# CHAPTER 23: Light: Geometric Optics

## Answers to Questions

1. (*a*) With a rough surface, the Moon would look just like it does now, because it has a rough surface. During the times of the month that we can see portions of the lit side, we see all parts of it reflecting back sunlight to us. Each and every spot is basically a point source of light, sending us reflections from any part that is lit by sunlight. This produces a relatively uniform color and intensity from all of the lit parts that we can see.

   (*b*) With a polished, mirror-like surface, the Moon would be mostly invisible to us, except for the one small spot that reflects an image of the Sun to us. We would also be able to see tiny bright dots scattered about on the Moon, as the light from stars is reflected back to us. We would also be able to see a small reflection of the lit portion of Earth at one spot. (This actually occurs right now in a diffuse way: Sunlight reflected from Earth to the Moon and then back to us gives an effect called "Earth shine," which can best be seen each month right after New Moon.)

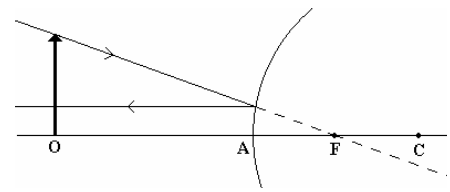
2. No, it is not reasonable to be able to start all of the ships in a harbor on fire at once with a spherical mirror. It may be possible to start them on fire one at a time, though. Archimedes could focus the Sun's rays down to a single point with a spherical mirror, which would get this one point hot enough to start a fire, but it is not possible to do this over a large area like an entire harbor. The other problem is that the spherical mirror he had would only have one focal length, so he would have to be the same distance from each ship that he started on fire. If one ship was farther away than another, then he would have to move closer to the harbor to start the farther ship on fire. This would take a long time and he would have to move his mirror over large distances just to start the ships on fire one at a time. This would also require a very precisely ground and polished mirror that is huge and has an extremely long focal length, which would have been difficult in Archimedes time.

3. The mirror actually doesn't reverse right and left, either, just as it doesn't reverse up and down. When you are looking in a flat mirror and move your right hand, it is the image of your right hand that moves in the mirror. When we see our image, though, we imagine it as if it is another person looking back at us. If we saw this other person raise their left hand, we would see the hand that is on the right side of their body (from our point of view) move.

4. Yes. When a concave mirror produces a real image of a real object, this means $d_i$ and $d_o$ are positive. The magnification equation $\left(m = -d_i/d_o\right)$ then says that $m$ is negative, which means the image is inverted. (If a virtual object is used, though, to make a real image, then $d_o$ is negative and $d_i$ is positive, which would give us a positive $m$ and an upright image.)

5. When an object is placed in front of a spherical mirror and the resulting magnification is -3.0, the image is real and inverted. A negative magnification tells us that the image is inverted, when compared to the object. For a real object, the object distance is positive. Since the magnification is negative, the magnification equation $\left(m = -d_i/d_o\right)$ says that the image distance is also positive. The mirror is concave. A convex mirror cannot make an image of a real object that is magnified. The image is located on the reflecting side of the mirror. When the image distance is positive, as it is in this situation, the image is always located on the reflecting (real) side of the mirror.

6. Ray 2 starts at the tip of the arrow (object) in front of the mirror and heads toward F (the focal point) on the other side of the mirror. When the ray encounters the reflecting surface of the mirror, it bounces back away from the mirror and parallel to the axis.

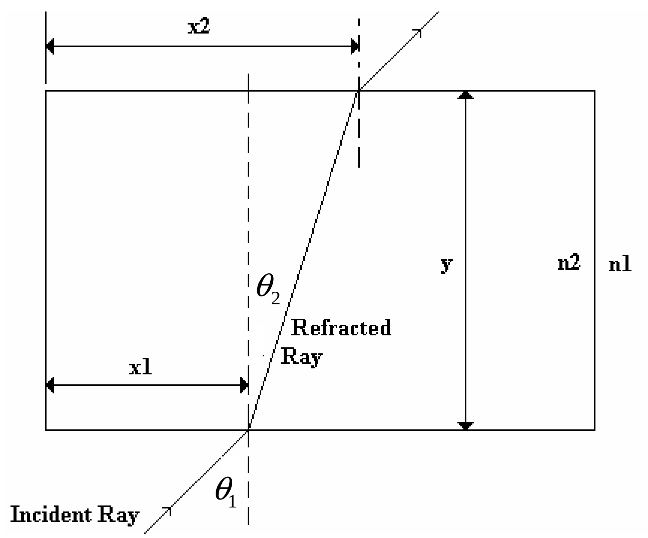



7. The focal length of a plane mirror is infinity. The magnification of a plane mirror is +1. As a spherical mirror gets a larger and larger radius, the front surface gets more and more flat. The ultimate limit is that as the radius (and focal length) of the spherical mirror goes to infinity, the front surface becomes perfectly flat. For this mirror, the image height and object height are identical (as are the image distance and object distance) and the image is virtual, with a magnification of $m = +1$.

8. This effect is similar to diffuse reflection off of a rough surface. The ripply sea provides many different surface orientations, and so the light from the Moon reflects from that surface at many different angles. This makes it look like there is one continuous reflection along the surface of the water between you and the Moon, instead of just one specular reflection spot.

9. When a light ray meets the boundary between two materials perpendicularly, both the angle of incidence and the angle of refraction are $0^\circ$. Snell's law shows this to be true for any combination of the indices of refraction of the two materials.

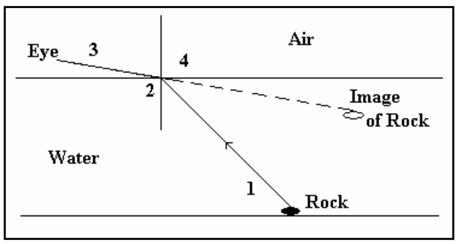
10. To determine the speed of light in a solid, transparent, rectangular object you would need to measure the incident and refracted angles. To do this, start by aiming a laser beam at a convenient angle at one side of the block $(\theta_1)$. Measure the distance from one end of the block to the point where the beam enters the block $(x_1)$, measure the thickness of the block $(y)$, and measure the distance from the same end of the block to the point where the beam exits the block $(x_2)$. Using geometry, the refracted angle is $\tan^{-1}\frac{x_2 - x_1}{y}$. Use Snell's law to find the index of refraction of the block: $n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2}$. The index of refraction for air $(n_1)$ is basically equal to 1. Then, we can finally use $v_2 = \frac{c}{n_2}$ to find the speed of light in the block.

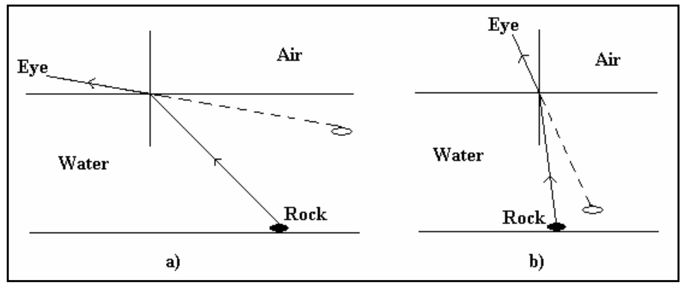



11. When you look down into a lake or pool, you will underestimate its depth. Light that is shining on the bottom of the pool will reflect off diffusely in all directions. When these light rays reach the surface, they will be bent away from the normal, according to Snell's law. Some of these rays go to your eyes, which means that you can "see" the bottom of the pool. Your brain interprets these rays as if they all traveled in straight lines only and did not bend, and so you "see" the bottom of the pool as shallower

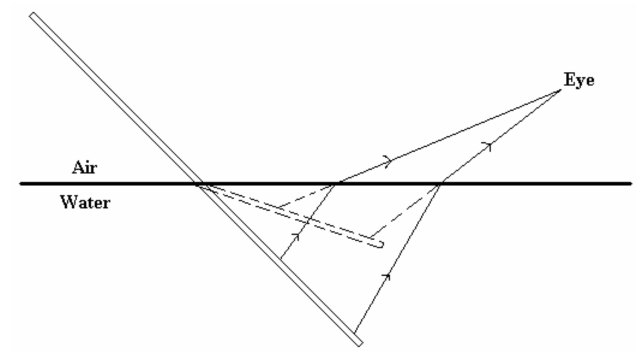
than it actually is. In the first diagram: 1 – Light leaves the rock on the bottom of the pool. 2 – Light bends away from the normal as it leaves the water and enters the air. 3 – The light enters your eye. 4 – Your brain recreates the ray's path back to a shallower point.

As the viewing angle gets large (when the angle of refraction increases), the pool appears to get more shallow. Think about viewing a submerged object from two different angles: a) from nearly straight above and b) from a large viewing angle, in the second diagram. Your brain "draws" a very shallow angle into the water when the viewing angle is large, as compared to when the viewing angle is small, due to Snell's law.

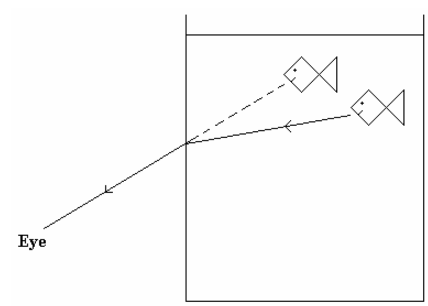



12. Your brain interprets the bending rays as if that the part of the stick that is underwater is shallower than it really is, so it looks bent. Each point on the stick that is underwater appears to you to be closer to the surface than it actually is.




13. The light rays leaving the fish bend away from the normal as they exit the tank and go into your eye. Thus, you see the image of the fish "above" (in this figure) the position of the real fish. Also, the image appears to be slightly closer to the wall of the aquarium. Since your brain doesn't know that light bends at the water/air interface, when your brain "follows" the ray back into the tank, it interprets the location of the fish as above its actual location.




14. One reason you can see a drop of transparent and colorless water on a surface is refraction. The light leaving the tabletop and going through the drop on the way to your eye will make that portion of the tabletop seem to be raised up, due to depth distortion and/or magnification due to the round drop. A second reason that you can see the drop is reflection. At certain angles, the light sources in the room will reflect off of the drop and give its location away.

15. When you are underwater and looking up out of the water at objects above, they all seem much smaller than they would look directly in air. As Conceptual Example 23-8 says, you would see the entire outside world compressed into a circle whose edge makes a $49^{o}$ angle with the vertical. This compression will make objects above the water look smaller than they really are. See Figure 23-25.

16. A spherical mirror can have negative object distance if some other system of lenses and/or mirrors forms an image that lies behind the spherical mirror – that is, the image of the other optics lies on the non-reflecting side of the spherical mirror. Then, since the object for our spherical mirror is on the back side of the mirror, the convention is to label this a negative object distance.

17. (*a*) The fact that light rays from stars (including our Sun) bend toward the vertical direction as they pass through Earth's atmosphere makes sense because the index of refraction of the atmosphere is slightly greater than that of the vacuum of space. Thus, as the light rays from stars enter the atmosphere, they slow down and bend toward the vertical, according to Snell's law.
    (*b*) The apparent positions of stars in the sky are too high when compared to actual positions. Since the light rays bend toward the vertical as they come into the atmosphere and then into our eyes, as we "follow" these rays back into space, which doesn't take the bending into account, the image of the stars appears higher than the actual position.

$\boxed{18.}$ To make a sharp image of an object that is very far away, the film of a camera must be placed at the focal length of the lens. Objects that are very far away have light rays coming into the camera that are basically parallel and these rays will be bent and focused to create an image at the focal point of the lens, which is where the film should be in order to record a focused image.

19. The mirror in Figure 23-46 is concave. The person is standing within the focal length of the mirror, which makes an upright and larger virtual image for a concave mirror. See Figure 23-16 and Example 23-3 to see a similar situation.

20. The camera lens moves farther from the film when the photographer refocuses. By moving closer to the object, the object distance has become smaller, which the thin lens equation says increases the image distance (and the image is now behind the film). To bring the image forward (and back to the film), you need to move the lens away from the film.

21. A diverging lens can actually form a real image if some other piece of optics forms a virtual object for the diverging lens. For a real object, though, a diverging lens cannot create a real image.

22. For a converging lens, with $d_o > f$, a real image will always be inverted, since the rays must cross the optical axis to create the image. See Figure 23-34 or Figure 23-37. For a converging lens, with $d_o < f$, a virtual image will always be upright, since the rays stay on the same side of the optical axis to create the image. See Figure 23-40. For a diverging lens, anywhere you put $d_o$, a virtual image will always be upright, since the rays stay on the same side of the optical axis to create the image. See Figure 23-36 or Figure 23-38.

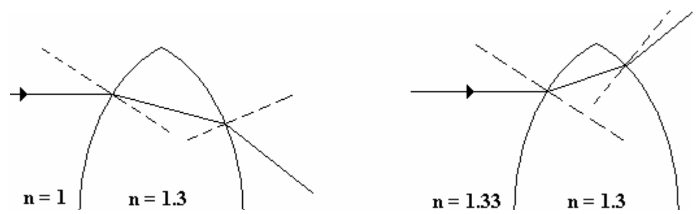
23. Yes, to say that light rays are "reversible" is consistent with the thin lens equation. Since $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$, if you interchange the image and object positions, the equation is unchanged.

24. Real images can be projected onto a screen. In this situation, real light rays actually focus at the position of the image, which can be seen on the screen. But virtual images cannot be projected onto a screen. In this situation, no real light rays are crossing at the position of the virtual image, and so they cannot be seen on a screen (recall a plane mirror's virtual image is actually behind the mirror, where no real light rays travel). Both real images and virtual images can be photographed. The lenses in the camera are

designed to focus either diverging or converging light rays down onto the film (similar to a human eye, which can also focus either diverging or converging light rays down onto our "film").

25. (*a*) Yes, as a thin converging lens is moved closer to a nearby object, the position of the real image changes. In this situation, the object distance decreases and the thin lens equation says that the image distance increases. Thus, the position of the image moves farther away from the lens.
    (*b*) Yes, as a thin converging lens is moved closer to a nearby object, the size of the real image changes. In this situation, the object distance decreases and the thin lens equation says that the image distance increases, which means that the magnification $\left(m = -d_i/d_o\right)$ gets larger. Thus, the size of the image increases.

26. No. If a lens with $n$ = 1.3 is a converging lens in air, it will become a diverging lens when placed in water, with $n$ = 1.33. The figure on the left shows that as parallel light rays enter the lens when it is in air, at the first surface the ray is bent toward the normal and at the second surface the ray is bent away from the normal, which is a net converging situation. The figure on the right shows that as parallel light rays enter the lens when it is in water, at the first surface the ray is bent away from the normal and at the second surface the ray is bent toward the normal, which is a net diverging situation.




27. The nose of the dog will be magnified more than the tail. The dog's nose is closer to the converging lens, so the thin lens equation says that this smaller object distance will create a larger image distance. The magnification equation $\left(m = -d_i/d_o\right)$, in turn, tells us that with a smaller $d_o$ and a larger $d_i$, the nose will have a larger magnification than the tail.

28. If the nose of the cat was closer to the converging lens than the focal point while the tail was farther away than the focal point, then the image of the nose would be virtual and the image of the tail would be real. The real image of the rear parts of the cat (from the tip of its tail to the focal point of the lens) will be spread out all the way to infinity on the other side of the lens. The front parts of the cat (from the focal point to the tip of its nose) will be spread out from infinity on the virtual side of the lens to nearly the focal point.

29. If the converging lens does not have a shorter focal length than the diverging lens, then the converging lens' image (which will be a virtual object for the diverging lens) will be too far away from the diverging lens and it will just form another virtual image. This situation does us no good in determining the focal length of the diverging lens, since we can't measure the distance from the lens to a virtual image (we can't project it on to a screen, for example).

30. You can have a virtual object any time that one lens creates a virtual image that another lens uses as an object.

31. No, the image point of an unsymmetrical lens does not change if you turn the lens around. The lensmaker's equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, is symmetrical (even if the lens is not). Turning the lens around interchanges $R_1$ and $R_2$, which does not change the focal length or the image point.

32. To make a lens fatter in the middle, make the radii of the two convex sides smaller. Consider the lensmaker's equation: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$. If $R_1$ and $R_2$ decrease, the focal length decreases.

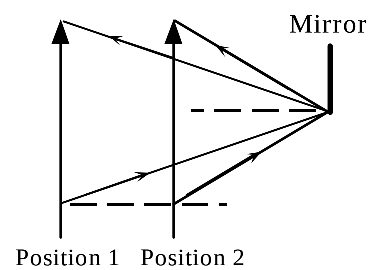
33. When an object is placed so that its image from one converging lens is placed at the focal point of a second converging lens, the final image is formed at infinity by the second lens. A real image is similar to a real object and when this image is placed at the focal point of another lens, the rays will head to the second lens in such a way as to be sent out on the other side of the lens in a direction parallel to the optical axis, which forms an image at infinity.

## Solutions to Problems

1. For a flat mirror the image is as far behind the mirror as the object is in front, so the distance from object to image is

$$d_o + d_i = 2.5\,\text{m} + 2.5\,\text{m} = \boxed{5.0\,\text{m.}}$$

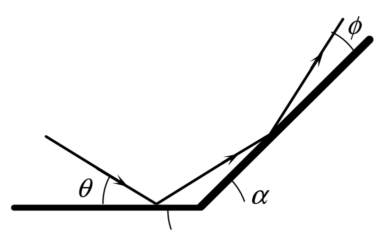
2. Because the angle of incidence must equal the angle of reflection, we see from the ray diagrams that the ray that reflects to your eye must be as far below the horizontal line to the reflection point on the mirror as the top is above the line, regardless of your position.




3. From the triangle formed by the mirrors and the first reflected ray, we have

$$\theta + \alpha + \phi = 180°;$$

$$40° + 135° + \phi = 180°, \text{ which gives } \phi = \boxed{5°.}$$

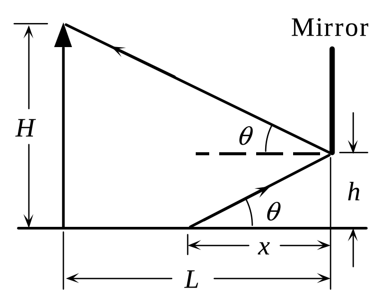



4. The angle of incidence is the angle of reflection. Thus we have

$$\tan\theta = \frac{(H-h)}{L} = \frac{h}{x};$$

$$\frac{(1.68\,\text{m} - 0.43\,\text{m})}{(2.20\,\text{m})} = \frac{(0.43\,\text{m})}{x},$$

which gives $x = 0.76\,\text{m} = \boxed{76\,\text{cm.}}$




5. Because the rays entering your eye are diverging from the image position behind the mirror, the diameter of the area on the mirror and the diameter of your pupil must subtend the same angle from the image:

$$\frac{D_{\text{mirror}}}{d_{\text{i}}} = \frac{D_{\text{pupil}}}{\left(d_{\text{o}} + d_{\text{i}}\right)};$$

$$\frac{D_{\text{mirror}}}{(90\,\text{cm})} = \frac{(5.5\,\text{mm})}{(90\,\text{cm} + 90\,\text{cm})}, \text{ which gives } D_{\text{mirror}} = 2.75\,\text{mm}.$$

Thus the area on the mirror is

$$A_{\text{mirror}} = \tfrac{1}{4}\pi D_{\text{mirror}}{}^2 = \tfrac{1}{4}\pi\left(2.75\times10^{-3}\,\text{m}\right)^2 = \boxed{5.9\times10^{-6}\,\text{m}^2.}$$

6. For the first reflection at *A* the angle of incidence $\theta_1$ is the angle of reflection. For the second reflection at *B* the angle of incidence $\theta_2$ is the angle of reflection. We can relate these angles to the angle at which the mirrors meet, $\phi$, by using the sum of the angles of the triangle *ABC*:

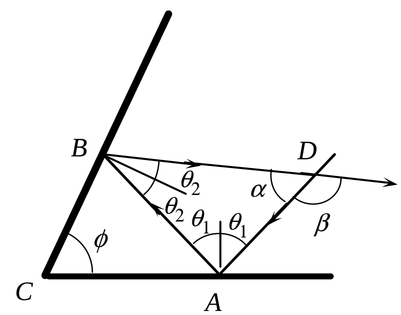
   $$\phi + \left(90° - \theta_1\right) + \left(90° - \theta_2\right) = 180°, \text{ or } \phi = \theta_1 + \theta_2.$$

   In the same way, for the triangle *ABD*, we have

   $$\alpha + 2\theta_1 + 2\theta_2 = 180°, \text{ or } \alpha = 180° - 2\left(\theta_1 + \theta_2\right) = 180° - 2\phi.$$

   At point *D* we see that the deflection is

   $$\beta = 180° - \alpha = 180° - \left(180° - 2\phi\right) = 2\phi.$$




7. The rays from the Sun will be parallel, so the image will be at the focal point. The radius is

   $$r = 2f = 2(18.0\,\text{cm}) = \boxed{36.0\,\text{cm.}}$$

8. To produce an image at infinity, the object will be at the focal point:

   $$d_{\text{o}} = f = \frac{r}{2} = \frac{(23.0\,\text{cm})}{2} = \boxed{11.5\,\text{cm.}}$$

$\boxed{9.}$ The ball is a convex mirror with a focal length

$$f = \frac{r}{2} = \frac{(-4.5\,\text{cm})}{2} = -2.25\,\text{cm}.$$

We locate the image from

$$\left(\frac{1}{d_{\text{o}}}\right) + \left(\frac{1}{d_{\text{i}}}\right) = \frac{1}{f};$$

$$\left[\frac{1}{(30.0\,\text{cm})}\right] + \left(\frac{1}{d_{\text{i}}}\right) = \frac{1}{(-2.25\,\text{cm})}, \text{ which gives } d_{\text{i}} = -2.09\,\text{cm}.$$

The image is $\boxed{\text{2.09 cm behind the surface, virtual.}}$

The magnification is

$$m = -\frac{d_{\text{i}}}{d_{\text{o}}} = \frac{-(-2.09\,\text{cm})}{(30.0\,\text{cm})} = +0.070.$$

Because the magnification is positive, the image is $\boxed{\text{upright.}}$

10. We find the image distance from the magnification:

$$m=\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}};$$

$$+3=\frac{-d_{\mathrm{i}}}{(1.4\,\mathrm{m})},\text{ which gives } d_{\mathrm{i}}=-4.2\,\mathrm{m}.$$

We find the focal length from

$$\left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(1.4\,\mathrm{m})}\right]+\left[\frac{1}{(-4.2\,\mathrm{m})}\right]=\frac{1}{f},\text{ which gives } f=2.1\mathrm{m}.$$

The radius of the concave mirror is

$$r=2f=2(2.1\mathrm{m})=\boxed{4.2\,\mathrm{m}.}$$

$\boxed{11.}$ We find the image distance from the magnification:

$$m=\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}};$$

$$+4.5=\frac{-d_{\mathrm{i}}}{(2.20\,\mathrm{cm})},\text{ which gives } d_{\mathrm{i}}=-9.90\,\mathrm{cm}.$$

We find the focal length from

$$\left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(2.20\,\mathrm{cm})}\right]+\left[\frac{1}{(-9.90\,\mathrm{cm})}\right]=\frac{1}{f},\text{ which gives } f=2.83\mathrm{cm}.$$

Because the focal length is positive, the mirror is $\boxed{\text{concave}}$ with a radius of

$$r=2f=2(2.83\mathrm{cm})=\boxed{5.7\,\mathrm{cm}.}$$

12. We find the image distance from the magnification:

$$m=\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}}.$$

$$+0.33=\frac{-d_{\mathrm{i}}}{(20.0\,\mathrm{m})}\text{ which gives } d_{\mathrm{i}}=-6.60\,\mathrm{m}.$$

We find the focal length from

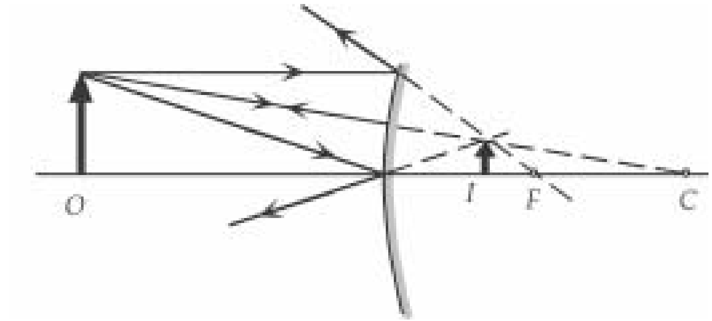
$$\left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(20.0\,\mathrm{m})}\right]+\left[\frac{1}{(-6.60\,\mathrm{m})}\right]=\frac{1}{f},\text{ which gives } f=-9.85\mathrm{m}.$$

Because the focal length is negative, the mirror is $\boxed{\text{convex}}$. (Only convex mirrors produce images that are right-side-up and smaller than the object. See also Example 23-4.) The radius is

$$r=2f=2(-9.85\,\mathrm{m})=\boxed{-19.7\,\mathrm{m}.}$$

13. (*a*) We see from the ray diagram that the image is behind the mirror, so it is virtual. We estimate the image distance as $\boxed{-7\,\text{cm.}}$




(*b*) If we use a focal length of $-10\,\text{cm}$, we locate the image from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(20\,\text{cm})}\right]+\left(\frac{1}{d_i}\right)=\frac{1}{(-10\,\text{cm})}, \text{ which gives } d_i=-6.7\,\text{cm}.$$

(*c*) We find the image size from the magnification:

$$m=\frac{h_i}{h_o}=\frac{-d_i}{d_o};$$

$$\frac{h_i}{(3.00\,\text{mm})}=-\frac{(-6.7\,\text{cm})}{(20\,\text{cm})}, \text{ which gives } h_i=\boxed{1.0\,\text{mm.}}$$

14. We find the image distance from the magnification, noting an upright image as in Fig. 23-46.

$$m=+0.5=\frac{h_i}{h_o}=\frac{-d_i}{d_o}=\frac{-d_i}{(3.0\,\text{m})}, \text{ which yields } d_i=\frac{(-3.0\,\text{m})}{(+0.5)}=-1.5\,\text{m}.$$

The thin lens equation gives the inverse focal length

$$\frac{1}{f}=\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\left[\frac{1}{(3.0\,\text{m})}\right]+\left[\frac{1}{(-1.5\,\text{m})}\right], \text{ so } f=-3.0\,\text{m}.$$

The radius of the mirror is $r=2f=2(-3.0\,\text{m})=\boxed{-6.0\,\text{m.}}$

15. (*a*) With $d_i=d_o$, we locate the object from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_o}\right)=\frac{1}{f}, \text{ which gives } d_o=2f=r.$$

The object should be placed at the $\boxed{\text{center of curvature.}}$

(*b*) Because the image is in front of the mirror, $d_i>0$, it is $\boxed{\text{real.}}$

(*c*) The magnification is

$$m=\frac{-d_i}{d_o}=\frac{-d_o}{d_o}=-1.$$

Because the magnification is negative, the image is $\boxed{\text{inverted.}}$

(*d*) As found in part (*c*), $m=\boxed{-1.}$

16. We take the object distance to be ∞, and find the focal length from

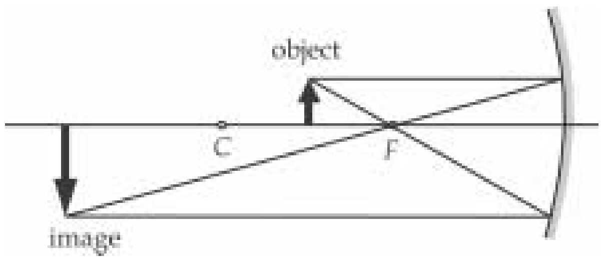
$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{\infty}\right)+\left[\frac{1}{(-18.0\,\text{cm})}\right]=\frac{1}{f}, \text{ which gives } f=-18.0\,\text{cm}.$$

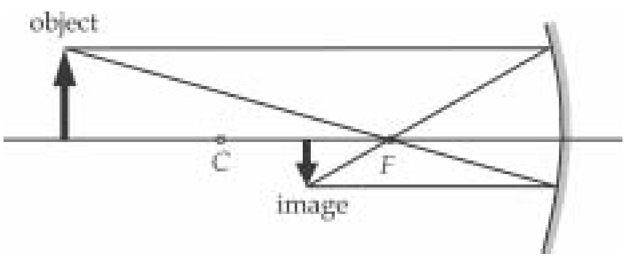
Because the focal length is negative, the mirror is $\boxed{\text{convex.}}$
The radius is

$$r=2f=2(-18.0\,\text{cm})=\boxed{-36.0\,\text{cm.}}$$

17.




$|h_i|>|h_o|$




$|h_i|<|h_o|$

We find the image distance from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f}=\frac{2}{r}, \text{ which we can write as } d_i=\frac{rd_o}{(2d_o-r)}.$$

The magnification is

$$m=\frac{-d_i}{d_o}=\frac{-r}{(2d_o-r)}.$$

If $d_o>r$, then $(2d_o-r)>r$, so

$$|m|=\frac{r}{(2d_o-r)}=\frac{r}{(>r)}<1.$$

If $d_o<r$, then $(2d_o-r)<r$, so

$$|m|=\frac{r}{(2d_o-r)}=\frac{r}{(<r)}>1.$$

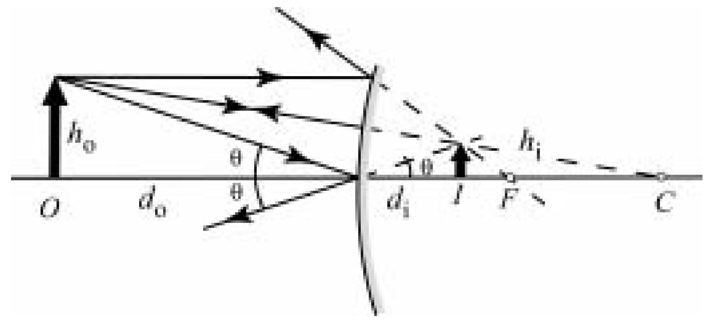
18. From the ray that reflects from the center of the mirror, we have

$$\tan\theta = \frac{h_o}{d_o} = \frac{h_i}{d_i};$$

$$|m| = \frac{h_o}{h_i} = \frac{d_i}{d_o}.$$

Because the image distance on the ray diagram is negative, we get

$$m = \frac{-d_i}{d_o}.$$




19. From the ray diagram, we see that

$$\tan\theta = \frac{h_o}{d_o} = \frac{h_i}{d_i};$$

$$\tan\alpha = \frac{h_o}{(d_o + r)} = \frac{h_i}{(r - d_i)}.$$

When we divide the two equations, we get

$$\frac{(d_o + r)}{d_o} = \frac{(r - d_i)}{d_i};$$

$$1 + \left(\frac{r}{d_o}\right) = \left(\frac{r}{d_i}\right) - 1, \text{ or}$$

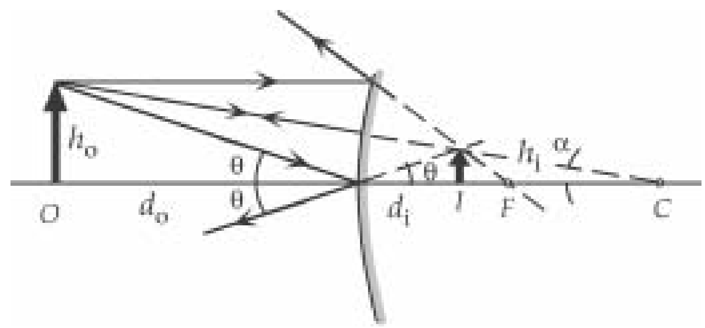
$$\left(\frac{r}{d_o}\right) - \left(\frac{r}{d_i}\right) = -2;$$

$$\left(\frac{1}{d_o}\right) - \left(\frac{1}{d_i}\right) = \frac{-2}{r} = \frac{-1}{f}, \text{ with } f = \frac{r}{2}.$$

From the ray diagram, we see that $d_i < 0.$

If we consider *f* to be negative, we have

$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f}.$$




20. We find the image distance from the magnification:

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o};$$

$$+0.65 = \frac{-d_i}{(2.2\,\text{m})}, \text{ which gives } d_i = -1.43\,\text{m}.$$

We find the focal length from

$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f};$$

$$\left[\frac{1}{(2.2\,\text{m})}\right] + \left[\frac{1}{(-1.43\,\text{m})}\right] = \frac{1}{f}, \text{ which gives } f = \boxed{-4.1\,\text{m.}}$$

21. (*a*) To produce a smaller image located behind the surface of the mirror requires a $\boxed{\text{convex mirror.}}$

(*b*) We find the image distance from the magnification:

$$m = \frac{h_\text{i}}{h_\text{o}} = \frac{-d_\text{i}}{d_\text{o}};$$

$$\frac{(3.5\,\text{cm})}{(4.5\,\text{cm})} = \frac{-d_\text{i}}{(28\,\text{cm})}, \text{ which gives } d_\text{i} = -21.8\,\text{cm}.$$

As expected, $d_\text{i} < 0$. The image is located $\boxed{\text{22 cm behind the surface.}}$

(*c*) We find the focal length from

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f};$$

$$\left[\frac{1}{(28\,\text{cm})}\right] + \left[\frac{1}{(-21.8\,\text{cm})}\right] = \frac{1}{f}, \text{ which gives } f = \boxed{-98\,\text{cm.}}$$

(*d*) The radius of curvature is

$$r = 2f = 2(-98\,\text{cm}) = \boxed{-196\,\text{cm.}}$$

22. (*a*) To produce a larger image requires a $\boxed{\text{concave mirror.}}$

(*b*) The image will be $\boxed{\text{erect and virtual.}}$

(*c*) We find the image distance from the magnification:

$$m = \frac{h_\text{i}}{h_\text{o}} = \frac{-d_\text{i}}{d_\text{o}};$$

$$1.33 = \frac{-d_\text{i}}{(20.0\,\text{cm})}, \text{ which gives } d_\text{i} = -26.6\,\text{cm}.$$

We find the focal length from

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f};$$

$$\left[\frac{1}{(20.0\,\text{cm})}\right] + \left[\frac{1}{(-26.6\,\text{cm})}\right] = \frac{1}{f}, \text{ which gives } f = 80.6\,\text{cm}.$$

The radius of curvature is

$$r = 2f = 2(80.6\,\text{cm}) = \boxed{161\,\text{cm.}}$$

23. (*a*) The speed in crown glass is

$$v = \frac{c}{n} = \frac{(3.00 \times 10^8\,\text{m/s})}{(1.52)} = \boxed{1.97 \times 10^8\,\text{m/s.}}$$

(*b*) The speed in Lucite is

$$v = \frac{c}{n} = \frac{(3.00 \times 10^8\,\text{m/s})}{(1.51)} = \boxed{1.99 \times 10^8\,\text{m/s.}}$$

(*c*) The speed in ethyl alcohol is $v = \dfrac{c}{n} = \dfrac{(3.00 \times 10^8\,\text{m/s})}{(1.36)} = \boxed{2.21 \times 10^8\,\text{m/s.}}$

24. We find the index of refraction from

$$v = \frac{c}{n};$$

$$2.29 \times 10^8\ \text{m/s} = \frac{(3.00 \times 10^8\ \text{m/s})}{n},\ \text{which gives } n = \boxed{1.31.}$$

25. We find the index of refraction from

$$v = \frac{c}{n};$$

$$0.89\, v_{\text{water}} = \frac{0.89c}{1.33} = \frac{c}{n},\ \text{which gives } n = \boxed{1.49.}$$

26. We find the angle of refraction in the glass from

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$(1.00)\sin 63° = (1.58)\sin\theta_2,\ \text{which gives } \theta_2 = \boxed{34°.}$$

27. We find the angle of refraction in the water from

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$(1.33)\sin 42.5° = (1.00)\sin\theta_2,\ \text{which gives } \theta_2 = \boxed{64.0°.}$$

28. We find the incident angle in the water from

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$(1.33)\sin\theta_1 = (1.00)\sin 66.0°,\ \text{which gives } \theta_1 = \boxed{43.4°.}$$

29. We find the incident angle in the air from

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$(1.00)\sin\theta_1 = (1.33)\sin 31.0°,\ \text{which gives } \theta_1 = 43.2°.$$

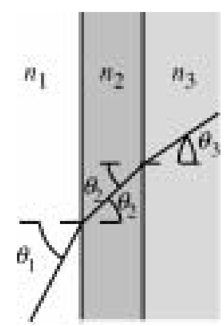
Thus the angle above the horizon is

$$90.0° - \theta_1 = 90.0° - 43.2° = \boxed{46.8°.}$$

30. (*a*) We find the angle in the glass from the refraction at the air–glass surface:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$(1.00)\sin 43.5° = (1.52)\sin\theta_2,\ \text{which gives } \theta_2 = \boxed{26.9°.}$$




(*b*) Because the surfaces are parallel, the refraction angle from the first surface is the incident angle at the second surface. We find the angle in the water from the refraction at the glass–water surface:

$$n_2 \sin\theta_2 = n_3 \sin\theta_3;$$

$$(1.52)\sin 26.9° = (1.33)\sin\theta_3,\ \text{which gives } \theta_3 = \boxed{31.2°.}$$

(*c*) If there were no glass, we would have

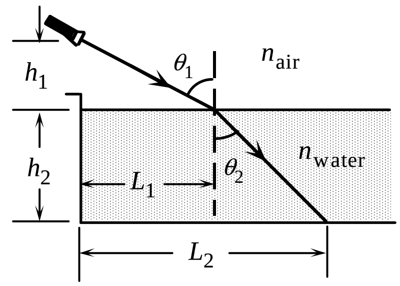
$$n_1 \sin\theta_1 = n_3 \sin\theta_3';$$

$$(1.00)\sin 43.5° = (1.33)\sin\theta_3',\ \text{which gives } \theta_3' = \boxed{31.2°.}$$

Note that, because the sides are parallel, $\theta_3$ is independent of the presence of the glass.

$\boxed{31.}$ We find the angle of incidence from the distances:

$$\tan\theta_1 = \frac{L_1}{h_1} = \frac{(2.7\,\mathrm{m})}{(1.3\,\mathrm{m})} = 2.076, \text{ so } \theta_1 = 64.3°.$$




For the refraction from air into water, we have

$$n_{\text{air}} \sin\theta_1 = n_{\text{water}} \sin\theta_2;$$

$$(1.00)\sin 64.3° = (1.33)\sin\theta_2, \text{ which gives } \theta_2 = 42.6°.$$

We find the horizontal distance from the edge of the pool from

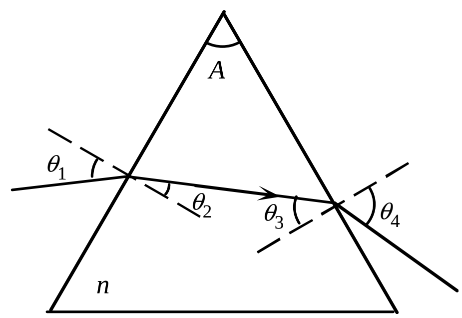
$$L = L_1 + L_2 = L_1 + h_2 \tan\theta_2$$

$$= 2.7\,\mathrm{m} + (2.1\,\mathrm{m})\tan 42.6° = \boxed{4.6\,\mathrm{m}.}$$

32. For the refraction at the first surface, we have

$$n_{\text{air}} \sin\theta_1 = n \sin\theta_2;$$

$$(1.00)\sin 45.0° = (1.58)\sin\theta_2, \text{ which gives } \theta_2 = 26.6°.$$




We find the angle of incidence at the second surface from

$$(90° - \theta_2) + (90° - \theta_3) + A = 180°, \text{ which gives}$$

$$\theta_3 = A - \theta_2 = 60° - 26.6° = 33.4°.$$

For the refraction at the second surface, we have

$$n \sin\theta_3 = n_{\text{air}} \sin\theta_4;$$

$$(1.58)\sin 33.4° = (1.00)\sin\theta_4, \text{ which gives}$$

$$\theta_4 = \boxed{60.5° \text{ from the normal.}}$$

33. The angle of reflection is equal to the angle of incidence:

$$\theta_{\text{ref1}} = \theta_1 = 2\theta_2.$$

For the refraction we have

$$n_{\text{air}} \sin\theta_1 = n_{\text{glass}} \sin\theta_2;$$

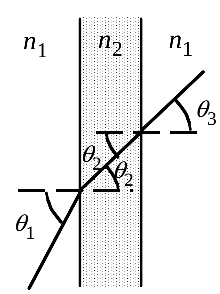
$$(1.00)\sin 2\theta_2 = (1.52)\sin\theta_2.$$

We use a trigonometric identity for the left-hand side:

$$\sin 2\theta_2 = 2\sin\theta_2 \cos\theta_2 = (1.52)\sin\theta_2, \text{ or } \cos\theta_2 = 0.760, \text{ so } \theta_2 = 40.5°.$$

Thus the angle of incidence is $\theta_1 = 2\theta_2 = \boxed{81.0°.}$

34. Because the surfaces are parallel, the angle of refraction from the first surface is the angle of incidence at the second. Thus for the refractions, we have




$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$n_2 \sin\theta_2 = n_3 \sin\theta_3.$$

When we add the two equations, we get

$$n_1 \sin\theta_1 = n_1 \sin\theta_3, \text{ which gives } \theta_3 = \theta_1.$$

Because the ray emerges in the same index of refraction, it is undeviated.

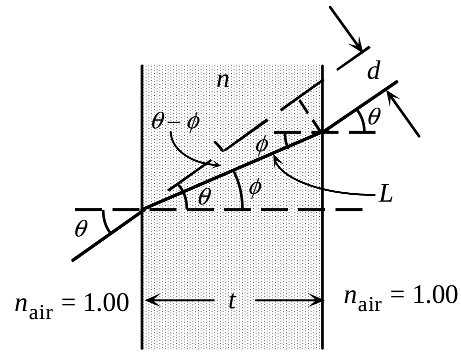
35. Because the glass surfaces are parallel, the exit beam will be traveling in the same direction as the original beam.
We find the angle inside the glass from

$$n_{\text{air}} \sin\theta = n\sin\phi.$$

If the angles are small, we use

$\cos\phi \approx 1,$ and $\sin\phi \approx \phi,$ where $\phi$ is in radians.

$$(1.00)\,\theta = n\phi, \text{ or } \phi = \frac{\theta}{n}.$$




We find the distance along the ray in the glass from

$$L = \frac{t}{\cos\phi} \approx t.$$

We find the perpendicular displacement from the original direction from

$$d = L\sin(\theta - \phi) \approx t(\theta - \phi) = t\left[\theta - \left(\frac{\theta}{n}\right)\right] = \frac{t\theta(n-1)}{n}.$$

36. When the light in the material with a higher index is incident at the critical angle, the refracted angle is 90°:

$$n_{\text{Lucite}} \sin\theta_1 = n_{\text{water}} \sin\theta_2;$$

$$(1.51)\sin\theta_1 = (1.33)\sin 90°, \text{which gives } \theta_1 = \boxed{61.7°.}$$

Because Lucite has the higher index, the light must start in $\boxed{\text{Lucite.}}$

37. When the light in the liquid is incident at the critical angle, the refracted angle is 90°:

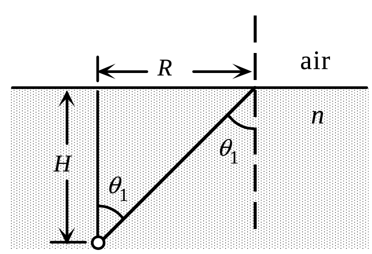
$$n_{\text{liquid}} \sin\theta_1 = n_{\text{air}} \sin\theta_2;$$

$$n_{\text{liquid}} \sin 47.7° = (1.00)\sin 90°, \text{ which gives } n_{\text{liquid}} = \boxed{1.35.}$$

38. We find the critical angle for light leaving the water:

$$n\sin\theta_1 = \sin\theta_2;$$

$$(1.33)\sin\theta_C = \sin 90°, \text{ which gives } \theta_C = 48.8°.$$




If the light is incident at a greater angle than this, it will totally reflect. We see from the diagram that

$$R > H\tan\theta_C = (62.0\,\text{cm})\tan 48.8° = \boxed{70.7\,\text{cm.}}$$

$\boxed{39.}$ We find the angle of incidence from the distances:

$$\tan\theta_1 = \frac{L}{h} = \frac{(7.0\,\text{cm})}{(8.0\,\text{cm})} = 0.875, \text{ so } \theta_1 = 41.2°.$$

For the maximum incident angle for the refraction from liquid into air, we have

$$n_{\text{liquid}} \sin\theta_1 = n_{\text{air}} \sin\theta_2;$$

$$n_{\text{liquid}} \sin\theta_{1\max} = (1.00)\sin 90°, \text{ which gives } \sin\theta_{1\max} = \frac{1}{n_{\text{liquid}}}.$$

Thus we have

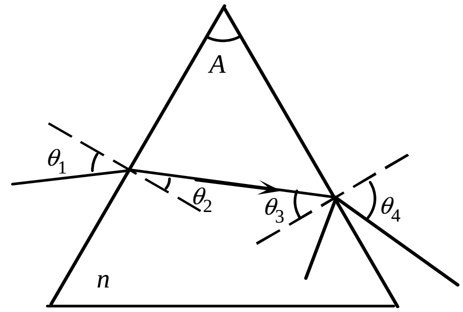
$$\sin\theta_1 \ge \sin\theta_{1\max} = \frac{1}{n_{\text{liquid}}};$$

$$\sin 41.2° = 0.659 \geq \frac{1}{n_{\text{liquid}}}, \text{ or } \boxed{n_{\text{liquid}} \geq 1.5.}$$

40. For the refraction at the first surface, we have

$$n_{\text{air}} \sin\theta_1 = n\sin\theta_2;$$

$$(1.00)\sin\theta_1 = n\sin\theta_2, \text{ which gives } \sin\theta_2 = \sin\frac{\theta_1}{n}.$$




We find the angle of incidence at the second surface from

$$(90° - \theta_2) + (90° - \theta_3) + A = 180°, \text{ which gives}$$

$$\theta_3 = A - \theta_2 = 75° - \theta_2.$$

For the refraction at the second surface, we have

$$n\sin\theta_3 = n_{\text{air}} \sin\theta_4 = (1.00)\sin\theta_4.$$

The maximum value of $\theta_4$ before internal reflection takes place at the second surface is 90°. Thus for internal reflection to occur, we have

$$n\sin\theta_3 = n\sin(A - \theta_2) \geq 1.$$

When we expand the left-hand side, we get

$$n(\sin A\cos\theta_2 - \cos A\sin\theta_2) \geq 1.$$

If we use the result from the first surface to eliminate $n$, we get

$$\frac{\sin\theta_1(\sin A\cos\theta_2 - \cos A\sin\theta_2)}{(\sin\theta_2)} = \sin\theta_1\left(\frac{\sin A}{\tan\theta_2 - \cos A}\right) \geq 1, \text{ or}$$

$$\frac{1}{\tan\theta_2} \geq \frac{\left[\left(\frac{1}{\sin\theta_1}\right) + \cos A\right]}{\sin A} = \frac{\left[\left(\frac{1}{\sin 45°}\right) + \cos 75°\right]}{\sin 75°} = 1.732, \text{ which gives}$$

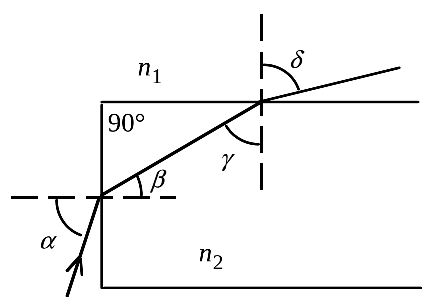
$$\tan\theta_2 \leq 0.577, \text{ so } \theta_2 \leq 30°.$$

From the result for the first surface, we have

$$n_{\min} = \frac{\sin\theta_1}{\sin\theta_{2\max}} = \frac{\sin 45°}{\sin 30°} = 1.414, \text{ so } \boxed{n \geq 1.414.}$$

41. For the refraction at the side of the rod, we have

$$n_2 \sin\gamma = n_1 \sin\delta.$$




The minimum angle for total reflection $\gamma_{\min}$ occurs when $\delta = 90°$:

$$n_2 \sin\gamma_{\min} = (1.00)(1) = 1, \text{ or } \sin\gamma_{\min} = \frac{1}{n_2}.$$

We find the maximum angle of refraction at the end of the rod from

$$\beta_{\max} = 90° - \gamma_{\min}.$$

Because the sine function increases with angle, for the refraction at the end of the rod, we have

$$n_1 \sin\alpha_{\max} = n_2 \sin\beta_{\max};$$

$$(1.00)\sin\alpha_{\max} = n_2 \sin(90° - \gamma_{\min}) = n_2 \cos\gamma_{\min}.$$

If we want total internal reflection to occur for any incident angle at the end of the fiber, the maximum value of $\alpha$ is 90°, so

$$n_2 \cos \gamma_{\min} = 1.$$

When we divide this by the result for the refraction at the side, we get $\tan \gamma_{\min} = 1$, or $\gamma_{\min} = 45°$. Thus we have

$$n_2 \geq \frac{1}{\sin \gamma_{\min}} = \frac{1}{\sin 45°} = 1.414.$$

42. (*a*) The ray enters normal to the first surface, so there is no deviation there. The angle of incidence is 45° at the second surface. When there is air outside the surface, we have

$$n_1 \sin \theta_1 = n_2 \sin \theta_2;$$
$$n_1 \sin 45° = (1.00) \sin \theta_2.$$

For total internal reflection to occur, $\sin \theta_2 \geq 1$, so we have

$$n_1 \geq \frac{1}{\sin 45°} = \boxed{1.414.}$$

(*b*) When there is water outside the surface, we have

$$n_1 \sin \theta_1 = n_2 \sin \theta_2;$$
$$(1.50) \sin 45° = (1.33) \sin \theta_2, \text{ which gives } \sin \theta_2 = 0.80.$$

Because $\sin \theta_2 < 1$, $\boxed{\text{the prism will not be totally reflecting.}}$

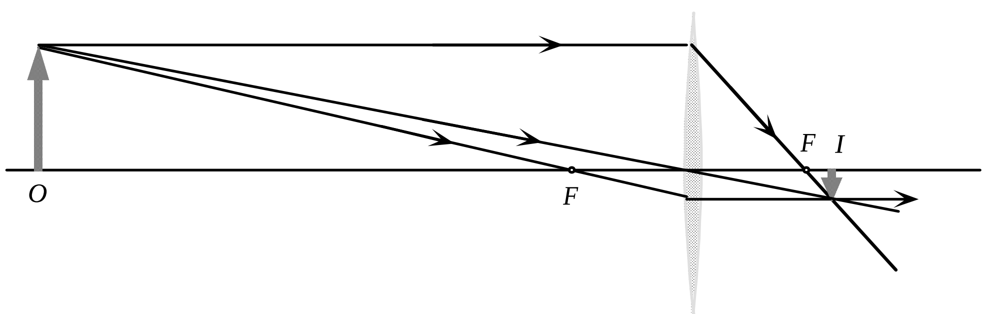
(*c*) For total reflection when there is water outside the surface, we have

$$n_1 \sin \theta_1 = n_2 \sin \theta_2;$$
$$n_1 \sin 45° = (1.33) \sin \theta_2.$$

For total internal reflection to occur, $\sin \theta_2 \geq 1$, so we have

$$n_1 \geq \frac{1.33}{\sin 45°} = \boxed{1.88.}$$

43. (*a*) From the ray diagram, the object distance is about six focal lengths, or $\boxed{\text{390 mm.}}$




(*b*) We find the object distance from

$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f};$$
$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{78.0\,\text{mm}}\right) = \frac{1}{65.0\,\text{mm}}, \text{ which gives } d_o = 390\,\text{mm} = \boxed{39.0\,\text{cm.}}$$

44. (*a*) To form a real image from parallel rays requires a $\boxed{\text{converging lens.}}$
(*b*) We find the power of the lens from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f}=P,\ \text{when } f \text{ is in meters;}$$

$$\left(\frac{1}{\infty}\right)+\left(\frac{1}{0.185\,\text{m}}\right)=P=\boxed{5.41\,\text{D.}}$$

45. To form a real image from a real object requires a $\boxed{\text{converging lens.}}$
We find the focal length of the lens from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{275\,\text{cm}}\right)+\left(\frac{1}{48.3\,\text{cm}}\right)=\frac{1}{f},\ \text{which gives } f=\boxed{+41.1\,\text{cm.}}$$

Because $d_i>0$, the image is $\boxed{\text{real.}}$

46. (*a*) The power of the lens is

$$P=\frac{1}{f}=\frac{1}{0.205\,\text{m}}=\boxed{4.88\,\text{D, converging.}}$$

(*b*) We find the focal length of the lens from

$$P=\frac{1}{f};$$

$$-6.25\,\text{D}=\frac{1}{f},\ \text{which gives } f=-0.160\,\text{m}=\boxed{-16.0\,\text{cm, diverging.}}$$

$\boxed{47.}$ (*a*) We locate the image from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{18\,\text{cm}}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{24\,\text{cm}},\ \text{which gives } d_i=-72\,\text{cm.}$$

The negative sign means the image is $\boxed{\text{72 cm behind the lens (virtual).}}$
(*b*) We find the magnification from

$$m=\frac{-d_i}{d_o}=-\frac{(-72\,\text{cm})}{(18\,\text{cm})}=\boxed{+4.0.}$$

48. We find the image distance from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{0.140\,\text{m}}\right)+\left(\frac{1}{d_i}\right)=-5.5\,\text{D},$$

which gives $d_i=-0.076\,\text{m}=\boxed{-7.9\text{ cm (virtual image behind the lens).}}$
We find the height of the image from

$$m = \frac{h_\mathrm{i}}{h_\mathrm{o}} = \frac{-d_\mathrm{i}}{d_\mathrm{o}};$$

$$\frac{h_\mathrm{i}}{(4.0\,\mathrm{mm})} = -\frac{(-7.9\,\mathrm{cm})}{(14.0\,\mathrm{cm})}, \text{ which gives } h_\mathrm{i} = \boxed{2.3\,\mathrm{mm}\,(\text{upright}).}$$

49. (*a*) We find the image distance from

$$\left(\frac{1}{d_\mathrm{o}}\right) + \left(\frac{1}{d_\mathrm{i}}\right) = \frac{1}{f};$$

$$\left(\frac{1}{10.0\times10^3\,\mathrm{mm}}\right) + \left(\frac{1}{d_\mathrm{i}}\right) = \frac{1}{80\,\mathrm{mm}}, \text{ which gives } d_\mathrm{i} = \boxed{81\,\mathrm{mm}.}$$

(*b*) For an object distance of 3.0 m, we have

$$\left(\frac{1}{3.0\times10^3\,\mathrm{mm}}\right) + \left(\frac{1}{d_\mathrm{i}}\right) = \frac{1}{80\,\mathrm{mm}}, \text{ which gives } d_\mathrm{i} = \boxed{82\,\mathrm{mm}.}$$

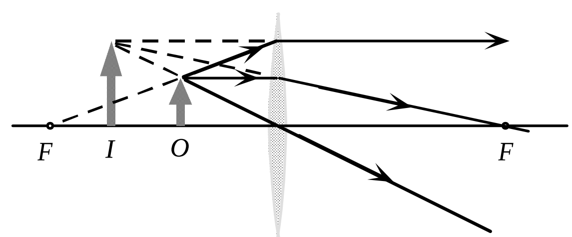
(*c*) For an object distance of 1.0 m, we have

$$\left(\frac{1}{1.0\times10^3\,\mathrm{mm}}\right) + \left(\frac{1}{d_\mathrm{i}}\right) = \frac{1}{80\,\mathrm{mm}}, \text{ which gives } d_\mathrm{i} = \boxed{87\,\mathrm{mm}.}$$

(*d*) We find the smallest object distance from

$$\left(\frac{1}{d_\mathrm{omin}}\right) + \left(\frac{1}{120\,\mathrm{mm}}\right) = \frac{1}{80\,\mathrm{mm}}, \text{ which gives } d_\mathrm{i} = 240\,\mathrm{mm} = \boxed{24\,\mathrm{cm}.}$$

50. (*a*) We see that the image is behind the lens, so it is $\boxed{\text{virtual.}}$




(*b*) From the ray diagram we see that we need a $\boxed{\text{converging lens.}}$

(*c*) We find the image distance from the magnification:

$$m = \frac{-d_\mathrm{i}}{d_\mathrm{o}};$$

$$+2.5 = \frac{-d_\mathrm{i}}{(8.0\,\mathrm{cm})}, \text{ which gives } d_\mathrm{i} = -20\,\mathrm{cm}.$$

We find power of the lens from

$$\left(\frac{1}{d_\mathrm{o}}\right) + \left(\frac{1}{d_\mathrm{i}}\right) = \frac{1}{f} = P, \text{ when } f \text{ is in meters;}$$

$$\left(\frac{1}{0.080\,\mathrm{m}}\right) + \left[\frac{1}{(-0.20\,\mathrm{m})}\right] = P = \boxed{7.5\,\mathrm{D}.}$$

51. We find the image distance from

$$\frac{1}{d_\mathrm{o}} + \frac{1}{d_\mathrm{i}} = \frac{1}{f};$$

$$\frac{1}{1.5\,\mathrm{m}} + \frac{1}{d_\mathrm{i}} = 8.0\,\mathrm{D}, \text{ which gives } d_\mathrm{i} = 0.1364\,\mathrm{m}.$$

(*a*) With $d_o = 0.5\,\text{m},$ the new image distance is determined by

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f};$$

$$\frac{1}{0.5\,\text{m}} + \frac{1}{d_i} = 8.0\,\text{D}, \text{ which gives } d_i = 0.1667\,\text{m}.$$

The image has moved $0.1667\,\text{m} - 0.1364\,\text{m} = 0.0303\,\text{m}$ or $\boxed{\text{3.0 cm away from the lens.}}$

(*b*) With $d_o = 2.5\,\text{m},$ the new image distance is determined by

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f};$$

$$\frac{1}{2.5\,\text{m}} + \frac{1}{d_i} = 8.0\,\text{D}, \text{ which gives } d_i = 0.1316\,\text{m}.$$

The image has moved $0.1316\,\text{m} - 0.1364\,\text{m} = -0.0048\,\text{m}$ or $\boxed{\text{0.5 cm toward the lens.}}$

52. $|h_i| = |h_o|$ when $d_i = d_o$. So we find $d_o$ from $\frac{1}{d_o} + \frac{1}{d_o} = \frac{1}{f},$ which gives $d_o = 2f$.

An object placed two focal lengths away from a converging lens will produce a real image that is also two focal lengths from the lens, inverted, and the same size as the object. With $f = 25\,\text{cm},$ the object should be placed a distance of $2f = \boxed{50\,\text{cm}}$ from the lens.

53. We can relate the image and object distance from the magnification:

$$m = \frac{-d_i}{d_o}, \text{ or } d_o = \frac{-d_i}{m}.$$

We use this in the lens equation:

$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f};$$

$$-\left(\frac{m}{d_i}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f}, \text{ which gives } d_i = (1-m)f.$$

(*a*) If the image is real, $d_i > 0$. With $f > 0,$ we see that $m < 1;$ thus $m = -2.00.$ The image distance is

$$d_i = [1 - (-2.00)](50.0\,\text{mm}) = 150\,\text{mm}.$$

The object distance is

$$d_o = \frac{-d_i}{m} = \frac{-(150\,\text{mm})}{(-2.00)} = \boxed{75.0\,\text{mm.}}$$

(*b*) If the image is virtual, $d_i < 0$. With $f > 0,$ we see that $m > 1;$ thus $m = +2.00.$ The image distance is

$$d_i = [1 - (+2.00)](50.0\,\text{mm}) = -50\,\text{mm}.$$

The object distance is

$$d_o = \frac{-d_i}{m} = \frac{-(-50\,\text{mm})}{(+2.00)} = \boxed{25.0\,\text{mm.}}$$

54. We can relate the image and object distance from the magnification:

$$m = \frac{-d_\mathrm{i}}{d_\mathrm{o}}, \text{ or } d_\mathrm{o} = \frac{-d_\mathrm{i}}{m}.$$

We use this in the lens equation:

$$\left(\frac{1}{d_\mathrm{o}}\right) + \left(\frac{1}{d_\mathrm{i}}\right) = \frac{1}{f};$$

$$-\left(\frac{m}{d_\mathrm{i}}\right) + \left(\frac{1}{d_\mathrm{i}}\right) = \frac{1}{f}, \text{ which gives } d_\mathrm{i} = (1-m)f.$$

(*a*) If the image is real, $d_\mathrm{i} > 0$. With $f < 0$, we see that $m > 1$; thus $m = +2.00$. The image distance is

$$d_\mathrm{i} = [1 - (+2.00)](-50.0\,\mathrm{mm}) = 50.0\,\mathrm{mm}.$$

The object distance is

$$d_\mathrm{o} = \frac{-d_\mathrm{i}}{m} = \frac{-(50.0\,\mathrm{mm})}{(+2.00)} = \boxed{-25.0\,\mathrm{mm}.}$$

The negative sign means the object is beyond the lens, so it would have to be an object formed by a preceding optical device.

(*b*) If the image is virtual, $d_\mathrm{i} < 0$. With $f < 0$, we see that $m < 1$; thus $m = -2.00$. The image distance is

$$d_\mathrm{i} = [1 - (-2.00)](-50.0\,\mathrm{mm}) = -150\,\mathrm{mm}.$$

The object distance is

$$d_\mathrm{o} = -\frac{d_\mathrm{i}}{m} = -\frac{(-150\,\mathrm{mm})}{(-2.00)} = \boxed{-75.0\,\mathrm{mm}.}$$

The negative sign means the object is beyond the lens, so it would have to be an object formed by a preceding optical device.

55. (*a*) We find the image distance from

$$\left(\frac{1}{d_\mathrm{o}}\right) + \left(\frac{1}{d_\mathrm{i}}\right) = \frac{1}{f};$$

$$\left(\frac{1}{1.20\times 10^3\,\mathrm{mm}}\right) + \left(\frac{1}{d_\mathrm{i}}\right) = \frac{1}{135\,\mathrm{mm}}, \text{ which gives } d_\mathrm{i} = \boxed{152\,\mathrm{mm}\,(\text{real, behind the lens}).}$$

We find the height of the image from

$$m = \frac{h_\mathrm{i}}{h_\mathrm{o}} = -\frac{d_\mathrm{i}}{d_\mathrm{o}};$$

$$\frac{h_\mathrm{i}}{2.00\,\mathrm{cm}} = \frac{-(152\,\mathrm{mm})}{(1.20\times 10^3\,\mathrm{mm})}, \text{ which gives } h_\mathrm{i} = \boxed{-0.254\,\mathrm{cm}\,(\text{inverted}).}$$

(*b*) We find the image distance from

$$\left(\frac{1}{d_\mathrm{o}}\right) + \left(\frac{1}{d_\mathrm{i}}\right) = \frac{1}{f};$$

$$\left(\frac{1}{1.20\times 10^3\,\mathrm{mm}}\right) + \left(\frac{1}{d_\mathrm{i}}\right) = \frac{1}{(-135\,\mathrm{mm})},$$

which gives $d_\text{i} = \boxed{-121\,\text{mm (virtual, in front of the lens)}}.$

We find the height of the image from

$$m = \frac{h_\text{i}}{h_\text{o}} = -\frac{d_\text{i}}{d_\text{o}};$$

$$\frac{h_\text{i}}{(2.00\,\text{cm})} = \frac{-(-121\,\text{mm})}{(1.20\times 10^3\,\text{mm})},\ \text{which gives } h_\text{i} = \boxed{+0.202\,\text{cm (upright)}.}$$

56. For a real object and image, both $d_\text{o}$ and $d_\text{i}$ must be positive, so the magnification will be negative:

$$m = \frac{-d_\text{i}}{d_\text{o}};$$

$$-2.5 = \frac{-d_\text{i}}{d_\text{o}},\ \text{or } d_\text{i} = 2.5 d_\text{o}.$$

We find the object distance from

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f};$$

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{2.5 d_\text{o}}\right) = \frac{1}{(+75\,\text{cm})},\ \text{which gives } d_\text{o} = 105\,\text{cm}.$$

The image distance is

$$d_\text{i} = 2.5 d_\text{o} = 2.5(105\,\text{cm}) = 262.5\,\text{cm}.$$

The distance between object and image is

$$L = d_\text{o} + d_\text{i} = 105\,\text{cm} + 262.5\,\text{cm} = \boxed{37\,\text{m.}}$$

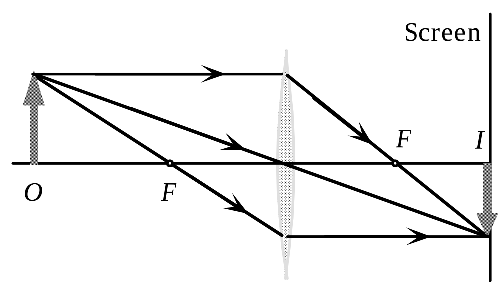
$\boxed{57.}$ The sum of the object and image distances must be the distance between object and screen:

$$d_\text{o} + d_\text{i} = L = 66.0\,\text{cm}.$$

For the lens we have

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f};$$

$$\left(\frac{1}{d_\text{o}}\right) + \left[\frac{1}{(66.0\,\text{cm} - d_\text{o})}\right] = \frac{1}{(12.5\,\text{cm})},$$




which gives a quadratic equation:

$$d_\text{o}^2 - (66.0\,\text{cm}) d_\text{o} + 825\,\text{cm}^2 = 0.$$

From the quadratic formula, $d_\text{o} = 49.2\,\text{cm}$ or $d_\text{o} = 16.8\,\text{cm}$, so the lens should be placed $\boxed{49.2\,\text{cm or } 16.8\,\text{cm}}$ from the object.

Note that in general the screen must be at least 4*f* from the object for an image to be formed on the screen.

58. We find the image formed by the refraction of the first lens:

$$\left(\frac{1}{d_\text{o1}}\right) + \left(\frac{1}{d_\text{i1}}\right) = \frac{1}{f_1};$$

$$\left(\frac{1}{36.0\,\text{cm}}\right)+\left(\frac{1}{d_{i1}}\right)=\frac{1}{28.0\,\text{cm}},\ \text{which gives } d_{i1}=+126\,\text{cm}.$$

This image is the object for the second lens. Because it is beyond the second lens, it has a negative object distance: $d_{o2}=16.5\,\text{cm}-126\,\text{cm}=-109.5\,\text{cm}.$

We find the image formed by the refraction of the second lens:

$$\left(\frac{1}{d_{o2}}\right)+\left(\frac{1}{d_{i2}}\right)=\frac{1}{f_2};$$

$$\left[\frac{1}{(-109.5\,\text{cm})}\right]+\left(\frac{1}{d_{i2}}\right)=\frac{1}{28.0\,\text{cm}},\ \text{which gives } d_{i2}=+22.3\,\text{cm}.$$

Thus the final image is $\boxed{\text{real, 22.3 cm beyond second lens.}}$

The total magnification is the product of the magnifications for the two lenses:

$$m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{d_{i1}d_{i2}}{d_{o1}d_{o2}}$$

$$=\frac{(+126\,\text{cm})(+22.3\,\text{cm})}{(+36.0\,\text{cm})(-109.5\,\text{cm})}=\boxed{-0.713(\text{inverted}).}$$

59. The image of an infinite object formed by the refraction of the first lens will be at the focal point:

$$d_{i1}=f_1=+20.0\,\text{cm}.$$

This image is the object for the second lens. Because it is beyond the second lens, it has a negative object distance: $d_{o2}=14.0\,\text{cm}-20.0\,\text{cm}=-6.0\,\text{cm}.$

We find the image formed by the refraction of the second lens:

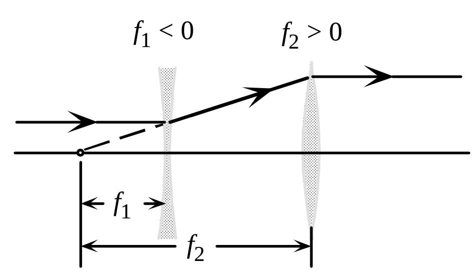
$$\left(\frac{1}{d_{o2}}\right)+\left(\frac{1}{d_{i2}}\right)=\frac{1}{f_2};$$

$$\left[\frac{1}{(-6.0\,\text{cm})}\right]+\left(\frac{1}{d_{i2}}\right)=\frac{1}{(-31.5\,\text{cm})},\ \text{which gives } d_{i2}=+7.4\,\text{cm}.$$

Thus the final image is $\boxed{\text{real, 7.4 cm beyond second lens.}}$

60. We see from the ray diagram that the image from the first lens will be a virtual image at its focal point. This is a real object for the second lens, and must be at the focal point of the second lens. If $L$ is the separation of the lenses, the focal length of the first lens is

$$f_1=L-f_2=21.0\,\text{cm}-31.0\,\text{cm}=\boxed{-10.0\,\text{cm}.}$$




61. (*a*) The position of the image formed by the first lens is still 30.0 cm behind the first lens, A. This image becomes the object for the second lens, B, at a distance $d_{oB}=20.0\,\text{cm}-30.0\,\text{cm}=-10.0\,\text{cm}.$

The image formed by lens B is located a distance $d_{iB}$ from lens B:

$$\frac{1}{d_{oB}}+\frac{1}{d_{iB}}=\frac{1}{f};$$

$$\frac{1}{-10.0\,\text{cm}}+\frac{1}{d_{iB}}=\frac{1}{25.0\,\text{cm}},$$

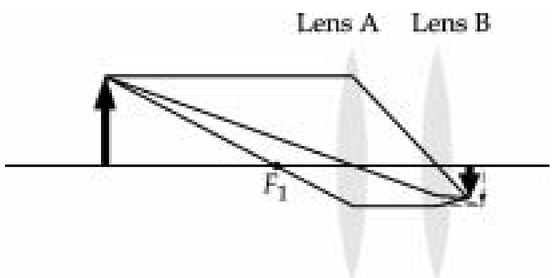
which gives $d_{iB} = \boxed{7.14\,\text{cm}}$ as the distance of the final image from lens B.

(*b*) The total magnification is the product of the magnifications for the two lenses:

$$m = m_1 m_2 = \left(-\frac{d_{iA}}{d_{oA}}\right)\left(-\frac{d_{iB}}{d_{oB}}\right) = \frac{d_{iA} d_{iB}}{d_{oA} d_{oB}}$$

$$= \frac{(30.0\,\text{cm})(7.14\,\text{cm})}{(60.0\,\text{cm})(-10.0\,\text{cm})} = \boxed{-0.357.}$$

(*c*)




62. Working backwards, we use

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2}$$

with $d_{i2} = -\frac{1}{2}(30.0\,\text{cm}) = -15.0\,\text{cm}$ and $f_2 = 20.0\,\text{cm}$:

$$\frac{1}{d_{o2}} + \frac{1}{-15.0\,\text{cm}} = \frac{1}{20.0\,\text{cm}} \text{ gives } d_{o2} = 8.57\,\text{cm}.$$

This means that $d_{i1} = 30.0\,\text{cm} - 8.57\,\text{cm} = 21.43\,\text{cm}.$ Now we find $d_{o1}$:

$$\frac{1}{d_{o1}} + \frac{1}{d_{o2}} = \frac{1}{f_1};$$

$$\frac{1}{d_{o1}} + \frac{1}{21.43\,\text{cm}} = \frac{1}{15.0\,\text{cm}}, \text{ which gives } d_{o1} = 50.0\,\text{cm}.$$

The original object is $\boxed{50.0\text{ cm}}$ to the left of the 15.0-cm-focal-length lens.

63. We find the image formed by the converging lens:

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1};$$

$$\frac{1}{33\,\text{cm}} + \frac{1}{d_{i1}} = \frac{1}{18\,\text{cm}}, \text{ which gives } d_{i1} = 39.6\,\text{cm}.$$

(*a*) The image from the first lens becomes the object for the second lens, with $d_{o2} = 12\,\text{cm} - 39.6\,\text{cm} = -27.6\,\text{cm}$ (27.6 cm to the right of the second lens). Now we find the image formed by the second lens:

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2};$$

$$\frac{1}{-27.6\,\text{cm}} + \frac{1}{d_{i2}} = \frac{1}{-14\,\text{cm}} \text{ gives } d_{i2} = -28.4\,\text{cm},$$ which means that the final image is 28 cm to the left of the second lens, or $28\,\text{cm} - 12\,\text{cm} = \boxed{16\,\text{cm to the left of the converging lens.}}$

(*b*) Now $d_{o2} = 38\,\text{cm} - 39.6\,\text{cm} = -1.6\,\text{cm}$, and

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \text{ yields}$$

$$\frac{1}{-1.6\,\text{cm}} + \frac{1}{d_{i2}} = \frac{1}{-14\,\text{cm}}, \text{ which gives } d_{i2} = 1.8\,\text{cm}.$$

Now the final image is $\boxed{\text{1.8 cm to the right of the diverging lens.}}$

64. (*a*) For the first lens, we have

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1};$$

$$\frac{1}{60.0\,\text{cm}} + \frac{1}{d_{i1}} = \frac{1}{20.0\,\text{cm}}, \text{ so that } d_{i1} = 30.0\,\text{cm}.$$

The image from the first lens becomes the object for the second lens, with $d_{o2} = 25.0\,\text{cm} - 30.0\,\text{cm} = -5.0\,\text{cm}$ (5.0 cm to the right of the diverging lens). Then for the second lens we have

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2};$$

$$\frac{1}{-5.0\,\text{cm}} + \frac{1}{d_{i2}} = \frac{1}{-10.0\,\text{cm}}, \text{ which gives } d_{i2} = 10\,\text{cm}.$$

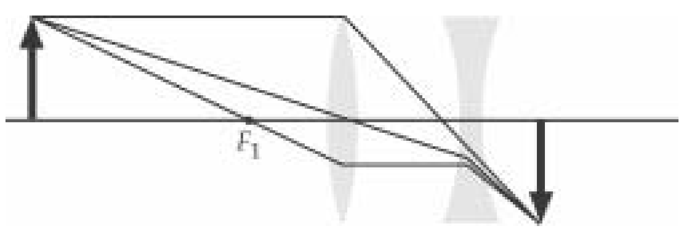
The final image is $\boxed{\text{10 cm to the right of the diverging lens.}}$

(*b*) The total magnification is the product of the magnifications for the two lenses:

$$m = m_1 m_2 = \left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right) = \frac{d_{i1}d_{i2}}{d_{o1}d_{o2}}$$

$$= \frac{(30.0\,\text{cm})(10\,\text{cm})}{(60.0\,\text{cm})(-5.0\,\text{cm})} = \boxed{-1.0.}$$

(*c*)




65. We find the focal length by finding the image distance for an object very far away.
For the first converging lens, we have

$$\left(\frac{1}{d_{o1}}\right) + \left(\frac{1}{d_{i1}}\right) = \frac{1}{f_C};$$

$$\left(\frac{1}{\infty}\right) + \left(\frac{1}{d_{i1}}\right) = \frac{1}{f_C}, \text{ or, as expected, } d_{i1} = f_C.$$

The first image is the object for the second lens. If the first image is real, the second object distance is negative:

$$d_{o2} = -d_{i1} = -f_C.$$

For the second diverging lens, we have

$$\left(\frac{1}{d_{o2}}\right) + \left(\frac{1}{d_{i2}}\right) = \frac{1}{f_D};$$

$$\left[\frac{1}{(-f_C)}\right] + \left(\frac{1}{d_{i2}}\right) = \frac{1}{f_D}.$$

Because the second image must be at the focal point of the combination, we have

$$\left(\frac{-1}{f_C}\right) + \left(\frac{1}{f_T}\right) = \frac{1}{f_D}, \text{ which gives } \frac{1}{f_D} = \left(\frac{1}{f_T}\right) - \left(\frac{1}{f_C}\right).$$

66. We find the focal length of the lens from

$$\frac{1}{f} = (n-1)\left[\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right)\right]$$

$$= (1.52-1)\left\{\left[\frac{1}{(-34.2\,\text{cm})}\right] + \left[\frac{1}{(-23.8\,\text{cm})}\right]\right\}, \text{ which gives } \boxed{f = -27.0\,\text{cm.}}$$

67. We find the index from the lensmaker's equation:

$$\frac{1}{f} = (n-1)\left[\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right)\right];$$

$$\frac{1}{28.9\,\text{cm}} = (n-1)\left[\left(\frac{1}{31.0\,\text{cm}}\right) + \left(\frac{1}{31.0\,\text{cm}}\right)\right], \text{ which gives } n = \boxed{1.54.}$$

68. We find the radius from the lensmaker's equation:

$$\frac{1}{f} = (n-1)\left[\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right)\right];$$

$$\frac{1}{(-23.4\,\text{cm})} = (1.50-1)\left[\left(\frac{1}{\infty}\right) + \left(\frac{1}{R_2}\right)\right], \text{ which gives } R_2 = \boxed{-11.7\,\text{cm.}}$$

The negative sign indicates concave.

69. We find the focal length from the lensmaker's equation, using $n = 1.51$ for Lucite:

$$\frac{1}{f} = (n-1)\left[\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right)\right];$$

$$\frac{1}{f} = (1.51-1)\left[\left(\frac{1}{\infty}\right) + \left(\frac{1}{-18.4\,\text{cm}}\right)\right], \text{ which gives } f = \boxed{-36.1\text{cm.}}$$

70. We find the radius from the lensmaker's equation, with $R_1 = R_2 = R$:

$$\frac{1}{f} = (n-1)\left[\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right)\right];$$

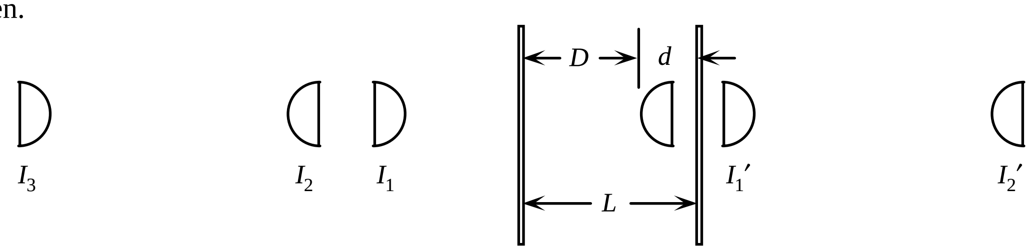
$$\frac{1}{25.0\,\text{cm}} = (1.52-1)\left(\frac{2}{R}\right), \text{ which gives } R = \boxed{26\,\text{cm.}}$$

71. We find the radius from the lensmaker's equation:

$$\frac{1}{f}=(n-1)\left[\left(\frac{1}{R_1}\right)+\left(\frac{1}{R_2}\right)\right];$$

$$+1.50\,\mathrm{D}=(1.56-1)\left[\left(\frac{1}{40.0\,\mathrm{cm}}\right)+\left(\frac{1}{R_2}\right)\right],\text{ which gives } R_2=\boxed{5.6\,\mathrm{m}\,(\text{convex}).}$$

72. For a plane mirror each image is as far behind the mirror as the object is in front. Each reflection produces a front-to-back reversal. We show the three images and the two intermediate images that are not seen.




(*a*) The first image is from a single reflection, so it is

$$d_1=2D=2(1.5\,\mathrm{m})=\boxed{3.0\,\mathrm{m}}\text{ away.}$$

The second image is from two reflections, so it is

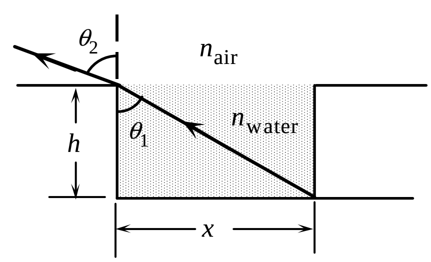
$$d_2=L+d+D=2.0\,\mathrm{m}+0.5\,\mathrm{m}+1.5\,\mathrm{m}=\boxed{4.0\,\mathrm{m}}\text{ away.}$$

The third image is from three reflections, so it is

$$d_3=2L+D+D=2(2.0\,\mathrm{m})+1.5\,\mathrm{m}+1.5\,\mathrm{m}=\boxed{7.0\,\mathrm{m}}\text{ away.}$$

(*b*) We see from the diagram that
the first image is facing $\boxed{\text{toward you;}}$
the second image is facing $\boxed{\text{away from you;}}$
the third image is facing $\boxed{\text{toward you.}}$

73. We find the angle of incidence for the refraction from water into air:




$$n_{\text{water}}\sin\theta_1=n_{\text{air}}\sin\theta_2;$$

$$(1.33)\sin\theta_1=(1.00)\sin(90°-14°),$$

which gives $\theta_1=47°$.

We find the depth of the pool from

$$\tan\theta_1=\frac{x}{h};$$

$$\tan 47°=\frac{(5.50\,\mathrm{m})}{h},\text{ which gives } h=\boxed{5.2\,\mathrm{m}.}$$

74. At the critical angle, the refracted angle is 90°. For the refraction from plastic to air, we have

$$n_{\text{plastic}}\sin\theta_{\text{plastic}}=n_{\text{air}}\sin\theta_{\text{air}};$$

$$n_{\text{plastic}}\sin 37.3°=(1.00)\sin 90°,\text{ which gives } n_{\text{plastic}}=1.65.$$

For the refraction from plastic to water, we have

$$n_{\text{plastic}}\sin\theta_{\text{plastic}}'=n_{\text{water}}\sin\theta_{\text{water}};$$

$$(1.65)\sin\theta_{\text{plastic}}'=(1.33)\sin 90°,\text{ which gives } \theta_{\text{plastic}}'=\boxed{53.7°.}$$

75. (*a*) As the radius of a sphere gets larger, the surface is flatter. The plane mirror can be considered a spherical mirror with an infinite radius, and thus $\boxed{f=\infty.}$

(*b*) When we use the mirror equation, we get

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

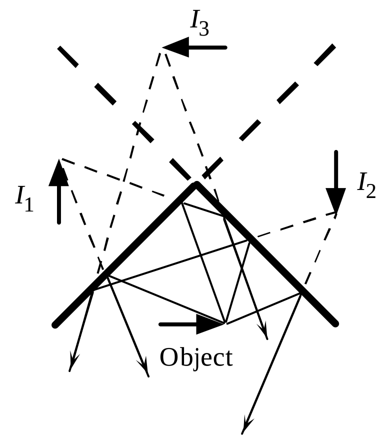
$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{\infty}=0,\text{ or }\boxed{d_i=-d_o.}$$

(*c*) For the magnification, we have

$$m=\frac{-d_i}{d_o}=\frac{-(-d_o)}{d_o}=\boxed{+1.}$$

(*d*) $\boxed{\text{Yes,}}$ these are consistent with the discussion on plane mirrors.

76. We show some of the rays from the tip of the arrow that form the three images. Single reflections form the two side images. Double reflections form the third image. The two reflections have reversed the orientation of the image.




77. We get an expression for the image distance from the lens equation:

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\frac{1}{d_i}=\left(\frac{1}{f}\right)-\left(\frac{1}{d_o}\right),\text{ or }d_i=\frac{fd_o}{(d_o-f)}.$$

If the lens is diverging, $f<0$. If we write $f=-|f|$, we get $d_i=\dfrac{-|f|d_o}{(d_o+|f|)}$.

For a real object, $d_o>0$.

All factors in the expression for $d_i$ are positive, thus $d_i<0$, so the image is always virtual.

We can have a real image, $d_i>0$, if $d_o<0$, and $|d_o|<|f|$, so the denominator is still positive. Thus to have a real image from a diverging lens, the condition is $\boxed{0<-d_o<-f.}$

78. The two students $\boxed{\text{chose different signs for the magnification,}}$ i.e., one upright and one inverted.

The focal length of the concave mirror is $f=\dfrac{R}{2}=\dfrac{(40\,\text{cm})}{2}=20\,\text{cm}.$

We relate the object and image distances from the magnification:

$$m = \frac{-d_\text{i}}{d_\text{o}};$$

$$\pm 3 = \frac{-d_\text{i}}{d_\text{o}}, \text{ which gives } d_\text{i} = \mp 3d_\text{o}.$$

When we use this in the mirror equation, we get

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f};$$

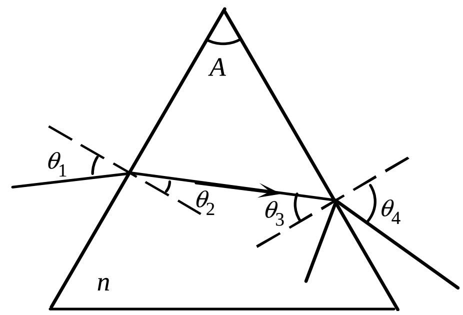
$$\left(\frac{1}{d_\text{o}}\right) + \left[\frac{1}{(\mp 3d_\text{o})}\right] = \frac{1}{f}, \text{ which gives } \boxed{d_\text{o} = \frac{2f}{3}, \frac{4f}{3} = 13.3\,\text{cm}, 26.7\,\text{cm}.}$$

The image distances are = – 40 cm (virtual, upright), and + 80 cm (real, inverted).

79. For the refraction at the second surface, we have

$$n \sin\theta_3 = n_\text{air} \sin\theta_4;$$

$$(1.50)\sin\theta_3 = (1.00)\sin\theta_4.$$




The maximum value of $\theta_4$ before internal reflection takes place at the second surface is 90°. Thus for internal reflection not to occur, we have

$$(1.50)\sin\theta_3 \le 1.00;$$

$$\sin\theta_3 \le 0.667, \text{ so } \theta_3 \le 41.8°.$$

We find the refraction angle at the second surface from

$$(90° - \theta_2) + (90° - \theta_3) + A = 180°, \text{ which gives}$$

$$\theta_2 = A - \theta_3 = 72° - \theta_3.$$

Thus $\theta_2 \ge 72° - 41.8° = 30.2°$.

For the refraction at the first surface, we have

$$n_\text{air} \sin\theta_1 = n \sin\theta_2;$$

$$(1.00)\sin\theta_1 = (1.50)\sin\theta_2; \text{ which gives } \sin\theta_1 = (1.50)\sin\theta_2.$$

For the limiting condition, we have

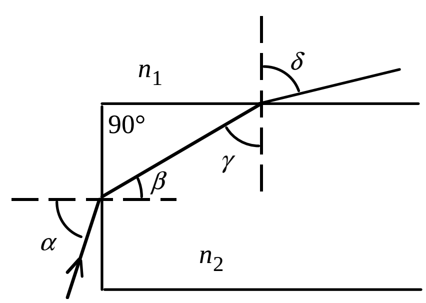
$$\sin\theta_1 \ge (1.50)\sin 30.2° = 0.754, \text{ so } \theta_1 \boxed{\ge 49°.}$$

80. If total internal reflection fails at all, it fails for $\alpha \approx 90°$. So assume $\alpha = 90°$. Then

$$n_1 \sin\alpha = n_2 \sin\beta$$

becomes

$$n_1 = n_2 \sin\beta.$$




We combine this with

$$n_2 \sin\gamma = n_1 \sin\delta$$

as follows. Since $\beta + \gamma = 90°$,

$$\sin\beta = \cos\gamma = \sqrt{1 - \sin^2\gamma} = \sqrt{1 - \left(\frac{n_1}{n_2}\sin\delta\right)^2},$$

and so

$$n_1 = n_2\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\delta};$$

$$1 = \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\delta};$$

$$\sin^2\delta = \left(\frac{n_2}{n_1}\right)^2 - 1.$$

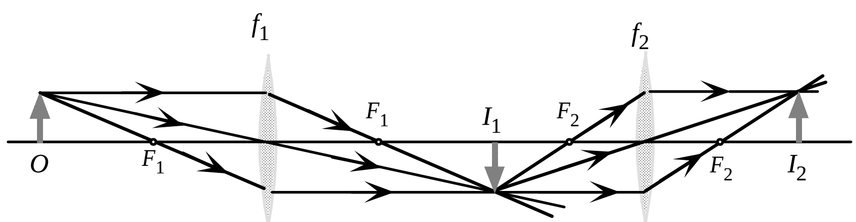
For total internal reflection to fail,

$$\sin\delta < 1;\ \sin^2\delta;$$

$$\left(\frac{n_2}{n_1}\right)^2 - 1 < 1;\ \left(\frac{n_2}{n_1}\right)^2 < 2.$$

This is not true when the rod $(n_2 = 1.54)$ is in air $(n_1 = 1.00)$, but it is when the rod is in water $(n_1 = 1.33)$. So with the rod in water, total internal reflection is not guaranteed.

81. (*a*)




We see that the image is real and upright, and estimate that it is 20 cm beyond the second lens.

(*b*) We find the image formed by the refraction of the first lens:

$$\left(\frac{1}{d_{o1}}\right) + \left(\frac{1}{d_{i1}}\right) = \frac{1}{f_1};$$

$$\left(\frac{1}{33\,\text{cm}}\right) + \left(\frac{1}{d_{i1}}\right) = \frac{1}{15\,\text{cm}},\ \text{which gives } d_{i1} = +27.5\,\text{cm}.$$

This image is the object for the second lens. Because it is in front of the second lens, it is a real object, with an object distance of $d_{o2} = 55\,\text{cm} - 27.5\,\text{cm} = 27.5\,\text{cm}$.

We find the image formed by the refraction of the second lens:

$$\left(\frac{1}{d_{o2}}\right) + \left(\frac{1}{d_{i2}}\right) = \frac{1}{f_2};$$

$$\left(\frac{1}{27.5\,\text{cm}}\right) + \left(\frac{1}{d_{i2}}\right) = \frac{1}{12\,\text{cm}},\ \text{which gives } d_{i2} = +21.3\,\text{cm}.$$

Thus the final image is $\boxed{\text{real, 21 cm beyond second lens.}}$

The total magnification is the product of the magnifications for the two lenses:

$$m = m_1 m_2 = \left(\frac{-d_{i1}}{d_{o1}}\right)\left(\frac{-d_{i2}}{d_{o2}}\right) = \frac{d_{i1}d_{i2}}{d_{o1}d_{o2}}$$

$$= \frac{(+27.5\,\text{cm})(+21.3\,\text{cm})}{(+33\,\text{cm})(+27.5\,\text{cm})} = +0.65.$$

Thus the final image is $\boxed{\text{0.65 times the size of the original object.}}$

82. (*a*) We use the lens equation with $d_\text{o} + d_\text{i} = d_\text{T}$:

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f};$$

$$\left(\frac{1}{d_o}\right) + \left[\frac{1}{(d_\text{T} - d_\text{o})}\right] = \frac{1}{f}.$$

When we rearrange this, we get a quadratic equation for $d_\text{o}$:

$d_\text{o}^2 - d_\text{T} d_\text{o} + d_\text{T} f = 0,$ which has the solution

$$d_\text{o} = \tfrac{1}{2}\left[d_\text{T} \pm \left(d_\text{T}^2 - 4 d_\text{T} f\right)^{\frac{1}{2}}\right].$$

If $d_\text{T} > 4f$, we see that the term inside the square root $d_\text{T}^2 - 4d_\text{T} f > 0$, and $\left(d_\text{T}^2 - 4d_\text{T} f\right)^{\frac{1}{2}} < d_\text{T}$, so we get two real, positive solutions for $d_\text{o}$.

(*b*) If $d_\text{T} < 4f$, we see that the term inside the square root $d_\text{T}^2 - 4d_\text{T} f < 0$, so there are no real solutions for $d_\text{o}$.

(*c*) When there are two solutions, the distance between them is

$$\Delta d = d_{\text{o}1} - d_{\text{o}2} = \tfrac{1}{2}\left[d_\text{T} + \left(d_\text{T}^2 - 4d_\text{T} f\right)^{\frac{1}{2}}\right] - \tfrac{1}{2}\left[d_\text{T} + \left(d_\text{T}^2 - 4d_\text{T} f\right)^{\frac{1}{2}}\right] = \boxed{\left(d_\text{T}^2 - 4d_\text{T} f\right)^{\frac{1}{2}}.}$$

The image positions are given by

$$d_\text{i} = d_\text{T} - d_\text{o} = \tfrac{1}{2}\left[d_\text{T} \mp \left(d_\text{T}^2 - 4d_\text{T} f\right)^{\frac{1}{2}}\right].$$

The ratio of image sizes is the ratio of magnifications:

$$m = \frac{m_2}{m_1} = \frac{\left(\dfrac{d_{\text{i}2}}{d_{\text{o}2}}\right)}{\left(\dfrac{d_{\text{i}1}}{d_{\text{o}1}}\right)} = \left(\frac{d_{\text{i}2}}{d_{\text{o}2}}\right)\left(\frac{d_{\text{o}1}}{d_{\text{i}1}}\right)$$

$$= \left\{\frac{\tfrac{1}{2}\left[d_\text{T} + \left(d_\text{T}^2 - 4d_\text{T} f\right)^{\frac{1}{2}}\right]}{\tfrac{1}{2}\left[d_\text{T} - \left(d_\text{T}^2 - 4d_\text{T} f\right)^{\frac{1}{2}}\right]}\right\}^2$$

$$= \boxed{\left\{\frac{\left[d_\text{T} + \left(d_\text{T}^2 - 4d_\text{T} f\right)^{\frac{1}{2}}\right]}{\left[d_\text{T} - \left(d_\text{T}^2 - 4d_\text{T} f\right)^{\frac{1}{2}}\right]}\right\}^2.}$$

83. We find the object distance from

$$\left(\frac{1}{d_\mathrm{o}}\right)+\left(\frac{1}{d_\mathrm{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{d_\mathrm{o}}\right)+\left(\frac{1}{8.00\times10^3\,\mathrm{mm}}\right)=\frac{1}{105\,\mathrm{mm}},\text{ which gives } d_\mathrm{o}=106\,\mathrm{mm}=\boxed{0.106\,\mathrm{m}.}$$

We find the size of the image from

$$m=\frac{h_\mathrm{i}}{h_\mathrm{o}}=\frac{-d_\mathrm{i}}{d_\mathrm{o}};$$

$$\frac{h_\mathrm{i}}{(0.036\,\mathrm{m})}=\frac{-(8.00\,\mathrm{m})}{(0.106\,\mathrm{m})},\text{ which gives } |h_\mathrm{i}|=\boxed{2.7\,\mathrm{m}.}$$

84. We find the object distance from the required magnification (which is negative for a real object and a real image):

$$m=\frac{h_\mathrm{i}}{h_\mathrm{o}}=\frac{-d_\mathrm{i}}{d_\mathrm{o}};$$

$$\frac{-(2.7\times10^3\,\mathrm{mm})}{(36\,\mathrm{mm})}=\frac{-(7.50\,\mathrm{m})}{d_\mathrm{o}},\text{ which gives } d_\mathrm{o}=0.10\,\mathrm{m}.$$

We find the focal length of the lens from

$$\left(\frac{1}{d_\mathrm{o}}\right)+\left(\frac{1}{d_\mathrm{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{0.10\,\mathrm{m}}\right)+\left(\frac{1}{7.50\,\mathrm{m}}\right)=\frac{1}{f},\text{ which gives } f=0.0987\,\mathrm{m}=\boxed{+9.9\,\mathrm{cm}.}$$

85. We get an expression for the image distance from the lens equation:

$$\left(\frac{1}{d_\mathrm{o}}\right)+\left(\frac{1}{d_\mathrm{i}}\right)=\frac{1}{f};$$

$$\frac{1}{d_\mathrm{i}}=\left(\frac{1}{f}\right)-\left(\frac{1}{d_\mathrm{o}}\right),\text{ or } d_\mathrm{i}=\frac{fd_\mathrm{o}}{(d_\mathrm{o}-f)}.$$

The magnification is

$$m=\frac{-d_\mathrm{i}}{d_\mathrm{o}}=\frac{-f}{(d_\mathrm{o}-f)}.$$

If the lens is converging, $f>0$.

For a real object, $d_\mathrm{o}>0$.

When $d_\mathrm{o}>f$, we have $(d_\mathrm{o}-f)>0$, so all factors in the expressions for $d_\mathrm{i}$ and $m$ are positive; thus $d_\mathrm{i}>0$ (real), and $m<0$ (inverted).

When $d_\mathrm{o}<f$, we have $(d_\mathrm{o}-f)<0$, so the denominator in the expressions for $d_\mathrm{i}$ and $m$ are negative;

thus $d_\mathrm{i}<0$ (virtual), and $m>0$ (upright).

For an object beyond the lens, $d_\text{o} > 0.$

When $-d_\text{o} > f$, we have $\left(d_\text{o} - f\right) < 0$, so both numerator and denominator in the expression for $d_\text{i}$ are negative; thus $d_\text{i} > 0$, so the image is $\boxed{\text{real.}}$ The numerator in the expression for *m* is negative; thus $m > 0$, so the image is $\boxed{\text{upright.}}$

When $0 < -d_\text{o} < f$, we have $\left(d_\text{o} - f\right) < 0$, so we get the same result: $\boxed{\text{real and upright.}}$

86. We will use two equations:

$$m = \frac{h_\text{i}}{h_\text{o}} = -\frac{d_\text{i}}{d_\text{o}} \qquad (1)$$

and

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f}. \qquad (2)$$

From (1),

$$-\frac{1}{m} = -\frac{h_\text{o}}{h_\text{i}} = \frac{d_\text{o}}{d_\text{i}},$$

and from (2),

$$\frac{1}{d_\text{i}} = \frac{1}{f} - \frac{1}{d_\text{o}}.$$

Therefore,

$$-\frac{h_\text{o}}{h_\text{i}} = d_\text{o}\left(\frac{1}{f} - \frac{1}{d_\text{o}}\right)$$
$$= \frac{d_\text{o}}{f} - 1; \text{ and so}$$
$$d_\text{o} = f\left(1 - \frac{h_\text{o}}{h_\text{i}}\right)$$
$$= \left(210\,\text{mm}\right)\left(1 - \frac{1.75\,\text{m}}{-0.00825\,\text{m}}\right)$$
$$= 44{,}755\,\text{mm}.$$

The reporter was standing $\boxed{45\text{ m}}$ from the subject.

87. (*a*) Because the Sun is very far away, the image will be at the focal point. We find the size of the image from

$$m = \frac{h_\text{i}}{h_\text{o}} = \frac{-d_\text{i}}{d_\text{o}};$$

$$\frac{h_\text{i}}{\left(1.4 \times 10^6\text{ km}\right)} = \frac{-\left(28\,\text{mm}\right)}{\left(1.5 \times 10^8\,\text{km}\right)}, \text{ which gives } h_\text{i} = \boxed{-0.26\,\text{mm.}}$$

(*b*) For a 50 mm lens, we have

$$\frac{h_\text{i}}{\left(1.4 \times 10^6\text{ km}\right)} = \frac{-\left(50\,\text{mm}\right)}{\left(1.5 \times 10^8\,\text{km}\right)}, \text{ which gives } h_\text{i} = \boxed{-0.47\,\text{mm.}}$$

(*c*) For a 135 mm lens, we have

$$\frac{h_{\text{i}}}{\left(1.4\times 10^{6}\text{ km}\right)}=\frac{-\left(135\text{ mm}\right)}{\left(1.5\times 10^{8}\text{ km}\right)},\text{ which gives } h_{\text{i}}=\boxed{-1.3\text{ mm.}}$$

(*d*) The equations show that image height is directly proportional to focal length. Relative to the 50 mm lens, the magnifications of the other two lenses are $\frac{28\text{ mm}}{50\text{ mm}}=\boxed{0.56}$ for the 28 mm lens and $\frac{135\text{ mm}}{50\text{ mm}}=\boxed{2.7}$ for the 135 mm lens.

88. (*a*) We find the focal length of the lens from

$$\left(\frac{1}{d_{\text{o}}}\right)+\left(\frac{1}{d_{\text{i}}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{34.5\text{ cm}}\right)+\left[\frac{1}{\left(-8.20\text{ cm}\right)}\right]=\frac{1}{f},\text{ which gives } f=\boxed{-10.8\text{ cm (diverging).}}$$

The image is in front of the lens, so it is $\boxed{\text{virtual.}}$

(*b*) We find the focal length of the lens from

$$\left(\frac{1}{d_{\text{o}}}\right)+\left(\frac{1}{d_{\text{i}}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{34.5\text{ cm}}\right)+\left[\frac{1}{\left(-41.5\text{ cm}\right)}\right]=\frac{1}{f},\text{ which gives } f=\boxed{+205\text{ cm (converging).}}$$

The image is in front of the lens, so it is virtual.

89. If we let $d_{\text{i}}$ represent the original image distance, then $d_{\text{i}}+10.0\text{ cm}$ represents the new image distance. From the general equation

$$\frac{1}{d_{\text{o}}}+\frac{1}{d_{\text{i}}}=\frac{1}{f}$$

we can then obtain two equations:

$$\frac{1}{60.0\text{ cm}}+\frac{1}{d_{\text{i}}}=\frac{1}{f}\text{ and}$$

$$\frac{1}{40.0\text{ cm}}+\frac{1}{d_{\text{i}}+10.0\text{ cm}}=\frac{1}{f}.$$

Setting the left sides equal, we solve

$$\frac{1}{60.0\text{ cm}}+\frac{1}{d_{\text{i}}}=\frac{1}{40.0\text{ cm}}+\frac{1}{d_{\text{i}}+10.0\text{ cm}},$$

which gives $d_{\text{i}}=-40.0\text{ cm or }30.0\text{ cm},$

where the latter is the solution corresponding to a real image. Now we solve

$$\frac{1}{60.0\text{ cm}}+\frac{1}{30.0\text{ cm}}=\frac{1}{f}$$

to find that $f=\boxed{20.0\text{ cm.}}$

90. Working backwards, we use

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2}$$

with $d_{i2}=17.0\,\text{cm}$ and $f_2=12.0\,\text{cm}$:

$$\frac{1}{d_{o2}}+\frac{1}{17.0\,\text{cm}}=\frac{1}{12.0\,\text{cm}} \text{ gives } d_{o2}=40.8\,\text{cm}.$$

This means that $d_{i1}=30.0\,\text{cm}-40.8\,\text{cm}=-10.8\,\text{cm}$ (10.8 to the left of the diverging lens). So for the diverging lens,

$$\left[\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1};\right]$$

$$\frac{1}{25.0\,\text{cm}}+\frac{1}{-10.8\,\text{cm}}=\frac{1}{f_1}, \text{ which gives } f_1=\boxed{19\,\text{cm.}}$$

91. From the given information,

$$m=-\frac{1}{2}=-\frac{d_i}{d_o};$$

$$d_o=2d_i.$$

So for $d_o=15\,\text{cm}$, $\boxed{d_i=7.5\,\text{cm.}}$ Now we find the focal length:

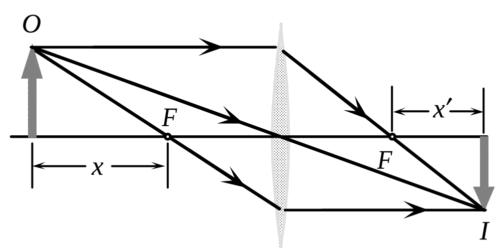
$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f};$$

$$\frac{1}{15\,\text{cm}}+\frac{1}{7.5\,\text{cm}}=\frac{1}{f}, \text{ which gives } f=5.0\,\text{cm}.$$

And since

$$f=\frac{r}{2}, \text{ it follows that } \boxed{r=10\,\text{cm.}}$$

92. (*a*) For the thin lens we have




$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(f+x)}\right]+\left[\frac{1}{(f+x')}\right]=\frac{1}{f},$$

which can be written as

$$2f+x+x'=\frac{(f+x)(f+x')}{f}$$

$$=f+(x+x')+\left(\frac{xx'}{f}\right), \text{ or } xx'=f^2.$$

(*b*) For the standard form we have

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{45.0\,\text{cm}}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{32.0\,\text{cm}}, \text{which gives } d_i=\boxed{+110.8\,\text{cm.}}$$

(*c*) For the Newtonian form we have

$$xx' = f^2;$$

$$(45.0\,\text{cm} - 32.0\,\text{cm})\,x' = (32.0\,\text{cm})^2, \text{ which gives } x' = 78.7\,\text{cm}.$$

Thus the distance from the lens is

$$d_\text{i} = x' + f = 78.7\,\text{cm} + 32.0\,\text{cm} = \boxed{110.8\,\text{cm}.}$$

93. We find the focal length by finding the image distance for an object very far away.
For the first lens, we have

$$\left(\frac{1}{d_{\text{o}1}}\right) + \left(\frac{1}{d_{\text{i}1}}\right) = \frac{1}{f_1};$$

$$\left(\frac{1}{\infty}\right) + \left(\frac{1}{d_{\text{i}1}}\right) = \frac{1}{(10.0\,\text{cm})}, \text{ or, as expected, } d_{\text{i}1} = 10.0\,\text{cm}.$$

The first image is the object for the second lens. The first image is real, so the second object has a negative object distance:

$$d_{\text{o}2} = -d_{\text{i}1} = -10.0\,\text{cm}.$$

For the second lens, we have

$$\left(\frac{1}{d_{\text{o}2}}\right) + \left(\frac{1}{d_{\text{i}2}}\right) = \frac{1}{f_2};$$

$$\left[\frac{1}{(-10.0\,\text{cm})}\right] + \left(\frac{1}{d_{\text{i}2}}\right) = \frac{1}{(-20.0\,\text{cm})}, \text{ which gives } d_{\text{i}2} = +20.0\,\text{cm}.$$

Because the second image must be at the focal point of the combination, we have

$$\boxed{f = +20.0\,\text{cm (converging).}}$$

94. (*a*) We find the focal length by finding the image distance for an object very far away.
For the first lens, we have

$$\left(\frac{1}{d_{\text{o}1}}\right) + \left(\frac{1}{d_{\text{i}1}}\right) = \frac{1}{f_1};$$

$$\left(\frac{1}{\infty}\right) + \left(\frac{1}{d_{\text{i}1}}\right) = \frac{1}{f_1}, \text{ or, as expected, } d_{\text{i}1} = f_1.$$

The first image is the object for the second lens. If the first image is real, the second object is virtual:

$$d_{\text{o}2} = -d_{\text{i}1} = -f_1.$$

For the second lens, we have

$$\left(\frac{1}{d_{\text{o}2}}\right) + \left(\frac{1}{d_{\text{i}2}}\right) = \frac{1}{f_2};$$

$$\left[\frac{1}{(-f_1)}\right] + \left(\frac{1}{d_{\text{i}2}}\right) = \frac{1}{f_2}.$$

Because the second image must be at the focal point of the combination, we have

$$\left(\frac{-1}{f_1}\right)+\left(\frac{1}{f_{\mathrm{T}}}\right)=\frac{1}{f_2}, \text{ which gives } \frac{1}{f_{\mathrm{T}}}=\left(\frac{1}{f_1}\right)+\left(\frac{1}{f_2}\right).$$

When we solve for $f_{\mathrm{T}}$, we get

$$f_{\mathrm{T}}=\frac{f_1 f_2}{(f_1+f_2)}.$$

(*b*) If we use the intermediate result $\frac{1}{f_{\mathrm{T}}}=\left(\frac{1}{f_1}\right)+\left(\frac{1}{f_2}\right)$, we see that

$$P=P_1+P_1.$$