## CHAPTER 25: Optical Instruments

## Answers to Questions

1. Stopping down a lens to a larger f-number means that the lens opening is smaller and only light rays coming through the central part of the lens are accepted. These rays form smaller circles of confusion, which means a greater range of object distances will be more sharply focused.
2. If a lens is stopped down too much (too big of an f-number), then diffraction will occur as light passes through the extremely small lens opening (the light will begin to "bend around" the edges of the stop). This diffraction will blur the image, especially around the edges of the film, and will lead to an image that is less sharp.
3. The lens equation $\left(\frac{1}{f}=\frac{1}{d_{\mathrm{o}}}+\frac{1}{d_{\mathrm{i}}}\right)$ says that as an object gets closer to the lens ( $d_{\mathrm{o}}$ decreases) and the focal length of the lens remains constant, the image distance $d_{\mathrm{i}}$ must get larger to create a focused image. Since we cannot move the film farther away from the lens, we need to move the lens farther away from the film. This is unlike the human eye, where the focal length of the lens is changed as the object distance changes.
4. As people get older, their eyes can no longer accommodate as well. It becomes harder for the muscles to change the shape of the lens, since the lens becomes less flexible with age. As people get older, their near point increases and becomes greater than the ideal value of 25 cm . They may still need the "regular" upper portion of their lenses so they can have a far point at infinity (most people need these at much younger ages), but now an older person will need a bifocal in the lower portion of their lenses to create a near point back at 25 cm for seeing close objects. Thus, as people get older, their far point is too small and their near point is too large. Bifocals can correct both of these problems.
5. No, a nearsighted person will not be able to see clearly if they wear their corrective lenses underwater. A nearsighted person has a far point that is closer than infinity and they wear corrective lenses to bring the image of a far away object to their far point so they can see it clearly. See the first pair of diagrams. The object is at infinity. In air, the image is at the far point. If the person's eyes and glasses are underwater, and since the index of refraction of glass is closer to that of water than to that of air, the glasses will not bend the light as much as they did in the air. Therefore, the image
 of the faraway object will now be at a position that is beyond the person's far point. The image will now be out of focus.
6. If a person's face appears narrower through the person's glasses, the lenses are diverging. If a person is wearing diverging lenses, then they are nearsighted. Since the rays from a person's ears will be bent away from the optical axis when they go through a diverging corrective lens, when that ray gets to our eyes, our brain follows that ray back through the lens without bending it (our brain doesn't know that light can bend) and places the ears closer to where the person's eyes actually are. Thus, diverging corrective lenses
 (for nearsighted people) make a person's head appear narrower when viewed through their glasses by an outside observer. In the diagram, a person viewing from the left would see an image of the ears about where the eyes should be.
7. To see far objects clearly, you want your eye muscles relaxed, which makes your lens relatively flat (large focal length). When you f -stop your eye down by closing your eyelids partially (squinting), you are only using the middle of your lens, where it is the most flat. This creates a smaller circle of confusion for the lens, which helps you see distant objects more clearly.
8. The images formed on our retinas of objects we look at are inverted and real. The implication of this is that our brains must flip this inverted image for us so that we can see things upright.
9. You can leave your eyes open while you are moving your head and still see clearly because your brain "refreshes" the image from the retina about 30 times a second. In other words, your brain and your retina work together in a manner similar to how a motion picture camera and film work together, which is not at all like how a still camera and film work together.
10. Yes, reading glasses are magnifiers. Similarity: they are both converging lenses. Difference: glasses try to put the image of the object at a person's near point so it is clear to see, while a magnifying lens tries to put the image of the object at infinity so it is incredibly large and it subtends as large of an angle as possible on our retina. Thus, the main difference is that the glasses need to have the object closer to the lens than the focal point, while the magnifying lens needs to have the object directly at the focal point.
11. A poor quality, inexpensive lens will not be correctly shaped to fix chromatic aberrations. Thus, the colors you see around the edges of these lenses are from all of the different colors of light getting focused at different points, instead of all being focused at the same point.
12. To minimize spherical aberrations of a planoconvex lens used to form a real image of an object at infinity, the convex side should face the object. In this situation, both surfaces bend the incoming parallel light rays a small amount each and nearly an equal amount each, which reduces the spherical aberrations. If the planar surface faced the object, then all of the bending of the light rays would take place only on the convex side where the rays emerge from the lens. Thus, with no bending on one face and all of the bending on the other face, the amount of bend is very different on the two faces and spherical aberrations will cause severe problems.


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13. Spherical aberrations can be present in a simple lens. To correct this in a simple lens, usually many lenses are used together in combination to minimize the bending at each of the surfaces. Your eye minimizes spherical aberrations by also bending the light at many different interfaces as it makes its way through the different parts of the eye (cornea, aqueous humor, lens, vitreous humor, etc.), each with their own $n$. Also, the cornea is less curved at the edges than it is at the center and the lens is less dense at the edges than at the center. Both of these reduce spherical aberrations in the eye since they cause the rays at the outer edges to be bent less strongly. Curvature of field occurs when the focal plane is not flat. Our curved retina helps with this distortion, whereas a flat piece of film in a camera, for example, wouldn't be able to fix this. Distortion is a result of the variation of the magnification at different distances from the optical axis. This is most common in wide-angle lenses, where it must be corrected for. This is compensated in the human eye because it is a very small lens and our retina is curved. Chromatic aberrations are mostly compensated for in the human eye because the lens absorbs shorter wavelengths and the retina is not very sensitive to most blue and violet wavelengths where most chromatic aberrations occur.
14. Chromatic aberrations in lenses occur due to dispersion, where different colors (or wavelengths) of light are bent different amounts due to the fact that the index of refraction of most materials varies with wavelength. So, when light goes through a lens (a curved transparent material), not all of the different colors come out of the lens at the exact same angle, but they are all focused at different positions. A mirror, in contrast, reflects light off of a smooth metallic surface. This surface reflects all different colors of light at the exact same angle, thus there is no refraction, no dispersion and no chromatic aberrations. Of course, in most mirrors, there is a piece of glass that covers the metallic reflector. Since the two faces of this piece of glass are parallel to each other, even though refraction and dispersion take place inside the piece of glass, when the light emerges from the parallel face, all of the different colors are once again going in the same direction, and there is no dispersion and no chromatic aberration.
15. The Rayleigh criterion gives us the resolution limit for two objects a distance $D$ apart when using light of wavelength $\lambda: \theta=\frac{1.22 \lambda}{D}$, which can be interpreted as the smaller the angle, the better the resolution. Looking at the two wavelengths given, $\theta_{\text {blue }}=\frac{1.22(450 \mathrm{~nm})}{D}$ and $\theta_{\text {red }}=\frac{1.22(700 \mathrm{~nm})}{D}$, we have $\frac{\theta_{\text {blue }}}{\theta_{\text {red }}}=\frac{\frac{1.22(450 \mathrm{~nm})}{D}}{\frac{1.22(700 \mathrm{~nm})}{D}}=\frac{450}{700}=0.64$. This could be expressed as saying that the resolution with blue light is $\frac{1}{0.64}=1.56$ times the resolution with red light.
16. A large reflecting mirror in a telescope has several advantages over a large refracting lens: One, the construction and grinding of a huge lens is very difficult (a mirror only needs one side to be ground and smoothed to perfection). Two, a huge lens is so heavy that it will sag under its own weight, especially since it needs to be held only along the edges of the lens in the telescope (mirrors can be well-supported along their entire back, non-reflecting, side). Three, a huge lens will have serious chromatic aberrations (a mirror has no chromatic aberrations).
17. To get the best resolution from a microscope, you should use blue/violet light. The Rayleigh criterion $\left(\theta=\frac{1.22 \lambda}{D}\right)$ shows that the shorter wavelengths of light have higher resolution. (Of course, you would need very special lenses, since most glass absorbs blue/violet light very strongly.)
18. No, visible light cannot be used to "see" individual atoms. It is not possible to resolve the detail of objects smaller than the wavelength of the radiation being used to look at the object. The shortest wavelength of visible light is approximately $400 \mathrm{~nm}=4 \times 10^{-5} \mathrm{~m}$, which is too big to use to see atoms that are $\sim 10^{-8} \mathrm{~m}$ in diameter. Looking at the Rayleigh criterion, it gives:
$\theta=\frac{1.22 \lambda}{D}=\frac{1.22\left(4 \times 10^{-5} \mathrm{~m}\right)}{10^{-8} \mathrm{~m}}=4880 \mathrm{rad}$. This is an impossible value, since our eyes have a value of about $5 \times 10^{-4} \mathrm{rad}$ and microscopes can only improve this by about 1000 X .

## Solutions to Problems

1. From the definition of the $f$-stop, we have

$$
\begin{aligned}
& f \text {-stop }=\frac{f}{D} ; \\
& 1.4=\frac{(55 \mathrm{~mm})}{D_{1.4}}, \text { which gives } D_{1.4}=39 \mathrm{~mm} ; \\
& 22=\frac{(55 \mathrm{~mm})}{D_{22}}, \text { which gives } D_{22}=2.5 \mathrm{~mm} .
\end{aligned}
$$

Thus the range of diameters is $2.5 \mathrm{~mm} \leq D \leq 39 \mathrm{~mm}$.
2. We find the $f$-number from

$$
f \text {-stop }=\frac{f}{D}=\frac{(14 \mathrm{~cm})}{(6.0 \mathrm{~cm})}=\frac{f}{2.3}
$$

3. The exposure is proportional to the area and the time:

$$
\frac{\text { Exposure } \propto A t \propto D^{2} t \propto t}{(f \text {-stop })^{2}}
$$

Because we want the exposure to be the same, we have

$$
\begin{aligned}
& \frac{t_{1}}{\left(f-\text { stop }_{1}\right)^{2}}=\frac{t_{2}}{\left(f-\text { stop }_{2}\right)^{2}} \\
& \frac{\left[\left(\frac{1}{250}\right) \mathrm{s}\right]}{(5.6)^{2}}=\frac{t_{2}}{(11)^{2}}, \text { which gives } t_{2}=\left(\frac{1}{60}\right) \mathrm{s} .
\end{aligned}
$$

4. The exposure is proportional to the area and the time:

$$
\frac{\text { Exposure } \propto A t \propto D^{2} t \propto t}{(f \text {-stop })^{2}}
$$

Because we want the exposure to be the same, we have

$$
\begin{aligned}
& \frac{t_{1}}{\left(f-\text { stop }_{1}\right)^{2}}=\frac{t_{2}}{\left(f-\text {-top }_{2}\right)^{2}} ; \\
& \frac{\left[\left(\frac{1}{125}\right) \mathrm{s}\right]}{(11)^{2}}=\frac{\left[\left(\frac{1}{1000}\right) \mathrm{s}\right]}{\left(f \text {-stop }_{2}\right)^{2}}, \text { which gives } f \text {-stop } 2=\frac{f}{4} .
\end{aligned}
$$

. For an object at infinity, the image will be in the focal plane, so we have $d_{1}=f=135 \mathrm{~mm}$.
When the object is at 1.2 m , we locate the image from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f} \\
& \left(\frac{1}{1200 \mathrm{~mm}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{135 \mathrm{~cm}}, \text { which gives } d_{\mathrm{i}}=152 \mathrm{~mm}
\end{aligned}
$$

Thus the distance from the lens to the film must change by

$$
d_{i}-f=152 \mathrm{~mm}-135 \mathrm{~mm}=17 \mathrm{~mm}
$$

6. We find the object distances from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f} ; \\
& \left(\frac{1}{d_{\mathrm{o} 1}}\right)+\left(\frac{1}{200.0 \mathrm{~mm}}\right)=\frac{1}{200 \mathrm{~mm}}, \text { which gives } d_{\mathrm{o} 1}=\infty ; \\
& \left(\frac{1}{d_{\mathrm{o} 2}}\right)+\left(\frac{1}{200.6 \mathrm{~mm}}\right)=\frac{1}{200 \mathrm{~mm}}, \text { which gives } d_{\mathrm{o} 1}=6.87 \times 10^{3} \mathrm{~mm}=6.87 \mathrm{~m} .
\end{aligned}
$$

Thus the range of object distances is $6.87 \mathrm{~m} \leq d_{\mathrm{o}} \leq \infty$.
7. The converging camera lens will form a real, inverted image. For the magnification, we have

$$
\begin{aligned}
& m=\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=-\frac{d_{\mathrm{i}}}{d_{\mathrm{o}}} \\
& -\frac{\left(24 \times 10^{-3} \mathrm{~m}\right)}{(28 \mathrm{~m})}=-\frac{d_{\mathrm{i}}}{(58 \mathrm{~m})}, \text { or } d_{\mathrm{i}}=4.97 \times 10^{-2} \mathrm{~m}
\end{aligned}
$$

We find the focal length of the lens from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f} \\
& \left(\frac{1}{50} \mathrm{~m}\right)+\left(\frac{1}{4.97} \times 10^{-2} \mathrm{~m}\right)=\frac{1}{f}, \text { which gives } f=5.0 \times 10^{-2} \mathrm{~m}=50 \mathrm{~mm} .
\end{aligned}
$$

8. From the similar triangles on the ray diagram, we see that

$$
\begin{aligned}
& \frac{H}{L_{1}}=\frac{h}{L_{2}} \\
& \frac{(2.0 \mathrm{~cm})}{(100 \mathrm{~cm})}=\frac{h}{(7.0 \mathrm{~cm})}
\end{aligned}
$$


which gives $h=0.014 \mathrm{~cm}=1.4 \mathrm{~mm}$.
To find the radius of each spot of the image, we consider the light going through a slit and find the distance from the central bright spot to the first dark spot. For destructive interference, the path-length difference to the first dark spot is

$$
d \sin \theta=!\lambda ;
$$

We find the locations on the screen from

$$
y=L_{2} \tan \theta .
$$

For small angles, we have
$\sin \theta \approx \tan \theta$, which gives

$$
y=L_{2}\left(\frac{!\lambda}{d}\right)=\frac{!L_{2} \lambda}{d}=\frac{\left(7.0 \times 10^{-2} \mathrm{~m}\right)\left(550 \times 10^{-9} \mathrm{~m}\right)}{\left(1.0 \times 10^{-3} \mathrm{~m}\right)}=1.9 \times 10^{-5} \mathrm{~m}=0.019 \mathrm{~mm} .
$$

Thus the diameter of the image spot is about 0.04 mm , which is much smaller than the separation of the spots, so they are easily resolvable.
9. We find the effective $f$-number for the pinhole:

$$
f-\text { stop }_{2}=\frac{f}{D}=\frac{(70 \mathrm{~mm})}{(1.0 \mathrm{~mm})}=\frac{f}{70} .
$$

The exposure is proportional to the area and the time:

$$
\frac{\text { Exposure } \propto A t \propto D^{2} t \propto t}{(f \text {-stop })^{2}} .
$$

Because we want the exposure to be the same, we have

$$
\begin{aligned}
& \frac{t_{1}}{\left(f-\text { stop }_{1}\right)^{2}}=\frac{t_{2}}{\left(f-\text { stop }_{2}\right)^{2}} ; \\
& \frac{\left[\left(\frac{1}{250}\right) \mathrm{s}\right]}{(11)^{2}}=\frac{t_{2}}{(70)^{2}}, \text { which gives } t_{2}=\left(\frac{1}{6}\right) \mathrm{s} .
\end{aligned}
$$

10. The length of the eyeball is the image distance for a far object, i.e., the focal length of the lens.

We find the $f$-number from

$$
f \text {-stop }=\frac{f}{D}=\frac{(20 \mathrm{~mm})}{(8.0 \mathrm{~mm})}=2.5 \text { or } \frac{f}{2.5} .
$$

11. With the contact lens, an object at infinity would have a virtual image at the far point of the eye.

We find the power of the lens from

$$
\left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f}=P \text {, when distances are in } \mathrm{m} \text {; }
$$

$$
\left(\frac{1}{\infty}\right)+\left(\frac{1}{-0.17 \mathrm{~m}}\right)=P=-5.9 \mathrm{D} .
$$

To find the new near point, we have

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f}=P \\
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{-0.12 \mathrm{~m}}\right)=-5.9 \mathrm{D}, \text { which gives } d_{\mathrm{o}}=0.41 \mathrm{~m}
\end{aligned}
$$

Glasses would be better, because they give a near point of 32 cm from the eye.
12. With the lens, the screen placed 55 cm from the eye, or 53.2 cm from the lens, is to produce a virtual image 115 cm from the eye, or 113.2 cm from the lens.
We find the power of the lens from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f}=P, \text { when distances are in } \mathrm{m} \\
& \left(\frac{1}{0.532 \mathrm{~m}}\right)+\left(\frac{1}{-1.132 \mathrm{~m}}\right)=P=+1.0 \mathrm{D} .
\end{aligned}
$$

13. With the glasses, an object at infinity would have its image

14 cm from the eye or $14 \mathrm{~cm}-2 \mathrm{~cm}=12 \mathrm{~cm}$ from the lens; $d_{\mathrm{i}}=-12 \mathrm{~cm}$.

$$
P=\frac{1}{f}=\left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\left(\frac{1}{\infty}\right)+\left(\frac{1}{-0.12 \mathrm{~m}}\right)=-8.3 \mathrm{D}
$$

With the contact lenses, $\mathrm{d}_{\mathrm{i}}=-14 \mathrm{~cm}$.

$$
P=\frac{1}{f}=\left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\left(\frac{1}{\infty}\right)+\left(\frac{1}{-0.14 \mathrm{~m}}\right)=-7.1 \mathrm{D}
$$

14. The actual near point of the person is 45 cm . With the lens, an object placed at the normal near point, 25 cm , or 23 cm from the lens, is to produce a virtual image 45 cm from the eye, or 43 cm from the lens.
We find the power of the lens from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f}=P, \text { when distances are in } \mathrm{m} \\
& \left(\frac{1}{0.23 \mathrm{~m}}\right)+\left(\frac{1}{-0.43 \mathrm{~m}}\right)=P=+2.0 \mathrm{D}
\end{aligned}
$$

15. (a) Since the diopter is negative, the lens is diverging, so it produces images closer than the object; thus the person is nearsighted.
(b) We find the far point by finding the image distance for an object at infinity:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f}=P \\
& \left(\frac{1}{\infty}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=-3.50 \mathrm{D}, \text { which gives } d_{\mathrm{i}}=-0.286 \mathrm{~m}=-28.6 \mathrm{~cm}
\end{aligned}
$$

Because this is the distance from the lens, the far point without glasses is

$$
28.6 \mathrm{~cm}+2.0 \mathrm{~cm}=30.6 \mathrm{~cm}
$$

16. (a) We find the power of the lens for an object at infinity:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f}=P \\
& \left(\frac{1}{\infty}\right)+\left(\frac{1}{-0.75 \mathrm{~m}}\right)=P=-1.3 \mathrm{D} .
\end{aligned}
$$

(b) To find the near point with the lens, we have

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f}=P \\
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{-0.25 \mathrm{~m}}\right)=-1.3 \mathrm{D}, \text { which gives } d_{\mathrm{o}}=0.37 \mathrm{~m}=37 \mathrm{~cm} .
\end{aligned}
$$

17. The 2.0 cm of a normal eye is the image distance for an object at infinity; thus it is the focal length of the lens of the eye. To find the length of the nearsighted eye, we find the image distance (distance from lens to retina) for an object at the far point of the eye:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f} \\
& \left(\frac{1}{17 \mathrm{~cm}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{2.0 \mathrm{~cm}}, \text { which gives } d_{\mathrm{i}}=2.27 \mathrm{~cm}
\end{aligned}
$$

Thus the difference is $2.27 \mathrm{~cm}-2.0 \mathrm{~cm}=0.3 \mathrm{~cm}$.
18. We find the far point of the eye by finding the image distance with the lens for an object at infinity:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o} 1}}\right)+\left(\frac{1}{d_{\mathrm{i} 1}}\right)=\frac{1}{f_{1}} \\
& \left(\frac{1}{\infty}\right)+\left(\frac{1}{d_{\mathrm{i} 1}}\right)=\frac{1}{-22.0 \mathrm{~cm}}, \text { which gives } d_{\mathrm{i} 1}=-22.0 \mathrm{~cm} \text { from the lens, or } 23.8 \mathrm{~cm} \text { from the eye. }
\end{aligned}
$$

We find the focal length of the contact lens from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o} 2}}\right)+\left(\frac{1}{d_{\mathrm{i} 2}}\right)=\frac{1}{f_{2}} \\
& \left(\frac{1}{\infty}\right)+\left(\frac{1}{-23.8 \mathrm{~cm}}\right)=\frac{1}{f_{2}}, \text { which gives } f_{2}=-23.8 \mathrm{~cm} .
\end{aligned}
$$

19. (a) We find the focal length of the lens for an object at infinity and the image on the retina:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o} 1}}\right)+\left(\frac{1}{d_{\mathrm{i} 1}}\right)=\frac{1}{f_{1}} \\
& \left(\frac{1}{\infty}\right)+\left(\frac{1}{2.0 \mathrm{~cm}}\right)=\frac{1}{f_{1}}, \text { which gives } f_{1}=2.0 \mathrm{~cm} .
\end{aligned}
$$

(b) With the object 30 cm from the eye, we have

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o} 2}}\right)+\left(\frac{1}{d_{\mathrm{i} 2}}\right)=\frac{1}{f_{2}} \\
& \left(\frac{1}{33 \mathrm{~cm}}\right)+\left(\frac{1}{2.0 \mathrm{~cm}}\right)=\frac{1}{f_{2}}, \text { which gives } f_{2}=1.9 \mathrm{~cm} .
\end{aligned}
$$

20. We find the object distance for an image at her eye's near point:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f}=P \\
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{-0.100 \mathrm{~m}}\right)=-4.0 \mathrm{D}, \text { which gives } d_{\mathrm{o}}=1.7 \times 10^{-2} \mathrm{~m}=17 \mathrm{~cm} .
\end{aligned}
$$

We find the object distance for an image at her eye's far point:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f}=P \\
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{-0.200 \mathrm{~m}}\right)=-4.0 \mathrm{D}, \text { which gives } d_{\mathrm{o}}=1.0 \mathrm{~m}=100 \mathrm{~cm} .
\end{aligned}
$$

21. The magnification with the image at infinity is

$$
M=\frac{N}{f}=\frac{(25 \mathrm{~cm})}{(12 \mathrm{~cm})}=2.1 \times .
$$

22. We find the focal length from

$$
\begin{aligned}
& M=\frac{N}{f} \\
& 3.5=\frac{(25 \mathrm{~cm})}{f}, \text { which gives } f=7.1 \mathrm{~cm} .
\end{aligned}
$$

23. (a) We find the focal length with the image at the near point from

$$
\begin{aligned}
& M=1+\frac{N}{f_{1}} \\
& 2.5=\frac{1+(25 \mathrm{~cm})}{f_{1}}, \text { which gives } f_{1}=17 \mathrm{~cm} .
\end{aligned}
$$

(b) If the eye is relaxed, the image is at infinity, so we have

$$
\begin{aligned}
& M=\frac{N}{f_{2}} \\
& 2.5=\frac{(25 \mathrm{~cm})}{f_{2}}, \text { which gives } f_{2}=10 \mathrm{~cm} .
\end{aligned}
$$

24. Maximum magnification is obtained with the image at the near point. We find the object distance from

$$
\left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f}
$$

$$
\left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{-25.0 \mathrm{~cm}}\right)=\frac{1}{9.00} \mathrm{~cm}, \text { which gives } d_{\mathrm{o}}=6.6 \mathrm{~cm} \text { from the lens. }
$$

The magnification is

$$
M=1+\frac{N}{f}=1+\frac{(25.0 \mathrm{~cm})}{(9.00 \mathrm{~cm})}=3.78 \times
$$

25. (a) The angular magnification with the image at the near point is

$$
M=1+\frac{N}{f}=1+\frac{(25.0 \mathrm{~cm})}{(9.50 \mathrm{~cm})}=3.63 \times
$$

(b) Because the object without the lens and the image with the lens are at the near point, the angular magnification is also the ratio of widths:

$$
\begin{aligned}
& M=\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}} \\
& 3.63=\frac{h_{\mathrm{i}}}{(3.30 \mathrm{~mm})}, \text { which gives } h_{\mathrm{i}}=12.0 \mathrm{~mm} .
\end{aligned}
$$


(c) We find the object distance from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f} \\
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{-25.0 \mathrm{~cm}}\right)=\frac{1}{9.50} \mathrm{~cm}
\end{aligned}
$$

which gives $d_{\mathrm{o}}=6.88 \mathrm{~cm}$ from the lens.
26. (a) We find the image distance from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f} \\
& \left(\frac{1}{5.55 \mathrm{~cm}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{6.00} \mathrm{~cm}
\end{aligned}
$$

which gives $d_{\mathrm{i}}=-74 \mathrm{~cm}$.
(b) From the diagram we see that the angular magnification is

$$
\begin{aligned}
M & =\frac{\theta^{\prime}}{\theta}=\frac{\left(\frac{h_{\mathrm{o}}}{d_{\mathrm{o}}}\right)}{\left(\frac{h_{\mathrm{o}}}{N}\right)}=\frac{N}{d_{\mathrm{o}}} \\
& =\frac{(25 \mathrm{~cm})}{(5.55 \mathrm{~cm})}=4.50 \times
\end{aligned}
$$

27. (a) We find the image distance from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f} \\
& \left(\frac{1}{7.5 \mathrm{~cm}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{8.5} \mathrm{~cm}, \text { which gives } d_{\mathrm{i}}=-64 \mathrm{~cm} .
\end{aligned}
$$

(b) The angular magnification is

$$
\begin{aligned}
M & =\frac{\theta^{\prime}}{\theta}=\frac{\left(\frac{h_{\mathrm{o}}}{d_{\mathrm{o}}}\right)}{\left(\frac{h_{\mathrm{o}}}{N}\right)}=\frac{N}{d_{\mathrm{o}}} \\
& =\frac{(25 \mathrm{~cm})}{(7.5 \mathrm{~cm})}=3.3 \times .
\end{aligned}
$$

28. We find the focal length of the lens from

$$
\begin{aligned}
& M=\frac{N}{f} \\
& 3.0=\frac{(25 \mathrm{~cm})}{f}, \text { which gives } f=8.3 \mathrm{~cm}
\end{aligned}
$$

(a) The magnification with the image at infinity is

$$
M_{1}=\frac{N_{1}}{f}=\frac{(55 \mathrm{~cm})}{(8.3 \mathrm{~cm})}=6.6 \times .
$$

(b) The magnification with the image at infinity is

$$
M_{2}=\frac{N_{2}}{f}=\frac{(16 \mathrm{~cm})}{(8.3 \mathrm{~cm})}=1.9 \times .
$$

Without the lens, the closest an object can be placed is the near point. A farther near point means a smaller angle subtended by the object without the lens, and thus greater magnification.
29. The magnification of the telescope is given by

$$
M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}}=-\frac{(76 \mathrm{~cm})}{(2.8 \mathrm{~cm})}=-27 \times
$$

For both object and image far away, the separation of the lenses is

$$
L=f_{\mathrm{o}}+f_{\mathrm{e}}=76 \mathrm{~cm}+2.8 \mathrm{~cm}=79 \mathrm{~cm} .
$$

30. We find the focal length of the eyepiece from the magnification:

$$
\begin{aligned}
& M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}} \\
& -25=-\frac{(78 \mathrm{~cm})}{f_{\mathrm{e}}}, \text { which gives } f_{\mathrm{e}}=3.1 \mathrm{~cm}
\end{aligned}
$$

For both object and image far away, the separation of the lenses is

$$
L=f_{\mathrm{o}}+f_{\mathrm{e}}=78 \mathrm{~cm}+3.1 \mathrm{~cm}=81 \mathrm{~cm}
$$

31. We find the focal length of the objective from the magnification:

$$
\begin{aligned}
& M=\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}} \\
& 8.0=\frac{f_{\mathrm{o}}}{(2.8 \mathrm{~cm})}, \text { which gives } f_{\mathrm{o}}=22 \mathrm{~cm} .
\end{aligned}
$$

32. We find the focal length of the eyepiece from the power:

$$
f_{\mathrm{e}}=\frac{1}{P}=\frac{1}{35} \mathrm{D}=0.029 \mathrm{~m}=2.9 \mathrm{~cm}
$$

The magnification of the telescope is given by

$$
M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}}=-\frac{(85 \mathrm{~cm})}{(2.9 \mathrm{~cm})}=-29 \times
$$

33. For both object and image far away, we find the focal length of the eyepiece from the separation of the lenses:

$$
\begin{aligned}
& L=f_{\mathrm{o}}+f_{\mathrm{e}} \\
& 75.2 \mathrm{~cm}=74.5 \mathrm{~cm}+f_{\mathrm{e}}, \text { which gives } f_{\mathrm{e}}=0.7 \mathrm{~cm}
\end{aligned}
$$

The magnification of the telescope is given by

$$
M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}}=-\frac{(75.2 \mathrm{~cm})}{(0.7 \mathrm{~cm})}=-110 \times
$$

34. For both object and image far away, we find the (negative) focal length of the eyepiece from the separation of the lenses:

$$
L=f_{\mathrm{o}}+f_{\mathrm{e}}
$$

$32.8 \mathrm{~cm}=36.0 \mathrm{~cm}+f_{\mathrm{e}}$, which gives $f_{\mathrm{e}}=-3.2 \mathrm{~cm}$.
The magnification of the telescope is given by

$$
M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}}=-\frac{(36.0 \mathrm{~cm})}{(-3.2 \mathrm{~cm})}=11 \times .
$$

35. The reflecting mirror acts as the objective, with a focal length

$$
f_{\mathrm{o}}=\frac{r}{2}=\frac{(6.0 \mathrm{~m})}{2}=3.0 \mathrm{~m}
$$

The magnification of the telescope is given by

$$
M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}}=-\frac{(300 \mathrm{~cm})}{(3.2 \mathrm{~cm})}=-94 \times .
$$

36. We find the focal length of the mirror from

$$
\begin{aligned}
& M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}} \\
& -120=-\frac{f_{\mathrm{o}}}{(3.2 \mathrm{~cm})}, \text { which gives } f_{\mathrm{o}}=3.8 \times 10^{2} \mathrm{~cm}=3.8 \mathrm{~m} .
\end{aligned}
$$

The radius is

$$
r=2 f_{\mathrm{o}}=2(3.8 \mathrm{~m})=7.6 \mathrm{~m}
$$

37. For the magnification we have

$$
M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}}=-170, \text { or } f_{\mathrm{o}}=170 f_{\mathrm{e}} .
$$

For both object and image far away, we have

$$
\begin{aligned}
& L=f_{\mathrm{o}}+f_{\mathrm{e}} ; \\
& 1.25 \mathrm{~m}=170 f_{\mathrm{e}}+f_{\mathrm{e}} \text {, which gives } f_{\mathrm{e}}=7.31 \times 10^{-3} \mathrm{~m}=7.31 \mathrm{~mm} .
\end{aligned}
$$

The focal length of the objective is

$$
f_{\mathrm{o}}=170 f_{\mathrm{e}}=170\left(7.31 \times 10^{-3} \mathrm{~m}\right)=1.24 \mathrm{~m} .
$$

38. First find the image distance of the star from the objective mirror. The star is off at infinity, so this will be at the focal point to the left of the objective mirror.

$$
d_{\mathrm{li}}=f_{1}=\frac{r_{1}}{2}=\frac{3.0 \mathrm{~m}}{2}=1.5 \mathrm{~m}
$$

Then the object distance of the image of the star from the eyepiece mirror is

$$
0.9 \mathrm{~m}-1.5 \mathrm{~m}=-0.6 \mathrm{~m}
$$

The negative sign indicates the direction to the left of the eyepiece mirror.
Next find the image distance of the image of the star from the eyepiece mirror.

$$
\begin{aligned}
& \frac{1}{d_{2 \mathrm{i}}}+\frac{1}{d_{2 \mathrm{o}}}=\frac{1}{f_{2}} \\
& \frac{1}{d_{2 \mathrm{i}}}=\frac{1}{f_{2}}-\frac{1}{d_{2 \mathrm{o}}}=\frac{1}{\left(\frac{r_{2}}{2}\right)}-\frac{1}{d_{2 \mathrm{o}}}=\frac{1}{(-0.75 \mathrm{~m})}-\frac{1}{(-0.6 \mathrm{~m})}
\end{aligned}
$$

Solve for $d_{2 i}$ to get

$$
d_{2 \mathrm{i}}=3 \mathrm{~m} .
$$

39. The magnification of the microscope is

$$
M=\frac{N_{l}}{f_{\mathrm{o}} f_{\mathrm{e}}}=\frac{(25 \mathrm{~cm})(17.5 \mathrm{~cm})}{(0.65 \mathrm{~cm})(1.40 \mathrm{~cm})}=480 \times .
$$

40. We find the focal length of the eyepiece from the magnification of the microscope:

$$
\begin{aligned}
& M=\frac{N_{l}}{f_{\mathrm{o}} f_{\mathrm{e}}} \\
& 620=\frac{(25 \mathrm{~cm})(17.5 \mathrm{~cm})}{(0.40 \mathrm{~cm}) f_{\mathrm{e}}}, \text { which gives } f_{\mathrm{e}}=1.8 \mathrm{~cm} .
\end{aligned}
$$

41. The total magnification is

$$
M=\frac{N_{l}}{f_{\mathrm{e}} f_{\mathrm{o}}}=\frac{(25 \mathrm{~cm})(17 \mathrm{~cm})}{(2.5 \mathrm{~cm})(0.28 \mathrm{~cm})}=610 \times .
$$

42. (a) The total magnification is

$$
M=M_{\mathrm{o}} M_{\mathrm{e}}=(59.0)(12.0)=708 \times .
$$

(b) With the final image at infinity, we find the focal length of the eyepiece from

$$
\begin{aligned}
& M_{\mathrm{e}}=\frac{N}{f_{\mathrm{e}}} \\
& 12.0=\frac{(25.0 \mathrm{~cm})}{f_{\mathrm{e}}}, \text { which gives } f_{\mathrm{e}}=2.08 \mathrm{~cm} .
\end{aligned}
$$

Because the image from the objective is at the focal point of the eyepiece, the image distance for the objective is

$$
d_{\mathrm{i}}=l-f_{\mathrm{e}}=20.0 \mathrm{~cm}-2.08 \mathrm{~cm}=17.9 \mathrm{~cm} .
$$

We find the object distance from the magnification:

$$
\begin{aligned}
& M_{\mathrm{o}}=\frac{d_{\mathrm{i}}}{d_{\mathrm{o}}} \\
& 59.0=\frac{(17.9 \mathrm{~cm})}{d_{\mathrm{o}}}, \text { which gives } d_{\mathrm{o}}=0.303 \mathrm{~cm} .
\end{aligned}
$$

We find the focal length of the objective from the lens equation:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f_{\mathrm{o}}} \\
& \left(\frac{1}{0.289 \mathrm{~cm}}\right)+\left(\frac{1}{17.9 \mathrm{~cm}}\right)=\frac{1}{f_{\mathrm{o}}}, \text { which gives } f_{\mathrm{o}}=0.298 \mathrm{~cm} .
\end{aligned}
$$

(c) We found the object distance: $d_{\mathrm{o}}=0.303 \mathrm{~cm}$.
43. (a) Because the image from the objective is at the focal point of the eyepiece, the image distance for the objective is

$$
d_{\mathrm{io}}=l-f_{\mathrm{e}}=16.0 \mathrm{~cm}-1.8 \mathrm{~cm}=14.2 \mathrm{~cm} .
$$

We find the object distance from the lens equation for the objective:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{oo}}}\right)+\left(\frac{1}{d_{\mathrm{io}}}\right)=\frac{1}{f_{\mathrm{o}}} ; \\
& \left(\frac{1}{d_{\mathrm{oo}}}\right)+\left(\frac{1}{14.2 \mathrm{~cm}}\right)=\frac{1}{0.80 \mathrm{~cm}}, \text { which gives } d_{\mathrm{oo}}=0.85 \mathrm{~cm} .
\end{aligned}
$$

(b) With the final image at infinity, the magnification of the eyepiece is

$$
M_{\mathrm{e}}=\frac{N}{f}=\frac{(25.0 \mathrm{~cm})}{(1.8 \mathrm{~cm})}=13.9 \times .
$$

The magnification of the objective is

$$
M_{\mathrm{o}}=\frac{d_{\mathrm{io}}}{d_{\mathrm{oo}}}=\frac{(14.2 \mathrm{~cm})}{(0.85 \mathrm{~cm})}=16.7 \times .
$$

The total magnification is

$$
M=M_{\mathrm{o}} M_{\mathrm{e}}=(16.7)(13.9)=230 \times .
$$

44. (a) We find the object distance from the lens equation for the eyepiece:

$$
\left(\frac{1}{d_{\mathrm{oe}}}\right)+\left(\frac{1}{d_{\mathrm{ie}}}\right)=\frac{1}{f_{\mathrm{e}}} ;
$$

$$
\left(\frac{1}{d_{\mathrm{oe}}}\right)+\left(\frac{1}{-25 \mathrm{~cm}}\right)=\frac{1}{1.8 \mathrm{~cm}} \text {, which gives } d_{\mathrm{oe}}=1.7 \mathrm{~cm} .
$$

The image distance for the objective is

$$
d_{\mathrm{io}}=l-d_{\mathrm{oc}}=16.0 \mathrm{~cm}-1.7 \mathrm{~cm}=14.3 \mathrm{~cm} .
$$

We find the object distance from the lens equation for the objective:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{oo}}}\right)+\left(\frac{1}{d_{\mathrm{io}}}\right)=\frac{1}{f_{\mathrm{o}}} ; \\
& \left(\frac{1}{d_{\mathrm{oo}}}\right)+\left(\frac{1}{14.3 \mathrm{~cm}}\right)=\frac{1}{0.80 \mathrm{~cm}}, \text { which gives } d_{\mathrm{oo}}=0.85 \mathrm{~cm} .
\end{aligned}
$$

(b) With the final image at the near point, the magnification of the eyepiece is

$$
M_{\mathrm{e}}=1+\frac{N}{f}=1+\frac{(25.0 \mathrm{~cm})}{(1.8 \mathrm{~cm})}=14.9 \times
$$

The magnification of the objective is

$$
M_{\mathrm{o}}=\frac{d_{\mathrm{io}}}{d_{\mathrm{oo}}}=\frac{(14.3 \mathrm{~cm})}{(0.85 \mathrm{~cm})}=16.9 \times .
$$

The total magnification is

$$
M=M_{\mathrm{o}} M_{\mathrm{e}}=(16.9)(14.9)=250 \times .
$$

45. (a) We find the image distance from the lens equation for the objective:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{oo}}}\right)+\left(\frac{1}{d_{\mathrm{io}}}\right)=\frac{1}{f_{\mathrm{o}}} \\
& \left(\frac{1}{0.790 \mathrm{~cm}}\right)+\left(\frac{1}{d_{\mathrm{io}}}\right)=\frac{1}{0.740} \mathrm{~cm}, \text { which gives } d_{\mathrm{io}}=11.7 \mathrm{~cm} .
\end{aligned}
$$

For the relaxed eye, the image from the objective is at the focal point of the eyepiece: $d_{\mathrm{oe}}=2.70 \mathrm{~cm}$. The distance between lenses is

$$
l=d_{\mathrm{io}}+d_{\mathrm{oe}}=11.7 \mathrm{~cm}-2.70 \mathrm{~cm}=14.4 \mathrm{~cm} .
$$

(b) With the final image at infinity, the magnification of the eyepiece is

$$
M_{\mathrm{e}}=\frac{N}{f}=\frac{(25.0 \mathrm{~cm})}{(2.70 \mathrm{~cm})}=9.26 \times .
$$

The magnification of the objective is

$$
M_{\mathrm{o}}=\frac{d_{\mathrm{io}}}{d_{\mathrm{oo}}}=\frac{(11.7 \mathrm{~cm})}{(0.790 \mathrm{~cm})}=14.8 \times .
$$

The total magnification is

$$
M=M_{\mathrm{o}} M_{\mathrm{e}}=(14.8)(9.26)=137 \times .
$$

46. (a) When lenses are in contact, the powers add:

$$
P=P_{1}+P_{2}=\left(\frac{1}{-0.28 \mathrm{~m}}\right)+\left(\frac{1}{0.23 \mathrm{~m}}\right)=+0.776 \mathrm{D} .
$$

It is a positive lens, and thus converging.
(b) The focal length is

$$
f=\frac{1}{P}=\frac{1}{0.776 \mathrm{D}}=1.3 \mathrm{~m} .
$$

47. (a) We find the incident angle from

$$
\sin \theta_{1}=\frac{h_{1}}{R}=\frac{(1.0 \mathrm{~cm})}{(12.0 \mathrm{~cm})}=0.0833 \text {, so } \theta_{1}=4.78^{\circ} \text {. }
$$

For the refraction at the curved surface, we have

$$
\begin{aligned}
& \sin \theta_{1}=n \sin \theta_{2} \\
& \sin 4.78^{\circ}=(1.50) \sin \theta_{2},
\end{aligned}
$$

which gives $\sin \theta_{2}=0.0556$, so $\theta_{2}=3.18^{\circ}$.
We see from the diagram that

$$
\theta_{3}=\theta_{1}-\theta_{2}=4.78^{\circ}-3.18^{\circ}=1.60^{\circ} .
$$

For the refraction at the flat face, we have

$$
\begin{aligned}
& n \sin \theta_{3}=\sin \theta_{4} \\
& (1.50) \sin 1.60^{\circ}=\sin \theta_{4},
\end{aligned}
$$

$$
\text { which gives } \sin \theta_{4}=0.0419 \text {, so } \theta_{4}=2.40^{\circ} \text {. }
$$

We see from the diagram that

$$
\begin{aligned}
h_{1}^{\prime}= & h_{1}-R \sin \theta_{3} \\
& =1.0 \mathrm{~cm}-(12.0 \mathrm{~cm}) \sin 1.60^{\circ}=0.665 \mathrm{~cm},
\end{aligned}
$$


so the distance from the flat face to the point where the ray crosses the axis is

$$
d_{1}=\frac{h_{1}{ }^{\prime}}{\tan \theta_{4}}=\frac{(0.665 \mathrm{~cm})}{\tan 2.40^{\circ}}=15.9 \mathrm{~cm} .
$$

(b) We find the incident angle from

$$
\sin \theta_{1}=\frac{h_{2}}{R}=\frac{(4.0 \mathrm{~cm})}{(12.0 \mathrm{~cm})}=0.333 \text {, so } \theta_{1}=19.47^{\circ} .
$$

For the refraction at the curved surface, we have

$$
\sin \theta_{1}=n \sin \theta_{2} ;
$$

$$
\sin 19.47^{\circ}=(1.50) \sin \theta_{2} \text {, which gives } \sin \theta_{2}=0.222 \text {, so } \theta_{2}=12.84^{\circ} \text {. }
$$

We see from the diagram that

$$
\theta_{3}=\theta_{1}-\theta_{2}=19.47^{\circ}-12.84^{\circ}=6.63^{\circ} .
$$

For the refraction at the flat face, we have

$$
n \sin \theta_{3}=\sin \theta_{4} ;
$$

$(1.50) \sin 6.63^{\circ}=\sin \theta_{4}$, which gives $\sin \theta_{4}=0.173$, so $\theta_{4}=9.97^{\circ}$.
We see from the diagram that

$$
h_{2}^{\prime}=h_{2}-R \sin \theta_{3}=4.0 \mathrm{~cm}-(12.0 \mathrm{~cm}) \sin 6.63^{\circ}=2.61 \mathrm{~cm},
$$

so the distance from the flat face to the point where the ray crosses the axis is

$$
d_{2}=\frac{h_{2}^{\prime}}{\tan \theta_{4}}=\frac{(2.61 \mathrm{~cm})}{\tan 9.97^{\circ}}=14.8 \mathrm{~cm} .
$$

(c) The separation of the "focal points" is

$$
\Delta d=d_{1}-d_{2}=15.9 \mathrm{~cm}-14.8 \mathrm{~cm}=1.1 \mathrm{~cm} .
$$

(d) When $h_{2}=4.0 \mathrm{~cm}$, the rays focus closer to the lens, so they will form a circle at the "focal point" for $h_{1}=1.0 \mathrm{~cm}$. We find the radius of this circle from similar triangles:

$$
\begin{aligned}
& \frac{h_{2}{ }^{\prime}}{d_{2}}=\frac{r}{\left(d_{1}-d_{2}\right)} \\
& \frac{(2.61 \mathrm{~cm})}{(14.8 \mathrm{~cm})}=\frac{r}{(1.1 \mathrm{~cm})}, \text { which gives } r=0.19 \mathrm{~cm} .
\end{aligned}
$$

48. The minimum angular resolution is

$$
\theta=\frac{1.22 \lambda}{D}=\frac{(1.22)\left(550 \times 10^{-9} \mathrm{~m}\right)}{(100 \mathrm{in})(0.0254 \mathrm{~m} / \mathrm{in})}=2.64 \times 10^{-7} \mathrm{rad}=\left(1.51 \times 10^{-5}\right)^{\circ} .
$$

49. The angular resolution of the eye, which is the required resolution using the telescope, is

$$
\theta_{\text {eye }}=\frac{d_{\text {eye }}}{L_{\text {eye }}}=\frac{\left(0.10 \times 10^{-3} \mathrm{~m}\right)}{\left(25 \times 10^{-2} \mathrm{~m}\right)}=4.0 \times 10^{-4} \mathrm{rad} .
$$

The resolution without the telescope is

$$
\theta=\frac{d}{L}=\frac{(7.0 \mathrm{~km})}{\left(3.84 \times 10^{5} \mathrm{~km}\right)}=1.82 \times 10^{-5} \mathrm{rad} .
$$

If we ignore the inverted image, the magnification is

$$
\begin{aligned}
& M=\frac{\theta_{\mathrm{eye}}}{\theta}=\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}} ; \\
& \frac{\left(4.0 \times 10^{-4} \mathrm{rad}\right)}{\left(1.82 \times 10^{-5} \mathrm{rad}\right)}=\frac{(2.0 \mathrm{~m})}{f_{\mathrm{e}}}, \text { which gives } f_{\mathrm{e}}=0.091 \mathrm{~m}=9.1 \mathrm{~cm} .
\end{aligned}
$$

The resolution limit is

$$
\theta=\frac{1.22 \lambda}{D}=\frac{(1.22)\left(5.5 \times 10^{-7} \mathrm{~m}\right)}{(0.11 \mathrm{~m})}=6.1 \times 10^{-6} \mathrm{rad} .
$$

50. The minimum angular resolution is

$$
\theta=\frac{1.22 \lambda}{D} .
$$

The distance between lines is the resolving power:

$$
\mathrm{RP}=f \theta=\frac{1.22 \lambda f}{D}=1.22 \lambda(f \text {-stop }) .
$$

For $\frac{f}{2}$ we have
$\mathrm{RP}_{2}=(1.22)\left(550 \times 10^{-9} \mathrm{~m}\right)(2)=1.34 \times 10^{-6} \mathrm{~m}=1.34 \times 10^{-3} \mathrm{~mm}$, so the resolution is

$$
\frac{1}{\mathrm{RP}_{2}}=\frac{1}{\left(1.22 \times 10^{-3} \mathrm{~mm}\right)}=746 \text { lines } / \mathrm{mm} .
$$

For $\frac{f}{16}$ we have

$$
\mathrm{RP}_{2}=(1.22)\left(550 \times 10^{-9} \mathrm{~m}\right)(16.7)=1.12 \times 10^{-5} \mathrm{~m}=1.12 \times 10^{-2} \mathrm{~mm} \text {, so the resolution is }
$$

$$
\frac{1}{\mathrm{RP}_{2}}=\frac{1}{\left(1.02 \times 10^{-2} \mathrm{~mm}\right)}=89 \text { lines } / \mathrm{mm} .
$$

51. The resolution of the telescope is

$$
\theta=\frac{1.22 \lambda}{D}=\frac{(1.22)\left(550 \times 10^{-9} \mathrm{~m}\right)}{(0.55 \mathrm{~m})}=1.22 \times 10^{-6} \mathrm{rad}
$$

The separation of the stars is

$$
d=L \theta=(151 \mathrm{y})\left(9.46 \times 10^{15} \mathrm{~m} / \mathrm{ly}\right)\left(1.22 \times 10^{-6} \mathrm{rad}\right)=1.7 \times 10^{11} \mathrm{~m} .
$$

52. (a) The resolution of the eye is

$$
\theta=\frac{1.22 \lambda}{D}=\frac{(1.22)\left(550 \times 10^{-9} \mathrm{~m}\right)}{\left(5.0 \times 10^{-3} \mathrm{~m}\right)}=1.34 \times 10^{-4} \mathrm{rad} .
$$

We find the maximum distance from

$$
\begin{aligned}
& d=L \theta ; \\
& 2.0 \mathrm{~m}=L\left(1.34 \times 10^{-4} \mathrm{rad}\right), \text { which gives } L=1.5 \times 10^{4} \mathrm{~m}=15 \mathrm{~km} .
\end{aligned}
$$

(b) The angular separation is the resolution:

$$
\theta=1.34 \times 10^{-4} \mathrm{rad}=\left(7.68 \times 10^{-3}\right)^{\circ}=0.46^{\prime}
$$

Our answer is less than the real resolution because of the presence of aberrations.
53. The separation is $s=400 \times 10^{6} \mathrm{~m}$. The distance away is $L=8 \times 10^{10} \mathrm{~m}$.

The angular separation is

$$
\theta=\frac{s}{L}=\frac{400 \times 10^{6} \mathrm{~m}}{8 \times 10^{10} \mathrm{~m}}=0.005 \mathrm{rad} .
$$

The angular resolution of the eye is

$$
\theta_{\mathrm{e}}=\frac{1.22 \lambda}{D}=\frac{(1.22)\left(550 \times 10^{-9} \mathrm{~m}\right)}{(0.05 \mathrm{~m})}=1.34 \times 10^{-5} \mathrm{rad} .
$$

Since $\theta>\theta_{\mathrm{e}}$, the answer is yes.
A person standing on Mars can resolve the Earth and its moon without a telescope.
54. For the diffraction from the crystal, we have

$$
m \lambda=2 d \sin \phi ; m=1,2,3, \ldots .
$$

For the first maximum, we get

$$
(1)(0.133 \mathrm{~nm})=2(0.280 \mathrm{~nm}) \sin \phi, \text { which gives } \phi=13.7^{\circ} .
$$

55. We find the spacing from

$$
\begin{aligned}
& m \lambda=2 d \sin \phi ; m=1,2,3, \ldots \\
& (2)(0.0973 \mathrm{~nm})=2 d \sin 23.4^{\circ}, \text { which gives } d=0.245 \mathrm{~nm} .
\end{aligned}
$$

56. (a) For the diffraction from the crystal, we have

$$
m \lambda=2 d \sin \phi ; m=1,2,3, \ldots
$$

When we form the ratio for the two orders, we get

$$
\begin{aligned}
& \frac{m_{2}}{m_{1}}=\frac{\left(\sin \phi_{2}\right)}{\left(\sin \phi_{1}\right)} \\
& \frac{2}{1}=\frac{\left(\sin \phi_{2}\right)}{\left(\sin 25.2^{\circ}\right)}, \text { which gives } \phi_{2}=58.4^{\circ} .
\end{aligned}
$$

(b) We find the wavelength from

$$
m_{1} \lambda=2 d \sin \phi_{1} ;
$$

(1) $\lambda=2(0.24 \mathrm{~nm}) \sin 25.2^{\circ}$, which gives $\lambda=0.20 \mathrm{~nm}$.
57. (a) Because X-ray images are shadows, the image will be the same size as the object, so the magnification is 1 .
(b) The rays from the point source will not refract, so we can use similar triangles to compare the image size to the object size for the front of the body:

$$
m_{1}=\frac{h_{\mathrm{i}}}{h_{\mathrm{o} 1}}=\frac{\left(d_{1}+d_{2}\right)}{d_{1}}=\frac{(15 \mathrm{~cm}+25 \mathrm{~cm})}{(15 \mathrm{~cm})}=2.7 .
$$

For the back of the body, the image and object have the same size, so the magnification is 1 .

58. (a) The focal length of the lens is

$$
f=\frac{1}{P}=\frac{1}{3.50 \mathrm{D}}=0.286 \mathrm{~m}=28.6 \mathrm{~cm} .
$$

(b) The lens produces a virtual image at his near point:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{ol}}}\right)+\left(\frac{1}{d_{\mathrm{il}}}\right)=\frac{1}{f}=P ; \\
& \left(\frac{1}{0.23 \mathrm{~m}}\right)+\left(\frac{1}{d_{\mathrm{i} 1}}\right)=+3.50 \mathrm{D}, \text { which gives } d_{\mathrm{i} 1}=-1.18 \mathrm{~m}, \text { so his near point is } 120 \mathrm{~cm} .
\end{aligned}
$$

(c) For Pam, we find the object distance that will have an image at her near point:

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o} 2}}\right)+\left(\frac{1}{d_{\mathrm{i} 2}}\right)=\frac{1}{f}=P \\
& \left(\frac{1}{d_{\mathrm{o} 2}}\right)+\left(\frac{1}{-0.23 \mathrm{~m}}\right)=+3.50 \mathrm{D}, \text { which gives } d_{\mathrm{o} 2}=-0.13 \mathrm{~m}, \text { which is } 15 \mathrm{~cm} \text { from her eyes. }
\end{aligned}
$$

59. The exposure is proportional to the intensity, the area and the time:

$$
\frac{\text { Exposure } \propto I A t \propto I D^{2} t \propto I t}{(f \text {-stop })^{2}} .
$$

With the same shutter speed, the time is constant.
Because we want the exposure to be the same, we have

$$
\frac{I_{1}}{\left(f-\text { stop }_{1}\right)^{2}}=\frac{I_{2}}{\left(f-\text { stop }_{2}\right)^{2}}
$$

$$
\frac{I_{1}}{(5.6)^{2}}=\frac{I_{2}}{(22)^{2}}, \text { which gives } I_{2}=16 I_{1} .
$$

Note that we have followed convention to use multiples of 2 .
60. When an object is very far away, the image will be at the focal point $d_{\mathrm{i}}=f$. Thus the magnification is

$$
m=-\frac{d_{\mathrm{i}}}{d_{\mathrm{o}}}=-\frac{f}{d_{\mathrm{o}}}, \text { that is, proportional to } f .
$$

61. For the magnification, we have

$$
m=-\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=-\frac{d_{\mathrm{i}}}{d_{\mathrm{o}}}=-1 \text {, so } d_{\mathrm{i}}=d_{\mathrm{o}} \text {. }
$$

We find the object distance from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{i}}}\right)=\frac{1}{f} \\
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\mathrm{o}}}\right)=\frac{1}{50 \mathrm{~mm}}, \text { which gives } d_{\mathrm{o}}=100 \mathrm{~mm} .
\end{aligned}
$$

The distance between the object and the film is

$$
d=d_{\mathrm{o}}+d_{\mathrm{i}}=100 \mathrm{~mm}+100 \mathrm{~mm}=200 \mathrm{~mm} .
$$

62. The actual far point of the person is 155 cm . With the lens, an object far away is to produce a virtual image 155 cm from the eye, or 153 cm from the lens.
We find the power of the lens from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{ol}}}\right)+\left(\frac{1}{d_{\mathrm{i} 1}}\right)=\frac{1}{f_{1}}=P_{1}, \text { when distances are in } \mathrm{m} ; \\
& \left(\frac{1}{\infty}\right)+\left(\frac{1}{-1.53 \mathrm{~m}}\right)=P_{1}=-0.65 \mathrm{D} \text { (upper part). }
\end{aligned}
$$

The actual near point of the person is 45 cm . With the lens, an object placed at the normal near point, 25 cm , or 23 cm from the lens, is to produce a virtual image 45 cm from the eye, or 43 cm from the lens. We find the power of the lens from

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o} 2}}\right)+\left(\frac{1}{d_{\mathrm{i} 2}}\right)=\frac{1}{f_{2}}=P_{2} \\
& \left(\frac{1}{0.23 \mathrm{~m}}\right)+\left(\frac{1}{-0.43 \mathrm{~m}}\right)=P_{2}=+2.0 \mathrm{D} \text { (lower part) } .
\end{aligned}
$$

63. The maximum magnification is achieved with the image at the near point:

$$
M_{1}=1+\frac{N_{1}}{f}=1+\frac{(15.0 \mathrm{~cm})}{(8.0 \mathrm{~cm})}=2.9 \times .
$$

For an adult we have

$$
M_{2}=1+\frac{N_{2}}{f}=1+\frac{(25.0 \mathrm{~cm})}{(8.0 \mathrm{~cm})}=4.1 \times .
$$

The person with the normal eye (adult) sees more detail.
64. The magnification for a relaxed eye is

$$
M=\frac{N}{f}=N P=(0.25 \mathrm{~m})(+4.0 D)=1.0 \times
$$

65. (a) The magnification of the telescope is given by

$$
M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}}=-\frac{P_{\mathrm{e}}}{P_{\mathrm{o}}}=-\frac{(4.5 \mathrm{D})}{(2.0 \mathrm{D})}=-2.25 \times .
$$

(b) To get a magnification greater than 1, for the eyepiece we use the lens with the smaller focal length, or greater power: 4.5 D.
66. To find the new near point, we have

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o} 1}}\right)+\left(\frac{1}{d_{\mathrm{i} 1}}\right)=\frac{1}{f_{1}}=P_{1}, \text { when distances are in } \mathrm{m} ; \\
& \left(\frac{1}{0.35 \mathrm{~m}}\right)+\left(\frac{1}{d_{\mathrm{i} 1}}\right)=+2.5 \mathrm{D}, \text { which gives } d_{\mathrm{i} 1}=-2.8 \mathrm{~m} .
\end{aligned}
$$

To give him a normal near point, we have

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o} 2}}\right)+\left(\frac{1}{d_{\mathrm{i} 2}}\right)=\frac{1}{f_{2}}=P_{2} \\
& \left(\frac{1}{0.25 \mathrm{~m}}\right)+\left(\frac{1}{-2.8 \mathrm{~m}}\right)=P_{2}=+3.6 \mathrm{D}
\end{aligned}
$$

67. For the minimum aperture the angle subtended at the lens by the smallest feature is the angular resolution:
$\theta=\frac{d}{L}=\frac{1.22 \lambda}{D} ;$

$$
\frac{\left(5 \times 10^{-2} \mathrm{~m}\right)}{\left(25 \times 10^{3} \mathrm{~m}\right)}=\frac{(1.22)\left(550 \times 10^{-9} \mathrm{~m}\right)}{D}, \text { which gives } D=0.34 \mathrm{~m}=34 \mathrm{~cm} .
$$

68. (a) When an object is very far away, the image will be at the focal point $d_{\mathrm{i} 1}=f$.

From the magnification, we have

$$
m=-\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=-\frac{d_{\mathrm{i}}}{d_{\mathrm{o}}} \text {, so we see that } h_{\mathrm{i}} \text { is proportional to } d_{\mathrm{i}}
$$

When the object and image are the same size, we get

$$
-\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=-1=-\frac{d_{\mathrm{i} 2}}{d_{\mathrm{o} 2}}, \text { so } d_{\mathrm{o} 2}=d_{\mathrm{i} 2}
$$

From the lens equation, we get

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o} 2}}\right)+\left(\frac{1}{d_{\mathrm{i} 2}}\right)=\frac{1}{f} \\
& \left(\frac{1}{d_{\mathrm{i} 2}}\right)+\left(\frac{1}{d_{\mathrm{i} 2}}\right)=\frac{1}{f}, \text { which gives } d_{\mathrm{i} 2}=2 f
\end{aligned}
$$

The required exposure time is proportional to the area of the image on the film:

$$
t \propto A \propto\left(h_{\mathrm{i}}\right)^{2} \propto\left(d_{\mathrm{i}}\right)^{2} .
$$

When we form the ratio, we get

$$
\frac{t_{2}}{t_{1}}=\left(\frac{d_{\mathrm{i} 2}}{d_{\mathrm{il}}}\right)^{2}=\left(\frac{2 f}{f}\right)^{2}=4 .
$$

(b) For the increased object distances, we have

$$
\begin{aligned}
& \left(\frac{1}{d_{\mathrm{o} 3}}\right)+\left(\frac{1}{d_{\mathrm{i} 3}}\right)=\frac{1}{f} ; \\
& \left(\frac{1}{4 f}\right)+\left(\frac{1}{d_{\mathrm{i} 3}}\right)=\frac{1}{f}, \text { which gives } d_{\mathrm{i} 3}=\frac{4 f}{3} ; \text { and } \frac{t_{3}}{t_{1}}=\left(\frac{4}{3}\right)^{2}=1.78 . \\
& \left(\frac{1}{d_{\mathrm{o} 4}}\right)+\left(\frac{1}{d_{\mathrm{i} 4}}\right)=\frac{1}{f} ; \\
& \left(\frac{1}{5 f}\right)+\left(\frac{1}{d_{\mathrm{i} 4}}\right)=\frac{1}{f}, \text { which gives } d_{\mathrm{i} 4}=\frac{5 f}{4} ; \text { and } \frac{t_{4}}{t_{1}}=\left(\frac{5}{4}\right)^{2}=1.56 .
\end{aligned}
$$

These increased exposures are less than the minimal adjustment on a typical camera, so they are negligible.
69. The focal length of the eyepiece is

$$
f_{\mathrm{e}}=\frac{1}{P_{\mathrm{e}}}=\frac{1}{23 \mathrm{D}}=4.3 \times 10^{-2} \mathrm{~m}=4.3 \mathrm{~cm} .
$$

For both object and image far away, we find the focal length of the objective from the separation of the lenses:
$L=f_{\mathrm{o}}+f_{\mathrm{e}} ;$
$85 \mathrm{~cm}=f_{\mathrm{o}}+4.3 \mathrm{~cm}$, which gives $f_{\mathrm{o}}=80.7 \mathrm{~cm}$.
The magnification of the telescope is given by

$$
M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}}=-\frac{(80.7 \mathrm{~cm})}{(4.3 \mathrm{~cm})}=-19 \times .
$$

70. The distance from the Earth to the Moon is $384 \times 10^{6} \mathrm{~m}$.

$$
\begin{aligned}
s=L \theta=\frac{L(1.22 \lambda)}{D} & =\frac{\left(384 \times 10^{6} \mathrm{~m}\right)(1.22)\left(550 \times 10^{-9} \mathrm{~m}\right)}{(2.4 \mathrm{~m})} \\
& =107 \mathrm{~m} .
\end{aligned}
$$

71. (a) The distance between the two lenses is $f_{\mathrm{o}}+f_{\mathrm{e}}=4.0 \mathrm{~cm}+44 \mathrm{~cm}=48 \mathrm{~cm}$.

The magnification is

$$
M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}}=-\frac{(44 \mathrm{~cm})}{(4.0 \mathrm{~cm})}=-11 \times .
$$

The eyepiece is the lens with $f=4.0 \mathrm{~cm}$.
(b) The magnification is

$$
M=-\frac{N l}{f_{\mathrm{e}} f_{\mathrm{o}}}=\frac{(25 \mathrm{~cm}) l}{(44 \mathrm{~cm})(4.0 \mathrm{~cm})}=25 .
$$

Solve for $l$ to get $l=180 \mathrm{~cm}$.
72. $M=-\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}}=-8.0$. Therefore, $f_{\mathrm{o}}=8.0 f_{\mathrm{e}}$.

Since $f_{\mathrm{o}}+f_{\mathrm{e}}=28 \mathrm{~cm}, 8.0 f_{\mathrm{e}}+f_{\mathrm{e}}=9.0 f_{\mathrm{e}}=28 \mathrm{~cm}$.
Solving, $f_{\mathrm{e}}=\frac{28 \mathrm{~cm}}{9.0}=3.1 \mathrm{~cm}$ and $f_{\mathrm{e}}=24.9 \mathrm{~cm}$.
73. The angular resolution must equal

$$
\theta=\frac{s}{L}=\frac{(0.05 \mathrm{~m})}{\left(130 \times 10^{3} \mathrm{~m}\right)}=3.85 \times 10^{-7} .
$$

Therefore,

$$
\theta=\frac{1.22 \lambda}{D}=\frac{(1.22)\left(550 \times 10^{-9} \mathrm{~m}\right)}{D}=3.8 \times 10^{-7} .
$$

Solve for $D$ to get $D=1.7 \mathrm{~m}$.
74. We find the focal lengths of the lens for the two colors:

$$
\begin{aligned}
\frac{1}{f_{\text {red }}} & =\left(n_{\text {red }}-1\right)\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right) \\
& =(1.5106-1)\left[\left(\frac{1}{18.4 \mathrm{~cm}}\right)+\left(\frac{1}{\infty}\right)\right] \text { which gives } f_{\text {red }}=36.04 \mathrm{~cm} . \\
\frac{1}{f_{\text {orange }}} & =\left(n_{\text {orange }}-1\right)\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right) \\
& =(1.5226-1)\left[\left(\frac{1}{18.4 \mathrm{~cm}}\right)+\left(\frac{1}{\infty}\right)\right] \text { which gives } f_{\text {orange }}=35.21 \mathrm{~cm} .
\end{aligned}
$$

We find the image distances from

$$
\begin{aligned}
& \left(\frac{1}{d_{o}}\right)+\left(\frac{1}{d_{\text {ired }}}\right)=\frac{1}{f_{\text {red }}} ; \\
& \left(\frac{1}{66.0 \mathrm{~cm}}\right)+\left(\frac{1}{\mathrm{~d}_{\text {ired }}}\right)=\frac{1}{36.04} \mathrm{~cm} \text {, which gives } d_{\text {ired }}=79.4 \mathrm{~cm} . \\
& \left(\frac{1}{d_{\mathrm{o}}}\right)+\left(\frac{1}{d_{\text {iorange }}}\right)=\frac{1}{f_{\text {orange }}} ; \\
& \left(\frac{1}{66.0 \mathrm{~cm}}\right)+\left(\frac{1}{d_{\text {iorange }}}\right)=\frac{1}{35.21 \mathrm{~cm}}, \text { which gives } d_{\text {iorange }}=75.5 \mathrm{~cm} .
\end{aligned}
$$

The images are 3.9 cm apart, an example of chromatic aberration.


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