CHAPTER 16: Electric Charge and Electric Field

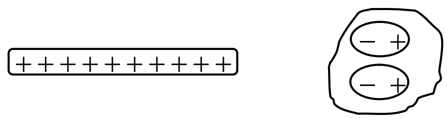
Answers to Questions

1. A plastic ruler is suspended by a thread and then rubbed with a cloth. As discussed in section 16-1, the ruler is negatively charged. Bring the charged comb close to the ruler. If the ruler is repelled by the comb, then the comb is negatively charged. If the ruler is attracted by the comb, then the comb is positively charged.

2. The clothing gets charged by frictional contact in the tumbling motion of the dryer. The air inside the dryer is dry, and so the clothes can sustain a relatively large static charge. That charged object will then polarize your clothing, and be attracted to you electrostatically.

3. Water is a polar molecule – it has a positive region and a negative region. Thus it is easily attracted to some other charged object, like an ion or electron in the air.

4. The positively charged rod slightly polarizes the molecules in the paper. The negative charges in the paper are slightly attracted to the part of the paper closest to the rod, while the positive charges in the paper are slightly repelled from the part of the paper closest to the rod. Since the opposite charges are now closer together and the like charges are now farther apart, there is a net attraction between the rod and the paper.



5. The plastic ruler has gained some electrons from the cloth and thus has a net negative charge. This charge polarizes the charge on the piece of paper, drawing positives slightly closer and repelling negatives slightly further away. This polarization results in a net attractive force on the piece of paper. A small amount of charge is able to create enough electric force to be stronger than gravity, and so the paper can be lifted.

   On a humid day this is more difficult because the water molecules in the air are polar. Those polar water molecules are able to attract some fraction of the free charges away from the plastic ruler. Thus the ruler has a smaller charge, the paper is less polarized, and there is not enough electric force to pick up the paper.

6. The net charge on the conductor is the unbalanced charge, or excess charge after neutrality has been established. The net charge is the sum of all of the positive and negative charges in the conductor. If a neutral conductor has extra electrons added to it, then the net charge is negative. If a neutral conductor has electrons removed from it, then the net charge is positive. If a neutral conductor has the same amount of positive and negative charge, then the net charge is zero.

   Free charges in a conductor refer to those electrons (usually 1 or 2 per atom) that are so loosely attracted to the nucleus that they are "free" to be moved around in the conductor by an external electric force. Neutral conductors have these free electrons.

7. For each atom in a conductor, only a small number of its electrons are free to move. For example, every atom of copper has 29 electrons, but only 1 or 2 from each atom are free to move easily. Also, not even all of the free electrons move. As electrons move toward a region, causing an excess of negative charge, that region then exerts a large repulsive force on other electrons, preventing them from all gathering in one place.

8. The force of gravity pulling down on the leaves, tending to return them to the vertical position.

9. The magnitude of the constant in Newton's law is very small, while the magnitude of the constant in Coulomb's law is quite large. Newton's law says the gravitational force is proportional to the product of the two masses, while Coulomb's law says the electrical force is proportional to the product of the two charges. Newton's law only produces attractive forces, since there is only one kind of gravitational mass. Coulomb's law produces both attractive and repulsive forces, since there are two kinds of electrical charge.

10. For the gravitational force, we don't notice it because the force is very weak, due to the very small value of *G*, the gravitational constant, and the small value of ordinary masses. For the electric force, we don't notice it because ordinary objects are electrically neutral to a very high degree. We notice our weight (the force of gravity) due to the huge mass of the Earth, making a significant gravity force. We notice the electric force when objects have a static charge (like static cling from the clothes dryer), creating a detectable electric force.

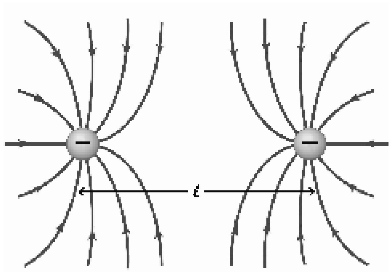
11. The electric force is conservative. You can "store" energy in it, and get the energy back. For example, moving a positive charge close to another stationary positive charge takes work (similar to lifting an object in the Earth's gravitational field), but if the positive charge is then released, it will gain kinetic energy and move away from the "stored energy" location (like dropping an object in the Earth's gravitational field). Another argument is that the mathematical form of Coulomb's law is identical to that of Newton's law of universal gravitation. We know that gravity is conservative, and so we would assume that the electric force is also conservative. There are other indications as well. If you move a charge around in an electric field, eventually returning to the starting position, the net work done will be 0 J. The work done in moving a charge around in an electric field is path independent – all that matters is the starting and ending locations. All of these are indications of a conservative force.

12. The charged plastic ruler has a negative charge residing on its surface. That charge polarizes the charge in the neutral paper, producing a net attractive force. When the piece of paper then touches the ruler, the paper can get charged by contact with the ruler, gaining a net negative charge. Then, since like charges repel, the paper is repelled by the comb.

13. The test charge creates its own electric field, and so the measured electric field is the sum of the original electric field plus the field of the test charge. By making the test charge small, the field that it causes is small, and so the actual measured electric field is not much different than the original field to measure.

14. A negative test charge could be used. For purposes of defining directions, the electric field might then be defined as the OPPOSITE of the force on the test charge, divided by the test charge. Equation (16-3) might be changed to $\vec{E} = -\vec{F}/q$, $q < 0$.

15.



16. The electric field is strongest to the right of the positive charge, because the individual fields from the positive charge and negative charge both are in the same direction (to the right) at that point, so they add to make a stronger field. The electric field is weakest to the left of the positive charge, because the individual fields from the positive charge and negative charge are in opposite directions at that point, and so they partially cancel each other. Another indication is the spacing of the field lines. The field lines are closer to each other to the right of the positive charge, and further apart to the left of the positive charge.

17. At point A, the net force on a positive test charge would be down and to the left, parallel to the nearby electric field lines. At point B, the net force on a positive test charge would be up and to the right, parallel to the nearby electric field lines. At point C, the net force on a positive test charge would be 0. In order of decreasing field strength, the points would be ordered A, B, C.

18. Electric field lines show the direction of the force on a test charge placed at a given location. The electric force has a unique direction at each point. If two field lines cross, it would indicate that the electric force is pointing in two directions at once, which is not possible.

19. From rule 1: A test charge would be either attracted directly towards or repelled directly away from a point charge, depending on the sign of the point charge. So the field lines must be directed either radially towards or radially away from the point charge.

From rule 2: The magnitude of the field due to the point charge only depends on the distance from the point charge. Thus the density of the field lines must be the same at any location around the point charge, for a given distance from the point charge.

From rule 3: If the point charge is positive, the field lines will originate from the location of the point charge. If the point charge is negative, the field lines will end at the location of the point charge.

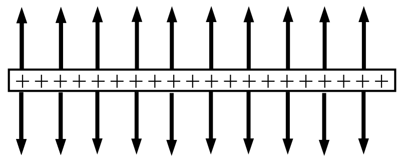
Based on rules 1 and 2, the lines are radial and their density is constant for a given distance. This is equivalent to saying that the lines must be symmetrically spaced around the point charge.

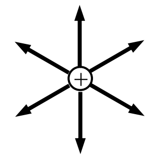
20. If the two charges are of opposite sign, then $\vec{\mathbf{E}} = 0$ at a point closer to the weaker charge, and on the opposite side of the weaker charge from the stronger charge. The fields due to the two charges are of opposite direction at such a point. If the distance between the two charges is *l*, then the point at which $\vec{\mathbf{E}} = 0$ is 2.41 *l* away from the weaker charge, and 3.41 *l* away from the stronger charge.

If the two charges are the same sign, then $\vec{\mathbf{E}} = 0$ at a point between the two charges, closer to the weaker charge. The point is 41% of the distance from the weaker charge to the stronger charge.

21. We assume that there are no other forces (like gravity) acting on the test charge. The direction of the electric field line gives the direction of the force on the test charge. The acceleration is always parallel to the force by Newton's 2nd law, and so the acceleration lies along the field line. If the particle is at rest initially and then released, the initial velocity will also point along the field line, and the particle will start to move along the field line. However, once the particle has a velocity, it will not follow the field line unless the line is straight. The field line gives the direction of the acceleration, or the direction of the change in velocity.

22. Since the line of charge is infinitely long, it has no preferred left or right direction. Thus by symmetry, the lines must point radially out away from the center of the line. A side view and an end view are shown for a line of positive charge. As seen from the end view, the field is not uniform. As you move further away from the line of charge, the field lines get further apart, indicating that the field gets weaker as you move away from the line of charge.





23. Just because the electric flux through a closed surface is zero, the field need not be zero on the surface. For example, consider a closed surface near an isolated point charge, and the surface does not enclose the charge. There will be electric field lines passing through the surface, but the total electric flux through the surface will be zero since the surface does not enclose any charge. The same number of field lines will enter the volume enclosed by the surface as leave the volume enclosed by the surface.

    On the contrary, if $\vec{\mathbf{E}} = 0$ at all points on the surface, then there are no electric field lines passing through the surface, and so the flux through the surface is zero.

24. The electric flux depends only on the charge enclosed by the gaussian surface, not on the shape of the surface. Φ_E will be the same for the cube as for the sphere.

Solutions to Problems

1. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(3.60\times10^{-6}\,\mathrm{C}\right)^2}{\left(9.3\times10^{-2}\,\mathrm{m}\right)^2} = 13.47\ \mathrm{N} \approx \boxed{13\,\mathrm{N}}$$

2. Use the charge per electron to find the number of electrons.

$$\left(-30.0\times10^{-6}\,\mathrm{C}\right)\left(\frac{1\ \text{electron}}{-1.602\times10^{-19}\,\mathrm{C}}\right) = \boxed{1.87\times10^{14}\ \text{electrons}}$$

3. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(1.602\times10^{-19}\,\mathrm{C}\right)\left(26\times1.602\times10^{-19}\,\mathrm{C}\right)}{\left(1.5\times10^{-12}\,\mathrm{m}\right)^2} = \boxed{2.7\times10^{-3}\,\mathrm{N}}$$

4. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(1.602\times10^{-19}\,\mathrm{C}\right)^2}{\left(5.0\times10^{-15}\,\mathrm{m}\right)^2} = \boxed{9.2\,\mathrm{N}}$$

5. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1 Q_2}{r^2} = \left(8.988\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left(25\times10^{-6}\,\text{C}\right)\left(3.0\times10^{-3}\,\text{C}\right)}{\left(3.5\times10^{-1}\,\text{m}\right)^2} = \boxed{5.5\times10^3\,\text{N}}$$

6. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^2}$, if the distance is multiplied by a factor of 1/8, the force will be multiplied by a factor of 64.

$$F = 64F_0 = 64\left(3.2\times10^{-2}\,\text{N}\right) = \boxed{2.0\ \text{N}}$$

7. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^2}$, if the force is tripled, the distance has been reduced by a factor of $\sqrt{3}$.

$$r = \frac{r_0}{\sqrt{3}} = \frac{8.45\ \text{cm}}{\sqrt{3}} = \boxed{4.88\ \text{cm}}$$

8. Use the charge per electron and the mass per electron.

$$\left(-42\times10^{-6}\,\text{C}\right)\left(\frac{1\ \text{electron}}{-1.602\times10^{-19}\,\text{C}}\right) = 2.622\times10^{14} \approx \boxed{2.6\times10^{14}\,\text{electrons}}$$

$$\left(2.622\times10^{14}\,\text{e}^-\right)\left(\frac{9.11\times10^{-31}\,\text{kg}}{1\ \text{e}^-}\right) = \boxed{2.4\times10^{-16}\,\text{kg}}$$

9. Convert the kg of H_2O to moles, then to atoms, then to electrons. Oxygen has 8 electrons per atom, and hydrogen has 1 electron per atom.

$$1.0\ \text{kg H}_2\text{O} = \left(1.0\ \text{kg H}_2\text{O}\right)\left(\frac{1\ \text{mole H}_2\text{O}}{1.8\times10^{-2}\,\text{kg}}\right)\left(\frac{6.02\times10^{23}\,\text{molec.}}{1\ \text{mole}}\right)\left(\frac{10\ \text{e}}{1\ \text{molec.}}\right)\left(\frac{-1.602\times10^{-19}\,\text{C}}{\text{e}}\right)$$

$$= \boxed{-5.4\times10^7\,\text{C}}$$

10. Take the ratio of the electric force divided by the gravitational force.

$$\frac{F_\text{E}}{F_\text{G}} = \frac{k\frac{Q_1Q_2}{r^2}}{G\frac{m_1m_2}{r^2}} = \frac{kQ_1Q_2}{Gm_1m_2} = \frac{\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.602\times10^{-19}\,\text{C}\right)^2}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.67\times10^{-27}\,\text{kg}\right)} = \boxed{2.3\times10^{39}}$$

The electric force is about 2.3×10^{39} times stronger than the gravitational force for the given scenario.

11. (*a*) Let one of the charges be q, and then the other charge is $Q_\text{T} - q$. The force between the charges is $F_\text{E} = k\frac{q\left(Q_\text{T}-q\right)}{r^2} = \frac{k}{r^2}\left(qQ_\text{T} - q^2\right) = \frac{k}{r^2}Q_\text{T}^2\left(\frac{q}{Q_\text{T}} - \left(\frac{q}{Q_\text{T}}\right)^2\right)$. If we let $x = \frac{q}{Q_\text{T}}$, then $F_\text{E} = \frac{k}{r^2}Q_\text{T}^2\left(x - x^2\right)$, where $0 \le x \le 1$. A graph of $f\left(x\right) = x - x^2$ between the limits of 0 and 1

shows that the maximum occurs at $x = 0.5$, or $\boxed{q = 0.5Q_\text{T}}$. Both charges are half of the total, and the actual maximized force is $F_\text{E} = 0.25\dfrac{k}{r^2}Q_\text{T}^2$.

(*b*) If one of the charges has all of the charge, and the other has no charge, then the force between them will be 0, which is the minimum possible force.

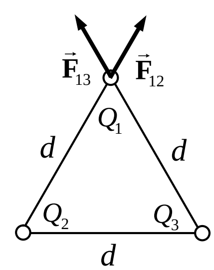
12. Let the right be the positive direction on the line of charges. Use the fact that like charges repel and unlike charges attract to determine the direction of the forces. In the following expressions, $k = 8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.

$$F_{+75} = -k\frac{(75\mu\text{C})(48\mu\text{C})}{(0.35\,\text{m})^2} + k\frac{(75\mu\text{C})(85\mu\text{C})}{(0.70\,\text{m})^2} = -147.2\,\text{N} \approx \boxed{-1.5\times10^2\,\text{N}}$$

$$F_{+48} = k\frac{(75\mu\text{C})(48\mu\text{C})}{(0.35\,\text{m})^2} + k\frac{(48\mu\text{C})(85\mu\text{C})}{(0.35\,\text{m})^2} = 563.5\,\text{N} \approx \boxed{5.6\times10^2\,\text{N}}$$

$$F_{-85} = -k\frac{(85\mu\text{C})(75\mu\text{C})}{(0.70\,\text{m})^2} - k\frac{(85\mu\text{C})(48\mu\text{C})}{(0.35\,\text{m})^2} = -416.3\,\text{N} \approx \boxed{-4.2\times10^2\,\text{N}}$$

13. The forces on each charge lie along a line connecting the charges. Let the variable *d* represent the length of a side of the triangle, and let the variable *Q* represent the charge at each corner. Since the triangle is equilateral, each angle is 60°.



$$F_{12} = k\frac{Q^2}{d^2} \rightarrow F_{12x} = k\frac{Q^2}{d^2}\cos 60^\circ,\ F_{12y} = k\frac{Q^2}{d^2}\sin 60^\circ$$

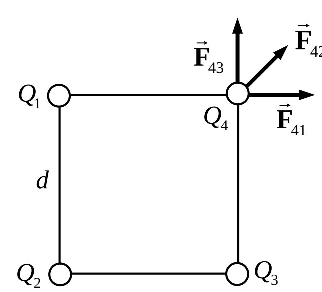
$$F_{13} = k\frac{Q^2}{d^2} \rightarrow F_{13x} = -k\frac{Q^2}{d^2}\cos 60^\circ,\ F_{13y} = k\frac{Q^2}{d^2}\sin 60^\circ$$

$$F_{1x} = F_{12x} + F_{13x} = 0 \qquad F_{1y} = F_{12y} + F_{13y} = 2k\frac{Q^2}{d^2}\sin 60^\circ = \sqrt{3}k\frac{Q^2}{d^2}$$

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \sqrt{3}k\frac{Q^2}{d^2} = \sqrt{3}\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(11.0\times10^{-6}\,\text{C}\right)^2}{(0.150\,\text{m})^2} = \boxed{83.7\,\text{N}}$$

The direction of $\vec{\mathbf{F}}_1$ is in the $\boxed{y\text{-direction}}$. Also notice that it lies along the bisector of the opposite side of the triangle. Thus the force on the lower left charge is of magnitude $\boxed{83.7\,\text{N}}$, and will point $\boxed{30^\circ \text{ below the } -x \text{ axis}}$. Finally, the force on the lower right charge is of magnitude $\boxed{83.7\,\text{N}}$, and will point $\boxed{30^\circ \text{ below the } +x \text{ axis}}$.

14. Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other three charges. The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable d represent the 0.100 m length of a side of the square, and let the variable Q represent the 6.00 mC charge at each corner.



$$F_{41} = k\frac{Q^2}{d^2} \;\rightarrow\; F_{41x} = k\frac{Q^2}{d^2}\,,\; F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \;\rightarrow\; F_{42x} = k\frac{Q^2}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q^2}{4d^2}\,,\; F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \;\rightarrow\; F_{43x} = 0\,,\; F_{43y} = k\frac{Q^2}{d^2}$$

Add the x and y components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4x} = F_{41x} + F_{42x} + F_{43x} = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 = k\frac{Q^2}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right) = F_{4y}$$

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k\frac{Q^2}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q^2}{d^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

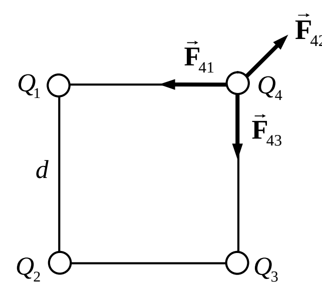
$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(6.00\times10^{-3}\,\text{C}\right)^2}{\left(0.100\,\text{m}\right)^2}\left(\sqrt{2} + \frac{1}{2}\right) = \boxed{6.19\times10^7\ \text{N}}$$

$$\theta = \tan^{-1}\frac{F_{4y}}{F_{4x}} = \boxed{45^\circ}$$ above the x-direction.

For each charge, the net force will be the magnitude determined above, and will lie along the line from the center of the square out towards the charge.

15. Determine the force on the upper right charge, and then the symmetry of the configuration says that the force on the lower left charge is the opposite of the force on the upper right charge. Likewise, determine the force on the lower right charge, and then the symmetry of the configuration says that the force on the upper left charge is the opposite of the force on the lower right charge.

The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable d represent the 0.100 m length of a side of the square, and let the variable Q represent the 6.00 mC charge at each corner.



$$F_{41} = k\frac{Q^2}{d^2} \;\rightarrow\; F_{41x} = -k\frac{Q^2}{d^2}\,,\; F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \;\rightarrow\; F_{42x} = k\frac{Q^2}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q^2}{4d^2}\,,\; F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \;\rightarrow\; F_{43x} = 0\,,\; F_{43y} = -k\frac{Q^2}{d^2}$$

Add the x and y components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4x} = F_{41x} + F_{42x} + F_{43x} = -k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 = k\frac{Q^2}{d^2}\left(-1 + \frac{\sqrt{2}}{4}\right) = -0.64645k\frac{Q^2}{d^2} = F_{4y}$$

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k\frac{Q^2}{d^2}\left(0.64645\right)\sqrt{2} = k\frac{Q^2}{d^2}\left(0.9142\right)$$

$$= \left(8.988\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(6.00\times10^{-3}\,\mathrm{C}\right)^2}{\left(0.100\,\mathrm{m}\right)^2}\left(0.9142\right) = \boxed{2.96\times10^{7}\ \mathrm{N}}$$

$\theta = \tan^{-1}\dfrac{F_{4y}}{F_{4x}} = \boxed{225^{\circ}}$ from the *x*-direction, or exactly towards the center of the square.

For each charge, the net force will be the magnitude of $\boxed{2.96\times10^{7}\ \mathrm{N}}$ and each net force will lie along the line from the charge inwards towards the center of the square.

16. Take the lower left hand corner of the square to be the origin of coordinates. Each charge will have a horizontal force on it due to one charge, a vertical force on it due to one charge, and a diagonal force on it due to one charge. Find the components of each force, add the components, find the magnitude of the net force, and the direction of the net force. At the conclusion of the problem is a diagram showing the net force on each of the two charges.

(*a*) $2Q:\ F_{2Qx} = k\dfrac{(2Q)Q}{l^2} + k\dfrac{(2Q)(4Q)}{2l^2}\cos 45^{\circ} = k\dfrac{Q^2}{l^2}\left(2+2\sqrt{2}\right) = 4.8284\dfrac{kQ^2}{l^2}$

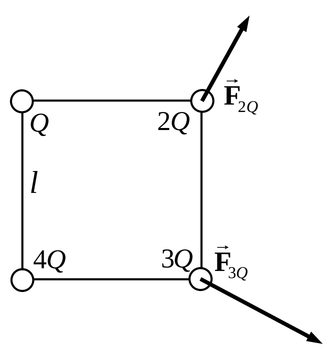
$$F_{2Qy} = k\frac{(2Q)(3Q)}{l^2} + k\frac{(2Q)(4Q)}{2l^2}\sin 45^{\circ} = k\frac{Q^2}{l^2}\left(6+2\sqrt{2}\right) = 8.8284\frac{kQ^2}{l^2}$$

$$F_{2Q} = \sqrt{F_{2Qx}^2 + F_{2Qy}^2} = \boxed{10.1\frac{kQ^2}{l^2}} \qquad \theta_{2Q} = \tan^{-1}\frac{F_{2y}}{F_{2x}} = \tan^{-1}\frac{8.8284}{4.8284} = \boxed{61^{\circ}}$$

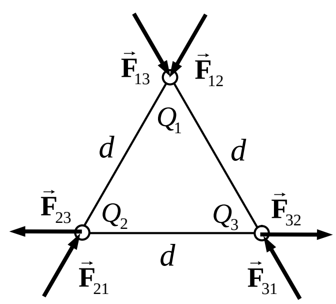
(*b*) $3Q:\ F_{3Qx} = k\dfrac{(3Q)(4Q)}{l^2} + k\dfrac{(3Q)Q}{2l^2}\cos 45^{\circ} = k\dfrac{Q^2}{l^2}\left(12+\dfrac{3}{4}\sqrt{2}\right) = 13.0607\dfrac{kQ^2}{l^2}$

$$F_{3Qy} = -k\frac{(3Q)(2Q)}{l^2} - k\frac{(3Q)Q}{2l^2}\sin 45^{\circ} = -k\frac{Q^2}{l^2}\left(6+\frac{3}{4}\sqrt{2}\right) = -7.0607\frac{kQ^2}{l^2}$$

$$F_{3Q} = \sqrt{F_{3Qx}^2 + F_{3Qy}^2} = \boxed{14.8\frac{kQ^2}{l^2}} \qquad \theta_{3Q} = \tan^{-1}\frac{F_{3y}}{F_{3x}} = \tan^{-1}\frac{-7.0607}{13.0607} = \boxed{332^{\circ}}$$



17. The forces on each charge lie along a line connecting the charges. Let the variable *d* represent the length of a side of the triangle. Since the triangle is equilateral, each angle is 60°. First calculate the magnitude of each individual force.



$$F_{12} = k\frac{|Q_1Q_2|}{d^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(4.0\times10^{-6}\,\text{C}\right)\left(8.0\times10^{-6}\,\text{C}\right)}{\left(1.20\,\text{m}\right)^2}$$
$$= 0.1997\,\text{N} = F_{21}$$

$$F_{13} = k\frac{|Q_1Q_3|}{d^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(4.0\times10^{-6}\,\text{C}\right)\left(6.0\times10^{-6}\,\text{C}\right)}{\left(1.20\,\text{m}\right)^2}$$
$$= 0.1498\,\text{N} = F_{31}$$

$$F_{23} = k\frac{|Q_2Q_3|}{d^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(8.0\times10^{-6}\,\text{C}\right)\left(6.0\times10^{-6}\,\text{C}\right)}{\left(1.20\,\text{m}\right)^2} = 0.2996\,\text{N} = F_{32}$$

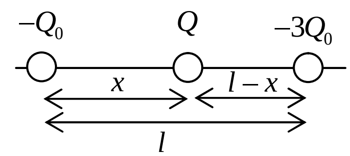
Now calculate the net force on each charge and the direction of that net force, using components.

$$F_{1x} = F_{12x} + F_{13x} = -\left(0.1997\,\text{N}\right)\cos 60^\circ + \left(0.1498\,\text{N}\right)\cos 60^\circ = -2.495\times10^{-2}\,\text{N}$$
$$F_{1y} = F_{12y} + F_{13y} = -\left(0.1997\,\text{N}\right)\sin 60^\circ - \left(0.1498\,\text{N}\right)\sin 60^\circ = -3.027\times10^{-1}\,\text{N}$$
$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \boxed{0.30\,\text{N}} \qquad \theta_1 = \tan^{-1}\frac{F_{1y}}{F_{1x}} = \tan^{-1}\frac{-3.027\times10^{-1}\,\text{N}}{-2.495\times10^{-2}\,\text{N}} = \boxed{265^\circ}$$
$$F_{2x} = F_{21x} + F_{23x} = \left(0.1997\,\text{N}\right)\cos 60^\circ - \left(0.2996\,\text{N}\right) = -1.998\times10^{-1}\,\text{N}$$
$$F_{2y} = F_{21y} + F_{23y} = \left(0.1997\,\text{N}\right)\sin 60^\circ + 0 = 1.729\times10^{-1}\,\text{N}$$
$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = \boxed{0.26\,\text{N}} \qquad \theta_2 = \tan^{-1}\frac{F_{2y}}{F_{2x}} = \tan^{-1}\frac{1.729\times10^{-1}\,\text{N}}{-1.998\times10^{-1}\,\text{N}} = \boxed{139^\circ}$$
$$F_{3x} = F_{31x} + F_{32x} = -\left(0.1498\,\text{N}\right)\cos 60^\circ + \left(0.2996\,\text{N}\right) = 2.247\times10^{-1}\,\text{N}$$
$$F_{3y} = F_{31y} + F_{32y} = \left(0.1498\,\text{N}\right)\sin 60^\circ + 0 = 1.297\times10^{-1}\,\text{N}$$
$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \boxed{0.26\,\text{N}} \qquad \theta_3 = \tan^{-1}\frac{F_{3y}}{F_{3x}} = \tan^{-1}\frac{1.297\times10^{-1}\,\text{N}}{2.247\times10^{-1}\,\text{N}} = \boxed{30^\circ}$$

18. Since the force is repulsive, both charges must be the same sign. Since the total charge is positive, both charges must be positive. Let the total charge be Q. Then if one charge is of magnitude q, then the other charge must be of magnitude $Q-q$. Write a Coulomb's law expression for one of the charges.

$$F = k\frac{q\left(Q-q\right)}{r^2} \rightarrow q^2 - Qq + \frac{Fr^2}{k} = 0 \rightarrow$$
$$q = \frac{Q \pm \sqrt{Q^2 - \dfrac{4Fr^2}{k}}}{2} = \frac{\left(560\times10^{-6}\,\text{C}\right) \pm \sqrt{\left(560\times10^{-6}\,\text{C}\right)^2 - \dfrac{4\left(22.8\,\text{N}\right)\left(1.10\,\text{m}\right)^2}{\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}}}{2}$$
$$= \boxed{5.54\times10^{-4}\,\text{C}\ ,\ 5.54\times10^{-6}\,\text{C}} \qquad Q - q = \boxed{5.54\times10^{-6}\,\text{C}\ ,\ 5.54\times10^{-4}\,\text{C}}$$

19. The negative charges will repel each other, and so the third charge must put an opposite force on each of the original charges. Consideration of the various possible configurations leads to the conclusion that the third charge must be positive and must be between the other two charges. See the diagram for the definition of variables. For each negative charge, equate the magnitudes of the two forces on the charge. Also note that $0 < x < l$.



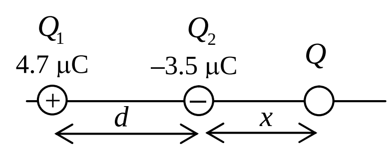
$$\text{left: } k\frac{Q_0Q}{x^2} = k\frac{3Q_0^2}{l^2} \qquad \text{right: } k\frac{3Q_0Q}{(l-x)^2} = k\frac{3Q_0^2}{l^2} \quad \rightarrow$$

$$k\frac{Q_0Q}{x^2} = k\frac{3Q_0Q}{(l-x)^2} \quad \rightarrow \quad x = \frac{l}{\sqrt{3}+1} = 0.366l$$

$$k\frac{Q_0Q}{x^2} = k\frac{3Q_0^2}{l^2} \quad \rightarrow \quad Q = 3Q_0\frac{x^2}{l^2} = Q_0\frac{3}{\left(\sqrt{3}+1\right)^2} = 0.402Q_0$$

Thus the charge should be of magnitude $\boxed{0.40Q_0}$, and a distance $\boxed{0.37l \text{ from } -Q_0 \text{ towards } -3Q_0}$.

20. Assume that the negative charge is d = 18.5 cm to the right of the positive charge, on the x-axis. To experience no net force, the third charge Q must be closer to the smaller magnitude charge (the negative charge). The third charge cannot be between the charges, because it would experience a force from each charge in the same direction, and so the net force could not be zero. And the third charge must be on the line joining the other two charges, so that the two forces on the third charge are along the same line. See the diagram. Equate the magnitudes of the two forces on the third charge, and solve for $x > 0$.



$$\left|\vec{\mathbf{F}}_1\right| = \left|\vec{\mathbf{F}}_2\right| \quad \rightarrow \quad k\frac{Q_1|Q|}{(d+x)^2} = k\frac{|Q_2||Q|}{x^2} \quad \rightarrow \quad x = d\frac{\sqrt{|Q_2|}}{\left(\sqrt{Q_1} - \sqrt{|Q_2|}\right)}$$

$$x = d\frac{\sqrt{|Q_2|}}{\left(\sqrt{Q_1} - \sqrt{|Q_2|}\right)} = (18.5\,\text{cm})\frac{\sqrt{3.5\times10^{-6}\,\text{C}}}{\left(\sqrt{4.7\times10^{-6}\,\text{C}} - \sqrt{3.5\times10^{-6}\,\text{C}}\right)} = \boxed{116\,\text{cm}}$$

21. (*a*) If the force is repulsive, both charges must be positive since the total charge is positive. Call the total charge Q.

$$Q_1 + Q_2 = Q \qquad F = \frac{kQ_1Q_2}{d^2} = \frac{kQ_1(Q-Q_1)}{d^2} \quad \rightarrow \quad Q_1^2 - QQ_1 + \frac{Fd^2}{k} = 0$$

$$Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2} = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}$$

$$= \frac{\left(90.0\times10^{-6}\,\text{C}\right) \pm \sqrt{\left(90.0\times10^{-6}\,\text{C}\right)^2 - 4\frac{(12.0\,\text{N})(1.06\,\text{m})^2}{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}}}{2}$$

$$= \boxed{69.9\times10^{-6}\,\text{C}\ ,\ 22.1\times10^{-6}\,\text{C}}$$

(*b*) If the force is attractive, then the charges are of opposite sign. The value used for F must then be negative. Other than that, the solution method is the same as for part (*a*).

$$Q_1 + Q_2 = Q \qquad F = \frac{kQ_1Q_2}{d^2} = \frac{kQ_1(Q - Q_1)}{d^2} \rightarrow Q_1^2 - QQ_1 + \frac{Fd^2}{k} = 0$$

$$Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2} = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}$$

$$= \frac{(90.0\times10^{-6}\,\text{C}) \pm \sqrt{(90.0\times10^{-6}\,\text{C})^2 - 4\frac{(-12.0\,\text{N})(1.06\,\text{m})^2}{(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)}}}{2}$$

$$= \boxed{104.4\times10^{-6}\,\text{C},\ -14.4\times10^{-6}\,\text{C}}$$

22. The spheres can be treated as point charges since they are spherical, and so Coulomb's law may be used to relate the amount of charge to the force of attraction. Each sphere will have a magnitude Q of charge, since that amount was removed from one sphere and added to the other, being initially uncharged.

$$F = k\frac{Q_1Q_2}{r^2} = k\frac{Q^2}{r^2} \rightarrow Q = r\sqrt{\frac{F}{k}} = (0.12\text{ m})\sqrt{\frac{1.7\times10^{-2}\,\text{N}}{8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}}$$

$$= 1.650\times10^{-7}\,\text{C}\left(\frac{1\text{ electron}}{1.602\times10^{-19}\,\text{C}}\right) = \boxed{1.0\times10^{12}\,\text{electrons}}$$

23. Use Eq. 16–3 to calculate the force.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} \rightarrow \vec{\mathbf{F}} = q\vec{\mathbf{E}} = (-1.602\times10^{-19}\,\text{C})(2360\,\text{N/C east}) = \boxed{3.78\times10^{-16}\,\text{N west}}$$

24. Use Eq. 16–3 to calculate the electric field.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{3.75\times10^{-14}\,\text{N south}}{1.602\times10^{-19}\,\text{C}} = \boxed{2.34\times10^5\,\text{N/C south}}$$

25. Use Eq. 16–3 to calculate the electric field.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{8.4\,\text{N down}}{-8.8\times10^{-6}\,\text{C}} = \boxed{9.5\times10^5\,\text{N/C up}}$$

26. Use Eq. 16–4a to calculate the electric field due to a point charge.

$$E = k\frac{Q}{r^2} = (8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{33.0\times10^{-6}\,\text{C}}{(2.00\times10^{-1}\,\text{m})^2} = \boxed{7.42\times10^6\,\text{N/C up}}$$

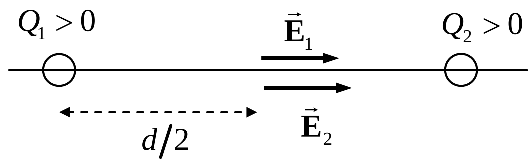
Note that the electric field points away from the positive charge.

27. Assuming the electric force is the only force on the electron, then Newton's 2nd law may be used to find the acceleration.

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} \quad \rightarrow \quad a = \frac{|q|}{m}E = \frac{\left(1.602\times10^{-19}\,\text{C}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)}750\,\text{N/C} = \boxed{1.32\times10^{14}\,\text{m/s}^2}$$

Since the charge is negative, the direction of the acceleration is $\boxed{\text{opposite to the field}}$.

28. The electric field due to the negative charge will point toward the negative charge, and the electric field due to the positive charge will point away from the positive charge. Thus both fields point in the same direction, towards the negative charge, and so can be added.

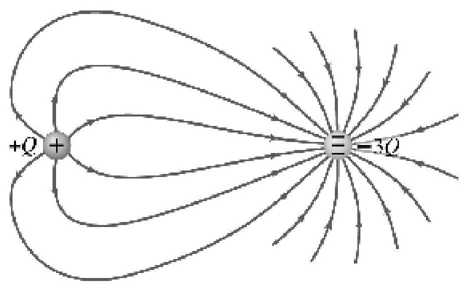


$$E = |E_1| + |E_2| = k\frac{|Q_1|}{r_1^2} + k\frac{|Q_2|}{r_2^2} = k\frac{|Q_1|}{(d/2)^2} + k\frac{|Q_2|}{(d/2)^2} = \frac{4k}{d^2}\left(|Q_1| + |Q_2|\right)$$

$$= \frac{4\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(8.0\times10^{-2}\,\text{m}\right)^2}\left(8.0\times10^{-6}\,\text{C} + 7.0\times10^{-6}\,\text{C}\right) = \boxed{8.4\times10^{7}\,\text{N/C}}$$

The direction is $\boxed{\text{towards the negative charge}}$.

29.



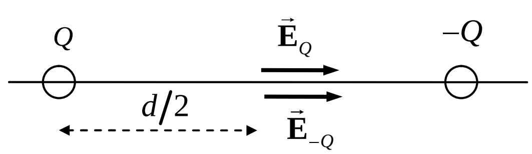
30. Assuming the electric force is the only force on the electron, then Newton's 2nd law may be used to find the electric field strength.

$$F_{net} = ma = qE \quad \rightarrow \quad E = \frac{ma}{q} = \frac{\left(1.67\times10^{-27}\,\text{kg}\right)\left(1\times10^{6}\right)\left(9.80\,\text{m/s}^2\right)}{\left(1.602\times10^{-19}\,\text{C}\right)} = 0.102\,\text{N/C} \approx \boxed{0.1\,\text{N/C}}$$

31. Since the electron accelerates from rest towards the north, the net force on it must be to the north. Assuming the electric force is the only force on the electron, then Newton's 2nd law may be used to find the electric field.

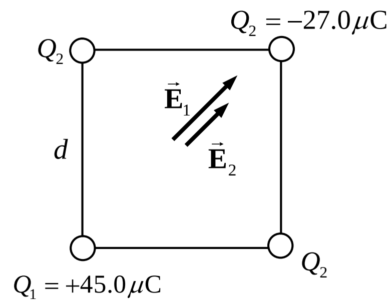
$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} \quad \rightarrow \quad \vec{\mathbf{E}} = \frac{m}{q}\vec{\mathbf{a}} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)}{\left(-1.602\times10^{-19}\,\text{C}\right)}\left(115\,\text{m/s}^2\text{ north}\right) = \boxed{6.54\times10^{-10}\,\text{N/C south}}$$

32. The field due to the negative charge will point towards the negative charge, and the field due to the positive charge will point towards the negative charge. Thus the magnitudes of the two fields can be added together to find the charges.



$$E_{\text{net}} = 2E_Q = 2k\frac{Q}{(d/2)^2} = \frac{8kQ}{d^2} \rightarrow Q = \frac{Ed^2}{8k} = \frac{(745\,\text{N/C})(1.60\times10^{-1}\,\text{m})}{8(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)} = \boxed{2.65\times10^{-10}\,\text{C}}$$

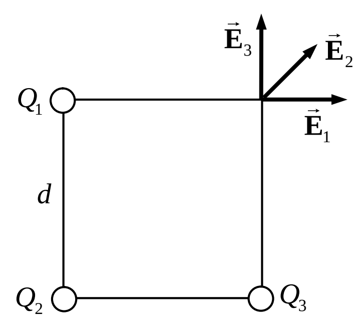
33. The field at the center due to the two negative charges on opposite corners (lower right and upper left in the diagram) will cancel each other, and so only the positive charge and the opposite negative charge need to be considered. The field due to the negative charge will point directly toward it, and the field due to the positive charge will point directly away from it. Accordingly, the two fields are in the same direction and can be added algebraically.



$$E = E_1 + E_2 = k\frac{Q_1}{d^2/2} + k\frac{|Q_2|}{d^2/2} = k\frac{Q_1 + |Q_2|}{d^2/2}$$

$$= (8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(47.0+27.0)\times10^{-6}\,\text{C}}{(0.525\,\text{m})^2/2} = \boxed{4.70\times10^6\,\text{N/C at }45^\circ}$$

34. The field at the upper right corner of the square is the vector sum of the fields due to the other three charges. Let the variable d represent the 1.0 m length of a side of the square, and let the variable Q represent the charge at each of the three occupied corners.



$$E_1 = k\frac{Q}{d^2} \rightarrow E_{1x} = k\frac{Q}{d^2},\; E_{1y} = 0$$

$$E_2 = k\frac{Q}{2d^2} \rightarrow E_{2x} = k\frac{Q}{2d^2}\cos45^\circ = k\frac{\sqrt{2}Q}{4d^2},\; E_{2y} = k\frac{\sqrt{2}Q}{4d^2}$$

$$E_3 = k\frac{Q}{d^2} \rightarrow E_{3x} = 0,\; E_{1y} = k\frac{Q}{d^2}$$

Add the x and y components together to find the total electric field, noting that $E_x = E_y$.

$$E_x = E_{1x} + E_{2x} + E_{3x} = k\frac{Q}{d^2} + k\frac{\sqrt{2}Q}{4d^2} + 0 = k\frac{Q}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right) = E_y$$

$$E = \sqrt{E_x^2 + E_y^2} = k\frac{Q}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q}{d^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$= (8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.25\times10^{-6}\,\text{C})}{(1.00\,\text{m})^2}\left(\sqrt{2} + \frac{1}{2}\right) = \boxed{3.87\times10^4\,\text{N/C}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \boxed{45^\circ}$$ from the x-direction.

35. Choose the rightward direction to be positive. Then the field due to $+Q$ will be positive, and the field due to $-Q$ will be negative.

$$E = k\frac{Q}{(x+a)^2} - k\frac{Q}{(x-a)^2} = kQ\left(\frac{1}{(x+a)^2} - \frac{1}{(x-a)^2}\right) = \boxed{\frac{-4kQxa}{(x^2-a^2)^2}}$$

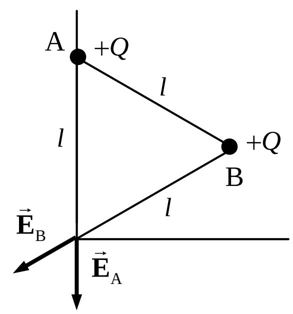
The negative sign means the field points to the $\boxed{\text{left}}$.

36. For the net field to be zero at point P, the magnitudes of the fields created by Q_1 and Q_2 must be equal. Also, the distance x will be taken as positive to the left of Q_1. That is the only region where the total field due to the two charges can be zero. Let the variable d represent the 12 cm distance, and note that $|Q_1| = \frac{1}{2}Q_2$.

$$\left|\vec{\mathbf{E}}_1\right| = \left|\vec{\mathbf{E}}_2\right| \rightarrow k\frac{|Q_1|}{x^2} = k\frac{Q_2}{(x+d)^2} \rightarrow$$

$$x = d\frac{\sqrt{|Q_1|}}{\left(\sqrt{Q_2} - \sqrt{|Q_1|}\right)} = d\frac{\sqrt{\frac{1}{2}Q_2}}{\left(\sqrt{Q_2} - \sqrt{\frac{1}{2}Q_2}\right)} = \frac{d}{\left(\sqrt{2}-1\right)} = \frac{12\,\text{cm}}{\sqrt{2}-1} = \boxed{29\,\text{cm}}$$

37. (*a*) The field due to the charge at A will point straight downward, and the field due to the charge at B will point along the line from A to the origin, 30° below the negative *x* axis.



$$E_{\mathrm{A}} = k\frac{Q}{l^2} \rightarrow E_{\mathrm{A}x} = 0,\; E_{\mathrm{A}x} = -k\frac{Q}{l^2}$$

$$E_{\mathrm{B}} = k\frac{Q}{l^2} \rightarrow E_{\mathrm{B}x} = -k\frac{Q}{l^2}\cos 30^\circ = -k\frac{\sqrt{3}Q}{2l^2}$$

$$E_{\mathrm{B}y} = -k\frac{Q}{l^2}\sin 30^\circ = -k\frac{Q}{2l^2}$$

$$E_x = E_{\mathrm{A}x} + E_{\mathrm{B}x} = -k\frac{\sqrt{3}Q}{2l^2} \qquad E_y = E_{\mathrm{A}y} + E_{\mathrm{B}y} = -k\frac{3Q}{2l^2}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\frac{3k^2Q^2}{4l^4} + \frac{9k^2Q^2}{4l^4}} = \sqrt{\frac{12k^2Q^2}{4l^4}} = \boxed{\frac{\sqrt{3}kQ}{l^2}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{-k\frac{3Q}{2l^2}}{-k\frac{\sqrt{3}Q}{2l^2}} = \tan^{-1}\frac{-3}{-\sqrt{3}} = \tan^{-1}\sqrt{3} = \boxed{240^\circ}$$

(*b*) Now reverse the direction of $\vec{\mathbf{E}}_{\mathrm{A}}$

$$E_{\mathrm{A}} = k\frac{Q}{l^2} \rightarrow E_{\mathrm{A}x} = 0,\; E_{\mathrm{A}x} = -k\frac{Q}{l^2}$$

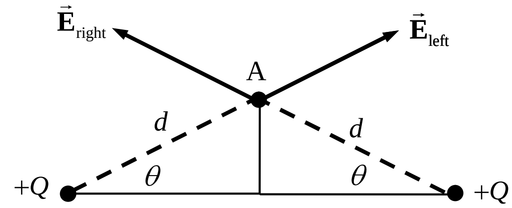
$$E_{\mathrm{B}} = k\frac{Q}{l^2} \rightarrow E_{\mathrm{B}x} = k\frac{Q}{l^2}\cos 30^\circ = k\frac{\sqrt{3}Q}{2l^2},\; E_{\mathrm{B}y} = k\frac{Q}{l^2}\sin 30^\circ = k\frac{Q}{2l^2}$$

$$E_x = E_{\mathrm{A}x} + E_{\mathrm{B}x} = k\frac{\sqrt{3}Q}{2l^2} \qquad E_y = E_{\mathrm{A}y} + E_{\mathrm{B}y} = -k\frac{Q}{2l^2}$$

$$E=\sqrt{E_x^2+E_y^2}=\sqrt{\frac{3k^2Q^2}{4l^4}+\frac{k^2Q^2}{4l^4}}=\sqrt{\frac{4k^2Q^2}{4l^4}}=\boxed{\frac{kQ}{l^2}}$$

$$\theta=\tan^{-1}\frac{E_y}{E_x}=\tan^{-1}\frac{k\dfrac{Q}{2l^2}}{-k\dfrac{\sqrt{3}Q}{2l^2}}=\tan^{-1}\frac{1}{-\sqrt{3}}=\boxed{330^\circ}$$

38. In each case, find the vector sum of the field caused by the charge on the left $\left(\vec{\mathbf{E}}_{\text{left}}\right)$ and the field caused by the charge on the right $\left(\vec{\mathbf{E}}_{\text{right}}\right)$



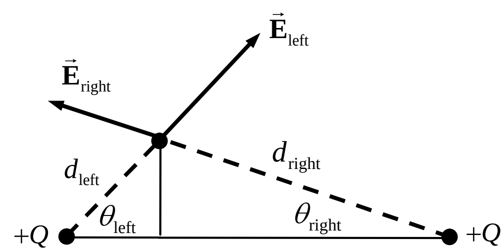
Point A: From the symmetry of the geometry, in calculating the electric field at point A only the vertical components of the fields need to be considered. The horizontal components will cancel each other.

$$\theta=\tan^{-1}\frac{5.0}{10.0}=26.6^\circ$$

$$d=\sqrt{(5.0\,\text{cm})^2+(10.0\,\text{cm})^2}=0.1118\,\text{m}$$

$$E_{\text{A}}=2\frac{kQ}{d^2}\sin\theta=2\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{7.0\times10^{-6}\,\text{C}}{(0.1118\,\text{m})^2}\sin 26.6^\circ=\boxed{4.5\times10^6\ \text{N/C}}\qquad \theta_{\text{A}}=\boxed{90^\circ}$$

Point B: Now the point is not symmetrically placed, and so horizontal and vertical components of each individual field need to be calculated to find the resultant electric field.



$$\theta_{\text{left}}=\tan^{-1}\frac{5.0}{5.0}=45^\circ\qquad \theta_{\text{left}}=\tan^{-1}\frac{5.0}{15.0}=18.4^\circ$$

$$d_{\text{left}}=\sqrt{(5.0\,\text{cm})^2+(5.0\,\text{cm})^2}=0.0707\,\text{m}$$

$$d_{\text{right}}=\sqrt{(5.0\,\text{cm})^2+(15.0\,\text{cm})^2}=0.1581\,\text{m}$$

$$E_x=\left(\vec{\mathbf{E}}_{\text{left}}\right)_x+\left(\vec{\mathbf{E}}_{\text{right}}\right)_x=k\frac{Q}{d_{\text{left}}^2}\cos\theta_{\text{left}}-k\frac{Q}{d_{\text{right}}^2}\cos\theta_{\text{right}}$$

$$=\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.0\times10^{-6}\,\text{C}\right)\left[\frac{\cos45^\circ}{(0.0707\,\text{m})^2}-\frac{\cos18.4^\circ}{(0.1581\,\text{m})^2}\right]=6.51\times10^6\ \text{N/C}$$

$$E_y=\left(\vec{\mathbf{E}}_{\text{left}}\right)_y+\left(\vec{\mathbf{E}}_{\text{right}}\right)_y=k\frac{Q}{d_{\text{left}}^2}\sin\theta_{\text{left}}+k\frac{Q}{d_{\text{right}}^2}\sin\theta_{\text{right}}$$

$$=\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.0\times10^{-6}\,\text{C}\right)\left[\frac{\sin45^\circ}{(0.0707\,\text{m})^2}+\frac{\sin18.4^\circ}{(0.1581\,\text{m})^2}\right]=9.69\times10^6\ \text{N/C}$$

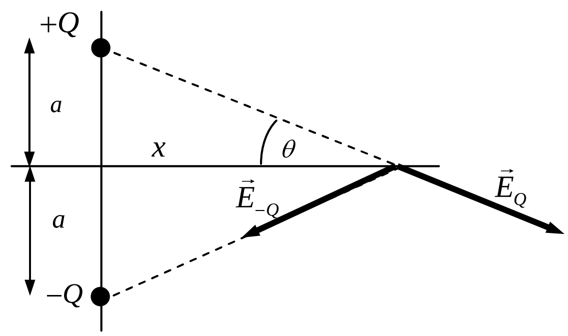
$$E_{\text{B}}=\sqrt{E_x^2+E_y^2}=\boxed{1.2\times10^7\ \text{N/C}}\qquad \theta_B=\tan^{-1}\frac{E_y}{E_x}=\boxed{56^\circ}$$

The results are consistent with Figure 16-31b. In the figure, the field at Point A points straight up, matching the calculations. The field at Point B should be to the right and vertical, matching the calculations. Finally, the field lines are closer together at Point B than at Point A, indicating that the field is stronger there, matching the calculations.

39. Both charges must be of the same sign so that the electric fields created by the two charges oppose each other, and so can add to zero. The magnitudes of the two electric fields must be equal.

$$E_1 = E_2 \;\rightarrow\; k\frac{Q_1}{(l/3)^2} = k\frac{Q_2}{(2l/3)^2} \;\rightarrow\; 9Q_1 = \frac{9Q_2}{4} \;\rightarrow\; \frac{Q_1}{Q_2} = \boxed{\frac{1}{4}}$$

40. From the diagram, we see that the x components of the two fields will cancel each other at the point P. Thus the net electric field will be in the negative y-direction, and will be twice the y-component of either electric field vector.



$$E_{\text{net}} = 2E\sin\theta = 2\frac{kQ}{x^2+a^2}\sin\theta$$

$$= \frac{2kQ}{x^2+a^2}\frac{a}{\left(x^2+a^2\right)^{1/2}}$$

$$= \boxed{\frac{2kQa}{\left(x^2+a^2\right)^{3/2}} \text{ in the negative } y \text{ direction}}$$

41. We assume that gravity can be ignored, which is proven in part (*b*).

(*a*) The electron will accelerate to the right. The magnitude of the acceleration can be found from setting the net force equal to the electric force on the electron. The acceleration is constant, so constant acceleration relationships can be used.

$$F_{\text{net}} = ma = |q|E \;\rightarrow\; a = \frac{|q|E}{m}$$

$$v^2 = v_0^2 + 2a\Delta x \;\rightarrow\; v = \sqrt{2a\Delta x} = \sqrt{2\frac{|q|E}{m}\Delta x}$$

$$= \sqrt{2\frac{\left(1.602\times10^{-19}\text{C}\right)\left(1.45\times10^{4}\text{ N/C}\right)}{\left(9.11\times10^{-31}\text{kg}\right)}\left(1.10\times10^{-2}\text{m}\right)} = \boxed{7.49\times10^{6}\text{ m/s}}$$

(*b*) The value of the acceleration caused by the electric field is compared to g.

$$a = \frac{|q|E}{m} = \frac{\left(1.602\times10^{-19}\text{C}\right)\left(1.45\times10^{4}\text{ N/C}\right)}{\left(9.11\times10^{-31}\text{kg}\right)} = 2.55\times10^{15}\text{ m/s}^2$$

$$\frac{a}{g} = \frac{2.55\times10^{15}\text{ m/s}^2}{9.80\text{ m/s}^2} = 2.60\times10^{14}$$

The acceleration due to gravity can be ignored compared to the acceleration caused by the electric field.

42. (*a*) The electron will experience a force in the opposite direction to the electric field. Since the electron is to be brought to rest, the electric field must be in the same direction as the initial velocity of the electron, and so is to the $\boxed{\text{right}}$.

(*b*) Since the field is uniform, the electron will experience a constant force, and therefore have a constant acceleration. Use constant acceleration relationships to find the field strength.

$$F = qE = ma \rightarrow a = \frac{qE}{m} \qquad v^2 = v_0^2 + 2a\Delta x = v_0^2 + 2\frac{qE}{m}\Delta x \rightarrow$$

$$E = \frac{m\left(v^2 - v_0^2\right)}{2q\Delta x} = \frac{-mv_0^2}{2q\Delta x} = -\frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(3.0\times10^{6}\,\text{m/s}\right)^2}{2\left(-1.602\times10^{-19}\,\text{C}\right)\left(4.0\times10^{-2}\,\text{m}\right)} = \boxed{6.4\times10^{2}\,\text{N/C}}$$

43. Use Gauss's law to determine the enclosed charge.

$$\Phi_E = \frac{Q_{\text{encl}}}{\varepsilon_o} \rightarrow Q_{\text{encl}} = \Phi_E\varepsilon_o = \left(1.45\times10^{3}\,\text{N}\cdot\text{m}^2/\text{C}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right) = \boxed{1.28\times10^{-8}\,\text{C}}$$

44. (*a*) $\Phi_E = E_\perp A = E\pi r^2 = \left(5.8\times10^{2}\,\text{N/C}\right)\pi\left(1.8\times10^{-1}\,\text{m}\right)^2 = \boxed{59\,\text{N}\cdot\text{m}^2/\text{C}}$

(*b*) $\Phi_E = E_\perp A = \left(E\cos45^\circ\right)\pi r^2 = \left(5.8\times10^{2}\,\text{N/C}\right)\left(\cos45^\circ\right)\pi\left(1.8\times10^{-1}\,\text{m}\right)^2 = \boxed{42\,\text{N}\cdot\text{m}^2/\text{C}}$

(*c*) $\Phi_E = E_\perp A = \left(E\cos90^\circ\right)\pi r^2 = \boxed{0}$

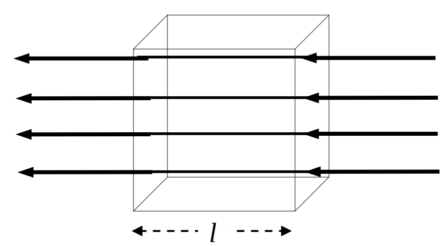
45. (*a*) Use Gauss's law to determine the electric flux.

$$\Phi_E = \frac{Q_{\text{encl}}}{\varepsilon_o} = \frac{-1.0\times10^{-6}\,\text{C}}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{-1.1\times10^{5}\,\text{N}\cdot\text{m}^2/\text{C}}$$

(*b*) Since there is no charge enclosed by surface A2, $\Phi_E = \boxed{0}$.

46. (*a*) Assuming that there is no charge contained within the cube, then the net flux through the cube is $\boxed{0}$. All of the field lines that enter the cube also leave the cube.

(*b*) There are four faces that have no flux through them, because none of the field lines pass through those faces. In the diagram shown, the left face has a positive flux and the right face has the opposite amount of negative flux.



$$\boxed{\begin{array}{l}\Phi_{\text{left}} = EA = El^2 = \left(6.50\times10^{3}\,\text{N/C}\right)l^2 \\ \Phi_{\text{right}} = -\left(6.50\times10^{3}\,\text{N/C}\right)l^2 \qquad \Phi_{\text{other}} = 0\end{array}}$$

47. Equation 16-10 applies.

$$E = \frac{Q/A}{\varepsilon_0} \rightarrow Q = \varepsilon_0 EA = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(130\,\text{N/C}\right)\left(1.0\,\text{m}\right)^2 = \boxed{1.15\times10^{-9}\,\text{C}}$$

48. The electric field can be calculated by Eq. 16-4a, and that can be solved for the charge.

$$E = k\frac{Q}{r^2} \rightarrow Q = \frac{Er^2}{k} = \frac{\left(2.75\times10^{2}\,\text{N/C}\right)\left(3.50\times10^{-2}\,\text{m}\right)^2}{8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{3.75\times10^{-11}\,\text{C}}$$

This corresponds to about 2×10^{8} electrons. Since the field points toward the ball, the charge must be $\boxed{\text{negative}}$.

49. See Example 16-11 for a detailed discussion related to this problem.
  (*a*) Inside a solid metal sphere the electric field is $\boxed{0}$.
  (*b*) Inside a solid metal sphere the electric field is $\boxed{0}$.
  (*c*) Outside a solid metal sphere the electric field is the same as if all the charge were concentrated at the center as a point charge.

$$|E| = k\frac{|Q|}{r^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(3.50\times10^{-6}\,\mathrm{C}\right)}{\left(3.10\,\mathrm{m}\right)^2} = \boxed{3.27\times10^3\ \mathrm{N/C}}$$

  (*d*) Same reasoning as in part (*c*).

$$|E| = k\frac{|Q|}{r^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(3.50\times10^{-6}\,\mathrm{C}\right)}{\left(6.00\,\mathrm{m}\right)^2} = \boxed{8.74\times10^2\ \mathrm{N/C}}$$

  (*e*) The answers would be $\boxed{\text{no different}}$ for a thin metal shell.

50. See Figure 16-33 in the text for additional insight into this problem.
  (*a*) Inside the shell, the field is that of the point charge, $\boxed{E = k\frac{Q}{r^2}}$.
  (*b*) There is no field inside the conducting material: $\boxed{E = 0}$.
  (*c*) Outside the shell, the field is that of the point charge, $\boxed{E = k\frac{Q}{r^2}}$.
  (*d*) The shell does not affect the field due to Q alone, except in the shell material, where the field is 0. The charge Q does affect the shell – it polarizes it. There will be an induced charge of $-Q$ uniformly distributed over the inside surface of the shell, and an induced charge of $+Q$ uniformly distributed over the outside surface of the shell.

51. (*a*) The net force between the thymine and adenine is due to the following forces.

O – H attraction: $$F_{\mathrm{OH}} = k\frac{(0.4e)(0.2e)}{\left(1.80\,\mathring{\mathrm{A}}\right)^2} = \frac{0.08ke^2}{\left(1.80\,\mathring{\mathrm{A}}\right)^2}$$

O – N repulsion: $$F_{\mathrm{ON}} = k\frac{(0.4e)(0.2e)}{\left(2.80\,\mathring{\mathrm{A}}\right)^2} = \frac{0.08ke^2}{\left(2.80\,\mathring{\mathrm{A}}\right)^2}$$

N – N repulsion: $$F_{\mathrm{NN}} = k\frac{(0.2e)(0.2e)}{\left(3.00\,\mathring{\mathrm{A}}\right)^2} = \frac{0.04ke^2}{\left(3.00\,\mathring{\mathrm{A}}\right)^2}$$

H – N attraction: $$F_{\mathrm{HN}} = k\frac{(0.2e)(0.2e)}{\left(2.00\,\mathring{\mathrm{A}}\right)^2} = \frac{0.04ke^2}{\left(2.00\,\mathring{\mathrm{A}}\right)^2}$$

$$F_{\mathrm{A\text{-}T}} = F_{\mathrm{OH}} - F_{\mathrm{ON}} - F_{\mathrm{NN}} + F_{\mathrm{HN}} = \left(\frac{0.08}{1.80^2} - \frac{0.08}{2.80^2} - \frac{0.04}{3.00^2} + \frac{0.04}{2.00^2}\right)\left(\frac{1}{1.0\times10^{-10}\,\mathrm{m}}\right)^2\frac{ke^2}{d^2}$$

$$= (.02004)\frac{\left(8.988\times 10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(1.602\times 10^{-19}\,\mathrm{C}\right)^2}{\left(1.0\times 10^{-10}\,\mathrm{m}\right)^2} = 4.623\times 10^{-10}\,\mathrm{N} \approx \boxed{4.6\times 10^{-10}\,\mathrm{N}}$$

(*b*) The net force between the cytosine and guanine is due to the following forces.

O – H attraction: $F_{\mathrm{OH}} = k\frac{(0.4e)(0.2e)}{\left(1.90\,\mathrm{\AA}\right)^2} = \frac{0.08ke^2}{\left(1.90\,\mathrm{\AA}\right)^2}$ (2 of these)

O – N repulsion: $F_{\mathrm{ON}} = k\frac{(0.4e)(0.2e)}{\left(2.90\,\mathrm{\AA}\right)^2} = \frac{0.08ke^2}{\left(2.90\,\mathrm{\AA}\right)^2}$ (2 of these)

H – N attraction: $F_{\mathrm{HN}} = k\frac{(0.2e)(0.2e)}{\left(2.00\,\mathrm{\AA}\right)^2} = \frac{0.04ke^2}{\left(2.00\,\mathrm{\AA}\right)^2}$

N – N repulsion: $F_{\mathrm{NN}} = k\frac{(0.2e)(0.2e)}{\left(3.00\,\mathrm{\AA}\right)^2} = \frac{0.04ke^2}{\left(3.00\,\mathrm{\AA}\right)^2}$

$$F_{\mathrm{C\text{-}G}} = 2F_{\mathrm{OH}} - 2F_{\mathrm{ON}} - F_{\mathrm{NN}} + F_{\mathrm{HN}} = \left(2\frac{0.08}{1.90^2} - 2\frac{0.08}{2.90^2} - \frac{0.04}{3.00^2} + \frac{0.04}{2.00^2}\right)\left(\frac{1}{1.0\times 10^{-10}\,\mathrm{m}}\right)^2\frac{ke^2}{d^2}$$

$$= (.03085)\frac{\left(8.988\times 10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(1.602\times 10^{-19}\,\mathrm{C}\right)^2}{\left(1.0\times 10^{-10}\,\mathrm{m}\right)^2} = 7.116\times 10^{-10}\,\mathrm{N} \approx \boxed{7.1\times 10^{-10}\,\mathrm{N}}$$

(*c*) For 10^5 pairs of molecules, we assume that half are A-T pairs and half are C-G pairs. We average the above results and multiply by 10^5.

$$F_{\mathrm{net}} = \tfrac{1}{2}10^5\left(F_{\mathrm{A\text{-}T}} + F_{\mathrm{C\text{-}G}}\right) = 10^5\left(4.623\times 10^{-10}\,\mathrm{N} + 7.116\times 10^{-10}\,\mathrm{N}\right)$$

$$= 5.850\times 10^{-5}\,\mathrm{N} \approx \boxed{6\times 10^{-5}\,\mathrm{N}}$$

52. Set the magnitude of the electric force equal to the magnitude of the force of gravity and solve for the distance.

$$F_{\mathrm{E}} = F_{\mathrm{G}} \quad\rightarrow\quad k\frac{e^2}{r^2} = mg \quad\rightarrow$$

$$r = e\sqrt{\frac{k}{mg}} = \left(1.602\times 10^{-19}\,\mathrm{C}\right)\sqrt{\frac{\left(8.988\times 10^{9}\ \mathrm{N\cdot m^2/C^2}\right)}{\left(9.11\times 10^{-31}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}} = \boxed{5.08\,\mathrm{m}}$$

53. Calculate the total charge on all electrons in 3.0 g of copper, and then compare the $38\mu\mathrm{C}$ to that value.

$$\text{Total electron charge} = 3.0\,\mathrm{g}\left(\frac{1\ \text{mole}}{63.5\,\mathrm{g}}\right)\left(\frac{6.02\times 10^{23}\,\text{atoms}}{\text{mole}}\right)\left(\frac{29\ \mathrm{e}}{\text{atoms}}\right)\left(\frac{1.602\times 10^{-19}\mathrm{C}}{1\ \mathrm{e}}\right)$$

$$= 1.32\times 10^{5}\,\mathrm{C}$$

$$\text{Fraction lost} = \frac{38\times10^{-6}\,\text{C}}{1.32\times10^{5}\,\text{C}} = \boxed{2.9\times10^{-10}}$$

54. Since the gravity force is downward, the electric force must be upward. Since the charge is positive, the electric field must also be upward. Equate the magnitudes of the two forces and solve for the electric field.

$$F_E = F_G \;\rightarrow\; qE = mg \;\rightarrow\; E = \frac{mg}{q} = \frac{(1.67\times10^{-27}\,\text{kg})(9.80\,\text{m/s}^2)}{(1.602\times10^{-19}\,\text{C})} = \boxed{1.02\times10^{-7}\,\text{N/C, up}}$$

55. Use Eq. 16-4a to calculate the magnitude of the electric charge on the Earth.

$$E = k\frac{Q}{r^2} \;\rightarrow\; Q = \frac{Er^2}{k} = \frac{(150\ \text{N/C})(6.38\times10^{6}\,\text{m})^2}{8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{6.8\times10^{5}\,\text{C}}$$

Since the electric field is pointing towards the Earth's center, the charge must be $\boxed{\text{negative}}$.

56. (*a*) From problem 55, we know that the electric field is pointed towards the Earth's center. Thus an electron in such a field would experience an upwards force of magnitude $F_E = eE$. The force of gravity on the electron will be negligible compared to the electric force.

$$F_E = eE = ma \;\rightarrow$$

$$a = \frac{eE}{m} = \frac{(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})}{(9.11\times10^{-31}\,\text{kg})} = 2.638\times10^{13}\,\text{m/s}^2 \approx \boxed{2.6\times10^{13}\,\text{m/s}^2\text{, up}}$$

(*b*) A proton in the field would experience a downwards force of magnitude $F_E = eE$. The force of gravity on the proton will be negligible compared to the electric force.

$$F_E = eE = ma \;\rightarrow$$

$$a = \frac{eE}{m} = \frac{(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})}{(1.67\times10^{-27}\,\text{kg})} = 1.439\times10^{10}\,\text{m/s}^2 \approx \boxed{1.4\times10^{10}\,\text{m/s}^2\text{, down}}$$

(*c*) For the electron: $\frac{a}{g} = \frac{2.638\times10^{13}\,\text{m/s}^2}{9.80\,\text{m/s}^2} \approx \boxed{2.7\times10^{12}}$

For the proton: $\frac{a}{g} = \frac{1.439\times10^{10}\,\text{m/s}^2}{9.80\,\text{m/s}^2} \approx \boxed{1.5\times10^{9}}$

57. For the droplet to remain stationary, the magnitude of the electric force on the droplet must be the same as the weight of the droplet. The mass of the droplet is found from its volume times the density of water. Let *n* be the number of excess electrons on the water droplet.

$$F_E = |q|E = mg \;\rightarrow\; neE = \tfrac{4}{3}\pi r^3\rho g \;\rightarrow$$

$$n = \frac{4\pi r^3\rho g}{3eE} = \frac{4\pi(1.8\times10^{-5}\,\text{m})^3(1.00\times10^{3}\,\text{kg/m}^3)(9.80\,\text{m/s}^2)}{3(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})} = 9.96\times10^{6} \approx \boxed{1.0\times10^{7}\,\text{electrons}}$$

58. There are four forces to calculate. Call the rightward direction the positive direction. The value of *k* is $8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$ and the value of *e* is $1.602\times10^{-19}\,\text{C}$.

$$F_{\text{net}} = F_{\text{CH}} + F_{\text{CN}} + F_{\text{OH}} + F_{\text{ON}} = \frac{k(0.40e)(0.20e)}{\left(1\times10^{-9}\,\text{m}\right)^2}\left[-\frac{1}{(0.30)^2}+\frac{1}{(0.40)^2}+\frac{1}{(0.18)^2}-\frac{1}{(0.28)^2}\right]$$

$$= 2.445\times10^{-10}\,\text{N} \approx \boxed{2.4\times10^{-10}\,\text{N}}$$

59. The electric force must be a radial force in order for the electron to move in a circular orbit.

$$F_{\text{E}} = F_{\text{radial}} \rightarrow k\frac{Q^2}{r_{\text{orbit}}^2} = \frac{mv^2}{r_{\text{orbit}}} \rightarrow$$

$$r_{\text{orbit}} = k\frac{Q^2}{mv^2} = \left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.602\times10^{-19}\,\text{C}\right)^2}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.1\times10^{6}\,\text{m/s}\right)^2} = \boxed{2.1\times10^{-10}\,\text{m}}$$

60. Set the Coulomb electrical force equal to the Newtonian gravitational force on one of the bodies (the Moon).

$$F_{\text{E}} = F_{\text{G}} \rightarrow k\frac{Q^2}{r_{\text{orbit}}^2} = G\frac{M_{\text{Moon}}M_{\text{Earth}}}{r_{\text{orbit}}^2} \rightarrow$$

$$Q = \sqrt{\frac{GM_{\text{Moon}}M_{\text{Earth}}}{k}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\,\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}} = \boxed{5.71\times10^{13}\,\text{C}}$$

61. (*a*) The electron will experience a force in the opposite direction to the electric field. Thus the acceleration is in the opposite direction to the initial velocity. The force is constant, and so constant acceleration equations apply. To find the stopping distance, set the final velocity to 0.

$$F = eE = ma \rightarrow a = \frac{eE}{m} \qquad v^2 = v_0^2 + 2a\Delta x \rightarrow$$

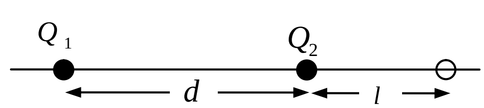
$$\Delta x = \frac{v^2 - v_0^2}{2a} = -\frac{mv_0^2}{2eE} = -\frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(21.5\times10^{6}\,\text{m/s}\right)^2}{2\left(-1.602\times10^{-19}\,\text{C}\right)\left(11.4\times10^{3}\,\text{N/C}\right)} = \boxed{0.115\,\text{m}}$$

(*b*) To return to the starting point, the velocity will reverse. Use that to find the time to return.

$$v = v_0 + at \rightarrow$$

$$t = \frac{v - v_0}{a} = \frac{-v_0 - v_0}{a} = -\frac{2mv_0}{qE} = -\frac{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(21.5\times10^{6}\,\text{m/s}\right)}{\left(-1.602\times10^{-19}\,\text{C}\right)\left(11.4\times10^{3}\,\text{N/C}\right)} = \boxed{2.14\times10^{-8}\,\text{s}}$$

62. Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge (Q_2). Also, in between the two charges, the fields due to the two charges are parallel to each other and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge, but not between them. In the diagram, this means that *x* must be positive.



$$E = -k\frac{|Q_2|}{l^2} + k\frac{Q_1}{(l+d)^2} = 0 \rightarrow |Q_2|(l+d)^2 = Q_1 l^2 \rightarrow$$

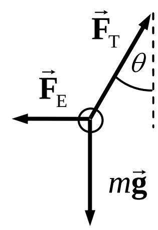
$$l=\frac{\sqrt{|Q_2|}}{\sqrt{Q_1}-\sqrt{|Q_2|}}d=\frac{\sqrt{5.0\times10^{-6}\,\mathrm{C}}}{\sqrt{2.5\times10^{-5}\,\mathrm{C}}-\sqrt{5.0\times10^{-6}\,\mathrm{C}}}(2.0\,\mathrm{m})=\boxed{\begin{array}{l}1.6\,\mathrm{m\ from\ }Q_2,\\2.6\,\mathrm{m\ from\ }Q_1\end{array}}$$

63. The sphere will oscillate sinusoidally about the equilibrium point, with an amplitude of 5.0 cm. The angular frequency of the sphere is given by $\omega=\sqrt{k/m}=\sqrt{126\,\mathrm{N/m}/0.800\,\mathrm{kg}}=12.5\,\mathrm{rad/s}$. The distance of the sphere from the table is given by $r=[0.150-0.050\cos(12.5t)]\,\mathrm{m}$. Use this distance and the charge to give the electric field value at the tabletop. That electric field will point upwards at all times, towards the negative sphere.

$$E=k\frac{|Q|}{r^2}=\frac{(8.988\times10^9\,\mathrm{N\cdot m^2/C^2})(3.00\times10^{-6}\,\mathrm{C})}{[0.150-0.050\cos(12.5t)]^2\,\mathrm{m^2}}=\frac{2.70\times10^4}{[0.150-0.050\cos(12.5t)]^2}\,\mathrm{N/C}$$

$$=\boxed{\frac{1.08\times10^7}{[3.00-\cos(12.5t)]^2}\,\mathrm{N/C},\ \mathrm{upwards}}$$

64. The wires form two sides of an equilateral triangle, and so the two charges are separated by a distance $d=78$ cm and are directly horizontal from each other. Thus the electric force on each charge is horizontal. From the free-body diagram for one of the spheres, write the net force in both the horizontal and vertical directions and solve for the electric force. Then write the electric force by Coulomb's law, and equate the two expressions for the electric force to find the charge.



$$\sum F_y=F_T\cos\theta-mg=0\ \rightarrow\ F_T=\frac{mg}{\cos\theta}$$

$$\sum F_x=F_T\sin\theta-F_E=0\ \rightarrow\ F_E=F_T\sin\theta=\frac{mg}{\cos\theta}\sin\theta=mg\tan\theta$$

$$F_E=k\frac{(Q/2)^2}{d^2}=mg\tan\theta\ \rightarrow\ Q=2d\sqrt{\frac{mg\tan\theta}{k}}$$

$$=2(7.8\times10^{-1}\,\mathrm{m})\sqrt{\frac{(24\times10^{-3}\,\mathrm{kg})(9.80\,\mathrm{m/s^2})\tan30^\circ}{(8.988\times10^9\,\mathrm{N\cdot m^2/C^2})}}=6.064\times10^{-6}\,\mathrm{C}\approx\boxed{6.1\times10^{-6}\,\mathrm{C}}$$

65. The electric field at the surface of the pea is given by Equation (16-4a). Solve that equation for the charge.

$$E=k\frac{Q}{r^2}\ \rightarrow\ Q=\frac{Er^2}{k}=\frac{(3\times10^6\,\mathrm{N/C})(3.75\times10^{-3}\,\mathrm{m})^2}{8.988\times10^9\,\mathrm{N\cdot m^2/C^2}}=\boxed{5\times10^{-9}\,\mathrm{C}}$$

This corresponds to about 3 billion electrons.

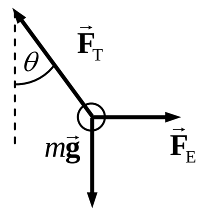
66. There will be a rightward force on Q_1 due to Q_2, given by Coulomb's law. There will be a leftward force on Q_1 due to the electric field created by the parallel plates. Let right be the positive direction.

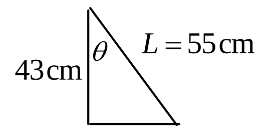
$$\sum F=k\frac{|Q_1Q_2|}{x^2}-|Q_1|E$$

$$= \left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(6.7\times10^{-6}\,\text{C}\right)\left(1.8\times10^{-6}\,\text{C}\right)}{\left(0.34\,\text{m}\right)^2} - \left(6.7\times10^{-6}\,\text{C}\right)\left(7.3\times10^{4}\ \text{N/C}\right)$$

$$= \boxed{0.45\,\text{N, right}}$$

67. Since the electric field exerts a force on the charge in the same direction as the electric field, the charge is positive. Use the free-body diagram to write the equilibrium equations for both the horizontal and vertical directions, and use those equations to find the magnitude of the charge.





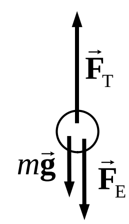
$$\theta = \cos^{-1}\frac{43}{55} = 38.6^\circ$$

$$\sum F_x = F_E - F_T\sin\theta = 0 \ \rightarrow\ F_E = F_T\sin\theta = QE$$

$$\sum F_y = F_T\cos\theta - mg = 0 \ \rightarrow\ F_T = \frac{mg}{\cos\theta} \ \rightarrow\ QE = mg\tan\theta$$

$$Q = \frac{mg\tan\theta}{E} = \frac{\left(1.0\times10^{-3}\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\tan 38.6^\circ}{\left(1.2\times10^{4}\ \text{N/C}\right)} = \boxed{6.5\times10^{-7}\,\text{C}}$$

68. The weight of the mass is only about 2 N. Since the tension in the string is more than that, there must be a downward electric force on the positive charge, which means that the electric field must be pointed down. Use the free-body diagram to write an expression for the magnitude of the electric field.



$$\sum F = F_T - mg - F_E = 0 \ \rightarrow\ F_E = QE = F_T - mg \ \rightarrow$$

$$E = \frac{F_T - mg}{Q} = \frac{5.67\,\text{N} - \left(0.210\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{3.40\times10^{-7}\text{C}} = \boxed{1.06\times10^{7}\ \text{N/C}}$$

69. To find the number of electrons, convert the mass to moles, the moles to atoms, and then multiply by the number of electrons in an atom to find the total electrons. Then convert to charge.

$$15\text{ kg Al} = \left(15\text{ kg Al}\right)\left(\frac{1\text{ mole Al}}{2.7\times10^{-2}\,\text{kg}}\right)\left(\frac{6.02\times10^{23}\,\text{atoms}}{1\text{ mole}}\right)\left(\frac{13\text{ electrons}}{1\text{ molecule}}\right)\left(\frac{-1.602\times10^{-19}\,\text{C}}{\text{electron}}\right)$$

$$= \boxed{-7.0\times10^{8}\,\text{C}}$$

The net charge of the bar is $\boxed{0\,\text{C}}$, since there are equal numbers of protons and electrons.

70. (*a*) The force of sphere B on sphere A is given by Coulomb's law.

$$F_{AB} = \boxed{\frac{kQ^2}{R^2},\ \text{away from B}}$$

(*b*) The result of touching sphere B to uncharged sphere C is that the charge on B is shared between the two spheres, and so the charge on B is reduced to $Q/2$. Again use Coulomb's law.

$$F_{AB} = k\frac{QQ/2}{R^2} = \boxed{\frac{kQ^2}{2R^2},\ \text{away from B}}$$

(c) The result of touching sphere A to sphere C is that the charge on the two spheres is shared, and so the charge on A is reduced to $3Q/4$. Again use Coulomb's law.

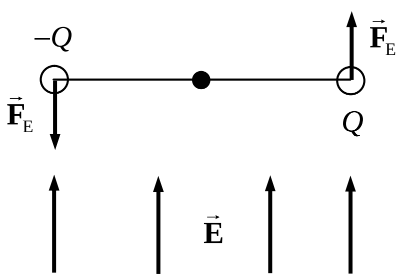
$$F_{AB} = k\frac{(3Q/4)(Q/2)}{R^2} = \boxed{\frac{3kQ^2}{8R^2}\text{, away from B}}$$

71. On the *x*-axis, the electric field can only be zero at a location closer to the smaller magnitude charge. Thus the field will never be zero to the left of the midpoint between the two charges. Also, in between the two charges, the field due to both charges will point to the left, and so the total field cannot be zero. Thus the only place on the x-axis where the field can be zero is to the right of the negative charge, and so *x* must be positive. Calculate the field at point *P* and set it equal to zero.

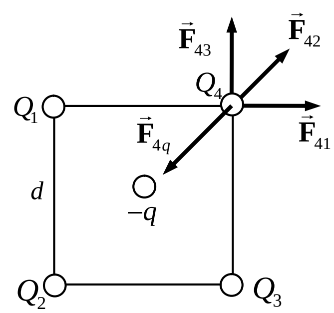
$$E = k\frac{(-Q/2)}{x^2} + k\frac{Q}{(x+d)^2} = 0 \;\rightarrow\; 2x^2 = (x+d)^2 \;\rightarrow\; x = \boxed{\frac{d}{\sqrt{2}-1} \approx 2.41d}$$

The field cannot be zero at any points off the *x*-axis. For any point off the *x*-axis, the electric fields due to the two charges will not be along the same line, and so they can never combine to give 0.

72. The electric field will put a force of magnitude $F_E = QE$ on each charge. The distance of each charge from the pivot point is $L/2$, and so the torque caused by each force is $\tau = F_E r_\perp = \frac{QEL}{2}$. Both torques will tend to make the rod rotate counterclockwise in the diagram, and so the net torque is $\tau_{net} = 2\left(\frac{QEL}{2}\right) = \boxed{QEL}$.



73. A negative charge must be placed at the center of the square. Let $Q = 8.0\,\mu C$ be the charge at each corner, let -*q* be the magnitude of negative charge in the center, and let $d = 9.2\,\text{cm}$ be the side length of the square. By the symmetry of the problem, if we make the net force on one of the corner charges be zero, the net force on each other corner charge will also be zero.



$$F_{41} = k\frac{Q^2}{d^2} \;\rightarrow\; F_{41x} = k\frac{Q^2}{d^2},\; F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \;\rightarrow\; F_{42x} = k\frac{Q^2}{2d^2}\cos45^\circ = k\frac{\sqrt{2}Q^2}{4d^2},\; F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \;\rightarrow\; F_{43x} = 0,\; F_{43y} = k\frac{Q^2}{d^2}$$

$$F_{4q} = k\frac{qQ}{d^2/2} \;\rightarrow\; F_{4qx} = -k\frac{2qQ}{d^2}\cos45^\circ = -k\frac{\sqrt{2}qQ}{d^2} = F_{4qy}$$

The net force in each direction should be zero.

$$\sum F_x = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 - k\frac{\sqrt{2}qQ}{d^2} = 0 \;\rightarrow\; q = Q\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = 7.66\times10^{-6}\,\text{C}$$

So the charge to be placed is $-q = \boxed{-7.66\times10^{-6}\,\text{C}}$.

This is an unstable equilibrium. If the center charge were slightly displaced, say towards the right, then it would be closer to the right charges than the left, and would be attracted more to the right. Likewise the positive charges on the right side of the square would be closer to it and would be attracted more to it, moving from their corner positions. The system would not have a tendency to return to the symmetric shape, but rather would have a tendency to move away from it if disturbed.