CHAPTER 30: Nuclear Physics and Radioactivity

Answers to Questions

1. Different isotopes of a given element have the same number of protons and electrons. Because they have the same number of electrons, they have almost identical chemical properties. Each isotope has a different number of neutrons from other isotopes of the same element. Accordingly, they have different atomic masses.

2. Identify the element based on the atomic number.
(*a*) Uranium (A = 92)
(*b*) Nitrogen (A = 7)
(*c*) Hydrogen (A = 1)
(*d*) Strontium (A = 38)
(*e*) Berkelium (A = 97)

3. The number of protons is the same as the atomic number, and the number of protons is the mass number minus the number of protons.
(*a*) Uranium: 92 protons, 140 neutrons
(*b*) Nitrogen: 7 protons, 11 neutrons
(*c*) Hydrogen: 1 proton, 0 neutrons
(*d*) Strontium: 38 protons, 44 neutrons
(*e*) Berkelium: 97 protons, 150 neutrons

4. With 88 nucleons and 50 neutrons, there must be 38 protons. This is the atomic number, and so the element is strontium. The nuclear symbol would be ${}^{88}_{38}\text{Sr}$.

5. The atomic mass of an element as shown in the periodic table is the average atomic mass of all naturally-occurring isotopes. For example, chlorine occurs as roughly 75% ${}^{35}_{17}\text{Cl}$ and 25% ${}^{36}_{17}\text{Cl}$, and so its atomic mass is about 35.5. This is the average atomic mass that a sample of naturally-occurring chlorine would have. Other smaller effects would include the fact that the masses of the nucleons are not exactly 1 atomic mass unit, and that some small fraction of the mass energy of the total set of nucleons is in the form of binding energy.

6. There must be some force holding the nucleus together. For all nuclei with atomic numbers greater than 1, there are protons packed very close to each other. If the only force present between the protons were the electrostatic force, the protons would repel each other and no nuclei would be stable. Since there are stable nuclei, there must be some other force stronger than the electrostatic force, holding the nucleus together – the strong nuclear force.

7. Similarities between the electromagnetic force and the strong nuclear force: They will both act on charged particles. Differences between the electromagnetic force and the strong nuclear force: The strong force is a short-range force, while the electromagnetic force is an infinite-range force. The strong force is only attractive, while the electromagnetic force can be either attractive or repulsive. The strong force acts between all nucleons no matter whether they are charged or neutral, while the electromagnetic force acts only on charged particles. The strong nuclear force is much stronger than the electromagnetic force.

8. Quoting from section 30-3, "… radioactivity was found in every case to be unaffected by the strongest physical and chemical treatments, including strong heating or cooling and the action of strong chemical reagents." Chemical reactions are a result of electron interactions, not nuclear processes. The absence of effects caused by chemical reactions is evidence that the radioactivity is not due to electron interactions. Another piece of evidence is the fact that the α-particle is much heavier than an electron and has a different charge than the electron, so it can't be an electron. Therefore it must be from the nucleus. Finally, the energies of the electrons or photons emitted from radioactivity are much higher than those corresponding to electron orbital transitions. All of these observations support radioactivity being a nuclear process.

9. For gamma decay: ${}^{64}_{29}\text{Cu} \rightarrow {}^{64}_{29}\text{Cu} + \gamma$. The resulting nuclide is still ${}^{64}_{29}\text{Cu}$.
   For beta emission: ${}^{64}_{29}\text{Cu} \rightarrow {}^{64}_{30}\text{Zn} + \text{e}^- + \bar{\nu}$. The resulting nuclide is ${}^{64}_{30}\text{Zn}$.
   For positron emission: ${}^{64}_{29}\text{Cu} \rightarrow {}^{64}_{28}\text{Ni} + \text{e}^+ + \nu$. The resulting nuclide is ${}^{64}_{28}\text{Ni}$.

10. The ${}^{238}_{92}\text{U}$ nucleus has 92 protons and 146 neutrons. It decays by α-decay (see appendix B), losing 2 protons and 2 neutrons. Thus the daughter nucleus has 144 neutrons. ${}^{238}_{92}\text{U} \rightarrow {}^{4}_{2}\text{He} + {}^{234}_{90}\text{Th}$. According to appendix B, it also can decay from an excited state via gamma radiation to a lower energy ${}^{238}_{92}\text{U}$, which has 146 neutrons.

11. Gamma rays are neutrally charged, they are made up of high energy photons (travel at *c*), and they have no mass. Alpha rays are made up of helium nuclei, they are the most massive of these three particles, and they have a charge of +2e. Beta rays are made up of electrons or positrons, they can be either positively or negatively charged, and the particles are accompanied by either a neutrino or an anti-neutrino upon decay.

12. (*a*) ${}^{24}_{11}\text{Na} \rightarrow {}^{24}_{12}\text{Mg} + \text{e}^- + \bar{\nu}$ Magnesium-24 is formed.
    (*b*) ${}^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + \text{e}^+ + \nu$ Neon-22 is formed.
    (*c*) ${}^{210}_{84}\text{Po} \rightarrow {}^{4}_{2}\text{He} + {}^{206}_{82}\text{Pb}$ Lead-206 is formed.

13. (*a*) ${}^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + \text{e}^- + \bar{\nu}$ Sulfur-32 is formed.
    (*b*) ${}^{35}_{16}\text{S} \rightarrow {}^{35}_{17}\text{Cl} + \text{e}^- + \bar{\nu}$ Chlorine-35 is formed.
    (*c*) ${}^{211}_{83}\text{Bi} \rightarrow {}^{4}_{2}\text{He} + {}^{207}_{81}\text{Tl}$ Thallium-207 is formed.

14. (*a*) ${}^{45}_{20}\text{Ca} \rightarrow {}^{45}_{21}\text{Sc} + \text{e}^- + \bar{\nu}$ Scandium-45 is the missing nucleus.
    (*b*) ${}^{58}_{29}\text{Cu} \rightarrow {}^{58}_{29}\text{Cu} + \gamma$ Copper-58 is the missing nucleus.
    (*c*) ${}^{46}_{24}\text{Cr} \rightarrow {}^{46}_{23}\text{V} + \text{e}^+ + \nu$ The positron and the neutrino are the missing particles.
    (*d*) ${}^{234}_{94}\text{Pu} \rightarrow {}^{230}_{92}\text{U} + \alpha$ Uranium-230 is the missing nucleus.
    (*e*) ${}^{239}_{93}\text{Np} \rightarrow {}^{239}_{94}\text{Pu} + \text{e}^- + \bar{\nu}$ The electron and the anti-neutrino are the missing particles.

15. The two "extra" electrons are no longer bound tightly to the nucleus (since the nucleus lost two positively charged protons), and so those extra electrons can move away from the nucleus, moving

towards a higher electric potential or some other positively charged object. They are not “emitted” during the decay, since they do not receive any of the kinetic energy from the decay. If they are close to each other, they would repel each other somewhat.

16. With β^- or β^+ decay, the number of protons in the nucleus changes. In β^- decay, the number of protons increases, and in β^+ decay, the number of protons decreases. Thus, the charge of the nucleus changes and when this occurs all of the energy levels of the atomic electrons change in size. Recall that both the energy values of the level and the Bohr radii are dependent on Z (the charge of the nucleus). Thus, after the decay, there is a lot of shuffling around of the electrons into the new and different energy levels. The electrons will change energies to occupy the new energy levels, and so many photons corresponding to those energy level changes will be emitted.

17. In alpha decay, assuming the energy of the parent nucleus is known, then the unknowns after the decay are the energies of the daughter nucleus and the alpha. These two values can be determined by energy and momentum conservation. Since there are two unknowns and two conditions, the values are uniquely determined. In beta decay, there are three unknown post-decay energies since there are three particles present after the decay. The conditions of energy and momentum conservation are not sufficient to exactly determine the energy of each particle, and so a range of possible values is possible.

18. In electron capture, the nucleus will effectively have a proton change to a neutron. This isotope will then lie to the left and above the original isotope. Since the process would only occur if it made the nucleus more stable, it must lie BELOW the line of stability in Fig. 30-2.

19. Neither hydrogen nor deuterium can emit an α particle. Hydrogen has only one nucleon (a proton) in its nucleus, and deuterium has only two nucleons (one proton and one neutron) in its nucleus. Neither one has the necessary four nucleons (two protons and two neutrons) to emit an α particle.

20. Many artificially produced radioactive isotopes are rare in nature because they have decayed away over time. If the half-lives of these isotopes are relatively short in comparison with the age of Earth (which is typical for these isotopes), then there won’t be any significant amount of these isotopes left to be found in nature.

21. After two months the sample will not have completely decayed. After one month half of the sample will remain, and after two months, one-fourth of the sample will remain. Each month half of the remaining atoms decay.

22. None of the elements with Z > 92 are stable because there is no number of neutrons that are capable of using the strong force in overcoming the electric repulsion of that many positive charges in such close proximity.

23. There are a total of 4 protons and 3 neutrons in the reactant particles. The α-particle has 2 protons and 2 neutrons, and so 2 protons and 1 neutron are in the other product particle. It must be Helium-3.

$$^{6}_{3}\text{Li} + ^{1}_{1}\text{p} \rightarrow ^{4}_{2}\alpha + ^{3}_{2}\text{He}$$

24. The technique of $^{14}_{6}\text{C}$ would not be used to measure the age of stone walls and tablets. Carbon-14 dating is only useful for measuring the age of objects that were living at some earlier time. Also, Carbon-14 dating is only useful for determining the age of objects less than about 60,000 years old. The stone from which the walls and tablets were made would be much older than 60,000 years.

25. In internal conversion, there is only one decay product (an electron) ejected from the nucleus, and so the electron would have a unique energy, equal to the kinetic energy that an emitted γ ray would have (minus the binding energy of the electron). In β decay, there are two decay products ejected from the nucleus (the electron and the neutrino), and the electron will have a range of possible energies, as discussed in question 17.

Solutions to Problems

Note: A factor that appears in the analysis of energies is

$$ke^2 = \left(9.00\times10^9\ \mathrm{N{\bullet}m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)^2 = 2.30\times10^{-28}\ \mathrm{J{\bullet}m} = 1.44\,\mathrm{MeV{\bullet}fm}.$$

1. We convert the units:

$$m = \frac{\left(139\,\mathrm{MeV}/c^2\right)}{\left(931.5\,\mathrm{MeV}/\mathrm{u}c^2\right)} = \boxed{0.149\,\mathrm{u}.}$$

2. The α particle is a helium nucleus:

$$r = \left(1.2\times10^{-15}\ \mathrm{m}\right)A^{\frac{1}{3}} = \left(1.2\times10^{-15}\ \mathrm{m}\right)(4)^{\frac{1}{3}} = \boxed{1.9\times10^{-15}\ \mathrm{m}} = 1.9\,\mathrm{fm}.$$

3. To find the rest mass of an α particle, we subtract the rest mass of the two electrons from the rest mass of a helium atom:

$$m_\alpha = m_{\mathrm{He}} - 2m_{\mathrm{e}}$$
$$= (4.002602\,\mathrm{u})\left(931.5\,\mathrm{MeV}/\mathrm{u}c^2\right) - 2\left(0.511\,\mathrm{MeV}/c^2\right) = \boxed{3727\,\mathrm{MeV}/c^2.}$$

4. (*a*) The radius of $^{64}\mathrm{Cu}$ is

$$r = \left(1.2\times10^{-15}\ \mathrm{m}\right)A^{\frac{1}{3}} = \left(1.2\times10^{-15}\ \mathrm{m}\right)(64)^{\frac{1}{3}} = \boxed{4.8\times10^{-15}\ \mathrm{m}} = 4.8\,\mathrm{fm}.$$

(*b*) We find the value of *A* from

$$r = \left(1.2\times10^{-15}\ \mathrm{m}\right)A^{\frac{1}{3}};$$

$$3.9\times10^{-15}\ \mathrm{m} = \left(1.2\times10^{-15}\ \mathrm{m}\right)A^{\frac{1}{3}},\ \text{which gives}\ A = \boxed{34.}$$

5. (*a*) The mass of a nucleus with mass number *A* is *A* u and its radius is

$$r = \left(1.2\times10^{-15}\ \mathrm{m}\right)A^{\frac{1}{3}}.$$

Thus the density is

$$\rho = \frac{m}{V}$$
$$= A\frac{\left(1.66\times10^{-27}\ \mathrm{kg/u}\right)}{\frac{4}{3}\pi r^3} = A\frac{\left(1.66\times10^{-27}\ \mathrm{kg/u}\right)}{\frac{4}{3}\pi\left(1.2\times10^{-15}\ \mathrm{m}\right)^3 A}$$
$$= \boxed{2.29\times10^{17}\ \mathrm{kg/m^3},}\ \text{independent of } A.$$

(*b*) We find the radius from

$$M = pV;$$

$$5.98\times 10^{24}\text{ kg} = \left(2.29\times 10^{17}\text{ kg/m}^3\right)\tfrac{4}{3}\pi R^3,\text{ which gives } R = \boxed{180\,\text{m.}}$$

(*c*) For equal densities, we have

$$\rho = \frac{M_{\text{Earth}}}{\tfrac{4}{3}\pi R_{\text{Earth}}{}^3} = \frac{m_{\text{U}}}{\tfrac{4}{3}\pi r_{\text{U}}{}^3};$$

$$\frac{\left(5.98\times 10^{24}\text{ kg}\right)}{\left(6.38\times 10^{6}\text{ m}\right)^3} = \frac{(238\,\text{u})\left(1.66\times 10^{-27}\text{ kg/u}\right)}{r_{\text{U}}{}^3},\text{ which gives } r_{\text{U}} = \boxed{2.58\times 10^{-10}\,\text{m.}}$$

6. (*a*) The fraction of mass is

$$\frac{m_{\text{p}}}{\left(m_{\text{p}} + m_{\text{e}}\right)} = \frac{\left(1.67\times 10^{-27}\text{ kg}\right)}{\left(1.67\times 10^{-27}\text{ kg} + 9.11\times 10^{-31}\text{ kg}\right)} = \boxed{0.99945.}$$

(*b*) The fraction of volume is

$$\left(\frac{r_{\text{nucleus}}}{r_{\text{atom}}}\right)^3 = \left[\frac{\left(1.2\times 10^{-15}\text{ m}\right)}{\left(0.53\times 10^{-10}\text{ m}\right)}\right]^3 = \boxed{1.2\times 10^{-14}.}$$

7. Electron mass is negligible compared to nucleon mass, and one nucleon weighs about 1.0 atomic mass unit. Therefore, in a 1.0-kg object,

$$N = \frac{(1.0\,\text{kg})\left(6.02\times 10^{26}\text{ u/kg}\right)}{1.0\,\text{u/nucleon}} \approx \boxed{6\times 10^{26}\,\text{nucleons.}}$$

It does $\boxed{\text{not}}$ matter what the element is, because $\boxed{\text{the mass of one nucleon is essentially the same for all elements.}}$

8. We find the radii of the two nuclei from

$$R = r_0 A^{\frac{1}{3}};$$

$$R_\alpha = (1.2\,\text{fm})(4)^{\frac{1}{3}} = 1.9\,\text{fm};$$

$$R_{\text{U}} = (1.2\,\text{fm})(238)^{\frac{1}{3}} = 7.4\,\text{fm}.$$

If the two nuclei are just touching, the Coulomb potential energy must be the initial kinetic energy of the α particle:

$$\text{KE} = \text{PE} = \frac{Z_\alpha Z_{\text{U}} k e^2}{\left(R_\alpha + R_{\text{U}}\right)}$$

$$= \frac{(2)(92)(1.44\,\text{MeV}\bullet\text{fm})}{(1.9\,\text{fm} + 7.4\,\text{fm})} = \boxed{28\,\text{MeV.}}$$

9. From Figure 30–1, we see that the average binding energy per nucleon at $A = 40$ is 8.6 MeV. Thus the total binding energy for ^{40}Ca is

$$(40)(8.6\,\text{MeV}) = \boxed{340\,\text{MeV.}}$$

10. (*a*) From Figure 30–1, we see that the average binding energy per nucleon at $A = 238$ is $7.5\,\text{MeV}$. Thus the total binding energy for ^{238}U is

$$(238)(7.5\,\text{MeV}) = \boxed{1.8\times10^3\ \text{MeV.}}$$

(*b*) From Figure 30–1, we see that the average binding energy per nucleon at $A = 84$ is $8.7\,\text{MeV}$. Thus the total binding energy for ^{84}Kr is

$$(84)(8.7\,\text{MeV}) = \boxed{730\,\text{MeV.}}$$

11. Deuterium consists of one proton and one neutron. We find the binding energy from the masses:

$$\text{Binding energy} = \left[m\left({}^1\text{H}\right) + m\left({}^1\text{n}\right) - m\left({}^2\text{H}\right)\right]c^2$$
$$= \left[(1.007825\,\text{u}) + (1.008665\,\text{u}) - (2.014102\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = \boxed{2.22\,\text{MeV.}}$$

$\boxed{12.}$ ^{14}N consists of seven protons and seven neutrons. We find the binding energy from the masses:

$$\text{Binding energy} = \left[7m\left({}^1\text{H}\right) + 7m\left({}^1\text{n}\right) - m\left({}^{14}\text{N}\right)\right]c^2$$
$$= \left[7(1.007825\,\text{u}) + 7(1.008665\,\text{u}) - (14.003074\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = 104.7\,\text{MeV}.$$

Thus the binding energy per nucleon is

$$\frac{(104.7\,\text{MeV})}{14} = \boxed{7.48\,\text{MeV.}}$$

13. We find the binding energy of the last neutron from the masses:

$$\text{Binding energy} = \left[m\left({}^{39}\text{K}\right) + m\left({}^1\text{n}\right) - m\left({}^{40}\text{K}\right)\right]c^2$$
$$= \left[(38.963707\,\text{u}) + (1.008665\,\text{u}) - (39.963999\,\text{u})\right]c^2\left(931.5\,\text{MeV}/c^2\right)$$
$$= \boxed{7.799\,\text{MeV.}}$$

14. (*a*) ^{6}Li consists of three protons and three neutrons. We find the binding energy from the masses:

$$\text{Binding energy} = \left[3m\left({}^1\text{H}\right) + 3m\left({}^1\text{n}\right) - m\left({}^6\text{Li}\right)\right]c^2$$
$$= \left[3(1.007825\,\text{u}) + 3(1.008665\,\text{u}) - (6.015121\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right)$$
$$= \boxed{32.0\,\text{MeV.}}$$

Thus the binding energy per nucleon is

$$\frac{(32.0\,\text{MeV})}{6} = \boxed{5.33\,\text{MeV.}}$$

(*b*) ^{208}Pb consists of 82 protons and 126 neutrons. We find the binding energy from the masses:

$$\text{Binding energy} = \left[82\,m\left({}^1\text{H}\right) + 126\,m\left({}^1\text{n}\right) - m\left({}^{208}\text{Pb}\right)\right]c^2$$
$$= \left[82(1.007825\,\text{u}) + 126(1.008665\,\text{u}) - (207.976627\,\text{u})\right]c^2\left(931.5\,\text{Mev/u}c^2\right)$$
$$= \boxed{1.64\,\text{GeV.}}$$

Thus the binding energy per nucleon is

$$\frac{1.64\,\text{GeV}}{208} = \boxed{7.87\,\text{MeV.}}$$

15. ^{23}Na consists of 11 protons and 12 neutrons. We find the binding energy from the masses:

$$\text{Binding energy} = \left[11m\left({}^{1}\text{H}\right) + 12\left({}^{1}\text{n}\right) - m\left({}^{23}\text{Na}\right)\right]c^2$$
$$= \left[11(1.007825\,\text{u}) + 12(1.008665\,\text{u}) - (22.989770\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right)$$
$$= 186.6\,\text{MeV}.$$

Then the average binding energy per nucleon is

$$\frac{186.6\,\text{MeV}}{23} = \boxed{8.11\,\text{MeV/nucleon.}}$$

Similarly, for ^{24}Na,

$$\text{Binding energy} = \left[11m\left({}^{1}\text{H}\right) + 12\left({}^{1}\text{n}\right) - m\left({}^{24}\text{Na}\right)\right]c^2$$
$$= \left[11(1.007825\,\text{u}) + 13(1.008665\,\text{u}) - (23.990963\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right)$$
$$= 193.5\,\text{MeV},$$

and the average binding force per nucleon is $\dfrac{193.5\,\text{MeV}}{24} = \boxed{8.06\,\text{MeV/nucleon.}}$

16. We find the required energy for separation from the masses.

(*a*) Removal of a proton creates an isotope of nitrogen:

$$\text{Energy(p)} = \left[m\left({}^{15}\text{N}\right) + m\left({}^{1}\text{H}\right) - m\left({}^{16}\text{O}\right)\right]c^2$$
$$= \left[(15.000108\,\text{u}) + (1.007825\,\text{u}) - (15.994915\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = \boxed{12.1\,\text{MeV.}}$$

(*b*) Removal of a neutron creates another isotope of oxygen:

$$\text{Energy(n)} = \left[m\left({}^{15}\text{O}\right) + m\left({}^{1}\text{n}\right) - m\left({}^{16}\text{O}\right)\right]c^2$$
$$= \left[(15.003065\,\text{u}) + (1.008665\,\text{u}) - (15.994915\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = \boxed{15.7\,\text{MeV.}}$$

The nucleons are held by the attractive strong nuclear force. It takes less energy to remove the proton because there is also the $\boxed{\text{repulsive electric force}}$ from the other protons.

17. (*a*) We find the binding energy from the masses:

$$\text{Binding energy} = \left[2m\left({}^{4}\text{He}\right) - m\left({}^{8}\text{Be}\right)\right]c^2$$
$$= \left[2(4.002602\,\text{u}) - (8.005305\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = -0.094\,\text{MeV}.$$

Because the binding energy is negative, the nucleus is unstable.

(*b*) We find the binding energy from the masses:

$$\text{Binding energy} = \left[3m\left({}^{4}\text{He}\right) - m\left({}^{12}\text{C}\right)\right]c^2$$
$$= \left[3(4.002602\,\text{u}) - (12.000000\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = +7.3\,\text{MeV}.$$

Because the binding energy is positive, the nucleus is $\boxed{\text{stable.}}$

18. The decay is ${}^{3}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{0}_{-1}\text{e} + \bar{\nu}$. When we add an electron to both sides to use atomic masses, we see that the mass of the emitted β particle is included in the atomic mass of ^{3}He.

Thus the energy released is

$$Q = \left[m\left({}^{3}\text{H}\right) - m\left({}^{3}\text{He}\right)\right]c^2 = \left[(3.016049\,\text{u}) - (3.016029\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = 0.0186\,\text{MeV}$$
$$= \boxed{18.6\,\text{keV.}}$$

19. The decay is ${}_{0}^{1}\text{n} \rightarrow {}_{1}^{1}\text{p} + {}_{-1}^{0}\text{e} + \bar{v}$. We take the electron mass to use the atomic mass of ${}^{1}\text{H}$. The kinetic energy of the electron will be maximum if no neutrino is emitted. If we ignore the recoil of the proton, the maximum kinetic energy is

$$\text{KE} = \left[m\left({}^{1}\text{n}\right) - m\left({}^{1}\text{H}\right)\right]c^2 = \left[(1.008665\,\text{u}) - (1.007825\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = \boxed{0.783\,\text{MeV.}}$$

20. For the decay ${}_{6}^{11}\text{C} \rightarrow {}_{5}^{10}\text{B} + {}_{1}^{1}\text{p}$, we find the difference of the initial and the final masses:

$$\Delta m = m\left({}^{11}\text{C}\right) - m\left({}^{10}\text{B}\right) - m\left({}^{1}\text{H}\right)$$
$$= (11.011433\,\text{u}) - (10.012936\,\text{u}) - (1.007825\,\text{u}) = -0.0099318\,\text{u}.$$

Thus some additional energy would have to be added.

21. If ${}_{11}^{22}\text{Na}$ were a β^- emitter, the resulting nucleus would be ${}_{12}^{22}\text{Mg}$, which has too few neutrons relative to the number of protons to be stable. Thus we have a $\boxed{\beta^+ \text{ emitter.}}$

For the reaction ${}_{11}^{22}\text{Na} \rightarrow {}_{10}^{22}\text{Ne} + \beta^+ + v$, if we add 11 electrons to both sides in order to use atomic masses, we see that we have two extra electron masses on the right. The kinetic energy of the β^+ will be maximum if no neutrino is emitted. If we ignore the recoil of the neon, the maximum kinetic energy is

$$\text{KE} = \left[m\left({}^{22}\text{Na}\right) - m\left({}^{22}\text{Ne}\right) - 2m(\text{e})\right]c^2$$
$$= \left[(21.994434\,\text{u}) - (21.991383\,\text{u}) - 2(0.00054858\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = \boxed{1.82\,\text{MeV.}}$$

22. For each decay, we find the difference of the initial and the final masses:

(*a*) $\Delta m = m\left({}^{236}\text{U}\right) - m\left({}^{235}\text{U}\right) - m\left({}^{1}\text{n}\right)$

$$= (236.045562\,\text{u}) - (235.043924\,\text{u}) - (1.008665\,\text{u}) = -0.00703\,\text{u}.$$

Because an increase in mass is required, the decay is $\boxed{\text{not possible.}}$

(*b*) $\Delta m = m\left({}^{16}\text{O}\right) - m\left({}^{15}\text{O}\right) - m\left({}^{1}\text{n}\right)$

$$= (15.994915\,\text{u}) - (15.003065\,\text{u}) - (1.008665\,\text{u}) = -0.0168\,\text{u}.$$

Because an increase in mass is required, the decay is $\boxed{\text{not possible.}}$

(*c*) $\Delta m = m\left({}^{23}\text{Na}\right) - m\left({}^{22}\text{Na}\right) - m\left({}^{1}\text{n}\right)$

$$= (22.989767\,\text{u}) - (21.994434\,\text{u}) - (1.008665\,\text{u}) = -0.0133\,\text{u}.$$

Because an increase in mass is required, the decay is $\boxed{\text{not possible.}}$

$\boxed{23.}$ We find the final nucleus by balancing the mass and charge numbers:

$$Z(X) = Z(\text{U}) - Z(\text{He}) = 92 - 2 = 90;$$
$$A(X) = A(\text{U}) - A(\text{He}) = 238 - 4 = 234, \text{ so the final nucleus is } {}_{90}^{234}\text{Th}.$$

If we ignore the recoil of the thorium, the kinetic energy of the α particle is

$$\text{KE} = \left[m\left({}^{238}\text{U}\right) - m\left({}^{234}\text{Th}\right) - m\left({}^{4}\text{He}\right)\right]c^2;$$
$$4.20\,\text{MeV} = \left[(238.050783\,\text{u}) - m\left({}^{234}\text{Th}\right) - (4.002602\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right),$$

which gives $m\left({}^{234}\text{Th}\right) = \boxed{234.04367\,\text{u.}}$

24. The kinetic energy of the electron will be maximum if no neutrino is emitted. If we ignore the recoil of the sodium, the maximum kinetic energy of the electron is

$$\text{KE} = \left[m\left({}^{23}\text{Ne}\right) - m\left({}^{23}\text{Na}\right)\right]c^2$$

$$= \left[(22.9945\,\text{u}) - (22.989767\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = \boxed{4.4\,\text{MeV.}}$$

When the neutrino has all of the kinetic energy, the minimum kinetic energy of the electron is $\boxed{0.}$ The sum of the kinetic energy of the electron and the energy of the neutrino must be from the mass difference, so the energy range of the neutrino will be $\boxed{0 \le E_\nu \le 4.4\,\text{MeV.}}$

25. We use conservation of momentum:

$$p\left({}^{4}\alpha\right) = p\left({}^{234}\chi\right);$$

$$\sqrt{2m_\alpha\left(\text{KE}_\alpha\right)} = \sqrt{2m_\chi\left(\text{KE}_\chi\right)};$$

$$\text{KE}_\chi = \frac{m_\alpha}{m_\chi}\left(\text{KE}_\alpha\right) = \frac{4}{234}(5.0\,\text{MeV}) = \boxed{0.085\,\text{MeV.}}$$

26. The reaction is ${}^{60}_{27}\text{Co} \to {}^{60}_{28}\text{Ni} + \beta^- + \nu$. The kinetic energy of the β^- will be maximum if no neutrino is emitted. If we ignore the recoil of the nickel, the maximum kinetic energy is

$$\text{KE} = \left[m\left({}^{60}\text{Co}\right) - m\left({}^{60}\text{Ni}\right) - m(\text{e})\right]c^2$$

$$= \left[(59.933822\,\text{u}) - (59.930791\,\text{u}) - (0.000549\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = \boxed{2.31\,\text{MeV.}}$$

27. (*a*) We find the final nucleus by balancing the mass and charge numbers:

$$Z(\chi) = Z(\text{P}) - Z(\text{e}) = 15 - (-1) = 16;$$

$$A(\chi) = A(\text{P}) - A(\text{e}) = 32 - 0 = 32, \text{ so the final nucleus is } \boxed{{}^{32}_{16}\text{S.}}$$

(*b*) If we ignore the recoil of the sulfur, the maximum kinetic energy of the electron is

$$\text{KE} = \left[m\left({}^{32}\text{P}\right) - m\left({}^{32}\text{S}\right)\right]c^2;$$

$$1.71\,\text{MeV} = \left[(31.973908\,\text{u}) - m\left({}^{32}\text{S}\right)\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right),$$

which gives $m\left({}^{32}\text{S}\right) = \boxed{31.97207\,\text{u.}}$

28. For alpha decay we have ${}^{218}_{84}\text{Po} \to {}^{214}_{82}\text{Pb} + {}^{4}_{2}\text{He}$. The Q value is

$$Q = \left[m\left({}^{218}\text{Po}\right) - m\left({}^{214}\text{Pb}\right) - m\left({}^{4}\text{He}\right)\right]c^2$$

$$= \left[(218.008965\,\text{u}) - (213.999798\,\text{u}) - (4.002602\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = \boxed{612\,\text{MeV.}}$$

For beta decay we have ${}^{218}_{84}\text{Po} \to {}^{218}_{85}\text{At} + {}^{0}_{-1}\text{e}$. The Q value is

$$Q = \left[m\left({}^{218}\text{Po}\right) - m\left({}^{218}\text{At}\right)\right]c^2$$

$$= \left[(218.008965\,\text{u}) - (218.00868\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = \boxed{0.27\,\text{MeV.}}$$

29. For the electron capture ${}^{7}_{4}\text{Be} + {}^{0}_{-1}\text{e} \to {}^{7}_{3}\text{Li} + \nu$, we see that if we add three electron masses to both sides to use the atomic mass for Li, we use the captured electron for the atomic mass of Be.

We find the Q value from

$$Q = \left[m\left({}^{7}\text{Be}\right) - m\left({}^{7}\text{Li}\right) \right] c^2$$

$$= \left[(7.016928\,\text{u}) - (7.016003\,\text{u}) \right] c^2 \left(931.5\,\text{MeV}/\text{u}c^2 \right) = \boxed{0.861\,\text{MeV.}}$$

30. We find the energy from

$$E = \frac{h_c}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(1.00\times10^{-13}\,\text{m}\right)\left(1.602\times10^{-19}\ \text{J/eV}\right)} = \boxed{12.4\,\text{MeV.}}$$

This is a γ ray from the nucleus. Electron transitions do not involve this much energy.

31. The kinetic energy of the β^+ particle will be maximum if no neutrino is emitted. If we ignore the recoil of the boron, the maximum kinetic energy is

$$\text{KE} = \left[m\left({}^{11}\text{C}\right) - m\left({}^{11}\text{B}\right) - 2m_\text{e} \right] c^2$$

$$= \left[(11.011433\,\text{u}) - (11.009305\,\text{u}) - 2(0.00054858) \right] c^2 \left(931.5\,\text{MeV}/\text{u}c^2 \right) = \boxed{0.960\,\text{MeV.}}$$

The sum of the kinetic energy of the β^+ particle and the energy of the neutrino must be from the mass difference, so the kinetic energy of the neutrino will range from $\boxed{\text{0.960 MeV to 0.}}$

32. We find the recoil energy using the fact of conservation of momentum:

$$p_\gamma = \frac{E_\gamma}{c} = p_\text{K};$$

$$\text{KE}_\text{K} = \frac{p_\text{K}^{\ 2}}{2m} = \frac{\left(\frac{E_\gamma}{c}\right)^2}{2m} = \frac{(1.46\,\text{MeV})^2}{2(40.0\,\text{u})\left(931.5\,\text{MeV}/\text{u}c^2\right)c^2} = 2.86\times10^{-5}\,\text{MeV} = \boxed{28.6\,\text{eV.}}$$

$\boxed{33.}$ If we ignore the recoil of the lead, the kinetic energy of the α particle is

$$\text{KE} = \left[m\left({}^{210}\text{Po}\right) - m\left({}^{206}\text{Pb}\right) - m\left({}^{4}\text{He}\right) \right] c^2$$

$$= \left[(209.982848\,\text{u}) - (205.974440\,\text{u}) - (4.002602\,\text{u}) \right] c^2 \left(931.5\,\text{MeV}/\text{u}c^2 \right) = \boxed{5.41\,\text{MeV.}}$$

34. The decay is ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$. If the uranium nucleus is at rest when it decays, for momentum conservation we have

$$p_\alpha = p_\text{Th}.$$

Thus the kinetic energy of the thorium nucleus is

$$\text{KE}_\text{Th} = \frac{p_\text{Th}^{\ 2}}{2m_\text{Th}} = \frac{p_\alpha^{\ 2}}{2m_\text{Th}} = \left(\frac{m_\alpha}{m_\text{Th}}\right)\left(\text{KE}_\alpha\right) = \left(\frac{4\,\text{u}}{234\,\text{u}}\right)(4.20\,\text{MeV}) = \boxed{0.0718\,\text{MeV.}}$$

The Q value is the total kinetic energy produced:

$$\text{Q} = \text{KE}_\alpha + \text{KE}_\text{Th} = 4.20\,\text{MeV} + 0.0718 = \boxed{4.27\,\text{MeV.}}$$

35. For the positron-emission process

$${}^{A}_{Z+1}\chi \rightarrow {}^{A}_{Z}\chi' + \text{e}^+ + v,$$

we need to add $Z+1$ electrons to the nuclear mass of χ to be able to use the atomic mass. On the right-hand side we use Z electrons to be able to use the atomic mass of χ'. Thus we have 1 electron mass and the β-particle mass, which means that we must include 2 electron masses on the right-hand side. The Q value will be

$$Q = \left[M_{\text{P}} - \left(M_{\text{D}} + 2m_{\text{e}}\right)\right]c^2 = \left(M_{\text{P}} - M_{\text{D}} - 2m_{\text{e}}\right)c^2.$$

36. We find the decay constant from

$$\lambda N = \lambda N_0 e^{-\lambda t};$$

$$320\,\text{decays/min} = \left(1280\,\text{decays/min}\right)e^{-\lambda(4.6\,\text{h})}, \text{ which gives } \lambda = 0.301/\text{h}.$$

Thus the half-life is

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{\left(0.301/\text{h}\right)} = \boxed{2.3\,\text{h.}}$$

37. (*a*) The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{\left(4.5\times10^{9}\text{ yr}\right)} = \boxed{1.5\times10^{-10}\,\text{yr.}}$$

(*b*) The half-life is

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{\left(8.2\times10^{-5}\text{ s}^{-1}\right)} = 8450\,\text{s} = \boxed{2.3\,\text{h.}}$$

38. The activity of the sample is

$$\frac{\Delta N}{\Delta t} = \lambda N = \left(\frac{0.693}{T_{\frac{1}{2}}}\right)N = \left[\frac{0.693}{\left(5730\,\text{yr}\right)\left(3.16\times10^{7}\text{ s/yr}\right)}\right]\left(3.1\times10^{20}\right) = \boxed{1.2\times10^{9}\text{ decays/s.}}$$

39. We find the fraction remaining from

$$N = N_0 e^{-\lambda t};$$

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-\left[\frac{(0.693)(3.0\,\text{yr})(12\,\text{mo/yr})}{9\,\text{mo}}\right]} = \boxed{0.0625.}$$

40. The fraction left is

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^n = \left(\tfrac{1}{2}\right)^6 = \boxed{0.015625.}$$

41. We find the number of nuclei from the activity of the sample:

$$\frac{\Delta N}{\Delta t} = \lambda N;$$

$$640\,\text{decays/s} = \left[\frac{(0.693)}{\left(4.468\times10^{9}\text{ yr}\right)\left(3.16\times10^{7}\text{ s/yr}\right)}\right]N, \text{ which gives } N = \boxed{1.30\times10^{20}\text{ nuclei.}}$$

42. Because only α particle decay changes the mass number (by 4), we have

$$N_\alpha = \frac{(235 - 207)}{4} = \boxed{7\,\alpha\text{ particles.}}$$

An α particle decreases the atomic number by 2, while a β^- particle increases the atomic number by 1, so we have

$$N_\beta = \frac{[92 - 82 - 7(2)]}{(-1)} = \boxed{4\beta^- \text{ particles.}}$$

43. The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{(8.02\,\text{days})(24\,\text{h/day})(3600\,\text{s/h})} = 1.000\times10^{-6}\,\text{s}^{-1}.$$

The initial number of nuclei is

$$N_0 = \left[\frac{(682\times10^{-6}\,\text{g})}{(131\,\text{g/mol})}\right](6.02\times10^{23}\,\text{atoms/mol}) = 3.134\times10^{18}\,\text{nuclei.}$$

(*a*) When $t = 0$, we get

$$\lambda N = \lambda N_0 e^{-\lambda t} = (1.000\times10^{-6}\,\text{s}^{-1})(3.134\times10^{18})e^0 = \boxed{3.13\times10^{12}\,\text{decays/s.}}$$

(*b*) When $t = 1.0\,\text{h}$, the exponent is

$$\lambda t = (1.000\times10^{-6}\,\text{s}^{-1})(1.0\,\text{h})(3600\,\text{s/h}) = 3.600\times10^{-3},$$

so we get

$$\lambda N = \lambda N_0 e^{-\lambda t} = (1.000\times10^{-6}\,\text{s}^{-1})(3.134\times10^{18})e^{-0.003600} = \boxed{3.12\times10^{12}\,\text{decays/s.}}$$

(*c*) When $t = 6$ months, the exponent is

$$\lambda t = (1.000\times10^{-6}\,\text{s}^{-1})(6\,\text{mo})(30.5\,\text{days/mo})(24\,\text{h/day})(3600\,\text{s/h}) = 15.81,$$

so we get

$$\lambda N = \lambda N_0 e^{-\lambda t} = (1.000\times10^{-6}\,\text{s}^{-1})(3.134\times10^{18})e^{-15.81} = \boxed{4.26\times10^{5}\,\text{decays/s.}}$$

44. The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{(30.8\,\text{s})} = 0.0225\,\text{s}^{-1}.$$

(*a*) The initial number of nuclei is

$$N_0 = \frac{(8.8\times10^{-6}\,\text{g})}{(124\,\text{g/mol})}(6.02\times10^{23}\,\text{atoms/mol}) = \boxed{4.3\times10^{16}\,\text{nuclei.}}$$

(*b*) When $t = 2.0\,\text{min}$, the exponent is

$$\lambda t = (0.0225\,\text{s}^{-1})(2.0\,\text{min})(60\,\text{s/min}) = 2.7,$$

so we get

$$N = N_0 e^{-\lambda t} = (4.3\times10^{16})e^{-2.7} = \boxed{2.9\times10^{15}\,\text{nuclei.}}$$

(*c*) The activity is

$$\lambda N = (0.0225\,\text{s}^{-1})(2.9\times10^{15}) = \boxed{6.5\times10^{13}\,\text{decays/s.}}$$

(*d*) We find the time from

$$\lambda N = \lambda N_0 e^{-\lambda t};$$

$$1\,\text{decay/s} = (0.0225\,\text{s}^{-1})(4.3\times10^{16})e^{-(0.0225/\text{s})t},$$ which gives $t = 1.53\times10^3\,\text{s} = \boxed{26\,\text{min.}}$

45. We find the number of nuclei from

$$\text{Activity} = \lambda N;$$

$$2.0\times10^{5}\ \text{decays/s} = \left[\frac{0.693}{\left(1.28\times10^{9}\ \text{yr}\right)\left(3.16\times10^{7}\ \text{s/yr}\right)}\right]N, \text{ which gives } N = 1.17\times10^{22}\ \text{nuclei.}$$

The mass is

$$m = \frac{\left(1.17\times10^{22}\ \text{nuclei}\right)\left(40.0\ \text{g/mol}\right)}{6.02\times10^{23}\ \text{nuclei/mol}} = \boxed{0.77\ \text{g.}}$$

46. The number of nuclei is

$$N = \left[\frac{\left(9.7\times10^{-6}\ \text{g}\right)}{\left(32\ \text{g/mol}\right)}\right]\left(6.02\times10^{23}\ \text{atoms/mol}\right) = 1.82\times10^{17}\ \text{nuclei.}$$

The activity is

$$\lambda N = \left[\frac{(0.693)}{\left(1.23\times10^{6}\ \text{s}\right)}\right]\left(1.82\times10^{17}\right) = \boxed{1.0\times10^{11}\ \text{decays/s.}}$$

47. We find the number of nuclei from

$$\text{Activity} = \lambda N;$$

$$2.65\times10^{5}\ \text{decays/s} = \left[\frac{(0.693)}{\left(7.55\times10^{6}\ \text{s}\right)}\right]N, \text{ which gives } N = 2.89\times10^{12}\ \text{nuclei.}$$

The mass is

$$m = \left[\frac{\left(2.89\times10^{12}\ \text{nuclei}\right)}{\left(6.02\times10^{23}\ \text{atoms/mol}\right)}\right]\left(35\ \text{g/mol}\right) = \boxed{1.68\times10^{-10}\ \text{g.}}$$

48. (*a*) The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{\left(1.59\times10^{5}\ \text{yr}\right)\left(3.16\times10^{7}\ \text{s/yr}\right)} = \boxed{1.38\times10^{-13}\ \text{s}^{-1}.}$$

(*b*) The activity is

$$\lambda N = \left(1.38\times10^{-13}\ \text{s}^{-1}\right)\left(7.50\times10^{19}\right) = 1.03\times10^{7}\ \text{decays/s} = \boxed{6.21\times10^{8}\ \text{decays/min.}}$$

49. We find the number of half-lives from

$$\frac{\left(\dfrac{\Delta N}{\Delta t}\right)}{\left(\dfrac{\Delta N}{\Delta t}\right)_0} = \left(\tfrac{1}{2}\right)^n;$$

$$\frac{1}{10} = \left(\tfrac{1}{2}\right)^n, \text{ or } n\log 2 = \log 10, \text{ which gives } n = 3.32.$$

Thus the half-life is

$$T_{\frac{1}{2}} = \frac{t}{n} = \frac{(8.6\ \text{min})}{3.32} = \boxed{2.6\ \text{min.}}$$

50. Because the fraction of atoms that are ^{14}C is so small, we use the atomic weight of ^{12}C to find the number of carbon atoms in 285 g:

$$N = \left[\frac{(285\,\text{g})}{(12\,\text{g/mol})}\right](6.02\times10^{23}\text{ atoms/mol}) = 1.43\times10^{25}\text{ atoms.}$$

The number of ^{14}C nuclei is

$$N_{14} = \left(\frac{1.3}{10^{12}}\right)(1.43\times10^{25}) = 1.86\times10^{13}\text{ nuclei.}$$

The activity is

$$\lambda N = \left[\frac{0.693}{(5730\,\text{yr})(3.16\times10^{7}\text{ s/yr})}\right](1.86\times10^{13}) = \boxed{71\text{decays/s.}}$$

51. We find the number of nuclei from

$$\text{Activity} = \lambda N;$$

$$6.70\times10^{2}\text{ decays/s} = \left[\frac{(0.693)}{(1.28\times10^{9}\text{ s})(3.16\times10^{7}\text{ s/yr})}\right]N,\ \text{which gives } N = 3.91\times10^{19}\text{ nuclei.}$$

The mass is

$$m = \left[\frac{(3.91\times10^{19}\text{ nuclei})}{(6.02\times10^{23}\text{ atoms/mol})}\right](40\,\text{g/mol}) = 2.60\times10^{-3}\text{ g} = \boxed{2.60\,\text{mg.}}$$

52. We assume that the elapsed time is much smaller than the half-life, so we can use a constant decay rate. Because ^{87}Sr is stable, and there was none present when the rocks were formed, every atom of ^{87}Rb that decayed is now an atom of ^{87}Sr. Thus we have

$$N_{\text{Sr}} = -\Delta N_{\text{Rb}} = \lambda N_{\text{Rb}}\,\Delta t,\ \text{or}$$

$$\frac{N_{\text{Sr}}}{N_{\text{Rb}}} = \left(\frac{0.693}{T_{\frac{1}{2}}}\right)\Delta t;$$

$$0.0160 = \left(\frac{0.693}{(4.75\times10^{10}\text{ yr})}\right)\Delta t,\ \text{which gives } \Delta t = \boxed{1.1\times10^{9}\text{ yr.}}$$

This is $\approx 2\%$ of the half-life, so our original assumption is valid.

53. The decay rate is

$$\frac{\Delta N}{\Delta t} = \lambda N.$$

If we assume equal numbers of nuclei decaying by α emission, we have

$$\frac{\left(\frac{\Delta N}{\Delta t}\right)_{218}}{\left(\frac{\Delta N}{\Delta t}\right)_{214}} = \frac{\lambda_{218}}{\lambda_{214}} = \frac{T_{\frac{1}{2},214}}{T_{\frac{1}{2},218}}$$

$$= \frac{\left(1.6\times10^{-4}\,\text{s}\right)}{\left(3.1\,\text{min}\right)\left(60\,\text{s/min}\right)} = \boxed{8.6\times10^{-7}.}$$

54. The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{\left(53\,\text{days}\right)} = \frac{0.0131}{\text{day}} = 1.52\times10^{-7}\,\text{s}^{-1}.$$

(*a*) We find the number of half-lives from

$$\frac{\left(\dfrac{\Delta N}{\Delta t}\right)}{\left(\dfrac{\Delta N}{\Delta t}\right)_0} = \left(\tfrac{1}{2}\right)^n;$$

$$\frac{\left(15\,\text{decays/s}\right)}{\left(450\,\text{decays/s}\right)} = \left(\tfrac{1}{2}\right)^n, \text{ or } n\log 2 = \log 30, \text{ which gives } n = 4.91.$$

Thus the elapsed time is

$$\Delta t = nT_{\frac{1}{2}} = \left(4.91\right)\left(53\,\text{days}\right) = \boxed{261\,\text{days}} \approx 8.5\,\text{months}.$$

(*b*) We find the number of nuclei from

$$\text{Activity} = \lambda N;$$

$$450\,\text{decays/s} = \left(1.52\times10^{-7}\,\text{s}^{-1}\right)N, \text{ which gives } N = 2.97\times10^{9}\text{ nuclei}.$$

The mass is

$$m = \left[\frac{\left(2.97\times10^{9}\text{ nuclei}\right)}{\left(6.02\times10^{23}\text{ atoms/mol}\right)}\right]\left(7\,\text{g/mol}\right) = \boxed{3.5\times10^{-17}\,\text{kg.}}$$

55. $\boxed{{}^{232}_{90}\text{Th}} \rightarrow {}^{228}_{88}\text{Ra} + {}^{4}_{2}\alpha, \boxed{{}^{228}_{88}\text{Ra}} \rightarrow {}^{228}_{89}\text{Ac} + \beta^-,$

$\boxed{{}^{228}_{89}\text{Ac}} \rightarrow {}^{228}_{90}\text{Th} + \beta^-, \boxed{{}^{228}_{90}\text{Th}} \rightarrow {}^{224}_{88}\text{Ra} + {}^{4}_{2}\alpha,$

$\boxed{{}^{224}_{88}\text{Ra}} \rightarrow \boxed{{}^{220}_{86}\text{Rn}} + {}^{4}_{2}\alpha, \boxed{{}^{235}_{92}\text{U}} \rightarrow {}^{231}_{90}\text{Th} + {}^{4}_{2}\alpha,$

$\boxed{{}^{231}_{90}\text{Th}} \rightarrow {}^{231}_{91}\text{Pa} + \beta^-, \boxed{{}^{231}_{91}\text{Pa}} \rightarrow {}^{227}_{89}\text{Ac} + {}^{4}_{2}\alpha,$

$\boxed{{}^{227}_{89}\text{Ac}} \rightarrow {}^{227}_{90}\text{Th} + \beta^-, \boxed{{}^{227}_{90}\text{Th}} \rightarrow \boxed{{}^{223}_{88}\text{Ra}} + {}^{4}_{2}\alpha.$

56. The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{\left(5730\,\text{yr}\right)} = 1.209\times10^{-4}\,/\,\text{yr}.$$

Because the fraction of atoms that are ${}^{14}\text{C}$ is so small, we use the atomic weight of ${}^{12}\text{C}$ to find the number of carbon atoms in 290 g:

$$N = \left[\frac{\left(290\,\text{g}\right)}{\left(12\,\text{g/mol}\right)}\right]\left(6.02\times10^{23}\text{ atoms/mol}\right) = 1.45\times10^{25}\text{ atoms},$$

so the number of ${}^{14}\text{C}$ nuclei in a sample from a living tree is

$N_{14} = \left(1.3 \times 10^{-12}\right)\left(1.45 \times 10^{25}\right) = 1.89 \times 10^{13}$ nuclei.

Because the carbon is being replenished in living trees, we assume that this number produced the activity when the club was made. We determine its age from

$\lambda N = \lambda N_{14} e^{-\lambda t}$;

$$8.0\,\text{decays/s} = \left[\frac{\left(1.209 \times 10^{-4}/\text{yr}\right)}{\left(3.16 \times 10^{7}\,\text{s/yr}\right)}\right]\left(1.89 \times 10^{13}\text{ nuclei}\right) e^{-\left(1.209 \times 10^{-4}/\text{yr}\right)t},$$

which gives $t = \boxed{1.8 \times 10^{4}\text{ yr.}}$

57. The number of radioactive nuclei decreases exponentially:

$N = N_0 e^{-\lambda t}$.

Every radioactive nucleus that decays becomes a daughter nucleus, so we have

$N_{\text{D}} = N_0 - N = \boxed{N_0\left(1 - e^{-\lambda t}\right).}$

58. We find the number of half-lives from

$$\frac{\left(\frac{\Delta N}{\Delta t}\right)}{\left(\frac{\Delta N}{\Delta t}\right)_0} = \left(\tfrac{1}{2}\right)^n;$$

$1.050 \times 10^{-2} = \left(\tfrac{1}{2}\right)^n$, or $n \log 2 = \log\left(\frac{1}{1.050 \times 10^{-2}}\right)$, which gives $n = 6.57$.

Thus the half-life is

$$T_{\frac{1}{2}} = \frac{t}{n} = \frac{(4.00\,\text{h})}{6.57} = 0.609\,\text{h} = 36.5\,\text{min}.$$

From the Appendix we see that the isotope is $\boxed{{}^{211}_{82}\text{Pb.}}$

59. Because the carbon is being replenished in living trees, we assume that the amount of ^{14}C is constant until the wood is cut, and then it decays. We find the number of half-lives from

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^n;$$

$0.060 = \left(\tfrac{1}{2}\right)^n$, or $n \log 2 = \log(16.7)$, which gives $n = 4.06$.

Thus the time is

$$t = nT_{\frac{1}{2}} = (4.06)(5730\,\text{yr}) = \boxed{2.3 \times 10^{4}\text{ yr.}}$$

60. (*a*) We find the mass number from its radius:

$$r = \left(1.2 \times 10^{-15}\text{ m}\right)A^{\frac{1}{3}};$$

$5.0 \times 10^{3}\text{ m} = \left(1.2 \times 10^{-15}\text{ m}\right)A^{\frac{1}{3}}$, which gives $A = \boxed{7.2 \times 10^{55}.}$

(*b*) The mass of the neutron star is

$$m = A\left(1.66\times10^{-27}\text{ kg/u}\right) = \left(7.2\times10^{55}\text{ u}\right)\left(1.66\times10^{-27}\text{ kg/u}\right) = \boxed{1.2\times10^{29}\text{ kg.}}$$

Note that this is about 6% of the mass of the Sun.

(*c*) The acceleration of gravity on the surface of the neutron star is

$$g = \frac{Gm}{r^2} = \frac{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(1.2\times10^{29}\text{ kg}\right)}{\left(5.0\times10^{3}\text{ m}\right)^2} = \boxed{3.2\times10^{11}\text{ m/s}^2.}$$

61. Because the tritium in water is being replenished, we assume that the amount is constant until the wine is made, and then it decays. We find the number of half-lives from

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^n;$$

$$0.10 = \left(\tfrac{1}{2}\right)^n, \text{ or } n\log 2 = \log(10), \text{ which gives } n = 3.32.$$

Thus the time is

$$t = nT_{\frac{1}{2}} = (3.32)(12.33\text{ yr}) = \boxed{41\text{ yr.}}$$

62. If we assume a body has 70 kg of water, the number of water molecules is

$$N_{\text{water}} = \left[\frac{\left(70\times10^3\text{ g}\right)}{(18\text{ g/mol})}\right]\left(6.02\times10^{23}\text{ atoms/mol}\right) = 2.34\times10^{27}\text{ molecules.}$$

The number of protons in a water molecule (H_2O) is $2 + 8 = 10,$ so the number of protons is

$$N_0 = 2.34\times10^{28}\text{ protons.}$$

If we assume that the time is much less than the half-life, the rate of decay is constant, so we have

$$\frac{\Delta N}{\Delta t} = \lambda N = \left(\frac{0.693}{T_{\frac{1}{2}}}\right)N;$$

$$\frac{(1\text{ proton})}{\Delta t} = \left[\frac{(0.693)}{\left(10^{28}\text{ yr}\right)}\right]\left(2.34\times10^{28}\text{ protons}\right), \text{ which gives } \Delta t = \boxed{6\times10^3\text{ yr.}}$$

63. We find the number of half-lives from

$$\frac{\left(\frac{\Delta N}{\Delta t}\right)}{\left(\frac{\Delta N}{\Delta t}\right)_0} = \left(\tfrac{1}{2}\right)^n;$$

$$1.00\times10^{-2} = \left(\tfrac{1}{2}\right)^n, \text{ or } n\log 2 = \log(100), \text{ which gives } n = 6.65, \text{ so the time is } \boxed{6.65T_{\frac{1}{2}}.}$$

64. We find the number of ^{40}K nuclei from

$$\text{Activity} = \lambda N_{40};$$

$$60\text{ decays/s} = \left[\frac{(0.693)}{\left(1.28\times10^9\text{ s}\right)\left(3.16\times10^7\text{ s/yr}\right)}\right]N_{40}, \text{ which gives } N_{40} = 3.5\times10^{18}\text{ nuclei.}$$

The mass of ^{40}K is

$$m_{40} = \left(3.5\times10^{18}\right)(40\text{ u})\left(1.66\times10^{-27}\text{ kg/u}\right) = 2.33\times10^{-7}\text{ kg} = \boxed{0.23\text{ mg.}}$$

From the Appendix we have

$N_{40} = (0.0117\%)N$, and $N_{39} = (93.2581\%)N$.

Thus the number of ^{39}K nuclei is

$$N_{39} = \left[\frac{(93.258\%)}{(0.0117\%)}\right](3.5\times10^{18}\text{ nuclei}) = 2.8\times10^{22}\text{ nuclei}.$$

The mass of ^{39}K is

$$m_{39} = (2.8\times10^{22})(39\,\text{u})(1.66\times10^{-27}\text{ kg/u}) = 1.81\times10^{-3}\text{ kg} = \boxed{1.8\,\text{g.}}$$

65. If the initial nucleus is at rest when it decays, for momentum conservation we have

$p_\alpha = p_\text{D}$.

Thus the kinetic energy of the daughter is

$$\text{KE}_\text{D} = \frac{P_\text{D}^2}{2m_\text{D}} = \frac{P_\alpha^2}{2m_\text{D}} = \left(\frac{m_\alpha}{m_\text{D}}\right)(\text{KE}_\alpha) = \left(\frac{A_\alpha}{A_\text{D}}\right)(\text{KE}_\alpha) = \left(\frac{4}{A_\text{D}}\right)(\text{KE}_\alpha).$$

Thus the fraction carried away by the daughter is

$$\frac{(\text{KE}_\text{D})}{(\text{KE}_\alpha + \text{KE}_\text{D})} = \frac{\left(\frac{4}{A_\text{D}}\right)(\text{KE}_\alpha)}{\left[(\text{KE}_\alpha) + \left(\frac{4}{A_\text{D}}\right)(\text{KE}_\alpha)\right]} = \frac{1}{\left[1 + \left(\frac{A_\text{D}}{4}\right)\right]}.$$

For the decay of ^{226}Ra, the daughter has $A_\text{D} = 222$, so we get

$$\text{fraction}_\text{D} = \frac{1}{\left[1 + \left(\frac{222}{4}\right)\right]} = 0.018.$$

Thus the α particle carries away $1 - 0.018 = 0.982 = \boxed{98.2\%.}$

66. We see from the periodic chart that Sr is in the same column as $\boxed{\text{calcium.}}$

If strontium is ingested, the body will treat it chemically as if it were calcium, which means it will be $\boxed{\text{stored by the body in bones.}}$

We find the number of half-lives to reach a 1% level from

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^n;$$

$0.01 = \left(\tfrac{1}{2}\right)^n$, or $n\log 2 = \log(100)$, which gives $n = 6.64$.

Thus the time is

$$t = nT_{\frac{1}{2}} = (6.64)(29\,\text{yr}) = \boxed{193\,\text{yr.}}$$

The decay reactions are

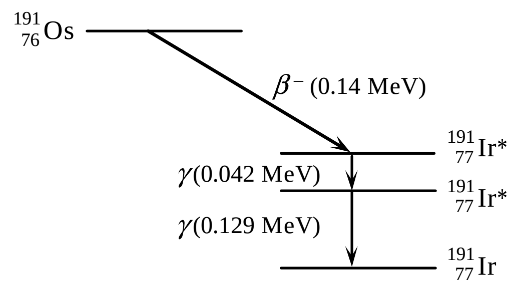
$$\boxed{\begin{array}{l} {}^{90}_{38}\text{Sr} \rightarrow {}^{90}_{39}\text{Y} + {}^{0}_{-1}\text{e} + \bar{v},\ {}^{90}_{39}Y \text{ is radioactive;} \\ {}^{90}_{39}\text{Y} \rightarrow {}^{90}_{40}\text{Zr} + {}^{0}_{-1}\text{e} + \bar{v},\ {}^{90}_{40}\text{Zr is stable.} \end{array}}$$

67. (*a*) We find the daughter nucleus by balancing the mass and charge numbers:

$$Z(X) = Z(\text{Os}) - Z(\text{e}^-) = 76 - (-1) = 77;$$

$$A(X) = A(\text{Os}) - A(\text{e}^-) = 191 - 0 = 191,$$

so the daughter nucleus is $\boxed{{}^{191}_{77}\text{Ir.}}$



(*b*) Because there is only one β energy, the β decay must be to the higher excited state.

68. (*a*) The number of nuclei is

$$N = \left(\frac{1.0\,\text{g}}{131\,\text{g/mol}}\right)(6.02 \times 10^{23}\text{ nuclei/mol}) = 4.60 \times 10^{21}\text{ nuclei.}$$

The activity is

$$\lambda N = \left[\frac{0.693}{(8.02\,\text{days})(8.64 \times 10^4\text{ s/day})}\right](4.60 \times 10^{21}) = \boxed{4.57 \times 10^{15}\text{ decays/s.}}$$

(*b*) The number of nuclei is

$$N = \left(\frac{1.0\,\text{g}}{238\,\text{g/mol}}\right)(6.02\times 10^{23}\text{ nuclei/mol}) = 2.53 \times 10^{21}\text{ nuclei.}$$

The activity is

$$\lambda N = \left[\frac{0.693}{(4.47 \times 10^9\text{ yr})(3.16 \times 10^7\text{ s/yr})}\right](2.53 \times 10^{21}) = \boxed{1.24\times 10^4\text{ decays/s.}}$$

69. From Figure 30–1, we see that the average binding energy per nucleon at $A = 29$ is 8.6 MeV. If we use the average atomic weight as the average number of nucleons for the two stable isotopes of copper, the total binding energy is

$$(63.5)(8.6\,\text{MeV}) = \boxed{550\,\text{MeV.}}$$

The number of atoms in a penny is

$$N = \left[\frac{(3\times 10^3\text{ g})}{(63.5\,\text{g/mol})}\right](6.02 \times 10^{23}\text{ atoms/mol}) = 2.84 \times 10^{22}\text{ atoms.}$$

Thus the total energy needed is

$$(2.84 \times 10^{22})(550\,\text{MeV}) = 1.57 \times 10^{25}\text{ MeV} = \boxed{2.5 \times 10^{12}\text{ J.}}$$

70. (*a*) $\Delta({}^4\text{He}) = m({}^4\text{He}) - A({}^4\text{He}) = 4.002602\,\text{u} - 4 = \boxed{0.002602\,\text{u}}$

$$= (0.002602\,\text{u})(931.5\,\text{MeV/u}c^2) = \boxed{2.424\,\text{Mev}/c^2.}$$

(*b*) $\Delta({}^{12}\text{C}) = m({}^{12}\text{C}) - A({}^{12}\text{C}) = 12.000000\,\text{u} - 12 = \boxed{0.}$

(*c*) $\Delta({}^{107}\text{Ag}) = m({}^{107}\text{Ag}) - A({}^{107}\text{Ag}) = 106.905091\,\text{u} - 107 = \boxed{-0.094909\,\text{u}}$

$$= (-0.094909\,\text{u})(931.5\,\text{MeV/u}c^2) = \boxed{-88.41\,\text{MeV}/c^2.}$$

(*d*) $\Delta({}^{235}\text{U}) = m({}^{235}\text{U}) - A({}^{235}\text{U}) = 235.043924\,\text{u} - 235 = \boxed{0.043924\,\text{u}}$

$$= (0.043924\,\text{u})(931.5\,\text{MeV/u}c^2) = \boxed{40.92\,\text{MeV}/c^2.}$$

(*e*) From the Appendix we see that

$$\boxed{\begin{array}{l}\Delta \ge 0 \text{ for } 0 \le Z \le 8 \text{ and } Z \ge 85;\\ \Delta < 0 \text{ for } 9 \le Z \le 84.\end{array}}$$

71. (*a*) The usual fraction of ^{14}C is 1.3×10^{-12}. Because the fraction of atoms that are ^{14}C is so small, we use the atomic weight of ^{12}C to find the number of carbon atoms in 92 g:

$$N = \left[\frac{(92\,\text{g})}{(12\,\text{g/mol})}\right](6.02\times10^{23}\,\text{atoms/mol}) = 4.62\times10^{24}\,\text{atoms.}$$

The number of ^{14}C nuclei in the sample is

$$N_{14} = (1.3\times10^{-12})(4.62\times10^{24}) = 6.00\times10^{12}\,\text{nuclei.}$$

We find the number of half-lives from

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^n;$$

$$\frac{1}{6.00\times10^{12}} = \left(\tfrac{1}{2}\right)^n, \text{ or } n\log 2 = \log(6.00\times10^{12}), \text{ which gives } n = 42.4.$$

Thus the time is

$$t = nT_{\frac{1}{2}} = (42.4)(5730\,\text{yr}) = \boxed{2.4\times10^5\,\text{yr.}}$$

(*b*) A similar calculation as above leads to $N_{14} = 1.83\times10^{13}$ nuclei, $n = 44.1$, and $t = 2.5\times10^5$ yr. Thus, sample amount has fairly little effect on the maximum age that can be measured, which is on the order of $\boxed{10^5\text{ yr.}}$

72. Because there are so many low-energy electrons available, this reaction would turn most of the protons into neutrons, which would eliminate chemistry, and thus life.
The *Q*-value of the reaction is

$$Q = \left[m(^1\text{H}) - m(^1\text{n})\right]c^2 = \left[(1.007825\,\text{u}) - (1.008665\,\text{u})\right]c^2(931.5\,\text{MeV/u}c^2) = -0.782\,\text{MeV.}$$

The percentage increase in the proton’s mass to make the *Q*-value = 0 is

$$\left(\frac{\Delta m}{m}\right)(100) = \left[\frac{(0.782\,\text{MeV}/c^2)}{(938.3\,\text{MeV}/c^2)}\right](100) = \boxed{0.083\%.}$$

73. The radius is given by

$$r = \frac{mv}{qB}$$

We set the two radii equal:

$$\frac{m_\alpha v_\alpha}{2eB} = \frac{m_\beta v_\beta}{eB};$$

$$m_\alpha v_\alpha = 2m_\beta v_\beta;$$

$$P_\alpha = 2P_\beta$$

The energies are given by

$$\text{KE} = \frac{p^2}{2m},$$

and therefore

$$\frac{\mathrm{KE}_\alpha}{\mathrm{KE}_\beta}=\frac{p_\alpha^2 m_\beta}{p_\beta^2 m_\alpha}=\frac{4m_\beta}{m_\alpha}=\frac{4(0.000549\,\mathrm{u})}{4.002603\,\mathrm{u}}=\boxed{\frac{5.49\times10^{-4}}{1}}.$$

74. We first find the number of $^{147}\mathrm{Sm}$ nuclei from the mass and proportion information:

$$N=\frac{(0.15)(1.00\,\mathrm{g})(6.02\times10^{23}\,\mathrm{nuclei/mol})}{147\,\mathrm{g/mol}}=6.14\times10^{20}\,\mathrm{nuclei}.$$

The activity level is determined by

Activity level $=\lambda N$;

$$120\,\mathrm{s}^{-1}=\left(\frac{0.693}{T_{\frac{1}{2}}}\right)(6.14\times10^{20}),\ \text{which gives } T_{\frac{1}{2}}=3.55\times10^{18}\,\mathrm{s}=\boxed{1.1\times10^{11}\,\mathrm{yr.}}$$

75. Since amounts are not specified, we may suppose that there are 0.72 g of $^{235}\mathrm{U}$ and $100.00-0.72=99.28\,\mathrm{g}$ of $^{238}\mathrm{U}$. Now we use $N=N_0e^{-\lambda t}=N_0e^{\frac{-0.693t}{T_{\frac{1}{2}}}}$.

(*a*) 1.0×10^9 years ago,

$$N_{0,235}=N_{235}e^{\frac{0.693t}{T_{\frac{1}{2}}}}$$

$$=(0.72\,\mathrm{g})e^{\frac{0.693(1.0\times10^9)}{(7.038\times10^8)}}=1.93\,\mathrm{g};$$

$$N_{0,238}=N_{238}e^{\frac{0.693t}{T_{\frac{1}{2}}}}$$

$$=(99.28\,\mathrm{g})e^{\frac{0.693(1.0\times10^9)}{(4.468\times10^9)}}=115.94\,\mathrm{g}.$$

The percentage of $^{235}\mathrm{U}$ was

$$\frac{1.93}{1.93+115.94}\times100\%=\boxed{1.6\%.}$$

(b) In 100×10^6 years,

$$N_{235}=N_{0,235}e^{\frac{0.693t}{T_{\frac{1}{2}}}}$$

$$=(0.72\,\mathrm{g})e^{\frac{-0.693(100\times10^6)}{(7.038\times10^8)}}=0.65\,\mathrm{g};$$

$$N_{238}=N_{0,238}e^{\frac{-0.693t}{T_{\frac{1}{2}}}}$$

$$=(99.28\,\mathrm{g})e^{\frac{-0.693(100\times10^6)}{(4.468\times10^9)}}=97.75\,\mathrm{g}.$$

The percentage of ^{235}U will be

$$\frac{0.65}{0.65 + 97.75} \times 100\% = \boxed{0.67\%.}$$

76. The mass of ^{40}K is

$$\left(400 \times 10^{-3}\,\text{g}\right)\left(0.000117\right) = 4.68 \times 10^{-5}\,\text{g}.$$

The number of nuclei is

$$N = \frac{\left(4.68 \times 10^{-5}\,\text{g}\right)\left(6.02 \times 10^{23}\,\text{atoms/mol}\right)}{40\,\text{g/mol}} = 7.04 \times 10^{17}\,\text{nuclei}.$$

The activity is

$$\lambda N = \frac{0.693\left(7.04 \times 10^{17}\right)}{\left(1.277 \times 10^{9}\,\text{yr}\right)\left(3.16 \times 10^{7}\,\text{s/yr}\right)} = \boxed{12.1\,\text{decays/s}.}$$

77. The mass of carbon 60,000 years ago was essentially 1.0 kg, for which the corresponding number of ^{12}C atoms would be

$$N_{12} = \frac{\left(1.0 \times 10^{3}\,\text{g}\right)\left(6.02 \times 10^{23}\,\text{atoms/mol}\right)}{12\,\text{g/mol}} = 5.02 \times 10^{25}\,\text{atoms}.$$

However, a small fraction will in fact be ^{14}C atoms, namely

$$N_{14} = \left(5.02 \times 10^{25}\right)\left(1.3 \times 10^{-12}\right) = 6.52 \times 10^{13}\,\text{atoms}.$$

The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{\left(5730\,\text{yr}\right)\left(3.16 \times 10^{7}\,\text{s/yr}\right)} = 3.83 \times 10^{-12}.$$

Activity today, finally, is given by

$$\text{Activity} = \lambda N_0 e^{-\lambda t} = \left(3.83 \times 10^{-12}\right)\left(6.52 \times 10^{13}\right) e^{-\left(3.83 \times 10^{-12}\right)\left(60{,}000\right)\left(3.16 \times 10^{7}\right)} = \boxed{0.18\,\text{decays/s}.}$$

78. The mass number changes only with an α decay for which the change is -4.

If the mass number is $4n$, then the new number is $4n - 4 = 4(n - 1) = 4n'$. Thus for each family, we have

$4n \rightarrow 4n - 4 \rightarrow 4n'$;

$4n + 1 \rightarrow 4n - 4 + 1 \rightarrow 4n' + 1$;

$4n + 2 \rightarrow 4n - 4 + 2 \rightarrow 4n' + 2$;

$4n + 3 \rightarrow 4n - 4 + 3 \rightarrow 4n' + 3$.

Thus the daughter nuclides are always in the same family.