CHAPTER 32: Elementary Particles

Answers to Questions

1. A reaction between two nucleons that would produce a π^- is: $p + n \rightarrow p + p + \pi^-$.

2. No, even if the proton has an extremely large kinetic energy (and, thus, an extremely large momentum), there is no way to conserve both energy and momentum during such a decay $\left(p \rightarrow n + \pi^+\right)$. Observed from the rest frame of the proton, the decay is energetically impossible, and so it is energetically impossible in every other reference frame as well. In the frame in which the proton is moving fast, the decay products must move very fast as well in order to conserve momentum, and with this constraint, there will still not be enough energy to make the decay energetically possible.

3. Antiatoms would be made up of antiprotons and antineutrons in the nucleus with positrons spinning around the nucleus in orbits. If antimatter and matter came into contact, all of the particle-antiparticle pairs would annihilate (convert matter and antimatter into energy) and release energy as gamma rays or other particles.

4. The photon is the particle that indicates the electromagnetic interaction.

5. Yes, if a neutrino is produced during a decay, the weak interaction is responsible. No, for example a weak interaction decay that does not produce a neutrino could produce a Z^0 instead.

6. Strong interaction forces are only seen at extremely short distances, but the rest of the time the weak interaction dominates. Also, the other decay products (electron and electron antineutrino) only interact via the weak force.

7. An electron takes part in the electromagnetic interaction (it is charged), the weak interaction, and the gravitational interaction (it has mass). A neutrino takes part in the weak interaction and the gravitational interaction (it has a small mass). A proton takes part in the strong interaction (baryon), the electromagnetic interaction (it is charged), the weak interaction, and the gravitational interaction (it has mass).

8. Charge and baryon number are conserved in the decays shown in Table 32-2. Here are a few examples.

$W^+ \rightarrow e^+ + \nu_e$:	All components have B = 0, and so baryon number is conserved. Charge is also conserved.
$\tau^- \rightarrow \mu^- + \overline{\nu}_\mu + \nu_\tau$:	All components have B = 0, and so baryon number is conserved. Since the neutrinos are not charged, charge is also conserved.
$n \rightarrow p + e^- + \overline{\nu}_e$:	The neutron and proton each have B = 1, and so baryon number is conserved. Since the charge on the left is equal to the charge on the right, charge is also conserved.

9. Since decays via the electromagnetic interaction are indicated by the production of photons, the decays in Table 32-2 that occur via the electromagnetic interaction are π^0, Σ^0, and η^0.
10. Since decays via the weak interaction are indicated by the production of neutrinos or W and Z bosons, the decays in Table 32-3 that occur via the weak interaction are: W, Z, muon, tau, pion, kaon, and neutron.

11. The decay times indicate that Σ^+ and Σ^- will decay via the weak interaction (although not to Λ^0). The decay times also indicate that the Σ^0 will decay via the electromagnetic interaction (verified by the production of a gamma).

12. Since the Δ baryon has B = 1, it must be made up of three quarks. Since the spin of the Δ baryon is 3/2, none of these quarks can be antiquarks. Thus, since the charges of quarks are either +2/3 or -1/3, the only charges we can create with this combination are q = -1 (= –1/3 – 1/3 – 1/3), 0 (= +2/3 – 1/3 – 1/3), +1 (= +2/3 + 2/3 – 1/3), and +2 (= +2/3 + 2/3 + 2/3).

13. Based on lifetimes shown in Table 32-4, the particle decays that occur via the electromagnetic interaction are J/ψ (3097) and Υ (9460).

14. All of the particles in Table 32-4, except for J/ψ (3097) and Υ (9460), decay via the weak interaction.

15. Baryons are made of a combination of three spin 1/2 quarks or spin -1/2 antiquarks. If a baryon has two quarks and one antiquark, or one quark and two antiquarks, then the magnitude of the spin will be 1/2. If a baryon has three quarks or three antiquarks, then the magnitude of the spin will be -1/2. If a meson has two quarks or two antiquarks, the magnitude of its spin will be 1. If a meson has a quark and an antiquark, the spin will be 0.

16. If a neutrinolet was massless it would not interact via the gravitation force; if it had no electrical charge it would not interact via the electromagnetic force; if it had no color charge it would not interact via the strong force; and if it does not interact via the weak force, then it would not interact with matter at all and it would be very difficult to say that it even exists at all. However, this neutrinolet would perhaps be identical to a photon.

17. (*a*) It is not possible for a particle to be both a lepton and a baryon. A lepton is an elementary particle, not composed of quarks, while baryons are made up of three quarks.
    (*b*) Yes, it is possible for a particle to be both a baryon and a hadron. All baryons are spin = 1/2 hadrons.
    (*c*) No, it is not possible for a particle to be both a meson and a quark. A meson is made up of two quarks.
    (*d*) No, it is not possible for a particle to be both a hadron and a lepton. A lepton is an elementary particle, while a hadron is made up of three quarks.

18. No, it is not possible to find a particle that is made up of two quarks and no antiquarks. First of all, this would give the particle a fractional charge, which has never been observed. Second, the "color" of the particle would not be "white". To have a "white" quark, requires a quark of one color and an antiquark with the corresponding anticolor.

    Yes, it is possible to find a particle that is made up of two quarks and two antiquarks. This combination of quarks/antiquarks would allow a particle to have a possible (non-fractional) charge and also allow it to be "white" in color. In Example 32-7, the comment is made that a π^0 is described as $\mathrm{u\bar{u}} + \mathrm{d\bar{d}}$.

19. In the nucleus, the strong interaction with the other nucleons does not allow the neutron to decay. When a neutron is free, the weak interaction is the dominant force and can cause the neutron to decay.

20. No, the reaction $e^- + p \rightarrow n + \bar{\nu}_e$ is not possible. The electron lepton number is not conserved: The reactants have $L_e = 1 + 0 = 1$, but the products have $L_e = 0 - 1 = -1$. Thus, this reaction is not possible.

21. The reaction $\Lambda^0 \rightarrow p^+ + e^- + \bar{\nu}_e$ proceeds via the weak force. We know this is the case since a neutrino is emitted.

Solutions to Problems

1. The total energy is given by Eq. 26-7a.

$$E = m_0c^2 + KE = 0.94\,GeV + 6.35\,GeV = \boxed{7.29\,GeV}$$

$\boxed{2.}$ Because the energy of the electrons is much greater than their rest mass, we have $KE = E = pc$. Combine that with Eq. 27-8 for the de Broglie wavelength.

$$E = pc\ ;\ p = \frac{h}{\lambda} \rightarrow E = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E} = \frac{\left(6.63\times10^{-34}\,J\bullet s\right)\left(3.0\times10^{8}\,m/s\right)}{\left(35\times10^{9}\,eV\right)\left(1.60\times10^{-19}\,J/eV\right)} = \boxed{3.6\times10^{-17}\,m}$$

$\boxed{3.}$ The frequency is related to the magnetic field in Eq. 32-2.

$$f = \frac{qB}{2\pi m} \rightarrow B = \frac{2\pi mf}{q} = \frac{2\pi\left(1.67\times10^{-27}\,kg\right)\left(2.8\times10^{7}\,Hz\right)}{1.60\times10^{-19}\,C} = \boxed{1.8\,T}$$

4. The time for one revolution is the period of revolution, which is the circumference of the orbit divided by the speed of the protons. Since the protons have very high energy, their speed is essentially the speed of light.

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(1.0\times10^{3}\,m\right)}{3.0\times10^{8}\,m/s} = \boxed{2.1\times10^{-5}\,s}$$

5. Use Eq. 32-2 to calculate the frequency.

$$f = \frac{qB}{2\pi m} = \frac{2\left(1.60\times10^{-19}\,C\right)\left(1.7\,T\right)}{2\pi\left[4\left(1.67\times10^{-27}\,kg\right)\right]} = \boxed{1.3\times10^{7}\,Hz} = 13\,MHz$$

6. (*a*) The maximum kinetic energy is $KE = \frac{q^2B^2R^2}{2m} = \frac{1}{2}mv^2$. Compared to Example 32-2, the charge has been doubled and the mass has been multiplied by 4. These two effects cancel each other in the equation, and so the maximum kinetic energy is unchanged.

$$\mathrm{KE}=\boxed{8.7\,\mathrm{MeV}}\qquad v=\sqrt{\frac{2\,\mathrm{KE}}{m}}=\sqrt{\frac{2\left(8.7\times10^{6}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}{4\left(1.66\times10^{-27}\,\mathrm{kg}\right)}}=\boxed{2.0\times10^{7}\,\mathrm{m/s}}$$

(*b*) The maximum kinetic energy is $\mathrm{KE}=\dfrac{q^2B^2R^2}{2m}=\frac{1}{2}mv^2$. Compared to Example 32-2, the charge is unchanged and the mass has been multiplied by 2. Thus the kinetic energy will be half of what it was in Example 32.2

$$\mathrm{KE}=\boxed{4.3\,\mathrm{MeV}}\qquad v=\sqrt{\frac{2\,\mathrm{KE}}{m}}=\sqrt{\frac{2\left(4.3\times10^{6}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}{\left(1.66\times10^{-27}\,\mathrm{kg}\right)}}=\boxed{2.0\times10^{7}\,\mathrm{m/s}}$$

The alpha and the deuteron have the same charge to mass ratio, and so move at the same speed.

(*c*) The frequency is given by $f=\dfrac{qB}{2\pi m}$. Since the charge to mass ratio of both the alpha and the deuteron is half that of the proton, the frequency for the alpha and the deuteron will both be half the frequency found in Example 32-2 for the proton.

$$\boxed{f=13\,\mathrm{MHz}}$$

7. From Eq. 30-1, the diameter of a nucleon is about $2.4\times10^{-15}\,\mathrm{m}$. The 30-MeV alpha particles and protons are not relativistic, so their momentum is given by $p=mv=\sqrt{2m\mathrm{KE}}$. The wavelength is given by Eq. 27-8, $\lambda=\dfrac{h}{p}=\dfrac{h}{\sqrt{2m\mathrm{KE}}}$.

$$\lambda_\alpha=\frac{h}{\sqrt{2m_\alpha\mathrm{KE}}}=\frac{6.63\times10^{-34}\,\mathrm{J{\bullet}s}}{\sqrt{2(4)\left(1.66\times10^{-27}\,\mathrm{kg}\right)\left(30\times10^{6}\,\mathrm{eV}\right)\left(1.6\times10^{-19}\,\mathrm{J/eV}\right)}}=2.63\times10^{-15}\,\mathrm{m}$$

$$\lambda_\mathrm{p}=\frac{h}{\sqrt{2m_\mathrm{p}\mathrm{KE}}}=\frac{6.63\times10^{-34}\,\mathrm{J{\bullet}s}}{\sqrt{2\left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(30\times10^{6}\,\mathrm{eV}\right)\left(1.6\times10^{-19}\,\mathrm{J/eV}\right)}}=5.24\times10^{-15}$$

The wavelength of the alpha particle is about the same as the diameter of a nucleon, while the wavelength of the proton is about twice the diameter of the nucleon. Thus the alpha particle will be better for picking out details in the nucleus.

8. The protons are accelerated twice during each revolution. During each acceleration the protons gain 55 keV of kinetic energy.

$$\frac{25\times10^{6}\,\mathrm{eV}}{2\left(55\times10^{3}\,\mathrm{eV/rev}\right)}=227\,\text{revolutions}\approx\boxed{230\,\text{revolutions}}$$

9. Because the energy of the protons is much greater than their rest mass, we have $\mathrm{KE}=E=pc$. Combine that with Eq. 27-8 for the de Broglie wavelength.

$$E=pc\ ;\ p=\frac{h}{\lambda}\ \rightarrow\ E=\frac{hc}{\lambda}\ \rightarrow\ \lambda=\frac{hc}{E}=\frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(3.0\times10^{8}\,\mathrm{m/s}\right)}{\left(7.0\times10^{12}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}=\boxed{1.8\times10^{-19}\,\mathrm{m}}$$

10. (*a*) The magnetic field is found from the maximum kinetic energy as derived in Example 32-2.

$$\text{KE}=\frac{q^2B^2R^2}{2m} \rightarrow B=\frac{\sqrt{2m\text{KE}}}{qR} \rightarrow$$

$$B=\frac{\sqrt{2(2.014)\left(1.66\times10^{-27}\text{kg}\right)\left(12\times10^{6}\text{eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}}{\left(1.60\times10^{-19}\text{C}\right)(1.0\text{m})}=0.7082\text{T}\approx\boxed{0.71\text{T}}$$

(*b*) The cyclotron frequency is given by Eq. 32-2.

$$f=\frac{qB}{2\pi m}=\frac{\left(1.60\times10^{-19}\text{C}\right)(0.7082\text{T})}{2\pi(2.014)\left(1.66\times10^{-27}\text{kg}\right)}=5.394\times10^{6}\text{Hz}\simeq\boxed{5.4\text{MHz}}$$

(*c*) The deuteron will be accelerated twice per revolution, and so will gain energy equal to twice its charge times the voltage on each revolution.

$$\text{number of revolutions}=n=\frac{12\times10^{6}\text{eV}}{2\left(1.60\times10^{-19}\text{C}\right)\left(22\times10^{3}\text{V}\right)}\left(1.60\times10^{-19}\text{ J/eV}\right)$$

$$=273\text{ revolutions}\approx\boxed{270\text{ revolutions}}$$

(*d*) The time is the number of revolutions divided by the frequency (which is revolutions per second).

$$\Delta t=\frac{n}{f}=\frac{273\text{ revolutions}}{5.394\times10^{6}\text{ rev/s}}=\boxed{5.1\times10^{-6}\text{s}}$$

(*e*) If we use an average radius of half the radius of the cyclotron, then the distance traveled is the average circumference times the number of revolutions.

$$\text{distance}=\tfrac{1}{2}2\pi rn=\pi(1.0\text{m})(273)=\boxed{860\text{m}}$$

11. Because the energy of the protons is much greater than their rest mass, we have $\text{KE}=E=pc$. A relationship for the magnetic field is given in section 32-1.

$$v=\frac{qBr}{m} \rightarrow mv=qBr \rightarrow p=qBr \rightarrow \frac{E}{c}=qBr \rightarrow$$

$$B=\frac{E}{qrc}=\frac{\left(7.0\times10^{15}\text{eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(4.25\times10^{3}\text{m}\right)\left(3.00\times10^{8}\text{ m/s}\right)}=\boxed{5.5\text{T}}$$

12. If the speed of the protons is *c*, then the time for one revolution is found from uniform circular motion. The number of revolutions is the total time divided by the time for one revolution. The energy per revolution is the total energy gained divided by the number of revolutions.

$$v=\frac{2\pi r}{T} \rightarrow T=\frac{2\pi r}{v}=\frac{2\pi r}{c} \qquad n=\frac{t}{T}=\frac{ct}{2\pi r}$$

$$\text{Energy/revolution}=\frac{\Delta E}{n}=\frac{(\Delta E)2\pi r}{ct}=\frac{\left(1.0\times10^{12}\text{eV}-150\times10^{9}\text{eV}\right)2\pi\left(1.0\times10^{3}\text{m}\right)}{\left(3.00\times10^{8}\text{ m/s}\right)(20\text{s})}$$

$$=8.9\times10^{5}\text{ eV/rev}=\boxed{0.89\text{MeV/rev}}$$

13. Start with an expression from section 32-1, with *q* replaced by *e*.

$$v=\frac{eBr}{m} \rightarrow mv=eBr \rightarrow p=eBr$$

In the relativistic limit, $p = E/c$ and so $\frac{E}{c} = eBr$. To put the energy in electron volts, divide the energy by the charge of the object.

$$\frac{E}{c} = eBr \rightarrow \boxed{\frac{E}{e} = Brc}$$

14. Because the energy of the protons is much greater than their rest mass, we have $\mathrm{KE} = E = pc$. Combine this with an expression from section 32-1.

$$v = \frac{qBr}{m} \rightarrow mv = qBr \rightarrow p = qBr \rightarrow \frac{E}{c} = qBr \rightarrow$$

$$B = \frac{E}{qrc} = \frac{\left(1.0\times10^{12}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(1.0\times10^{3}\,\mathrm{m}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)} = \boxed{3.3\,\mathrm{T}}$$

15. The energy released is the difference in the mass energy between the products and the reactant.

$$\Delta E = m_{\pi^+}c^2 - m_{\mu^+}c^2 - m_{\nu_\mu}c^2 = 139.6\,\mathrm{MeV} - 105.7\,\mathrm{MeV} - 0 = \boxed{33.9\,\mathrm{MeV}}$$

16. The energy released is the difference in the mass energy between the products and the reactant.

$$\Delta E = m_{\Lambda^0}c^2 - m_n c^2 - m_{\pi^0}c^2 = 1115.7\,\mathrm{MeV} - 939.6\,\mathrm{MeV} - 135.0\,\mathrm{MeV} = \boxed{41.1\,\mathrm{MeV}}$$

17. The energy required is the mass energy of the two particles.

$$E = 2m_n c^2 = 2\left(939.6\,\mathrm{MeV}\right) = \boxed{1879.2\,\mathrm{MeV}}$$

18. Use Eq. 32-3 to estimate the range of the force based on the mass of the mediating particle.

$$mc^2 \approx \frac{hc}{2\pi d} \rightarrow d \approx \frac{hc}{2\pi mc^2} = \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(3.0\times10^{8}\,\mathrm{m/s}\right)}{2\pi\left(497.7\times10^{6}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)} \approx \boxed{4.0\times10^{-16}\,\mathrm{m}}$$

19. Because the two protons are heading towards each other with the same speed, the total momentum of the system is 0. The minimum kinetic energy for the collision would result in all three particles at rest, and so the minimum kinetic energy of the collision must be equal to the mass energy of the π^0. Each proton will have half of that kinetic energy. From Table 32-2, the mass of the π^0 is $135.0\,\mathrm{MeV}/c^2$.

$$2\left(\mathrm{KE}_{\mathrm{proton}}\right) = m_{\pi^0}c^2 = 135.0\,\mathrm{MeV} \rightarrow \mathrm{KE}_{\mathrm{proton}} = \boxed{67.5\,\mathrm{MeV}}$$

20. Because the two neutrons are heading towards each other with the same speed, the total momentum of the system is 0. The minimum kinetic energy for the collision would result in all four particles at rest, and so the minimum kinetic energy of the collision must be equal to the mass energy of the $\mathrm{K^+K^-}$ pair. Each neutron will have half of that kinetic energy. From Table 32-2, the mass of each of the $\mathrm{K^+}$ and the $\mathrm{K^-}$ is $493.7\,\mathrm{MeV}/c^2$.

$$2\left(\mathrm{KE}_{\mathrm{neutron}}\right) = 2m_{\mathrm{K}}c^2 \rightarrow \mathrm{KE}_{\mathrm{neutron}} = m_{\mathrm{K}}c^2 = \boxed{493.7\,\mathrm{MeV}}$$

21. We use the average mass of $85\,\mathrm{GeV}/c^2$.

$$mc^2 \approx \frac{hc}{2\pi d} \rightarrow d \approx \frac{hc}{2\pi mc^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.0\times10^{8}\,\text{m/s}\right)}{2\pi\left(85\times10^{9}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} \approx \boxed{2.3\times10^{-18}\,\text{m}}$$

22. The energy of the two photons (assumed to be equal so that momentum is conserved) must be the combined rest mass energy of the proton and antiproton.

$$2m_0c^2 = 2hf = 2h\frac{c}{\lambda} \rightarrow \lambda = \frac{hc}{m_0c^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(938.3\times10^{6}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} \approx \boxed{1.32\times10^{-15}\,\text{m}}$$

23. (*a*) $\Lambda^0 \rightarrow \text{n} + \pi^-$ Charge conservation is violated, since $0 \neq 0-1$.
Strangeness is violated, since $-1 \neq 0+0$

(*b*) $\Lambda^0 \rightarrow \text{p} + \text{K}^-$ Energy conservation is violated, since
$1115.7\,\text{MeV}/c^2 < 938.3\,\text{MeV}/c^2 + 493.7\,\text{MeV}/c^2$

(*c*) $\Lambda^0 \rightarrow \pi^+ + \pi^-$ Baryon number conservation is violated, since $1 \neq 0+0$
Strangeness is violated, since $-1 \neq 0+0$

24. (*a*) The *Q*-value is the mass energy of the reactants minus the mass energy of the products.

$$Q = m_{\Lambda^0}c^2 - \left(m_\text{p}c^2 + m_{\pi^-}c^2\right) = 1115.7\,\text{MeV} - \left(938.3\,\text{MeV} + 139.6\,\text{MeV}\right) = \boxed{37.8\,\text{MeV}}$$

(*b*) Energy conservation for the decay gives the following.

$$m_{\Lambda^0}c^2 = E_\text{p} + E_{\pi^-} \rightarrow E_{\pi^-} = m_{\Lambda^0}c^2 - E_\text{p}$$

Momentum conservation says that the magnitudes of the momenta of the two products are equal. Then convert that relationship to energy using $E^2 = p^2c^2 + m_0^2c^4$, with energy conservation.

$$p_\text{p} = p_{\pi^-} \rightarrow \left(p_\text{p}c\right)^2 = \left(p_{\pi^-}c\right)^2 \rightarrow$$

$$E_\text{p}^2 - m_\text{p}^2c^4 = E_{\pi^-}^2 - m_{\pi^-}^2c^4 = \left(m_{\Lambda^0}c^2 - E_\text{p}\right)^2 - m_{\pi^-}^2c^4$$

$$E_\text{p}^2 - m_\text{p}^2c^4 = \left(m_{\Lambda^0}^2c^4 - 2E_\text{p}m_{\Lambda^0}c^2 + E_\text{p}^2\right) - m_{\pi^-}^2c^4 \rightarrow$$

$$E_\text{p} = \frac{m_{\Lambda^0}^2c^4 + m_\text{p}^2c^4 - m_{\pi^-}^2c^4}{2m_{\Lambda^0}c^2} = \frac{\left(1115.7\,\text{MeV}\right)^2 + \left(938.3\,\text{MeV}\right)^2 - \left(139.6\,\text{MeV}\right)^2}{2\left(1115.7\,\text{MeV}\right)} = 943.7\,\text{MeV}$$

$$E_{\pi^-} = m_{\Lambda^0}c^2 - E_\text{p} = 1115.7\,\text{MeV} - 943.7\,\text{MeV} = 172.0\,\text{MeV}$$

$$\text{KE}_\text{p} = E_\text{p} - m_\text{p}c^2 = 943.7\,\text{MeV} - 938.3\,\text{MeV} = \boxed{5.4\,\text{MeV}}$$

$$\text{KE}_{\pi^-} = E_{\pi^-} - m_{\pi^-}c^2 = 172.0\,\text{MeV} - 139.6\,\text{MeV} = \boxed{32.4\,\text{MeV}}$$

25. (*a*) We work in the rest frame of the isolated electron, so that it is initially at rest. Energy conservation gives the following.

$$m_\text{e}c^2 = \text{KE}_\text{e} + m_\text{e}c^2 + E_\gamma \rightarrow \text{KE}_\text{e} = -E_\gamma \rightarrow \text{KE}_\text{e} = E_\gamma = 0$$

Since the photon has no energy, it does not exist, and so has not been emitted.

(*b*) For the photon exchange in Figure 32-7, the photon exists for such a short time that the uncertainty principle allows energy to not be conserved during the exchange.

26. The total momentum of the electron and positron is 0, and so the total momentum of the two photons must be 0. Thus each photon has the same momentum, and so each photon also has the same energy. The total energy of the photons must be the total energy of the electron / positron pair.

$$E_{e^+/e^-\text{ pair}} = E_{\text{photons}} \rightarrow 2\left(m_0c^2 + \text{KE}\right) = 2hf = 2h\frac{c}{\lambda} \rightarrow$$

$$\lambda = \frac{hc}{m_0c^2 + \text{KE}} = \frac{\left(6.63\times10^{-34}\,\text{J•s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(0.511\times10^{6}\,\text{eV} + 420\times10^{3}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = 1.335\times10^{-12}\,\text{m} \approx \boxed{1.34\times10^{-12}\,\text{m}}$$

27. Since the pion decays from rest, the momentum before the decay is zero. Thus the momentum after the decay is also zero, and so the magnitudes of the momenta of the positron and the neutrino are equal. We also treat the neutrino as massless. Use energy and momentum conservation along with the relativistic relationship between energy and momentum.

$$m_{\pi^+}c^2 = E_{e^+} + E_\nu \ ; \ p_{e^+} = p_\nu \rightarrow \left(p_{e^+}^2c^2\right) = \left(p_\nu^2c^2\right) \rightarrow E_{e^+}^2 - m_{e^+}^2c^4 = E_\nu^2$$

$$E_{e^+}^2 - m_{e^+}^2c^4 = \left(m_{\pi^+}c^2 - E_{e^+}\right)^2 = m_{\pi^+}^2c^4 - 2E_{e^+}m_{\pi^+}c^2 + E_{e^+}^2 \rightarrow 2E_{e^+}m_{\pi^+}c^2 = m_{\pi^+}^2c^4 + m_{e^+}^2c^4$$

$$E_{e^+} = \tfrac{1}{2}m_{\pi^+}c^2 + \frac{m_{e^+}^2c^2}{2m_{\pi^+}} \rightarrow \text{KE}_{e^+} + m_{e^+}c^2 = \tfrac{1}{2}m_{\pi^+}c^2 + \frac{m_{e^+}^2c^2}{2m_{\pi^+}} \rightarrow$$

$$\text{KE}_{e^+} = \tfrac{1}{2}m_{\pi^+}c^2 - m_{e^+}c^2 + \frac{m_{e^+}^2c^2}{2m_{\pi^+}} = \tfrac{1}{2}\left(139.6\,\text{MeV}\right) - 0.511\,\text{MeV} + \frac{\left(0.511\,\text{MeV}/c^2\right)\left(0.511\,\text{MeV}\right)}{2\left(139.6\,\text{MeV}/c^2\right)}$$

$$= \boxed{69.3\,\text{MeV}}$$

28. (*a*) For the reaction $\pi^- + \text{p} \rightarrow \text{n} + \eta^0$, the conservation laws are as follows.

Charge: $-1+1=0+0$	Charge is conserved.
Baryon number: $0+1=1+0$	Baryon number is conserved.
Lepton number: $0+0=0+0$	Lepton number is conserved.
Strangeness: $0+0=0+0$	Strangeness is conserved.

The reaction is possible.

(*b*) For the reaction $\pi^+ + \text{p} \rightarrow \text{n} + \pi^0$, the conservation laws are as follows.

Charge: $1+1\neq0+0$	Charge is NOT conserved.

The reaction is forbidden, because charge is not conserved.

(*c*) For the reaction $\pi^+ + \text{p} \rightarrow \text{p} + \text{e}^+$, the conservation laws are as follows.

Charge: $1+1=1+1$	Charge is conserved.
Baryon number: $0+1=1+0$	Baryon number is conserved.
Lepton number: $0+0\neq0+1$	Lepton number is NOT conserved.

The reaction is forbidden, because lepton number is not conserved.

(*d*) For the reaction $\text{p} \rightarrow \text{e}^+ + \nu_e$, the conservation laws are as follows.

Charge: $1=1+0$	Charge is conserved.
Baryon number: $1\neq0+0$	Baryon number NOT conserved.

The reaction is forbidden, because baryon number is not conserved.

(*e*) For the reaction $\mu^+ \rightarrow \text{e}^+ + \overline{\nu}_\mu$, the conservation laws are as follows.

Charge: $1 = 1 + 0$ Charge is conserved.
Baryon number: $0 = 0 + 0$ Baryon number is conserved.
Electron lepton number: $0 \neq -1 + 0$ Lepton number is NOT conserved.

The reaction is forbidden, because lepton number is not conserved.

(*f*) For the reaction $\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$, the conservation laws are as follows.

Mass Energy: $938.3\,\mathrm{MeV}/c^2 < 939.6\,\mathrm{MeV}/c^2 + 0.511\,\mathrm{MeV}/c^2$

Mass energy is NOT conserved.

The reaction is forbidden, because energy is not conserved.

29. Since the Ξ^- decays from rest, the momentum before the decay is zero. Thus the momentum after the decay is also zero, and so the momenta of the Λ^0 and π^- are equal in magnitude. Use energy and momentum conservation along with the relativistic relationship between energy and momentum.

$$m_{\Xi^-}c^2 = E_{\Lambda^0} + E_{\pi^-} \rightarrow E_{\pi^-} = m_{\Xi^-}c^2 - E_{\Lambda^0}$$

$$p_{\Lambda^0} = p_{\pi^-} \rightarrow \left(p_{\Lambda^0}c\right)^2 = \left(p_{\pi^-}c\right)^2 \rightarrow$$

$$E_{\Lambda^0}^2 - m_{\Lambda^0}^2c^4 = E_{\pi^-}^2 - m_{\pi^-}^2c^4 = \left(m_{\Xi^-}c^2 - E_{\Lambda^0}\right)^2 - m_{\pi^-}^2c^4$$

$$E_{\Lambda^0}^2 - m_{\Lambda^0}^2c^4 = \left(m_{\Xi^-}^2c^4 - 2E_{\Lambda^0}m_{\Xi^-}c^2 + E_{\Lambda^0}^2\right) - m_{\pi^-}^2c^4 \rightarrow$$

$$E_{\Lambda^0} = \frac{m_{\Xi^-}^2c^4 + m_{\Lambda^0}^2c^4 - m_{\pi^-}^2c^4}{2m_{\Xi^-}c^2} = \frac{(1321.3\,\mathrm{MeV})^2 + (1115.7\,\mathrm{MeV})^2 - (139.6\,\mathrm{MeV})^2}{2(1321.3\,\mathrm{MeV})} = 1124.3\,\mathrm{MeV}$$

$$E_{\pi^-} = m_{\Xi^-}c^2 - E_{\Lambda^0} = 1321.3\,\mathrm{MeV} - 1124.3\,\mathrm{MeV} = 197.0\,\mathrm{MeV}$$

$$\mathrm{KE}_{\Lambda^0} = E_{\Lambda^0} - m_{\Lambda^0}c^2 = 1124.3\,\mathrm{MeV} - 1115.7\,\mathrm{MeV} = \boxed{8.6\,\mathrm{MeV}}$$

$$\mathrm{KE}_{\pi^-} = E_{\pi^-} - m_{\pi^-}c^2 = 197.0\,\mathrm{MeV} - 139.6\,\mathrm{MeV} = \boxed{57.4\,\mathrm{MeV}}$$

30. A π^+ could NOT be produced in the reaction $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{n} + \pi^+$. The pion has a mass energy of 139.6 MeV, and so the extra 100 MeV of energy could not create it. The *Q*-value for the reaction is $Q = 2m_{\mathrm{p}}c^2 - \left(2m_{\mathrm{p}}c^2 + m_{\pi^+}c^2\right) = -139.6\,\mathrm{MeV}$, and so more than 139.6 MeV of kinetic energy is needed. The minimum initial kinetic energy would produce the particles all moving together at the same speed, having the same total momentum as the incoming proton. We consider the products to be one mass $M = m_{\mathrm{p}} + m_{\mathrm{n}} + m_{\pi^+}$ since they all move together with the velocity. We use energy and momentum conservation, along with their relativistic relationship, $E^2 = p^2c^2 + m_0^2c^4$.

$$E_p + m_p c^2 = E_M \quad ; \quad p_p = p_M \rightarrow (p_p c)^2 = (p_M c)^2 \rightarrow E_p^2 - m_{\pi^-}^2 c^4 = E_M^2 - M^2 c^4 \rightarrow$$

$$E_p^2 - m_p^2 c^4 = (E_p + m_p c^2)^2 - M^2 c^4 = E_p^2 + 2E_p m_p c^2 + m_p^2 c^4 - M^2 c^4 \rightarrow$$

$$E_p = \frac{M^2 c^4 - 2m_p^2 c^4}{2m_p c^2} = \mathrm{KE}_p + m_p c^2 \rightarrow$$

$$\mathrm{KE}_{\pi^-} = \frac{M^2 c^4 - 2m_p^2 c^4}{2m_p c^2} - m_p c^2 = \frac{M^2 c^4}{2m_p c^2} - 2m_p c^2$$

$$= \frac{(938.3\,\mathrm{MeV} + 939.6\,\mathrm{MeV} + 139.6\,\mathrm{MeV})^2}{2(938.3\,\mathrm{MeV})} - 2(938.3\,\mathrm{MeV}) = \boxed{292.4\,\mathrm{MeV}}$$

31. The two neutrinos must move in the opposite direction of the electron in order for the electron to have the maximum kinetic energy, and thus the total momentum of the neutrinos will be equal in magnitude to the momentum of the electron. Since a neutrino is (essentially) massless, we have that $E_\nu = p_\nu c$. We assume that the muon is at rest when it decays. Use conservation of energy and momentum, along with their relativistic relationship.

$$p_{e^-} = p_{\bar{\nu}_e} + p_{\nu_\mu}$$

$$m_{\mu^-} c^2 = E_{e^-} + E_{\bar{\nu}_e} + E_{\nu_\mu} = E_{e^-} + p_{\bar{\nu}_e} c + p_{\nu_\mu} c = E_{e^-} + (p_{\bar{\nu}_e} + p_{\nu_\mu}) c = E_{e^-} + p_{e^-} c \rightarrow$$

$$m_{\mu^-} c^2 - E_{e^-} = p_{e^-} c \rightarrow (m_{\mu^-} c^2 - E_{e^-})^2 = (p_{e^-} c)^2 = E_{e^-}^2 - m_{e^-}^2 c^4 \rightarrow$$

$$m_{\mu^-}^2 c^4 - 2m_{\mu^-} c^2 E_{e^-} + E_{e^-}^2 = E_{e^-}^2 - m_{e^-}^2 c^4 \rightarrow E_{e^-} = \frac{m_{\mu^-}^2 c^4 + m_{e^-}^2 c^4}{2m_{\mu^-} c^2} = \mathrm{KE}_{e^-} + m_{e^-} c^2 \rightarrow$$

$$\mathrm{KE}_{e^-} = \frac{m_{\mu^-}^2 c^4 + m_{e^-}^2 c^4}{2m_{\mu^-} c^2} - m_{e^-} c^2 = \frac{(105.7\,\mathrm{MeV})^2 + (0.511\,\mathrm{MeV})^2}{2(105.7\,\mathrm{MeV})} - (0.511\,\mathrm{MeV}) = \boxed{52.3\,\mathrm{MeV}}$$

32. The width of the peak was measured with a ruler and found to be about 9.5 mm, while the 200 MeV spacing on the graph was measured to be about 14 mm. Use a proportion to estimate the energy width, and then use the uncertainty principle to estimate the lifetime.

$$\frac{\Delta E}{200\,\mathrm{MeV}} = \frac{9.5\,\mathrm{mm}}{14\,\mathrm{mm}} \rightarrow \Delta E = (200\,\mathrm{MeV})\left(\frac{9.5\,\mathrm{mm}}{14\,\mathrm{mm}}\right) = \boxed{136\,\mathrm{MeV}}$$

$$\Delta E \approx \frac{h}{2\pi\Delta t} \rightarrow \Delta t \approx \frac{h}{2\pi\Delta E} = \frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{2\pi(136\times10^6\,\mathrm{eV})\left(\frac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right)} = \boxed{4.85\times10^{-24}\,\mathrm{s}}$$

33. Apply the uncertainty principle, which says that $\Delta E \approx \frac{h}{2\pi\Delta t}$.

$$\Delta E \approx \frac{h}{2\pi\Delta t} \rightarrow \Delta t \approx \frac{h}{2\pi\Delta E} = \frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{2\pi(88\times10^3\,\mathrm{eV})\left(\frac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right)} = \boxed{7.5\times10^{-21}\,\mathrm{s}}$$

34. We estimate the lifetime from the energy width and the uncertainty principle.

$$\Delta E \approx \frac{h}{2\pi\Delta t} \rightarrow \Delta t \approx \frac{h}{2\pi\Delta E} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{2\pi\left(277\times10^{3}\,\text{eV}\right)\left(\frac{1.60\times10^{-19}\,\text{J}}{1\,\text{eV}}\right)} = \boxed{2.38\times10^{-21}\,\text{s}}$$

35. We find the energy width from the lifetime on Table 32-2 and the uncertainty principle.

(*a*) $\Delta t = 5\times10^{-19}\,\text{s} \quad \Delta E \approx \frac{h}{2\pi\Delta t} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{2\pi\left(5\times10^{-19}\,\text{s}\right)\left(\frac{1.60\times10^{-19}\,\text{J}}{1\,\text{eV}}\right)} = 1319\,\text{eV} \approx \boxed{1.3\,\text{keV}}$

(*b*) $\Delta t = 7.4\times10^{-20}\,\text{s} \quad \Delta E \approx \frac{h}{2\pi\Delta t} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{2\pi\left(7.4\times10^{-20}\,\text{s}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = 8912\,\text{eV} \approx \boxed{8.9\,\text{keV}}$

36. (*a*) For $\text{B}^- = \text{b}\,\overline{\text{u}}$, we have

Charge:	$-1 = -\frac{1}{3} - \frac{2}{3}$	Spin:	$0 = \frac{1}{2} - \frac{1}{2}$
Baryon number:	$0 = \frac{1}{3} - \frac{1}{3}$	Strangeness:	$0 = 0 + 0$
Charm:	$0 = 0 + 0$	Bottomness:	$-1 = -1 + 0$
Topness:	$0 = 0 + 0$		

(*b*) Because B^+ is the antiparticle of B^-, $\boxed{\text{B}^+ = \overline{\text{b}}\,\text{u}}$. The B^0 still must have a bottom quark, but must be neutral. Therefore $\boxed{\text{B}^0 = \text{b}\,\overline{\text{d}}}$. Because $\overline{\text{B}}^0$ is the antiparticle to B^0, we must have $\boxed{\overline{\text{B}}^0 = \overline{\text{b}}\,\text{d}}$.

37. (*a*) Charge: $(0) = (+1) + (-1)$ Charge is conserved.

Energy: $1314.9\,\text{Mev}/c^2 > 1189.4\,\text{Mev}/c^2 + 139.6\,\text{Mev}/c^2 = 1329\,\text{Mev}/c^2$

Energy is not conserved.

[The decay is not possible, because energy is not conserved.]

(*b*) Charge: $(-1) = (0) + (-1) + (0)$ Charge is conserved.

Energy: $1672.5\,\text{Mev}/c^2 > 1192.6\,\text{Mev}/c^2 + 139.6\,\text{Mev}/c^2 + 0\,\text{Mev}/c^2 = 1332.2\,\text{Mev}/c^2$

Energy is conserved.

Lepton number: $(0) = (0) + (0) + (1)$ Lepton number is not conserved.

[The decay is not possible, because lepton number is not conserved.]

(*c*) Charge: $(0) = (0) + (0) + (0)$ Charge is conserved.

Energy: $1192.6\,\text{Mev}/c^2 > 1115.7\,\text{Mev}/c^2 + 0\,\text{Mev}/c^2 + 0\,\text{Mev}/c^2 = 1115.7\,\text{Mev}/c^2$

Energy is conserved.

Lepton number: $(0) = (0) + (0) + (0)$ Lepton number is conserved.

Baryon number: $(0) = (0) + (0) + (0)$ Baryon number is conserved.

[The decay is possible.]

38. (*a*) The neutron has a baryon number of 1, so there must be three quarks. The charge must be 0, as

must be the strangeness, the charm, the bottomness, and the topness. Thus $\boxed{n = \mathrm{u\,u\,d}}$.

(*b*) The antineutron is the anti particle of the neutron, so $\boxed{\overline{n} = \overline{\mathrm{u}}\,\overline{\mathrm{u}}\,\overline{\mathrm{d}}}$.

(*c*) The Λ^0 has a strangeness of -1, so it must contain an "s" quark. It is a baryon, so it must contain three quarks. And it must have charge, charm, bottomness, and topness equal to 0. Thus $\boxed{\Lambda^0 = \mathrm{u\,d\,s}}$.

(*d*) The $\overline{\Sigma}^0$ has a strangeness of +1, so it must contain an $\overline{\mathrm{s}}$ quark. It is a baryon, so it must contain three quarks. And it must have charge, charm, bottomness, and topness equal to 0. Thus $\boxed{\overline{\Sigma}^0 = \overline{\mathrm{u}}\,\overline{\mathrm{d}}\,\overline{\mathrm{s}}}$.

39. (*a*) The combination $\mathrm{u\,u\,d}$ has charge = +1, baryon number = +1, and strangeness, charm, bottomness, and topness all equal to 0. Thus $\boxed{\mathrm{u\,u\,d} = \mathrm{p}}$.

(*b*) The combination $\overline{\mathrm{u}}\,\overline{\mathrm{u}}\,\overline{\mathrm{s}}$ has charge = -1, baryon number = -1, strangenss = +1, and charm, bottomness, and topness all equal to 0. Thus $\boxed{\overline{\mathrm{u}}\,\overline{\mathrm{u}}\,\overline{\mathrm{s}} = \overline{\Sigma}^-}$.

(*c*) The combination $\overline{\mathrm{u}}\,\mathrm{s}$ has charge = -1, baryon number = 0, strangeness = -1, and charm, bottomness, and topness all equal to 0. Thus $\boxed{\overline{\mathrm{u}}\,\mathrm{s} = \mathrm{K}^-}$.

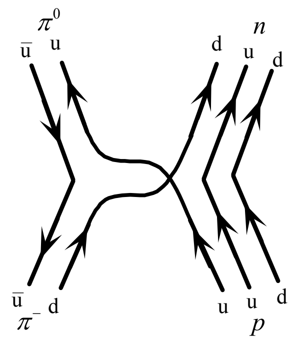
(*d*) The combination $\mathrm{d}\,\overline{\mathrm{u}}$ has charge = -1, baryon number = 0, and strangeness, charm, bottomness, and topness all equal to 0. Thus $\boxed{\mathrm{d}\,\overline{\mathrm{u}} = \pi^-}$

(*e*) The combination $\overline{\mathrm{c}}\,\mathrm{s}$ has charge = -1, baryon number = 0, strangeness = -1, charm = -1, and bottomness and topness of 0. Thus $\boxed{\overline{\mathrm{c}}\,\mathrm{s} = \mathrm{D}_S^-}$

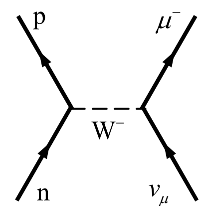
40. To form the D^0 meson, we must have a total charge of 0, a baryon number of 0, a strangeness of 0, and a charm of +1. We assume that there is no topness or bottomness. To get the charm, we must have a "c" quark, with a charge of $+\frac{2}{3}e$. To have a neutral meson, there must be another quark with a charge of $-\frac{2}{3}e$. To have a baryon number of 0, that second quark must be an antiquark. The only candidate with those properties is an anti-up quark. Thus $\boxed{D^0 = \mathrm{c}\,\overline{\mathrm{u}}}$.

41. To form the D_S^+ meson, we must have a total charge of +1, a baryon number of 0, a strangeness of +1, and a charm of +1. We assume that there is no topness or bottomness. To get the charm, we must have a "c" quark, with a charge of $+\frac{2}{3}e$. To have a total charge of +1, there must be another quark with a charge of $+\frac{1}{3}e$. To have a baryon number of 0, that second quark must be an antiquark. To have a strangeness of +1, the other quark must be an anti-strange. Thus $\boxed{D_S^+ = \mathrm{c}\,\overline{\mathrm{s}}}$.

42.



43. Since leptons are involved, the reaction $\mathrm{n} + \nu_\mu \rightarrow \mathrm{p} + \mu^-$ is a weak interaction. Since there is a charge change in the lepton, a W boson must be involved in the interaction.



44. The total energy is the sum of the kinetic energy and the mass energy. The wavelength is found from the relativistic momentum.

$$E = \mathrm{KE} + m_0c^2 = 25\times10^9\,\mathrm{eV} + 938\times10^6\,\mathrm{eV} = 2.59\times10^{10}\,\mathrm{eV} \approx \boxed{26\,\mathrm{GeV}}$$

$$\lambda = \frac{h}{p} = \frac{h}{\dfrac{\sqrt{E^2-\left(m_0c^2\right)^2}}{c}} = \frac{hc}{\sqrt{E^2-\left(m_0c^2\right)^2}}$$

$$= \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(3.00\times10^8\,\mathrm{m/s}\right)}{\sqrt{\left(2.59\times10^{10}\,\mathrm{eV}\right)^2-\left(938\times10^6\,\mathrm{eV}\right)^2}}\frac{1}{\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)} = \boxed{4.8\times10^{-17}\,\mathrm{m}}$$

45. (*a*) At an energy of 1.0 TeV, the protons are moving at practically the speed of light. From uniform circular motion we find the time for the protons to complete one revolution of the ring. Then the total charge that passes any point in the ring during that time is the charge of the entire group of stored protons. The current is then the total charge divided by the period.

$$v = \frac{2\pi R}{T} \rightarrow T = \frac{2\pi R}{v} = \frac{2\pi R}{c}$$

$$I = \frac{Ne}{T} = \frac{Nec}{2\pi R} = \frac{\left(5.0\times10^{13}\,\mathrm{protons}\right)\left(1.60\times10^{-19}\,\mathrm{C/proton}\right)\left(3.0\times10^8\,\mathrm{m/s}\right)}{2\pi\left(1.0\times10^3\,\mathrm{m}\right)} = \boxed{0.38\,\mathrm{A}}$$

(*b*) The 1.0 TeV is equal to the KE of the proton beam.

$$\mathrm{KE}_{\mathrm{beam}} = \mathrm{KE}_{\mathrm{car}} \rightarrow \mathrm{KE}_{\mathrm{beam}} = \tfrac{1}{2}mv^2 \rightarrow$$

$$v = \sqrt{\frac{2\mathrm{KE}_{\mathrm{beam}}}{m}} = \sqrt{\frac{2\left(1.0\times10^{12}\,\mathrm{eV/proton}\right)\left(5.0\times10^{13}\,\mathrm{protons}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}{1500\,\mathrm{kg}}} = 103\,\mathrm{m/s}$$

$$\approx \boxed{1.0\times10^2\,\mathrm{m/s}}$$

46. These protons will be moving at essentially the speed of light for the entire time of acceleration. The number of revolutions is the total gain in energy divided by the energy gain per revolution. Then the distance is the number of revolutions times the circumference of the ring, and the time is the distance of travel divided by the speed of the protons.

$$\text{N}=\frac{\Delta E}{\Delta E/\text{rev}}=\frac{\left(1.0\times10^{12}\,\text{eV}-150\times10^{9}\,\text{eV}\right)}{2.5\times10^{6}\,\text{eV/rev}}=3.4\times10^{5}\,\text{rev}$$

$$d=N\left(2\pi R\right)=\left(3.4\times10^{5}\right)2\pi\left(1.0\times10^{3}\,\text{m}\right)=2.136\times10^{9}\,\text{m}\approx\boxed{2.1\times10^{9}\,\text{m}}$$

$$t=\frac{d}{c}=\frac{2.136\times10^{9}\,\text{m}}{3.00\times10^{8}\,\text{m/s}}=\boxed{7.1\,\text{s}}$$

47. By assuming that the kinetic energy is approximately 0, the total energy released is the rest mass energy of the annihilating pair of particles.

(*a*) $E_{\text{total}}=2m_0c^2=2\left(0.511\,\text{MeV}\right)=\boxed{1.022\,\text{MeV}}$

(*b*) $E_{\text{total}}=2m_0c^2=2\left(938.3\,\text{MeV}\right)=\boxed{1876.6\,\text{MeV}}$

48. (*a*) For the reaction $\pi^- + \text{p} \rightarrow \text{K}^+ + \Sigma^-$, the conservation laws are as follows.

Charge: $-1+1=1-1$ Charge is conserved.
Baryon number: $0+1=0+1$ Baryon number is conserved.
Lepton number: $0+0=0+0$ Lepton number is conserved.
Strangeness: $0+0=1-1$ Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*b*) For the reaction $\pi^+ + \text{p} \rightarrow \text{K}^+ + \Sigma^+$, the conservation laws are as follows.

Charge: $1+1=1+1$ Charge is conserved.
Baryon number: $0+1=0+1$ Baryon number is conserved.
Lepton number: $0+0=0+0$ Lepton number is conserved.
Strangeness: $0+0=1-1$ Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*c*) For the reaction $\pi^- + \text{p} \rightarrow \Lambda^0 + \text{K}^0 + \pi^0$, the conservation laws are as follows.

Charge: $-1+1=0+0+0$ Charge is conserved.
Baryon number: $0+1=1+0+0$ Baryon number is conserved.
Lepton number: $0+0=0+0+0$ Lepton number is conserved.
Strangeness: $0+0=-1+1+0$ Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*d*) For the reaction $\pi^+ + \text{p} \rightarrow \Sigma^0 + \pi^0$, the conservation laws are as follows.

Charge: $1+1\neq0+0$ Charge is NOT conserved.

The reaction is not possible, because charge is not conserved.

(*e*) For the reaction $\pi^- + \text{p} \rightarrow \text{p} + \text{e}^- + \overline{\nu}_\text{e}$, the conservation laws are as follows.

Charge: $-1+1=1-1+0$ Charge is conserved.
Baryon number: $0+1=1+0+0$ Baryon number is conserved.
Lepton number: $0+0=0+1-1$ Lepton number is conserved.
Strangeness: $0+0=0+0+0$ Strangeness is conserved.

The reaction is possible, via the weak interaction.

49. (*a*) For the reaction $\pi^- + \mathrm{p} \rightarrow \mathrm{K}^0 + \mathrm{p} + \pi^0$, the conservation laws are as follows.

Charge: $-1+1 \neq 0+1+0$ — Charge is NOT conserved.

The reaction is not possible, because charge is not conserved.

(*b*) For the reaction $\mathrm{K}^- + \mathrm{p} \rightarrow \Lambda^0 + \pi^0$, the conservation laws are as follows.

Charge: $-1+1 = 0+0$ — Charge is conserved.
Spin: $0+\frac{1}{2} = \frac{1}{2}+0$ — Spin is conserved.
Baryon number: $0+1 = 1+0$ — Baryon number is conserved.
Lepton number: $0+0 = 0+0$ — Lepton number is conserved.
Strangeness: $-1+0 = -1+0$ — Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*c*) For the reaction $\mathrm{K}^+ + \mathrm{n} \rightarrow \Sigma^+ + \pi^0 + \gamma$, the conservation laws are as follows.

Charge: $1+0 = 1+0+0$ — Charge is conserved.
Spin: $0+\frac{1}{2} = -\frac{1}{2}+0+1$ — Spin is conserved.
Baryon number: $0+1 = 1+0+0$ — Baryon number is conserved.
Lepton number: $0+0 = 0+0+0$ — Lepton number is conserved.
Strangeness: $1+0 \neq -1+0+0$ — Strangeness is NOT conserved.

The reaction is not possible via the strong interaction because strangeness is not conserved. It is possible via the weak interaction.

(*d*) For the reaction $\mathrm{K}^+ \rightarrow \pi^0 + \pi^0 + \pi^+$, the conservation laws are as follows.

Charge: $1 = 0+0+1$ — Charge is conserved.
Spin: $0 = 0+0+0$ — Spin is conserved.
Baryon number: $0 = 0+0+0$ — Baryon number is conserved.
Lepton number: $0 = 0+0+0$ — Lepton number is conserved.
Strangeness: $1 \neq 0+0+0$ — Strangeness is NOT conserved.

The reaction is not possible via the strong interaction because strangeness is not conserved. It is possible via the weak interaction.

(*e*) For the reaction $\pi^+ \rightarrow \mathrm{e}^+ + \nu_\mathrm{e}$, the conservation laws are as follows.

Charge: $1 = 1+0$ — Charge is conserved.
Spin: $0 = -\frac{1}{2}+\frac{1}{2}$ — Spin is conserved.
Baryon number: $0 = 0+0$ — Baryon number is conserved.
Lepton number: $0 = -1+1$ — Lepton number is conserved.
Strangeness: $0+0 = 0+0+0$ — Strangeness is conserved.

The reaction is possible, via the weak interaction.

50. The π^- is the anti-particle of the π^+, so the reaction is $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$. The conservation rules are as follows.

Charge: $-1 = -1+0$ — Charge is conserved.
Baryon number: $0 = 0+0$ — Baryon number is conserved.
Lepton number: $0 = 1-1$ — Lepton number is conserved.
Strangeness: $0 = 0+0$ — Strangeness is conserved.

Spin: $0 = \frac{1}{2} - \frac{1}{2}$ Spin is conserved

51. Use Eq. 32-3 to estimate the mass of the particle based on the given distance.

$$mc^2 \approx \frac{hc}{2\pi d} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.0\times10^{8}\,\text{m/s}\right)}{2\pi\left(10^{-18}\,\text{m}\right)}\left(\frac{1}{1.60\times10^{-19}\,\text{J/eV}}\right) = 1.98\times10^{11}\,\text{eV} \approx \boxed{200\,\text{GeV}}$$

This value is of the same order of magnitude as the mass of the $W^{\pm}$.

52. We assume that the interaction happens essentially at rest, so that there is no initial kinetic energy or momentum. Thus the momentum of the neutron and the momentum of the π^0 will have the same magnitude. From energy conservation we find the total energy of the π^0.

$$m_{\pi^-}c^2 + m_{\text{p}}c^2 = E_{\pi^0} + m_{\text{n}}c^2 + \text{KE}_{\text{n}} \rightarrow$$

$$E_{\pi^0} = m_{\pi^-}c^2 + m_{\text{p}}c^2 - \left(m_{\text{n}}c^2 + \text{KE}_{\text{n}}\right) = 139.6\,\text{MeV} + 938.3\,\text{MeV} - \left(939.6\,\text{MeV} + 0.60\,\text{MeV}\right)$$

$$= 137.7\,\text{MeV}$$

From momentum conservation, we can find the mass energy of the π^0. We utilize Eq. 26-10 to relate momentum and energy.

$$p_{\text{n}} = p_{\pi^0} \rightarrow \left(p_{\text{n}}c\right)^2 = \left(p_{\pi^0}c\right)^2 \rightarrow E_{\text{n}}^2 - m_{\text{n}}^2c^4 = E_{\pi^0}^2 - m_{\pi^0}^2c^4 \rightarrow m_{\pi^0}^2c^4 = E_{\pi^0}^2 - E_{\text{n}}^2 + m_{\text{n}}^2c^4 \rightarrow$$

$$m_{\pi^0}c^2 = \sqrt{E_{\pi^0}^2 - E_{\text{n}}^2 + m_{\text{n}}^2c^4} = \left[\left(137.7\,\text{MeV}\right)^2 - \left(939.6\,\text{MeV} + 0.60\,\text{MeV}\right)^2 + \left(939.6\,\text{MeV}\right)^2\right]^{1/2}$$

$$= 133.5\,\text{MeV} \rightarrow m_{\pi^0} = \boxed{133.5\,\text{MeV}/c^2}$$

The reference value is $133.5\,\text{MeV}$.

53. The Q-value is the mass energy of the reactants minus the mass energy of the products.
For the first reaction, $\text{p} + \text{p} \rightarrow \text{p} + \text{p} + \pi^0$:

$$Q = 2m_{\text{p}}c^2 - \left(2m_{\text{p}}c^2 + m_{\pi^0}c^2\right) = -m_{\pi^0}c^2 = \boxed{-135.0\,\text{MeV}}$$

For the second reaction, $\text{p} + \text{p} \rightarrow \text{p} + \text{n} + \pi^+$:

$$Q = 2m_{\text{p}}c^2 - \left(m_{\text{p}}c^2 + m_{\text{n}}c^2 + m_{\pi^+}c^2\right) = m_{\text{p}}c^2 - m_{\text{n}}c^2 - m_{\pi^+}c^2a$$

$$= 938.3\,\text{MeV} - 939.6\,\text{MeV} - 139.6\,\text{MeV} = \boxed{-140.9\,\text{MeV}}$$

54. The Q-value is the mass energy of the reactants minus the mass energy of the products.

$$Q = m_{\pi^-}c^2 + m_{\text{p}}c^2 - \left(m_{\Lambda^0}c^2 + m_{\text{K}^0}c^2\right) = 139.6\,\text{MeV} + 938.3\,\text{MeV} - \left(1115.7\,\text{MeV} + 497.7\,\text{MeV}\right)$$

$$= \boxed{-535.5\,\text{MeV}}$$

We consider the products to be one mass $M = m_{\Lambda^0} + m_{\text{K}^0} = 1613.4\,\text{MeV}/c^2$ since they both have the same speed. Energy conservation gives the following: $E_{\pi^-} + m_{\text{p}}c^2 = E_M$. Momentum conservation says that the incoming momentum is equal to the outgoing momentum. Then convert that relationship to energy using the relativistic relationship that $E^2 = p^2c^2 + m_0^2c^4$.

$$p_{\pi^-} = p_{\mathrm{M}} \rightarrow \left(p_{\pi^-}c\right)^2 = \left(p_{\mathrm{M}}c\right)^2 \rightarrow E_{\pi^-}^2 - m_{\pi^-}^2 c^4 = E_M^2 - M^2c^4 \rightarrow$$

$$E_{\pi^-}^2 - m_{\pi^-}^2 c^4 = \left(E_{\pi^-} + m_{\mathrm{p}}c^2\right)^2 - M^2c^4 = E_{\pi^-}^2 + 2E_{\pi^-}m_{\mathrm{p}}c^2 + m_{\mathrm{p}}^2c^4 - M^2c^4 \rightarrow$$

$$E_{\pi^-} = \frac{M^2c^4 - m_{\pi^-}^2c^4 - m_{\mathrm{p}}^2c^4}{2m_{\mathrm{p}}c^2} = \mathrm{KE}_{\pi^-} + m_{\pi^-}c^2 \rightarrow$$

$$\mathrm{KE}_{\pi^-} = \frac{M^2c^4 - m_{\pi^-}^2c^4 - m_{\mathrm{p}}^2c^4}{2m_{\mathrm{p}}c^2} - m_{\pi^-}c^2$$

$$= \frac{\left(1613.4\,\mathrm{MeV}\right)^2 - \left(139.6\,\mathrm{MeV}\right)^2 - \left(938.3\,\mathrm{MeV}\right)^2}{2\left(938.3\,\mathrm{MeV}\right)} - \left(139.6\,\mathrm{MeV}\right) = \boxed{768.0\,\mathrm{MeV}}$$

55. The fundamental fermions are the quarks and electrons. In a water molecule there are 2 hydrogen atoms consisting of one electron and one proton each, and 1 oxygen atom, consisting of 8 electrons, 8 protons, and 8 neutrons. Thus there are 18 nucleons, consisting of 3 quarks each, and 10 electrons. The total number of fermions is thus $18\times3+10=\boxed{64\text{ fermions}}$.

56. Since there is no initial momentum, the final momentum must add to zero. Thus each of the pions must have the same momentum, and therefore the same kinetic energy. Use energy conservation to find the kinetic energy of each pion.

$$2m_{\mathrm{p}}c^2 = 2\mathrm{KE}_\pi + 2m_\pi c^2 \rightarrow \mathrm{KE}_\pi = m_{\mathrm{p}}c^2 - m_\pi c^2 = 938.3\,\mathrm{MeV} - 139.6\,\mathrm{MeV} = \boxed{798.7\,\mathrm{MeV}}$$

57. (*a*) First, from the uncertainty principle, Eq. 28-1. The energy is so high that we assume $E = pc$.

$$\Delta x\Delta p \approx \frac{h}{2\pi} \rightarrow \Delta x\frac{\Delta E}{c} \approx \frac{h}{2\pi} \rightarrow$$

$$\Delta E \approx \frac{hc}{2\pi\Delta x} = \frac{\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)}{2\pi\left(10^{-32}\,\mathrm{m}\right)}\frac{\left(1\,\mathrm{GeV}/10^9\,\mathrm{eV}\right)}{\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)} = \boxed{2.0\times10^{16}\,\mathrm{GeV}}$$

Second, from de Broglie's wavelength formula. We take the de Broglie wavelength as the unification distance.

$$\lambda = \frac{h}{p} = \frac{h}{Ec} \rightarrow$$

$$E = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)}{\left(10^{-32}\,\mathrm{m}\right)}\frac{\left(1\,\mathrm{GeV}/10^9\,\mathrm{eV}\right)}{\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)} = \boxed{3.1\times10^{15}\,\mathrm{GeV}}$$

Both energies are reasonably close to $10^{16}\,\mathrm{GeV}$. This energy is the amount that could be violated in conservation of energy if the universe were the size of the unification distance.

(*b*) From Eq. 13-8, we have $E = \frac{3}{2}kT$.

$$E = \tfrac{3}{2}kT \rightarrow T = \frac{2E}{3k} = \frac{2\left(10^{25}\,\mathrm{eV}\right)\left(1.6\times10^{-19}\,\mathrm{J/eV}\right)}{3\left(1.38\times10^{-23}\,\mathrm{J/K}\right)} = 7.7\times10^{28}\,\mathrm{K} \cong \boxed{10^{29}\,\mathrm{K}}$$

58. The *Q*-value is the energy of the reactants minus the energy of the products. We assume that one of the initial protons is at rest, and that all four final particles have the same speed and therefore the

same kinetic energy, since they all have the same mass. We consider the products to be one mass $M = 4m_p$ since they all have the same speed.

$$Q = 2m_pc^2 - 4m_pc^2 = 2m_pc^2 - Mc^2 = -2m_pc^2$$

Energy conservation gives the following, where KE_{th} is the threshold energy.

$$\left(\text{KE}_{th} + m_pc^2\right) + m_pc^2 = E_M = \text{KE}_M + Mc^2$$

Momentum conservation says that the incoming momentum is equal to the outgoing momentum. Then convert that relationship to energy using the relativistic relationship that $E^2 = p^2c^2 + m_0^2c^4$.

$$p_p = p_M \rightarrow \left(p_pc\right)^2 = \left(p_Mc\right)^2 \rightarrow \left(\text{KE}_{th} + m_pc^2\right)^2 - m_p^2c^4 = \left(\text{KE}_M + Mc^2\right)^2 - M^2c^4 \rightarrow$$

$$\text{KE}_{th}^2 + 2\text{KE}_{th}m_pc^2 + m_p^2c^4 - m_p^2c^4 = \text{KE}_{th}^2 + 4\text{KE}_{th}m_pc^2 + 4m_p^2c^4 - \left(4m_p\right)^2c^4 \rightarrow$$

$$2\text{KE}_{th}m_pc^2 = 4\text{KE}_{th}m_pc^2 + 4m_p^2c^4 - 16m_p^2c^4 \rightarrow 2\text{KE}_{th}m_pc^2 = 12m_p^2c^4 \rightarrow$$

$$\text{KE}_{th} = 6m_pc^2 = 3|Q|$$

59. To find the length in the lab, we need to know the speed of the particle which is moving relativistically. Start with Eq. 26-6.

$$\text{KE} = m_0c^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) \rightarrow v = c\sqrt{1 - \frac{1}{\left(\frac{\text{KE}}{m_0c^2} + 1\right)^2}} = c\sqrt{1 - \frac{1}{\left(\frac{450\times10^6\,\text{eV}}{1777\times10^6\,\text{eV}} + 1\right)^2}} = 0.603c$$

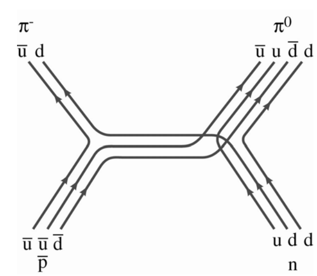
$$\Delta t_{lab} = \frac{\Delta t_0}{\sqrt{1-v^2/c^2}} = \frac{2.91\times10^{-13}\,\text{s}}{\sqrt{1-(0.603)^2}} = 3.65\times10^{-13}\,\text{s}$$

$$\Delta x_{lab} = v\Delta t_{lab} = (0.603)\left(3.00\times10^8\,\text{m/s}\right)\left(3.65\times10^{-13}\,\text{s}\right) = \boxed{6.60\times10^{-5}\,\text{m}}$$

60. (*a*) To conserve charge, the missing particle must be neutral. To conserve baryon number, the missing particle must be a meson. To conserve strangeness, charm, topness, and bottomness, the missing particle must be made of up and down quarks and antiquarks only. With all this information, the missing particle is $\boxed{\pi^0 = \text{u}\,\overline{\text{u}} + \text{d}\,\overline{\text{d}}}$.

(*b*) This is a weak interaction since one product is a lepton. To conserve charge, the missing particle must be neutral. To conserve the muon lepton number, the missing particle must be an antiparticle in the muon family. With this information, the missing particle is $\boxed{\overline{\nu}_\mu}$.

61. One $\overline{\text{u}}$ quark from the antiproton joins with a d quark from the neutron to make the π^-. The other four quarks form the π^0.



62. A relationship between total energy and speed is given by Eq. 26-8.

$$E=\frac{m_0c^2}{\sqrt{1-v^2/c^2}} \rightarrow \sqrt{1-v^2/c^2}=\frac{m_0c^2}{E} \rightarrow 1-\frac{v^2}{c^2}=\left(\frac{m_0c^2}{E}\right)^2 \rightarrow$$

$$\frac{v}{c}=\sqrt{1-\left(\frac{m_0c^2}{E}\right)^2}=\sqrt{1-\left(\frac{9.38\times10^8\,\text{eV}}{7.0\times10^{12}\text{eV}}\right)^2}=1\ (\text{to 8 digits}) \rightarrow \boxed{v=c}$$

Use the binomial expansion to express the answer differently.

$$\frac{v}{c}=\sqrt{1-\left(\frac{9.38\times10^8\,\text{eV}}{7.0\times10^{12}\text{eV}}\right)^2}\cong1-\tfrac{1}{2}\left(\frac{9.38\times10^8\,\text{eV}}{7.0\times10^{12}\text{eV}}\right)^2=1-9.0\times10^9 \rightarrow v=c\left(1-9.0\times10^9\right)$$