CHAPTER 28: Quantum Mechanics of Atoms

Answers to Questions

1. (*a*) Both the matter wave Ψ and the string wave describe/represent the wave amplitude with respect to time and position. A matter wave Ψ is three-dimensional, but a string wave is two-dimensional. The string wave tells you the displacement of any part of the string from its equilibrium position at any given time, but for a matter wave the $|\Psi|^2$ represents the probability of finding the mass in a particular volume. The wave speed on the string depends on the string tension and mass density, but the Ψ wave speed depends on the energy.
   (*b*) Both the matter wave Ψ and the EM wave describe/represent the wave amplitude with respect to time and position, although the EM wave describes the amplitudes of the electric and magnetic fields (their strengths). Again, for the matter wave the $|\Psi|^2$ represents the probability of finding the mass in a particular volume. An EM wave can be polarized, but a matter wave cannot. The EM wave represents a vector field, but a matter wave is a scalar. The speed of the EM wave is *c*, but the matter wave's speed depends on the energy.

2. Bohr's model of the atom has the electrons in particular orbits with precise values of radii, but quantum mechanics gives a wide range of possible radii for each electron and only gives the probability of finding the electron at a particular radius. The uncertainty principle also says that both the position and the momentum of an electron cannot be known precisely, but Bohr's model does predict exact values for both position and momentum.

3. As a particle becomes more massive, the uncertainty of its momentum $(\Delta p = m\Delta v)$ becomes larger, which reduces the uncertainty of its position (due to $\Delta x = \hbar / \Delta p$). Thus, with a small uncertainty of its position, we can do a better job of predicting its future position.

4. Because of the very small value of $\hbar$, the uncertainties in both a baseball's momentum and position can be very small compared to typical macroscopic positions and momenta without being close to the limit imposed by the uncertainty principle. For instance, uncertainty in the baseball's position could be 10^{-10} m, and the uncertainty in the baseball's momentum $10^{-10}\ \mathrm{kg{\bullet}m/s}$, and still easily satisfy the uncertainty principle. Yet we never try to measure the position or momentum of a baseball to that precision. For an electron, however, typical values of uncertainty in position and momentum can be close to the uncertainty principle limit, and so the relative uncertainty can be much higher for that small object.

5. No, it would not be possible to balance a very sharp needle precisely on its point. Both the center of mass of the needle and the point of the needle cannot be precisely known, especially when speed and momentum are small. If we don't know exactly where they are, but we need the center of mass directly above the point to balance, it is not possible. Also, the speed and momentum of both the center of mass and the point cannot be precisely known, especially if we think we know their positions exactly. Thus, if we do know that the center of mass and the point lined up exactly, then we couldn't also know that they are both not moving, and it would fall over.

6. No. The hot soup must warm up the thermometer when they come in contact, and so the final temperature of the thermometer and soup system will be less than the initial temperature of the soup. The thermometer reading will be lower than the original temperature of the soup. This is another

situation where making the measurement affects the physical situation and causes error or uncertainty to enter in the measurement.

7. Yes. The uncertainty principle tells us that the precision of a position measurement is dependent on the precision of the momentum ($\Delta x \Delta p = \hbar$). Thus, the uncertainty principle limits our ability to make a measurement of position. If it was possible to allow the uncertainty of the momentum to go to ∞ , then we could make an exact measurement of *x*.

8. If $\Delta x = 0$, then $\Delta p = \infty$. Thus, the uncertainty principle tells us that if we knew the position of an object precisely, then we would not know anything about the momentum.

9. Yes, air usually escapes when you check the air pressure in a tire, since some of the air needs to flow into the device you are using to check the pressure. The relationship to the uncertainty principle is that by making the measurement, you have affected the state of the system.

10. Yes, this is consistent with the uncertainty principle for energy $\left(\Delta E \Delta t = \hbar\right)$. The ground state energy can be fairly precisely known because the electrons remain in that state for a very long time. Thus, as $\Delta t \to \infty$, then $\Delta E \to 0$. For the excited states, the electrons do not remain in them for very long, thus the Δt in this case is relatively small, making the ΔE , the energy width, fairly large.

11. The quantum mechanical model predicts that the electron in the hydrogen atom will spend more time near the nucleus. The Bohr model predicts that the electron is always at some certain distance from the nucleus and that it can't get any closer if it is in the lowest energy level. Quantum mechanics predicts that the electron spends time both closer and farther from the nucleus (compared to Bohr), since it can be described as an "electron cloud" that is centered upon the Bohr radius. Thus, the quantum mechanical model predicts that the electron will spend more time near the nucleus. See Figure 28-6 for a good visual of this situation.

12. As the number of electrons goes up, so does the number of protons in the nucleus, which increases the attraction of the electrons to the center of the atom. Even though the outer electrons are partially screened by the inner electrons, they are all pulled closer to the very positive nuclei of the large Z atoms when compared to the low Z atoms. This keeps the atom from expanding in a "linear" fashion as larger numbers of electrons are added to the large Z atoms.

13. In helium, the electron falling down to the lowest energy level is interacting with another electron (which doesn't happen in hydrogen) and the electron is interacting with a more positive nucleus than in hydrogen. Thus, the emission spectrum in helium is very different than in hydrogen. The excited electrons are in very different environments, thus they have very different energy levels with different spacings. So, even though both atoms have such a ground state (n = 1, ℓ = 0, m_ℓ = 0), there are very different states above that ground state in which the electrons can exist. This then creates different emission spectral lines when the electrons return to the ground state.

14 The ground state configurations could only have half the number of electrons in each shell: $\ell = 0$ shells would only have 1 state, $\ell = 1$ shells would have 3 states, $\ell = 2$ shells would have 5 states, etc. With no spin number, the Pauli Exclusion Principle says that only one electron is allowed per state. Thus, we would need more shells to hold all of the electrons and the elements would need to be "stacked" differently in the periodic table, since 1 electron would fill the *s* shell and 3 electrons would fill the *p* shell and 5 electrons would fill the *d* shell, etc. The periodic table would look something like this:

$^{1}H_{1s}1$								
$^{2}He_{2s}1$						$^{3}Li_{2p}1$	$^{4}Be_{2p}2$	$^{5}B_{2p}3$
$^{6}C_{3s}1$						$^{7}N_{3p}1$	$^{8}O_{3p}2$	$^{9}F_{3p}3$
$^{10}Ne_{4s}1$	$^{11}Na_{3d}1$	$^{12}Mg_{3d}2$	$^{13}Al_{3d}3$	$^{14}Si_{3d}4$	$^{15}P_{3d}5$	$^{16}S_{4p}1$	$^{17}Cl_{4p}2$	$^{18}Ar_{4p}3$
$^{19}K_{5s}1$	$^{20}Ca_{4d}1$							

15. (*a*) Allowed.
    (*b*) Not allowed. Cannot have 8 electrons in the *p* shell.
    (*c*) Allowed.

16. Uranium: $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}\ 5p^6\ 6s^2\ 5d^{10}\ 6p^6\ 4f^{14}\ 7s^2\ 5f^3\ 6d^1$.

17. (*a*) Group II. Outer electrons are $3s^2$: magnesium.
    (*b*) Group VIII. Outer electrons are $3s^2\ 3p^6$: argon.
    (*c*) Group I. Outer electron is $4s^1$: potassium.
    (*d*) Group VII. Outer electrons are $2s^2\ 2p^5$: fluorine.

18. Both chlorine and iodine are in the same column of the periodic table (Group VII). Thus, both of their outer electron configurations are p^5 (5 electrons in their outer *p* shell), which makes them both halogens. Halogens lack one electron from a filled shell, so both of their outer electron orbit shapes are similar and can readily accept an additional electron from another atom. This makes their chemical reaction properties extremely similar.

19. Both sodium and potassium are in the same column of the periodic table (Group I). Thus, both of their outer electron configurations are s^1 (1 electron in their outer *s* shell), which makes them both alkali metals. Alkali metals have one lone electron in their outermost shell, so both of their outer electron orbit shapes are similar and they can readily share this electron with another atom (especially since this outermost electron spends a considerable amount of time very far away from the nucleus). This makes their chemical reaction properties extremely similar.

20. Neon is a “closed shell” atom (a noble gas) where all of the electron shells are completely full and spherically symmetric, and so its electrons are not readily ionized. Sodium is a Group I atom (an alkali metal) that has one lone electron in the outermost shell, and this electron spends a considerable amount of time far away from the nucleus and also shielded from the nucleus. The result is that sodium can be ionized much easier than neon, which the two ionization energies demonstrate very well.

21. As electrons are decelerated in their interactions with positive nuclei in a material, they radiate energy (called bremsstrahlung radiation). Because energy is conserved, the energy of the emitted photon must equal the loss of kinetic energy of the electron. An electron may lose all or part of its kinetic energy in such a collision. The continuous X-ray spectrum of Figure 28-11 is explained as being due to such bremsstrahlung collisions where varying amounts of energy are lost by the electrons. The shortest wavelength X-ray (λ_o) must be due to an electron that gives up all of its kinetic energy to produce one photon in a single collision. If X-rays were not photons but particles with mass, some of the kinetic energy would have to go into creating the mass of this particle, but since the λ_o corresponds directly to the kinetic energy of the incoming electrons, X-rays must be massless photons. Thus, this result is evidence that the photon theory of light is valid.

22. When we use the Bohr theory to predict the X-ray line wavelengths, we need to estimate the amount that the electron that is changing to a lower level is partially shielded by the other electron(s) in those lower

levels. So, for example, if we are looking at an electron changing into the $n = 1$ level, we estimate that the falling electron "sees" a nucleus of charge $Z - 1$, instead of just Z. The problem with this estimate is that the lower level electron(s) does not do a perfect job of shielding the changing electron, thus our calculated X-ray line wavelengths are slightly different from the measured ones.

23. To figure out which lines in an X-ray spectrum correspond to which transitions, you would use the Bohr model to estimate the differences in the sizes of the energy level jumps that the falling electrons will make. Then, you just need to match the $n = 2 \rightarrow n = 1$ energy to the K_α line and the $n = 3 \rightarrow n = 1$ energy to the K_β line and the $n = 3 \rightarrow n = 2$ jump energy to the L_α line and so on.

24. The difference in energy between $n = 1$ and $n = 2$ levels is much bigger than between any other combination of energy levels. Thus, electron transitions between the $n = 1$ and $n = 2$ levels produce photons of extremely high energy and frequency, which means these photons have very short wavelengths (since frequency and wavelength are inversely proportional to each other).

25. Spontaneous emission occurs when an electron is in an excited state of an atom and it spontaneously (with no external stimulus) drops back down to a lower energy level. To do this, it emits a photon to carry away the excess energy. Stimulated emission occurs when an electron is in an excited state of an atom but a photon strikes the atom and causes or stimulates the electron to make its transition to a lower energy level sooner than it would have done so spontaneously. The stimulating photon has to have the same energy as the difference in energy levels of the transition.

26. Different: Laser light is coherent (all of the photons are in the same phase) and ordinary light is not coherent (all of the photons have random phases). Laser light is nearly a perfect plane wave, while ordinary light is spherically symmetric (which means that the intensity of laser light remains nearly constant as it moves away from the source, while the intensity of ordinary light drops off as $1/r^2$). Laser light is always monochromatic, while ordinary light can be monochromatic, but it usually is not. Similar: Both travel at c and both display wave-particle duality of photons.

27. The 0.0005 W laser beam's intensity is nearly constant as it travels away from its source due to it being approximately a plane wave. The street lamp's intensity, though, drops off as $1/r^2$ as it travels away from its source due to it being a spherically symmetric wave. So, at a distance, the intensity of the lamp light that is actually reaching the camera is much less than 1000 W. Also, the street lamp's intensity is spread out over many wavelengths, whereas the laser's intensity is all at one wavelength, and film can be more sensitive to certain wavelengths.

28. No, the intensity of light from a laser beam does not drop off as the inverse square of the distance. Laser light is much closer to being a plane wave rather than a spherically symmetric wave, and so its intensity is nearly constant along the entire beam.

Problem Solutions

1. We find the wavelength of the neutron from

$$\lambda = \frac{h}{p} = \frac{h}{\left[2m_0\left(\text{KE}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{\left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left[2\left(1.67\times10^{-27}\ \text{kg}\right)\left(0.025\,\text{eV}\right)\left(1.6\times10^{-19}\ \text{J/eV}\right)\right]^{\frac{1}{2}}} = 1.81\times10^{-10}\ \text{m}.$$

The peaks of the interference pattern are given by

$$d\sin\theta = n\lambda,\ n = 1,\ 2, \ldots .$$

and the positions on the screen are

$$y = L\tan\theta.$$

For small angles, $\sin\theta = \tan\theta$, so we have

$$y = \frac{nL\lambda}{d}.$$

Thus the separation is

$$\Delta y = \frac{L\lambda}{d} = \frac{\left(1.0\,\text{m}\right)\left(1.81\times10^{-10}\ \text{m}\right)}{\left(0.50\,\text{m}\right)} = \boxed{3.6\times10^{-7}\ \text{m.}}$$

2. We find the wavelength of the bullet from

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{\left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(3.0\times10^{-3}\ \text{kg}\right)\left(220\,\text{m/s}\right)} = 1.0\times10^{-33}\ \text{m}.$$

The half-angle for the central circle of the diffraction pattern is given by

$$\sin\theta = \frac{1.22\lambda}{D}.$$

For small angles, $\sin\theta \approx \tan\theta$, so we have

$$r = L\tan\theta \approx L\sin\theta = \frac{1.22L\lambda}{D};$$

$$0.50\times10^{-2}\ \text{m} = \frac{1.22L\left(1.0\times10^{-33}\ \text{m}\right)}{\left(3.0\times10^{-3}\ \text{m}\right)},\ \text{which gives } L = \boxed{1.2\times10^{28}\ \text{m.}}$$

Diffraction effects are negligible for macroscopic objects.

3. We find the uncertainty in the momentum:

$$\Delta p = m\Delta v = \left(1.67\times10^{-27}\ \text{kg}\right)\left(0.024\times10^{5}\ \text{m/s}\right) = 4.00\times10^{-24}\ \text{kg}\bullet\text{m/s}.$$

We find the uncertainty in the proton's position from

$$\Delta x \geq \frac{\hbar}{\Delta p} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(4.00\times10^{-24}\ \text{kg}\bullet\text{m/s}\right)} = 2.6\times10^{-11}\ \text{m}.$$

Thus the accuracy of the position is $\boxed{\pm1.3\times10^{-11}\,\text{m.}}$

4. We find the uncertainty in the momentum:

$$\Delta p \geq \frac{\hbar}{\Delta x} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(2.0\times10^{-8}\ \text{m}\right)} = 5.28\times10^{-27}\ \text{kg}\bullet\text{m/s}.$$

We find the uncertainty in the velocity from

$$\Delta p = m\Delta v;$$

$$5.28\times10^{-27}\ \text{kg}\bullet\text{m/s} = \left(9.11\times10^{-31}\ \text{kg}\right)\Delta v,\ \text{which gives}\ \Delta v = \boxed{5.8\times10^{3}\ \text{m/s}.}$$

5. We find the minimum uncertainty in the energy of the state from

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(10^{-8}\ \text{s}\right)} = 1.1\times10^{-26}\ \text{J} = \boxed{6.6\times10^{-8}\ \text{eV}.}$$

6. We find the lifetime of the particle from

$$\Delta t \geq \frac{\hbar}{\Delta E} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(2.49\,\text{GeV}\right)\left(1.60\times10^{-10}\ \text{J/GeV}\right)} = \boxed{2.65\times10^{-25}\ \text{s}.}$$

7. We find the uncertainty in the energy of the muon from

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(2.2\times10^{-6}\ \text{s}\right)} = 4.8\times10^{-29}\ \text{J} = \boxed{3.0\times10^{-10}\ \text{eV}.}$$

Thus the uncertainty in the mass is

$$\Delta m = \frac{\Delta E}{c^2} = \boxed{3.0\times10^{-10}\ \text{eV}/c^2.}$$

$\boxed{8.}$ We find the uncertainty in the energy of the free neutron from

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(900\,\text{s}\right)} = 1.17\times10^{-37}\ \text{J}.$$

Thus the uncertainty in the mass is

$$\Delta m = \frac{\Delta E}{c^2} = \frac{\left(1.17\times10^{-37}\ \text{J}\right)}{\left(3.00\times10^{8}\ \text{m/s}\right)^2} = \boxed{1.30\times10^{-54}\ \text{kg}.}$$

9. The uncertainty in the velocity is

$$\Delta v = \left(\frac{0.055}{100}\right)\left(150\,\text{m/s}\right) = 0.0825\,\text{m/s}.$$

For the electron, we have

$$\Delta x \geq \frac{\hbar}{m\Delta v} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.0825\,\text{m/s}\right)} = \boxed{1.4\times10^{-3}\ \text{m}.}$$

For the baseball, we have

$$\Delta x \geq \frac{\hbar}{m\Delta v} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(0.140\,\text{kg}\right)\left(0.0825\,\text{m/s}\right)} = \boxed{9.1\times10^{-33}\ \text{m}.}$$

The uncertainty for the electron is greater by a factor of 1.5×10^{29}.

10. We use the radius as the uncertainty in position for the neutron. We find the uncertainty in the momentum from

$$\Delta p \geq \frac{\hbar}{\Delta x} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(1.0\times10^{-15}\ \text{m}\right)} = \boxed{1.055\times10^{-19}\ \text{kg}\bullet\text{m/s}.}$$

If we assume that the lowest value for the momentum is the least uncertainty, we estimate the lowest possible kinetic energy as

$$E = \frac{(\Delta p)^2}{2m} = \frac{\left(1.055\times10^{-19}\ \text{kg}\bullet\text{m/s}\right)^2}{2\left(1.67\times10^{-27}\ \text{kg}\right)} = 3.33\times10^{-12}\ \text{J} = \boxed{21\text{MeV.}}$$

11. We use the radius as the uncertainty in position for the electron. We find the uncertainty in the momentum from

$$\Delta p \geq \frac{\hbar}{\Delta x} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(1.0\times10^{-15}\ \text{m}\right)} = 1.055\times10^{-19}\ \text{kg}\bullet\text{m/s}.$$

If we assume that the lowest value for the momentum is the least uncertainty, we estimate the lowest possible energy as

$$E = (\text{KE}) + m_0c^2 = \left(p^2c^2 + m_0{}^2c^4\right)^{\frac{1}{2}} = \left[(\Delta p)^2c^2 + m_0{}^2c^4\right]^{\frac{1}{2}}$$

$$= \left[\left(1.055\times10^{-19}\ \text{kg}\bullet\text{m/s}\right)^2\left(3.00\times10^8\ \text{m/s}\right)^2 + \left(9.11\times10^{-31}\ \text{kg}\right)^2\left(3.00\times10^8\ \text{m/s}\right)^4\right]^{\frac{1}{2}}$$

$$= 3.175\times10^{-11}\ \text{J} \approx \boxed{200\,\text{MeV.}}$$

12. The momentum of the electron is

$$p = \left[2m(\text{KE})\right]^{\frac{1}{2}} = \left[2\left(9.11\times10^{-31}\ \text{kg}\right)\left(3.00\,\text{keV}\right)\left(1.60\times10^{-16}\ \text{J/keV}\right)\right]^{\frac{1}{2}} = 2.96\times10^{-23}\ \text{kg}\bullet\text{m/s}.$$

When the energy is increased by 1.00%, the new momentum is

$$p' = \left[2m(1.0100\text{KE})\right]^{\frac{1}{2}} = \left[2m(\text{KE})\right]^{\frac{1}{2}}(1 + 0.0100)^{\frac{1}{2}} \approx p\left[1 + \tfrac{1}{2}(0.0100)\right].$$

Thus the uncertainty in momentum is

$$\Delta p = p' - p = p\tfrac{1}{2}(0.0100) = \left(2.96\times10^{-23}\ \text{kg}\bullet\text{m/s}\right)^{\frac{1}{2}}(0.0100) = 1.48\times10^{-25}\ \text{kg}\bullet\text{m/s}.$$

We find the uncertainty in the electron's position from

$$\Delta x \geq \frac{\hbar}{\Delta p} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(1.48\times10^{-25}\ \text{kg}\bullet\text{m/s}\right)} = \boxed{7.13\times10^{-10}\ \text{m.}}$$

13. The value of ℓ can range from 0 to $n-1$. Thus for n = 6, we have $\boxed{\ell = 0, 1, 2, 3, 4, 5.}$

14. The value of m_ℓ can range from $-\ell$ to $+\ell$. Thus for $\ell = 3$, we have $\boxed{m_\ell = -3, -2, -1, 0, 1, 2, 3.}$

The possible values of m_s are $\boxed{-\tfrac{1}{2}, +\tfrac{1}{2}.}$

15. The number of electrons in the subshell is determined by the value of ℓ. For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$, or $2\ell+1$ values. For each of these there are two values of m_s. Thus the total number for $\ell=3$ is

$$N = 2(2\ell+1) = 2\left[2(3)+1\right] = \boxed{14 \text{ electrons.}}$$

16. The value of ℓ can range from 0 to $n-1$. Thus for $n=4$, we have

$$\ell = 0,1,2,3.$$

For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$, or $2\ell+1$ values. For each of these there are two values of m_s. Thus the total number for each ℓ is $2(2\ell+1)$.

The number of states is

$$N = 2(0+1) + 2(2+1) + 2(4+1) + 2(6+1) = \boxed{32 \text{ states.}}$$

We start with $\ell=0$, and list the quantum numbers in the order (n,ℓ,m_ℓ,m_s);

$(4, 0, 0, -\frac{1}{2})$, $(4, 0, 0, +\frac{1}{2})$, $(4, 1, -1, -\frac{1}{2})$, $(4, 1, -1, +\frac{1}{2})$, $(4, 1, 0, -\frac{1}{2})$, $(4, 1, 0, +\frac{1}{2})$,
$(4, 1, 1, -\frac{1}{2})$, $(4, 1, 1, +\frac{1}{2})$, $(4, 2, -2, -\frac{1}{2})$, $(4, 2, -2, +\frac{1}{2})$, $(4, 2, -1, -\frac{1}{2})$, $(4, 2, -1, +\frac{1}{2})$,
$(4, 2, 0, -\frac{1}{2})$, $(4, 2, 0, +\frac{1}{2})$, $(4, 2, 1, -\frac{1}{2})$, $(4, 2, 1, +\frac{1}{2})$, $(4, 2, 2, -\frac{1}{2})$, $(4, 2, 2, +\frac{1}{2})$,
$(4, 3, -3, -\frac{1}{2})$, $(4, 3, -3, +\frac{1}{2})$, $(4, 3, -2, -\frac{1}{2})$, $(4, 3, -2, +\frac{1}{2})$, $(4, 3, -1, -\frac{1}{2})$, $(4, 3, -1, +\frac{1}{2})$,
$(4, 3, 0, -\frac{1}{2})$, $(4, 3, 0, +\frac{1}{2})$, $(4, 3, 1, -\frac{1}{2})$, $(4, 3, 1, +\frac{1}{2})$, $(4, 3, 2, -\frac{1}{2})$, $(4, 3, 2, +\frac{1}{2})$,
$(4, 3, 3, -\frac{1}{2})$, $(4, 3, 3, +\frac{1}{2})$.

17. We start with the $n=1$ shell, and list the quantum numbers in the order (n,ℓ,m_ℓ,m_s);

(*a*) $(1, 0, 0, -\frac{1}{2})$, $(1, 0, 0, +\frac{1}{2})$, $(2, 0, 0, -\frac{1}{2})$, $(2, 0, 0, +\frac{1}{2})$, $(2, 1, -1, -\frac{1}{2})$, $(2, 1, -1, +\frac{1}{2})$.

Note that, without additional information, there are other possibilities for the last three electrons.

(*b*) $(1, 0, 0, -\frac{1}{2})$, $(1, 0, 0, +\frac{1}{2})$, $(2, 0, 0, -\frac{1}{2})$, $(2, 0, 0, +\frac{1}{2})$, $(2, 1, -1, -\frac{1}{2})$, $(2, 1, -1, +\frac{1}{2})$,
$(2, 1, 0, -\frac{1}{2})$, $(2, 1, 0, +\frac{1}{2})$, $(2, 1, -1, -\frac{1}{2})$, $(2, 1, -1, +\frac{1}{2})$, $(3, 0, 0, -\frac{1}{2})$, $(3, 0, 0, +\frac{1}{2})$.

18. We start with the $n=1$ shell, and list the quantum numbers in the order (n,ℓ,m_ℓ,m_s);

$(1, 0, 0, -\frac{1}{2})$, $(1, 0, 0, +\frac{1}{2})$, $(2, 0, 0, -\frac{1}{2})$, $(2, 0, 0, +\frac{1}{2})$, $(2, 1, -1, -\frac{1}{2})$, $(2, 1, -1, +\frac{1}{2})$,
$(2, 1, 0, -\frac{1}{2})$.

Note that, without additional information, there are other possibilities for the last three electrons.

19. The value of ℓ can range from 0 to $n-1$. Thus for $\ell=4$, we have $\boxed{n \geq 5.}$

For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$: $\boxed{m_\ell = -4, -3, -2, -1, 0, 1, 2, 3, 4.}$

There are two values of m_s: $\boxed{m_s = -\frac{1}{2}, +\frac{1}{2}.}$

20. The magnitude of the angular momentum depends on ℓ only:

$$L = \hbar\left[\ell(\ell+1)\right]^{\frac{1}{2}} = \left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)\left[(3)(3+1)\right]^{\frac{1}{2}} = \boxed{3.66\times10^{-34}\ \text{kg}\bullet\text{m}^2/\text{s.}}$$

21. The value of m_ℓ can range from $-\ell$ to $+\ell$, so we have $\boxed{\ell \geq 3.}$

The value of ℓ can range from 0 to $n-1$. Thus we have $\boxed{n \geq \ell + 1 \text{(minimum 4)}.}$

There are two values of m_s: $\boxed{m_s = -\frac{1}{2}, +\frac{1}{2}.}$

22. From *spdfg*, we see that the "*g*" subshell has $\ell = 4$, so the number of electrons is

$$N = 2(2\ell + 1) = 2\left[2(4) + 1\right] = \boxed{18 \text{ electrons.}}$$

23. (*a*) We start with hydrogen and fill the levels as indicated in the periodic table:

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^7 4s^2.}$$

Note that the $4s^2$ level is filled before the 3*d* level is started.

(*b*) For $Z = 36$ we have

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6.}$$

(*c*) For $Z = 38$ we have

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 5s^2.}$$

Note that the $5s^2$ level is filled before the 4*d* level is started.

24. (*a*) Selenium has $Z = 34$:

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^4.}$$

(*b*) Gold has $Z = 79$:

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^1.}$$

(*c*) Radium has $Z = 88$:

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2 6p^6 7s^2.}$$

25. (*a*) The principal quantum number is $n = \boxed{6.}$

(*b*) The energy of the state is

$$E_6 = -\frac{(13.6\,\text{eV})}{n^2} = -\frac{(13.6\,\text{eV})}{6^2} = \boxed{-0.378\,\text{eV}.}$$

(*c*) The "*s*" subshell has $\ell = \boxed{0.}$ The magnitude of the angular momentum depends on ℓ only:

$$L = \hbar\left[\ell(\ell + 1)\right]^{\frac{1}{2}} = \left(1.055 \times 10^{-34}\,\text{J}\bullet\text{s}\right)\left[(0)(0 + 1)\right]^{\frac{1}{2}} = \boxed{0.}$$

(*d*) For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$: $\boxed{m_\ell = 0.}$

26. The third electron in lithium is in the 2*s* subshell, which is outside the more tightly bound filled 1*s* shell. This makes it appear as if there is a "nucleus" with a net charge of $+1e$. Thus we use the energy of the hydrogen atom:

$$E_2 = -\frac{(13.6\,\text{eV})}{n^2} = -\frac{(13.6\,\text{eV})}{2^2} = -3.4\,\text{eV},$$

so the binding energy is $\boxed{3.4 \text{ eV.}}$

Our assumption of complete shielding of the nucleus by the 2*s* electrons is probably not correct.

27. In a filled subshell, we have $2(2\ell+1)$ electrons. All of the m_ℓ values

$$-\ell, -\ell+1, \ldots, 0, \ldots, \ell-1, \ell$$

are filled, so their sum is zero. For each m_ℓ value, both values of m_s are filled, so their sum is also zero. Thus the total angular momentum is zero.

28. (*a*) The $4p \rightarrow 3p$ transition is forbidden, because $\Delta\ell \neq \pm 1$.
(*b*) The $2p \rightarrow 1s$ transition is allowed, $\Delta\ell = -1$.
(*c*) The $3d \rightarrow 2d$ transition is forbidden, because $\Delta\ell \neq \pm 1$.
(*d*) The $4d \rightarrow 3s$ transition is forbidden, because $\Delta\ell \neq \pm 1$.
(*e*) The $4s \rightarrow 3p$ transition is allowed, $\Delta\ell = +1$.

29. Photon emission means a jump to a lower state, so $n = 1, 2, 3, 4,$ or 5. For a *d* subshell, $\ell = 2$, and because $\Delta\ell = \pm 1$, the new value of ℓ must be 1 or 3.
(*a*) $\ell = 1$ corresponds to a *p* subshell, and $\ell = 3$ corresponds to an *f* subshell. Keeping in mind that $0 \le \ell \le n-1$, we find the following possible destination states: $2p, 3p, 4p, 5p, 4f, 5f$.
(*b*) In a hydrogen atom, ℓ has no appreciable effect on energy, and so for energy purposes there are four possible destination states, corresponding to *n* = 2, 3, 4, and 5. Thus there are four different photon wavelengths corresponding to four possible changes in energy.

30. The shortest wavelength X-ray has the most energy, which is the maximum kinetic energy of the electron in the tube:

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J}\bullet\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(33.5\times10^{3}\ \text{eV})} = 3.70\times10^{-11}\,\text{m} = 0.0370\,\text{nm}.$$

The longest wavelength of the continuous spectrum would be at the limit of the X-ray region of the electromagnetic spectrum, generally on the order of 1 nm.

31. The shortest wavelength X-ray has the most energy, which is the maximum kinetic energy of the electron in the tube:

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\bullet\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(0.030\times10^{-9}\,\text{m})} = 4.1\times10^{4}\ \text{eV} = 41\,\text{keV}.$$

Thus the operating voltage of the tube is 41 kV.

32. The energy of the photon with the shortest wavelength must equal the maximum kinetic energy of an electron:

$$hf_0 = \frac{hc}{\lambda_0} = eV, \text{ or}$$

$$\lambda_0 = \frac{hc}{eV} = \frac{(6.63\times10^{-34}\ \text{J}\bullet\text{s})(3\times10^{8}\ \text{m/s})(10^{9}\ \text{nm/m})}{(1.60\times10^{-19}\ \text{J/eV})(1\text{e})V}$$

$$= \frac{(1.24\times10^{3}\ \text{V}\bullet\text{nm})}{V}.$$

33. With the shielding provided by the remaining $n=1$ electron, we use the energies of the hydrogen atom with *Z* replaced by $Z-1$. The energy of the photon is

$$hf = \Delta E = -(13.6\,\text{eV})(24-1)^2\left[\left(\frac{1}{2^2}\right)-\left(\frac{1}{1^2}\right)\right] = 6.90\times10^3\,\text{eV}.$$

The wavelength of the photon is

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^8\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(6.90\times10^3\,\text{eV})} = 1.8\times10^{-10}\,\text{m} = \boxed{0.18\,\text{nm.}}$$

34. With the shielding provided by the remaining $n=1$ electron, we use the energies of the hydrogen atom with *Z* replaced by $Z-1$. The energy of the photon is

$$hf = \Delta E = -(13.6\,\text{eV})(26-1)^2\left[\left(\frac{1}{2^2}\right)-\left(\frac{1}{1^2}\right)\right] = 6.40\times10^3\,\text{eV}.$$

The wavelength of the photon is

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^8\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(6.40\times10^3\,\text{eV})} = 1.9\times10^{-10}\,\text{m} = \boxed{0.19\,\text{nm.}}$$

35. If we assume that the shielding is provided by the remaining $n=1$ electron, we use the energies of the hydrogen atom with *Z* replaced by $Z-1$. The energy of the photon is

$$hf = \Delta E = -(13.6\,\text{eV})(42-1)^2\left[\left(\frac{1}{3^2}\right)-\left(\frac{1}{1^2}\right)\right] = 2.03\times10^4\,\text{eV}.$$

The wavelength of the photon is

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^8\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(2.03\times10^4\,\text{eV})} = 6.1\times10^{-11}\,\text{m} = \boxed{0.061\,\text{nm.}}$$

We do not expect perfect agreement because there is some $\boxed{\text{partial shielding provided by the } n=2 \text{ shell,}}$ which was ignored when we replaced *Z* by $Z-1$.

36. The K_α line is from the $n=2$ to $n=1$ transition. We use the energies of the hydrogen atom with *Z* replaced by $Z-1$. Thus we have

$$hf = \Delta E \propto (Z-1)^2, \text{ so } \lambda \propto \frac{1}{(Z-1)^2}.$$

When we form the ratio for the two materials, we get

$$\frac{\lambda_\chi}{\lambda_{\text{iron}}} = \frac{(Z_{\text{iron}}-1)^2}{(Z_\chi-1)^2};$$

$$\frac{(229\,\text{nm})}{(194\,\text{nm})} = \frac{(26-1)^2}{(Z_\chi-1)^2}, \text{ which gives } Z_\chi = 24,$$

so the material is $\boxed{\text{chromium.}}$

37. The energy of a pulse is

$$E = P\,\Delta t = (0.68\,\text{W})(28\times10^{-3}\,\text{s}) = \boxed{0.019\,\text{J.}}$$

The energy of a photon is

$$hf = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\bullet\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(640\times10^{-9}\,\text{m})} = 1.94\,\text{eV}.$$

Thus the number of photons in a pulse is

$$N = \frac{E}{hf} = \frac{(0.019\,\text{J})}{(1.94\,\text{eV})(1.6\times10^{-19}\ \text{J/eV})} = \boxed{6.1\times10^{16}\ \text{photons.}}$$

38. Intensity equals power per area.

(*a*) $I = \frac{P}{S} = \frac{P}{\pi r^2} = \frac{0.50\times10^{-3}\,\text{W}}{\pi(1.5\times10^{-3}\text{m})^2} = \boxed{71\,\text{W/m}^2.}$

(*b*) $I = \frac{P}{S} = \frac{P}{4\pi r^2} = \frac{40\,\text{W}}{4\pi(2.0\,\text{m})^2} = \boxed{0.80\,\text{W/m}^2.}$

The laser beam is more intense by a factor of $\frac{71\,\text{W/m}^2}{0.80\,\text{W/m}^2} = \boxed{89.}$

39. We find the angular half width of the beam from

$$\Delta\theta = \frac{1.22\lambda}{d} = \frac{1.22(694\times10^{-9}\ \text{m})}{(3.0\times10^{-3}\ \text{m})} = 2.8\times10^{-4}\ \text{rad},$$

so the angular width is $\theta = \boxed{5.6\times10^{-4}\ \text{rad.}}$

(*a*) The diameter of the beam when it reaches the satellite is

$$D = r\theta = (300\times10^{3}\ \text{m})(5.6\times10^{-4}\ \text{rad}) = \boxed{1.7\times10^{2}\ \text{m.}}$$

(*b*) The diameter of the beam when it reaches the Moon is

$$D = r\theta = (384\times10^{6}\ \text{m})(5.6\times10^{-4}\ \text{rad}) = \boxed{2.2\times10^{5}\ \text{m.}}$$

40. Transition from the E_3' state to the E_2' state releases photons with energy 1.96 eV. The wavelength is determined by

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\ \text{J}\bullet\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(1.96\,\text{eV})} = 6.34\times10^{-7}\,\text{m} = \boxed{634\,\text{nm.}}$$

41. We can relate the momentum to the radius of the orbit from the quantum condition:

$$L = mvr = pr = n\hbar,\ \text{so}\ p = \frac{n\hbar}{r} = \frac{\hbar}{r_1}\ \text{for the ground state.}$$

If we assume that this is the uncertainty of the momentum, the uncertainty of the position is

$$\Delta x \geq \frac{\hbar}{\Delta p} = \frac{\hbar}{\left(\frac{\hbar}{r_1}\right)} = \boxed{r_1,\ \text{which is the Bohr radius.}}$$

42. (*a*) We find the minimum uncertainty in the energy of the state from

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{(1.055\times 10^{-34}\ \text{J}\bullet\text{s})}{(10^{-8}\ \text{s})} = 1.1\times 10^{-26}\ \text{J} = \boxed{6.6\times 10^{-8}\ \text{eV.}}$$

Note that, because the ground state is stable, we associate the uncertainty with the excited state.

(*b*) The transition energy is

$$E = -(13.6\,\text{eV})\left[\left(\frac{1}{2^2}\right) - \left(\frac{1}{1^2}\right)\right] = 10.2\,\text{eV},$$

so we have

$$\frac{\Delta E}{E} = \frac{(6.6\times 10^{-8}\ \text{eV})}{(10.2\,\text{eV})} = \boxed{6.5\times 10^{-9}.}$$

(*c*) The wavelength of the line is

$$\lambda = \frac{hc}{E} = \frac{(6.63\times 10^{-34}\ \text{J}\bullet\text{s})(3.00\times 10^{8}\ \text{m/s})}{(1.60\times 10^{-19}\ \text{J/eV})(10.2\,\text{eV})} = 1.22\times 10^{-7}\ \text{m} = \boxed{122\,\text{nm.}}$$

We find the width of the line from

$$\lambda + \Delta\lambda = \frac{hc}{(E + \Delta E)}$$

$$= \frac{\left[\frac{hc}{E}\right]}{\left[1 + \left(\frac{\Delta E}{E}\right)\right]} \approx \lambda\left[1 - \left(\frac{\Delta E}{E}\right)\right].$$

If we ignore the sign, we get

$$\Delta\lambda = \lambda\left(\frac{\Delta E}{E}\right) = (122\,\text{nm})(6.5\times 10^{-9}) = \boxed{7.9\times 10^{-7}\ \text{nm.}}$$

$\boxed{43.}$ The value of ℓ can range from 0 to $n-1$. Thus for $n=5$, we have $\ell \leq 4$.

The smallest value of L is $\boxed{0.}$

The largest magnitude of L is

$$L = \hbar\left[\ell(\ell+1)\right]^{\frac{1}{2}} = (1.055\times 10^{-34}\ \text{J}\bullet\text{s})\left[(4)(4+1)\right]^{\frac{1}{2}} = \boxed{4.72\times 10^{-34}\ \text{kg}\bullet\text{m}^2/\text{s}.}$$

44. (*a*) We find the quantum number for the orbital angular momentum from

$$L = M_{\text{earth}} vR = \frac{M_{\text{earth}} 2\pi R^2}{T} = \hbar\left[\ell(\ell+1)\right]^{\frac{1}{2}};$$

$$\frac{(5.98\times 10^{24}\ \text{kg})2\pi(1.50\times 10^{11}\ \text{m})^2}{(3.16\times 10^{7}\ \text{s})} = (1.055\times 10^{-34}\ \text{J}\bullet\text{s})\left[\ell(\ell+1)\right]^{\frac{1}{2}},$$

which gives $\ell = \boxed{2.5\times 10^{74}.}$

(*b*) The value of m_ℓ can range from $-\ell$ to $+\ell$, or $2\ell+1$ values, so the number of orientations is

$$N = 2\ell + 1 = 2(2.5\times 10^{74}) + 1 = \boxed{5.0\times 10^{74}.}$$

45. (*a*) We find the wavelength of the bullet from

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{(6.63 \times 10^{-34}\ \text{J•s})}{(12 \times 10^{-3}\ \text{kg})(180\ \text{m/s})} = \boxed{3.1 \times 10^{-34}\ \text{m.}}$$

(*b*) We find the uncertainty in the momentum component perpendicular to the motion:

$$\Delta p_y \geq \frac{\hbar}{\Delta y} = \frac{(1.055 \times 10^{-34}\ \text{J•s})}{(0.60 \times 10^{-8}\ \text{m})} = \boxed{1.8 \times 10^{-32}\ \text{kg•m/s.}}$$

46. From the Bohr formula for the radius, we see that

$$r \propto \frac{1}{Z},\ \text{so}\ r_{\text{uranium}} = \frac{r_1}{Z} = \frac{(0.529 \times 10^{-10}\ \text{m})}{(92)} = \boxed{5.8 \times 10^{-13}\ \text{m.}}$$

The innermost electron would "see" a nucleus with charge *Ze*.
Thus we use the energy of the hydrogen atom:

$$E_1 = -\frac{(13.6\,\text{eV})Z^2}{n^2} = -\frac{(13.6\,\text{eV})(92)^2}{1^2} = -1.15 \times 10^5\ \text{eV},$$

so the binding energy is $\boxed{115\ \text{keV.}}$

47. If we assume that the rate at which heat is produced is 100% of the electrical power input, we have

$$P = IV = \frac{Q}{t} = mc\left(\frac{\Delta T}{t}\right);$$

$$\frac{(25\,\text{mA})(95\,\text{kV})(60\,\text{s/min})}{(4186\,\text{J/kcal})} = (0.085\,\text{kg})(0.11\text{kcal/kg}\,\text{C°})\left(\frac{\Delta T}{t}\right),$$

which gives $\frac{\Delta T}{t} = \boxed{3.6 \times 10^3\ \text{C°/min.}}$

48. (*a*) Boron has $Z = 4$, so the outermost electron has $n = 2$. We use the Bohr result with an effective *Z*:

$$E_2 = -\frac{(13.6\,\text{eV})(Z_{\text{eff}})^2}{n^2};$$

$$-8.26\,\text{eV} = -\frac{(13.6\,\text{eV})(Z_{\text{eff}})^2}{2^2},\ \text{which gives}\ Z_{\text{eff}} = \boxed{1.56.}$$

Note that this indicates some shielding by the second electron in the $n = 2$ shell.

(*b*) We find the average radius from

$$r = \frac{n^2 r_1}{Z_{\text{eff}}} = \frac{2^2(0.529 \times 10^{-10}\ \text{m})}{(1.56)} = \boxed{1.4 \times 10^{-10}\ \text{m.}}$$

49. The wavelength of the photon emitted for the transition from n' to n is

$$\frac{1}{\lambda} = \left(\frac{2\pi^2 Z^2 e^4 m k^2}{h^3 c}\right)\left[\left(\frac{1}{n'^2}\right) - \left(\frac{1}{n^2}\right)\right] = RZ^2\left[\left(\frac{1}{n'^2}\right) - \left(\frac{1}{n^2}\right)\right].$$

The K_α line is from the $n' = 2$ to $n = 1$ transition, and the other $n = 1$ electron shields the nucleus, so the effective Z is $Z - 1$:

$$\frac{1}{\lambda} = RZ_{\text{eff}}{}^2\left[\left(\frac{1}{n'^2}\right) - \left(\frac{1}{n^2}\right)\right] = R(Z-1)^2\left[\left(\frac{1}{2^2}\right) - \left(\frac{1}{1^2}\right)\right] = \tfrac{3}{4}R(Z-1)^2.$$

Thus we have

$\dfrac{1}{\sqrt{\lambda}} = \left(\tfrac{3}{4}R\right)^{\frac{1}{2}}(Z-1),$ which is the equation of a straight line, as in the Moseley plot, with $b = 1$.

The value of a is

$$a = \left(\tfrac{3}{4}R\right)^{\frac{1}{2}} = \left[\tfrac{3}{4}\left(1.0974\times10^{7}\ \text{m}^{-1}\right)\right]^{\frac{1}{2}} = \boxed{2869\,\text{m}^{-\frac{1}{2}}.}$$

50. (*a*) For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$, or $2\ell + 1$ values. For each of these there are two values of m_s. Thus the total number of states in a subshell is

$$N = 2(2\ell + 1).$$

(*b*) For $\ell = 0, 1, 2, 3, 4, 5,$ and 6, $N = 2, 6, 10, 14, 18, 22,$ and 26, respectively.

51. For a given n, $0 \le \ell \le n - 1$. Since for each ℓ the number of possible states is $2(2\ell + 1)$, the number of possible states for a given n is

$$\sum_{\ell=0}^{n-1} 2(2\ell+1) = 4\sum_{\ell=0}^{n-1}\ell + \sum_{\ell=0}^{n-1} 2 = 4\left(\frac{n(n-1)}{2}\right) + 2n = 2n^2.$$

52. We find the wavelength of the electron from

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_0(\text{KE})}}$$

$$= \frac{6.63\times10^{-34}\ \text{J}\bullet\text{s}}{\sqrt{2\left(9.11\times10^{-31}\text{kg}\right)\left(45\times10^{3}\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}} = 5.79\times10^{-12}\,\text{m}.$$

The peaks of the interference pattern are given by

$$d\sin\theta = n\lambda,\ n = 1, 2, \ldots$$

and the positions on the screen are

$$y = L\tan\theta.$$

For small angles, $\sin\theta = \tan\theta$, so we have

$$y = \frac{nL\lambda}{d}.$$

Thus the separation is

$$\Delta y = \frac{L\lambda}{d} = \frac{(0.350\,\text{m})\left(5.79\times10^{-12}\,\text{m}\right)}{2.0\times10^{-6}\,\text{m}} = 1.0\times10^{-6}\,\text{m} = \boxed{1.0\,\mu\text{m}.}$$

53. According to the Bohr model,

$$L_{\text{Bohr}} = n\frac{h}{2\pi} = \boxed{2\hbar.}$$

According to quantum mechanics,

$L_{\text{QM}} = \sqrt{\ell(\ell+1)}\,\hbar_{\text{p}}$ and with $n = 2$, it follows that $\ell = 0$ or $\ell = 1$, so that

$$L_{QM} = \sqrt{0(0+1)}\,\hbar = \boxed{0} \text{ or}$$
$$L_{QM} = \sqrt{1(1+1)}\,\hbar = \boxed{\sqrt{2}\,\hbar.}$$

54. We find the uncertainty in the position from

$$\Delta x = \frac{\hbar}{m\Delta v} = \frac{1.055\times10^{-34}\,\text{J•s}}{(1100\,\text{kg})(0.22\,\text{m/s})} = \boxed{4.4\times10^{-37}\,\text{m.}}$$

55. The difference in measured energies is

$$\Delta E = E_2 - E_1 = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1}$$
$$= (6.63\times10^{-34}\,\text{J•s})(3.00\times10^8\,\text{m/s})\left(\frac{1}{489\times10^{-9}\,\text{m}} - \frac{1}{487\times10^{-9}\,\text{m}}\right) = 1.67\times10^{-21}\,\text{J.}$$

Then the lifetime of the excited state is determined by

$$\Delta E\Delta t \approx \hbar;$$
$$(1.67\times10^{-21}\,\text{J})\Delta t \approx 1.055\times10^{-34}\,\text{J•s, which gives } \Delta t = \boxed{6.3\times10^{-14}\,\text{s.}}$$

56. We find the wavelength of the proton from

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_0(\text{KE})}}$$
$$= \frac{6.63\times10^{-34}\,\text{J•s}}{\sqrt{2(1.67\times10^{-27}\,\text{kg})(550\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}} = 1.22\times10^{-12}\,\text{m.}$$

The peaks of the interference pattern are given by

$$d\sin\theta = n\lambda,\ n = 1, 2, \ldots$$

and the positions on the screen are

$$y = L\tan\theta.$$

For small angles, $\sin\theta = \tan\theta$, so we have

$$y = \frac{nL\lambda}{d}.$$

Thus the separation is

$$\Delta y = \frac{L\lambda}{d} = \frac{(28\,\text{m})(1.22\times10^{-12}\,\text{m})}{0.70\times10^{-3}\,\text{m}} = 4.89\times10^{-8}\,\text{m} = \boxed{48.9\,\text{nm.}}$$

57. The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_0(\text{KE})}},$$

so that for a given KE,

$$\Delta x = \lambda \propto \frac{1}{\sqrt{m_0}}.$$

Since $(\Delta x)(\Delta p)$ is constant,

$$\Delta p \propto \sqrt{m_0};$$

$$\frac{\Delta p_{\text{proton}}}{\Delta p_{\text{electron}}} = \frac{\sqrt{m_{\text{p}}}}{\sqrt{m_{\text{e}}}} = \frac{\sqrt{1.67\times 10^{-27}\,\text{kg}}}{\sqrt{9.11\times 10^{-31}\,\text{kg}}} = 42.8$$

$$\Delta p_{\text{proton}} : \Delta p_{\text{electron}} = 43:1.$$

58. Limiting the number of electron shells to six would mean that the periodic table stops with radon (Rn), since the next element, francium (Fr), begins filling the seventh shell. Including all elements up through radon means $\boxed{86}$ elements.

59. The de Broglie wavelength is determined by

$$\lambda = \frac{h}{mv};$$

$$0.50\,\text{m} = \frac{6.63\times 10^{-34}\,\text{J•s}}{(75.0\,\text{kg})v}, \text{ which gives } v = \boxed{1.8\times 10^{-35}\,\text{m/s.}}$$

Since λ is comparable to the width of a typical doorway, $\boxed{\text{yes,}}$ you would notice diffraction effects. However, assuming that walking through the doorway requires travel through a distance of 0.2 m, the time Δt required will be determined by

$$v\Delta t = d$$

$$(1.8\times 10^{-35}\,\text{m/s})\Delta t = 0.2\,\text{m}, \text{ which gives } \Delta t \approx \boxed{10^{34}\,\text{s.}}$$

60. Since

$$\frac{1}{\lambda} \propto Z^2\left(\frac{1}{n'^2} - \frac{1}{n^2}\right),$$

we seek Z and n'_{x} so that for every fourth value of n_{x} there is an n_{h} such that

$$\frac{1}{1} - \frac{1}{n_{\text{h}}^2} = Z^2\left(\frac{1}{n'^{\,2}_{\text{x}}} - \frac{1}{n_{\text{x}}^2}\right).$$

This condition is met when $Z = 4$ and $n'_{\text{x}} = 4$. The resulting spectral lines for $n_{\text{x}} = 4, 8, 12, \ldots$ match the hydrogen Lyman series lines for $n = 1, 2, 3, \ldots$. The element is $\boxed{\text{beryllium.}}$

61. The K_α line is from the $n = 2$ to $n = 1$ transition. We use the energies of the hydrogen atom with Z replaced by $Z - 1$. Thus we have

$$\frac{1}{\lambda} = \left(\frac{2\pi^2 e^4 mk^2}{h^3 c}\right)(Z-1)^2\left(\frac{1}{n'^2} - \frac{1}{n^2}\right);$$

$$\frac{1}{0.154\times 10^{-9}\,\text{m}} = (1.097\times 10^{7}\,\text{m}^{-1})(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right), \text{ which gives } Z \approx 29.$$

The element is $\boxed{\text{copper.}}$

62. We find the uncertainty in the momentum component perpendicular to the motion, when the width of the beam is constrained to a dimension D:

$$\Delta p_y \ge \frac{\left(\frac{h}{2\pi}\right)}{\Delta y} = \frac{\left(\frac{h}{2\pi}\right)}{D}.$$

The half angle of the beam is given by the direction of the velocity:

$$\sin\theta = \frac{v_y}{c}.$$

We assume that the angle is small: $\sin\theta \approx \theta$, and we take the minimum uncertainty to be the perpendicular momentum, so we have

$$\theta \approx \frac{v_y}{v_x} = \frac{p_y}{p} = \frac{\left[\dfrac{\left(\dfrac{h}{2\pi}\right)}{D}\right]}{\left(\dfrac{h}{\lambda}\right)} = \frac{\lambda}{2\pi D}.$$

The angular spread is

$$2\theta \approx \frac{\lambda}{\pi D} \approx \frac{\lambda}{D}.$$