CHAPTER 19: DC Circuits

Answers to Questions

1. The birds are safe because they are not grounded. Both of their legs are essentially at the same voltage (the only difference being due to the small resistance of the wire between their feet), and so there is no current flow through their bodies since the potential difference across their legs is very small. If you lean a metal ladder against the power line, you are making essentially a short circuit from the high potential wire to the low potential ground. A large current will flow at least momentarily, and that large current will be very dangerous to anybody touching the ladder.

2. If the lights are connected in parallel, if one bulb burns out, the rest of the string stays lit. That makes it easy to tell which light has gone out. A parallel string is more complicated to assemble than a series string, since two wires must be attached from bulb to bulb.

   If the lights are connected in series, if one bulb burns out, all of the bulbs will go out. That makes it difficult to tell which light has gone out. A series string is simpler to assemble than one in parallel, since only one wire must be attached from bulb to bulb. A "blinker bulb" can make the entire string flash on and off by cutting off the current.

3. If 20 of the 6-V lamps were connected in series and then connected to the 120 V line, there would be a voltage drop of 6 V for each of the lamps, and they would not burn out due to too much voltage. Being in series, if one of the bulbs went out for any reason, they would all turn off.

4. If the lightbulbs are in series, each will have the same current. The power dissipated by the bulb as heat and light is given by $P = I^2 R$. Thus the bulb with the higher resistance (R_2) will be brighter. If the bulbs are in parallel, each will have the same voltage. The power dissipated by the bulb as heat and light is given by $P = V^2 / R$. Thus the bulb with the lower resistance (R_1) will be brighter.

5. The outlets are connected in parallel to each other, because you can use one outlet without using the other. If they were in series, both outlets would have to be used at the same time to have a completed circuit. Also, both outlets supply the same voltage to whatever device is plugged in to the outlet, which indicates that they are wired in parallel to the voltage source.

6. The power output from a resistor is given by $P = V^2 / R$. To maximize this value, the voltage needs to be as large as possible and the resistance as small as possible. That can be accomplished by putting the two batteries in series, and then connecting the two resistors in parallel to each other, across the full 2-battery voltage.

7. The power supplied by the battery is the product of the battery voltage times the total current flowing from the battery. With the two resistors in series, the current is half that with a single resistor. Thus the battery has to supply half the power for the two series resistors than for the single resistor.

8. There is more current flowing in the room's wiring when both bulbs are on, yet the voltage remains the same. Thus the resistance of the room's circuit must have decreased. Also, since the bulbs are in parallel, adding two resistors in parallel always results in a net resistance that is smaller than either of the individual resistances.

9. No, the sign of the battery's emf does not depend on the direction of the current through the battery. The sign of the battery's emf depends on the direction you go through the battery in applying the loop rule. If you go from negative pole to positive pole, the emf is added. If you go from positive pole to negative pole, the emf is subtracted.

   But the terminal voltage does depend on the direction of the current through the battery. If current is flowing through the battery in the normal orientation (leaving the positive terminal, flowing through the circuit, and arriving at the negative terminal) then there is a voltage drop across the internal resistance, and the terminal voltage is less than the emf. If the current flows in the opposite sense (as in charging the battery), then there is a voltage rise across the terminal resistance, and the terminal voltage is higher than the emf.

10. (*a*) stays the same
    (*b*) increases
    (*c*) decreases
    (*d*) increases
    (*e*) increases
    (*f*) decreases
    (*g*) decreases
    (*h*) increases
    (*i*) stays the same

11. Batteries are connected in series to increase the voltage available to a device. For instance, if there are two 1.5-V batteries in series in a flashlight, the potential across the bulb will be 3.0 V. The batteries need not be nearly identical.

    Batteries are connected in parallel to increase the total amount of current available to a device. The batteries need to be nearly identical. If they are not, the larger voltage batteries will recharge the smaller voltage batteries.

12. The terminal voltage of a battery can exceed its emf if the battery is being charged – if current is passing through the battery "backwards" from positive pole to negative pole. Then the terminal voltage is the emf of the battery plus the voltage drop across the internal resistance.

13. Refer to Figure 19-2. Connect the battery to a known resistance *R*, and measure the terminal voltage V_{ab}. The current in the circuit is given by Ohm's law to be $I = \frac{V_{\text{ab}}}{R}$. It is also true that $V_{\text{ab}} = \mathcal{E} - Ir$ and so the internal resistance can be calculated by $r = \frac{\mathcal{E} - V_{\text{ab}}}{I} = R\frac{\mathcal{E} - V_{\text{ab}}}{V_{\text{ab}}}$.

14. The formulas are "opposite" each other in a certain sense. When connected in series, resistors add linearly but capacitors add reciprocally, according to the following rules.

$$R_{\substack{\text{eq}\\ \text{series}}} = R_1 + R_2 + R_3 + \cdots \qquad \frac{1}{C_{\substack{\text{eq}\\ \text{series}}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

    The series resistance is always larger than any one component resistance, and the series capacitance is always smaller than any one component capacitance. When connected in parallel, resistors add reciprocally but capacitors add linearly, according to the following rules.

$$\frac{1}{R_{\text{eq parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad C_{\text{eq parallel}} = C_1 + C_2 + C_3 + \cdots$$

The parallel resistance is always smaller than any one component resistance, and the parallel capacitance is always larger than any one component capacitance.

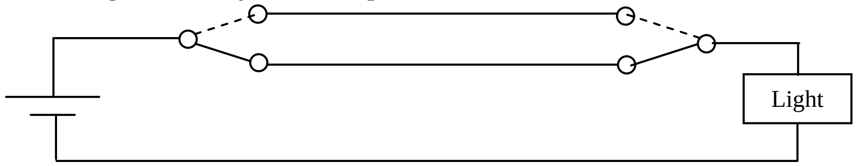
One way to consider the source of this difference is that the voltage across a resistor is proportional to the resistance, by $V = IR$, but the voltage across a capacitor is inversely proportion to the capacitance, by $V = Q/C$.

15. The energy stored in a capacitor network can be calculated by $PE = \frac{1}{2}CV^2$. Since the voltage for the capacitor network is the same in this problem for both configurations, the configuration with the highest equivalent capacitance will store the most energy. The parallel combination has the highest equivalent capacitance, and so stores the most energy. Another way to consider this is that the total stored energy is the sum of the quantity $PE = \frac{1}{2}CV^2$ for each capacitor. Each capacitor has the same capacitance, but in the parallel circuit, each capacitor has a larger voltage than in the series circuit. Thus the parallel circuit stores more energy.

16. The soles of your shoes are made of a material which has a relatively high resistance, and there is relatively high resistance flooring material between your shoes and the literal ground (the Earth). With that high resistance, a malfunctioning appliance would not be able to cause a large current flow through your body. The resistance of bare feet is much less than that of shoes, and the feet are in direct contact with the ground, so the total resistance is much lower and so a larger current would flow through your body.

17. As the sound wave is incident on the diaphragm, the diaphragm will oscillate with the same frequency as the sound wave. That oscillation will change the gap between the capacitor plates, causing the capacitance to change. As the capacitance changes, the charge state of the capacitor will change. For instance, if the plates move together, the capacitance will increase, and charge will flow to the capacitor. If the plates move apart, the capacitance will decrease, and charge will flow away from the capacitor. This current will change the output voltage. Thus if the wave has a frequency of 200 Hz, the capacitance and thus the current and thus the output voltage will all change with a frequency of 200 Hz.

18. The following is one way to accomplish the desired task.



In the current configuration, the light would be on. If either switch is moved, the light will go out. But then if either switch were moved again, the light would come back on.

19. The total energy supplied by the battery is greater than the total energy stored by the capacitor. The extra energy was dissipated in the form of heat in the resistor while current was flowing. That energy will not be "recovered" during the discharging process.

20. An analog voltmeter has a high value resistor in series with the galvanometer, so that the actual voltage drop across the galvanometer is small. An analog ammeter has a very low value resistor in parallel with the galvanometer, so that the actual current through the galvanometer is small. Another significant difference is how they are connected to a circuit. A voltmeter goes in parallel with the component being measured, while an ammeter goes in series with the component being measured.

21. If you mistakenly use an ammeter where you intend to use a voltmeter, you are inserting a short in parallel with some resistance. That means that the resistance of the entire circuit has been lowered, and all of the current will flow through the low-resistance ammeter. Ammeters usually have a fairly small current limit, and so the ammeter might very likely get damaged in such a scenario. Also, if the ammeter is inserted across a voltage source, the source will provide a large current, and again the meter will almost certainly be damaged, or at least disabled by burning out a fuse.

22. An ideal ammeter would have zero resistance so that there was no voltage drop across it, and so it would not affect a circuit into which it was placed in parallel. A zero resistance will not change the resistance of a circuit if placed in series, and hence would not change the current in the circuit. An ideal voltmeter would have infinite resistance so that it would draw no current, and thus would not affect a circuit into which it was placed in parallel. An infinite resistance will not change the resistance of a circuit if placed in parallel, and hence would not change the current in the circuit.

23. The behavior of the circuit is the same in either case. The same current will flow in the circuit with the elements rearranged, because the current in a series circuit is the same for all elements in the circuit.

24. When the voltmeter is connected to the circuit, it reduces the resistance of that part of the circuit. That will make the (resistor + voltmeter) combination a smaller fraction of the total resistance of the circuit than the resistor was alone, which means that it will have a smaller fraction of the total voltage drop across it.

25. The voltmeter, if it has a particularly high internal resistance, will measure the emf of the battery, which is 1.5 V. But if the battery has a high internal resistance (indicating that the battery is "used up", then its terminal voltage, which is the voltage supplied to the flashlight bulb, can be quite low when more current is drawn from the battery by a low resistance like the bulb.

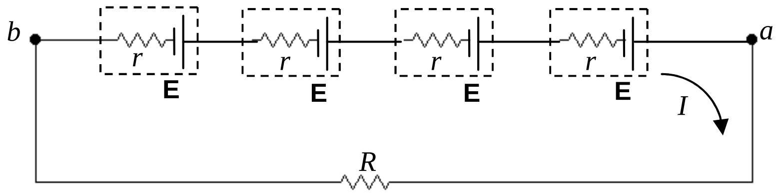
Solutions to Problems

1. See Figure 19-2 for a circuit diagram for this problem. Using the same analysis as in Example 19-1, the current in the circuit is $I = \frac{\mathcal{E}}{R+r}$. Use Eq. 19-1 to calculate the terminal voltage.

(*a*) $V_{ab} = \mathcal{E} - Ir = \mathcal{E} - \left(\frac{\mathcal{E}}{R+r}\right)r = \frac{\mathcal{E}(R+r) - \mathcal{E}r}{R+r} = \mathcal{E}\frac{R}{R+r} = (8.50\,\text{V})\frac{81.0\,\Omega}{(81.0+0.900)\,\Omega} = \boxed{8.41\,\text{V}}$

(*b*) $V_{ab} = \mathcal{E}\frac{R}{R+r} = (8.50\,\text{V})\frac{810\,\Omega}{(810+0.900)\,\Omega} = \boxed{8.49\,\text{V}}$

2. See the circuit diagram below. The current in the circuit is I. The voltage V_{ab} is given by Ohm's law to be $V_{ab} = IR$. That same voltage is the terminal voltage of the series EMF.



$$V_{ab} = (\mathcal{E} - Ir) + (\mathcal{E} - Ir) + (\mathcal{E} - Ir) + (\mathcal{E} - Ir) = 4(\mathcal{E} - Ir) \quad \text{and} \quad V_{ab} = IR$$

$$4(\mathcal{E} - Ir) = IR \rightarrow r = \frac{\mathcal{E} - \frac{1}{4}IR}{I} = \frac{(1.5\,\text{V}) - \frac{1}{4}(0.45\,\text{A})(12\,\Omega)}{0.45\,\text{A}} = \boxed{0.33\,\Omega}$$

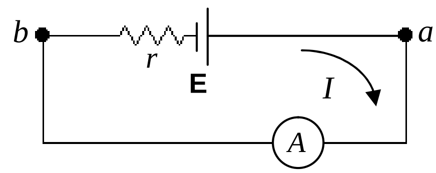
3. See Figure 19-2 for a circuit diagram for this problem. Use Eq. 19-1.

$$V_{ab} = \mathcal{E} - Ir \rightarrow r = \frac{\mathcal{E} - V_{ab}}{I} = \frac{12.0\,\text{V} - 8.4\,\text{V}}{75\,\text{A}} = \boxed{0.048\,\Omega}$$

$$V_{ab} = IR \rightarrow R = \frac{V_{ab}}{I} = \frac{8.4\,\text{V}}{75\,\text{A}} = \boxed{0.11\,\Omega}$$

4. We take the low-resistance ammeter to have no resistance. The circuit is shown. The terminal voltage will be 0 volts.

$$V_{ab} = \mathcal{E} - Ir = 0 \rightarrow r = \frac{\mathcal{E}}{I} = \frac{1.5\,\text{V}}{22\,\text{A}} = \boxed{0.068\,\Omega}$$



5. For the resistors in series, use Eq. 19-3.

$$R_{eq} = R_1 + R_2 + R_3 + R_4 = 4(240\,\Omega) = \boxed{960\,\Omega}$$

For the resistors in parallel, use Eq. 19-4.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{4}{240\,\Omega} \rightarrow R_{eq} = \frac{240\,\Omega}{4} = \boxed{60\,\Omega}$$

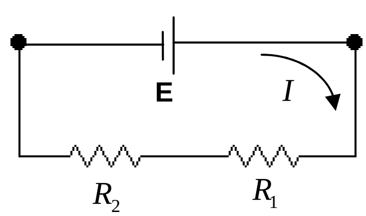
6. (*a*) For the resistors in series, use Eq. 19-3, which says the resistances add linearly.

$$R_{eq} = 3(45\,\Omega) + 3(75\,\Omega) = \boxed{360\,\Omega}$$

(*b*) For the resistors in parallel, use Eq. 19-4, which says the resistances add reciprocally.

$$\frac{1}{R_{eq}} = \frac{3}{45\,\Omega} + \frac{3}{75\,\Omega} = \frac{3(75\,\Omega) + 3(45\,\Omega)}{(75\,\Omega)(45\,\Omega)} \rightarrow R_{eq} = \frac{(75\,\Omega)(45\,\Omega)}{3(75\,\Omega) + 3(45\,\Omega)} = \boxed{9.4\,\Omega}$$

7. The equivalent resistance is the sum of the two resistances: $R_{eq} = R_1 + R_2$. The current in the circuit is then the voltage divided by the equivalent resistance: $I = \frac{V}{R_{eq}} = \frac{V}{R_1 + R_2}$. The voltage across the 2200-Ω resistor is given by Ohm's law.



$$V_{2200} = IR_2 = \frac{V}{R_1 + R_2} R_2 = V \frac{R_2}{R_1 + R_2} = (12.0\,\text{V}) \frac{2200\,\Omega}{650\,\Omega + 2200\,\Omega} = \boxed{9.3\,\text{V}}$$

8. The possible resistances are each resistor considered individually, the series combination, and the parallel combination.

series: $R_{eq} = R_1 + R_2 = 25\,\Omega + 35\,\Omega = 60\,\Omega$

parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(25\,\Omega)(35\,\Omega)}{60\,\Omega} = 15\,\Omega$

$\boxed{25\,\Omega\text{ , }35\,\Omega\text{ , }60\,\Omega\text{ , }15\,\Omega}$

9. (*a*) The maximum resistance is made by combining the resistors in series.

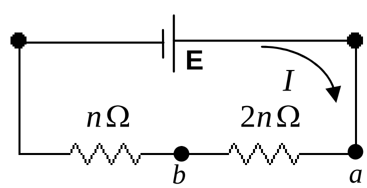
$$R_{eq} = R_1 + R_2 + R_3 = 680\,\Omega + 940\,\Omega + 1200\,\Omega = \boxed{2820\,\Omega}$$

(*b*) The minimum resistance is made by combining the resistors in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \rightarrow$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{680\,\Omega} + \frac{1}{940\,\Omega} + \frac{1}{1200\,\Omega}\right)^{-1} = \boxed{3.0\times10^{2}\,\Omega}$$

10. Connecting $3n$ of the resistors in series, where n is an integer, will enable you to make a voltage divider with a 4.0 V output.



$$R_{eq} = 3n(1.0\,\Omega) \qquad I = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{3n}$$

$$V_{ab} = 2n(1.0\,\Omega) I = 2n(1.0\,\Omega)\frac{\mathcal{E}}{3n} = \frac{2}{3}\mathcal{E} = \frac{2}{3}(6.0\,\text{V}) = 4.0\,\text{V}$$

11. The resistors can all be connected in series.

$$R_{eq} = R + R + R = 3(240\,\Omega) = \boxed{720\,\Omega}$$

The resistors can all be connected in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \rightarrow R_{eq} = \left(\frac{3}{R}\right)^{-1} = \frac{R}{3} = \frac{240\,\Omega}{3} = \boxed{80\,\Omega}$$

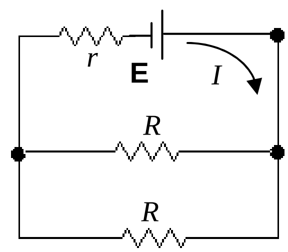
Two resistors in series can be placed in parallel with the third.

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R+R} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \rightarrow R_{eq} = \frac{2R}{3} = \frac{2(240\,\Omega)}{3} = \boxed{160\,\Omega}$$

Two resistors in parallel can be placed in series with the third.

$$R_{eq} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = R + \frac{R}{2} = \frac{3}{2}(240\,\Omega) = \boxed{360\,\Omega}$$

12. The resistance of each bulb can be found from its power rating.



$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(12.0\,\text{V})^2}{3.0\,\text{W}} = 48\,\Omega$$

Find the equivalent resistance of the two bulbs in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \rightarrow R_{eq} = \frac{R}{2} = \frac{48\,\Omega}{2} = 24\,\Omega$$

The terminal voltage is the voltage across this equivalent resistance. Use that to find the current drawn from the battery.

$$V_{ab} = IR_{eq} \rightarrow I = \frac{V_{ab}}{R_{eq}} = \frac{V_{ab}}{R/2} = \frac{2V_{ab}}{R}$$

Finally, use the terminal voltage and the current to find the internal resistance, as in Eq. 19-1.

$$V_{ab} = \mathsf{E} - Ir \rightarrow r = \frac{\mathsf{E} - V_{ab}}{I} = \frac{\mathsf{E} - V_{ab}}{\left(\frac{2V_{ab}}{R}\right)} = R\frac{\mathsf{E} - V_{ab}}{2V_{ab}} = (48\,\Omega)\frac{12.0\,\text{V} - 11.8\,\text{V}}{2(11.8\,\text{V})} = \boxed{0.4\,\Omega}$$

13. (*a*) Each bulb should get one-eighth of the total voltage, but let us prove that instead of assuming it. Since the bulbs are identical, the net resistance is $R_{eq} = 8R$. The current flowing through the bulbs is then $V_{tot} = IR_{eq} \rightarrow I = \frac{V_{tot}}{R_{eq}} = \frac{V_{tot}}{8R}$. The voltage across one bulb is found from Ohm's law.

$$V = IR = \frac{V_{tot}}{8R}R = \frac{V_{tot}}{8} = \frac{110\,\text{V}}{8} = 13.75\,\text{V} \approx \boxed{14\,\text{V}}$$

(*b*) $I = \frac{V_{tot}}{8R} \rightarrow R = \frac{V_{tot}}{8I} = \frac{110\,\text{V}}{8(0.50\,\text{A})} = 27.5\,\Omega \approx \boxed{28\,\Omega}$

$$P = I^2R = (0.50\,\text{A})^2(27.5\,\Omega) = 6.875\,\text{W} \approx \boxed{6.9\,\text{W}}$$

14. We model the resistance of the long leads as a single resistor *r*. Since the bulbs are in parallel, the total current is the sum of the current in each bulb, and so $I = 8I_R$. The voltage drop across the long leads is $V_{leads} = Ir = 8I_R r = 8(0.24\,\text{A})(1.6\,\Omega) = 3.072\,\text{V}$. Thus the voltage across each of the parallel resistors is $V_R = V_{tot} - V_{leads} = 110\,\text{V} - 3.072\,\text{V} = 106.9\,\text{V}$. Since we have the current through each resistor, and the voltage across each resistor, we calculate the resistance using Ohm's law.

$$V_R = I_R R \rightarrow R = \frac{V_R}{I_R} = \frac{106.9\,\text{V}}{0.24\,\text{A}} = 445.4\,\Omega = \boxed{450\,\Omega}$$

The total power delivered is $P = V_{tot}I$, and the "wasted" power is I^2r. The fraction wasted is the ratio of those powers.

$$\text{fraction wasted} = \frac{I^2r}{IV_{tot}} = \frac{Ir}{V_{tot}} = \frac{8(0.24\,\text{A})(1.6\,\Omega)}{110\,\text{V}} = \boxed{0.028}$$

So about 3% of the power is wasted.

15. Each bulb will get one-eighth of the total voltage, and so $V_{\text{bulb}} = \frac{V_{\text{tot}}}{8}$. Use that voltage and the power dissipated by each bulb to calculate the resistance of a bulb.

$$P_{\text{bulb}} = \frac{V_{\text{bulb}}^2}{R} \quad \rightarrow \quad R = \frac{V_{\text{bulb}}^2}{P} = \frac{V_{\text{tot}}^2}{64P} = \frac{(110\,\text{V})^2}{64(7.0\,\text{W})} = \boxed{27\,\Omega}$$

16. To fix this circuit, connect another resistor in parallel with the 480-Ω resistor so that the equivalent resistance is the desired 320 Ω.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \rightarrow \quad R_2 = \left(\frac{1}{R_{\text{eq}}} - \frac{1}{R_1}\right)^{-1} = \left(\frac{1}{320\,\Omega} - \frac{1}{480\,\Omega}\right)^{-1} = \boxed{960\,\Omega}$$

17. (*a*) The equivalent resistance is found by combining the 820 Ω and 680 Ω resistors in parallel, and then adding the 470 Ω resistor in series with that parallel combination.

$$R_{\text{eq}} = \left(\frac{1}{820\,\Omega} + \frac{1}{680\,\Omega}\right)^{-1} + 470\,\Omega = 372\,\Omega + 470\,\Omega = 842\,\Omega \approx \boxed{840\,\Omega}$$

(*b*) The current delivered by the battery is $I = \frac{V}{R_{\text{eq}}} = \frac{12.0\,\text{V}}{842\,\Omega} = 1.425 \times 10^{-2}\,\text{A}$. This is the current in the 470 Ω resistor. The voltage across that resistor can be found by Ohm's law.

$$V_{470} = IR = \left(1.425 \times 10^{-2}\,\text{A}\right)\left(470\,\Omega\right) = \boxed{6.7\,\text{V}}$$

Thus the voltage across the parallel combination must be $12.0\,\text{V} - 6.7\,\text{V} = \boxed{5.3\,\text{V}}$. This is the voltage across both the 820 Ω and 680 Ω resistors, since parallel resistors have the same voltage across them. Note that this voltage value could also be found as follows.

$$V_{\text{parallel}} = IR_{\text{parallel}} = \left(1.425 \times 10^{-2}\,\text{A}\right)\left(372\,\Omega\right) = 5.3\,\text{V}$$

$\boxed{18.}$ The resistance of each bulb can be found by using Eq. 18-6, $P = V^2/R$. The two individual resistances are combined in parallel. We label the bulbs by their wattage.

$$P = V^2/R \quad \rightarrow \quad \frac{1}{R} = \frac{P}{V^2}$$

$$R_{\text{eq}} = \left(\frac{1}{R_{75}} + \frac{1}{R_{40}}\right)^{-1} = \left(\frac{75\,\text{W}}{(110\,\text{V})^2} + \frac{40\,\text{W}}{(110\,\text{V})^2}\right)^{-1} = 105.2\,\Omega \approx \boxed{110\,\Omega}$$

19. (*a*) Note that adding resistors in series always results in a larger resistance, and adding resistors in parallel always results in a smaller resistance. Closing the switch adds another resistor in parallel with R_3 and R_4, which lowers the net resistance of the parallel portion of the circuit, and thus lowers the equivalent resistance of the circuit. That means that more current will be delivered by the battery. Since R_1 is in series with the battery, its voltage will increase. Because of that increase, the voltage across R_3 and R_4 must decrease so that the total voltage drops around the loop are equal to the battery voltage. Since there was no voltage across R_2 until the switch was closed, its voltage will increase. To summarize:

$\boxed{V_1 \text{ and } V_2 \text{ increase ; } V_3 \text{ and } V_4 \text{ decrease}}$

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance. Thus

$\boxed{I_1 \text{ and } I_2 \text{ increase ; } I_3 \text{ and } I_4 \text{ decrease}}$

(*c*) Since the battery voltage does not change and the current delivered by the battery increases, the power delivered by the battery, found by multiplying the voltage of the battery by the current delivered, $\boxed{\text{increases}}$.

(*d*) Before the switch is closed, the equivalent resistance is R_3 and R_4 in parallel, combined with R_1 in series.

$$R_{\text{eq}} = R_1 + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 125\,\Omega + \left(\frac{2}{125\,\Omega}\right)^{-1} = 187.5\,\Omega$$

The current delivered by the battery is the same as the current through R_1.

$$I_{\text{total}} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{22.0\,\text{V}}{187.5\,\Omega} = 0.1173\,\text{A} = I_1$$

The voltage across R_1 is found by Ohm's law.

$$V_1 = IR_1 = (0.1173\,\text{A})(125\,\Omega) = 14.66\,\text{V}$$

The voltage across the parallel resistors is the battery voltage less the voltage across R_1.

$$V_{\text{p}} = V_{\text{battery}} - V_1 = 22.0\,\text{V} - 14.66\,\text{V} = 7.34\,\text{V}$$

The current through each of the parallel resistors is found from Ohm's law.

$$I_3 = \frac{V_{\text{p}}}{R_2} = \frac{7.34\,\text{V}}{125\,\Omega} = 0.0587\,\text{A} = I_4$$

Notice that the current through each of the parallel resistors is half of the total current, within the limits of significant figures. The currents before closing the switch are as follows.

$$\boxed{I_1 = 0.117\,\text{A} \quad I_3 = I_4 = 0.059\,\text{A}}$$

After the switch is closed, the equivalent resistance is R_2, R_3, and R_4 in parallel, combined with R_1 in series. Do a similar analysis.

$$R_{\text{eq}} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 125\,\Omega + \left(\frac{3}{125\,\Omega}\right)^{-1} = 166.7\,\Omega$$

$$I_{\text{total}} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{22.0\,\text{V}}{166.7\,\Omega} = 0.1320\,\text{A} = I_1 \qquad V_1 = IR_1 = (0.1320\,\text{A})(125\,\Omega) = 16.5\,\text{V}$$

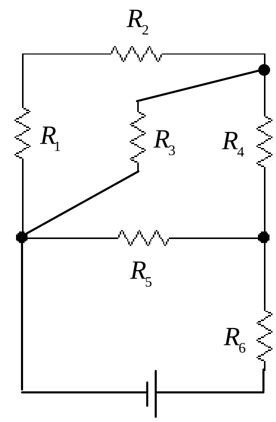
$$V_{\text{p}} = V_{\text{battery}} - V_1 = 22.0\,\text{V} - 16.5\,\text{V} = 5.5\,\text{V} \qquad I_2 = \frac{V_{\text{p}}}{R_2} = \frac{5.5\,\text{V}}{125\,\Omega} = 0.044\,\text{A} = I_3 = I_4$$

Notice that the current through each of the parallel resistors is one third of the total current, within the limits of significant figures. The currents after closing the switch are as follows.

$$\boxed{I_1 = 0.132\,\text{A} \quad I_2 = I_3 = I_4 = 0.044\,\text{A}}$$

$\boxed{\text{Yes}}$, the predictions made in part (*b*) are all confirmed.

20. The resistors have been numbered in the accompanying diagram to help in the analysis. R_1 and R_2 are in series with an equivalent resistance of $R_{12} = R + R = 2R$. This combination is in parallel with R_3, with an equivalent resistance of $R_{123} = \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = \frac{2}{3}R$. This combination is in series with R_4, with an equivalent resistance of $R_{1234} = \frac{2}{3}R + R = \frac{5}{3}R$. This combination is in parallel with R_5, with an equivalent resistance of $R_{12345} = \left(\frac{1}{R} + \frac{3}{5R}\right)^{-1} = \frac{5}{8}R$. Finally, this combination is in series with R_6, and we calculate the final equivalent resistance.



$$R_{eq} = \tfrac{5}{8}R + R = \tfrac{13}{8}R = \tfrac{13}{8}(2.8\,\text{k}\Omega) = 4.55\,\text{k}\Omega \approx \boxed{4.6\,\text{k}\Omega}$$

21. We label identical resistors from left to right as R_{left}, R_{middle}, and R_{right}. When the switch is opened, the equivalent resistance of the circuit increases from $\frac{3}{2}R + r$ to $2R + r$. Thus the current delivered by the battery decreases, from $\frac{\mathcal{E}}{\frac{3}{2}R + r}$ to $\frac{\mathcal{E}}{2R + r}$. Note that this is LESS than a 50% decrease.

(*a*) Because the current from the battery has decreased, the voltage drop across R_{left} will decrease, since it will have less current than before. The voltage drop across R_{right} decreases to 0, since no current is flowing in it. The voltage drop across R_{middle} will increase, because even though the total current has decreased, the current flowing through R_{middle} has increased since before the switch was opened, only half the total current was flowing through R_{middle}. To summarize:

$$\boxed{V_{\text{left}}\text{ decreases ; }V_{\text{middle}}\text{ increases ; }V_{\text{right}}\text{ goes to 0}}.$$

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance.

$$\boxed{I_{\text{left}}\text{ decreases ; }I_{\text{middle}}\text{ increases ; }I_{\text{right}}\text{ goes to 0}}$$

(*c*) Since the current from the battery has decreased, the voltage drop across *r* will decrease, and thus the terminal voltage increases.

(*d*) With the switch closed, the equivalent resistance is $\frac{3}{2}R + r$. Thus the current in the circuit is $I_{\text{closed}} = \frac{\mathcal{E}}{\frac{3}{2}R + r}$, and the terminal voltage is given by Eq. 19-1.

$$V_{\substack{\text{terminal}\\ \text{closed}}} = \mathcal{E} - I_{\text{closed}}r = \mathcal{E} - \frac{\mathcal{E}}{\frac{3}{2}R + r}r = \mathcal{E}\left(1 - \frac{r}{\frac{3}{2}R + r}\right) = (15.0\,\text{V})\left(1 - \frac{0.50\,\Omega}{\frac{3}{2}(5.50\,\Omega) + 0.50\,\Omega}\right)$$

$$= \boxed{14.1\,\text{V}}$$

(*e*) With the switch open, the equivalent resistance is $2R + r$. Thus the current in the circuit is $I_{\text{closed}} = \frac{\mathcal{E}}{2R + r}$, and again the terminal voltage is given by Eq. 19-1.

$$V_{\substack{\text{terminal}\\\text{closed}}} = \mathsf{E} - I_{\text{closed}} r = \mathsf{E} - \frac{\mathsf{E}}{2R+r} r = \mathsf{E}\left(1 - \frac{r}{2R+r}\right) = (15.0\,\text{V})\left(1 - \frac{0.50\,\Omega}{2(5.50\,\Omega)+0.50\,\Omega}\right)$$
$$= \boxed{14.3\,\text{V}}$$

22. Find the maximum current and resulting voltage for each resistor under the power restriction.

$$P = I^2 R = \frac{V^2}{R} \quad \to \quad I = \sqrt{\frac{P}{R}}\ ,\ V = \sqrt{RP}$$

$$I_{1800} = \sqrt{\frac{0.5\,\text{W}}{1.8\times10^3\,\Omega}} = 0.0167\,\text{A} \qquad V_{1800} = \sqrt{(0.5\,\text{W})(1.8\times10^3\,\Omega)} = 30\,\text{V}$$

$$I_{2800} = \sqrt{\frac{0.5\,\text{W}}{2.8\times10^3\,\Omega}} = 0.0134\,\text{A} \qquad V_{2800} = \sqrt{(0.5\,\text{W})(2.8\times10^3\,\Omega)} = 37.4\,\text{V}$$

$$I_{2100} = \sqrt{\frac{0.5\,\text{W}}{2.1\times10^3\,\Omega}} = 0.0154\,\text{A} \qquad V_{2100} = \sqrt{(0.5\,\text{W})(2.1\times10^3\,\Omega)} = 32.4\,\text{V}$$

The parallel resistors have to have the same voltage, and so the voltage across that combination is limited to 32.4 V. That would require a current given by Ohm's law and the parallel combination of the two resistors.

$$I_{\text{parallel}} = \frac{V_{\text{parallel}}}{R_{\text{parallel}}} = V_{\text{parallel}}\left(\frac{1}{R_{2800}} + \frac{1}{R_{2100}}\right) = (32.4\,\text{V})\left(\frac{1}{2800\,\Omega} + \frac{1}{2100\,\Omega}\right) = 0.027\,\text{A}$$

This is more than the maximum current that can be in R_{1800}. Thus the maximum current that R_{1800} can carry, $0.0167\,\text{A}$, is the maximum current for the circuit. The maximum voltage that can be applied across the combination is the maximum current times the equivalent resistance. The equivalent resistance is the parallel combination of R_{2800} and R_{2100} added to R_{1800}.

$$V_{\max} = I_{\max} R_{\text{eq}} = I_{\max}\left[R_{1800} + \left(\frac{1}{R_{2800}} + \frac{1}{R_{2100}}\right)^{-1}\right] = (0.0167\,\text{A})\left[1800\,\Omega + \left(\frac{1}{2800\,\Omega} + \frac{1}{2100\,\Omega}\right)^{-1}\right]$$
$$= (0.0167\,\text{A})\left[1800\,\Omega + \left(\frac{1}{2800\,\Omega} + \frac{1}{2100\,\Omega}\right)^{-1}\right] = \boxed{50\,\text{V}}$$

23. All of the resistors are in series, so the equivalent resistance is just the sum of the resistors. Use Ohm's law then to find the current, and show all voltage changes starting at the negative pole of the battery and going counterclockwise.

$$I = \frac{\mathsf{E}}{R_{\text{eq}}} = \frac{9.0\,\text{V}}{(8.0+12.0+2.0)\,\Omega} = 0.409\,\text{A} \approx \boxed{0.41\,\text{A}}$$

$$\sum \text{voltages} = 9.0\,\text{V} - (8.0\,\Omega)(0.409\,\text{A}) - (12.0\,\Omega)(0.409\,\text{A}) - (2.0\,\Omega)(0.409\,\text{A})$$
$$= 9.0\,\text{V} - 3.27\,\text{V} - 4.91\,\text{V} - 0.82\,\text{V} = \boxed{0.00\,\text{V}}$$

24. Apply Kirchhoff's loop rule to the circuit starting at the upper left corner of the circuit diagram, in order to calculate the current. Assume that the current is flowing clockwise.

$$-I(1.0\,\Omega)+18\,\text{V}-I(6.6\,\Omega)-12\,\text{V}-I(2.0\,\Omega)=0 \;\rightarrow\; I=\frac{6\,\text{V}}{9.6\,\Omega}=0.625\text{A}$$

The terminal voltage for each battery is found by summing the potential differences across the internal resistance and EMF from left to right. Note that for the 12 V battery, there is a voltage gain going across the internal resistance from left to right.

$$18\,\text{V battery: } V_{\text{terminal}}=-I(1.0\,\Omega)+18\,\text{V}=-(0.625\,\text{A})(1.0\,\Omega)+18\,\text{V}=\boxed{17.4\,\text{V}}$$

$$12\,\text{V battery: } V_{\text{terminal}}=I(2.0\,\Omega)+12\,\text{V}=(0.625\,\text{A})(2.0\,\Omega)+12\,\text{V}=\boxed{13.3\,\text{V}}$$

25. From Example 19-8, we have $I_1=-0.87\,\text{A}$, $I_2=2.6\,\text{A}$, $I_3=1.7\,\text{A}$. If another significant figure had been kept, the values would be $I_1=-0.858\,\text{A}$, $I_2=2.58\,\text{A}$, $I_3=1.73\,\text{A}$. We use those results.

(*a*) To find the potential difference between points a and d, start at point a and add each individual potential difference until reaching point d. The simplest way to do this is along the top branch.

$$V_{\text{ad}}=V_{\text{d}}-V_{\text{a}}=-I_1(30\,\Omega)=-(0.858\,\text{A})(30\,\Omega)=\boxed{-25.7\,\text{V}}$$

Slight differences will be obtained in the final answer depending on the branch used, due to rounding. For example, using the bottom branch, we get the following.

$$V_{\text{ad}}=V_{\text{d}}-V_{\text{a}}=\mathcal{E}_1-I_2(21\Omega)=80\,\text{V}-(2.58\,\text{A})(21\Omega)=-25.8\,\text{V}$$

(*b*) For the 80-V battery, the terminal voltage is the potential difference from point g to point e. For the 45-V battery, the terminal voltage is the potential difference from point d to point b.

$$80\,\text{V battery: } V_{\text{terminal}}=\mathcal{E}_1-I_2r=80\,\text{V}-(2.58\,\text{A})(1.0\,\Omega)=\boxed{77.4\,\text{V}}$$

$$45\,\text{V battery: } V_{\text{terminal}}=E_2-I_3r=45\,\text{V}-(1.73\,\text{A})(1.0\,\Omega)=\boxed{43.3\,\text{V}}$$

26. To find the potential difference between points a and b, the current must be found from Kirchhoff's loop law. Start at point a and go counterclockwise around the entire circuit, taking the current to be counterclockwise.

$$-IR+\mathcal{E}-IR-IR+\mathcal{E}-IR=0 \;\rightarrow\; I=\frac{\mathcal{E}}{2R}$$

$$V_{\text{ab}}=V_{\text{a}}-V_{\text{b}}=-IR+\mathcal{E}-IR=\mathcal{E}-2IR=\mathcal{E}-2\frac{\mathcal{E}}{2R}R=\boxed{0\,\text{V}}$$

27. Because there are no resistors in the bottom branch, it is possible to write Kirchhoff loop equations that only have one current term, making them easier to solve. To find the current through R_1, go around the outer loop counterclockwise, starting at the lower left corner.

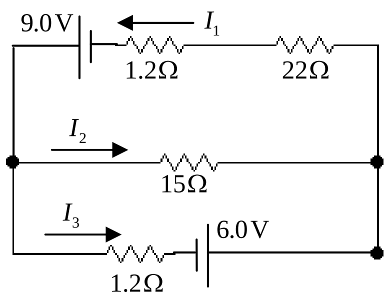
$$V_3-I_1R_1+V_1=0 \;\rightarrow\; I_1=\frac{V_3+V_1}{R_1}=\frac{6.0\,\text{V}+9.0\,\text{V}}{22\,\Omega}=\boxed{0.68\,\text{A, left}}$$

To find the current through R_2, go around the lower loop counterclockwise, starting at the lower left corner.

$$V_3-I_2R_2=0 \;\rightarrow\; I_2=\frac{V_3}{R_2}=\frac{6.0\,\text{V}}{15\,\Omega}=\boxed{0.40\,\text{A, left}}$$

28. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the left of the circuit.

$$I_1 = I_2 + I_3$$



Another equation comes from Kirchhoff's loop rule applied to the outer loop, starting at the lower left corner, and progressing counterclockwise.

$$-I_3(1.2\,\Omega) + 6.0\,\text{V} - I_1(22\,\Omega) - I_1(1.2\,\Omega) + 9.0\,\text{V} = 0 \quad \rightarrow$$

$$15 = 23.2I_1 + 1.2I_3$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the lower left corner, and progressing counterclockwise.

$$-I_3(1.2\,\Omega) + 6.0\,\text{V} + I_2(15\,\Omega) = 0 \quad \rightarrow \quad 6 = -15I_2 + 1.2I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$15 = 23.2I_1 + 1.2I_3 = 23.2(I_2 + I_3) + 1.2I_3 = 23.2I_2 + 24.4I_3 \;\; ; \;\; 6 = -15I_2 + 1.2I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

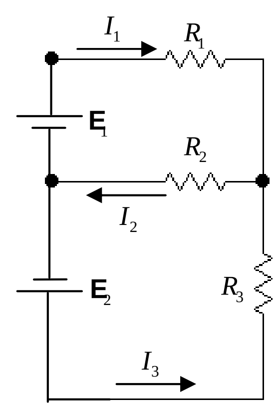
$$6 = -15I_2 + 1.2I_3 \quad \rightarrow \quad I_2 = \frac{-6 + 1.2I_3}{15}$$

$$15 = 23.2I_2 + 24.4I_3 = 23.2\left(\frac{-6 + 1.2I_3}{15}\right) + 24.4I_3 \quad \rightarrow \quad 225 = -138 + 27.84I_3 + 366I_3 \quad \rightarrow$$

$$I_3 = \frac{363}{393.84} = 0.9217\,\text{A} \;\; ; \;\; I_2 = \frac{-6 + 1.2I_3}{15} = \frac{-6 + 1.2(0.9217)}{15} = -0.3263\,\text{A} \approx \boxed{0.33\,\text{A}\text{, left}}$$

$$I_1 = I_2 + I_3 = 0.5954\,\text{A} \approx \boxed{0.60\,\text{A}\text{, left}}$$

29. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the right of the circuit.



$$I_2 = I_1 + I_3 \quad \rightarrow \quad I_1 = I_2 - I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise.

$$E_1 - I_1R_1 - I_2R_2 = 0 \quad \rightarrow \quad 9 = 25I_1 + 18I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$E_2 - I_3R_3 - I_2R_2 = 0 \quad \rightarrow \quad 12 = 35I_3 + 18I_2$$

Substitute $I_1 = I_2 - I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$9 = 25I_1 + 18I_2 = 25(I_2 - I_3) + 18I_2 = 43I_2 - 25I_3 \;\; ; \;\; 12 = 35I_3 + 18I_2$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

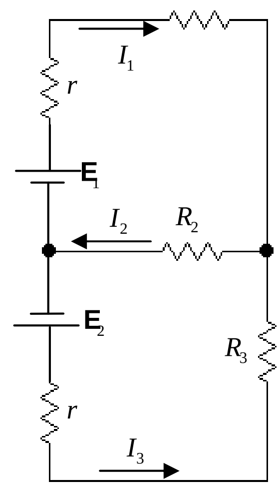
$$12 = 35I_3 + 18I_2 \quad \rightarrow \quad I_2 = \frac{12 - 35I_3}{18}$$

$$9 = 43I_2 - 25I_3 = 43\left(\frac{12 - 35I_3}{18}\right) - 25I_3 \quad \rightarrow \quad 162 = 516 - 1505I_3 - 450I_3 \quad \rightarrow$$

$$I_3 = \frac{354}{1955} = 0.1811\,\text{A} \approx \boxed{0.18\,\text{A , up}} \;\; ; \;\; I_2 = \frac{12 - 35I_3}{18} = 0.3145\,\text{A} \approx \boxed{0.31\,\text{A , left}}$$

$$I_1 = I_2 - I_3 = 0.1334\,\text{A} \approx \boxed{0.13\,\text{A , right}}$$

30. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the right of the circuit.



$$I_2 = I_1 + I_3 \quad \rightarrow \quad I_1 = I_2 - I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise.

$$\mathcal{E}_1 - I_1 r - I_1 R_1 - I_2 R_2 = 0 \quad \rightarrow \quad 9 = 26I_1 + 18I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$\mathcal{E}_2 - I_3 r - I_3 R_3 - I_2 R_2 = 0 \quad \rightarrow \quad 12 = 36I_3 + 18I_2$$

Substitute $I_1 = I_2 - I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$9 = 26I_1 + 18I_2 = 26\left(I_2 - I_3\right) + 18I_2 = 44I_2 - 25I_3 \;\; ; \;\; 12 = 36I_3 + 18I_2$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$12 = 36I_3 + 18I_2 \quad \rightarrow \quad I_2 = \frac{12 - 36I_3}{18} = \frac{2 - 6I_3}{3}$$

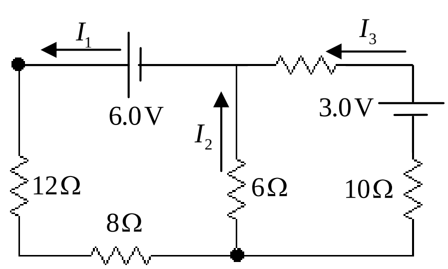
$$9 = 44I_2 - 25I_3 = 44\left(\frac{2 - 6I_3}{3}\right) - 25I_3 \quad \rightarrow \quad 27 = 88 - 264I_3 - 75I_3 \quad \rightarrow$$

$$I_3 = \frac{61}{339} = 0.1799\,\text{A} \approx \boxed{0.18\,\text{A , up}} \;\; ; \;\; I_2 = \frac{2 - 6I_3}{3} = 0.3069\,\text{A} \approx \boxed{0.31\,\text{A left}}$$

$$I_1 = I_2 - I_3 = 0.127\,\text{A} \approx \boxed{0.13\,\text{A , right}}$$

To two significant figures, the currents do not change from problem 29 with the addition of internal resistances.

31. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches at the top center of the circuit.



$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the left loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$6.0\,\text{V} - I_1(12\,\Omega) - I_1(8\,\Omega) - I_2(6\,\Omega) = 0 \;\rightarrow\; 6 = 20I_1 + 6I_2 \;\rightarrow\; 3 = 10I_1 + 3I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$3.0\,\text{V} - I_3(2\,\Omega) + I_2(6.0\,\Omega) - I_3(10\,\Omega) = 0 \;\rightarrow\; 3 = -6I_2 + 12I_3 \;\rightarrow\; 1 = -2I_2 + 4I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$3 = 10I_1 + 3I_2 = 10(I_2 + I_3) + 3I_2 = 13I_2 + 10I_3 \;;\; 1 = -2I_2 + 4I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$1 = -2I_2 + 4I_3 \;\rightarrow\; I_2 = \frac{4I_3 - 1}{2}$$

$$3 = 13I_2 + 10I_3 = 13\left(\frac{4I_3 - 1}{2}\right) + 10I_3 \;\rightarrow\; 6 = 52I_3 - 13 + 20I_3 \;\rightarrow$$

$$I_3 = \frac{19}{72} = 0.2639\,\text{A} \approx 0.26\,\text{A} \;;\; I_2 = \frac{4I_3 - 1}{2} = 0.0278\,\text{A} \approx 0.028\,\text{A}$$

$$I_1 = I_2 + I_3 = 0.2917\,\text{A} \approx 0.29\,\text{A}$$

The current in each resistor is as follows:

$$\boxed{2\,\Omega\text{: } 0.26\text{ A} \quad 6\,\Omega\text{: } 0.028\text{ A} \quad 8\,\Omega\text{: } 0.29\text{ A} \quad 10\,\Omega\text{: } 0.26\text{ A} \quad 12\,\Omega\text{: } 0.29\text{ A}}$$

32. Since there are three currents to determine, there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction near the negative terminal of the middle battery.

$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the middle battery, and progressing counterclockwise.

$$12.0\,\text{V} - I_2(1.0\,\Omega) - I_2(10\,\Omega) - I_1(12\,\Omega) + 12.0\,\text{V} - I_2(1.0\,\Omega) - I_1(8.0\,\Omega) = 0 \;\rightarrow$$

$$24 = 11I_2 + 21I_1 = 0$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the middle battery, and progressing clockwise.

$$12.0\,\text{V} - I_2(1.0\,\Omega) - I_2(10\,\Omega) + I_3(18\,\Omega) + I_3(1.0\,\Omega) - 6.0\,\text{V} + I_3(15\,\Omega) = 0 \;\rightarrow$$

$$6 = 11I_2 - 34I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$24 = 11I_2 + 21I_1 = 11I_2 + 21(I_2 + I_3) = 32I_2 + 21I_3 \;;\; 6 = 11I_2 - 34I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$6 = 11I_2 - 34I_3 \;\rightarrow\; I_2 = \frac{6 + 34I_3}{11}$$

$$24 = 32I_2 + 21I_3 = 32\left(\frac{6 + 34I_3}{11}\right) + 21I_3 \;\rightarrow\; 264 = 192 + 1088I_3 + 231I_3 \;\rightarrow\; 72 = 1319I_3 \;\rightarrow$$

$$I_3 = \frac{72}{1319} = 0.05459\,\text{A} \approx \boxed{0.055\,\text{A}} \;;\; I_2 = \frac{6 + 34I_3}{11} = 0.714\,\text{A} \approx \boxed{0.71\,\text{A}} \;;\; I_1 = I_2 + I_3 = \boxed{0.77\,\text{A}}$$

Also find the terminal voltage of the 6.0-V battery.

$$V_{\text{terminal}} = \mathsf{E} - I_3 r = 6.0\,\text{V} - (0.0546\,\text{A})(1.0\,\Omega) = \boxed{5.9\,\text{V}}$$

33. Since there are three currents to determine, there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction near the negative terminal of the middle battery.

$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the middle battery, and progressing counterclockwise.

$$12.0\,\text{V} - I_2(1.0\,\Omega) - I_2(10\,\Omega) + 12.0\,\text{V} - I_2(1.0\,\Omega) - I_1(8.0\,\Omega) = 0 \quad \rightarrow$$

$$24 = 11 I_2 + 9 I_1 = 0$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the middle battery, and progressing clockwise.

$$12.0\,\text{V} - I_2(1.0\,\Omega) - I_2(10\,\Omega) + I_3(18\,\Omega) + I_3(1.0\,\Omega) - 6.0\,\text{V} + I_3(15\,\Omega) = 0 \quad \rightarrow$$

$$6 = 11 I_2 - 34 I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$24 = 11 I_2 + 9 I_1 = 11 I_2 + 9(I_2 + I_3) = 20 I_2 + 9 I_3 \ ; \ 6 = 11 I_2 - 34 I_3$$

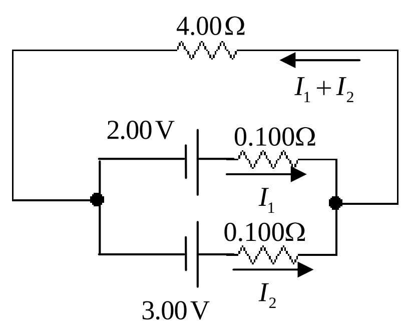
Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$6 = 11 I_2 - 34 I_3 \quad \rightarrow \quad I_2 = \frac{6 + 34 I_3}{11}$$

$$24 = 20 I_2 + 9 I_3 = 20\left(\frac{6 + 34 I_3}{11}\right) + 9 I_3 \quad \rightarrow \quad 264 = 120 + 680 I_3 + 99 I_3 \quad \rightarrow \quad 144 = 779 I_3 \quad \rightarrow$$

$$I_3 = \frac{144}{779} = 0.1849\,\text{A} \ ; \ I_2 = \frac{6 + 34 I_3}{11} = 1.1170\,\text{A} \ ; \ I_1 = I_2 + I_3 = \boxed{1.30\,\text{A}}$$

34. Define I_1 to be the current to the right through the 2.00 V battery, and I_2 to be the current to the right through the 3.00 V battery. At the junction, they combine to give current $I = I_1 + I_2$ to the left through the top branch. Apply Kirchhoff's loop rule first to the upper loop, and then to the outer loop, and solve for the currents.



$$2.00\,\text{V} - I_1(0.100\,\Omega) - (I_1 + I_2)(4.00\,\Omega) = 0 \quad \rightarrow$$

$$2.00 - 4.100 I_1 - 4.00 I_2 = 0$$

$$3.00\,\text{V} - I_2(0.100\,\Omega) - (I_1 + I_2)(4.00\,\Omega) = 0 \quad \rightarrow$$

$$3.00 - 4.00 I_1 - 4.100 I_2 = 0$$

Solve the first equation for I_2, and substitute into the second equation to solve for the currents.

$$2.00 - 4.100I_1 - 4.00I_2 = 0 \quad \rightarrow \quad I_2 = \frac{2.00 - 4.100I_1}{4.00}$$

$$3.00 - 4.00I_1 - -4.100I_2 = 3.00 - 4.00I_1 - 4.100\left(\frac{2.00 - 4.100I_1}{4.00}\right) = 0$$

Multiply by 4

$$12.00 - 16.00I_1 - 4.100\left(2.00 - 4.100I_1\right) = 12.00 - 16.00I_1 - 8.20 + 16.81I_1 = 0$$

$$I_1 = -\frac{3.80}{0.81} = -4.691\,\text{A} \qquad I_2 = \frac{2.00 - 4.100I_1}{4.00} = \frac{2.00 - 4.100(-4.691)}{4.00} = 5.308\,\text{A}$$

The voltage across R is its resistance times $I = I_1 + I_2$.

$$V_R = R(I_1 + I_2) = (4.00\,\Omega)(-4.691\,\text{A} + 5.308\,\text{A}) = 2.468\,\text{V} \approx \boxed{2.47\,\text{V}}$$

Note that the top battery is being charged – the current is flowing through it from positive to negative.

35. (*a*) Capacitors in parallel add according to Eq. 19-5.

$$C_{\text{eq}} = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 = 6\left(4.7\times10^{-6}\,\text{F}\right) = \boxed{2.82\times10^{-5}\,\text{F}} = 28.2\,\mu\text{F}$$

(*b*) Capacitors in series add according to Eq. 19-6.

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \frac{1}{C_5} + \frac{1}{C_6}\right)^{-1} = \left(\frac{6}{4.7\times10^{-6}\,\text{F}}\right)^{-1} = \frac{4.7\times10^{-6}\,\text{F}}{6} = \boxed{7.8\times10^{-7}\,\text{F}}$$

$$= 0.78\,\mu\text{F}$$

36. The maximum capacitance is found by connecting the capacitors in parallel.

$$C_{\max} = C_1 + C_2 + C_3 = 3.2\times10^{-9}\,\text{F} + 7.5\times10^{-9}\,\text{F} + 1.00\times10^{-8}\,\text{F} = \boxed{2.07\times10^{-8}\,\text{F in parallel}}$$

The minimum capacitance is found by connecting the capacitors in series.

$$C_{\min} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \left(\frac{1}{3.2\times10^{-9}\,\text{F}} + \frac{1}{7.5\times10^{-9}\,\text{F}} + \frac{1}{1.00\times10^{-8}\,\text{F}}\right)^{-1} = \boxed{1.83\times10^{-9}\,\text{F in series}}$$

37. The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} + C_3 = \left(\frac{1}{3.00\times10^{-6}\,\text{F}} + \frac{1}{4.00\times10^{-6}\,\text{F}}\right)^{-1} + 2.00\times10^{-6}\,\text{F} = \boxed{3.71\times10^{-6}\,\text{F}} = 3.71\,\mu\text{F}$$

38. The full voltage is across the $2.00\,\mu\text{F}$ capacitor, and so $\boxed{V_{2.00} = 26.0\,\text{V}}$. To find the voltage across the two capacitors in series, find their equivalent capacitance and the charge stored. That charge will be the same for both of the series capacitors. Finally, use that charge to determine the voltage on each capacitor. Notice that the sum of the voltages across the series capacitors is 26.0 V.

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{3.00\times10^{-6}\,\text{F}} + \frac{1}{4.00\times10^{-6}\,\text{F}}\right)^{-1} = 1.714\times10^{-6}\,\text{F}$$

$$Q_{\text{eq}} = C_{\text{eq}}V = \left(1.714\times10^{-6}\,\text{F}\right)\left(26.0\,\text{V}\right) = 4.456\times10^{-5}\,\text{C}$$

$$V_{3.00} = \frac{Q_{\text{eq}}}{C} = \frac{4.456\times10^{-5}\,\text{C}}{3.00\times10^{-6}\,\text{F}} = \boxed{14.9\,\text{V}} \qquad V_{4.00} = \frac{Q_{\text{eq}}}{C} = \frac{4.456\times10^{-5}\,\text{C}}{4.00\times10^{-6}\,\text{F}} = \boxed{11.1\,\text{V}}$$

39. To reduce the net capacitance, another capacitor must be added in series.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow \frac{1}{C_2} = \frac{1}{C_{eq}} - \frac{1}{C_1} = \frac{C_1 - C_{eq}}{C_1 C_{eq}} \rightarrow$$

$$C_2 = \frac{C_1 C_{eq}}{C_1 - C_{eq}} = \frac{(4.8\times10^{-9}\,\mathrm{F})(2.9\times10^{-9}\,\mathrm{F})}{(4.8\times10^{-9}\,\mathrm{F}) - (2.9\times10^{-9}\,\mathrm{F})} = \boxed{7.3\times10^{-9}\,\mathrm{F}}$$

Yes, an existing connection needs to be broken in the process. One of the connections of the original capacitor to the circuit must be disconnected in order to connect the additional capacitor in series.

40. Capacitors in parallel add linearly, and so adding a capacitor in parallel will increase the net capacitance without removing the $5.0\,\mu\mathrm{F}$ capacitor.

$$5.0\,\mu\mathrm{F} + C = 16\mu\mathrm{F} \rightarrow C = \boxed{11.0\mu\mathrm{F}\text{ connected in parallel}}$$

41. The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$C_{eq} = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = C_1 + \left(\frac{C_3}{C_2 C_3} + \frac{C_2}{C_2 C_3}\right)^{-1} = C_1 + \left(\frac{C_2 + C_3}{C_2 C_3}\right)^{-1} = \boxed{C_1 + \frac{C_2 C_3}{C_2 + C_3}}$$

42. For each capacitor, the charge is found by multiplying the capacitance times the voltage. For C_1, the full 45.0 V is across the capacitance, so $Q_1 = C_1 V = (22.6\times10^{-6}\,\mathrm{F})(45.0\,\mathrm{V}) = 1.02\times10^{-3}\,\mathrm{C}$. The equivalent capacitance of the series combination of C_2 and C_3 has the full 45.0 V across it, and the charge on the series combination is the same as the charge on each of the individual capacitors.

$$C_{eq} = \left(\frac{1}{C} + \frac{1}{C/2}\right)^{-1} = \frac{C}{3} \qquad Q_{eq} = C_{eq} V = \tfrac{1}{3}(22.6\times10^{-6}\,\mathrm{F})(45.0\,\mathrm{V}) = 3.39\times10^{-4}\,\mathrm{C}$$

To summarize: $\boxed{Q_1 = 1.02\times10^{-3}\,\mathrm{C} \qquad Q_2 = Q_3 = 3.39\times10^{-4}\,\mathrm{C}}$

43. Capacitors in series have the same charge, so $Q_3 = 24.0\mu\mathrm{C}$. The voltage on a capacitor is the charge on the capacitor divided by the capacitance.

$$V_2 = \frac{Q_2}{C_2} = \frac{24.0\,\mu\mathrm{C}}{16.0\,\mu\mathrm{F}} = 1.50\,\mathrm{V} \;;\; V_3 = \frac{Q_3}{C_3} = \frac{24.0\,\mu\mathrm{C}}{16.0\,\mu\mathrm{F}} = 1.50\,\mathrm{V}$$

Adding the two voltages together gives $V = V_2 + V_3 = 3.00\,\mathrm{V}$. This is also V_1. The charge on C_1 is found from the capacitance and the voltage: $Q_1 = C_1 V_1 = (16.0\,\mu\mathrm{F})(3.00\,\mathrm{V}) = 48.0\,\mu\mathrm{C}$.

$$\boxed{\begin{array}{l} Q_1 = 48.0\,\mu\mathrm{C},\ V_1 = 3.00\,\mathrm{V} \qquad Q_2 = 24.0\,\mu\mathrm{C},\ V_2 = 1.50\,\mathrm{V} \qquad Q_3 = 24.0\,\mu\mathrm{C},\ V_3 = 1.50\,\mathrm{V} \\ V = 3.00\,\mathrm{V} \end{array}}$$

44. Find the equivalent capacitance, and then calculate the stored energy using Eq. 17-10, $\mathrm{PE} = \tfrac{1}{2}CV^2$. The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$C_{eq} = C + \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} = \tfrac{3}{2}C \qquad \mathrm{PE} = \tfrac{1}{2}C_{eq}V^2 = \tfrac{1}{2}\tfrac{3}{2}(7.2\times10^{-6}\,\mathrm{F})(78\,\mathrm{V})^2 = \boxed{3.3\times10^{-2}\,\mathrm{J}}$$

45. When the capacitors are connected in series, they each have the same charge as the net capacitance.

(*a*) $Q_1 = Q_2 = Q_{\text{eq}} = C_{eq}V = \left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1} V = \left(\frac{1}{0.40\times10^{-6}\,\mu\text{F}}+\frac{1}{0.60\times10^{-6}\,\mu\text{F}}\right)^{-1}(9.0\,\text{V})$

$= 2.16\times10^{-6}\,\text{C}$

$V_1 = \frac{Q_1}{C_1} = \frac{2.16\times10^{-6}\,\text{C}}{0.40\times10^{-6}\,\text{F}} = \boxed{5.4\,\text{V}} \qquad V_2 = \frac{Q_2}{C_2} = \frac{2.16\times10^{-6}\,\text{C}}{0.60\times10^{-6}\,\text{F}} = \boxed{3.6\,\text{V}}$

(*b*) $Q_1 = Q_2 = Q_{\text{eq}} = 2.16\times10^{-6}\,\text{C} \approx \boxed{2.2\times10^{-6}\,\text{C}}$

When the capacitors are connected in parallel, they each have the full potential difference.

(*c*) $V_1 = \boxed{9.0\,\text{V}} \qquad V_2 = \boxed{9.0\,\text{V}} \qquad Q_1 = C_1V_1 = (0.40\times10^{-6}\,\text{F})(9.0\,\text{V}) = \boxed{3.6\times10^{-6}\,\text{C}}$

$Q_2 = C_2V_2 = (0.60\times10^{-6}\,\text{F})(9.0\,\text{V}) = \boxed{5.4\times10^{-6}\,\text{C}}$

46. (*a*) The two capacitors are in parallel. Both capacitors have their high voltage plates at the same potential (the middle plate), and both capacitors have their low voltage plates at the same potential (the outer plates, which are connected).

(*b*) The capacitance of two capacitors in parallel is the sum of the individual capacitances.

$$C = C_1 + C_2 = \frac{\varepsilon_0 A}{d_1} + \frac{\varepsilon_0 A}{d_2} = \boxed{\varepsilon_0 A\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}$$

47. The energy stored by a capacitor is given by Eq. 17-10, $\text{PE} = \frac{1}{2}CV^2$.

$\text{PE}_{\text{final}} = 3\text{PE}_{\text{initial}} \quad\rightarrow\quad \frac{1}{2}C_{\text{final}}V_{\text{final}}^2 = 3\frac{1}{2}C_{\text{initial}}V_{\text{initial}}^2$

One simple way to accomplish this is to have $C_{\text{final}} = 3C_{\text{initial}}$ and $V_{\text{final}} = V_{\text{initial}}$. In order to keep the voltage the same for both configurations, any additional capacitors must be connected in $\boxed{\text{parallel}}$ to the original capacitor. In order to triple the capacitance, we recognize that capacitors added in parallel add linearly. Thus if a capacitor of value $2C = \boxed{500\,\text{pF}}$ were connected in parallel to the original capacitor, the final capacitance would be 3 times the original capacitance with the same voltage, and so the potential energy would triple.

48. Capacitors in series each store the same amount of charge, and so the capacitance on the unknown capacitor is 125 pC. The voltage across the 185-pF capacitor is $V_{185} = \frac{Q_{185}}{C_{185}} = \frac{125\times10^{-12}\,\text{C}}{185\times10^{-12}\,\text{F}} = 0.676\,\text{V}$.

Thus the voltage across the unknown capacitor is $25\,\text{V} - V_{185} = 25\,\text{V} - 0.676\text{V} = 24.324\,\text{V}$. The capacitance can be calculated from the voltage across and charge on that capacitor.

$$C = \frac{Q}{V} = \frac{125\times10^{-12}\,\text{C}}{24.324\,\text{V}} = \boxed{5.14\,\text{pF}}$$

49. From Eq. 19-7, the product RC is equal to the time constant.

$$\tau = RC \quad\rightarrow\quad R = \frac{\tau}{C} = \frac{3.0\,\text{s}}{3.0\times10^{-6}\,\text{F}} = \boxed{1.0\times10^{6}\,\Omega}$$

50. (*a*) From Eq. 19-7. the product *RC* is equal to the time constant.

$$\tau = RC \rightarrow C = \frac{\tau}{R} = \frac{35.0\times10^{-6}\,\mathrm{s}}{15.0\times10^{3}\,\Omega} = \boxed{2.33\times10^{-9}\,\mathrm{F}}$$

(*b*) Since the battery has an EMF of 24.0 V, if the voltage across the resistor is 16.0 V, the voltage across the capacitor will be 8.0 V as it charges. Use the expression for the voltage across a charging capacitor.

$$V_C = \mathcal{E}\left(1-e^{-t/\tau}\right) \rightarrow e^{-t/\tau} = \left(1-\frac{V_C}{\mathcal{E}}\right) \rightarrow -\frac{t}{\tau} = \ln\left(1-\frac{V_C}{\mathcal{E}}\right) \rightarrow$$

$$t = -\tau\ln\left(1-\frac{V_C}{\mathcal{E}}\right) = -\left(35.0\times10^{-6}\,\mathrm{s}\right)\ln\left(1-\frac{8.0\,\mathrm{V}}{24.0\,\mathrm{V}}\right) = \boxed{1.42\times10^{-5}\,\mathrm{s}}$$

51. The voltage of the discharging capacitor is given by $V_C = V_0 e^{-t/RC}$. The capacitor voltage is to be $0.010V_0$.

$$V_C = V_0 e^{-t/RC} \rightarrow 0.010V_0 = V_0 e^{-t/RC} \rightarrow 0.010 = e^{-t/RC} \rightarrow \ln(0.010) = -\frac{t}{RC} \rightarrow$$

$$t = -RC\ln(0.010) = -\left(6.7\times10^{3}\,\Omega\right)\left(3.0\times10^{-6}\,\mathrm{F}\right)\ln(0.010) = \boxed{9.3\times10^{-2}\,\mathrm{s}}$$

52. (*a*) With the switch open, the resistors are in series with each other. Apply the loop rule clockwise around the left loop, starting at the negative terminal of the source, to find the current.

$$V - IR_1 - IR_2 = 0 \rightarrow I = \frac{V}{R_1+R_2} = \frac{24\,\mathrm{V}}{8.8\,\Omega+4.4\,\Omega} = 1.818\,\mathrm{A}$$

The voltage at point a is the voltage across the $4.4\,\Omega$-resistor.

$$V_a = IR_2 = (1.818\,\mathrm{A})(4.4\,\Omega) = \boxed{8.0\,\mathrm{V}}$$

(*b*) With the switch open, the capacitors are in series with each other. Find the equivalent capacitance. The charge stored on the equivalent capacitance is the same value as the charge stored on each capacitor in series.

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1}+\frac{1}{C_2} \rightarrow C_{\mathrm{eq}} = \frac{C_1C_2}{C_1+C_2} = \frac{(0.48\mu\mathrm{F})(0.24\mu\mathrm{F})}{(0.48\mu\mathrm{F}+0.24\mu\mathrm{F})} = 0.16\mu\mathrm{F}$$

$$Q_{\mathrm{eq}} = VC_{\mathrm{eq}} = (24.0\,\mathrm{V})(0.16\mu\mathrm{F}) = 3.84\mu\mathrm{C} = Q_1 = Q_2$$

The voltage at point b is the voltage across the $0.24\mu\mathrm{F}$-capacitor.

$$V_b = \frac{Q_2}{C_2} = \frac{3.84\mu\mathrm{C}}{0.24\mu\mathrm{F}} = \boxed{16\,\mathrm{V}}$$

(*c*) The switch is now closed. After equilibrium has been reached a long time, there is no current flowing in the capacitors, and so the resistors are again in series, and the voltage of point a must be 8.0 V. Point b is connected by a conductor to point a, and so point b must be at the same potential as point a, $\boxed{8.0\,\mathrm{V}}$. This also means that the voltage across C_2 is 8.0 V, and the voltage across C_1 is 16 V.

(*d*) Find the charge on each of the capacitors, which are no longer in series.

$$Q_1 = V_1C_1 = (16\,\mathrm{V})(0.48\mu\mathrm{F}) = 7.68\mu\mathrm{C}$$

$$Q_2 = V_2C_2 = (8.0\,\mathrm{V})(0.24\mu\mathrm{F}) = 1.92\mu\mathrm{C}$$

When the switch was open, point b had a net charge of 0, because the charge on the negative plate of C_1 had the same magnitude as the charge on the positive plate of C_2. With the switch closed, these charges are not equal. The net charge at point b is the sum of the charge on the negative plate of C_1 and the charge on the positive plate of C_2.

$$Q_b = -Q_1 + Q_2 = -7.68\mu\text{C} + 1.92\mu\text{C} = -5.76\mu\text{C}$$

Thus $\boxed{5.76\mu\text{C}}$ of charge has passed through the switch, from right to left.

53. The resistance is the full-scale voltage multiplied by the sensitivity.

$$R = V_{\substack{\text{full-}\\\text{scale}}}(\text{sensitivity}) = (250\,\text{V})(30{,}000\,\Omega/V) = \boxed{7.5\times10^{6}\Omega}$$

54. The full-scale current is the reciprocal of the sensitivity.

$$I_{\substack{\text{full-}\\\text{scale}}} = \frac{1}{20{,}000\,\Omega/V} = \boxed{5\times10^{-5}\text{A}} \text{ or } 50\mu\text{A}$$

55. (*a*) To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. See Figure 19-30 for a circuit diagram.

$$V_{\text{shunt}} = V_G \;\rightarrow\; (I_{\text{full}} - I_G)R_{\text{shunt}} = I_G R_G \;\rightarrow$$

$$R_{\text{shunt}} = \frac{I_G R_G}{(I_{\text{full}} - I_G)} = \frac{(50\times10^{-6}\,\text{A})(30\,\Omega)R_G}{(30\,\text{A} - 50\times10^{-6}\,\text{A})} = \boxed{5.0\times10^{-5}\,\Omega}$$

(*b*) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full-scale voltage corresponds to the full scale current of the galvanometer. See Figure 19-31 for a circuit diagram.

$$V_{\text{full scale}} = I_G(R_{\text{ser}} + R_G) \;\rightarrow\; R_{\text{ser}} = \frac{V_{\text{full scale}}}{I_G} - R_G = \frac{250\,\text{V}}{50\times10^{-6}\,\text{A}} - 30\,\Omega = \boxed{5.0\times10^{6}\,\Omega}$$

56. (*a*) The current for full-scale deflection of the galvanometer is

$$I_G = \frac{1}{\text{sensitivity}} = \frac{1}{35{,}000\,\Omega/\text{V}} = 2.857\times10^{-5}\,\text{A}$$

To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. The total current is to be 2.0 A. See Figure 19-30 for a circuit diagram.

$$I_G r_G = I_s R_s \;\rightarrow$$

$$R_s = \frac{I_G}{I_s} r_G = \frac{I_G}{I_{\text{full}} - I_G} r_G = \frac{2.857\times10^{-5}\,\text{A}}{2.0\text{A} - 2.857\times10^{-5}\,\text{A}}(20\,\Omega) = 2.857\times10^{-4}\,\Omega$$

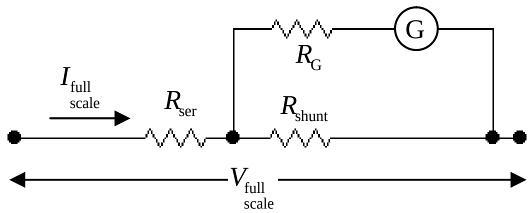
$$\approx \boxed{2.9\times10^{-4}\,\Omega \text{ in parallel}}$$

(*b*) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full scale voltage corresponds to the full scale current of the galvanometer. See Figure 19-31 for a circuit diagram. The total current must be the full-scale deflection current.

$$V_{\text{full}} = I_G(r_G + R) \quad \rightarrow$$

$$R = \frac{V_{\text{full}}}{I_G} - r_G = \frac{1.0\text{V}}{2.857\times10^{-5}\text{A}} - 20.0\,\Omega = 34981\,\Omega \approx \boxed{35\,\text{k}\Omega \text{ in series}}$$

57. To make a voltmeter, a resistor R_{ser} must be placed in series with the existing meter so that the desired full scale voltage corresponds to the full scale current of the galvanometer. We know that 10 mA produces full scale deflection of the galvanometer, so the voltage drop across the total meter must be 10 V when the current through the meter is 10 mA.



$$V_{\substack{\text{full}\\\text{scale}}} = I_{\substack{\text{full}\\\text{scale}}} R_{\text{eq}} = I_{\substack{\text{full}\\\text{scale}}}\left[R_{\text{ser}} + \left(\frac{1}{R_G} + \frac{1}{R_{\text{shunt}}}\right)^{-1}\right] \quad \rightarrow$$

$$R_{\text{ser}} = \frac{V_{\substack{\text{full}\\\text{scale}}}}{I_{\substack{\text{full}\\\text{scale}}}} - \left(\frac{1}{R_G} + \frac{1}{R_{\text{shunt}}}\right)^{-1} = \frac{10\,\text{V}}{10\times10^{-3}\text{A}} - \left(\frac{1}{30\,\Omega} + \frac{1}{0.2\,\Omega}\right)^{-1} = 999.8\,\Omega \approx \boxed{1.0\times10^3\,\Omega}$$

The sensitivity is $\frac{1.0\times10^3\,\Omega}{10\,\text{V}} = \boxed{100\,\Omega/\text{V}}$

58. If the voltmeter were ideal, then the only resistance in the circuit would be the series combination of the two resistors. The current can be found from the battery and the equivalent resistance, and then the voltage across each resistor can be found.

$$R_{\text{tot}} = R_1 + R_2 = 38\,\text{k}\Omega + 27\,\text{k}\Omega = 65\,\text{k}\Omega \qquad I = \frac{V}{R_{\text{tot}}} = \frac{45\,\text{V}}{65\times10^3\,\Omega} = 6.923\times10^{-4}\,\text{A}$$

$$V_1 = IR_1 = (6.923\times10^{-4}\,\text{A})(38\times10^3\,\Omega) = 26.31\,\text{V}$$

$$V_2 = IR_2 = (6.923\times10^{-4}\,\text{A})(27\times10^3\,\Omega) = 18.69\,\text{V}$$

Now put the voltmeter in parallel with the $38\text{ k}\Omega$ resistor. Find its equivalent resistance, and then follow the same analysis as above.

$$R_{\text{eq}} = \left(\frac{1}{38\,\text{k}\Omega} + \frac{1}{95\,\text{k}\Omega}\right)^{-1} = 27.1\,\text{k}\Omega$$

$$R_{\text{tot}} = R_{\text{eq}} + R_2 = 27.1\,\text{k}\Omega + 27\,\text{k}\Omega = 54.1\,\text{k}\Omega \qquad I = \frac{V}{R_{\text{tot}}} = \frac{45\,\text{V}}{54.1\times10^3\,\Omega} = 8.318\times10^{-4}\,\text{A}$$

$$V_1 = V_{\text{eq}} = IR_{\text{eq}} = (8.318\times10^{-4}\,\text{A})(27.1\times10^3\,\Omega) = 22.54\,\text{V} \approx \boxed{22.5\,\text{V}}$$

$$\%\text{ error} = \frac{26.31\,\text{V} - 22.54\,\text{V}}{26.31\,\text{V}}\times100 = \boxed{14\%\text{ too low}}$$

And now put the voltmeter in parallel with the $27\text{ k}\Omega$ resistor, and repeat the process.

$$R_{eq} = \left(\frac{1}{27\,k\Omega} + \frac{1}{95\,k\Omega}\right)^{-1} = 21.0\,k\Omega$$

$$R_{tot} = R_{eq} + R_1 = 21.0\,k\Omega + 38\,k\Omega = 59.0\,k\Omega \qquad I = \frac{V}{R_{tot}} = \frac{45\,V}{59.0\times10^3\,\Omega} = 7.627\times10^{-4}\,A$$

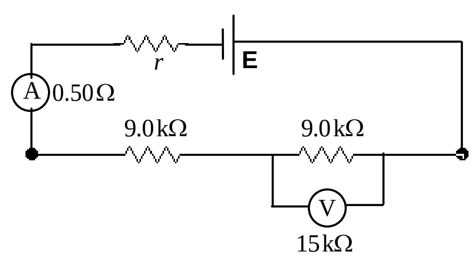
$$V_2 = V_{eq} = IR_{eq} = \left(7.627\times10^{-4}\,A\right)\left(21.0\times10^3\,\Omega\right) = 16.02\,V \approx \boxed{16.0\,V}$$

$$\%\text{ error} = \frac{18.69\,V - 16.02\,V}{18.69\,V}\times100 = \boxed{14\%\text{ too low}}$$

59. The total resistance with the ammeter present is $R_{eq} = 1293\,\Omega$. The voltage supplied by the battery is found from Ohm's law to be $V_{battery} = IR_{eq} = \left(5.25\times10^{-3}\,A\right)\left(1293\,\Omega\right) = 6.788\,V$. When the ammeter is removed, we assume that the battery voltage does not change. The equivalent resistance changes to $R'_{eq} = 1230\,\Omega$, and the new current is again found from Ohm's law.

$$I = \frac{V_{battery}}{R'_{eq}} = \frac{6.788\,V}{1230\,\Omega} = \boxed{5.52\times10^{-3}\,A}$$

60. Find the equivalent resistance for the entire circuit, and then find the current drawn from the source. That current will be the ammeter reading. The ammeter and voltmeter symbols in the diagram below are each assumed to have resistance.



$$R_{eq} = 1.0\,\Omega + 0.50\,\Omega + 9000\,\Omega + \frac{(9000\,\Omega)(15000\,\Omega)}{(9000\,\Omega + 15000\,\Omega)}$$

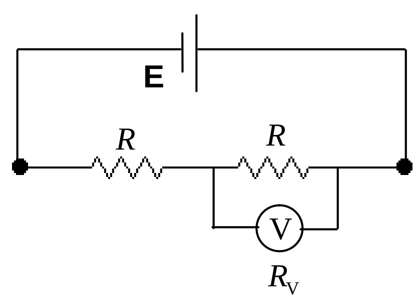
$$= 14626.5\,\Omega \approx 14630\,\Omega$$

$$I_{source} = \frac{\mathbf{E}}{R_{eq}} = \frac{12.0\,V}{14630\,\Omega} = \boxed{8.20\times10^{-4}\,A}$$

The voltmeter reading will be the source current times the equivalent resistance of the resistor–voltmeter combination.

$$V_{meter} = I_{source}R_{eq} = \left(8.20\times10^{-4}\,A\right)\frac{(9000\,\Omega)(15000\,\Omega)}{(9000\,\Omega + 15000\,\Omega)} = \boxed{4.61\,V}$$

61. The sensitivity of the voltmeter is 1000 ohms per volt on the 3.0 volt scale, so it has a resistance of 3000 ohms. The circuit is shown in the diagram. Find the equivalent resistance of the meter–resistor parallel combination and the entire circuit.



$$R_p = \left(\frac{1}{R} + \frac{1}{R_V}\right)^{-1} = \frac{R_V R}{R_V + R} = \frac{(3000\,\Omega)(9400\,\Omega)}{3000\,\Omega + 9400\,\Omega} = 2274\,\Omega$$

$$R_{eq} = R + R_p = 2274\,\Omega + 9400\,\Omega = 11674\,\Omega$$

Using the meter reading of 2.0 volts, calculate the current into the parallel combination, which is the current delivered by the battery. Use that current to find the EMF of the battery.

$$I = \frac{V}{R_p} = \frac{2.0\,\text{V}}{2274\,\Omega} = 8.795\times10^{-4}\,\text{A}$$

$$\mathcal{E} = IR_{eq} = \left(8.795\times10^{-4}\,\text{A}\right)\left(11674\,\Omega\right) = 10.27\,\text{V} \approx \boxed{10\,\text{V}}$$

62. We know from Example 19-15 that the voltage across the resistor without the voltmeter connected is 4.0 V. Thus the minimum acceptable voltmeter reading is 97% of that: (0.97)(4.0 V) = 3.88 V. The voltage across the other resistor would then be 4.12 V, which is used to find the current in the circuit.

$$I = \frac{V_2}{R_2} = \frac{4.12\,\text{V}}{15,000\,\Omega} = 2.747\times10^{-4}\,\text{A}$$

This current is used with the voltmeter reading to find the equivalent resistance of the meter–resistor combination, from which the voltmeter resistance can be found.

$$R_{comb} = \frac{V_{comb}}{I} = \frac{3.88\,\text{V}}{2.747\times10^{-4}\,\text{A}} = 14,120\,\Omega$$

$$\frac{1}{R_{comb}} = \frac{1}{R_1} + \frac{1}{R_{meter}} \quad \rightarrow \quad \frac{1}{R_{meter}} = \frac{1}{R_{comb}} - \frac{1}{R_1} \quad \rightarrow$$

$$R_{meter} = \frac{R_1 R_{comb}}{R_1 - R_{comb}} = \frac{\left(15,000\,\Omega\right)\left(14,120\,\Omega\right)}{15,000\,\Omega - 14,120\,\Omega} = 240,700\,\Omega \approx \boxed{240\,\text{k}\Omega}$$

63. By calling the voltmeter "high resistance," we can assume it has no current passing through it. Write Kirchhoff's loop rule for the circuit for both cases, starting with the negative pole of the battery and proceeding counterclockwise.

Case 1: $V_{meter} = V_1 = I_1 R_1 \qquad \mathcal{E} - I_1 r - I_1 R_1 = 0 \;\rightarrow\; \mathcal{E} = I_1\left(r + R_1\right) = \frac{V_1}{R_1}\left(r + R_1\right)$

Case 2: $V_{meter} = V_2 = I_2 R_2 \qquad \mathcal{E} - I_2 r - I_2 R_2 = 0 \;\rightarrow\; \mathcal{E} = I_2\left(r + R_2\right) = \frac{V_2}{R_2}\left(r + R_2\right)$

Solve these two equations for the two unknowns of $\mathcal{E}$ and r.

$$\mathcal{E} = \frac{V_1}{R_1}\left(r + R_1\right) = \frac{V_2}{R_2}\left(r + R_2\right) \quad \rightarrow$$

$$r = R_1 R_2 \left(\frac{V_2 - V_1}{V_1 R_2 - V_2 R_1}\right) = \left(35\,\Omega\right)\left(9.0\,\Omega\right)\left(\frac{8.1\,\text{V} - 9.7\,\text{V}}{\left(9.7\,\text{V}\right)\left(9.0\,\Omega\right) - \left(8.1\,\text{V}\right)\left(35\,\Omega\right)}\right) = 2.569\,\Omega \approx \boxed{2.6\,\Omega}$$

$$\mathcal{E} = \frac{V_1}{R_1}\left(r + R_1\right) = \frac{9.7\,\text{V}}{35\,\Omega}\left(2.569\,\Omega + 35\,\Omega\right) = 10.41\,\text{V} \approx \boxed{10.4\,\text{V}}$$

64. We can use a voltage divider circuit, as discussed in Example 19-3. The current to be passing through the body is given by Ohm's law.

$$I = \frac{V_{body}}{R_{body}} = \frac{0.25\,\text{V}}{2,000\,\Omega} = 1.25\times10^{-4}\,\text{A}$$

This is the current in the entire circuit. Use this to find the resistor to put in series with the body.

$$V_{battery} = I\left(R_{body} + R_{series}\right) \quad \rightarrow$$

$$R_{\text{series}} = \frac{V_{\text{battery}}}{I} - R_{\text{body}} = \frac{9.0\,\text{V}}{1.25\times10^{-4}\,\text{A}} - 2000\,\Omega = 70000\,\Omega = \boxed{70\,\text{k}\Omega}$$

65. (*a*) Since $P = V^2/R$ and the voltage is the same for each combination, the power and resistance are inversely related to each other. So for the $\boxed{\text{50 W output, use the higher-resistance filament}}$. For the $\boxed{\text{100 W output, use the lower-resistance filament}}$. For the $\boxed{\text{150 W output, use the filaments in parallel}}$.

(*b*) $P = V^2/R \;\rightarrow\; R = \dfrac{V^2}{P}$ $\quad R_1 = \dfrac{(120\,\text{V})^2}{50\,\text{W}} = 288\,\Omega \approx \boxed{290\,\Omega}$ $\quad R_2 = \dfrac{(120\,\text{V})^2}{100\,\text{W}} = 144\,\Omega \approx \boxed{140\,\Omega}$

As a check, the parallel combination of the resistors gives the following.

$$R_{\text{p}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(288\,\Omega)(144\,\Omega)}{288\,\Omega + 144\,\Omega} = 96\,\Omega \qquad P = \frac{V^2}{R} = \frac{(120\,\text{V})^2}{96\,\Omega} = 150\,\text{W}\,.$$

66. The voltage drop across the two wires is the 3.0 A current times their total resistance.

$$V_{\text{wires}} = IR_{\text{wires}} = (3.0\,\text{A})(0.0065\,\Omega/\text{m})(190\,\text{m})\,R_{\text{p}} = \boxed{3.7\,\text{V}}$$

Thus the voltage applied to the apparatus is $V = V_{\text{source}} - V_{\text{wires}} = 120\,\text{V} - 3.7\,\text{V} = 116.3\,\text{V} \approx \boxed{116\,\text{V}}$.

67. The equivalent resistance is the series sum of all the resistances. The current is found from Ohm's law.

$$I = \frac{\mathbf{E}}{R_{\text{eq}}} = \frac{220\,\text{V}}{3\times10^{4}\,\Omega} = 0.00733\,\text{A} \approx \boxed{7\times10^{-3}\text{A}}$$

This is about 7 milliamps, and 10 milliamps is considered to be a dangerous level in that it can cause sustained muscular contraction. The 7 milliamps could certainly be felt by the patient, and could be painful.

68. (*a*) When the capacitors are connected in parallel, their equivalent capacitance is found from Eq. 19-5. The stored energy is given by Eq. 17-10.

$$C_{\text{eq}} = C_1 + C_2 = 1.00\,\mu\text{F} \;;\; E = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}\left(1.00\times10^{-6}\,\text{F}\right)(45\,\text{V})^2 = \boxed{1.01\times10^{-3}\text{J}}$$

(*b*) When the capacitors are connected in parallel, their equivalent capacitance is found from Eq. 19-6. The stored energy is again given by Eq. 17-10.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \;\rightarrow\; C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(0.40\,\mu\text{F})(0.60\,\mu\text{F})}{1.00\,\mu\text{F}} = 0.24\,\mu\text{F}$$

$$E = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}\left(0.24\times10^{-6}\,\text{F}\right)(45\,\text{V})^2 = \boxed{2.43\times10^{-4}\text{J}}$$

(*c*) The charge is found from Eq. 17.7, $Q = CV$.

$$Q_a = \left(C_{\text{eq}}\right)_a V = (1.00\,\mu\text{F})(45\,\text{V}) = \boxed{45\,\mu\text{C}}$$

$$Q_b = \left(C_{\text{eq}}\right)_b V = (0.24\,\mu\text{F})(45\,\text{V}) = 10.8\,\mu\text{C} \approx \boxed{11\,\mu\text{C}}$$

69. The capacitor will charge up to 63% of its maximum value, and then discharge. The charging time is the time for one heartbeat.

$$t_{\text{beat}} = \frac{1\,\text{min}}{72\,\text{beats}} \times \frac{60\,\text{s}}{1\,\text{min}} = 0.8333\,\text{s}$$

$$V = V_0\left(1 - e^{-\frac{t}{RC}}\right) \rightarrow 0.63V_0 = V_0\left(1 - e^{-\frac{t_{\text{beat}}}{RC}}\right) \rightarrow e^{-\frac{t_{\text{beat}}}{RC}} = 0.37 \rightarrow \left(-\frac{t_{\text{beat}}}{RC}\right) = \ln(0.37) \rightarrow$$

$$R = -\frac{t_{\text{beat}}}{C\ln(0.37)} = -\frac{0.8333\,\text{s}}{(7.5\times10^{-6}\,\text{F})(-0.9943)} = \boxed{1.1\times10^{-5}\,\Omega}$$

70. (*a*) Apply Ohm's law to find the current.

$$I = \frac{V_{\text{body}}}{R_{\text{body}}} = \frac{110\,\text{V}}{950\,\Omega} = 0.116\,\text{A} \approx \boxed{0.12\,\text{A}}$$

(*b*) The description of "alternative path to ground" is a statement that the 45Ω path is in parallel with the body. Thus the full 110 V is still applied across the body, and so the current is the same: $\boxed{0.12\,\text{A}}$.

(*c*) If the current is limited to a total of 1.5 A, then that current will get divided between the person and the parallel path. The voltage across the body and the parallel path will be the same, since they are in parallel.

$$V_{\text{body}} = V_{\text{alterante}} \rightarrow I_{\text{body}}R_{\text{body}} = I_{\text{alternate}}R_{\text{alternate}} = (I_{\text{total}} - I_{\text{body}})R_{\text{alternate}} \rightarrow$$

$$I_{\text{body}} = I_{\text{total}}\frac{R_{\text{alternate}}}{(R_{\text{body}} + R_{\text{alternate}})} = (1.5\,\text{A})\frac{45\,\Omega}{950\,\Omega + 45\,\Omega} = 0.0678\,\text{A} \approx \boxed{68\,\text{mA}}$$

This is still a very dangerous current.

71. (*a*) If the ammeter shows no current with the closing of the switch, then points B and D must be at the same potential, because the ammeter has some small resistance. Any potential difference between points B and D would cause current to flow through the ammeter. Thus the potential drop from A to B must be the same as the drop from A to D. Since points B and D are at the same potential, the potential drop from B to C must be the same as the drop from D to C. Use these two potential relationships to find the unknown resistance.

$$V_{\text{BA}} = V_{\text{DA}} \rightarrow I_3R_3 = I_1R_1 \rightarrow \frac{R_3}{R_1} = \frac{I_1}{I_3}$$

$$V_{\text{CB}} = V_{\text{CD}} \rightarrow I_3R_x = I_1R_2 \rightarrow R_x = R_2\frac{I_1}{I_3} = \boxed{R_2\frac{R_3}{R_1}}$$

(*b*) $R_x = R_2\frac{R_3}{R_1} = (972\,\Omega)\left(\frac{42.6\,\Omega}{630\,\Omega}\right) = \boxed{65.7\,\Omega}$

72. From the solution to problem 71, the unknown resistance is given by $R_x = R_2 \dfrac{R_3}{R_1}$. We use that with Eq. 18-3 to find the length of the wire.

$$R_x = R_2 \frac{R_3}{R_1} = \frac{\rho L}{A} = \frac{\rho L}{\pi (d/2)^2} = \frac{4\rho L}{\pi d^2} \rightarrow$$

$$L = \frac{R_2 R_3 \pi d^2}{4 R_1 \rho} = \frac{(46.0\,\Omega)(3.48\,\Omega)\pi (9.20\times 10^{-4}\,\text{m})^2}{4(38.0\,\Omega)(10.6\times 10^{-8}\,\Omega\bullet\text{m})} = \boxed{26.4\,\text{m}}$$

73. There are eight values of effective capacitance that can be obtained from the four capacitors.

All four in parallel: $C_{eq} = C + C + C + C = \boxed{4C}$

All four in series: $C_{eq} = \left(\frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}\right)^{-1} = \boxed{\tfrac{1}{4}C}$

(Three in parallel) in series with one:

$$\frac{1}{C_{eq}} = \frac{1}{C+C+C} + \frac{1}{C} = \frac{1}{3C} + \frac{1}{C} = \frac{4}{3C} \rightarrow C_{eq} = \boxed{\tfrac{3}{4}C}$$

(Two in parallel) in series with (two in parallel):

$$\frac{1}{C_{eq}} = \frac{1}{C+C} + \frac{1}{C+C} = \frac{1}{2C} + \frac{1}{2C} = \frac{2}{2C} \rightarrow C_{eq} = \boxed{C}$$

(Two in parallel) in series with (two in series)

$$\frac{1}{C_{eq}} = \frac{1}{C+C} + \frac{1}{C} + \frac{1}{C} = \frac{1}{2C} + \frac{2}{C} = \frac{5}{2C} \rightarrow C_{eq} = \boxed{\tfrac{2}{5}C}$$

(Two in series) in parallel with (two in series)

$$C_{eq} = \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} + \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} = \left(\frac{2}{C}\right)^{-1} + \left(\frac{2}{C}\right)^{-1} = \frac{C}{2} + \frac{C}{2} = C \quad \text{not a new value}$$

(Three in series) in parallel with one.

$$C_{eq} = \left(\frac{1}{C} + \frac{1}{C} + \frac{1}{C}\right)^{-1} + C = \left(\frac{3}{C}\right)^{-1} + C = \boxed{\tfrac{4}{3}C}$$

(Two in series) in parallel with (two in parallel)

$$C_{eq} = \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} + C + C = \left(\frac{2}{C}\right)^{-1} + 2C = \boxed{\tfrac{5}{2}C}$$

((Two in series) in parallel with one) in series with one

$$\frac{1}{C_{eq}} = \left[\left(\frac{1}{C} + \frac{1}{C}\right)^{-1} + C\right]^{-1} + \frac{1}{C} = \left[\frac{C}{2} + C\right]^{-1} + \frac{1}{C} = \frac{2}{3C} + \frac{1}{C} = \frac{5}{3C} \rightarrow C_{eq} = \boxed{\tfrac{3}{5}C}$$

74. (*a*) From the diagram, we see that one group of 4 plates is connected together, and the other group of 4 plates is connected together. This common grouping shows that the capacitors are connected $\boxed{\text{in parallel}}$.

(*b*) Since they are connected in parallel, the equivalent capacitance is the sum of the individual capacitances. The variable area will change the equivalent capacitance.

$$C_{eq} = 7C = 7\varepsilon_0 \frac{A}{d}$$

$$C_{min} = 7\varepsilon_0 \frac{A_{min}}{d} = 7\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\frac{\left(2.0\times10^{-4}\,\text{m}^2\right)}{\left(1.5\times10^{-3}\,\text{m}\right)} = 8.3\times10^{-12}\,\text{F}$$

$$C_{max} = 7\varepsilon_0 \frac{A_{max}}{d} = 7\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\frac{\left(9.5\times10^{-4}\,\text{m}^2\right)}{\left(1.5\times10^{-3}\,\text{m}\right)} = 3.9\times10^{-11}\,\text{F}$$

And so the range is $\boxed{\text{from } 8.3\,\text{pF to } 39\,\text{pF}}$.

75. The terminal voltage and current are given for two situations. Apply Eq. 19-1 to both of these situations, and solve the resulting two equations for the two unknowns.

$$V_1 = \mathcal{E} - I_1 r \;;\; V_2 = \mathcal{E} - I_2 r \;\rightarrow\; \mathcal{E} = V_1 + I_1 r = V_2 + I_2 r \;\rightarrow$$

$$r = \frac{V_2 - V_1}{I_1 - I_2} = \frac{47.3\,\text{V} - 40.8\,\text{V}}{7.40\,\text{A} - 2.20\,\text{A}} = \boxed{1.25\,\Omega} \;;\; \mathcal{E} = V_1 + I_1 r = 40.8\,\text{V} + \left(7.40\,\text{A}\right)\left(1.25\,\Omega\right) = \boxed{50.1\,\text{V}}$$

76. One way is to connect N resistors in series. If each resistor watt can dissipate 0.5 W, then it will take 7 resistors in series to dissipate 3.5 W. Since the resistors are in series, each resistor will be 1/7 of the total resistance.

$$R = \frac{R_{eq}}{7} = \frac{2200\,\Omega}{7} = 314\,\Omega \approx 310\,\Omega$$

So connect $\boxed{7\;\; 310\,\Omega \text{ resistors, each rated at ½ W, in series}}$.

Or, the resistors could be connected in parallel. Again, if each resistor watt can dissipate 0.5 W, then it will take 7 resistors in parallel to dissipate 3.5 W. Since the resistors are in parallel, the equivalent resistance will be 1/7 of each individual resistance.

$$\frac{1}{R_{eq}} = 7\left(\frac{1}{R}\right) \;\rightarrow\; R = 7R_{eq} = 7\left(2200\,\Omega\right) = 15.4\,\text{k}\Omega$$

So connect $\boxed{7\;\; 15.4\,\text{k}\Omega \text{ resistors, each rated at ½ W, in parallel}}$.

77. There are two answers because it is not known which direction the given current is flowing through the $4.0\,\text{k}\Omega$ resistor.

Assume the current is to the right. The voltage across the $4.0\,\text{k}\Omega$ resistor is given by Ohm's law as $V = IR = \left(3.50\times10^{-3}\,\text{A}\right)\left(4000\,\Omega\right) = 14\,\text{V}$. The voltage drop across the $8.0\,\text{k}\Omega$ must be the same, and the current through it is $I = \frac{V}{R} = \frac{14\,\text{V}}{8000\,\Omega} = 1.75\times10^{-3}\,\text{A}$. The total current in the circuit is the sum of the two currents, and so $I_{tot} = 5.25\times10^{-3}\,\text{A}$. That current can be used to find the terminal voltage of the battery. Write a loop equation, starting at the negative terminal of the unknown battery and going clockwise.

$$V_{ab} - \left(5000\,\Omega\right)I_{tot} - 14.0\,\text{V} - 12.0\,\text{V} - \left(1.0\,\Omega\right)I_{tot} \;\rightarrow$$

$$V_{ab} = 26.0\text{V} + \left(5001\Omega\right)\left(5.25\times10^{-3}\,\text{A}\right) = \boxed{52.3\,\text{V}}$$

If the current is to the left, then the voltage drop across the parallel combination of resistors is still 14.0 V, but with the opposite orientation. Again write a loop equation, starting at the negative terminal of the unknown battery and going clockwise. The current is now to the left.

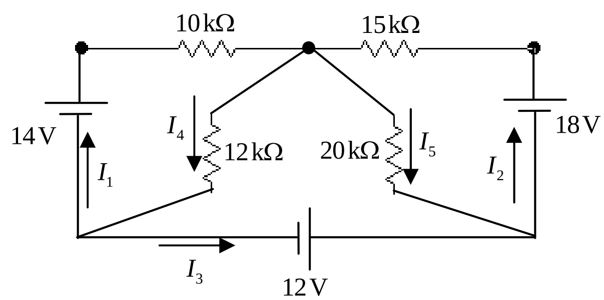
$$V_{ab} + (5000\,\Omega) I_{tot} + 14.0\,\text{V} - 12.0\,\text{V} + (1.0\,\Omega) I_{tot} \rightarrow$$
$$V_{ab} = -2.0\text{V} - (5001\Omega)(5.25\times10^{-3}\,\text{A}) = \boxed{-28.3\,\text{V}}$$

78. The potential difference is the same on each half of the capacitance, so it can be treated as two capacitors in parallel. Let the original plate area be *A* and the distance between the plates be *d*. Each of the "new" capacitors has half the area of the original capacitor.

$$C_0 = \varepsilon_0 \frac{A}{d} \qquad C_1 = K_1\varepsilon_0 \frac{A/2}{d} \qquad C_2 = K_2\varepsilon_0 \frac{A/2}{d}$$

$$C_{new} = C_1 + C_2 = K_1\varepsilon_0 \frac{A/2}{d} + K_2\varepsilon_0 \frac{A/2}{d} = \varepsilon_0 \frac{A}{d}\left(\frac{K_1}{2} + \frac{K_2}{2}\right) = \boxed{C_0\left(\frac{K_1+K_2}{2}\right)}$$

79. There are three loops, and so there are three loop equations necessary to solve the circuit. We identify five currents in the diagram, and so we need two junction equations to complete the analysis.



Lower left junction: $I_4 = I_1 + I_3$

Lower right junction: $I_2 = I_3 + I_5$

Left loop, clockwise: $14\,\text{V} - 10000 I_1 - 12000 I_4 = 0$

Right loop, counterclockwise: $18\,\text{V} - 15000 I_2 - 20000 I_5 = 0$

Bottom loop, counterclockwise: $12\,\text{V} + 20000 I_5 - 12000 I_4 = 0$

$$14 - 10000 I_1 - 12000(I_1 + I_3) = 0 \rightarrow 14 - 22000 I_1 - 12000 I_3 = 0$$
$$18 - 15000(I_3 + I_5) - 20000 I_5 = 0 \rightarrow 18 - 15000 I_3 - 35000 I_5 = 0$$
$$12 + 20000 I_5 - 12000(I_1 + I_3) = 0 \rightarrow 12 - 12000 I_1 - 12000 I_3 + 20000 I_5 = 0$$

$$I_1 = \frac{14 - 12000 I_3}{22000} \qquad I_5 = \frac{18 - 15000 I_3}{35000}$$

$$12 - 12000\left[\frac{14 - 12000 I_3}{22000}\right] - 12000 I_3 + 20000\left[\frac{18 - 15000 I_3}{35000}\right] = 0$$

$$12 - \tfrac{12}{22}(14 - 12000 I_3) - 12000 I_3 + \tfrac{20}{35}[18 - 15000 I_3] = 0$$

$$12 - \tfrac{6}{11}14 + \tfrac{4}{7}18 = \left(12000 + \tfrac{4}{7}15000 - \tfrac{6}{11}12000\right) I_3$$

$$14.65 = 14026 I_3 \rightarrow I_3 = \frac{14.65}{14026} = 1.044\times10^{-3}\,\text{A}$$

(*a*) $I_1 = \dfrac{14 - 12000 I_3}{22000} = \dfrac{14 - 12000(1.044\times10^{-3})}{22000} = \boxed{6.7\times10^{-5}\,\text{A, upward}}$

(*b*) Reference all voltages to the lower left corner of the circuit diagram. Then $V_a = 14\,\text{V}$, $V_b = 12\,\text{V} + 18\,\text{V} = 30\,\text{V}$, and $V_a - V_b = 14\,\text{V} - 30\,\text{V} = \boxed{-16\,\text{V}}$.

80. To build up a high voltage, the cells will have to be put in series. 120 V is needed from a series of 0.80 V cells. Thus $\frac{120\,\text{V}}{0.80\,\text{V/cell}} = 150\text{ cells}$ are needed to provide the desired voltage. Since these cells are all in series, their current will all be the same at 350 mA. To achieve the higher current desired, banks made of 150 cells each can be connected in parallel. Then their voltage will still be at 120 V, but the currents would add making a total of $\frac{1.00\,\text{A}}{350\times10^{-3}\,\text{A/bank}} = 2.86\text{ banks} \approx 3\text{ banks}$. So the total number of cells is $\boxed{450\text{ cells}}$. The panel area is $450\text{ cells}\left(9.0\times10^{-4}\,\text{m}^2/\text{cell}\right) = \boxed{0.405\,\text{m}^2}$. The cells should be wired in $\boxed{\text{3 banks of 150 cells in series per bank, with the banks in parallel}}$. This will produce 1.05 A at 120 V. To optimize the output, $\boxed{\text{always have the panel pointed directly at the sun}}$.

81. (*a*) If the terminal voltage is to be 3.0 V, then the voltage across R_1 will be 9.0 V. This can be used to find the current, which then can be used to find the value of R_2.

$$V_1 = IR_1 \quad \rightarrow \quad I = \frac{V_1}{R_1} \qquad V_2 = IR_2 \quad \rightarrow$$

$$R_2 = \frac{V_2}{I} = R_1\frac{V_2}{V_1} = (10.0\,\Omega)\frac{3.0\,\text{V}}{9.0\,\text{V}} = 3.333\,\Omega \approx \boxed{3.3\,\Omega}$$

(*b*) If the load has a resistance of $7.0\,\Omega$, then the parallel combination of R_2 and the load must be used to analyze the circuit. The equivalent resistance of the circuit can be found and used to calculate the current in the circuit. Then the terminal voltage can be found from Ohm's law, using the parallel combination resistance.

$$R_{2+\text{load}} = \frac{R_2R_{\text{load}}}{R_2+R_{\text{load}}} = \frac{(3.33\,\Omega)(7.0\,\Omega)}{10.33\,\Omega} = 2.26\,\Omega \qquad R_{eq} = 2.26\,\Omega + 10.0\,\Omega = 12.26\,\Omega$$

$$I = \frac{V}{R_{\text{eq}}} = \frac{12.0\,\text{V}}{12.26\,\Omega} = 0.9788\,\text{A} \qquad V_{\text{T}} = IR_{2+\text{load}} = (0.9788\,\text{A})(2.26\,\Omega) = 2.21\,\text{V} \approx \boxed{2.2\,\text{V}}$$

The presence of the load has affected the terminal voltage significantly.

82. (*a*) The light will first flash when the voltage across the capacitor reaches 90.0 V.

$$V = \mathcal{E}_0\left(1 - e^{-\frac{t}{RC}}\right)$$

$$t = -RC\ln\left(1 - \frac{V}{\mathcal{E}_0}\right) = -\left(2.35\times10^6\,\Omega\right)\left(0.150\times10^{-6}\,\text{F}\right)\ln\left(1 - \frac{90}{105}\right) = \boxed{0.686\,\text{s}} \quad \rightarrow$$

(*b*) We see from the equation that $t \propto R$, and so if R increases, the $\boxed{\text{time will increase}}$.

(*c*) The capacitor discharges through a very low resistance (the lamp), and so the discharge time constant is very short. Thus the flash is very brief.

(*d*) Once the lamp has flashed, the stored energy in the capacitor is gone, and there is no source of charge to maintain the lamp current. The lamp "goes out", the lamp resistance increases, and the capacitor starts to recharge. It charges again for about 0.686 seconds, and the process will repeat.

83. If the switches are both open, then the circuit is a simple series circuit. Use Kirchhoff's loop rule to find the current in that case.

$$6.0\text{ V} - I\left(50\,\Omega + 20\,\Omega + 10\,\Omega\right) = 0 \quad \rightarrow \quad I = \frac{6.0\text{ V}}{80\,\Omega} = 0.075\text{ A}$$

If the switches are both closed, the 20-Ω resistor is in parallel with R. Apply Kirchhoff's loop rule to the outer loop of the circuit, with the 20-Ω resistor having the current found previously.

$$6.0\text{ V} - I\left(50\,\Omega\right) - \left(0.075\text{ A}\right)\left(20\,\Omega\right) = 0 \quad \rightarrow \quad I = \frac{6.0\text{ V} - \left(0.075\text{ A}\right)\left(20\,\Omega\right)}{50\,\Omega} = 0.090\text{ A}$$

This is the current that flows into the parallel combination. Since 0.075 A is in the 20-Ω resistor, 0.015 A must be in R. The voltage drops across R and the 20-Ω resistor must be the same, since they are in parallel.

$$V_{20} = V_R \quad \rightarrow \quad I_{20}R_{20} = I_R R \quad \rightarrow \quad R = R_{20}\frac{I_{20}}{I_R} = \left(20\,\Omega\right)\frac{0.075\text{ A}}{0.015\text{ A}} = \boxed{100\,\Omega}$$

84. The current in the circuit can be found from the resistance and the power dissipated. Then the product of that current and the equivalent resistance is equal to the battery voltage.

$$P = I^2R \quad \rightarrow \quad I = \sqrt{\frac{P_{33}}{R_{33}}} = \sqrt{\frac{0.50\text{ W}}{33\,\Omega}} = 0.123\text{ A}$$

$$R_{\text{eq}} = 33\,\Omega + \left(\frac{1}{68\,\Omega} + \frac{1}{75\,\Omega}\right)^{-1} = 68.88\,\Omega \qquad V = IR_{\text{eq}} = \left(0.123\text{ A}\right)\left(68.88\,\Omega\right) = 8.472\text{ V} \approx \boxed{8.5\text{ V}}$$

85. (*a*) The 12-Ω and the 30-Ω resistors are in parallel, with a net resistance $R_{1\text{-}2}$ as follows.

$$R_{1\text{-}2} = \left(\frac{1}{12\,\Omega} + \frac{1}{30\,\Omega}\right)^{-1} = 8.57\,\Omega$$

$R_{1\text{-}2}$ is in series with the 4.5-Ω resistor, for a net resistance $R_{1\text{-}2\text{-}3}$ as follows.

$$R_{1\text{-}2\text{-}3} = 4.5\,\Omega + 8.57\,\Omega = 13.07\,\Omega$$

That net resistance is in parallel with the 18-Ω resistor, for a final equivalent resistance as follows.

$$R_{\text{eq}} = \left(\frac{1}{13.07\,\Omega} + \frac{1}{18\,\Omega}\right)^{-1} = 7.57\,\Omega \approx \boxed{7.6\,\Omega}$$

(*b*) Find the current in the 18-Ω resistor by using Kirchhoff's loop rule for the loop containing the battery and the 18-Ω resistor.

$$\mathsf{E} - I_{18}R_{18} = 0 \quad \rightarrow \quad I_{18} = \frac{\mathsf{E}}{R_{18}} = \frac{6.0\text{ V}}{18\,\Omega} = \boxed{0.33\text{ A}}$$

(*c*) Find the current in $R_{1\text{-}2}$ and the 4.5-Ω resistor by using Kirchhoff's loop rule for the outer loop containing the battery and the resistors $R_{1\text{-}2}$ and the 4.5-Ω resistor.

$$\mathsf{E} - I_{1\text{-}2}R_{1\text{-}2} - I_{1\text{-}2}R_{4.5} = 0 \quad \rightarrow \quad I_{1\text{-}2} = \frac{\mathsf{E}}{R_{1\text{-}2} + R_{4.5}} = \frac{6.0\text{ V}}{13.07\,\Omega} = 0.459\text{ A}$$

This current divides to go through the 12-Ω and 30-Ω resistors in such a way that the voltage drop across each of them is the same. Use that to find the current in the 12-Ω resistor.

$$I_{1\text{-}2} = I_{12} + I_{30} \quad \rightarrow \quad I_{30} = I_{1\text{-}2} - I_{12}$$

$$V_{R_{12}} = V_{R_{30}} \quad \rightarrow \quad I_{12}R_{12} = I_{30}R_{30} = \left(I_{1\text{-}2} - I_{12}\right)R_{30} \quad \rightarrow$$

$$I_{12} = I_{1\text{-}2}\frac{R_{30}}{\left(R_{12} + R_{30}\right)} = \left(0.459\,\text{A}\right)\frac{30\,\Omega}{42\,\Omega} = \boxed{0.33\,\text{A}}$$

(*d*) The current in the 4.5-Ω resistor was found above to be $I_{1\text{-}2} = 0.459\,\text{A}$. Find the power accordingly.

$$P_{4.5} = I_{1\text{-}2}^2 R_{4.5} = \left(0.459\,\text{A}\right)^2\left(4.5\,\Omega\right) = \boxed{0.95\,\text{W}}$$

86. (*a*) We assume that the ammeter is ideal and so has 0 resistance, but that the voltmeter has resistance R_V. Then apply Ohm's law, using the equivalent resistance. We also assume the voltmeter is accurate, and so it is reading the voltage across the battery.

$$V = IR_\text{eq} = I\frac{1}{\frac{1}{R} + \frac{1}{R_\text{V}}} \quad \rightarrow \quad V\left(\frac{1}{R} + \frac{1}{R_\text{V}}\right) = I \quad \rightarrow \quad \frac{1}{R} + \frac{1}{R_\text{V}} = \frac{I}{V} \quad \rightarrow \quad \boxed{\frac{1}{R} = \frac{I}{V} - \frac{1}{R_\text{V}}}$$

(*b*) We now assume the voltmeter is ideal, and so has an infinite resistance, but that the ammeter has resistance R_A. We also assume that the voltmeter is accurate and so is reading the voltage across the battery.

$$V = IR_\text{eq} = I\left(R + R_\text{A}\right) \quad \rightarrow \quad R + R_\text{A} = \frac{V}{I} \quad \rightarrow \quad \boxed{R = \frac{V}{I} - R_\text{A}}$$

87. Write Kirchhoff's loop rule for the circuit, and substitute for the current and the bulb resistance based on the bulb ratings.

$$P_\text{bulb} = \frac{V_\text{bulb}^2}{R_\text{bulb}} \quad \rightarrow \quad R_\text{bulb} = \frac{V_\text{bulb}^2}{P_\text{bulb}} \qquad P_\text{bulb} = I_\text{bulb}V_\text{bulb} \quad \rightarrow \quad I_\text{bulb} = \frac{P_\text{bulb}}{V_\text{bulb}}$$

$$\mathsf{E} - I_\text{bulb}R - I_\text{bulb}R_\text{bulb} = 0 \quad \rightarrow$$

$$R = \frac{\mathsf{E}}{I_\text{bulb}} - R_\text{bulb} = \frac{\mathsf{E}}{P_\text{bulb}/V_\text{bulb}} - \frac{V_\text{bulb}^2}{P_\text{bulb}} = \frac{V_\text{bulb}}{P_\text{bulb}}\left(\mathsf{E} - V_\text{bulb}\right) = \frac{3.0\,\text{V}}{2.5\,\text{W}}\left(9.0\,\text{V} - 3.0\,\text{V}\right) = \boxed{7.2\,\Omega}$$