy.  $\frac{1}{2}mv^2 = 21 \text{ J}$ , so  $v = \sqrt{2(21 \text{ J})/(5.4 \times 10^{-10} \text{ kg})} = 2.8 \times 10^5 \text{ m/s}$ .
- 3. (E) is correct.** Electric field is given by  $E = V_{ba}/d = 10 \text{ V}/0.25 \text{ m} = 40 \text{ V/m}$ , a scalar quantity.
- 4. (A) is correct.** First, the potential difference of the plates is given by  $\Delta PE/q = V_{ba} = (70 \text{ J})/(3.5 \text{ C}) = 20 \text{ V}$ . Next, the quotient of voltage and distance gives the electric field,  $E = V_{ba}/d = 20 \text{ V}/0.008 \text{ m} = 2500 \text{ V/m}$ .
- 5. (A) is correct.** The relation between work and potential difference is given by  $W = qV_{ba} = q(kQ/r_b - kQ/r_a) = qk(Q/r_b - Q/r_a)$   
 $= (1.5 \times 10^{-5} \text{ C})(9.0 \times 10^9 \text{ Nm}^2/\text{C}^2)(-2.0 \times 10^{-3} \text{ C})[1/0.25 \text{ m} - 1/1 \text{ m}]$   
 $= (1.5 \times 10^{-5} \text{ C})(9.0 \times 10^9 \text{ Nm}^2/\text{C}^2)(-2.0 \times 10^{-3} \text{ C})[3 \text{ m}^{-1}] = -8.1 \text{ J}$ .
- 6. (C) is correct.**  $V_A = V_{A1} + V_{A2} = kQ_1/r_1 + kQ_2/r_2$   
 $= (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(-2.0 \times 10^{-5} \text{ C})/0.15$   
 $+ (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(+2.0 \times 10^{-5} \text{ C})/0.05$   
 $= 3.6 \times 10^6 \text{ V} - 1.2 \times 10^6 \text{ V} = 2.4 \times 10^6 \text{ V}$ .  
This actually represents a dipole, where  $\theta = 0^\circ$ , but  $r$  is not large with respect to  $l$ , so the dipole formula is not valid.
- 7. (B) is correct.** The potential is given by  $V = kp \cos \theta/r^2$   
 $= (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(5.0 \times 10^{-30} \text{ C}\cdot\text{m}) \cos 60^\circ/(3.0 \times 10^{-9} \text{ m})^2$   
 $= 2.5 \times 10^{-3} \text{ V}$ .
- 8. (E) is the correct answer.** Because the potential is given by  $V = kp \cos \theta/r^2$ , without entering any values other than  $\cos \theta$ , it is clear that the numerator will be zero, and therefore that the potential will be zero.
- 9. (B) is the correct answer.** For this capacitor design, capacitance is given by  $C = \epsilon_0 A/d = (8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(9 \times 10^{-4} \text{ m}^2)/(2 \times 10^{-3} \text{ m})$   
 $= 4 \times 10^{-12} \text{ F}$ . Next, using the relation of charge to voltage,  $Q = CV$   
 $= (4 \times 10^{-12} \text{ F})(9 \text{ V}) = 3.6 \times 10^{-11} \text{ C}$ .
- 10. (D) is correct.** All three units are equivalent. Unit I and unit II are equivalent, since  $N = \text{J/m}$ , and units II and III are equivalent, because  $V = \text{J/C}$ . Units I and III are thus logically equivalent, also.

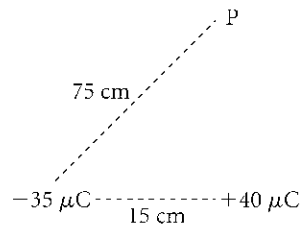
### Free-Response Questions

- (a) The dipole moment,  $p$ , can be isolated from the standard equation,  
 $V = kp \cos \theta/r^2$ , such that  $p = Vr^2/k \cos \theta$   
 $= (15 \text{ V})(1.24 \times 10^{-10} \text{ m})^2/(9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(\cos 41^\circ)$   
 $= 3.4 \times 10^{-31} \text{ C}\cdot\text{m}$ .

- (b)  $Q = p/l = 3.4 \times 10^{-31} \text{ C}\cdot\text{m}/6.8 \times 10^{-11} \text{ m} = 5.0 \times 10^{-21} \text{ C}$ , so one pole has a charge of  $+5.0 \times 10^{-21} \text{ C}$ , and the other has a charge of  $-5.0 \times 10^{-21} \text{ C}$ .
- (c) The potential is zero at any distance much larger than the separation distance of the charges at a  $90^\circ$  angle from the negative charge. This happens along the line perpendicular to the line of the dipole charges. There are an infinite number of places where this happens, either above or below at a relatively large distance.

*This response correctly rearranges the dipole potential formula to solve for the dipole moment in part a. The response to part b not only finds the charge based on the definition of the dipole moment, but also it states that there are two opposite charges and gives their values. The response to part c correctly points out that there are an infinite number of points, at a large enough distance away, where the potential is zero. Note that in any plane containing the dipole line there are two places ("either above or below") for a specific potential.*

2. A sketch of the situation is shown below.



Without calculations, it is apparent that  $P$  must be located somewhere along a circle centered at the negative charge,  $Q_1$ , with a radius of 75 cm, and that it must form a triangle with the two point charges (unless all are collinear). The voltage at point  $P$ ,  $V_P = 40,000 \text{ V}$ , where

$$V_P = V_{1P} + V_{2P} = kQ_1/r_1 + kQ_2/r_2.$$

$$40,000 \text{ V} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(-3.5 \times 10^{-5} \text{ C})/0.75 + (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4.0 \times 10^{-5} \text{ C})/r_2.$$

Solving for  $r_2$ , the distance is approximately 78 cm. It is possible, because the 15, 75, and 78 are possible dimensions for a triangle.

*This response features a sketch demonstrating that the points must form a triangle. The correct signs are taken into account when determining the scalar quantity. Finally, the distance necessary for the given electric potential is isolated and fits the necessary dimensional criteria.*