CHAPTER 26: The Special Theory of Relativity

Answers to Questions

1. No. Since the windowless car in an exceptionally smooth train moving at a constant velocity is an inertial reference frame and all of the laws of physics are the same in any inertial frame, there is no way for you to tell if you are moving or not.

2. There is no way for you to tell the difference between absolute and relative motion. You always think that you are at rest and that other things around you are moving, but you really can't tell if maybe you are moving and everything else is at rest.

3. The ball will land on the roof of the railroad car (ignoring air resistance and assuming that the velocity of the train is constant). Both the ball and the car are already moving forward, so when the ball is thrown straight up into the air with respect to the car, it will continue to move forward at the same rate as the car and fall back down to land on the roof.

4. Whether you say the Earth goes around the Sun or the Sun goes around the Earth depends on your reference frame. It is valid to say either one, depending on which frame you choose. The laws of physics, though, won't be the same in each of these reference frames, since the Earth is accelerating as it goes around the Sun. The Sun is nearly an inertial reference frame, but the Earth is not.

5. If you were in a spaceship traveling at 0.5*c* away from a star, its starlight would pass you at a speed of *c*. The speed of light is a constant in any reference frame, according to the 2nd postulate of special relativity.

6. The clocks are not at fault and they are functioning properly. Time itself is actually measured to pass more slowly in moving reference frames when compared to a rest frame.

7. Time actually passes more slowly in the moving reference frame. It is not just that it seems this way, it has actually been measured to pass more slowly, as predicted by special relativity.

8. If the young-looking astronaut has been traveling at extremely high speeds (relative to Earth) for many Earth years, since her clock will run slow compared to the clocks at rest on Earth, it is possible that her son could now be much older than she is after her trip. This is a twist on the twin paradox.

9. You would not notice a change in your own heartbeat, mass, height or waistline. No matter how fast you are moving relative to Earth, you are at rest in your own reference frame. Thus, you would not notice any changes in your own characteristics. To observers on Earth, you are moving away at 0.5*c*, which gives $\gamma = 0.87$. To these observers, it would appear that your heartbeat has slowed by a factor of 0.87, that your mass has increased by a factor of 1/0.87 = 1.15, and that your waistline has decreased by a factor of 0.87 (all due to the relativity equations for time dilation, mass increase, and length contraction), but that your height would be unchanged (since there is no relative motion between you and Earth in that direction).

10. Time dilation and length contraction do occur at 90 km/h, but the effects are so small that they are not measurable.

11. If the speed of light was infinite, then time dilation and length contraction would not occur. The factor found in these equations, $\sqrt{1-v^2/c^2}$, would always be equal to zero at any speed *v*.
12. If the speed of light was 25 m/s, then we would see relativistic effects all the time, something like the opening Chapter figure or Figure 26-11 with question 14. Everything moving would look length contracted and time dilation and mass increase would have to be taken into account in all experiments. One of the most unusual changes for today's modern inhabitants of Earth would be that nothing would be able to move faster than 25 m/s, which is only about 56 mi/h.

13. As *v* gets very close to *c*, an outside observer should be able to show that $L = L_o\sqrt{1-v^2/c^2}$ is getting smaller and smaller and that the limit as $v \to c$ is that $L \to 0$. This would show that *c* is a limiting speed, since nothing can get smaller than having a length of 0. Also, as *v* gets very close to *c*, an outside observer should be able to show that $\Delta t = \dfrac{\Delta t_o}{\sqrt{1-v^2/c^2}}$ is getting longer and longer and that the limit as $v \to c$ is that $\Delta t \to \infty$. This would show that *c* is a limiting speed, since the slowest that time can pass is that it comes to a stop.

14. Mr. Tompkins and his bike would look very thin to the people standing on the sidewalk if *c* = 20 mi/h, due to the extreme length contraction that would occur. He would be the same height, though, since there is no relative motion in that direction between Mr. Tompkins and the people on the sidewalk.

15. No, the speed limit of *c* for an electron does not put an upper limit on the electron's momentum. As $v \to c$ in $p = \dfrac{m_o v}{\sqrt{1-v^2/c^2}}$ then $p \to \infty$, which can be attributed to the fact that $m = \dfrac{m_o}{\sqrt{1-v^2/c^2}}$ goes to ∞ as $v \to c$ (the relativistic mass increase).

16. No, it is not possible for a non-zero rest mass particle to attain a speed of *c*. This is because its relativistic mass would increase to ∞ as $v \to c$, which means it would take an infinite amount of energy to increase the speed of this infinite mass particle to *c*.

17. No, $E = mc^2$ does not conflict with the conservation of energy, it actually completes it. Since this equation shows us that mass and energy are interconvertible, it says it is now necessary to include mass as a form of energy in the analysis of physical processes.

18. Yes. One way to describe the energy stored in the compressed spring is to say it is a mass increase (although it would be so small that it could not be measured). This mass will convert back to energy when the spring is uncompressed.

19. Matter and energy are interconvertible (matter can be converted into energy and energy can be converted into matter), thus we should say "energy can neither be created nor destroyed."

20. No, our intuitive notion that velocities simply add is not completely wrong. Our intuition is based on our everyday experiences and at these everyday speeds our intuition is correct regarding how velocities add. Our intuition does break down, though, at very high speeds, where we have to take into account relativistic effects. Relativity does not contradict classical mechanics, but it is a more general theory where classical mechanics is a limiting case.

Solutions to Problems

1. You measure the contracted length. We find the rest length from

$$L = L_0\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}};$$

$$28.2\,\text{m} = L_0\left[1-(0.750)^2\right]^{\frac{1}{2}}, \text{ which gives } L_0 = \boxed{42.6\,\text{m.}}$$

2. We find the lifetime at rest from

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$4.76\times10^{-6}\,\text{s} = \frac{\Delta t_0}{\left\{1-\left[\frac{(2.70\times10^8\,\text{m/s})}{(3.00\times10^8\,\text{m/s})}\right]^2\right\}^{\frac{1}{2}}}, \text{ which gives } \Delta t_0 = \boxed{2.07\times10^{-6}\,\text{s.}}$$

3. (*a*) $$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left\{1-\left[\frac{(20{,}000\,\text{m/s})}{(3.00\times10^8\,\text{m/s})}\right]^2\right\}^{\frac{1}{2}} = \boxed{1.00.}$$

(*b*) $$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left[1-(0.020)^2\right]^{\frac{1}{2}} = \boxed{0.9998.}$$

(*c*) $$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left[1-(0.200)^2\right]^{\frac{1}{2}} = \boxed{0.980.}$$

(*d*) $$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left[1-(0.95)^2\right]^{\frac{1}{2}} = \boxed{0.312.}$$

(*e*) $$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left[1-(0.98)^2\right]^{\frac{1}{2}} = \boxed{0.199.}$$

(*f*) $$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left[1-(0.999)^2\right]^{\frac{1}{2}} = \boxed{0.0447.}$$

4. You measure the contracted length:

$$L = L_0\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}$$

$$=(125\,\text{ly})\left\{1-\left[\frac{(2.50\times10^{8}\ \text{m/s})}{(3.00\times10^{8}\ \text{m/s})}\right]^{2}\right\}^{\frac{1}{2}}=\boxed{55.3\,\text{ly.}}$$

5. We determine the speed from the time dilation:

$$\Delta t=\frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$4.10\times10^{-8}\ \text{s}=\frac{(2.60\times10^{-8}\ \text{s})}{\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}}},\ \text{which gives } v=\boxed{0.773c.}$$

6. We determine the speed from the length contraction:

$$L=L_0\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}};$$

$$35\,\text{ly}=(82\,\text{ly})\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}},\ \text{which gives } v=\boxed{0.90c.}$$

7. We determine the speed from the length contraction:

$$L=L_0\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}};$$

$$25\,\text{ly}=(85\,\text{ly})\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}},\ \text{which gives } v=0.956c.$$

We then find the time from the speed and distance:

$$t=\frac{d}{v}=\frac{(25\,\text{y})c}{0.956c}=\boxed{26\,\text{y.}}$$

8. We determine the speed from the length contraction:

$$L=L_0\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}};$$

$$0.900\,\text{m}=(1.00\,\text{m})\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}},\ \text{which gives } v=\boxed{0.436c.}$$

9. We convert the speed: $\dfrac{(40{,}000\,\text{km/h})}{(3.6\,\text{ks/h})}=1.11\times10^{4}\ \text{m/s}.$

The length contraction is given by

$$L = L_0\sqrt{1-\left(\frac{v}{c}\right)^2};$$

$$1-\frac{L}{L_0} = 1-\sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$= 1-\sqrt{1-\left[\frac{(1.11\times10^4\ \text{m/s})}{(3.00\times10^8\ \text{m/s})}\right]^2}$$

$$= 7\times10^{-10}\ \text{or}\ \boxed{7\times10^{-8}\ \%.}$$

10. For a 1.00 percent change, the factor in the expressions for time dilation and length contraction must equal $1-0.0100 = 0.9900$:

$$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = 0.9900,\ \text{which gives}\ v = \boxed{0.141c.}$$

$\boxed{11.}$ In the Earth frame, the clock on the *Enterprise* will run slower.

(*a*) We find the elapsed time on the ship from

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$5.0\ \text{yr} = \frac{\Delta t_0}{\left[1-(0.84)^2\right]^{\frac{1}{2}}},\ \text{which gives}\ \Delta t_0 = \boxed{2.7\ \text{yr.}}$$

(*b*) We find the elapsed time on the Earth from

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{(5.0\ \text{yr})}{\left[1-(0.84)^2\right]^{\frac{1}{2}}} = \boxed{9.2\ \text{yr.}}$$

12. (*a*) To an observer on Earth, 10.6 ly is the rest length, so the time will be

$$t_{\text{Earth}} = \frac{L_0}{v} = \frac{(10.6\ \text{ly})}{0.960c} = \boxed{11.0\ \text{yr.}}$$

(*b*) We find the dilated time on the spacecraft from

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$11.0\,\text{yr} = \frac{\Delta t_0}{\left[1-(0.960)^2\right]^{\frac{1}{2}}},\ \text{which gives}\ \Delta t_0 = \boxed{3.09\,\text{yr.}}$$

(*c*) To the spacecraft observer, the distance to the star is contracted:

$$L = L_0\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = (10.6\,\text{ly})\left[1-(0.960)^2\right]^{\frac{1}{2}} = \boxed{2.97\,\text{ly.}}$$

(*d*) To the spacecraft observer, the speed of the spacecraft is

$$v = \frac{L}{\Delta t} = \frac{(2.97\,\text{ly})}{3.09\,\text{yr}} = \boxed{0.960c,}\ \text{as expected.}$$

13. (*a*) You measure the contracted length. We find the rest length from

$$L = L_0\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}};$$

$$4.80\,\text{m} = L_0\left[1-(0.660)^2\right]^{\frac{1}{2}},\ \text{which gives}\ L_0 = \boxed{6.39\,\text{m.}}$$

Distances perpendicular to the motion do not change, so the rest height is $\boxed{1.25\ \text{m.}}$

(*b*) We find the dilated time in the spacecraft from

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$20.0\,\text{s} = \frac{\Delta t_0}{\left[1-(0.660)^2\right]^{\frac{1}{2}}},\ \text{which gives}\ \Delta t_0 = \boxed{15.0\,\text{s.}}$$

(*c*) To your friend, you moved at the same relative speed: $\boxed{0.660c.}$

(*d*) She would measure the same time dilation: $\boxed{15.0\ \text{s.}}$

14. In the Earth frame, the average lifetime of the pion will be dilated:

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}.$$

The speed as a fraction of the speed of light is

$$\frac{v}{c} = \frac{d}{c}\Delta t = \frac{d\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}{c}\Delta t_0;$$

$$\frac{v}{c} = \frac{(15\,\text{m})\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}{\left(3.00\times10^8\ \text{m/s}\right)\left(2.60\times10^{-8}\ \text{s}\right)},$$

which gives $v = \boxed{0.887c = 2.66\times10^8\ \text{m/s.}}$

15. The momentum of the proton is

$$p = \frac{m_0 v}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}} = \frac{\left(1.67 \times 10^{-27}\ \text{kg}\right)(0.85)\left(3.00 \times 10^8\ \text{m/s}\right)}{\left[1 - (0.85)^2\right]^{\frac{1}{2}}} = \boxed{8.1 \times 10^{-19}\ \text{kg} \cdot \text{m/s}.}$$

16. We find the speed from

$$m = \frac{m_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$2m_0 = \frac{m_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}, \text{ which gives } v = \boxed{0.866c.}$$

17. The initial momentum of the particle is

$$p = \frac{m_0 v}{\left[1 - \frac{v^2}{c^2}\right]^{\frac{1}{2}}};$$

$$= \frac{m_0 (0.20c)}{\left[1 - (0.20)^2\right]^{\frac{1}{2}}}.$$

We now find v which doubles the momentum:

$$\frac{m_0 v}{\left[1 - \frac{v^2}{c^2}\right]^{\frac{1}{2}}} = 2\left(\frac{m_0 (0.20c)}{\left[1 - (0.20)^2\right]^{\frac{1}{2}}}\right);$$

$$\frac{v^2}{1 - \frac{v^2}{c^2}} = \frac{c^2}{6}, \text{ which gives } v = \boxed{0.38c.}$$

18. (*a*) The momentum of the particle is

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{m_0 (0.10c)}{\sqrt{1 - (0.10)^2}} = 0.10050cm_0.$$

The classical formula gives $p = m_0 v = 0.10cm_0$.

The error is

$$\frac{0.10cm_0 - 0.10050cm_0}{0.10050cm_0} = -0.0050 = \boxed{-0.50\%.}$$

(*b*) The momentum of the particle is

$$p = \frac{m_0(0.50c)}{\sqrt{1-(0.50)^2}} = 0.57735cm_0.$$

The classical formula gives $0.50cm_0$.

The error is

$$\frac{0.50cm_0 - 0.57735cm_0}{0.57735cm_0} = -0.13 = \boxed{-13\%.}$$

19. The proton's momenta are found using

$$p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}};$$

$$p_1 = \frac{m_0(0.45c)}{\sqrt{1-(0.45)^2}} = 0.5039cm_0;$$

$$p_2 = \frac{m_0(0.90c)}{\sqrt{1-(0.90)^2}} = 2.065cm_0;$$

$$p_3 = \frac{m_0(0.98c)}{\sqrt{1-(0.98)^2}} = 4.925cm_0.$$

(*a*) $$\frac{p_2 - p_1}{p_1} = \frac{2.065cm_0 - 0.5039cm_0}{0.5039cm_0} = 3.10 = \boxed{310\%.}$$

(*b*) $$\frac{p_3 - p_2}{p_2} = \frac{4.925cm_0 - 2.065cm_0}{2.065cm_0} = 1.40 = \boxed{140\%.}$$

20. We find the increase in mass from

$$\Delta m = \frac{\Delta E}{c^2} = \frac{(4.82\times 10^4\text{ J})}{(3.00\times 10^8\text{ m/s})^2} = \boxed{5.36\times 10^{-13}\text{ kg.}}$$

Note that this is so small, most chemical reactions are considered to have mass conserved.

21. We find the loss in mass from

$$\Delta m = \frac{\Delta E}{c^2} = \frac{(200\,\text{MeV})(1.60\times 10^{-13}\text{ J/MeV})}{(3.00\times 10^8\text{ m/s})^2} = \boxed{3.56\times 10^{-28}\text{ kg.}}$$

22. The rest energy of the electron is

$$E = m_0c^2 = (9.11\times 10^{-31}\text{ kg})(3.00\times 10^8\text{ m/s})^2 = \boxed{8.20\times 10^{-14}\text{ J}}$$

$$= \frac{(8.20\times 10^{-14}\text{ J})}{(1.60\times 10^{-13}\text{ J/MeV})} = \boxed{0.511\text{MeV.}}$$

23. The rest mass of the proton is

$$m_0 = \frac{E}{c^2} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(2.9979\times10^{8}\text{ m/s}\right)^2}{\left(1.60\times10^{-13}\text{ J/MeV}\right)c^2} = \frac{938\text{MeV}}{c^2}.$$

24. We find the necessary mass conversion from

$$\Delta m = \frac{\Delta E}{c^2} = \frac{\left(8\times10^{19}\text{ J}\right)}{\left(3.00\times10^{8}\text{ m/s}\right)^2} = \boxed{9\times10^{2}\text{ kg.}}$$

25. We find the energy equivalent of the mass from

$$E = mc^2 = \left(1.0\times10^{-3}\text{ kg}\right)\left(3.00\times10^{8}\text{ m/s}\right)^2 = \boxed{9.0\times10^{13}\text{ J.}}$$

If this energy increases the gravitational energy, we have

$E = mgh$;

$9.0\times10^{13}\text{ J} = m\left(9.80\text{ m/s}^2\right)\left(0.25\times10^{3}\text{ m}\right)$, which gives $m = \boxed{3.7\times10^{10}\text{ kg.}}$

26. If the kinetic energy is equal to the rest energy, we have

$\text{KE} = (m - m_0)c^2 = m_0c^2$, or $m = 2m_0$.

We find the speed from

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$2m_0 = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},$$ which gives $v = 0.866c.$

27. If the kinetic energy is 25% of the rest energy, we have

$\text{KE} = (m - m_0)c^2 = 0.25m_0c^2$, or $m = 1.25m_0$.

We find the speed from

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}};$$

$$1.25m_0 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},$$ which gives $v = \boxed{0.60c.}$

28. (*a*) We find the work required from

$$W = \Delta\text{KE} = (m - m_0)c^2 = m_0c^2\left(\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\} - 1\right)$$

$$= (938\,\text{MeV})\left(\left\{\frac{1}{\left[1-(0.997)^2\right]^{\frac{1}{2}}}\right\} - 1\right) = 11.2\times10^3\,\text{MeV} = \boxed{11.2\,\text{GeV}\left(2.23\times10^{-9}\,\text{J}\right).}$$

(*b*) The momentum of the proton is

$$p = mv = \frac{m_0v}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}$$

$$= \frac{\left(1.67\times10^{-27}\,\text{kg}\right)(0.997)\left(3.00\times10^8\,\text{m/s}\right)}{\left[1-(0.997)^2\right]^{\frac{1}{2}}} = \boxed{6.46\times10^{-18}\,\text{kg}\bullet\text{m/s}.}$$

29. The speed of the proton is

$$v = \frac{\left(2.60\times10^8\,\text{m/s}\right)}{\left(3.00\times10^8\,\text{m/s}\right)} = 0.867c.$$

The kinetic energy is

$$\text{KE} = (m - m_0)c^2 = m_0c^2\left(\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\} - 1\right)$$

$$= (938\,\text{MeV})\left(\left\{\frac{1}{\left[1-(0.867)^2\right]^{\frac{1}{2}}}\right\} - 1\right) = \boxed{942\,\text{MeV}\left(1.51\times10^{-10}\,\text{J}\right).}$$

The momentum of the proton is

$$p = mv = m_0v\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\}$$

$$= \left(1.67\times10^{-27}\text{ kg}\right)\left(2.60\times10^{8}\text{ m/s}\right)\left\{\frac{1}{\left[1-\left(0.867\right)^{2}\right]^{\frac{1}{2}}}\right\} = \boxed{8.70\times10^{-19}\text{ kg}\bullet\text{m/s}\left(\frac{1.63\times10^{3}\text{ MeV}}{c}\right).}$$

30. The total energy of the proton is

$$E = \text{KE} + m_0c^2 = 750\text{ MeV} + 938\text{ MeV} = 1688\text{ MeV}.$$

The relation between the momentum and energy is

$$\left(pc\right)^2 = E^2 - \left(m_0c^2\right)^2;$$

$$p^2\left(3.00\times10^{8}\text{ m/s}\right)^2 = \left[\left(1688\text{ MeV}\right)^2 - \left(938\text{ MeV}\right)^2\right]\left(1.60\times10^{-13}\text{ J/MeV}\right)^2,$$

which gives $p = \boxed{7.48\times10^{-19}\text{ kg}\bullet\text{m/s}.}$

31. The kinetic energy acquired by the proton is

$$\text{KE} = qV = \left(1\text{e}\right)\left(105\text{ MV}\right) = 105\text{ MeV}.$$

The mass of the proton is

$$m = m_0 + \frac{\text{KE}}{c^2} = \frac{938\text{ MeV}}{c^2} + \frac{\left(105\text{ MeV}\right)}{c^2} = \frac{1043\text{ MeV}}{c^2}.$$

We find the speed from

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$\frac{1043\text{MeV}}{c^2} = \frac{\frac{938\text{MeV}}{c^2}}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},$$ which gives $v = \boxed{0.437c.}$

32. The mass of the electron is

$$m = m_0 + \frac{\text{KE}}{c^2} = \frac{0.511\text{MeV}}{c^2} + \frac{\left(1.00\text{ MeV}\right)}{c^2} = \frac{1.51\text{MeV}}{c^2}.$$

We find the speed from

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$\frac{1.51\text{MeV}}{c^2} = \frac{\frac{0.511\text{MeV}}{c^2}}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},$$ which gives $v = \boxed{0.941c.}$

33. The kinetic energy acquired by the electron is

$$\text{KE} = qV = (1\text{e})(0.025\,\text{MV}) = 0.025\,\text{MeV}.$$

The mass of the electron is

$$m = m_0 + \frac{\text{KE}}{c^2} = \frac{0.511\text{MeV}}{c^2} + \frac{(0.025\,\text{MeV})}{c^2} = \frac{0.536\,\text{MeV}}{c^2}.$$

We find the speed from

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$\frac{0.536\text{MeV}}{c^2} = \frac{\left(\frac{0.511\text{MeV}}{c^2}\right)}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},\ \text{which gives } v = \boxed{0.30c.}$$

34. If M_0 is the rest mass of the new particle, for conservation of energy we have

$$2mc^2 = \frac{2m_0c^2}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}} = M_0c^2,\ \text{which gives } M_0 = \boxed{\frac{2m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}}.$$

Because energy is conserved, there was $\boxed{\text{no loss.}}$

The final particle is at rest, so the kinetic energy loss is the initial kinetic energy of the two colliding particles:

$$\text{KE}_{\text{loss}} = 2(m - m_0)c^2 = \boxed{2m_0c^2\left(\left\{\frac{1}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}\right\} - 1\right)}.$$

$\boxed{35.}$ (*a*) The total energy of the proton is

$$E = mc^2 = \text{KE} + m_0c^2 = \tfrac{1}{2}mc^2 + m_0c^2,\ \text{which gives } m = 2m_0.$$

We find the speed from

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$2m_0 = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},\ \text{which gives } v = \boxed{0.866c.}$$

(*b*) $E = \text{KE} + m_0c^2 = \tfrac{1}{2}m_0c^2 + m_0c^2$, which gives $m = \tfrac{3}{2}m_0$.

We find the speed from

$$\tfrac{3}{2}m_0 = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},\ \text{which gives } v = \boxed{0.745c.}$$

36. The total energy of the electron is

$$E = mc^2 = \text{KE} + m_0c^2 = m_0c^2 + m_0c^2 = 2m_0c^2,\ \text{which gives } m = 2m_0.$$

We find the speed from

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$2m_0 = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},\ \text{which gives } \left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}} = \tfrac{1}{2},\ \text{so } v = \boxed{0.866c.}$$

The momentum of the electron is

$$p = mv = \frac{m_0 v}{\left[1-\left(\frac{v}{c^2}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{(9.11\times10^{-31}\text{ kg})(0.866)(3.00\times10^{8}\text{ m/s})}{\left(\frac{1}{2}\right)} = \boxed{4.73\times10^{-22}\text{ kg•m/s.}}$$

37. (*a*) The kinetic energy is

$$\text{KE} = (m - m_0)c^2 = m_0c^2\left(\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\} - 1\right)$$

$$= (27,000\text{ kg})(3.00\times10^{8}\text{ m/s})^2\left(\left\{\frac{1}{\left[1-(0.21)^2\right]^{\frac{1}{2}}}\right\} - 1\right) = \boxed{5.5\times10^{19}\text{ J.}}$$

(*b*) When we use the classical expression, we get

$$\text{KE}_c = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(27,000\text{ kg})\left[(0.21)(3.00\times10^{8}\text{ m/s})\right]^2 = 5.36\times10^{19}\text{ J.}$$

The error is

$$\frac{(5.36 - 5.5)}{(5.5)} = -0.033 = \boxed{-3.3\%.}$$

38. The speed of the proton is

$$v = \frac{(7.35 \times 10^7 \text{ m/s})}{(3.00 \times 10^8 \text{ m/s})} = 0.245c.$$

The kinetic energy is

$$\text{KE} = (m - m_0)c^2 = m_0c^2 \left(\left\{ \frac{1}{\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}} \right\} - 1 \right)$$

$$= (938\,\text{MeV}) \left(\left\{ \frac{1}{\left[1 - (0.245)^2\right]^{\frac{1}{2}}} \right\} - 1 \right) = \boxed{29.5\,\text{MeV}\,(4.7 \times 10^{-12}\text{ J}).}$$

The momentum of the proton is

$$p = mv = \frac{m_0 v}{\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}$$

$$= (1.67 \times 10^{-27}\text{ kg})(7.35 \times 10^7\text{ m/s}) \left\{ \frac{1}{\left[1 - (0.245)^2\right]^{\frac{1}{2}}} \right\} = \boxed{1.3 \times 10^{-19}\text{ kg}\bullet\text{m/s.}}$$

From the classical expressions, we get

$\text{KE}_c = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27}\text{ kg})(7.35 \times 10^7\text{ m/s})^2 = 4.51 \times 10^{-12}\text{ J}$, with an error of

$$\frac{(4.51 - 4.7)}{(4.7)} = -0.05 = \boxed{-5\%.}$$

$p = mv = (1.67 \times 10^{-27}\text{ kg})(7.35 \times 10^7\text{ m/s}) = 1.2 \times 10^{-19}\text{ kg}\bullet\text{m/s}$, with an error of

$$\frac{(1.2 - 1.3)}{(1.3)} = -0.03 = \boxed{-3\%.}$$

39. If we ignore the recoil of the neptunium nucleus, the increase in kinetic energy is the kinetic energy of the alpha particle;

$$\text{KE}_\alpha = \left[m_{\text{Am}} - (m_{\text{Np}} + m_\alpha)\right]c^2;$$

$$5.5\,\text{MeV} = \left[241.05682\,\text{u} - (m_{\text{Np}} + 4.00260\,\text{u})\right]c^2 \left(\frac{931.5\text{MeV}}{\text{u}c^2}\right),$$

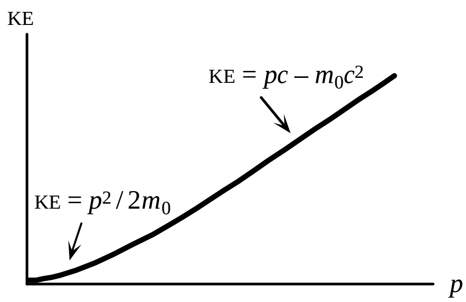
which gives $m_{\text{Np}} = \boxed{237.04832\,\text{u.}}$

40. The increase in kinetic energy comes from the decrease in potential energy:

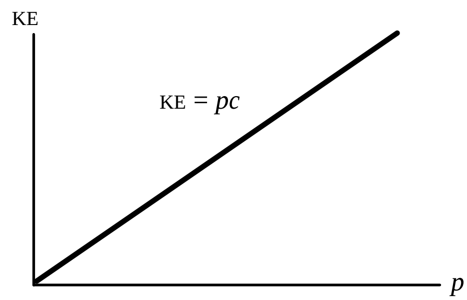
$$\text{KE} = (m - m_0)c^2 = m_0c^2\left(\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\}-1\right);$$

$$6.60\times10^{-14}\text{ J} = (9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})^2\left(\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\}-1\right),\text{ which gives } v = \boxed{0.833c.}$$

41. (*a*)



(*b*)



42. The total energy of the proton is

$$E = mc^2 = \text{KE} + m_0c^2 = 998\,\text{GeV} + 0.938\,\text{GeV} = 999\,\text{GeV},\text{ so the mass is }\frac{999\text{GeV}}{c^2}.$$

We find the speed from

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$\frac{999\text{GeV}}{c^2} = \frac{\left(\frac{0.938\text{GeV}}{c^2}\right)}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},\text{ which gives }\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}} = 9.39\times10^{-4},\text{ so } v = 1.00c.$$

The magnetic force provides the radial acceleration:

$$qvB = \frac{mv^2}{r},\text{ or}$$

$$B = \frac{mv}{qr} = \frac{m_0v}{qr\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}$$

$$=\frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(1.6\times10^{-19}\text{ C}\right)\left(1.0\times10^{3}\text{ m}\right)\left(9.39\times10^{-4}\right)}=\boxed{3.3\text{T.}}$$

Note that the mass is constant during the revolution.

43. We find the speed in the frame of the Earth from

$$u=\frac{\left(v+u'\right)}{\left(1+\dfrac{vu'}{c^2}\right)}=\frac{\left(0.50c+0.50c\right)}{\left[1+\left(0.50\right)\left(0.50\right)\right]}=\boxed{0.80c.}$$

44. (*a*) In the reference frame of the second spaceship, the Earth is moving at 0.50*c*, and the first spaceship is moving at 0.50*c* relative to the Earth. Thus the speed of the first spaceship relative to the second is

$$u=\frac{\left(v+u'\right)}{\left(1+\dfrac{vu'}{c^2}\right)}=\frac{\left(0.50c+0.50c\right)}{\left[1+\left(0.50\right)\left(0.50\right)\right]}=\boxed{0.80c.}$$

(*b*) In the reference frame of the first spaceship, the Earth is moving at $-0.50c$, and the second spaceship is moving at $-0.50c$ relative to the Earth. Thus the speed of the second spaceship relative to the first is

$$u=\frac{\left(v+u'\right)}{\left(1+\dfrac{vu'}{c^2}\right)}=\frac{\left[-0.50c+\left(-0.50c\right)\right]}{\left[1+\left(-0.50\right)\left(-0.50\right)\right]}=\boxed{-0.80c,}\text{ as expected.}$$

45. We take the positive direction in the direction of the first spaceship.

(*a*) In the reference frame of the Earth, the first spaceship is moving at $+0.71c$, and the second spaceship is moving at $+0.87c$ relative to the first. Thus the speed of the second spaceship relative to the Earth is

$$u=\frac{\left(v+u'\right)}{\left(1+\dfrac{vu'}{c^2}\right)}=\frac{\left(+0.71c+0.87c\right)}{\left[1+\left(0.71\right)\left(0.87\right)\right]}=\boxed{0.98c.}$$

(*b*) In the reference frame of the Earth, the first spaceship is moving at $+0.71c$, and the second spaceship is moving at $-0.87c$ relative to the first. Thus the speed of the second spaceship relative to the Earth is

$$u=\frac{\left(v+u'\right)}{\left(1+\dfrac{vu'}{c^2}\right)}=\frac{\left[+0.71c+\left(-0.87c\right)\right]}{\left[1+\left(0.71\right)\left(-0.87\right)\right]}=\boxed{-0.42c.}$$

46. We take the positive direction in the direction of the *Enterprise*. In the reference frame of the alien vessel, the Earth is moving at $-0.60c$, and the *Enterprise* is moving at $+0.90c$ relative to the Earth. Thus the speed of the *Enterprise* relative to the alien vessel is

$$u=\frac{\left(v+u'\right)}{\left(1+\dfrac{vu'}{c^2}\right)}=\frac{\left(-0.60c+0.90c\right)}{\left[1+\left(-0.60\right)\left(+0.90\right)\right]}=\boxed{0.65c.}$$

Note that the relative speed of the two vessels as seen on Earth is $0.90c-0.60c=0.30c$.

47. From the first pod's point of view, the spaceship and the second pod are moving in the same direction. Thus the speed of the second pod relative to the first is

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}} = \frac{0.60c + 0.70c}{1 + (0.60)(0.70)} = \boxed{0.92c.}$$

48. The speed of rocket B relative to Earth can be written as

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}},$$

where v is the speed of Rocket A relative to Earth and u' is the speed of Rocket B relative to Rocket A. With the given values, then,

$$0.95c = \frac{0.75c + u'}{1 + \frac{0.75u'}{c}}, \text{ which gives } u' = \boxed{0.70c.}$$

49. (*a*) To travelers on the spacecraft, the distance to the star is contracted:

$$L = L_0\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = (4.3\,\text{ly})\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}.$$

Because the star is moving toward the spacecraft, to cover this distance in 4.0 yr, the speed of the star must be

$$v = \frac{L}{t} = \left(\frac{4.3\,\text{ly}}{4.0\,\text{yr}}\right)\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = (1.075c)\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}, \text{ which gives } v = 0.73c.$$

Thus relative to the Earth-star system, the speed of the spacecraft is $\boxed{0.73c.}$

(*b*) According to observers on Earth, clocks on the spacecraft run slow:

$$t_{\text{Earth}} = \frac{t}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}} = \frac{(4.0\,\text{yr})}{\left[1 - (0.73)^2\right]^{\frac{1}{2}}} = \boxed{5.9\,\text{yr.}}$$

Note that this agrees with the time found from distance and speed:

$$t_{\text{Earth}} = \frac{L_0}{v} = \frac{(4.3\,\text{ly})}{0.73c} = 5.9\,\text{yr}.$$

50. The electrostatic force provides the radial acceleration:

$$\frac{ke^2}{r^2} = \frac{mv^2}{r}.$$

Thus we find the speed from

$$v^2 = \frac{(9.0\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{(9.11\times10^{-31}\ \text{kg})(0.5\times10^{-10}\ \text{m})},$$

which gives $v = 2\times10^6\ \text{m/s}$.

Because this is less than $0.1c$, the electron is $\boxed{\text{not relativistic.}}$

51. (*a*) We find the speed from

$$m = \frac{m_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$14{,}000 m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2, \text{ which gives}$$

$$\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}} = 7.14 \times 10^{-5}, \text{ or } \left(\frac{v}{c}\right)^2 = 1 - 5.10 \times 10^{-9}.$$

When we take the square root, we get

$$\frac{v}{c} = \left(1 - 5.10 \times 10^{-9}\right)^{\frac{1}{2}} \approx 1 - \tfrac{1}{2}\left(5.10 \times 10^{-9}\right) = 1 - 2.55 \times 10^{-9}.$$

Thus the speed is $\boxed{0.77\,\text{m/s less than } c.}$

(*b*) The contracted length of the tube is

$$L = L_0 \left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = (3.0\,\text{km})\left(7.14 \times 10^{-5}\right) = 2.1 \times 10^{-4}\,\text{km} = \boxed{21\,\text{cm.}}$$

52. We find the mass change from the required energy:

$$E = Pt = m_0 c^2;$$

$$(100\,\text{W})\left(3.16 \times 10^7\,\text{s}\right) = m_0 \left(3.00 \times 10^8\,\text{m/s}\right)^2, \text{ which gives } m_0 = \boxed{3.5 \times 10^{-8}\,\text{kg.}}$$

53. The minimum energy is required to produce the pair at rest:

$$E = 2m_0 c^2 = 2(0.511\,\text{MeV}) = \boxed{1.02\,\text{MeV}\left(1.64 \times 10^{13}\,\text{J}\right).}$$

54. The magnetic force provides the radial acceleration:

$$qvB = \frac{mv^2}{r}, \text{ or}$$

$$r = \frac{mv}{qB} = \frac{m_0 v}{qB\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{\left(9.11 \times 10^{-31}\,\text{kg}\right)(0.92)\left(3.00 \times 10^8\,\text{m/s}\right)}{\left(1.6 \times 10^{-19}\,\text{C}\right)(1.8\,\text{T})\left[1 - (0.92)^2\right]^{\frac{1}{2}}}$$

$$= 2.2 \times 10^{-3}\,\text{m} = \boxed{2.2\,\text{mm.}}$$

55. Because the total energy of the muons becomes electromagnetic energy, we have

$$E = m_1c^2 + m_2c^2 = \frac{m_0}{\left[1-\left(\frac{v_1^2}{c^2}\right)\right]^{\frac{1}{2}}} + \frac{m_0}{\left[1-\left(\frac{v_2^2}{c^2}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{\left(\frac{105.7\text{MeV}}{c^2}\right)(c^2)}{\left[1-(0.33)^2\right]^{\frac{1}{2}}} + \frac{\left(\frac{105.7\text{MeV}}{c^2}\right)(c^2)}{\left[1-(0.50)^2\right]^{\frac{1}{2}}} = \boxed{234\,\text{MeV.}}$$

56. The kinetic energy comes from the decrease in rest mass:

$$\text{KE} = \left[m_\text{n} - \left(m_\text{p} + m_e + m_\nu\right)\right]c^2$$

$$= \left[1.008665\,\text{u} - (1.00728\,\text{u} + 0.000549\,\text{u} + 0)\right]c^2\left(\frac{931.5\text{MeV}}{\text{u}c^2}\right) = \boxed{0.78\,\text{MeV.}}$$

57. (*a*) We find the rate of mass loss from

$$\frac{\Delta m}{\Delta t} = \frac{\left(\frac{\Delta E}{\Delta t}\right)}{c^2}$$

$$= \frac{\left(4\times10^{26}\ \text{W}\right)}{\left(3\times10^8\ \text{m/s}\right)^2} = \boxed{4.4\times10^9\ \text{kg/s.}}$$

(*b*) We find the time from

$$\Delta t = \frac{\Delta m}{\text{rate}} = \frac{\left(5.98\times10^{24}\ \text{kg}\right)}{\left(4.4\times10^9\ \text{kg/s}\right)\left(3.16\times10^7\ \text{s/yr}\right)} = \boxed{4.3\times10^7\ \text{yr.}}$$

(*c*) We find the time for the Sun to lose all of its mass at this rate from

$$\Delta t = \frac{\Delta m}{\text{rate}} = \frac{\left(2.0\times10^{30}\ \text{kg}\right)}{\left(4.4\times10^9\ \text{kg/s}\right)\left(3.16\times10^7\ \text{s/yr}\right)} = \boxed{1.4\times10^{13}\ \text{yr.}}$$

58. The speed of the particle is

$$v = \frac{\left(2.24\times10^7\ \text{m/s}\right)}{\left(3.00\times10^8\ \text{m/s}\right)} = 0.747c.$$

We use the momentum to find the rest mass:

$$p = mv = \frac{m_0v}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}};$$

$$3.0\times10^{-22}\,\text{kg}\bullet\text{m/s} = \frac{m_0(0.747)\left(3.00\times10^8\ \text{m/s}\right)}{\left[1-(0.747)^2\right]^{\frac{1}{2}}},$$

which gives $m_0 = 9.11\times10^{-31}$ kg.

Because the particle has a negative charge, it is $\boxed{\text{an electron.}}$

59. The binding energy is the energy required to provide the increase in rest mass:

$$\text{KE} = \left[\left(2m_{\text{p}} + 2m_{e}\right) - m_{\text{He}}\right]c^2$$

$$= \left[2(1.00783\,\text{u}) + 2(1.00867\,\text{u}) - 4.00260\,\text{u}\right]c^2\left(\frac{931.5\,\text{MeV}}{\text{u}c^2}\right) = \boxed{28.3\,\text{MeV.}}$$

60. We convert the speed: $\dfrac{(110\,\text{km/h})}{(3.6\,\text{ks/h})} = 30.6\,\text{m/s}.$

Because this is much smaller than c, the mass of the car is

$$m = \frac{m_0}{\left[1 - \left(\dfrac{v^2}{c^2}\right)\right]^{\frac{1}{2}}} \approx m_0\left[1 + \tfrac{1}{2}\left(\frac{v}{c}\right)^2\right].$$

The fractional change in mass is

$$\frac{(m - m_0)}{m_0} = \left[1 + \tfrac{1}{2}\left(\frac{v}{c}\right)^2\right] - 1 = \tfrac{1}{2}\left(\frac{v}{c}\right)^2$$

$$= \tfrac{1}{2}\left[\frac{(30.6\,\text{m/s})}{(3.00\times 10^8\,\text{m/s})}\right]^2 = 5.19\times 10^{-15} = \boxed{5.19\times 10^{-13}\%.}$$

61. (*a*) The magnitudes of the momenta are equal:

$$p = mv = \frac{m_0 v}{\left[1 - \left(\dfrac{v}{c}\right)^2\right]^{\frac{1}{2}}}$$

$$= \frac{(1.67\times 10^{-27}\,\text{kg})(0.935)(3.00\times 10^8\,\text{m/s})}{\left[1 - (0.935)^2\right]^{\frac{1}{2}}} = \boxed{1.32\times 10^{-18}\,\text{kg}\bullet\text{m/s.}}$$

(*b*) Because the protons are moving in opposite directions, the sum of the momenta is $\boxed{0.}$

(*c*) In the reference frame of one proton, the laboratory is moving at 0.935c. The other proton is moving at $+0.935c$ relative to the laboratory. Thus the speed of the other proton relative to the first is

$$u = \frac{(v + u')}{\left(1 + \dfrac{vu'}{c^2}\right)} = \frac{\left[+0.935c + (+0.935c)\right]}{\left[1 + (+0.935)(+0.935)\right]} = 0.998c.$$

The magnitude of the momentum of the other proton is

$$p = mv = \frac{m_0 v}{\left[1 - \left(\dfrac{v}{c}\right)^2\right]^{\frac{1}{2}}}$$

$$= \frac{(1.67\times 10^{-27}\,\text{kg})(0.998)(3.00\times 10^8\,\text{m/s})}{\left[1 - (0.998)^2\right]^{\frac{1}{2}}} = \boxed{7.45\times 10^{-18}\,\text{kg}\bullet\text{m/s.}}$$

62. The relation between energy and momentum is

$$E = \left(m_0^{\,2}c^4 + p^2c^2\right)^{\frac{1}{2}} = c\left(m_0^{\,2}c^2 + p^2\right)^{\frac{1}{2}}.$$

For the momentum, we have

$$p = mv = \frac{Ev}{c^2}, \text{ or}$$

$$v = \frac{pc^2}{E} = \frac{pc}{\left(m_0^{\,2}c^2 + p^2\right)^{\frac{1}{2}}}.$$

63. The kinetic energy that must be supplied is

$$\text{KE} = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0c^2$$

$$= \frac{m_0c^2}{\sqrt{1 - (0.10)^2}} - m_0c^2 = 0.00504m_0c^2.$$

This is the amount of energy that would be liberated by converting $0.00504m_0$ of mass into energy, or

$$0.00504\left(5\times10^9\text{ kg}\right) = \boxed{2.5\times10^7\text{ kg.}}$$

64. (*a*) If the kinetic energy is 5.00 times as great as the rest energy, we have

$$\text{KE} = (m - m_0)c^2 = 5.00m_0c^2, \text{ or } m = 6.00m_0.$$

We find the speed from

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}};$$

$$6.00m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \text{ which gives } v = \boxed{0.986c.}$$

(*b*) If the kinetic energy is 999 times as great as the rest energy,

$$\text{KE} = (m - m_0)c^2 = 999m_0c^2, \text{ or } m = 1000m_0.$$

Then

$$1000m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ gives } v = \boxed{\left(1 - 5\times10^{-7}\right)c.}$$

65. To an observer in the barn reference frame, if the boy runs fast enough, the measured contracted length will be less than 12.0 m, so the observer can say that the two ends of the pole were simultaneously inside the barn. We find the necessary speed from

$$L = L_0\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}};$$

$$12.0\,\text{m} = (15.0\,\text{m})\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}},\ \text{which gives } v = 0.60c.$$

To the boy, the barn is moving and thus the length of the barn as he would measure it is less than the length of the pole:

$$L = L_0\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = (12.0\,\text{m})\left[1-(0.60)^2\right]^{\frac{1}{2}} = 7.68\,\text{m}.$$

However, simultaneity is relative. Thus when the two ends are simultaneously inside the barn to the barn observer, those two events are not simultaneous to the boy. Thus he would claim that the observer in the barn determined that the ends of the pole were inside the barn at different times, which is also what the boy would say. It is $\boxed{\text{not possible}}$ in the boy's frame to have both ends of the pole inside the barn simultaneously.

66. We find the loss in mass from

$$\Delta m = \frac{\Delta E}{c^2} = \frac{484\times10^3\ \text{J}}{\left(3.00\times10^8\ \text{m/s}\right)^2} = \boxed{5.38\times10^{-12}\ \text{kg.}}$$

One mole of hydrogen weighs $1\,\text{g} = 0.001$ kg.

One mole of oxygen weighs $16\,\text{g} = 0.016$ kg.

So the total mass of the reactants is 0.018 kg, and Δm as a percentage of that is

$$\frac{5.38\times10^{-12}\ \text{kg}}{0.018\ \text{kg}} = 3.0\times10^{-10} = \boxed{3.0\times10^{-8}\,\%.}$$

67. From one particle's point of view, the laboratory and the other particle are moving in the same direction. Thus the speed of one particle relative to the other is

$$u = \frac{(v+u')}{\left(1+\frac{vu'}{c^2}\right)} = \frac{(0.75c+0.75c)}{\left[1+(0.75)(0.75)\right]} = \boxed{0.96c.}$$

68. (*a*) Earth observers see the ship as contracted, as given by the equation

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = (25\,\text{m})\sqrt{1-(0.75)^2} = \boxed{17\,\text{m.}}$$

(*b*) Earth observers see the launch as dilated (lengthened) in time, as given by the equation

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{23\,\text{min}}{\sqrt{1-(0.75)^2}} = \boxed{35\,\text{min}.}$$

69. Every observer will measure the speed of a beam of light to be $\boxed{c.}$

Check: In the reference frame of the Earth, you are moving at $+0.85c$ and the beam is traveling at $-c$ relative to you. Thus the beam's speed relative to Earth is

$$u = \frac{v+u'}{1+vu'} = \frac{0.85c+(-c)}{1+(0.85)(-1)} = -c.$$

The beam's speed as a positive value, relative to Earth, is c.

70. The kinetic energy that must be supplied is

$$\text{KE} = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$$

$$= \frac{m_0c^2}{\sqrt{1-(0.60)^2}} - m_0c^2 = 0.25m_0c^2.$$

This is the amount that would be liberated by converting $0.25m_0$ of mass into energy, or

$$0.25(150{,}000\,\text{kg}) = \boxed{37{,}500\,\text{kg.}}$$

From Earth's point of view the trip will take

$$\Delta t = \frac{d}{v} = \frac{(25\,\text{y})c}{0.60c} = 41.7\,\text{y.}$$

According to the astronauts, it will take less time:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}};$$

$$41.7\,\text{y} = \frac{\Delta t_0}{\sqrt{1-(0.60)^2}},\ \text{which gives}\ \Delta t_0 = \boxed{33\,\text{y.}}$$

71. The kinetic energy is

$$\text{KE} = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$$

$$= \frac{(12{,}500\,\text{kg})(3.00\times10^8\,\text{m/s})^2}{\sqrt{1-(0.99)^2}} - (12{,}500\,\text{kg})(3.00\times10^8\,\text{m/s})^2 = \boxed{6.8\times10^{21}\,\text{J.}}$$

We compare this with annual U.S. energy consumption:

$$\frac{6.8\times10^{21}\,\text{J}}{10^{20}\,\text{J}} = 68.$$

The spaceship's kinetic energy is about 68 times as great.

72. The passengers will see the trip distance shrunken as given by

$$L = L_0\sqrt{1-\frac{v^2}{c^2}},$$

and they will measure their speed to be

$$v = \frac{L}{\Delta t_0},\ \text{which means that}$$

$$v = \frac{L_0}{\Delta t_0}\sqrt{1-\frac{v^2}{c^2}}$$

$$= \frac{(6.0\,\text{y})c}{1.0}\sqrt{1-\frac{v^2}{c^2}},\ \text{which gives}\ v = 0.986c.$$

The work required to supply the necessary kinetic energy is

$$\text{KE} = \frac{m_0c^2}{\sqrt{1-\dfrac{v^2}{c^2}}} - m_0c^2$$

$$= \frac{(42{,}000\,\text{kg})(3.00\times10^{-8}\,\text{m/s})^2}{\sqrt{1-(0.986)^2}} - (42{,}000\,\text{kg})(3.00\times10^{8}\,\text{m/s})^2 = \boxed{1.82\times10^{22}\,\text{J.}}$$

73. (*a*) Because the spring is at rest on the spaceship, its period is

$$T = 2\pi\left(\frac{m}{k}\right)^{\frac{1}{2}} = 2\pi\left[\frac{(1.68\,\text{kg})}{(48.7\,\text{N/m})}\right]^{\frac{1}{2}} = \boxed{1.17\,\text{s.}}$$

(*b*) The oscillating mass is a clock. According to observers on Earth, clocks on the spacecraft run slow:

$$T_{\text{Earth}} = \frac{T}{\left[1-\left(\dfrac{v^2}{c^2}\right)\right]^{\frac{1}{2}}} = \frac{(1.17\,\text{s})}{\left[1-(0.900)^2\right]^{\frac{1}{2}}} = \boxed{2.68\,\text{s.}}$$

74. The neutrino has no rest mass, so we have

$$E_\nu = \left(p_\nu{}^2c^2 + m_\nu{}^2c^4\right)^{\frac{1}{2}} = p_\nu c.$$

Because the pi meson decays at rest, momentum conservation tells us that the muon and neutrino have equal and opposite momenta:

$$p_\mu = p_\nu = p.$$

For energy conservation, we have

$$E_\pi = E_\mu + E_\nu;$$

$$m_\pi c^2 = \left(p_\mu{}^2c^2 + m_\mu{}^2c^4\right)^{\frac{1}{2}} + p_\nu c = \left(p^2c^2 + m_\mu{}^2c^4\right)^{\frac{1}{2}} + pc.$$

If we rearrange and square, we get

$$\left(m_\pi c^2 - pc\right)^2 = m_\pi{}^2c^4 - 2m_\pi c^2pc + p^2c^2 = p^2c^2 + m_\mu{}^2c^4,\ \text{or}\ \ pc = \frac{\left(m_\pi{}^2c^2 - m_\mu{}^2c^2\right)}{2m_\pi}.$$

The kinetic energy of the muon is

$$\text{KE}_\mu = E_\mu - m_\mu c^2 = \left(m_\pi c^2 - pc\right) - m_\mu c^2 = m_\pi c^2 - m_\mu c^2 - \frac{\left(m_\pi{}^2c^2 - m_\mu{}^2c^2\right)}{2m_\pi}$$

$$= \frac{\left(2m_\pi{}^2 - 2m_\mu m_\pi - m_\pi{}^2 + m_\mu{}^2\right)c^2}{2m_\pi} = \frac{\left(m_\pi{}^2 - 2m_\mu m_\pi + m_\mu{}^2\right)c^2}{2m_\pi} = \frac{\left(m_\pi - m_\mu\right)^2c^2}{2m_\pi}.$$