## The Lens Equation

The relation between the focal length of a converging lens, $f$, and the image and object distances is given by the **lens equation,** $1/d_o + 1/d_i = 1/f$, which is derived from the relation $h_i/h_o = d_i/d_o$. For a diverging lens, the relation is such that $1/d_o - 1/d_i = -1/f$. As the distance from the lens to the object approaches infinity, the distance from the lens to the image approaches the focal length.

The first lens equation can serve for both types of lenses when the appropriate **sign conventions** are followed, as explained below.

1. For a converging lens, $f$ has a positive value. For a diverging lens, $f$ has a negative value.
2. For an object on the same side of the lens from which the light originates (which is usually the case), $d_o$ has a positive value. For an object on the opposite of the lens from which the light originates, $d_o$ has a negative value.
3. For an image on the opposite side from where the light originates, $d_i$ has a positive value. For an image on the same of the lens as where the light originates, $d_i$ has a negative value. This gives a positive $d_i$ for real images and a negative $d_i$ for virtual images.
4. $h_o$ is always taken to be positive. $h_i$ is positive if the image is upright relative to the object, and $h_i$ is negative if the image is inverted relative to the object.

The ratio of $h_i/h_o = -d_i/d_o = m$, called the **lateral magnification** of a lens which has a positive value if the image is upright and a negative value if it is inverted relative to the object.

In agreement with convention #1 above, converging lenses are also called **positive lenses,** and diverging lenses are called **negative lenses.**

## Problem Solving for Lenses; Problem Solving for Combinations of Lenses

A **virtual image** will be formed when an object is between the focal point of a converging lens and the lens itself, whereas a **real image** will occur when an object is placed at a distance beyond its focal length.

To solve for image formation by multiple lenses in combination, the image produced by the first lens can be considered the object for the next lens, and so on, to determine the characteristics of the final image.

### *For Additional Review*

Consider problem-solving situations involving light rays traveling through combinations of both thin lens and spherical mirrors.

### *Multiple-Choice Questions*

1. What is the height of an image produced by a 5 cm high object placed 15 cm from the mirrored surface of a concave spherical mirror whose focal length is 10 cm?
   (A) 30 cm, upright
   (B) 10 cm, upright
   (C) 10 cm, inverted
   (D) 15 cm, inverted
   (E) 30 cm, inverted

2. What is the height of an object that produces an 18 cm high inverted image when it is placed 8 cm from the mirrored surface of a concave spherical mirror and has an image distance of 24 cm?
(A) 6 cm, upright
(B) 52 cm, upright
(C) 52 cm, inverted
(D) 18 cm, inverted
(E) 6 cm, inverted

3. What is the ratio of the radius of curvature of a spherical convex mirror to its focal length?
(A) $-1/2$
(B) $1/2$
(C) $-1$
(D) 2
(E) $-2$

4. For a giant spherical convex mirror with a focal length of 17 m, what would be the image distance of an object that is placed 9 m from the center of the mirror along the principle axis?
(A) 5.9 m, on the same side of the mirror
(B) 5.9 m, on the opposite side of the mirror
(C) 19 m, on the same side of the mirror
(D) 19 m, on the opposite side of the mirror
(E) No image will be formed.

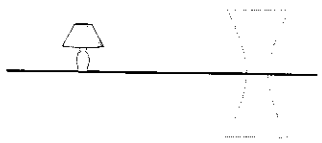
5. A ray of light travels from one substance into another medium at 33.0° from the normal and is refracted at 38° from the normal. What is the associated critical angle for a light ray traveling between these substances?
(A) 45°
(B) 62°
(C) 74°
(D) 85°
(E) 90°

6. At which of the following angles from the normal will total internal reflection occur for a light ray traveling between two substances in which the incident substance has twice the index of refraction as the refracting substance.
(A) 21°
(B) 24°
(C) 27°
(D) 30°
(E) 33°

7. What is the absolute height and type of the image produced when an 11 cm high object is placed 10 cm from a converging lens of focal length 5 cm?
(A) 11 cm, real
(B) 10 cm, virtual
(C) 10 cm, real
(D) 11 cm, virtual
(E) 5 cm, real

*Questions 8 & 9* refer to the diagram shown below.

An upright object placed 25 cm away from a diverging lens produces an image that is 10 cm from the center of the lens and on the same side of the lens.



8. What would be the focal length of the diverging lens?
(A) −8 cm
(B) −11 cm
(C) −17 cm
(D) −21 cm
(E) −35 cm

9. What is the orientation and type of image formed by the diverging lens?
(A) Upright and real
(B) Upright and virtual
(C) Inverted and real
(D) Inverted and virtual
(E) Not enough information is provided to determine the orientation and type of image.

10. Which of the following is NOT an accurate sign convention for geometric optics when dealing with thin lenses?
    (A) Focal length is considered negative for a diverging lens.
    (B) The image distance is positive when on the same side from which light is approaching.
    (C) The object distance is positive when on the same side from which light is approaching.
    (D) The image distance is always positive for a real image.
    (E) The height of the object is always considered positive.

***Free-Response Questions***

1. A person is standing 2.35 meters from a full-length plane mirror. His eye level is 1.55 meters from the ground. An object with a height of 25 cm is 1 meter away from the mirror. What is the perceived distance and the actual distance from the person's eye to the top of the object? Draw a diagram demonstrating your solution method.
2. Two converging lenses are placed 28 cm from each other collinear along their axes. Their focal lengths are both 8 cm. A 6 cm high object is placed 12 cm from the center of one of the two lenses, on the far side of both lenses.
    (a) What is the resulting image distance from the original object when both lenses are used together to produce the final image?
    (b) What will be the height of the final image produced, and will it be inverted or upright?
    (c) Will the FINAL image be real or virtual? Why?