CHAPTER 24: The Wave Nature of Light

Answers to Questions

1. Huygens' principle applies both to sound waves and water waves. Huygens' principle applies to all waves that form a wave crest. Both sounds and water waves can be represented in this way.

2. A piece of evidence that light is energy is that you can focus the light from the Sun with a magnifying glass on to a sheet of paper and burn a hole in it. You have added so much energy to the paper that its temperature rises to the point where it ignites.

3. There are certain situations where describing light as rays works well (for example, lenses) and there are other situations where describing light as waves works well (for example, diffraction). Actually, the ray model doesn't work at all when describing diffraction. Thus, there are always limitations to the "models" we use to describe nature and we need to realize what these are.

4. The main reason that we can hear sounds around corners, but not see around corners, is diffraction. Sound waves have very long wavelengths when compared to light waves, which makes diffraction effects much more obvious. Diffraction effects are very noticeable once the size of the object that the wave is diffracting around is about the same size as the wavelength of the wave. The wavelength of sound is on the order of 1 m, while the wavelength of light is on the order of 0.1 μm. A secondary reason is reflection. Sounds waves reflect off of walls very well in a specular manner, and so can bounce around corners, but light reflects off of the walls in a very diffuse manner.

$\boxed{5.}$ The wavelength of light in a medium such as water is decreased when compared to the wavelength in air. Thus, $d \sin\theta = m\lambda$ says that θ is decreased for a particular *m* and *d*. This means that the bright spots on the screen are more closely packed together in water than in air.

6. As red light is switched to blue light, the wavelength of the light is decreased. Thus, $d \sin\theta = m\lambda$ says that θ is decreased for a particular *m* and *d*. This means that the bright spots on the screen are more closely packed together with blue light than with red light.

$\boxed{7.}$ Destructive interference occurs when the path lengths of two rays of light from the same source differ by odd half-integers of the wavelength ($\lambda/2$, $3\lambda/2$, $5\lambda/2$, $\left(m+\frac{1}{2}\right)\lambda$, etc.). Under these conditions, the wave crests from one ray match up with the wave troughs from the other ray and cancellation occurs (destructive interference).

8. One reason was that the double-slit experiment allowed scientists to measure an actual wavelength, which was something that could not be done at the time with diffraction observations. Another reason was that using a particle model, you could explain diffraction in a qualitative way by talking about the particles bouncing off the edges of the diffracting object or the two edges of a single slit. When a second slit is added, the particle model had a much more difficult time explaining why there are now dark spots where the particles used to be able to strike with only one slit open.

9. Similarities between doing a double-slit experiment with sound and light: the sources must be coherent for the interference pattern to be observed; they both produce a pattern of high and low intensity at some distance away from the double slit (bright and dark for the light and loud and quiet for the sound); they both work best with a single-frequency source. Differences between doing a double-slit experiment with

sound and light: The slits for light must be extremely close together when compared to sound; you don't actually need slits for sound (just use two speakers).

$\boxed{10.}$ The reason you do not get an interference pattern from the two headlights of a distant car is that they are not coherent light sources (they have random phases). Thus, you cannot produce zones of destructive and constructive interference where the crests and troughs match up or the crests and crests match up. Also, the headlights are far enough apart that even if they were coherent, the interference pattern would be so tightly packed that it would not be observable with the unaided eye.

11. Basically there would be two overlapping single-slit diffraction patterns, with one being blue and the other being red. Wherever the two different patterns overlapped, there would be a purplish (red + blue) bright spot. There would not be a central bright spot, due to the different wavelengths of the two colors.

12. The two faces of the pane of glass are parallel to each other, so the white light rays that were separated into colors as they entered the front face all come out of the second face at the same angle as each other. When these rays reach your eyes, they all combine back together into white light. The separated colors in the prism reach the other face of the prism at all different angles (since the second face of the prism is not parallel to the first face), which means that all of these different colors leave the prism at all different angles. These light rays are still separated into colors when they reach your eyes, which allows you to see a rainbow of colors at different angles.

13. Since red light is bent less than violet light in glass (the index of refraction for red light in glass is less than the index of refraction for violet light in glass), the focal length of both a converging lens and a diverging lens is longer for red light and shorter for violet light.

14. By looking at the direction and the relative amount that the light rays bend at each interface, we can infer the relative sizes of the indices of refraction in the different materials (bends toward normal = faster material to slower material or smaller n material to larger n material; bends away from normal = slower material to faster material or larger n material to smaller n material). From the first material to the second material the ray bends toward the normal, thus it slows down and $n_1 < n_2$. From the second material to the third material the ray bends away from the normal, thus it speeds up and $n_2 > n_3$. Careful inspection shows that the ray in the third material does not bend back away from the normal as far as the ray was in the first material, thus the speed in the first material is the faster than in the third material and $n_1 < n_3$. Thus, the overall ranking of indices of refraction is: $n_1 < n_3 < n_2$.

15. As you squeeze your fingers together, you start to see vertical bright and dark bands that are aligned parallel to your fingers. The dark bands get wider as you continue to bring your fingers closer together, until at one point, when your fingers are still not actually touching, the dark bands seem to quickly jump in and darken the entire gap.

16. (*a*) When you immerse a single-slit diffraction apparatus in water, the wavelength of light gets smaller, due to the increase in the index of refraction, and the diffraction pattern gets more closely spaced. The equation $\sin\theta = \frac{\lambda}{D}$ for the half-width of the central maximum says that θ is decreased for a particular D when the wavelength decreases. And the equation for the locations of the minima, $\sin\theta = \frac{m\lambda}{D}$, also indicates that θ is decreased for a particular m and D when the wavelength decreases. This means that the bright spots on the screen are more closely spaced in water than in air.

(*b*) When you perform a single-slit diffraction experiment in vacuum, the wavelength of light gets slightly larger, due to the decrease in the index of refraction, and the diffraction pattern spreads farther apart. The equation $\sin\theta = \dfrac{\lambda}{D}$ for the half-width of the central maximum says that θ is increased for a particular *D* when the wavelength increases. And the equation for the locations of the minima, $\sin\theta = \dfrac{m\lambda}{D}$, also indicates that θ is increased for a particular *m* and *D* when the wavelength increases. This means that the bright spots on the screen are spread farther apart in vacuum than in air.

17. (*a*) When you increase the slit width in a single-slit diffraction experiment, the spacing of the fringes decreases. The equation for the location of the minima, $\sin\theta = \dfrac{m\lambda}{D}$, indicates that θ is decreased for a particular *m* and λ when the width *D* increases. This means that the bright spots on the screen are more closely packed together for a wider slit.
(*b*) When you increase the wavelength of light used in a single-slit diffraction experiment, the spacing of the fringes increases. The equation for the location of the minima, $\sin\theta = \dfrac{m\lambda}{D}$, indicates that θ is increased for a particular *m* and *D* when the wavelength increases. This means that the bright spots on the screen are spread further apart for a longer wavelength.

18. The interference pattern created by the diffraction grating with 10^4 lines/cm has bright maxima that are more sharply defined and narrower than the interference pattern created by the two slits 10^{-4} cm apart.

19. (*a*) The advantage of having many slits in a diffraction grating is that this makes the bright maxima in the interference pattern more sharply defined, brighter, and narrower.
(*b*) The advantage of having closely spaced slits in a diffraction grating is that this spreads out the bright maxima in the interference pattern and makes them easier to measure.

20. (*a*) The color at the top of the rainbow for the diffraction grating is violet. The equation $d\sin\theta = m\lambda$ says that θ is smallest (thus, the deviation from horizontal is smallest) for the shortest wavelength, for a given *d* and *m*. The wavelength of violet light (450 nm) is shorter than that of red light (700 nm).
(*b*) The color at the top of a rainbow for the prism is red. The index of refraction for transparent materials (like the glass that makes up the prism) is smaller for long (red) wavelengths and larger for short (violet) wavelengths. Since the red light encounters a smaller index of refraction as it goes through the prism, it doesn't slow as much as the violet light, which also means that it doesn't bend as much as the violet. If the red light is bent away from the horizontal direction least, it will appear at the top of the rainbow.

21. For the red and violet colors from different orders to overlap, the angles in the equation $d\sin\theta = m\lambda$ would need to be equal. Mathematically, this means: $\dfrac{(\mathrm{m}+1)(400\text{ nm})}{\mathrm{d}} < \dfrac{\mathrm{m}(700\text{ nm})}{\mathrm{d}}$.
In other words, the m^{th} order 700 nm line overlaps the $(m+1)^{\text{th}}$ order 400 nm line. Reducing this equation, we get $m > 4/3$. So, starting with the 2^{nd} order 700 nm line and the 3^{rd} order 400 nm line, the observed spectra will always overlap at this and higher orders. This answer does not depend on *d*. As you can see from the above equation, the slit spacing cancels out.

22. Once the thickness of the film becomes more than a few wavelengths thick, several interference patterns become mixed together, and it is hard to see any individual effects. When the thickness of the film is only about 1 λ thick, then the reflections from the top and bottom surfaces of the film for each color have path differences of just one constructive interference (path difference = $\lambda/2$) and one destructive interference (path difference = λ) patterns. It is easy for our eyes to pick out these widely spaced bright colors that are separated by dark areas on the film. Once the film gets very thick, though, there are many constructive ($\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc.) and many destructive (λ, 2λ, 3λ, etc.) path differences allowed. The resulting interference patterns are all closely spaced and overlapping, making it difficult for our eyes to distinguish between the bright and dark areas. As the film gets even thicker, the larger amount of overlap causes all the colors to run together, making it impossible to see the individual interference patterns.

23. There are many, many circular tracks on a CD and each track is made up of a series of pits and high spots. Light reflects very well off of the high spots and not the low spots. Thus, when you shine white light on a CD, each track is a slightly different distance from you, and as the light reflects off of each track to you, they each have a different path length. Thus, you'll basically see a colorful diffraction grating pattern. If a monochromatic light is used, you will see a single-color interference pattern. In other words, instead of seeing the full rainbow of colors spreading out from the center of the CD (as shown in Figure 24-56), there would just be several "spokes" of the same color as your source spreading out from the center of the CD. The spacings of these "spokes" can be used just like a diffraction grating to determine that the track spacing on the CD is approximately 1600 nm.

24. As you move farther away from the center of the curved piece of glass on top, the path differences change more rapidly due to the curvature. Thus, you get higher order interference patterns more closely spaced together. An air wedge has equally spaced interference patterns because as you move farther from the contact point of the flat piece of glass on top, the path differences change linearly.

25. If yellow/green light is getting reflected back to us, then the coating must be designed to transmit violet, blue, orange, and red light completely.

26. At the edge of the oil drop, the film is so thin that the path difference between the light reflecting off of the top surface and the light going through the oil and reflecting off of the bottom surface is so small that we can consider it to be zero. Thus, the two different rays of light must be in phase when they reach our eyes. We know that the phase of the light being reflected off of the top surface of the oil must have been flipped 180°, since the index of refraction of oil is greater than that of air. Thus, the reflection off of the bottom surface of the film (where it touches the water), must also have flipped the phase of the light 180°. This tells us that the index of refraction of the water is higher than that of the oil. Thus, we know that the index of refraction of the oil is greater than that of air and less than that of water: $1.00 < n < 1.33$.

$\boxed{27.}$ Polarization tells us that light is a transverse wave. Longitudinal waves cannot be polarized.

28. Polarized sunglasses completely (100%) block horizontally polarized glare and block all other polarizations of light 50%. Regular sunglasses just block 50-75% of all light coming in. The advantage of polarized sunglasses is the total elimination of glare. Even if regular sunglasses block a glare at 75%, the glare is so intense that it still makes it difficult for our eyes.

$\boxed{29.}$ To determine if a pair of sunglasses is polarizing or not, first find the glare (strong reflection) from a light source on a horizontal non-metallic surface (water, tiled floor, polished tabletop, top of car). Then look at the reflection through the sunglasses and rotate them. If they are polarizing sunglasses, then there should be orientations of the glasses that almost completely block out the glare and other orientations that let 50% of the glare through. If they are not polarizing sunglasses, the glare will be slightly diminished by the same amount at all orientations of the glasses, but it will never completely go away.

30. The first sheet of polarizer will diminish the intensity of the incoming non-polarized light by 50%: $I_1 = \frac{1}{2}I_0$. The next polarizer will diminish the light again, but this time only by a factor of $\cos^2\theta$: $I_1 = I_1\cos^2 30^\circ = \frac{3}{4}I_1 = \frac{3}{8}I_0$. The next polarizer will diminish the light by the same factor: $I_3 = I_2\cos^2 30^\circ = \frac{3}{4}I_2 = \frac{9}{32}I_0$. The last polarizer will again diminish the light by the same factor: $I_4 = I_3\cos^2 30^\circ = \frac{3}{4}I_3 = \frac{27}{128}I_0 = 0.211I_0$. Thus, about 21% of the light gets through these four polarizers when each one is rotated by 30°.

31. If Earth had no atmosphere, the "color" of the sky would be black (and dotted with stars and planets) at all times. This is the condition of the sky that the astronauts found on the Moon, which has no atmosphere. The reason the sky is blue for Earth, is that the air molecules scatter light from the Sun in all directions, and preferentially scatter blue light down to the surface. If there were no air molecules to scatter the light from the Sun, the only light we would see would be from the stars/planets and directly from the Sun and the rest would be black.

32. If the atmosphere were 50% more dense, sunlight would be much redder than it is now. As the atmosphere increased in density, more and more of the blue light would be scattered away in all directions, making the light that reaches the ground very red. Think of the color of a deep red sunset, but this would be the color even at noon.

Solutions to Problems

1. For constructive interference, the path difference is a multiple of the wavelength:
   $$d\sin\theta = m\lambda,\quad m = 0, 1, 2, 3, \ldots.$$
   For the fifth order, we have
   $$\left(1.6\times10^{-5}\,\text{m}\right)\sin 8.8^\circ = (5)\lambda, \text{ which gives } \lambda = \boxed{4.9\times10^{-7}\,\text{m.}}$$

2. For constructive interference, the path difference is a multiple of the wavelength:
   $$d\sin\theta = m\lambda,\quad m = 0, 1, 2, 3, \ldots.$$
   For the third order, we have
   $$d\sin 18^\circ = (3)\left(610\times10^{-9}\,\text{m}\right), \text{ which gives } d = 5.9\times10^{-6}\,\text{m} = \boxed{5.9\,\mu\text{m.}}$$

3. For constructive interference, the path difference is a multiple of the wavelength:
   $$d\sin\theta = m\lambda,\quad m = 0, 1, 2, 3, \ldots.$$
   We find the location on the screen from
   $$y = L\tan\theta.$$
   For small angles, we have
   $$\sin\theta \approx \tan\theta, \text{ which gives}$$
   $$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For adjacent fringes, $\Delta m = 1$, so we have

$$\Delta y = \frac{L\lambda\,\Delta m}{d};$$

$$0.065\,\text{m} = \frac{(5.00\,\text{m})\lambda(1)}{(0.048\times 10^{-3}\,\text{m})}, \text{ which gives } \lambda = \boxed{6.2\times 10^{-7}\,\text{m.}}$$

The frequency is

$$f = \frac{c}{\lambda} = \frac{(3.00\times 10^{8}\,\text{m/s})}{(6.24\times 10^{-7}\,\text{m})} = \boxed{4.8\times 10^{14}\,\text{Hz.}}$$

4. For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\quad m = 0, 1, 2, 3, \ldots .$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta, \text{ which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For adjacent fringes, $\Delta m = 1$, so we have

$$\Delta y = \frac{L\lambda\,\Delta m}{d};$$

$$= \frac{(3.6\,\text{m})(656\times 10^{-9}\,\text{m})(1)}{(0.060\times 10^{-3}\,\text{m})} = 3.9\times 10^{-2}\,\text{m} = \boxed{3.9\text{ cm.}}$$

$\boxed{5.}$ For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\quad m = 0, 1, 2, 3, \ldots .$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta, \text{ which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For the fourth order we have

$$48\times 10^{-3}\,\text{m} = \frac{(1.5\,\text{m})(680\times 10^{-9}\,\text{m})(4)}{d}, \text{ which gives } d = 8.5\times 10^{-5}\,\text{m} = \boxed{0.085\text{ mm.}}$$

6. For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\quad m = 0, 1, 2, 3, \ldots .$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta, \text{ which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For the second order of the two wavelengths, we have

$$y_a = \frac{L\lambda_a m}{d} = \frac{(1.6\,\text{m})(480\times10^{-9}\,\text{m})(2)}{(0.54\times10^{-3}\,\text{m})} = 2.84\times10^{-3}\,\text{m} = 2.84\,\text{mm};$$

$$y_b = \frac{L\lambda_b m}{d} = \frac{(1.6\,\text{m})(620\times10^{-9}\,\text{m})(2)}{(0.54\times10^{-3}\,\text{m})} = 3.67\times10^{-3}\,\text{m} = 3.67\,\text{mm}.$$

Thus the two fringes are separated by $3.67\,\text{mm} - 2.84\,\text{mm} = \boxed{0.8\text{ mm.}}$

7. For constructive interference of the second order for the blue light, we have

$$d\sin\theta = m\lambda_b = (2)(460\,\text{nm}) = 920\,\text{nm}.$$

For destructive interference of the other light, we have

$$d\sin\theta = \left(m' + \tfrac{1}{2}\right)\lambda,\ m' = 0, 1, 2, 3, \dots .$$

When the two angles are equal, we get

$$920\,\text{nm} = \left(m' + \tfrac{1}{2}\right)\lambda,\ m' = 0, 1, 2, 3, \dots .$$

For the first three values of m', we get

$$920\,\text{nm} = \left(0 + \tfrac{1}{2}\right)\lambda,\ \text{which gives}\ \lambda = 1.84\times10^{3}\ \text{nm};$$

$$920\,\text{nm} = \left(1 + \tfrac{1}{2}\right)\lambda,\ \text{which gives}\ \lambda = 613\,\text{nm};$$

$$920\,\text{nm} = \left(2 + \tfrac{1}{2}\right)\lambda,\ \text{which gives}\ \lambda = 368\,\text{nm}.$$

The only one of these that is visible light is $\boxed{613\text{ nm.}}$

8. For destructive interference, the path difference is

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, 3, \dots;\ \text{or}$$

$$\sin\theta = \frac{\left(m + \frac{1}{2}\right)(2.5\,\text{cm})}{(5.0\,\text{cm})} = \left(m + \tfrac{1}{2}\right)(0.50),\ m = 0, 1, 2, 3, \dots .$$

The angles for the first three regions of complete destructive interference are

$$\sin\theta_0 = \frac{\left(m + \frac{1}{2}\right)\lambda}{d} = \left(0 + \tfrac{1}{2}\right)(0.50) = 0.25,\ \theta_0 = 15^\circ;$$

$$\sin\theta_1 = \frac{\left(m + \frac{1}{2}\right)\lambda}{d} = \left(1 + \tfrac{1}{2}\right)(0.50) = 0.75,\ \theta_1 = 49^\circ;$$

$$\sin\theta_2 = \frac{\left(m + \frac{1}{2}\right)\lambda}{d} = \left(2 + \tfrac{1}{2}\right)(0.50) = 1.25,\ \text{therefore, no third region.}$$

We find the locations at the end of the tank from

$$y = L\tan\theta;$$

$$y_0 = (2.0\,\text{m})\tan 15^\circ = 0.52\,\text{m};$$

$$y_1 = (2.0\,\text{m})\tan 49^\circ = 2.3\,\text{m}.$$

Thus you could stand

$\boxed{\text{0.52 m, or 2.3 m away from the line perpendicular to the board midway between the openings.}}$

9. The 180° phase shift produced by the glass is equivalent to a path length of $\frac{1}{2}\lambda$. For constructive interference on the screen, the total path difference is a multiple of the wavelength:

$$\tfrac{1}{2}\lambda + d\sin\theta_{\max} = m\lambda,\ m = 0, \pm 1, \pm 2, \pm 3, \ldots;\ \text{or}\ d\sin\theta = \left(m - \tfrac{1}{2}\right)\lambda,\ m = 0, \pm 1, \pm 2, \pm 3, \ldots.$$

For destructive interference on the screen, the total path difference is

$$\tfrac{1}{2}\lambda + d\sin\theta_{\max} = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, \pm 1, \pm 2, \pm 3, \ldots;\ \text{or}\ d\sin\theta = m\lambda,\ m = 0, \pm 1, \pm 2, \pm 3, \ldots.$$

Thus the pattern is just the $\boxed{\text{reverse of the usual double-slit pattern.}}$

10. For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots.$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta,\ \text{which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For the third order we have

$$12\times 10^{-3}\,\text{m} = \frac{(3)(1.6\,\text{m})\left(500\times 10^{-9}\,\text{m}\right)}{d},\ \text{which gives}\ d = 2.0\times 10^{-4}\,\text{m}.$$

With the new wavelength, then, the second-order maximum is located a distance of

$$y = \frac{mL\lambda}{d} = \frac{(2)(1.6\,\text{m})\left(650\times 10^{-9}\,\text{m}\right)}{2.0\times 10^{-4}\,\text{m}} = 0.010\,\text{m} = \boxed{10\,\text{mm}}\ \text{from the central maximum.}$$

11. For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots.$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta,\ \text{which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For adjacent fringes, $\Delta m = 1$, so we have

$$\Delta y = \frac{L\lambda\,\Delta m}{d} = \frac{(5.0\,\text{m})\left(544\times 10^{-9}\,\text{m}\right)(1)}{1.0\times 10^{-3}\,\text{m}} = 2.7\times 10^{-3}\,\text{m} = \boxed{2.7\,\text{mm.}}$$

12. The presence of the water changes the wavelength: $\lambda_{\text{water}} = \dfrac{\lambda}{n_{\text{water}}} = \dfrac{480\,\text{nm}}{1.33} = 361\,\text{nm}.$

For constructive interference, the path difference is a multiple of the wavelength in the water:

$$d\sin\theta = m\lambda_{\text{water}},\ m = 0,1,2,3,\ldots.$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$\sin\theta \approx \tan\theta$, which gives

$$y = L\left(\frac{m\lambda_{\text{water}}}{d}\right) = \frac{mL\lambda_{\text{water}}}{d}.$$

For adjacent fringes, $\Delta m = 1$, so we have

$$\Delta y = \frac{L\lambda_{\text{water}}\,\Delta m}{d}$$

$$= \frac{(0.400\,\text{m})(361\times10^{-9}\,\text{m})(1)}{(6.00\times10^{-5}\,\text{m})} = 2.41\times10^{-3}\,\text{m} = \boxed{2.41\,\text{mm.}}$$

13. To change the center point from constructive interference to destructive interference, the phase shift produced by the introduction of the plastic must be an odd multiple of half a wavelength, corresponding to the change in the number of wavelengths in the distance equal to the thickness of the plastic. The minimum thickness will be for a shift of a half wavelength:

$$N = \left(\frac{t}{\lambda_{\text{plastic}}}\right) - \left(\frac{t}{\lambda}\right) = \left(\frac{tn_{\text{plastic}}}{\lambda}\right) - \left(\frac{t}{\lambda}\right) = \left(\frac{t}{\lambda}\right)(n_{\text{plastic}} - 1) = \tfrac{1}{2};$$

$$\left[\frac{t}{(640\,\text{nm})}\right](1.60 - 1) = \tfrac{1}{2},\ \text{which gives } t = \boxed{533\,\text{nm.}}$$

14. We find the speed of light from the index of refraction, $v = \dfrac{c}{n}$. For the change, we have

$$\frac{(v_{\text{red}} - v_{\text{violet}})}{v_{\text{violet}}} = \frac{\left[\left(\dfrac{c}{n_{\text{red}}}\right) - \left(\dfrac{c}{n_{\text{violet}}}\right)\right]}{\left(\dfrac{c}{n_{\text{violet}}}\right)}$$

$$= \frac{(n_{\text{violet}} - n_{\text{red}})}{n_{\text{red}}} = \frac{(1.665 - 1.617)}{(1.617)} = 0.030 = \boxed{3.0\%.}$$

$\boxed{15.}$ We find the angles of refraction in the glass from

$$n_1\sin\theta_1 = n_2\sin\theta_2;$$

$$(1.00)\sin 60.00^\circ = (1.4820)\sin\theta_{2,450},\ \text{which gives } \theta_{2,450} = 35.76^\circ;$$

$$(1.00)\sin 60.00^\circ = (1.4742)\sin\theta_{2,700},\ \text{which gives } \theta_{2,700} = 35.98^\circ.$$

Thus the angle between the refracted beams is

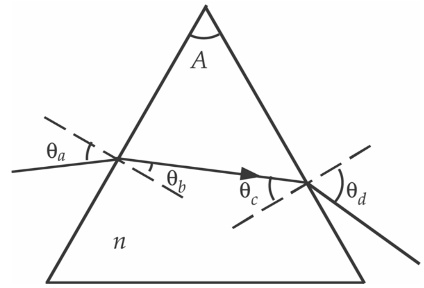
$$\theta_{2,700} - \theta_{2,450} = 35.98^\circ - 35.76^\circ = \boxed{0.22^\circ.}$$

16. For the refraction at the first surface, we have

$n_{\text{air}} \sin\theta_a = n\sin\theta_b;$

$(1.00)\sin 45° = (1.642)\sin\theta_{b1},$ which gives $\theta_{b1} = 25.51°;$

$(1.00)\sin 45° = (1.619)\sin\theta_{b2},$ which gives $\theta_{b2} = 25.90°.$



We find the angle of incidence at the second surface from

$(90° - \theta_b) + (90° - \theta_c) + A = 180°,$ which gives

$\theta_{c1} = A - \theta_{b1} = 60.00° - 25.51° = 34.49°;$

$\theta_{c2} = A - \theta_{b2} = 60.00° - 25.90° = 34.10°.$

For the refraction at the second surface, we have

$n\sin\theta_c = n_{\text{air}}\sin\theta_d;$

$(1.642)\sin 34.49° = (1.00)\sin\theta_{d1},$ which gives $\boxed{\theta_{d1} = 68.4° \text{ from the normal;}}$

$(1.619)\sin 34.10° = (1.00)\sin\theta_{d2},$ which gives $\boxed{\theta_{d2} = 65.2° \text{ from the normal.}}$

17. We find the angle to the first minimum from

$$\sin\theta_{1\min} = \frac{m\lambda}{D} = \frac{(1)(580\times10^{-9}\,\text{m})}{(0.0440\times10^{-3}\,\text{m})} = 0.0132, \text{ so } \theta_{1\min} = 0.755°.$$

Thus the angular width of the central diffraction peak is

$\Delta\theta_1 = 2\theta_{1\min} = 2(0.755°) = \boxed{1.51°.}$

$\boxed{18.}$ The angle from the central maximum to the first minimum is 17.5°.
We find the wavelength from

$D\sin\theta_{1\min} = m\lambda;$

$(2.60\times10^{-6}\,\text{m})\sin(17.5°) = (1)\lambda,$ which gives $\lambda = 7.82\times10^{-7}\,\text{m} = \boxed{782\,\text{nm.}}$

19. For constructive interference from the single slit, the path difference is

$D\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 1, 2, 3, \ldots.$

For the first fringe away from the central maximum, we have

$(3.20\times10^{-6}\,\text{m})\sin\theta_1 = \left(\tfrac{3}{2}\right)(520\times10^{-9}\,\text{m}),$ which gives $\theta_1 = 14.1°.$

We find the distance on the screen from

$y_1 = L\tan\theta_1 = (10.0\,\text{m})\tan 14.1° = \boxed{2.51\,\text{m.}}$

20. We find the angle to the first minimum from

$$\sin\theta_{1\min} = \frac{m\lambda}{D} = \frac{(1)(450\times10^{-9}\,\text{m})}{1.0\times10^{-3}\,\text{m}} = 0.00045.$$

We find the distance on the screen from

$y = L\tan\theta.$

For small angles, we have

$\sin\theta \approx \tan\theta,$ which gives

$y = L\sin\theta = (5.0\,\text{m})(0.00045) = 0.00225\,\text{m}.$

Thus the width of the central maximum is

$$2y = 0.0045\,\text{m} = \boxed{0.45\,\text{cm.}}$$

21. The angle from the central maximum to the first bright fringe is $16°$.
For constructive interference from the single slit, the path difference is

$$D\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 1,\ 2,\ 3,\ \ldots.$$

For the first fringe away from the central maximum, we have

$$D\sin(16°) = \left(\tfrac{3}{2}\right)\left(653\times10^{-9}\,\text{m}\right),\ \text{which gives } D = 3.6\times10^{-6}\,\text{m} = \boxed{3.6\,\mu\text{m.}}$$

22. We find the angle to the first minimum from

$$\sin\theta_{1\min} = \frac{m\lambda}{D} = \frac{(1)\left(589\times10^{-9}\,\text{m}\right)}{\left(0.0348\times10^{-3}\,\text{m}\right)} = 0.0169,\ \text{so } \theta_{1\min} = 0.970°.$$

We find the distance on the screen from

$$y_1 = L\tan\theta_1 = (2.30\,\text{m})\tan 0.970° = 3.89\times10^{-2}\,\text{m} = 3.89\,\text{cm.}$$

Thus the width of the peak is

$$\Delta y_1 = 2y_1 = 2(3.89\,\text{cm}) = \boxed{7.79\,\text{cm.}}$$

23. We find the angular half-width θ of the central maximum from

$$\sin\theta = \frac{\lambda}{D};$$

$$\sin\left(\frac{55.0°}{2}\right) = \frac{440\times10^{-9}\,\text{m}}{D},\ \text{which gives } D = \boxed{9.53\times10^{-7}\,\text{m.}}$$

24. We find the angle to the first minimum from the distances:

$$\tan\theta_{1\min} = \tfrac{1}{2}\frac{(9.20\,\text{cm})}{(255\,\text{cm})} = 0.0180 = \sin\theta_{1\min},\ \text{because the angle is small.}$$

We find the slit width from

$$D\sin\theta_{1\min} = m\lambda;$$

$$D(0.0180) = (1)\left(415\times10^{-9}\,\text{m}\right),\ \text{which gives } D = 2.30\times10^{-5}\,\text{m} = \boxed{0.0230\,\text{mm.}}$$

25. Because the angles are small, we have

$$\tan\theta_{1\min} = \tfrac{1}{2}\frac{(\Delta y_1)}{L} = \sin\theta_{1\min}.$$

The condition for the first minimum is

$$D\sin\theta_{1\min} = \tfrac{1}{2}D\frac{\Delta y_1}{L} = \lambda.$$

If we form the ratio of the expressions for the two wavelengths, we get

$$\frac{\Delta y_{1b}}{\Delta y_{1a}} = \frac{\lambda_b}{\lambda_a};$$

$$\frac{\Delta y_{1b}}{(4.0\,\text{cm})} = \frac{(420\,\text{nm})}{(650\,\text{nm})},\ \text{which gives } \Delta y_{1b} = \boxed{2.6\,\text{cm.}}$$

26. There will be no diffraction minima if the angle for the first minimum is greater than $90°$. Thus the limiting condition is

$$D\sin\theta_{1\min} = m\lambda;$$

$$D_{\max}\sin 90° = (1)\lambda, \text{ or } \boxed{D_{\max} = \lambda.}$$

27. We find the angle for the second order from

$$d\sin\theta = m\lambda;$$

$$\left(1.45\times10^{-5}\,\text{m}\right)\sin\theta = (2)\left(560\times10^{-9}\,\text{m}\right), \text{ which gives } \sin\theta = 0.0772, \text{ so } \theta = \boxed{4.43°.}$$

28. We find the wavelength from

$$d\sin\theta = m\lambda;$$

$$\left[\frac{1}{(3500\,\text{lines/cm})}\right]\left(10^{-2}\,\text{m/cm}\right)\sin 28.0° = 3\lambda, \text{ which gives } \lambda = 4.47\times10^{-7}\,\text{m} = \boxed{447\,\text{nm.}}$$

29. We find the slit separation from

$$d\sin\theta = m\lambda;$$

$$d\sin 18.0° = (3)\left(630\times10^{-9}\,\text{m}\right), \text{ which gives } d = 6.12\times10^{-6}\,\text{m} = 6.12\times10^{-4}\,\text{cm}.$$

The number of lines/cm is

$$\frac{1}{d} = \frac{1}{\left(6.12\times10^{-4}\,\text{cm}\right)} = \boxed{1.64\times10^{3}\,\text{lines/cm.}}$$

30. Because the angle increases with wavelength, to have a complete order we use the largest wavelength. The maximum angle is 90°, so we have

$$d\sin\theta = m\lambda;$$

$$\left[\frac{1}{(8300\,\text{lines/cm})}\right]\left(10^{-2}\,\text{m/cm}\right)\sin 90° = m\left(700\times10^{-9}\,\text{m}\right), \text{ which gives } m = 1.70.$$

Thus only $\boxed{\text{one full order}}$ can be seen on each side of the central white line.

31. We find the slit separation from

$$d\sin\theta = m\lambda;$$

$$d\sin 15.5° = (1)\left(589\times10^{-9}\,\text{m}\right), \text{ which gives } d = 2.20\times10^{-6}\,\text{m} = \boxed{2.20\,\mu\text{m.}}$$

We find the angle for the fourth order from

$$d\sin\theta = m\lambda;$$

$$\left(2.20\times10^{-6}\,\text{m}\right)\sin\theta_4 = (4)\left(589\times10^{-9}\,\text{m}\right), \text{ which gives } \sin\theta_4 = 1.069, \text{ so there is } \boxed{\text{no fourth order.}}$$

32. We find the angles for the second order from

$$d\sin\theta = m\lambda \text{ with } m = 2.$$

$$\frac{1}{6.0\times10^{5}\,\text{lines/m}}\sin\theta_1 = 2\left(7.0\times10^{-7}\,\text{m}\right) \text{ gives } \sin\theta_1 = 0.84, \text{ so } \theta_1 = 57.1°.$$

$$\frac{1}{6.0\times10^{5}\,\text{lines/m}}\sin\theta_2 = 2\left(4.5\times10^{-7}\,\text{m}\right) \text{ gives } \sin\theta_2 = 0.54, \text{ so } \theta_2 = 32.7°.$$

Therefore, $\Delta\theta = 57.1° - 32.7° = \boxed{24°.}$

33. We find the wavelengths from

$$d\sin\theta = m\lambda;$$

$$\left[\frac{1}{(9700\,\text{lines/cm})}\right](10^{-2}\text{ m/cm})\sin 31.2° = (1)\lambda_1,\ \text{which gives }\ \lambda_1 = 5.34\times10^{-7}\text{ m} = \boxed{534\,\text{nm;}}$$

$$\left[\frac{1}{(9700\,\text{lines/cm})}\right](10^{-2}\text{ m/cm})\sin 36.4° = (1)\lambda_2,\ \text{which gives }\ \lambda_2 = 6.12\times10^{-7}\text{ m} = \boxed{612\,\text{nm;}}$$

$$\left[\frac{1}{(9700\,\text{lines/cm})}\right](10^{-2}\text{ m/cm})\sin 47.5° = (1)\lambda_3,\ \text{which gives }\ \lambda_3 = 7.60\times10^{-7}\text{ m} = \boxed{760\,\text{nm.}}$$

$\boxed{34.}$ The maximum angle is 90°, so we have

$$d\sin\theta = m\lambda;$$

$$\left[\frac{1}{(6000\,\text{lines/cm})}\right](10^{-2}\text{ m/cm})\sin 90° = m(633\times10^{-9}\text{ m}),\ \text{which gives }\ m = 2.63.$$

Thus $\boxed{\text{two orders}}$ can be seen on each side of the central white line.

35. Because the angle increases with wavelength, to have a full order we use the largest wavelength. The maximum angle is 90°, so we find the minimum separation from

$$d\sin\theta = m\lambda;$$

$$d_{\min}\sin 90° = (2)(750\times10^{-9}\text{ m}),\ \text{which gives }\ d_{\min} = 1.50\times10^{-6}\text{ m} = 1.50\times10^{-4}\text{ cm}.$$

The maximum number of lines/cm is

$$\frac{1}{d_{\min}} = \frac{1}{(1.50\times10^{-4}\text{ cm})} = \boxed{6.67\times10^{3}\text{ lines/cm.}}$$

36. We find the angles for the first order from

$$d\sin\theta = m\lambda = \lambda;$$

$$\left[\frac{1}{(8500\,\text{lines/cm})}\right](10^{-2}\text{ m/cm})\sin\theta_{410} = (410\times10^{-9}\text{ m}),\ \text{which gives}$$

$$\sin\theta_{410} = 0.3485,\ \text{so}\ \theta_{410} = 20.4°;$$

$$\left[\frac{1}{(8500\,\text{lines/cm})}\right](10^{-2}\text{ m/cm})\sin\theta_{750} = (750\times10^{-9}\text{ m}),\ \text{which gives}$$

$$\sin\theta_{750} = 0.6375,\ \text{so}\ \theta_{750} = 39.6°.$$

The distances from the central white line on the screen are

$$y_{410} = L\tan\theta_{410} = (2.30\text{ m})\tan 20.4° = 0.855\text{ m};$$

$$y_{750} = L\tan\theta_{750} = (2.30\text{ m})\tan 39.6° = 1.90\text{ m}.$$

Thus the width of the spectrum is

$$y_{750} - y_{410} = 1.90\text{ m} - 0.855\text{ m} = \boxed{1.05\text{ m.}}$$

37. We find the angle for the first order from

$$d\sin\theta = m\lambda = \lambda;$$

$$d\sin 21.5° = 6.328\times10^{-7}\text{ m},\ \text{which gives }\ d = 1.73\times10^{-6}\text{ m}.$$

The number of lines per meter is

$$\frac{1}{d} = \frac{1}{1.73 \times 10^{-6}\,\text{m}} = \boxed{5.79 \times 10^5\ \text{lines/m.}}$$

38. Because the angles on each side of the central line are not the same, the incident light is not normal to the grating. We use the average angles:

$$\theta_1 = \frac{(26°38' + 26°48')}{2} = 26°43' = 26.72°;$$

$$\theta_2 = \frac{(41°08' + 41°19')}{2} = 41°14' = 41.23°.$$

We find the wavelengths from

$$d \sin\theta = m\lambda;$$

$$\left[\frac{1}{(9500\,\text{lines/cm})}\right](10^{-2}\ \text{m/cm})\sin 26.72° = (1)\lambda_1,\ \text{which gives}\ \lambda_1 = 4.73 \times 10^{-7}\,\text{m} = \boxed{473\,\text{nm;}}$$

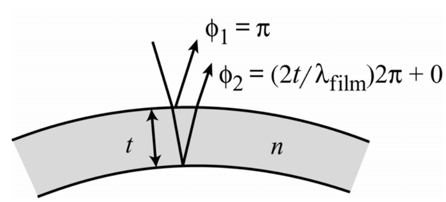
$$\left[\frac{1}{(9500\,\text{lines/cm})}\right](10^{-2}\ \text{m/cm})\sin 41.23° = (1)\lambda_2,\ \text{which gives}\ \lambda_2 = 6.94 \times 10^{-7}\,\text{m} = \boxed{694\,\text{nm.}}$$

Note that the second wavelength is not visible.

39. We equate a path difference of one wavelength with a phase difference of 2π. With respect to the incident wave, the wave that reflects at the top surface from the higher index of the soap bubble has a phase change of

$$\phi_1 = \pi.$$

With respect to the incident wave, the wave that reflects from the air at the bottom surface of the bubble has a phase change due to the additional path-length but no phase change on reflection:



$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + 0.$$

For constructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi - \pi = m2\pi,\ m = 0, 1, 2, ...,\ \text{or}\ t = \tfrac{1}{2}\lambda_{\text{film}}\left(m + \tfrac{1}{2}\right),\ m = 0, 1, 2, ...$$

The wavelengths in air that produce strong reflection are given by

$$\lambda = n\lambda_{\text{film}} = \frac{2nt}{\left(m + \frac{1}{2}\right)} = \frac{4(1.34)(120\,\text{nm})}{(2m + 1)} = \frac{(643\,\text{nm})}{(2m + 1)}.$$

Thus we see that, for the light to be in the visible spectrum, the only value of m is 0:

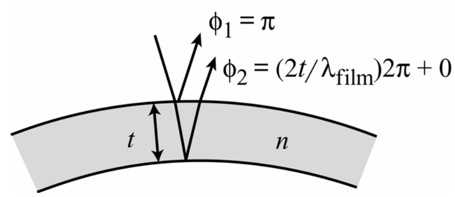
$$\lambda = \frac{(643\,\text{nm})}{(0 + 1)} = 643\,\text{nm},\ \text{which is an}\ \boxed{\text{orange-red.}}$$

40. Between the 25 dark lines there are 24 intervals. When we add the half-interval at the wire end, we have 24.5 intervals, so the separation is

$$\frac{26.5\,\text{cm}}{24.5\,\text{intervals}} = \boxed{1.08\,\text{cm.}}$$

41. We equate a path difference of one wavelength with a phase difference of 2π. With respect to the incident wave, the wave that reflects at the top surface from the higher index of the soap bubble has a phase change of

$$\phi_1 = \pi.$$



With respect to the incident wave, the wave that reflects from the air at the bottom surface of the bubble has a phase change due to the additional path-length but no phase change on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + 0.$$

For destructive interference, the net phase change is

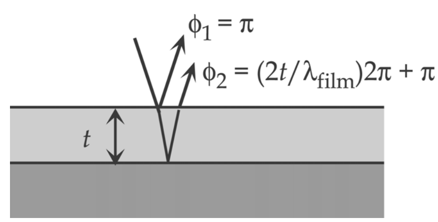
$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi - \pi = \left(m - \tfrac{1}{2}\right)2\pi,\ m = 0, 1, 2, ..., \text{ or } t = \tfrac{1}{2}\lambda_{\text{film}}m = \tfrac{1}{2}\left(\frac{\lambda}{n}\right)m,\ m = 0, 1, 2, ...$$

The minimum non-zero thickness is

$$t_{\text{min}} = \tfrac{1}{2}\left[\frac{(480\,\text{nm})}{(1.42)}\right](1) = \boxed{169\,\text{nm.}}$$

$\boxed{42.}$ With respect to the incident wave, the wave that reflects from the top surface of the coating has a phase change of

$$\phi_1 = \pi.$$



With respect to the incident wave, the wave that reflects from the glass $(n \approx 1.5)$ at the bottom surface of the coating has a phase change due to the additional path-length and a phase change of π on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi.$$

For constructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi - \pi = m2\pi,\ m = 1, 2, 3, ..., \text{ or } t = \tfrac{1}{2}\lambda_{\text{film}}m = \tfrac{1}{2}\left(\frac{\lambda}{n_{\text{film}}}\right)m,\ m = 1, 2, 3, ...$$

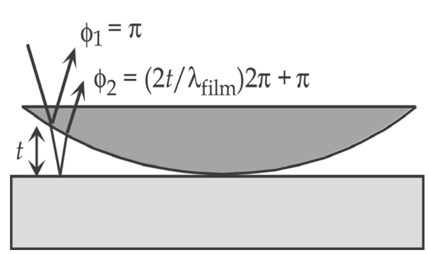
The minimum non-zero thickness occurs for $m = 1$:

$$t_{\text{min}} = \frac{\lambda}{2n_{\text{film}}} = \frac{(570\,\text{nm})}{2(1.25)} = \boxed{228\,\text{nm.}}$$

570 nm is in the middle of the visible spectrum. The transmitted light will be stronger in the wavelengths at the ends of the spectrum, so the lens would emphasize the red and violet wavelengths.

43. The phase difference for the reflected waves from the path-length difference and the reflection at the bottom surface is

$$\phi = \left(\frac{2t}{\lambda}\right)2\pi + \pi.$$



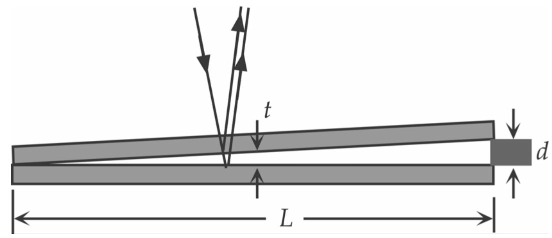
For the dark rings, this phase difference must be an odd multiple of π, so we have

$$\phi = \left(\frac{2t}{\lambda}\right)2\pi + \pi = (2m+1)\pi,\ m = 0, 1, 2, ...,\ \text{or}$$
$$t = \tfrac{1}{2}m\lambda,\ m = 0, 1, 2, ...$$

Because $m = 0$ corresponds to the dark center, m represents the number of the ring. Thus the thickness of the lens is the thickness of the air at the edge of the lens:

$$t = \tfrac{1}{2}(31)(550\,\text{nm}) = 8.5 \times 10^3\ \text{nm} = \boxed{8.5\,\mu\text{m}.}$$

44. There is a phase difference for the reflected waves from the path-length difference, $\left(\frac{2t}{\lambda}\right)2\pi,$ and the reflection at the bottom surface, π. For destructive interference, this phase difference must be an odd multiple of π, so we have



$$\phi = \left(\frac{2t}{\lambda}\right)2\pi + \pi = (2m+1)\pi,\ m = 0, 1, 2, ..., \text{or}$$
$$t = \tfrac{1}{2}m\lambda,\ m = 0, 1, 2, ...$$

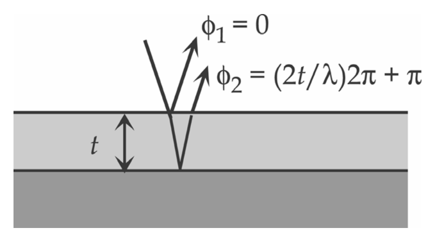
Because $m = 0$ corresponds to the edge where the glasses touch, $m + 1$ represents the number of the fringe. Thus the thickness of the foil is

$$d = \tfrac{1}{2}(27)(670\,\text{nm}) = 9.05 \times 10^3\ \text{nm} = \boxed{9.05\,\mu\text{m}.}$$

45. With respect to the incident wave, the wave that reflects from the air at the top surface of the air layer has a phase change of

$$\phi_1 = 0.$$

With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the air layer has a phase change due to the additional path-length and a change on reflection:



$$\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi.$$

For constructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda}\right)2\pi + \pi - 0 = m2\pi,\ m = 1, 2, 3, ..., \text{or } t = \tfrac{1}{2}\lambda\left(m - \tfrac{1}{2}\right),\ m = 1, 2, 3, ...$$

The minimum thickness is

$$t_{\text{min}} = \tfrac{1}{2}(450\,\text{nm})\left(1 - \tfrac{1}{2}\right) = \boxed{113\,\text{nm}.}$$

For destructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda}\right)2\pi + \pi - 0 = (2m+1)\pi,\ m = 0, 1, 2, ... ,\ \textit{or}\ t = \tfrac{1}{2}m\lambda,\ m = 0, 1, 2, ...$$

The minimum non-zero thickness is

$$t_{\text{min}} = \tfrac{1}{2}(450\,\text{nm})(1) = \boxed{225\,\text{nm}.}$$

46. The polarizing angle θ_{p} is found using

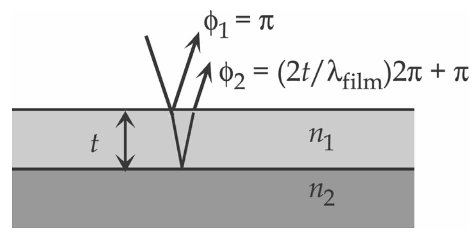
$$\tan\theta_{\text{p}} = \frac{n_2}{n_1}.$$

For an oil–diamond interface, $\tan\theta_p = \dfrac{2.42}{1.43}$, which gives $\theta_p = 59.4°$.

The material $\boxed{\text{does appear to be diamond.}}$

47. With respect to the incident wave, the wave that reflects from the top surface of the alcohol has a phase change of

$$\phi_1 = \pi.$$



With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the alcohol has a phase change due to the additional path-length and a phase change of π on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{film}}\right)2\pi + \pi.$$

For constructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{1film}}\right)2\pi + \pi - \pi = m_1 2\pi,\ m_1 = 1, 2, 3, \ldots,\ \text{or}\ t = \tfrac{1}{2}\lambda_{1film}(m_1) = \tfrac{1}{2}\left(\frac{\lambda_1}{n_{film}}\right)(m_1),\ m_1 = 1, 2, 3, \ldots.$$

For destructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{2film}}\right)2\pi + \pi - \pi = (2m_2 + 1)\pi,\ m_2 = 0, 1, 2, \ldots,\ \text{or}\ t = \tfrac{1}{4}\left(\frac{\lambda_2}{n_{film}}\right)(2m_2 + 1),\ m_2 = 0, 1, 2, \ldots.$$

When we combine the two equations, we get

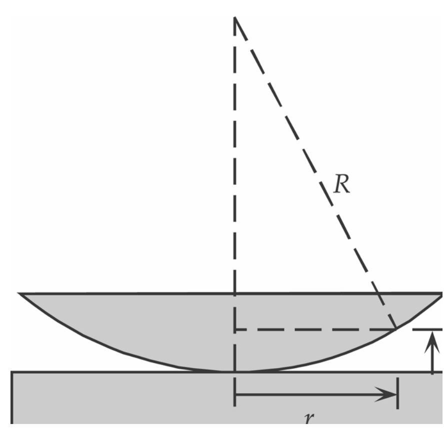
$$\tfrac{1}{2}\left(\frac{\lambda_1}{n_{film}}\right)(m_1) = \tfrac{1}{4}\left(\frac{\lambda_2}{n_{film}}\right)(2m_2 + 1),\ \text{or}\ \frac{(2m_2 + 1)}{2m_1} = \frac{\lambda_1}{\lambda_2} = \frac{(640\,\text{nm})}{(512\,\text{nm})} = 1.25 = \frac{5}{4}.$$

Thus we see that $m_1 = m_2 = 2$, and the thickness of the film is

$$t = \tfrac{1}{2}\left(\frac{\lambda_1}{n_{film}}\right)(m_1) = \tfrac{1}{2}\left[\frac{(640\,\text{nm})}{(1.36)}\right](2) = \boxed{471\,\text{nm.}}$$

48. At a distance r from the center of the lens, the thickness of the air space is y, and the phase difference for the reflected waves from the path-length difference and the reflection at the bottom surface is

$$\phi = \left(\frac{2y}{\lambda}\right)2\pi + \pi.$$



For the dark rings, we have

$$\phi = \left(\frac{2y}{\lambda}\right)2\pi + \pi = (2m + 1)\pi,\ m = 0, 1, 2, \ldots,\ \text{or}$$

$$y = \tfrac{1}{2}m\lambda,\ m = 0, 1, 2, \ldots.$$

Because $m = 0$ corresponds to the dark center, m represents the number of the ring. From the triangle in the diagram, we have

$$r^2 + (R - y)^2 = R^2,\ \text{or}\ r^2 = 2yR - y^2 \approx 2yR,\ \text{when}\ y \ll R,$$

which becomes

$$r^2 = 2\left(\tfrac{1}{2}m\lambda\right)R = m\lambda R,\ m = 0, 1, 2, \ldots.$$

When the apparatus is immersed in the liquid, the same analysis holds, if we use the wavelength in the liquid. If we form the ratio for the two conditions, we get

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{\lambda_1}{\lambda_2} = n, \text{ so}$$

$$n = \left(\frac{2.92\,\text{cm}}{2.48\,\text{cm}}\right)^2 = \boxed{1.39.}$$

49. One fringe shift corresponds to a change in path length of λ. The number of fringe shifts produced by a mirror movement of ΔL is

$$N = 2\frac{\Delta L}{\lambda};$$

$$644 = \frac{2\left(0.225 \times 10^{-3}\,\text{m}\right)}{\lambda}, \text{ which gives } \lambda = 6.99 \times 10^{-7}\,\text{m} = \boxed{699\,\text{nm.}}$$

50. One fringe shift corresponds to a change in path length of λ. The number of fringe shifts produced by a mirror movement of ΔL is

$$N = 2\frac{\Delta L}{\lambda};$$

$$272 = 2\frac{\Delta L}{\left(589\,\text{nm}\right)}, \text{ which gives } \Delta L = 8.01 \times 10^{4}\,\text{nm} = \boxed{80.1\,\mu\text{m.}}$$

$\boxed{51.}$ One fringe shift corresponds to a change in path length of λ. The number of fringe shifts produced by a mirror movement of ΔL is

$$N = 2\frac{\Delta L}{\lambda};$$

$$850 = 2\frac{\Delta L}{\left(589 \times 10^{-9}\,\text{m}\right)}, \text{ which gives } \Delta L = 2.50 \times 10^{-4}\,\text{m} = \boxed{0.250\,\text{mm.}}$$

52. One fringe shift corresponds to an effective change in path length of λ. The actual distance has not changed, but the number of wavelengths in the depth of the cavity has. If the cavity has a depth *d*, the number of wavelengths in vacuum is $\frac{d}{\lambda}$, and the number with the gas present is $\frac{d}{\lambda_{\text{gas}}} = \frac{n_{\text{gas}} d}{\lambda}$. Because the light passes through the cavity twice, the number of fringe shifts is

$$N = 2\left[\left(\frac{n_{\text{gas}} d}{\lambda}\right) - \left(\frac{d}{\lambda}\right)\right] = 2\left(\frac{d}{\lambda}\right)\left(n_{\text{gas}} - 1\right);$$

$$236 = 2\left[\frac{\left(1.30 \times 10^{-2}\,\text{m}\right)}{\left(610 \times 10^{-9}\,\text{m}\right)}\right]\left(n_{\text{gas}} - 1\right), \text{ which gives } n_{\text{gas}} = \boxed{1.00554.}$$

53. If the initial intensity is I_0, through the two sheets we have

$$I_1 = \tfrac{1}{2} I_0,$$

$$I_2 = I_1 \cos^2\theta = \tfrac{1}{2} I_0 \cos^2\theta, \text{ which gives}$$

$$\frac{I_2}{I_0} = \tfrac{1}{2}\cos^2\theta = \tfrac{1}{2}\cos^2 65° = \boxed{0.089.}$$

54. Because the light is coming from air to glass, we find the angle from the vertical from

$$\tan\theta_p = n_{glass} = 1.52, \text{ which gives } \boxed{\theta_p = 56.7°.}$$

55. Because the light is coming from water to diamond, we find the angle from the vertical from

$$\tan\theta_p = \frac{n_{diamond}}{n_{water}} = \frac{2.42}{1.33} = 1.82, \text{ which gives } \boxed{\theta_p = 61.2°.}$$

56. If I_0 is the intensity passed by the first Polaroid, the intensity passed by the second will be I_0 when the two axes are parallel. To reduce the intensity by half, we have

$$I = I_0\cos^2\theta = \tfrac{1}{2}I_0, \text{ which gives } \theta = \boxed{45°.}$$

57. If the initial intensity is I_0, through the two sheets we have

$$I_1 = \tfrac{1}{2}I_0,$$
$$I_2 = I_1\cos^2\theta; \text{ which means}$$
$$\frac{I_2}{I_0} = \tfrac{1}{2}\cos^2\theta.$$

(*a*) For $\dfrac{I_2}{I_0} = \tfrac{1}{3}$,

$$\tfrac{1}{3} = \tfrac{1}{2}\cos^2\theta \text{ gives } \theta = \boxed{35.3°.}$$

(*b*) For $\dfrac{I_2}{I_0} = \tfrac{1}{10}$,

$$\tfrac{1}{10} = \tfrac{1}{2}\cos^2\theta \text{ gives } \theta = \boxed{63.4°.}$$

58. If the initial intensity is I_0, through the two sheets we have

$$I_1 = I_0\cos^2\theta_1;$$
$$I_2 = I_1\cos^2\theta_2 = I_0\cos^2\theta_1\cos^2\theta_2;$$
$$0.15I_0 = I_0\cos^2\theta_1\cos^2 40°, \text{ which gives } \theta_1 = \boxed{60°.}$$

59. Through the successive sheets we have

$$I_1 = I_0\cos^2\theta_1,$$
$$I_2 = I_1\cos^2\theta_2, \text{ which gives}$$
$$I_2 = I_0\cos^2\theta_1\cos^2\theta_2 = I_0\left(\cos^2 19.0°\right)\left(\cos^2 38.0°\right) = 0.555I_0.$$

Thus the reduction is $\boxed{44.5\%.}$

60. If the light is coming from water to air, we find Brewster's angle from

$$\tan\theta_p = \frac{n_{air}}{n_{water}} = \frac{1.00}{1.33} = 0.752, \text{ which gives } \boxed{\theta_p = 36.9°.}$$

For the refraction at the critical angle from water to air, we have

$$n_{air}\sin\theta_1 = n_{water}\sin\theta_2;$$
$$(1.00)\sin 90° = (1.33)\sin\theta_c, \text{ which gives } \boxed{\theta_c = 48.8°.}$$

If the light is coming from air to water, we find Brewster's angle from

$$\tan\theta_{\text{p}}' = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1.00} = 1.33, \text{ which gives } \boxed{\theta_{\text{p}}' = 53.1°.}$$

Thus $\theta_{\text{p}} + \theta_{\text{p}}' = 90.0°.$

61. When plane-polarized light hits a sheet oriented at an angle θ,

$$I_2 = I_1\cos^2\theta.$$

For $\theta = 45°$,

$$\frac{I_2}{I_1} = \cos^2 45° = \tfrac{1}{2}.$$

So sheets two through five will each reduce the intensity by $\tfrac{1}{2}$.

Since the first sheet will reduce the intensity of the unpolarized incident light by $\tfrac{1}{2}$ as well, the intensity of the transmitted beam will be

$$I = I_0\left(\tfrac{1}{2}\right)^5 = \boxed{0.031 I_0.}$$

62. (*a*) For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda, \; m = 0, 1, 2, 3, \ldots .$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta, \text{ which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For adjacent bands, $\Delta m = 1$, so we have

$$\Delta y = \frac{L\lambda\Delta m}{d};$$

$$2.0\times10^{-2}\text{ m} = \frac{(4.0\text{ m})(5.0\times10^{-7}\text{ m})(1)}{d}, \text{ which gives } d = 1.0\times10^{-4}\text{ m} = \boxed{0.10\text{ mm.}}$$

(*b*) For destructive interference, the path difference is given by

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda, \; m = 0, 1, 2, 3, \ldots .$$

Again we find the location on the screen from

$$y = L\tan\theta,$$

and again we use $\sin\theta \approx \tan\theta$, this time to obtain

$$y = \frac{\left(m + \tfrac{1}{2}\right)L\lambda}{d}.$$

We are told that

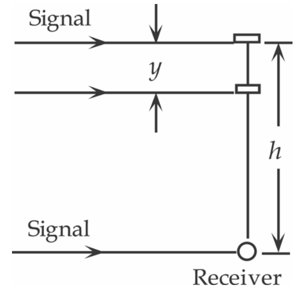
$$\frac{\left(5 + \tfrac{1}{2}\right)L\lambda_2}{d} = \frac{\left(4 + \tfrac{1}{2}\right)L\lambda_1}{d};$$

$$\left(5 + \tfrac{1}{2}\right)\lambda_2 = \left(4 + \tfrac{1}{2}\right)\left(5.0\times10^{-7}\text{ m}\right),$$

which gives $\lambda_2 = \boxed{4.1\times10^{-7}\text{ m.}}$

63. The wavelength of the signal is

$$\lambda = \frac{v}{f} = \frac{(3.00 \times 10^8 \text{ m/s})}{(75 \times 10^6 \text{ Hz})} = 4.00 \text{m}.$$



(*a*) There is a phase difference between the direct and reflected signals from the path difference, $\left(\frac{h}{\lambda}\right)2\pi$, and the reflection, π.
The total phase difference is

$$\phi = \left(\frac{h}{\lambda}\right)2\pi + \pi = \left[\frac{(118 \text{m})}{(4.00 \text{m})}\right]2\pi + \pi = 30(2\pi).$$

Thus the interference is $\boxed{\text{constructive.}}$

(*b*) When the plane is 22 m closer to the receiver, the phase difference is

$$\phi = \left[\frac{(h-y)}{\lambda}\right]2\pi + \pi$$

$$= \left[\frac{(118 \text{m} - 22 \text{m})}{(4.00 \text{m})}\right]2\pi + \pi = 24(2\pi) + \pi.$$

Thus the interference is $\boxed{\text{destructive.}}$

64. We find the angles for the first order from

$d \sin\theta = m\lambda = \lambda;$

$$\frac{1}{3.00 \times 10^5 \text{ lines/m}} \sin\theta_{\text{h}} = 6.56 \times 10^{-7} \text{ m, which gives } \sin\theta_{\text{h}} = 0.197, \text{ so } \boxed{\theta_{\text{h}} = 11.3°;}$$

$$\frac{1}{3.00 \times 10^5 \text{ lines/m}} \sin\theta_{\text{n}} = 6.50 \times 10^{-7} \text{ m, which gives } \sin\theta_{\text{n}} = 0.195, \text{ so } \boxed{\theta_{\text{n}} = 11.2°;}$$

$$\frac{1}{3.00 \times 10^5 \text{ lines/m}} \sin\theta_{\text{a}} = 6.97 \times 10^{-7} \text{ m, which gives } \sin\theta_{\text{a}} = 0.209, \text{ so } \boxed{\theta_{\text{a}} = 12.1°.}$$

65. For constructive interference, the path difference is a multiple of the wavelength:

$d \sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots .$

We find the location on the screen from

$y = L \tan\theta.$

For small angles, we have

$\sin\theta \approx \tan\theta,$ which gives

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For the second-order fringes we have

$$y_1 = \frac{2L\lambda_1}{d};$$

$$y_2 = \frac{2L\lambda_2}{d}.$$

When we subtract the two equations, we get

$$\Delta y = y_1 - y_2 = \left(\frac{2L}{d}\right)(\lambda_1 - \lambda_2);$$

$$1.33 \times 10^{-3}\ \text{m} = \left[\frac{2(1.70\,\text{m})}{(0.60 \times 10^{-3}\ \text{m})}\right](590\,\text{nm} - \lambda_2),\ \text{which gives } \lambda_2 = \boxed{355\,\text{nm.}}$$

66. The wavelength of the signal is

$$\lambda = \frac{v}{f} = \frac{(3.00 \times 10^8\ \text{m/s})}{(102.1 \times 10^6\ \text{Hz})} = 2.94\,\text{m}.$$

Because measurements are made far from the antennae, we can use the analysis for the double slit. For constructive interference, the path difference is a multiple of the wavelength:

$$d \sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots;$$

$$(8.0\,\text{m})\sin\theta_{1\max} = (1)(2.94\,\text{m}),\ \text{which gives } \theta_{1\max} = 22°;$$

$$(8.0\,\text{m})\sin\theta_{2\max} = (2)(2.94\,\text{m}),\ \text{which gives } \theta_{2\max} = 47°;$$

$$(8.0\,\text{m})\sin\theta_{3\max} = (3)(2.94\,\text{m}),\ \text{which gives } \sin\theta_{3\max} > 1,\ \text{so there is no third maximum.}$$

Because the interference pattern will be symmetrical above and below the midline and on either side of the antennae, the angles for maxima are $\boxed{\text{22°, 47°, 133°, 158° above and below the midline.}}$

For destructive interference, the path difference is an odd multiple of half a wavelength:

$$d \sin\theta = (m - \tfrac{1}{2})\lambda,\ m = 1, 2, 3, \ldots;\ \text{or}$$

$$(8.0\,\text{m})\sin\theta_{1\min} = (1 - \tfrac{1}{2})(2.94\,\text{m}),\ \text{which gives } \theta_{1\min} = 11°;$$

$$(8.0\,\text{m})\sin\theta_{2\min} = (2 - \tfrac{1}{2})(2.94\,\text{m}),\ \text{which gives } \theta_{2\min} = 33°;$$

$$(8.0\,\text{m})\sin\theta_{3\min} = (3 - \tfrac{1}{2})(2.94\,\text{m}),\ \text{which gives } \theta_{3\min} = 67°;$$

$$(8.0\,\text{m})\sin\theta_{4\min} = (4 - \tfrac{1}{2})(2.94\,\text{m}),\ \text{which gives } \sin\theta_{4\min} > 1,\ \text{so there is no fourth minimum.}$$

Because the interference pattern will be symmetrical above and below the midline and on either side of the antennae, the angles for minima are $\boxed{\text{11°, 33°, 67°, 113°, 147°, 169° above and below the midline.}}$

67. The wavelength of the sound is

$$\lambda = \frac{v}{f} = \frac{(343\,\text{m/s})}{(750\,\text{Hz})} = 0.457\,\text{m}.$$

We find the angles of the minima from

$$D \sin\theta = m\lambda,\ m = 1, 2, 3, \ldots;$$

$$(0.88\,\text{m})\sin\theta_1 = (1)(0.457\,\text{m}),\ \text{which gives } \sin\theta_1 = 0.520,\ \text{so } \theta_1 = 31°;$$

$$(0.88\,\text{m})\sin\theta_2 = (2)(0.457\,\text{m}),\ \text{which gives} \sin\theta_2 = 1.04,\ \text{so there is no } \theta_2.$$

Thus the whistle would not be heard clearly at angles of $\boxed{\text{31° on either side of the normal.}}$

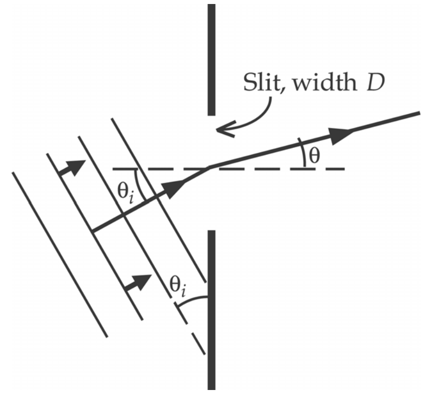
68. The path difference between the top and bottom of the slit for the incident wave is

$D\sin\theta_i$.

The path difference between the top and bottom of the slit for the diffracted wave is

$D\sin\theta$.

When $\theta = \theta_i$, the net path difference is zero, and there will be constructive interference. $\boxed{\text{There is a central maximum at } \theta = 30°.}$



When the net path difference is a multiple of a wavelength, there will be minima given by

$(D\sin\theta_i) - (D\sin\theta) = m\lambda,\ m = \pm1, \pm2, ...,$ or

$$\boxed{\sin\theta = \sin 30° - \left(\frac{m\lambda}{D}\right), \text{ where } m = \pm1, \pm2, ...}$$

69. The lines act like a grating. Assuming the first order, we find the separation of the lines from

$d\sin\theta = m\lambda;$

$d\sin 51° = (1)(460\times10^{-9}\,\text{m}),$ which gives $d = 5.9\times10^{-7}\,\text{m} = \boxed{590\,\text{nm.}}$

70. Because the angle increases with wavelength, to miss a complete order we use the smallest wavelength. The maximum angle is 90°. We find the slit separation from

$d\sin\theta = m\lambda;$

$d\sin 90° = (2)(400\times10^{-9}\,\text{m}),$ which gives $d = 8.00\times10^{-7}\,\text{m} = 8.00\times10^{-5}\,\text{cm}.$

The number of lines/cm is

$$\frac{1}{d} = \frac{1}{(8.00\times10^{-5}\,\text{cm})} = \boxed{12{,}500 \text{ lines/cm.}}$$

71. Because the angle increases with wavelength, we compare the maximum angle for the second order with the minimum angle for the third order:

$d\sin\theta = m\lambda,$ or $\sin\theta = \dfrac{\lambda}{d};$

$$\sin\theta_{2\max} = \frac{(2)(750\,\text{nm})}{d};$$

$$\sin\theta_{3\min} = \frac{(3)(400\,\text{nm})}{d}.$$

When we divide the two equations, we get

$$\frac{\sin\theta_{3\min}}{\sin\theta_{2\max}} = \frac{(1200\,\text{nm})}{(1500\,\text{nm})} = 0.8.$$

Because the value of the sine increases with angle, this means $\theta_{3\min} < \theta_{2\max}$, so the orders overlap.

To determine the overlap, we find the second-order wavelength that coincides with $\theta_{3\min}$:

$(2)\lambda_2 = (3)(400\,\text{nm}),$ which gives $\lambda_2 = 600\,\text{nm}.$

We find the third-order wavelength that coincides with $\theta_{2\max}$ from

$(2)(750\,\text{nm}) = (3)\lambda_3,$ which gives $\lambda_3 = 500\,\text{nm}.$

$\boxed{\text{Thus 600 nm to 750 nm of the second order overlaps with 400 nm to 500 nm of the third order.}}$

72. We find the angles for the first order from the distances:

$$\tan\theta_1 = \frac{y_1}{L} = \frac{(3.32\,\text{cm})}{(60.0\,\text{cm})} = 0.0553, \text{ so } \theta_1 = 3.17°;$$

$$\tan\theta_2 = \frac{y_2}{L} = \frac{(3.71\,\text{cm})}{(60.0\,\text{cm})} = 0.0618, \text{ so } \theta_2 = 3.54°.$$

We find the separation of lines from

$$d\sin\theta_1 = m\lambda_1;$$

$$d\sin 3.17° = (1)(589\times10^{-9}\text{ m}), \text{ which gives } d = 1.066\times10^{-5}\text{ m} = 1.066\times10^{-3}\text{ cm}.$$

For the second wavelength we have

$$d\sin\theta_2 = m\lambda_2;$$

$$(1.06\times10^{-5}\text{ m})\sin 3.54° = (1)\lambda_2, \text{ which gives } \lambda_2 = 6.58\times10^{-7}\text{ m} = \boxed{658\,\text{nm.}}$$

The number of lines/cm is

$$\frac{1}{d} = \frac{1}{(1.066\times10^{-3}\text{ cm})} = \boxed{938 \text{ lines/cm.}}$$

73. We find the angles for the first order from

$$d\sin\theta = m\lambda = \lambda;$$

$$\left[\frac{1}{(8600\text{ lines/cm})}\right](10^{-2}\text{ m/cm})\sin\theta_1 = 4.6\times10^{-7}\text{ m}, \text{ which gives } \sin\theta_1 = 0.396, \text{ so } \theta_1 = 23.3°;$$

$$\left[\frac{1}{(8600\text{ lines/cm})}\right](10^{-2}\text{ m/cm})\sin\theta_2 = 6.8\times10^{-7}\text{ m}, \text{ which gives } \sin\theta_2 = 0.585, \text{ so } \theta_2 = 35.8°.$$

The distances from the central white line on the screen are

$$y_1 = L\tan\theta_1 = (2.5\,\text{m})\tan 23.3° = 1.1\,\text{m};$$

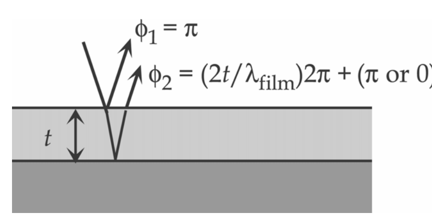
$$y_2 = L\tan\theta_2 = (2.5\,\text{m})\tan 35.8° = 1.8\,\text{m}.$$

Thus the separation of the lines is

$$y_2 - y_1 = 1.8\,\text{m} - 1.1\,\text{m} = \boxed{0.7\,\text{m.}}$$

74. We equate a path difference of one wavelength with a phase difference of 2π. With respect to the incident wave, the wave that reflects at the top surface of the film has a phase change of

$$\phi_1 = \pi.$$



If we assume that the film has an index less than glass, the wave that reflects from the glass has a phase change due to the additional path-length and a phase change on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi.$$

For destructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi - \pi = \left(m - \tfrac{1}{2}\right)2\pi,\ m = 1, 2, \ldots, \text{ or } t = \tfrac{1}{2}\lambda_{\text{film}}\left(m - \tfrac{1}{2}\right) = \tfrac{1}{2}\left(\frac{\lambda}{n}\right)\left(m - \tfrac{1}{2}\right),\ m = 1, 2, \ldots.$$

For the minimum thickness, $m = 1$, we have

$$150\,\text{nm} = \tfrac{1}{2}\left[\frac{(600\,\text{nm})}{(n)}\right]\left(1 - \tfrac{1}{2}\right),\ \text{which gives } n = 1,\ \text{so } \boxed{\text{no film with } n < n_{\text{glass}} \text{ is possible.}}$$

If we assume that the film has an index greater than glass, the wave that reflects from the glass has a phase change due to the additional path-length and no phase change on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + 0.$$

For destructive interference, the net phase change is

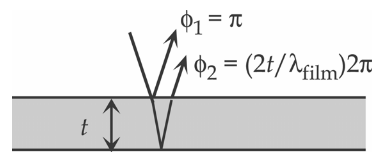
$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi - \pi = \left(m - \tfrac{1}{2}\right)2\pi,\ m = 1, 2, \ldots,\ \text{or } t = \tfrac{1}{2}\lambda_{\text{film}}m = \tfrac{1}{2}\left(\frac{m\lambda}{n}\right),\ m = 1, 2, \ldots.$$

For the minimum thickness, $m = 1$, we have

$$150\,\text{nm} = \tfrac{1}{2}\left(\frac{(1)(600\,\text{nm})}{n}\right),\ \text{which gives } n = \boxed{2.00.}$$

75. With respect to the incident wave, the wave that reflects at the top surface of the film has a phase change of

$$\phi_1 = \pi.$$



The wave that reflects from the bottom surface has a phase change due to the additional path-length and no phase change on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + 0.$$

For destructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi - \pi = \left(m - \tfrac{1}{2}\right)2\pi,\ m = 1, 2, \ldots,\ \text{or } t = \tfrac{1}{2}\lambda_{\text{film}}m = \tfrac{1}{2}\left(\frac{m\lambda}{n}\right),\ m = 1, 2, \ldots.$$

For the two wavelengths we have

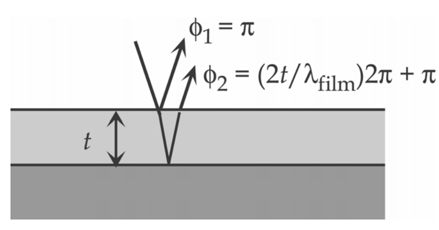
$$t = \tfrac{1}{2}\left(\frac{m_1\lambda_1}{n}\right) = \tfrac{1}{2}\left(\frac{m_2\lambda_2}{n}\right),\ \text{or } \frac{m_1}{m_2} = \frac{\lambda_2}{\lambda_1} = \frac{640\,\text{nm}}{512\,\text{nm}} = 1.25 = \frac{5}{4}.$$

Thus $m_1 = 5$, and $m_2 = 4$. For the thickness we have

$$t = \tfrac{1}{2}\left(\frac{m_1\lambda_1}{n}\right) = \tfrac{1}{2}\left[\frac{(5)(512\,\text{nm})}{1.58}\right] = \boxed{810\,\text{nm.}}$$

76. With respect to the incident wave, the wave that reflects from the top surface of the coating has a phase change of

$$\phi_1 = \pi.$$



With respect to the incident wave, the wave that reflects from the glass $(n \approx 1.5)$ at the bottom surface of the coating has a phase change due to the additional path-length and a phase change of π on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi.$$

For destructive interference, this phase difference must be an odd multiple of π, so we have

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi - \pi = (2m+1)\pi,\ m = 0, 1, 2, ...,\ \text{or}\ t = \tfrac{1}{4}(2m+1)\lambda_{\text{film}},\ m = 0, 1, 2, ...$$

Thus the minimum thickness is

$$t_{\min} = \tfrac{1}{4}\frac{\lambda}{n}.$$

(*a*) For the blue light we get

$$t_{\min} = \tfrac{1}{4}\frac{(450\,\text{nm})}{(1.38)} = \boxed{81.5\,\text{nm.}}$$

(*b*) For the red light we get

$$t_{\min} = \tfrac{1}{4}\frac{(700\,\text{nm})}{(1.38)} = \boxed{127\,\text{nm.}}$$

77. As explained in Example 24-8, the $\frac{1}{2}$-cycle phase change at the lower surface means that maximum destructive interference will occur when the thickness *t* is such that

$$2t = m\lambda,\ m = 0, 1, 2, ...$$

We set $m = 1$ to find the smallest nonzero value of *t*:

$$t = \frac{\lambda}{2} = \frac{640\,\text{nm}}{2} = \boxed{320\,\text{nm.}}$$

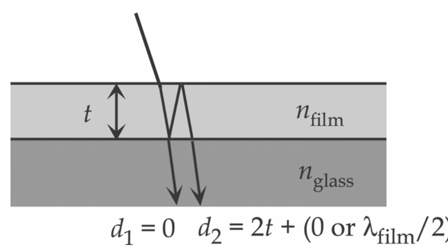
As also explained in Example 24-8, maximum constructive interference will occur when

$$2t = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, ...$$

We set $m = 0$ to find the smallest value of *t*:

$$t = \frac{\lambda}{4} = \frac{640\,\text{nm}}{4} = \boxed{160\,\text{nm.}}$$

78. If we consider the two rays shown in the diagram, we see that the second ray has reflected twice. If $n_{\text{film}} < n_{\text{glass}}$, the first reflection from the glass produces a shift equivalent to $\frac{1}{2}\lambda_{\text{film}}$, while the second reflection from the air produces no shift. When we compare the two rays at the film-glass surface, we see that the second ray has a total shift of



$$d_2 - d_1 = 2t + \tfrac{1}{2}\lambda_{\text{film}}.$$

For maxima, we have

$$2t + \tfrac{1}{2}\lambda_{\text{film}} = m\lambda_{\text{film}},\ m = 1, 2, 3, ...,\ \text{or}\ \boxed{t = \frac{\frac{1}{2}\left(m - \frac{1}{2}\right)\lambda}{n_{\text{film}}},\ m = 1, 2, 3, ...}$$

For minima, we have

$$2t + \tfrac{1}{2}\lambda_{\text{film}} = \left(m + \tfrac{1}{2}\right)\lambda_{\text{film}},\ m = 0, 1, 2, 3, ...,\ \text{or}\ \boxed{t = \frac{\frac{1}{2}m\lambda}{n_{\text{film}}},\ m = 0, 1, 2, 3, ...}$$

We see that for a film of zero thickness, that is, $t \ll \lambda_{\text{film}}$, there will be a minimum.

If $n_{\text{film}} > n_{\text{glass}}$, the first reflection from the glass produces no shift, while the second reflection from the air also produces no shift. When we compare the two rays at the film-glass surface, we see that the second ray has a total shift of

$$d_2 - d_1 = 2t.$$

For maxima, we have

$$2t = m\lambda_{\text{film}},\ m = 0, 1, 2, 3, \ldots,\ \text{or}\ \boxed{t = \frac{\frac{1}{2}m\lambda}{n_{\text{film}}},\ m = 0, 1, 2, 3, \ldots .}$$

For minima, we have

$$2t = \left(m - \tfrac{1}{2}\right)\lambda_{\text{film}},\ m = 1, 2, 3, \ldots,\ \text{or}\ \boxed{t = \frac{\frac{1}{2}\left(m - \frac{1}{2}\right)\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \ldots .}$$

We see that for a film of zero thickness, that is, $t \ll \lambda_{\text{film}}$, there will be a maximum.

79. Because the light is coming from air to water, we find the angle from the vertical from

$$\tan\theta_{\text{p}} = n_{\text{water}} = 1.33, \text{ which gives } \theta_{\text{p}} = 53.1°.$$

Thus the angle above the horizon is $90.0° - 53.1° = \boxed{36.9°.}$

80. If the original intensity is I_0, the first Polaroid sheet will reduce the intensity of the original beam to

$$I_1 = \tfrac{1}{2}I_0.$$

If the axis of the second Polaroid sheet is oriented at an angle θ, the intensity is

$$I_2 = I_1 \cos^2\theta.$$

(*a*) $I_2 = I_1 \cos^2\theta = 0.25 I_1$, which gives $\theta = \boxed{60°.}$

(*b*) $I_2 = I_1 \cos^2\theta = 0.10 I_1$, which gives $\theta = \boxed{72°.}$

(*c*) $I_2 = I_1 \cos^2\theta = 0.010 I_1$, which gives $\theta = \boxed{84°.}$

81. (*a*) $I_1 = \dfrac{I_0}{2};$

$$I_2 = I_1 \cos^2\theta_1 = \left(\frac{I_0}{2}\right)\cos^2 62°;$$

$$I_3 = I_2 \cos^2\theta_2 = \left(\frac{I_0}{2}\right)\cos^2 62° \cos^2 28° = \boxed{0.086 I_0.}$$

(*b*) If the third polarizer is placed in front of the other two, the last polarizer blocks whatever light comes through the middle one. Thus $\boxed{\text{no light gets transmitted.}}$

82. (*a*) Through the successive polarizers we have

$$I_1 = \tfrac{1}{2}I_0;$$

$$I_2 = I_1 \cos^2\theta_2 = \tfrac{1}{2}I_0 \cos^2\theta_2;$$

$$I_3 = I_2 \cos^2\theta_3 = \tfrac{1}{2}I_0 \cos^2\theta_2 \cos^2\theta_3;$$

$$I_4 = I_3 \cos^2\theta_4 = \tfrac{1}{2}I_0 \cos^2\theta_2 \cos^2\theta_3 \cos^2\theta_4 = \tfrac{1}{2}I_0 \cos^2 30° \cos^2 30° \cos^2 30° = \boxed{0.21 I_0.}$$

(*b*) If we remove the second polarizer, we get

$$I_1 = \tfrac{1}{2}I_0;$$

$$I_3 = I_1 \cos^2\theta_3' = \tfrac{1}{2}I_0 \cos^2\theta_3';$$

$$I_4 = I_3 \cos^2\theta_4 = \tfrac{1}{2}I_0 \cos^2\theta_3' \cos^2\theta_4 = \tfrac{1}{2}I_0 \cos^2 60° \cos^2 30° = 0.094 I_0.$$

Thus we can decrease the intensity by removing either the $\boxed{\text{second or third polarizer.}}$

(*c*) If we remove the $\boxed{\text{second and third polarizers,}}$ we will have two polarizers with their axes perpendicular, so no light will be transmitted.

83. We will find the width of the central maximum by first finding the angular half-width θ, using

$$\sin\theta = \frac{\lambda}{D} = \frac{630\times10^{-9}\,\text{m}}{0.010\,\text{m}} = 6.30\times10^{-5}.$$

We find the distance on the Moon's surface from

$$y = L\tan\theta.$$

For small angles, we have

$\sin\theta \approx \tan\theta$, which gives

$$y = L\sin\theta = (3.80\times10^{8}\,\text{m})(6.30\times10^{-5})2.394\times10^{4}\,\text{m}.$$

Thus the width of the central maximum is

$$2y = 4.788\times10^{4}\,\text{m} = \boxed{47.9\,\text{km.}}$$

84. If the original intensity is I_0, the first polarizers will reduce the intensity to

$$I_1 = \tfrac{1}{2}I_0.$$

Each subsequent polarizer oriented at an angle θ to the preceding one will reduce the intensity as given by the equation

$$\text{I}_{\text{i}+1} = \text{I}_{\text{i}}\cos^2\theta.$$

So for n polarizers (including the first one),

$$\text{I}_{\text{f}} = \left(\tfrac{1}{2}\text{I}_0\right)\left(\cos^2\theta\right)^{\text{n}-1}.$$

We seek n such that

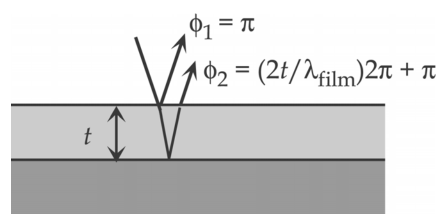
$$\tfrac{1}{4}I_0 = \left(\tfrac{1}{2}I_0\right)\left(\cos^2 10^\circ\right)^{n-1};$$

$$\tfrac{1}{2} = \left(\cos 10^\circ\right)^{2n-2};$$

$-\ln 2 = \left(2n-1\right)\ln\left(\cos 10^\circ\right)$, which gives $n = 23.6$. So $\boxed{24}$ polarizers are needed for the intensity to drop below $\tfrac{1}{4}$ of its original value.

85. With respect to the incident wave, the wave that reflects from the top surface of the film has a phase change of

$$\phi_1 = \pi.$$



With respect to the incident wave, the wave that reflects from the glass $(n = 1.52)$ at the bottom surface of the film has a phase change due to the additional path-length and a phase change of π on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi.$$

For constructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi - \pi = m2\pi,\ m = 1, 2, 3, \ldots,\ \text{or}\ t = \tfrac{1}{2}\lambda_{\text{film}}m = \tfrac{1}{2}\left(\frac{\lambda}{n_{\text{film}}}\right)m,\ m = 1, 2, 3, \ldots.$$

The minimum non-zero thickness occurs for $m = 1$:

$$t_{\text{min}} = \frac{\lambda}{2n_{film}} = \frac{643\,\text{nm}}{2(1.34)} = \boxed{240\,\text{nm.}}$$

86. Destructive interference occurs when the path difference equals an odd number of half-wavelengths.

We write the equation

$$\sqrt{y^2 + (175\,\text{m})^2} - y = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, 3, \ldots .$$

The wavelength λ is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\ \text{m/s}}{6.0 \times 10^6\ \text{Hz}} = 50\,\text{m}.$$

Solving:

$$\sqrt{y^2 + (175\,\text{m})^2} = y + \left(m + \tfrac{1}{2}\right)(50\,\text{m});$$

$$y^2 + (175\,\text{m})^2 = y^2 + 2\left(m + \tfrac{1}{2}\right)(50\,\text{m})\,y + \left[\left(m + \tfrac{1}{2}\right)(50\,\text{m})\right]^2;$$

$$y = \frac{(175\,\text{m})^2 - \left[\left(m + \tfrac{1}{2}\right)(50\,\text{m})\right]^2}{2\left(m + \tfrac{1}{2}\right)(50\,\text{m})}$$

For $m = 0, 1, 2,$ and 3, y equals 600 m, 167 m, 60 m, and 0, respectively. The first three points on the y axis at which there is destructive interference are at $\text{y} = \boxed{0, 60\,\text{m, and } 167\,\text{m.}}$

87. (*a*) For the refraction at the first surface, we have

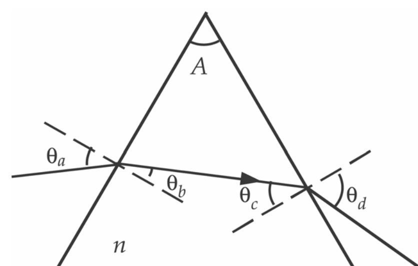
$$n_{\text{air}} \sin\theta_a = n \sin\theta_b;$$

$$(1.00)\sin 45° = (1.652)\sin\theta_{b1},\ \text{which gives}$$

$$\theta_{b1} = 25.34°;$$

$$(1.00)\sin 45° = (1.619)\sin\theta_{b2},\ \text{which gives}$$

$$\theta_{b2} = 25.90°.$$



We find the angle of incidence at the second surface from

$$(90° - \theta_b) + (90° - \theta_c) + A = 180°,\ \text{which gives}$$

$$\theta_{c1} = A - \theta_{b1} = 60.00° - 25.34° = 34.66°;$$

$$\theta_{c2} = A - \theta_{b2} = 60.00° - 25.90° = 34.10°$$

For the refraction at the second surface, we have

$$n \sin\theta_c = n_{\text{air}} \sin\theta_d;$$

$$(1.652)\sin 34.66° = (1.00)\sin\theta_{d1},\ \text{which gives}\ \theta_{d1} = 69.96°;$$

$$(1.619)\sin 34.10° = (1.00)\sin\theta_{d2},\ \text{which gives}\ \theta_{d2} = 65.20°.$$

The angle between the emerging beams is $69.96° - 65.20° = \boxed{4.8°.}$

(*b*) We find the angles for the first order from

$$d \sin\theta = m\lambda = \lambda;$$

$$\frac{1}{6200\,\text{lines/cm}}\left(10^{-2}\ \text{m/cm}\right)\sin\theta_1 = 420 \times 10^{-9}\,\text{m},\ \text{which gives}\ \sin\theta_1 = 0.2604,\ \text{so}\ \theta_1 = 15.09°.$$

$$\frac{1}{6200\,\text{lines/cm}}\left(10^{-2}\ \text{m/cm}\right)\sin\theta_2 = 650 \times 10^{-9}\,\text{m},\ \text{which gives}\ \sin\theta_2 = 0.4030,\ \text{so}\ \theta_2 = 23.77°.$$

The angle between the first-order maxima is $23.77° - 15.09° = \boxed{8.7°.}$

88. We use the lensmaker's equation to find the two focal lengths:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

With $R_1 = \infty$ and $R_2 = 18.4\,\text{cm}$, this gives

$$f = \frac{18.4\,\text{cm}}{n-1};$$

$$f_{\text{red}} = \frac{18.4\,\text{cm}}{1.5106-1} = 36.04\,\text{cm};$$

$$f_{\text{yellow}} = \frac{18.4\,\text{cm}}{1.5226-1} = 35.21\,\text{cm}.$$

Now we find the two image distances.

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f};$$

$$\frac{1}{66.0\,\text{cm}} + \frac{1}{d_{\text{i-red}}} = \frac{1}{36.04\,\text{cm}}, \text{ which gives } \boxed{d_{\text{i-red}} = 79.4\,\text{cm}.}$$

$$\frac{1}{66.0\,\text{cm}} + \frac{1}{d_{\text{i-yellow}}} = \frac{1}{35.21\,\text{cm}}, \text{ which gives } \boxed{d_{\text{i-yellow}} = 75.5\,\text{cm}.}$$