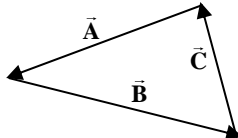# CHAPTER 3: Kinematics in Two Dimensions; Vectors

Answers to Questions

1. Their velocities are NOT equal, because the two velocities have different directions.

2. (*a*) During one year, the Earth travels a distance equal to the circumference of its orbit, but has a displacement of 0 relative to the Sun.
   (*b*) The space shuttle travels a large distance during any flight, but the displacement from one launch to the next is 0.
   (*c*) Any kind of cross country "round trip" air travel would result in a large distance traveled, but a displacement of 0.
   (*d*) The displacement for a race car from the start to the finish of the Indy 500 auto race is 0.

3. The displacement can be thought of as the "straight line" path from the initial location to the final location. The length of path will always be greater than or equal to the displacement, because the displacement is the shortest distance between the two locations. Thus the displacement can never be longer than the length of path, but it can be less. For any path that is not a single straight line segment, the length of path will be longer than the displacement.

4. Since both the batter and the ball started their motion at the same location (where the ball was hit) and ended their motion at the same location (where the ball was caught), the displacement of both was the same.

5. The magnitude of the vector sum need not be larger than the magnitude of either contributing vector. For example, if the two vectors being added are the exact opposite of each other, the vector sum will have a magnitude of 0. The magnitude of the sum is determined by the angle between the two contributing vectors.

6. If the two vectors are in the same direction, the magnitude of their sum will be a maximum, and will be 7.5 km. If the two vectors are in the opposite direction, the magnitude of their sum will be a minimum, and will be 0.5 km. If the two vectors are oriented in any other configuration, the magnitude of their sum will be between 0.5 km and 7.5 km.

7. Two vectors of unequal magnitude can never add to give the zero vector. However, three vectors of unequal magnitude can add to give the zero vector. If their geometric sum using the tail-to-tip method gives a closed triangle, then the vector sum will be zero. See the diagram, in which $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$

8. (*a*) The magnitude of a vector can equal the length of one of its components if the other components of the vector are all 0; i.e. if the vector lies along one of the coordinate axes.
   (*b*) The magnitude of a vector can never be less than one of its components, because each component contributes a positive amount to the overall magnitude, through the Pythagorean relationship. The square root of a sum of squares is never less than the absolute value of any individual term.

9. A particle with constant speed can be accelerating, if its direction is changing. Driving on a curved roadway at constant speed would be an example. However, a particle with constant velocity cannot be accelerating – its acceleration must be zero. It has both constant speed and constant direction.

10. To find the initial speed, use the slingshot to shoot the rock directly horizontally (no initial vertical speed) from a height of 1 meter. The vertical displacement of the rock can be related to the time of flight by Eq. 2-11b. Take downward to be positive.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}at^2 \quad \rightarrow \quad 1\,\text{m} = \tfrac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{2(1\,\text{m})\big/(9.8\,\text{m/s}^2)} = 0.45\,\text{s}\,.$$

Measure the horizontal range R of the rock with the meter stick. Then, if we measure the horizontal range R, we know that $R = v_x t = v_x(0.45\,\text{s})$, and so $v_x = R/0.45\,\text{s}$. The only measurements are the height of fall and the range, both of which can be measured by a meter stick.

11. Assume that the bullet was fired from behind and below the airplane. As the bullet rose in the air, its vertical speed would be slowed by both gravity and air resistance, and its horizontal speed would be slowed by air resistance. If the altitude of the airplane was slightly below the maximum height of the bullet, then at the altitude of the airplane, the bullet would be moving quite slowly in the vertical direction. If the bullet's horizontal speed had also slowed enough to approximately match the speed of the airplane, then the bullet's velocity relative to the airplane would be small. With the bullet moving slowly, it could safely be caught by hand.

12. The moving walkway will be moving at the same speed as the "car". Thus, if you are on the walkway, you are moving the same speed as the car. Your velocity relative to the car is 0, and it is easy to get into the car. But it is very difficult to keep your balance while trying to sit down into a moving car from a stationary platform. It is easier to keep your balance by stepping on to the moving platform while walking, and then getting into the car with a velocity of 0 relative to the car.

13. Your reference frame is that of the train you are riding. If you are traveling with a relatively constant velocity (not over a hill or around a curve or drastically changing speed), then you will interpret your reference frame as being at rest. Since you are moving forward faster than the other train, the other train is moving backwards relative to you. Seeing the other train go past your window from front to rear makes it look like the other train is going backwards. This is similar to passing a semi truck on the interstate – out of a passenger window, it looks like the truck is going backwards.

14. When you stand still under the umbrella in a vertical rain, you are in a cylinder-shaped volume in which there is no rain. The rain has no horizontal component of velocity, and so the rain cannot move from outside that cylinder into it. You stay dry. But as you run, you have a forward horizontal velocity relative to the rain, and so the rain has a backwards horizontal velocity relative to you. It is the same as if you were standing still under the umbrella but the rain had some horizontal component of velocity towards you. The perfectly vertical umbrella would not completely shield you.

15. (*a*) The ball lands at the same point from which it was thrown inside the train car – back in the thrower's hand.
    (*b*) If the car accelerates, the ball will land behind the point from which it was thrown.
    (*c*) If the car decelerates, the ball will land in front of the point from which it was thrown.
    (*d*) If the car rounds a curve (assume it curves to the right), then the ball will land to the left of the point from which it was thrown.
    (*e*) The ball will be slowed by air resistance, and so will land behind the point from which it was thrown.

16. Both rowers need to cover the same "cross river" distance. The rower with the greatest speed in the "cross river" direction will be the one that reaches the other side first. The current has no bearing on the problem because the current doesn't help either of the boats move across the river. Thus the rower heading straight across will reach the other side first. All of his rowing effort has gone into

crossing the river. For the upstream rower, some of his rowing effort goes into battling the current, and so his "cross river" speed will be only a fraction of his rowing speed.

17. The baseball is hit and caught at approximately the same height, and so the range formula of $R = v_0^2 \sin 2\theta_0 / g$ is particularly applicable. Thus the baseball player is judging the initial speed of the ball and the initial angle at which the ball was hit.

18. The arrow should be aimed above the target, because gravity will deflect the arrow downward from a horizontal flight path. The angle of aim (above the horizontal) should increase as the distance from the target increases, because gravity will have more time to act in deflecting the arrow from a straight-line path. If we assume that the arrow when shot is at the same height as the target, then the range formula is applicable: $R = v_0^2 \sin 2\theta_0 / g \rightarrow \theta = \frac{1}{2}\sin^{-1}\left(Rg / v_0^2\right)$. As the range and hence the argument of the inverse sine function increases, the angle increases.

19. The horizontal component of the velocity stays constant in projectile motion, assuming that air resistance is negligible. Thus the horizontal component of velocity 1.0 seconds after launch will be the same as the horizontal component of velocity 2.0 seconds after launch. In both cases the horizontal velocity will be given by $v_x = v_0 \cos \theta = \left(30\,\text{m/s}\right)\left(\cos 30^\circ\right) = 26\,\text{m/s}$.

20. (*a*) Cannonball A, with the larger angle, will reach a higher elevation. It has a larger initial vertical velocity, and so by Eq. 2-11c, will rise higher before the vertical component of velocity is 0.
     (*b*) Cannonball A, with the larger angle, will stay in the air longer. It has a larger initial vertical velocity, and so takes more time to decelerate to 0 and start to fall.
     (*c*) The cannonball with a launch angle closest to 45° will travel the farthest. The range is a maximum for a launch angle of 45°, and decreases for angles either larger or smaller than 45°.
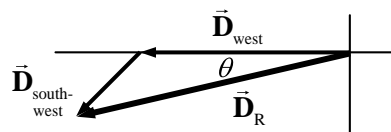
Solutions to Problems

1. The resultant vector displacement of the car is given by $\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_{\text{west}} + \vec{\mathbf{D}}_{\text{south-west}}$. The westward displacement is $215 + 85 \cos 45^\circ = 275.1\,\text{km}$ and the south displacement is $85 \sin 45^\circ = 60.1\,\text{km}$. The resultant displacement has a magnitude of $\sqrt{275.1^2 + 60.1^2} = 281.6\,\text{km}$ $\approx \boxed{282\,\text{km}}$. The direction is $\theta = \tan^{-1} 60.1 / 275.1 = 12.3^\circ \approx \boxed{12^\circ \text{ south of west}}$.

2. The truck has a displacement of $18 + \left(-16\right) = 2$ blocks north and 10 blocks east. The resultant has a magnitude of $\sqrt{2^2 + 10^2} = \boxed{10 \text{ blocks}}$ and a direction of $\tan^{-1} 2 / 10 = \boxed{11^\circ \text{ north of east}}$.
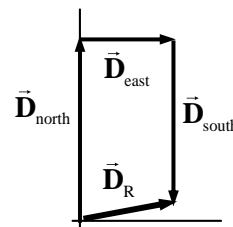
3. Label the "INCORRECT" vector as vector $\vec{\mathbf{X}}$. Then Fig. 3-6 (c) illustrates the relationship $\vec{\mathbf{V}}_1 + \vec{\mathbf{X}} = \vec{\mathbf{V}}_2$ via the tail-to-tip method. Thus $\vec{\mathbf{X}} = \boxed{\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1}$.
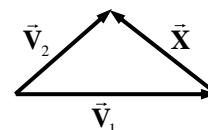
4. Given that $V_x = 6.80$ units and $V_y = -7.40$ units, the magnitude of $\vec{V}$ is

given by $V = \sqrt{V_x^2 + V_y^2} = \sqrt{6.80^2 + (-7.40)^2} = \boxed{10.0 \text{ units}}$. The direction

is given by an angle of $\theta = \tan^{-1}\dfrac{-7.40}{6.80} = \boxed{-47°}$, or $47°$ below the positive x-

axis.
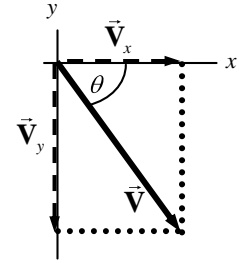
5. The vectors for the problem are drawn approximately to scale. The resultant has a length of $\boxed{58 \text{ m}}$ and a direction $\boxed{48°}$ north of east. If calculations are done, the actual resultant should be 57.4 m at $47.5°$ north of east.
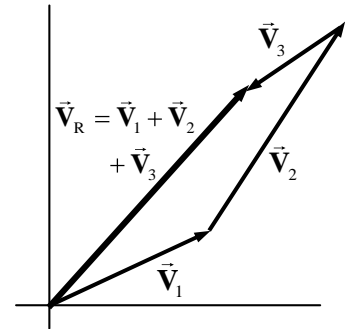
6. The sum is found by adding the components of vectors $\vec{V}_1$ and $\vec{V}_2$

$$\vec{V} = \vec{V}_1 + \vec{V}_2 = (8.0, -3.7, 0.0) + (3.9, -8.1, -4.4) = \boxed{(11.9, -11.8, -4.4)}$$
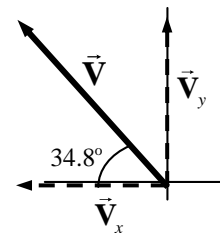
$$V = |\vec{V}| = \sqrt{(11.9)^2 + (-11.8)^2 + (-4.4)^2} = \boxed{17.3}$$

7. (*a*) See the accompanying diagram

   (*b*) $V_x = -14.3\cos 34.8° = \boxed{-11.7 \text{ units}}$ $V_y = 14.3\sin 34.8° = \boxed{8.16 \text{ units}}$

   (*c*) $V = \sqrt{V_x^2 + V_y^2} = \sqrt{(-11.7)^2 + (8.16)^2} = \boxed{14.3 \text{ units}}$

   $\theta = \tan^{-1}\dfrac{8.16}{11.7} = \boxed{34.8° \text{ above the } -x \text{ axis}}$

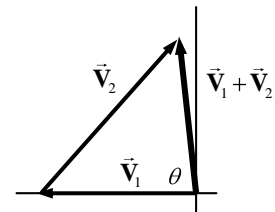8. (*a*) $V_{1x} = \boxed{-6.6 \text{ units}}$ $V_{1y} = \boxed{0 \text{ units}}$

   $V_{2x} = 8.5\cos 45° = \boxed{6.0 \text{ units}}$ $V_{2y} = 8.5\sin 45° = \boxed{6.0 \text{ units}}$

   (*b*) $\vec{V}_1 + \vec{V}_2 = (V_{1x} + V_{2x}, V_{1y} + V_{2y}) = (-0.6, 6.0)$

   $|\vec{V}_1 + \vec{V}_2| = \sqrt{(-0.6)^2 + (6.0)^2} = 6.0 \text{ units}$ $\theta = \tan^{-1}\dfrac{6.0}{0.6} = 84°$

   The sum has a magnitude of $\boxed{6.0 \text{ units}}$, and is $\boxed{84° \text{ clockwise from the } - \text{negative x-axis}}$, or $96°$ counterclockwise from the positive x-axis.

9. (*a*) $v_{\text{north}} = (735 \text{ km/h})(\cos 41.5°) = \boxed{550 \text{ km/h}}$ $v_{\text{west}} = (735 \text{ km/h})(\sin 41.5°) = \boxed{487 \text{ km/h}}$

   (*b*) $\Delta d_{\text{north}} = v_{\text{north}} t = (550 \text{ km/h})(3.00 \text{ h}) = \boxed{1650 \text{ km}}$

   $\Delta d_{\text{west}} = v_{\text{west}} t = (487 \text{ km/h})(3.00 \text{ h}) = \boxed{1460 \text{ km}}$

10. $A_x = 44.0 \cos 28.0° = 38.85$ $A_y = 44.0 \sin 28.0° = 20.66$

    $B_x = -26.5 \cos 56.0° = -14.82$ $B_y = 26.5 \sin 56.0° = 21.97$

    $C_x = 31.0 \cos 270° = 0.0$ $C_y = 31.0 \sin 270° = -31.0$

    (a) $\left(\vec{A} + \vec{B} + \vec{C}\right)_x = 38.85 + (-14.82) + 0.0 = 24.03 = \boxed{24.0}$

        $\left(\vec{A} + \vec{B} + \vec{C}\right)_y = 20.66 + 21.97 + (-31.0) = 11.63 = \boxed{11.6}$

    (b) $\left|\vec{A} + \vec{B} + \vec{C}\right| = \sqrt{(24.03)^2 + (11.63)^2} = \boxed{26.7}$ $\theta = \tan^{-1}\dfrac{11.63}{24.03} = \boxed{25.8°}$

11. $A_x = 44.0 \cos 28.0° = 38.85$ $A_y = 44.0 \sin 28.0° = 20.66$

    $C_x = 31.0 \cos 270° = 0.0$ $C_y = 31.0 \sin 270° = -31.0$

    $\left(\vec{A} - \vec{C}\right)_x = 38.85 - 0.0 = 38.85$ $\left(\vec{A} - \vec{C}\right)_y = 20.66 - (-31.0) = 51.66$

    $\left|\vec{A} - \vec{C}\right| = \sqrt{(38.85)^2 + (51.66)^2} = \boxed{64.6}$ $\theta = \tan^{-1}\dfrac{51.66}{38.85} = \boxed{53.1°}$

12. $A_x = 44.0 \cos 28.0° = 38.85$ $A_y = 44.0 \sin 28.0° = 20.66$

    $B_x = -26.5 \cos 56.0° = -14.82$ $B_y = 26.5 \sin 56.0° = 21.97$

    (a) $\left(\vec{B} - \vec{A}\right)_x = (-14.82) - 38.85 = -53.67$ $\left(\vec{B} - \vec{A}\right)_y = 21.97 - 20.66 = 1.31$

        Note that since the *x* component is negative and the *y* component is positive, the vector is in the 2nd quadrant.

        $\left|\vec{B} - \vec{A}\right| = \sqrt{(-53.67)^2 + (1.31)^2} = \boxed{53.7}$ $\theta_{B-A} = \tan^{-1}\dfrac{1.31}{-53.67} = \boxed{1.4° \text{ above } -x \text{ axis}}$

    (b) $\left(\vec{A} - \vec{B}\right)_x = 38.85 - (-14.82) = 53.67$ $\left(\vec{A} - \vec{B}\right)_y = 20.66 - 21.97 = -1.31$

        Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

        $\left|\vec{A} - \vec{B}\right| = \sqrt{(53.67)^2 + (-1.31)^2} = \boxed{53.7}$ $\theta = \tan^{-1}\dfrac{-1.31}{53.7} = \boxed{1.4° \text{ below } +x \text{ axis}}$

        Comparing the results shows that $\vec{B} - \vec{A}$ is the opposite of $\vec{A} - \vec{B}$.

13. $A_x = 44.0 \cos 28.0° = 38.85$ $A_y = 44.0 \sin 28.0° = 20.66$

    $B_x = -26.5 \cos 56.0° = -14.82$ $B_y = 26.5 \sin 56.0° = 21.97$

    $C_x = 31.0 \cos 270° = 0.0$ $C_y = 31.0 \sin 270° = -31.0$

    (a) $\left(\vec{A} - \vec{B} + \vec{C}\right)_x = 38.85 - (-14.82) + 0.0 = 53.67$

        $\left(\vec{A} - \vec{B} + \vec{C}\right)_y = 20.66 - 21.97 + (-31.0) = -32.31$

        Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

        $\left|\vec{A} - \vec{B} + \vec{C}\right| = \sqrt{(53.67)^2 + (-32.31)^2} = \boxed{62.6}$ $\theta = \tan^{-1}\dfrac{-32.31}{53.67} = \boxed{31.0° \text{ below } +x \text{ axis}}$

(b) $\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}}\right)_x = 38.85+\left(-14.82\right)-0.0 = 24.03$

$\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}}\right)_y = 20.66+21.97-\left(-31.0\right) = 73.63$

$\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}}\right| = \sqrt{\left(24.03\right)^2+\left(73.63\right)^2} = \boxed{77.5}$ $\theta = \tan^{-1}\dfrac{73.63}{24.03} = \boxed{71.9°}$

(c) $\left(\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)_x = 0.0-38.85-\left(-14.82\right) = -24.03$

$\left(\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)_y = -31.0-20.66-21.97 = -73.63$

Note that since both components are negative, the vector is in the 3rd quadrant.

$\left|\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}}\right| = \sqrt{\left(-24.03\right)^2+\left(-73.63\right)^2} = \boxed{77.5}$ $\theta = \tan^{-1}\dfrac{-73.63}{-24.03} = \boxed{71.9° \text{ below} - x \text{ axis}}$

Note that the answer to (c) is the exact opposite of the answer to (b).

14. $A_x = 44.0\cos 28.0° = 38.85$ $A_y = 44.0\sin 28.0° = 20.66$

$B_x = -26.5\cos 56.0° = -14.82$ $B_y = 26.5\sin 56.0° = 21.97$

$C_x = 31.0\cos 270° = 0.0$ $C_y = 31.0\sin 270° = -31.0$

(a) $\left(\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right)_x = -14.82-2\left(38.85\right) = -92.52$ $\left(\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right)_y = 21.97-2\left(20.66\right) = -19.35$

Note that since both components are negative, the vector is in the 3rd quadrant.

$\left|\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right| = \sqrt{\left(-92.52\right)^2+\left(-19.35\right)^2} = \boxed{94.5}$ $\theta = \tan^{-1}\dfrac{-19.35}{-92.52} = \boxed{11.8° \text{ below} - x \text{ axis}}$

(b) $\left(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right)_x = 2\left(38.85\right)-3\left(-14.82\right)+2\left(0.0\right) = 122.16$

$\left(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right)_y = 2\left(20.66\right)-3\left(21.97\right)+2\left(-31.0\right) = -86.59$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$\left|2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right| = \sqrt{\left(122.16\right)^2+\left(-86.59\right)^2} = \boxed{149.7}$ $\theta = \tan^{-1}\dfrac{-86.59}{122.16} = \boxed{35.3° \text{ below} + x \text{ axis}}$

15. The *x* component is negative and the *y* component is positive, since the summit is to the west of north. The angle measured counterclockwise from the positive *x* axis would be 122.4°. Thus the components are found to be

$x = -4580\sin 32.4° = -2454$ m $y = 4580\cos 32.4° = 3867$ m $z = 2450$ m

$\boxed{\vec{\mathbf{r}} = \left(-2450 \text{ m}, 3870 \text{ m}, 2450 \text{ m}\right)}$ $\left|\vec{\mathbf{r}}\right| = \sqrt{\left(-2454\right)^2+\left(4580\right)^2+\left(2450\right)^2} = \boxed{5190 \text{ m}}$

16. $70.0 = \sqrt{x^2+\left(-55.0\right)^2}$ → $4900 = x^2+3025$ → $x^2 = 1875$ → $x = \boxed{\pm 43.3 \text{ units}}$

17. Choose downward to be the positive *y* direction. The origin will be at the point where the tiger leaps from the rock. In the horizontal direction, $v_{x0} = 3.5$ m/s and $a_x = 0$. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80$ m/s^2, $y_0 = 0$, and the final location $y = 6.5$ m. The time for the tiger to reach the ground is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_yt^2 \quad \rightarrow \quad 6.5\text{m} = 0 + 0 + \tfrac{1}{2}\left(9.8\,\text{m}/\text{s}^2\right)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2\left(6.5\text{m}\right)}{9.8\,\text{m}/\text{s}^2}} = 1.15 \text{ sec}$$

The horizontal displacement is calculated from the constant horizontal velocity.

$$\Delta x = v_x t = \left(3.5\,\text{m}/\text{s}\right)\left(1.15 \text{ sec}\right) = \boxed{4.0 \text{ m}}$$

18. Choose downward to be the positive y direction. The origin will be at the point where the diver dives from the cliff. In the horizontal direction, $v_{x0} = 1.8\,\text{m}/\text{s}$ and $a_x = 0$. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\text{m}/\text{s}^2$, $y_0 = 0$, and the time of flight is $t = 3.0 \text{ s}$. The height of the cliff is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_yt^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}\left(9.80\,\text{m}/\text{s}^2\right)\left(3.0\,\text{s}\right)^2 = \boxed{44 \text{ m}}$$

The distance from the base of the cliff to where the diver hits the water is found from the horizontal motion at constant velocity:

$$\Delta x = v_x t = \left(1.8\,\text{m}/\text{s}\right)\left(3 \text{ s}\right) = \boxed{5.4 \text{ m}}$$
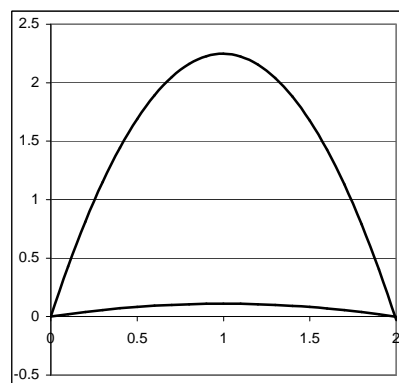
19. Apply the range formula from Example 3-8.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow$$

$$\sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{\left(2.0 \text{ m}\right)\left(9.8\,\text{m}/\text{s}^2\right)}{\left(6.8\,\text{m}/\text{s}\right)^2} = 0.4239$$

$$2\theta_0 = \sin^{-1} 0.4239 \quad \rightarrow \quad \theta_0 = \boxed{13^\circ, 77^\circ}$$

There are two angles because each angle gives the same range. If one angle is $\theta = 45^\circ + \delta$, then $\theta = 45^\circ - \delta$ is also a solution. The two paths are shown in the graph.

20. Choose upward to be the positive y direction. The origin is the point from which the pebbles are released. In the vertical direction, $a_y = -9.80\,\text{m}/\text{s}^2$, the velocity at the window is $v_y = 0$, and the vertical displacement is 4.5 m. The initial y velocity is found from Eq. 2-11c.

$$v_y^2 = v_{y0}^2 + 2a_y\left(y - y_0\right) \quad \rightarrow$$

$$v_{y0} = \sqrt{v_y^2 - 2a_y\left(y - y_0\right)} = \sqrt{0 - 2\left(-9.80\,\text{m}/\text{s}^2\right)\left(4.5 \text{ m}\right)} = 9.39\,\text{m}/\text{s}$$

Find the time for the pebbles to travel to the window from Eq. 2-11a.

$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{0 - 9.4\,\text{m}/\text{s}}{-9.8\,\text{m}/\text{s}^2} = 0.958 \text{ s}$$

Find the horizontal speed from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x / t = 5.0 \text{ m} / 0.958 \text{ s} = \boxed{5.2\,\text{m}/\text{s}}$$

This is the speed of the pebbles when they hit the window.

$\boxed{21.}$ Choose downward to be the positive y direction. The origin will be at the point where the ball is thrown from the roof of the building. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\text{m}/\text{s}^2$, $y_0 = 0$, and the displacement is 45.0 m. The time of flight is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_yt^2 \quad \rightarrow \quad 45.0 \text{ m} = \tfrac{1}{2}\left(9.80 \text{ m/s}^2\right)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2\left(45.0 \text{ m}\right)}{9.80 \text{ m/s}^2}} = 3.03 \text{ sec}$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity:

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x / t = 24.0 \text{ m}/3.03 \text{ s} = \boxed{7.92 \text{ m/s}}\,.$$

22. Choose the point at which the football is kicked the origin, and choose upward to be the positive y direction. When the football reaches the ground again, the y displacement is 0. For the football, $v_{y0} = \left(18.0\sin 35.0°\right) \text{m/s}$, $a_y = -9.80 \text{ m/s}^2$ and the final y velocity will be the opposite of the starting y velocity (reference problem 3-28). Use Eq. 2-11a to find the time of flight.

$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{\left(-18.0\sin 35.0°\right)\text{m/s} - \left(18.0\sin 35.0°\right)\text{m/s}}{-9.80 \text{ m/s}^2} = \boxed{2.11 \text{ s}}$$

23. Choose downward to be the positive y direction. The origin is the point where the ball is thrown from the roof of the building. In the vertical direction, $v_{y0} = 0$, $y_0 = 0$, and $a_y = 9.80 \text{ m/s}^2$. The initial horizontal velocity is 22.2 m/s and the horizontal range is 36.0 m. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad t = \Delta x / v_x = 36.0 \text{ m}/22.2 \text{ m/s} = 1.62 \text{ s}$$

The vertical displacement, which is the height of the building, is found by applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_yt^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}\left(9.80 \text{ m/s}^2\right)\left(1.62 \text{ s}\right)^2 = \boxed{12.9 \text{ m}}$$

$\boxed{24.}$ (*a*) Use the "Level horizontal range" formula from Example 3-8.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow \quad v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{\left(7.80 \text{ m}\right)\left(9.80 \text{ m/s}^2\right)}{\sin 2\left(28.0°\right)}} = \boxed{9.60 \text{ m/s}}$$

(*b*) Now increase the speed by 5.0% and calculate the new range. The new speed would be $9.60 \text{ m/s}\left(1.05\right) = 10.1 \text{ m/s}$ and the new range would be

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{\left(10.1 \text{ m/s}\right)^2 \sin 2\left(28.0°\right)}{9.80 \text{ m/s}^2} = 8.60 \text{ m}\,,$$

an increase of $\boxed{0.80 \text{ m} \left(10\% \text{ increase}\right)}$.

25. Calculate the range as derived in Example 3-8: $R = \dfrac{v_0^2 \sin 2\theta_0}{g}$. If the launching speed and angle are held constant, the range is inversely proportional to the value of g. The acceleration due to gravity on the Moon is $1/6^{\text{th}}$ that on Earth.

$$R_{\text{Earth}} = \frac{v_0^2 \sin 2\theta_0}{g_{\text{Earth}}} \qquad R_{\text{Moon}} = \frac{v_0^2 \sin 2\theta_0}{g_{\text{Moon}}} \quad \rightarrow \quad R_{\text{Earth}}\,g_{\text{Earth}} = R_{\text{Moon}}\,g_{\text{Moon}}$$

$$R_{\text{Moon}} = R_{\text{Earth}}\frac{g_{\text{Earth}}}{g_{\text{Moon}}} = 6R_{\text{Earth}}$$

Thus on the Moon, the person can jump $\boxed{\text{6 times farther}}$.

26. (a) Choose downward to be the positive y direction. The origin is the point where the bullet leaves the gun. In the vertical direction, $v_{y0} = 0$, $y_0 = 0$, and $a_y = 9.80 \text{ m/s}^2$. In the horizontal direction, $\Delta x = 75.0 \text{ m}$ and $v_x = 180 \text{ m/s}$. The time of flight is found from the horizontal motion at constant velocity.
$$\Delta x = v_x t \quad \rightarrow \quad t = \Delta x / v_x = 75.0 \text{ m} / 180 \text{ m/s} = 0.4167 \text{ s}$$
This time can now be used in Eq. 2-11b to find the vertical drop of the bullet.
$$y = y_0 + v_{y0} t + \tfrac{1}{2} a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2} \left(9.80 \text{ m/s}^2 \right) \left(0.4167 \text{ s} \right)^2 = \boxed{0.851 \text{ m}}$$

(b) For the bullet to hit the target at the same level, the level horizontal range formula of Example 3-8 applies. The range is 75.0 m, and the initial velocity is 180 m/s. Solving for the angle of launch results in the following.
$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow \quad \sin 2\theta_0 = \frac{Rg}{v_0^2} \quad \rightarrow \quad \theta_0 = \frac{1}{2} \sin^{-1} \frac{(75.0 \text{ m})(9.80 \text{ m/s}^2)}{(180 \text{ m/s})^2} = \boxed{0.650^\circ}$$
Because of the symmetry of the range formula, there is also an answer of the complement of the above answer, which would be 89.35°. That is an unreasonable answer from a practical physical viewpoint – it is pointing the gun almost straight up.

27. Choose downward to be the positive y direction. The origin is the point where the supplies are dropped. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80 \text{ m/s}^2$, $y_0 = 0$, and the final position is $y = 160 \text{ m}$. The time of flight is found from applying Eq. 2-11b to the vertical motion.
$$y = y_0 + v_{y0} t + \tfrac{1}{2} a_y t^2 \quad \rightarrow \quad 160 \text{ m} = 0 + 0 + \tfrac{1}{2} \left(9.80 \text{ m/s}^2 \right) t^2 \quad \rightarrow$$
$$t = \sqrt{\frac{2(160 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{5.71 \text{ s}}$$
Note that the speed of the airplane does not enter into this calculation.

28. The horizontal component of the speed does not change during the course of the motion, and so $v_{xf} = v_{x0}$. The net vertical displacement is 0 if the firing level equals the landing level. Eq. 2-11c then gives $v_{yf}^2 = v_{y0}^2 + 2a_y \Delta y = v_{y0}^2$. Thus $v_{yf}^2 = v_{y0}^2$, and from the horizontal $v_{xf}^2 = v_{x0}^2$. The initial speed is $v_0 = \sqrt{v_{y0}^2 + v_{x0}^2}$. The final speed is $v_f = \sqrt{v_{yf}^2 + v_{xf}^2} = \sqrt{v_{y0}^2 + v_{x0}^2} = v_0$. Thus $\boxed{v_f = v_0}$.

29. Choose upward to be the positive y direction. The origin is point from which the football is kicked. The initial speed of the football is $v_0 = 20.0 \text{ m/s}$. We have $v_{y0} = v_0 \sin 37.0^\circ = 12.04 \text{ m/s}$, $y_0 = 0$, and $a_y = -9.80 \text{ m/s}^2$. In the horizontal direction, $v_x = v_0 \cos 37.0^\circ = 15.97 \text{ m/s}$, and $\Delta x = 36.0 \text{ m}$. The time of flight to reach the goalposts is found from the horizontal motion at constant speed:
$$\Delta x = v_x t \quad \rightarrow \quad t = \Delta x / v_x = 36.0 \text{ m} / 15.97 \text{ m/s} = 2.254 \text{ s} .$$
Now use this time with the vertical motion data and Eq. 2-11b to find the height of the football when it reaches the horizontal location of the goalposts.
$$y = y_0 + v_{y0} t + \tfrac{1}{2} a_y t^2 = 0 + (12.04 \text{ m/s})(2.254 \text{ s}) + \tfrac{1}{2} \left(-9.80 \text{ m/s}^2 \right) (2.254 \text{ s})^2 = 2.24 \text{ m}$$

Since the ball's height is less than 3.00 m, the football does not clear the bar. It is 0.76 m too low when it reaches the horizontal location of the goalposts.

30. Choose the origin to be where the projectile is launched, and upwards to be the positive y direction. The initial velocity of the projectile is v_0, the launching angle is θ_0, $a_y = -g$, and $v_{y0} = v_0 \sin \theta_0$.

    (a) The maximum height is found from Eq. 2-11c, $v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$, with $v_y = 0$ at the maximum height.

    $$y_{max} = 0 + \frac{v_y^2 - v_{y0}^2}{2a_y} = \frac{-v_0^2 \sin^2 \theta_0}{-2g} = \frac{v_0^2 \sin^2 \theta_0}{2g} = \frac{(65.2 \,\text{m/s})^2 \sin^2 34.5^\circ}{2(9.80 \,\text{m/s}^2)} = \boxed{69.6 \,\text{m}}$$

    (b) The total time in the air is found from Eq. 2-11b, with a total vertical displacement of 0 for the ball to reach the ground.

    $$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = v_0 \sin \theta_0 t - \tfrac{1}{2}gt^2 \quad \rightarrow$$

    $$t = \frac{2v_0 \sin \theta_0}{g} = \frac{2(65.2 \,\text{m/s}) \sin 34.5^\circ}{(9.80 \,\text{m/s}^2)} = \boxed{7.54 \,\text{s}} \text{ and } t = 0$$

    The time of 0 represents the launching of the ball.

    (c) The total horizontal distance covered is found from the horizontal motion at constant velocity.

    $$\Delta x = v_x t = (v_0 \cos \theta_0)t = (65.2 \,\text{m/s})(\cos 34.5^\circ)(7.54 \,\text{s}) = \boxed{405 \,\text{m}}$$

    (d) The velocity of the projectile 1.50 s after firing is found as the vector sum of the horizontal and vertical velocities at that time. The horizontal velocity is a constant $v_0 \cos \theta_0 = (65.2 \,\text{m/s})(\cos 34.5^\circ) = 53.7 \,\text{m/s}$. The vertical velocity is found from Eq. 2-11a.

    $$v_y = v_{y0} + at = v_0 \sin \theta_0 - gt = (65.2 \,\text{m/s}) \sin 34.5^\circ - (9.80 \,\text{m/s}^2)(1.50 \,\text{s}) = 22.2 \,\text{m/s}$$

    Thus the speed of the projectile is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{53.7^2 + 22.2^2} = \boxed{58.1 \,\text{m/s}}$.

    The direction above the horizontal is given by $\theta = \tan^{-1} \dfrac{v_y}{v_x} = \tan^{-1} \dfrac{22.2}{53.7} = \boxed{22.5^\circ}$.

31. Choose the origin to be at ground level, under the place where the projectile is launched, and upwards to be the positive y direction. For the projectile, $v_0 = 65.0 \,\text{m/s}$, $\theta_0 = 37.0^\circ$, $a_y = -g$, $y_0 = 125$, and $v_{y0} = v_0 \sin \theta_0$

    (a) The time taken to reach the ground is found from Eq. 2-11b, with a final height of 0.

    $$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = 125 + v_0 \sin \theta_0 t - \tfrac{1}{2}gt^2 \quad \rightarrow$$

    $$t = \frac{-v_0 \sin \theta_0 \pm \sqrt{v_0^2 \sin^2 \theta_0 - 4(-\tfrac{1}{2}g)(125)}}{2(-\tfrac{1}{2}g)} = \frac{-39.1 \pm 63.1}{-9.8} = 10.4 \,\text{s} \,, \; -2.45 \,\text{s} = \boxed{10.4 \,\text{s}}$$

    Choose the positive sign since the projectile was launched at time $t = 0$.

    (b) The horizontal range is found from the horizontal motion at constant velocity.

    $$\Delta x = v_x t = (v_0 \cos \theta_0)t = (65.0 \,\text{m/s}) \cos 37.0^\circ (10.4 \,\text{s}) = \boxed{541 \,\text{m}}$$

    (c) At the instant just before the particle reaches the ground, the horizontal component of its velocity is the constant $v_x = v_0 \cos \theta_0 = (65.0 \,\text{m/s}) \cos 37.0^\circ = \boxed{51.9 \,\text{m/s}}$. The vertical component is found from Eq. 2-11a.

$$v_y = v_{y0} + at = v_0 \sin \theta_0 - gt = (65.0 \text{ m/s}) \sin 37.0° - (9.80 \text{ m/s}^2)(10.4 \text{ s})$$

$$= \boxed{-63.1 \text{ m/s}}$$

(*d*) The magnitude of the velocity is found from the *x* and *y* components calculated in part c) above.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(51.9 \text{ m/s})^2 + (-63.1 \text{ m/s})^2} = \boxed{81.7 \text{ m/s}}$$

(*e*) The direction of the velocity is $\theta = \tan^{-1} \dfrac{v_y}{v_x} = \tan^{-1} \dfrac{-63.1}{51.9} = -50.6°$, and so the object is

moving $\boxed{50.6° \text{ below the horizon}}$.

(*f*) The maximum height above the cliff top reached by the projectile will occur when the y-velocity is 0, and is found from Eq. 2-11c.

$$v_y^2 = v_{y0}^2 + 2a_y (y - y_0) \quad \rightarrow \quad 0 = v_0^2 \sin^2 \theta_0 - 2gy_{\text{max}}$$

$$y_{\text{max}} = \frac{v_0^2 \sin^2 \theta_0}{2g} = \frac{(65.0 \text{ m/s})^2 \sin^2 37.0°}{2(9.80 \text{ m/s}^2)} = \boxed{78.1 \text{ m}}$$

32. Choose the origin to be the point of release of the shot put. Choose upward to be the positive *y* direction. Then $y_0 = 0$, $v_{y0} = (15.5 \sin 34.0°) \text{ m/s} = 8.67 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, and $y = -2.20$ m at the end of the motion. Use Eq. 2-11b to find the time of flight.

$$y = y_0 + v_{y0}t + \tfrac{1}{2} a_y t^2 \quad \rightarrow \quad \tfrac{1}{2} a_y t^2 + v_{y0}t - y = 0 \quad \rightarrow$$

$$t = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 - 4\left(\tfrac{1}{2} a_y\right)(-y)}}{2 \tfrac{1}{2} a_y} = \frac{-8.67 \pm \sqrt{(8.67)^2 - 2(-9.80)(2.20)}}{-9.80} = 1.99 \text{ s}, -0.225 \text{ s}$$

Choose the positive result since the time must be greater than 0. Now calculate the horizontal distance traveled using the horizontal motion at constant velocity.

$$\Delta x = v_x t = (15.5 \cos 34°) \text{ m/s} (1.99 \text{ s}) = \boxed{25.6 \text{ m}}$$

33. Choose the origin to be where the projectile is launched, and upwards to be the positive *y* direction. The initial velocity of the projectile is v_0, the launching angle is θ_0, $a_y = -g$, and $v_{y0} = v_0 \sin \theta_0$.

The range of the projectile is given by the range formula from Example 3-8, $R = \dfrac{v_0^2 \sin 2\theta_0}{g}$. The

maximum height of the projectile will occur when its vertical speed is 0. Apply Eq. 2-11c.

$$v_y^2 = v_{y0}^2 + 2a_y (y - y_0) \quad \rightarrow \quad 0 = v_0^2 \sin^2 \theta_0 - 2gy_{\text{max}} \quad \rightarrow \quad y_{\text{max}} = \frac{v_0^2 \sin^2 \theta_0}{2g}$$

Now find the angle for which $R = y_{\text{max}}$.

$$R = y_{\text{max}} \quad \rightarrow \quad \frac{v_0^2 \sin 2\theta_0}{g} = \frac{v_0^2 \sin^2 \theta_0}{2g} \quad \rightarrow \quad \sin 2\theta_0 = \frac{\sin^2 \theta_0}{2} \quad \rightarrow$$

$$2 \sin \theta_0 \cos \theta_0 = \frac{\sin^2 \theta_0}{2} \quad \rightarrow \quad 4 \cos \theta_0 = \sin \theta_0 \quad \rightarrow \quad \tan \theta_0 = 4 \quad \rightarrow \quad \theta_0 = \tan^{-1} 4 = \boxed{76°}$$

34. Choose the origin to be the location from which the balloon is
    fired, and choose upward as the positive *y* direction. Assume
    the boy in the tree is a distance *H* up from the point at which
    the balloon is fired, and that the tree is a distance *D* horizontally
    from the point at which the balloon is fired. The equations of
    motion for the balloon and boy are as follows, using constant
    acceleration relationships.
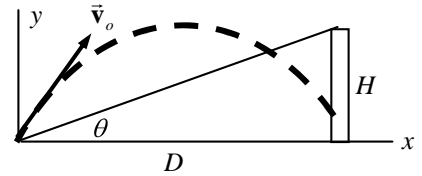
$$x_{\text{Balloon}} = v_0 \cos \theta_0 t \qquad y_{\text{Balloon}} = 0 + v_0 \sin \theta_0 t - \tfrac{1}{2} g t^2 \qquad y_{\text{Boy}} = H - \tfrac{1}{2} g t^2$$

Use the horizontal motion at constant velocity to find the elapsed time after the balloon has traveled
D to the right.

$$D = v_0 \cos \theta_0 t_D \qquad \rightarrow \qquad t_D = \frac{D}{v_0 \cos \theta_0}$$

Where is the balloon vertically at that time?

$$y_{\text{Balloon}} = v_0 \sin \theta_0 t_D - \tfrac{1}{2} g t_D^2 = v_0 \sin \theta_0 \frac{D}{v_0 \cos \theta_0} - \tfrac{1}{2} g \left(\frac{D}{v_0 \cos \theta_0} \right)^2 = D \tan \theta_0 - \tfrac{1}{2} g \left(\frac{D}{v_0 \cos \theta_0} \right)^2$$

Where is the boy vertically at that time? Note that $H = D \tan \theta_o$.

$$y_{\text{Boy}} = H - \tfrac{1}{2} g t_D^2 = H - \tfrac{1}{2} g \left(\frac{D}{v_0 \cos \theta_0} \right)^2 = D \tan \theta_0 - \tfrac{1}{2} g \left(\frac{D}{v_0 \cos \theta_0} \right)^2$$

The boy and the balloon are at the same height and the same horizontal location at the same time.
Thus they collide!

35. Choose the origin to be the location on the ground directly below the airplane at the time the supplies
    are dropped, and choose upward as the positive *y* direction. For the supplies, $y_0 = 235 \text{ m}$, $v_{y0} = 0$,
    $a_y = -g$, and the final *y* location is $y = 0 \text{ m}$. The initial (and constant) *x* velocity of the supplies is
    $v_x = 69.4 \text{ m/s}$.
    (*a*) The time for the supplies to reach the ground is found from Eq. 2-11b.

$$y = y_0 + v_{y0} t + \tfrac{1}{2} a_y t^2 \qquad \rightarrow \qquad 0 = y_0 + 0 + \tfrac{1}{2} a t^2 \qquad \rightarrow$$

$$t = \sqrt{\frac{-2 y_0}{a}} = \sqrt{\frac{-2 (235 \text{ m})}{\left(-9.80 \text{ m/s}^2\right)}} = 6.93 \text{ s}$$

Then the horizontal distance of travel for the package is found from the horizontal motion at
constant velocity.

$$\Delta x = v_x t = (69.4 \text{ m/s})(6.93 \text{ s}) = \boxed{481 \text{ m}}$$

(*b*) Now the supplies have to travel a horizontal distance of only 425 m. Thus the time of flight will
    be less, and is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \qquad \rightarrow \qquad t = \Delta x / v_x = 425 \text{ m} / 69.4 \text{ m/s} = 6.124 \text{ s} .$$

The *y* motion must satisfy Eq. 2-11b for this new time, but the same vertical displacement and
acceleration.

$$y = y_0 + v_{y0} t + \tfrac{1}{2} a_y t^2 \qquad \rightarrow$$

$$v_{y0} = \frac{y - y_0 - \tfrac{1}{2} a_y t^2}{t} = \frac{0 - 235 \text{ m} - \tfrac{1}{2} \left(-9.80 \text{ m/s}^2\right)(6.124 \text{ s})^2}{6.124 \text{ s}} = \boxed{-8.37 \text{ m/s}}$$

Notice that since this is a negative velocity, the object must be projected DOWN.

(*c*) The horizontal component of the speed of the supplies upon landing is the constant horizontal speed of 69.4 m/s. The vertical speed is found from Eq. 2-11a.

$$v_y = v_{y0} + a_y t = -8.37 \text{ m/s} + \left(-9.80 \text{ m/s}^2\right)\left(6.124 \text{ s}\right) = 68.4 \text{ m/s}$$

Thus the speed is given by

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(69.4 \text{ m/s}\right)^2 + \left(68.4 \text{ m/s}\right)^2} = \boxed{97.4 \text{ m/s}}$$

36. Call the direction of the boat relative to the water the positive direction.

(*a*) $\vec{\mathbf{v}}_{\substack{\text{jogger} \\ \text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{jogger} \\ \text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.} \\ \text{water}}} = 2.2 \text{ m/s} + 7.5 \text{ m/s}$

$= \boxed{9.7 \text{ m/s in the direction the boat is moving}}$

(*b*) $\vec{\mathbf{v}}_{\substack{\text{jogger} \\ \text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{jogger} \\ \text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.} \\ \text{water}}} = -2.2 \text{ m/s} + 7.5 \text{ m/s}$
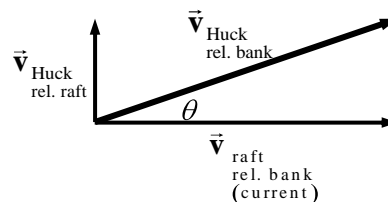
$= \boxed{5.3 \text{ m/s in the direction the boat is moving}}$

37. Call the direction of the flow of the river the *x* direction, and the direction of Huck walking relative to the raft the *y* direction.

$\vec{\mathbf{v}}_{\substack{\text{Huck} \\ \text{rel. bank}}} = \vec{\mathbf{v}}_{\substack{\text{Huck} \\ \text{rel. raft}}} + \vec{\mathbf{v}}_{\substack{\text{raft rel.} \\ \text{bank}}} = (0, 0.60) \text{ m/s} + (1.70, 0) \text{ m/s}$

$= (1.70, 0.60) \text{ m/s}$

Magnitude: $v_{\substack{\text{Huck} \\ \text{rel. bank}}} = \sqrt{1.70^2 + 0.60^2} = \boxed{1.80 \text{ m/s}}$
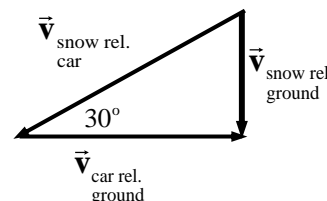
Direction: $\theta = \tan^{-1} \dfrac{0.60}{1.70} = \boxed{19° \text{ relative to river}}$

38. We have $v_{\substack{\text{car rel.} \\ \text{ground}}} = 25 \text{ m/s}$. Use the diagram, illustrating $\vec{\mathbf{v}}_{\substack{\text{snow rel.} \\ \text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{snow rel.} \\ \text{car}}} + \vec{\mathbf{v}}_{\substack{\text{car rel.} \\ \text{ground}}}$, to calculate the other speeds.

$\cos 30° = \dfrac{v_{\substack{\text{car rel.} \\ \text{ground}}}}{v_{\substack{\text{snow rel.} \\ \text{car}}}} \rightarrow v_{\substack{\text{snow rel.} \\ \text{car}}} = 25 \text{ m/s} / \cos 30° = \boxed{29 \text{ m/s}}$

$\tan 30° = \dfrac{v_{\substack{\text{snow rel.} \\ \text{ground}}}}{v_{\substack{\text{car rel.} \\ \text{ground}}}} \rightarrow v_{\substack{\text{snow rel.} \\ \text{ground}}} = (25 \text{ m/s}) \tan 30° = \boxed{14 \text{ m/s}}$

39. Call the direction of the flow of the river the *x* direction, and the direction the boat is headed the *y* direction.

(*a*) $v_{\substack{\text{boat rel.} \\ \text{shore}}} = \sqrt{v_{\substack{\text{water rel.} \\ \text{shore}}}^2 + v_{\substack{\text{boat rel.} \\ \text{water}}}^2} = \sqrt{1.20^2 + 2.30^2} = \boxed{2.59 \text{ m/s}}$

$\theta = \tan^{-1} \dfrac{1.20}{2.30} = 27.6° , \phi = 90° - \theta = \boxed{62.4° \text{ relative to shore}}$
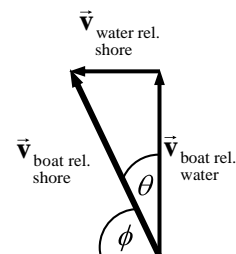
(*b*)   The position of the boat after 3.00 seconds is given by

$$\Delta d = v_{\substack{\text{boat rel.}\\\text{shore}}}t = \left[(1.20, 2.30)\,\text{m/s}\right](3.00\,\text{sec})$$

$$= \boxed{(3.60\text{ m downstream}, 6.90\text{ m across the river})}$$

As a magnitude and direction, it would be 7.8 m away from the starting point, at an angle of 62.4° relative to the shore.

40. If each plane has a speed of 785 km/hr, then their relative speed of approach is 1570 km/hr. If the planes are 11 km apart, then the time for evasive action is found from

$$\Delta d = vt \quad \rightarrow \quad t = \frac{\Delta d}{v} = \left(\frac{11.0\text{ km}}{1570\,\text{km/hr}}\right)\left(\frac{3600\text{ sec}}{1\text{ hr}}\right) = \boxed{25.2\text{ s}}$$

41. Call east the positive *x* direction and north the positive *y* direction. Then the following vector velocity relationship exists.

(*a*) $\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}}$

$$= (0, -600)\,\text{km/h} + \left(100\cos 45.0^\circ, 100\sin 45.0^\circ\right)\text{km/h}$$

$$= (70.7, -529)\,\text{km/h}$$

$$v_{\substack{\text{plane rel.}\\\text{ground}}} = \sqrt{(70.7\,\text{km/h})^2 + (-529\,\text{km/h})^2} = \boxed{540\,\text{km/h}}$$

$$\theta = \tan^{-1}\frac{70.7}{529} = \boxed{7.6^\circ \text{ east of south}}$$

(*b*)   The plane is away from its intended position by the distance the air has caused it to move. The wind speed is 100 km/h, so after 10 min (1/6 h), the plane is off course by $\Delta x = v_x t = (100\,\text{km/h})\left(\tfrac{1}{6}\text{h}\right) = \boxed{17\text{ km}}$.
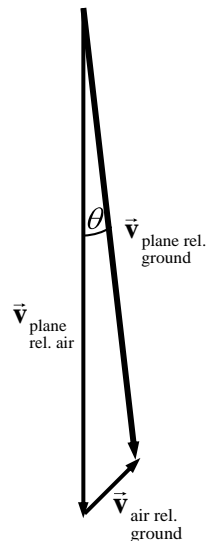
42. Call east the positive *x* direction and north the positive *y* direction. Then the following vector velocity relationship exists.

$$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}} \quad \rightarrow$$

$$\left(0, -v_{\substack{\text{plane rel.}\\\text{ground}}}\right) = (-600\sin\theta, 600\cos\theta)\,\text{km/h}$$

$$+ \left(100\cos 45.0^\circ, 100\sin 45.0^\circ\right)\text{km/h}$$

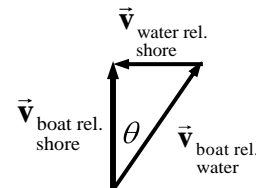Equate *x* components in the above equation.

$$0 = -600\sin\theta + 100\cos 45.0^\circ \quad \rightarrow$$

$$\theta = \sin^{-1}\frac{100\cos 45.0^\circ}{600} = \boxed{6.77^\circ, \text{ west of south}}$$
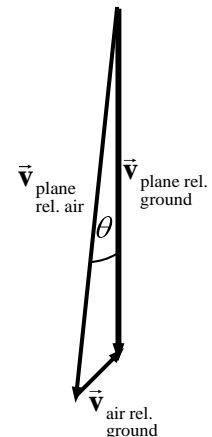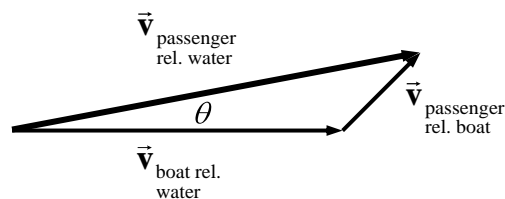
43. From the diagram in figure 3-29, it is seen that

$$v_{\substack{\text{boat rel.}\\\text{shore}}} = v_{\substack{\text{boat rel.}\\\text{water}}}\cos\theta = (1.85\,\text{m/s})\cos 40.4^\circ = \boxed{1.41\,\text{m/s}}.$$

44. Call the direction of the boat relative to the water the x direction, and upward the y direction. Also see the diagram.

$$\vec{v}_{\substack{\text{passenger} \\ \text{rel. water}}} = \vec{v}_{\substack{\text{passenger} \\ \text{rel. boat}}} + \vec{v}_{\substack{\text{boat rel.} \\ \text{water}}}$$

$$= \left(0.50\cos 45°, 0.50\sin 45°\right)\text{m/s}$$

$$+ \left(1.50, 0\right)\text{m/s} = \left(1.854, 0.354\right)\text{m/s}$$

$$v_{\substack{\text{passenger} \\ \text{rel. water}}} = \sqrt{1.854^2 + 0.354^2} = \boxed{1.89\,\text{m/s}}$$
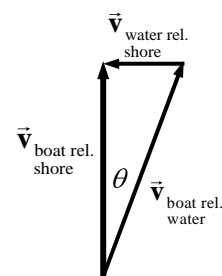
45. Call the direction of the flow of the river the x direction, and the direction straight across the river the y direction. The boat is traveling straight across the river. The boat is headed at $\theta = 28.5°$ upstream, at a speed of $v_{\substack{\text{boat rel.} \\ \text{water}}} = 2.60\,\text{m/s}$.

(*a*) $\sin\theta = v_{\substack{\text{water rel.} \\ \text{shore}}} \Big/ v_{\substack{\text{boat rel.} \\ \text{water}}} \quad\rightarrow\quad v_{\substack{\text{water rel.} \\ \text{shore}}} = \left(2.60\,\text{m/s}\right)\sin 28.5° = \boxed{1.24\,\text{m/s}}$

(*b*) $\cos\theta = v_{\substack{\text{boat rel.} \\ \text{shore}}} \Big/ v_{\substack{\text{boat rel.} \\ \text{water}}} \quad\rightarrow\quad v_{\substack{\text{boat rel.} \\ \text{shore}}} = \left(2.60\,\text{m/s}\right)\cos 28.5° = \boxed{2.28\,\text{m/s}}$
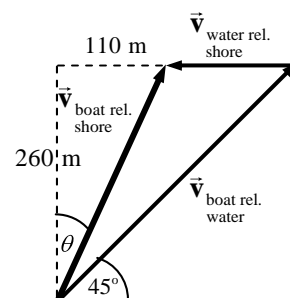
46. Call the direction of the flow of the river the x direction, and the direction straight across the river the y direction. From the diagram, $\theta = \tan^{-1} 110\,\text{m}/260\,\text{m} = 22.9°$. Equate the vertical components of the velocities to find the speed of the boat relative to the shore.

$$v_{\substack{\text{boat rel.} \\ \text{shore}}}\cos\theta = v_{\substack{\text{boat rel.} \\ \text{water}}}\sin 45° \quad\rightarrow$$

$$v_{\substack{\text{boat rel.} \\ \text{shore}}} = \left(1.70\,\text{m/s}\right)\frac{\sin 45°}{\cos 22.9°} = 1.305\,\text{m/s}$$

Equate the horizontal components of the velocities.

$$v_{\substack{\text{boat rel.} \\ \text{shore}}}\sin\theta = v_{\substack{\text{boat rel.} \\ \text{water}}}\cos 45° - v_{\substack{\text{water} \\ \text{rel. shore}}} \quad\rightarrow$$

$$v_{\substack{\text{water} \\ \text{rel. shore}}} = v_{\substack{\text{boat rel.} \\ \text{water}}}\cos 45° - v_{\substack{\text{boat rel.} \\ \text{shore}}}\sin\theta$$

$$= \left(1.70\,\text{m/s}\right)\cos 45° - \left(1.305\,\text{m/s}\right)\sin 22.9° = 0.69\,\text{m/s} \approx \boxed{0.7\,\text{m/s}}$$
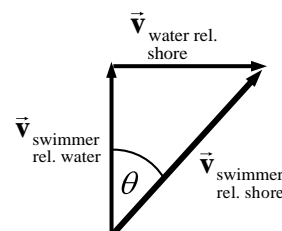
47. Call the direction of the flow of the river the x direction, and the direction straight across the river the y direction. Call the location of the swimmer's starting point the origin.

$$\vec{v}_{\substack{\text{swimmer} \\ \text{rel. shore}}} = \vec{v}_{\substack{\text{swimmer} \\ \text{rel. water}}} + \vec{v}_{\substack{\text{water rel.} \\ \text{shore}}} = \left(0, 0.45\,\text{m/s}\right) + \left(0.40\,\text{m/s}, 0\right)$$

$$= \left(0.40, 0.45\right)\text{m/s}$$

(*a*) Since the swimmer starts from the origin, the distances covered in the x and y directions will be exactly proportional to the speeds in those directions.

$$\frac{\Delta x}{\Delta y} = \frac{v_x t}{v_y t} = \frac{v_x}{v_y} \quad\rightarrow\quad \frac{\Delta x}{75\,\text{m}} = \frac{0.40\,\text{m/s}}{0.45\,\text{m/s}} \quad\rightarrow\quad \Delta x = \boxed{67\,\text{m}}$$
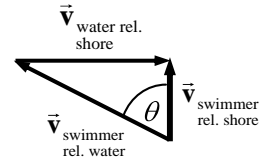
(*b*) The time is found from the constant velocity relationship for either the *x* or *y* directions.

$$\Delta y = v_y t \quad \rightarrow \quad t = \frac{\Delta y}{v_y} = \frac{75 \text{ m}}{0.45 \text{ m/s}} = \boxed{170 \text{ s}}$$

48. (*a*) Call the direction of the flow of the river the *x* direction, and the direction straight across the river the *y* direction.

$$\sin \theta = \frac{v_{\text{water rel.}}^{\text{shore}}}{v_{\text{swimmer}}^{\text{rel. water}}} = \frac{0.40 \text{ m/s}}{0.45 \text{ m/s}} \quad \rightarrow \quad \theta = \sin^{-1} \frac{0.40}{0.45} = 62.73^\circ = \boxed{62^\circ}$$

(*b*) From the diagram her speed with respect to the shore is

$$v_{\text{swimmer}}^{\text{rel. shore}} = v_{\text{swimmer}}^{\text{rel. water}} \cos \theta = (0.45 \text{ m/s}) \cos 62.73^\circ = 0.206 \text{ m/s}$$

The time to cross the river can be found from the constant velocity relationship.

$$\Delta x = vt \quad \rightarrow \quad t = \frac{\Delta x}{v} = \frac{75 \text{ m}}{0.206 \text{ m/s}} = 364 \text{ s} = \boxed{3.6 \times 10^2 \text{ s} = 6.1 \text{ min}}$$

49. Call east the positive *x* direction and north the positive *y* direction. The following is seen from the diagram. Apply the law of sines to the triangle formed by the three vectors.

$$\frac{v_{\text{plane}}^{\text{rel. air}}}{\sin 125^\circ} = \frac{v_{\text{air rel.}}^{\text{ground}}}{\sin \theta} \quad \rightarrow \quad \sin \theta = \frac{v_{\text{air rel.}}^{\text{ground}}}{v_{\text{plane}}^{\text{rel. air}}} \sin 125^\circ \quad \rightarrow$$

$$\theta = \sin^{-1} \left(\frac{95}{620} \sin 125^\circ \right) = 7.211^\circ$$
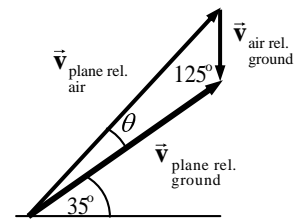
So the plane should head in a direction of $35.0^\circ + 7.2^\circ = \boxed{42.2^\circ \text{ north of east}}$.

50. Take the origin to be the location at which the speeder passes the police car, in the reference frame of the unaccelerated police car. The speeder is traveling at $145 \text{ km/h} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 40.28 \text{ m/s}$

relative to the ground, and the policeman is traveling at $95 \text{ km/h} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 26.39 \text{ m/s}$ relative

to the ground. Relative to the unaccelerated police car, the speeder is traveling at $13.89 \text{ m/s} = v_s$, and the police car is not moving. Do all of the calculations in the frame of reference of the unaccelerated police car.

The position of the speeder in the chosen reference frame is given by $\Delta x_s = v_s t$. The position of the policeman in the chosen reference frame is given by $\Delta x_p = \frac{1}{2} a_p (t-1)^2, t > 1$. The police car overtakes the speeder when these two distances are the same.; i.e., $\Delta x_s = \Delta x_p$.

$$\Delta x_s = \Delta x_p \quad \rightarrow \quad v_s t = \frac{1}{2} a_p (t-1)^2 \quad \rightarrow \quad (13.89 \text{ m/s})t = \frac{1}{2} (2 \text{ m/s}^2)(t^2 - 2t + 1) = t^2 - 2t + 1$$

$$t^2 - 15.89t + 1 = 0 \quad \rightarrow \quad t = \frac{15.89 \pm \sqrt{15.89^2 - 4}}{2} = 0.0632 \text{ s}, 15.83 \text{ s}$$

Since the police car doesn't accelerate until $t = 1.00$ s, the correct answer is $\boxed{t = 15.8 \text{ s}}$.

51. Take the origin to be the location at which the speeder passes the police car. The speed of the speeder is v_s. The position of the speeder after the 7.00 seconds is $\Delta x_s = v_s t = v_s (7.00 \text{ s})$. The position of the police car is calculated based on the fact that the car traveled 1 second at the original velocity, and then 6 seconds under acceleration. Note that the police car's velocity must have the units changed.

$$v_p = (95 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 26.39 \text{ m/s} \qquad a_p = 2.00 \text{ m/s}^2$$

$$\Delta x_p = v_p (1.00 \text{ s}) + v_p (6.00 \text{ s}) + \tfrac{1}{2} a_p (6.00 \text{ s})^2 = 220.7 \text{ m}$$

The police car overtakes the speeder when these two distances are the same; i.e., $\Delta x_s = \Delta x_p$.

$$v_s (7 \text{ s}) = 220.7 \text{ m} \quad \rightarrow \quad v_s = \frac{220.7 \text{ m}}{7 \text{ s}} \left(\frac{3.6 \text{ km/h}}{1 \text{ m/s}} \right) = \boxed{114 \text{ km/h}}$$

52. Call east the positive x direction and north the positive y direction. From the first diagram, this relative velocity relationship is seen.
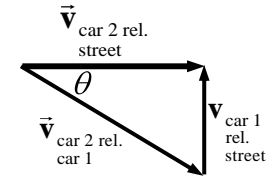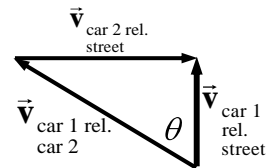
$$\vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{street}}} = \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{car 2}}} + \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{street}}} \quad \rightarrow \quad v_{\substack{\text{car 1 rel.}\\\text{car 2}}} = \sqrt{(-55)^2 + (35)^2} = \boxed{65 \text{ km/h}}$$

$$\theta = \tan^{-1} 55/35 = \boxed{58° \text{ West of North}}$$

For the other relative velocity relationship:

$$\vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{street}}} = \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{car 1}}} + \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{street}}} \quad \rightarrow \quad v_{\substack{\text{car 2 rel.}\\\text{car 1}}} = \sqrt{(55)^2 + (-35)^2} = \boxed{65 \text{ km/h}}$$

$$\theta = \tan^{-1} 35/55 = \boxed{32° \text{ South of East}}$$

Notice that the two relative velocities are opposites of each other: $\vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{car 1}}} = -\vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{car 2}}}$

53. Since the arrow will start and end at the same height, use the range formula derived in Example 3-8. The range is 27 m, and the initial speed of the arrow is 35 m/s.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow \quad \sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{(27 \text{ m})(9.80 \text{ m/s}^2)}{(35 \text{ m/s})^2} = 0.216$$
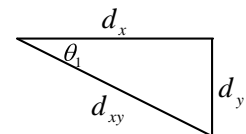
$$\theta_0 = \tfrac{1}{2} \sin^{-1} 0.216 = 6.2°, 83.8°$$

Only the first answer is practical, so the result is $\boxed{\theta_0 = 6.2°}$.

54. The plumber's displacement in component notation is $\boxed{\vec{\mathbf{d}} = (50 \text{ m}, -25 \text{ m}, -10 \text{ m})}$. Since this is a 3-dimensional problem, it requires 2 angles to determine his location (similar to latitude and longitude on the surface of the Earth). In the x-y plane, this follows.

$$\theta_1 = \tan^{-1} \frac{d_y}{d_x} = \tan^{-1} \frac{25}{50} = 27° \text{ South of East}$$

$$d_{xy} = \sqrt{d_x^2 + d_y^2} = \sqrt{(50)^2 + (-25)^2} = 55.9 \text{ m}$$

For the vertical motion, consider another right triangle, made up of d_{xy} as one leg, and the vertical displacement d_z as the other leg. See the second figure, and the following calculations.
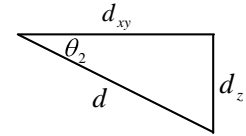
$$\theta_2 = \tan^{-1}\frac{d_z}{d_{xy}} = \tan^{-1}\frac{10 \text{ m}}{55.9 \text{ m}} = 10° \text{ Below the Horizontal}$$

$$d = \sqrt{d_{xy}^2 + d_z^2} = \sqrt{d_x^2 + d_y^2 + d_z^2} = \sqrt{(50)^2 + (-25)^2 + (-10)^2} = 57 \text{ m}$$

The result is that the displacement is $\boxed{57 \text{ m}}$, at an angle of $\boxed{27° \text{ South of East}}$, and $\boxed{10° \text{ Below the Horizontal}}$.

55. Assume a constant upward slope, and so the deceleration is along a straight line. The starting velocity along that line is $120 \text{ km/h}\left(\dfrac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 33.3 \text{ m/s}$. The ending velocity is 0 m/s. The acceleration is found from Eq. 2-11a.

$$v = v_0 + at \quad \rightarrow \quad 0 = 33.3 \text{ m/s} + a(6.0 \text{ s}) \quad \rightarrow \quad a = -\frac{33.3 \text{ m/s}}{6.0 \text{ s}} = -5.56 \text{ m/s}^2$$

The horizontal acceleration is $a_{\text{horiz}} = a\cos\theta = -5.56 \text{ m/s}^2(\cos 32°) = \boxed{-4.7 \text{ m/s}^2}$.
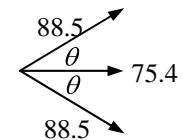
The vertical acceleration is $a_{\text{vert}} = a\sin\theta = -5.56 \text{ m/s}^2(\sin 30°) = \boxed{-2.8 \text{ m/s}^2}$

The horizontal acceleration is to the left in the textbook diagram, and the vertical acceleration is down.

56. Magnitude $= \sqrt{75.4^2 + y^2} = 88.5 \quad \rightarrow \quad y = \pm\sqrt{88.5^2 - 75.4^2} = \pm 46.34 = \boxed{\pm 46.3}$

Direction $= \tan^{-1}\dfrac{46.34}{75.4} = \boxed{31.6° \text{ relative to } x \text{ axis}}$

See the diagram for the two possible answers.

57. Choose the x direction to be the direction of train travel (the direction the passenger is facing) and choose the y direction to be up. This relationship exists among the velocities: $\vec{v}_{\substack{\text{rain rel.}\\\text{ground}}} = \vec{v}_{\substack{\text{rain rel.}\\\text{train}}} + \vec{v}_{\substack{\text{train rel.}\\\text{ground}}}$. From the diagram, find the expression for the speed of the raindrops.
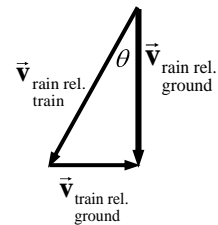
$$\tan\theta = \frac{v_{\substack{\text{train rel.}\\\text{ground}}}}{v_{\substack{\text{rain rel.}\\\text{ground}}}} = \frac{v_T}{v_{\substack{\text{rain rel.}\\\text{ground}}}} \quad \rightarrow \quad \boxed{v_{\substack{\text{rain rel.}\\\text{ground}}} = \frac{v_T}{\tan\theta}}$$

58. Call east the positive x direction and north the positive y direction. Then this relative velocity relationship follows (see the accompanying diagram).

$$\vec{v}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{v}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{v}_{\substack{\text{air rel.}\\\text{ground}}}$$

Equate the x components of the velocity vectors.
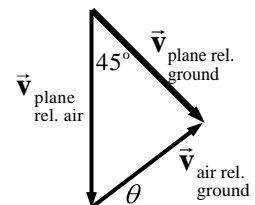
$$\left(125\,\text{km/h}\right)\cos 45^\circ = 0 + v_{\text{wind }x} \quad \rightarrow \quad v_{\text{wind }x} = 88.4\,\text{km/h}\ .$$

From the *y* components of the above equation:

$$-125\sin 45^\circ = -155 + v_{\text{wind-y}} \quad \rightarrow \quad v_{\text{wind-y}} = 155 - 125\sin 45^\circ = 66.6\,\text{km/h}$$

The magnitude of the wind velocity is

$$v_{\text{wind}} = \sqrt{v_{\text{wind-x}}^2 + v_{\text{wind-y}}^2} = \sqrt{\left(88.4\,\text{km/h}\right)^2 + \left(66.6\,\text{km/h}\right)^2} = \boxed{111\,\text{km/h}}\ .$$

The direction of the wind is $\theta = \tan^{-1}\dfrac{v_{\text{wind-y}}}{v_{\text{wind-x}}} = \tan^{-1}\dfrac{66.6}{88.4} = \boxed{37.0^\circ \ \text{north of east}}$.

59. Work in the frame of reference in which the train is at rest. Then, relative to the train, the car is moving at 20 km/h. The car has to travel 1 km in that frame of reference to pass the train, and so the time to pass can be found from the constant horizontal velocity relationship.

$$\Delta x = v_x t \quad \rightarrow \quad t_{\substack{\text{same}\\\text{direction}}} = \frac{\Delta x}{\left(v_x\right)_{\substack{\text{same}\\\text{direction}}}} = \frac{1\,\text{km}}{20\,\text{km/h}} = 0.05\,\text{h}\left(\frac{3600\,\text{s}}{1\,\text{h}}\right) = \boxed{180\,\text{s}}$$

The car travels 1 km in the frame of reference of the stationary train, but relative to the ground, the car is traveling at 95 km/hr and so relative to the ground the car travels this distance:

$$\Delta x = v_x t_{\substack{\text{same}\\\text{direction}}} = \left(95\,\text{km/h}\right)\left(0.05\,\text{h}\right) = \boxed{4.8\,\text{km}}$$

If the car and train are traveling in opposite directions, then the velocity of the car relative to the train will be 170 km/h. Thus the time to pass will be

$$t_{\substack{\text{opposite}\\\text{direction}}} = \frac{\Delta x}{\left(v_x\right)_{\substack{\text{opposite}\\\text{direction}}}} = \frac{1\,\text{km}}{170\,\text{km/h}} = \left(\frac{1}{170}\,\text{h}\right)\left(\frac{3600\,\text{s}}{1\,\text{h}}\right) = \boxed{21.2\,\text{s}}\ .$$

The distance traveled by the car relative to the ground will be

$$\Delta x = v_x t_{\substack{\text{opposite}\\\text{direction}}} = \left(95\,\text{km/h}\right)\left(\frac{1}{170}\,\text{h}\right) = \boxed{0.56\,\text{km}}\ .$$

60. The time of flight is found from the constant velocity relationship for horizontal motion.

$$\Delta x = v_x t = \quad \rightarrow \quad t = \Delta x / v_x = 8.0\,\text{m}/9.1\,\text{m/s} = \boxed{0.88\,\text{s}}$$

The *y* motion is symmetric in time – it takes half the time of flight to rise, and half to fall. Thus the time for the jumper to fall from his highest point to the ground is 0.44 sec. His vertical speed is zero at the highest point. From this time, starting vertical speed, and the acceleration of gravity, the maximum height can be found. Call upward the positive *y* direction. The point of maximum height is the starting position y_0, the ending position is $y = 0$, the starting vertical speed is 0, and $a = -g$. Use Eq. 2-11b to find the height.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = y_0 + 0 - \tfrac{1}{2}\left(9.8\,\text{m/s}^2\right)\left(0.44\,\text{s}\right)^2 \quad \rightarrow \quad y_0 = \boxed{0.95\,\text{m}}\ .$$

61. Assume that the golf ball takes off and lands at the same height, so that the range formula derived in Example 3-8 can be applied. The only variable is to be the acceleration due to gravity.

$$R_{\text{Earth}} = v_0^2 \sin 2\theta_0 / g_{\text{Earth}} \qquad R_{\text{Moon}} = v_0^2 \sin 2\theta_0 / g_{\text{Moon}}$$

$$\frac{R_{\text{Earth}}}{R_{\text{Moon}}} = \frac{v_0^2 \sin 2\theta_0 / g_{\text{Earth}}}{v_0^2 \sin 2\theta_0 / g_{\text{Moon}}} = \frac{1/g_{\text{Earth}}}{1/g_{\text{Moon}}} = \frac{g_{\text{Moon}}}{g_{\text{Earth}}} = \frac{35\,\text{m}}{180\,\text{m}} = 0.19 \quad \rightarrow$$

$$g_{\text{Moon}} = 0.19\,g_{\text{Earth}} \approx \boxed{1.9\,\text{m/s}^2}$$

62. The minimum speed will be that for which the ball just clears the fence; i.e., the ball has a height of 7.5 m when it is 95 m horizontally from home plate. The origin is at home plate, with upward as the positive y direction. For the ball, $y_0 = 1.0$ m, $y = 7.5$ m, $a_y = -g$, $v_{y0} = v_0 \sin\theta_0$, $v_x = v_0 \cos\theta_0$, and $\theta_0 = 38°$. See the diagram (not to scale). For the horizontal motion at constant velocity,
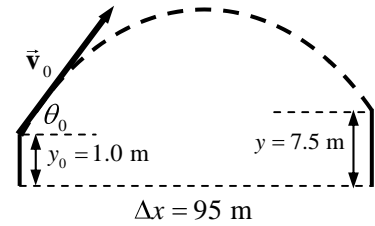
$\Delta x = v_x t = v_0 \cos\theta_0 t$, and so $t = \dfrac{\Delta x}{v_0 \cos\theta_0}$. For the vertical motion, apply Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = y_0 + v_0\left(\sin\theta_0\right)t - \tfrac{1}{2}gt^2$$

Substitute the value of the time of flight for the first occurrence only in the above equation, and then solve for the time.

$$y = y_0 + v_0 t \sin\theta_0 - \tfrac{1}{2}gt^2 \;\rightarrow\; y = y_0 + v_0 \sin\theta_0 \dfrac{\Delta x}{v_0 \cos\theta_0} - \tfrac{1}{2}gt^2 \;\rightarrow\;$$

$$t = \sqrt{2\left(\dfrac{y_0 - y + \Delta x \tan\theta_0}{g}\right)} = \sqrt{2\left(\dfrac{1.0\text{ m} - 7.5\text{ m} + (95\text{ m})\tan 38°}{9.80\,\text{m}/\text{s}^2}\right)} = 3.718\text{ s}$$

Finally, use the time with the horizontal range to find the initial speed.

$$\Delta x = v_0 \cos\theta_0 t \;\rightarrow\; v_0 = \dfrac{\Delta x}{t\cos\theta_0} = \dfrac{95\text{ m}}{(3.718\text{ s})\cos 38°} = \boxed{32\,\text{m}/\text{s}}$$

63. Choose downward to be the positive y direction. The origin is at the point from which the divers push off the cliff. In the vertical direction, the initial velocity is $v_{y0} = 0$, the acceleration is $a_y = 9.80\,\text{m}/\text{s}^2$, and the displacement is 35 m. The time of flight is found from Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 35\text{ m} = 0 + 0 + \tfrac{1}{2}\left(9.8\,\text{m}/\text{s}^2\right)t^2 \;\rightarrow\; t = \sqrt{\dfrac{2(35\text{ m})}{9.8\,\text{m}/\text{s}^2}} = \boxed{2.7\text{ s}}$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \;\rightarrow\; v_x = \Delta x / t = 5.0\text{ m}/2.7\text{ s} = \boxed{1.9\,\text{m}/\text{s}}$$

64. Choose the origin to be the location on the ground directly underneath the ball when served, and choose upward as the positive y direction. Then for the ball, $y_0 = 2.50$ m, $v_{y0} = 0$, $a_y = -g$, and the y location when the ball just clears the net is $y = 0.90$ m. The time for the ball to reach the net is calculated from Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 0.90\text{ m} = 2.50\text{ m} + 0 + \tfrac{1}{2}\left(-9.80\,\text{m}/\text{s}^2\right)t^2 \;\rightarrow\;$$

$$t_{\substack{\text{to}\\\text{net}}} = \sqrt{\dfrac{2(-1.60\text{ m})}{-9.80\,\text{m}/\text{s}^2}} = 0.57143\text{ s}$$

The x velocity is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \;\rightarrow\; v_x = \dfrac{\Delta x}{t} = \dfrac{15.0\text{ m}}{0.57143\text{ s}} = 26.25 \approx \boxed{26.3\,\text{m}/\text{s}}\;.$$

This is the minimum speed required to clear the net.

To find the full time of flight of the ball, set the final y location to be $y = 0$, and again use Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0.0 \text{ m} = 2.50 \text{ m} + \tfrac{1}{2}\left(-9.80 \text{ m/s}^2\right)t^2 \quad \rightarrow$$

$$t_{\text{total}} = \sqrt{\frac{2(-2.50 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.7143 \approx \boxed{0.714 \text{ s}}$$

The horizontal position where the ball lands is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (26.25 \text{ m/s})(0.7143 \text{ s}) = 18.75 \approx \boxed{18.8 \text{ m}}.$$

Since this is between 15.0 and 22.0 m, $\boxed{\text{the ball lands in the "good" region}}$.

65. Work in the frame of reference in which the car is at rest at ground level. In this reference frame, the helicopter is moving horizontally with a speed of

$$215 \text{ km/h} - 155 \text{ km/h} = 60 \text{ km/h}\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 16.67 \text{ m/s} .$$

For the vertical motion, choose the level of the helicopter to be the origin, and downward to be positive. Then the package's y displacement is $y = 78.0 \text{ m}$, $v_{y0} = 0$, and $a_y = g$. The time for the package to fall is calculated from Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 78.0 \text{ m} = \tfrac{1}{2}\left(9.80 \text{ m/s}^2\right)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(78.0 \text{ m})}{9.80 \text{ m/s}^2}} = 3.99 \text{ sec}$$

The horizontal distance that the package must move, relative to the "stationary" car, is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (16.67 \text{ m/s})(3.99 \text{ s}) = 66.5 \text{ m}$$

Thus the angle under the horizontal for the package release will be

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{78.0 \text{ m}}{66.5 \text{ m}}\right) = 49.55° \approx \boxed{50°} \text{ (to 2 significant figures).}$$

66. (*a*) For the upstream trip, the boat will cover a distance of $D/2$ with a net speed of $v - u$, so the time is $t_1 = \dfrac{D/2}{v-u} = \dfrac{D}{2(v-u)}$. For the downstream trip, the boat will cover a distance of $D/2$ with a net speed of $v + u$, so the time is $t_2 = \dfrac{D/2}{v+u} = \dfrac{D}{2(v+u)}$. Thus the total time for the round trip will be $t = t_1 + t_2 = \dfrac{D}{2(v-u)} + \dfrac{D}{2(v+u)} = \boxed{\dfrac{Dv}{(v^2-u^2)}}$.

(*b*) For the boat to go directly across the river, it must be angled against the current in such a way that the net velocity is straight across the river, as in the picture. This equation must be satisfied:

$$\underset{\substack{\text{shore}}}{\vec{V}_{\text{boat rel.}}} = \underset{\substack{\text{water}}}{\vec{V}_{\text{boat rel.}}} + \underset{\substack{\text{shore}}}{\vec{V}_{\text{water rel.}}} = \vec{V} + \vec{u} .$$

Thus $\underset{\substack{\text{shore}}}{v_{\text{boat rel.}}} = \sqrt{v^2 - u^2}$, and the time to go a distance $D/2$ across
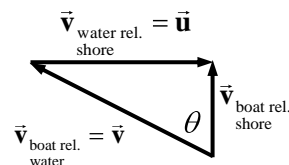
the river is $t_1 = \dfrac{D/2}{\sqrt{v^2 - u^2}} = \dfrac{D}{2\sqrt{v^2 - u^2}}$. The same relationship would be in effect for crossing

back, so the time to come back is given by $t_2 = t_1$ and the total time is $t = t_1 + t_2 = \boxed{\dfrac{D}{\sqrt{v^2 - u^2}}}$.

The speed v must be greater than the speed u . The velocity of the boat relative to the shore when going upstream is $v - u$. If $v < u$, the boat will not move upstream at all, and so the first part of the trip would be impossible. Also, in part b, we see that v is longer than u in the triangle, since v is the hypotenuse.

67. Choose the origin to be the point from which the projectile is launched, and choose upward as the positive y direction. The y displacement of the projectile is 155 m, and the horizontal range of the projectile is 195 m. The acceleration in the y direction is $a_y = -g$, and the time of flight is 7.6 s.

The horizontal velocity is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \frac{\Delta x}{t} = \frac{195 \text{ m}}{7.6 \text{ s}} = 25.7 \text{ m/s}$$

Calculate the initial y velocity from the given data and Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 155 \text{ m} = v_{y0}(7.6 \text{ s}) + \tfrac{1}{2}\left(-9.80 \text{ m/s}^2\right)(7.6 \text{ s})^2 \quad \rightarrow \quad v_{y0} = 57.6 \text{ m/s}$$

Thus the initial velocity and direction of the projectile are:

$$v_0 = \sqrt{v_x^2 + v_{y0}^2} = \sqrt{(25.7 \text{ m/s})^2 + (57.6 \text{ m/s})^2} = \boxed{63 \text{ m/s}}$$

$$\theta = \tan^{-1}\frac{v_{y0}}{v_x} = \tan^{-1}\frac{57.6 \text{ m/s}}{25.7 \text{ m/s}} = \boxed{66^\circ}$$

68. Choose downward to be the positive y direction for this problem.
  (*a*) The vertical component of her acceleration is directed downward, and its magnitude will be given by $a_y = a \sin\theta = \left(1.80 \text{ m/s}^2\right)\sin 30.0^\circ = \boxed{0.900 \text{ m/s}^2}$.
  (*b*) The time to reach the bottom of the hill is calculated from Eq. 2-11b, with a y displacement of 335 m, $v_{y0} = 0$, and $a_y = 0.900 \text{ m/s}^2$.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 335 \text{ m} = 0 + 0 + \tfrac{1}{2}\left(0.900 \text{ m/s}^2\right)(t)^2 \quad \rightarrow$$

$$t = \sqrt{\frac{2(335 \text{ m})}{\left(0.900 \text{ m/s}^2\right)}} = \boxed{27.3 \text{ s}}$$

69. The proper initial speeds will be those for which the ball has traveled a horizontal distance somewhere between 10.78 m and 11.22 m while it changes height from 2.10 m to 2.60 m with a shooting angle of 38.0° . Choose the origin to be at the shooting location of the basketball, with upward as the positive y direction. Then the vertical displacement is $y = 0.5 \text{ m}$, $a_y = -9.80 \text{ m/s}^2$, $v_{y0} = v_0 \sin\theta_0$, and the (constant) x velocity is $v_x = v_0 \cos\theta_0$. See the diagram (not to scale).

For the horizontal motion at constant velocity,

$$\Delta x = v_x t = v_0 \cos\theta_0 t \text{ and so } t = \frac{\Delta x}{v_0 \cos\theta_0} .$$

θ_0

$\Delta y = 0.5 \text{ m}$

$\Delta x = 10.78 \text{ m} - 11.22 \text{ m}$
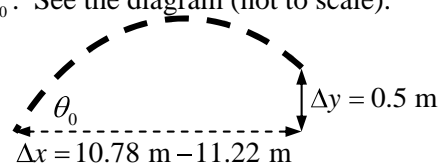
For the vertical motion, applying Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = v_0 \sin\theta t - \tfrac{1}{2}gt^2$$

Substitute the expression for the time of flight and solve for the initial velocity.

$$y = v_0 \sin\theta t - \tfrac{1}{2}gt^2 = v_0 \sin\theta \frac{\Delta x}{v_0 \cos\theta_0} - \tfrac{1}{2}g\left(\frac{\Delta x}{v_0 \cos\theta_0}\right)^2 = \Delta x \tan\theta - \frac{g(\Delta x)^2}{2v_0^2 \cos^2\theta_0}$$

$$v_0 = \sqrt{\frac{g(\Delta x)^2}{2\cos^2\theta_0(-y + \Delta x \tan\theta)}}$$

For $\Delta x = 10.78$ m , the shortest shot:

$$v_0 = \sqrt{\frac{(9.80\,\text{m/s}^2)(10.78\text{ m})^2}{2\cos^2 38.0°\left[(-0.5\text{ m}+(10.78\text{ m})\tan 38.0°)\right]}} = \boxed{10.8\,\text{m/s}}.$$

For $\Delta x = 11.22$ m , the longest shot:

$$v_0 = \sqrt{\frac{(9.80\,\text{m/s}^2)(11.22\text{ m})^2}{2\cos^2 38.0°\left[(-0.5\text{ m}+(11.22\text{ m})\tan 38.0°)\right]}} = \boxed{11.0\,\text{m/s}}.$$

70. Choose the origin to be the location at water level directly underneath the diver when she left the board. Choose upward as the positive y direction. For the diver, $y_0 = 5.0$ m , the final y position is $y = 0.0$ m (water level), $a_y = -g$, the time of flight is $t = 1.3$ s , and the horizontal displacement is $\Delta x = 3.0$ m .

(a) The horizontal velocity is determined from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \frac{\Delta x}{t} = \frac{3.0\text{ m}}{1.3\text{ s}} = 2.31\,\text{m/s}$$

The initial y velocity is found using Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \rightarrow 0\text{ m} = 5.0\text{ m} + v_{y0}(1.3\text{ s}) + \tfrac{1}{2}(-9.80\,\text{m/s}^2)(1.3\text{ s})^2 \rightarrow$$

$$v_{y0} = 2.52\,\text{m/s}$$

The magnitude and direction of the initial velocity is

$$v_0 = \sqrt{v_x^2 + v_{y0}^2} = \sqrt{(2.31\,\text{m/s})^2 + (2.52\,\text{m/s})^2} = \boxed{3.4\,\text{m/s}}$$

$$\theta = \tan^{-1}\frac{v_{y0}}{v_x} = \tan^{-1}\frac{2.52\,\text{m/s}}{2.31\,\text{m/s}} = \boxed{48°\text{ above the horizontal}}$$

(b) The maximum height will be reached when the y velocity is zero. Use Eq. 2-11c.

$$v_y^2 = v_{y0}^2 + 2a\Delta y \rightarrow 0 = (2.52\,\text{m/s})^2 + 2(-9.80\,\text{m/s}^2)(y_{max} - 5.0\text{ m}) \rightarrow$$

$$y_{max} = \boxed{5.3\text{ m}}$$

(c) To find the velocity when she enters the water, the horizontal velocity is the (constant) value of $v_x = 2.31\,\text{m/s}$. The vertical velocity is found from Eq. 2-11a.

$$v_y = v_{y0} + at = 2.52\,\text{m/s} + (-9.80\,\text{m/s}^2)(1.3\text{ s}) = -10.2\,\text{m/s} .$$

The magnitude and direction of this velocity is given by

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(2.31\,\text{m/s})^2 + (-10.2\,\text{m/s})^2} = 10.458\,\text{m/s} \approx \boxed{10\,\text{m/s}}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{-10.2\,\text{m/s}}{2.31\,\text{m/s}} = \boxed{-77^\circ \;(\text{below the horizontal})}$$

71. (*a*) Choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive *y* direction. At the end of its flight over the 8 cars, the car must be at $y = -1.5$ m .

Also for the car, $v_{y0} = 0$, $a_y = -g$, $v_x = v_0$, and $\Delta x = 20$ m . The time of flight is found from the horizontal motion at constant velocity: $\Delta x = v_x t \;\rightarrow\; t = \Delta x/v_0$. That expression for the time is used in Eq. 2-11b for the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; y = 0 + 0 + \tfrac{1}{2}(-g)\left(\frac{\Delta x}{v_0}\right)^2 \;\rightarrow\;$$

$$v_0 = \sqrt{\frac{-g\,(\Delta x)^2}{2(y)}} = \sqrt{\frac{-(9.80\,\text{m/s}^2)(20\text{ m})^2}{2(-1.5\text{ m})}} = \boxed{36\,\text{m/s}}$$

(*b*) Again choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive *y* direction. The *y* displacement of the car at the end of its flight over the 8 cars must again be $y = -1.5$ m . For the car, $v_{y0} = v_0 \sin\theta_0$, $a_y = -g$, $v_x = v_0 \cos\theta_0$, and $\Delta x = 20$ m . The launch angle is $\theta_0 = 10^\circ$. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \;\rightarrow\; t = \frac{\Delta x}{v_0 \cos\theta_0}$$

That expression for the time is used in Eq. 2-11b for the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; y = v_0 \sin\theta_0 \frac{\Delta x}{v_0 \cos\theta_0} + \tfrac{1}{2}(-g)\left(\frac{\Delta x}{v_0 \cos\theta_0}\right)^2 \;\rightarrow\;$$

$$v_0 = \sqrt{\frac{g\,(\Delta x)^2}{2(\Delta x \tan\theta_0 - y)\cos^2\theta_0}} = \sqrt{\frac{(9.80\,\text{m/s}^2)(20\text{ m})^2}{2((20\text{ m})\tan 10^\circ + 1.5\text{ m})\cos^2 10^\circ}} = \boxed{20\,\text{m/s}}$$

72. Choose the origin to be the point at ground level directly below where the ball was hit. Call upwards the positive *y* direction. For the ball, we have $v_0 = 28\,\text{m/s}$, $\theta_0 = 61^\circ$, $a_y = -g$, $y_0 = 0.9$ m , and $y = 0.0$ m .

(*a*) To find the horizontal displacement of the ball, the horizontal velocity and the time of flight are needed. The (constant) horizontal velocity is given by $v_x = v_0 \cos\theta_0$. The time of flight is found from Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 0 = y_0 + v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \;\rightarrow\;$$

$$t = \frac{-v_0 \sin\theta_0 \pm \sqrt{v_0^2 \sin^2\theta_0 - 4\left(-\tfrac{1}{2}g\right)y_0}}{2\left(-\tfrac{1}{2}g\right)}$$

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$$= \frac{-(28\,\text{m/s})\sin 61° \pm \sqrt{(28\,\text{m/s})^2 \sin^2 61° - 4\left(-\frac{1}{2}\right)\left(9.80\,\text{m/s}^2\right)(0.9\,\text{m})}}{2\left(-\frac{1}{2}\right)\left(9.80\,\text{m/s}^2\right)}$$

$= 5.034\,\text{s}, -0.0365\,\text{s}$

Choose the positive time, since the ball was hit at $t = 0$. The horizontal displacement of the ball will be found by the constant velocity relationship for horizontal motion.

$$\Delta x = v_x t = v_0 \cos\theta_0 t = (28\,\text{m/s})(\cos 61°)(5.034\text{s}) = 68.34\,\text{m} \approx \boxed{68\,\text{m}}$$

(b) The center fielder catches the ball right at ground level. He ran $105\,\text{m} - 68.34\,\text{m} = 36.66\,\text{m}$ to catch the ball, so his average running speed would be

$$v_{avg} = \frac{\Delta d}{t} = \frac{36.66\,\text{m}}{5.034\,\text{s}} = 7.282\,\text{m/s} \approx \boxed{7.3\,\text{m/s}}$$

73. Since the ball is being caught at the same height from which it was struck, use the range formula to find the horizontal distance the ball travels.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(32\,\text{m/s})^2 \sin(2 \times 55°)}{9.8\,\text{m/s}^2} = 98.188\,\text{m}$$

Then as seen from above, the location of home plate, the point where the ball must be caught, and the initial location of the outfielder are shown in the diagram. The dark arrow shows the direction in which the outfielder must run. The length of that distance is found from the law of cosines as applied to the triangle.

$$x = \sqrt{a^2 + b^2 - 2ab \cos\theta}$$

$$= \sqrt{98.188^2 + 85^2 - 2(98.188)(85)\cos 22°} = 37.27\,\text{m}$$

The angle θ at which the outfielder should run is found from the law of sines.

$$\frac{\sin 22°}{x} = \frac{\sin\theta}{98\,\text{m}} \quad \rightarrow \quad \theta = \sin^{-1}\left(\frac{98.188}{37.27}\sin 22°\right) = 81° \text{ or } 99°$$

Since $98.188^2 > 85^2 + 37.27^2$, the angle must be obtuse, so we choose $\theta = 97°$.
Assume that the outfielder's time for running is the same as the time of flight of the ball. The time of flight of the ball is found from the horizontal motion of the ball at constant velocity.

$$R = v_x t = v_0 \cos\theta_0 t \quad \rightarrow \quad t = \frac{R}{v_0 \cos\theta_0} = \frac{98.188\,\text{m}}{(32\,\text{m/s})\cos 55°} = 5.35\,\text{s}$$

Thus the average velocity of the outfielder must be $v_{avg} = \frac{\Delta d}{t} = \frac{37.27\,\text{m}}{5.35\,\text{s}} = \boxed{7.0\,\text{m/s}}$ at an angle of

$\boxed{97°}$ relative to the outfielder's line of sight to home plate.
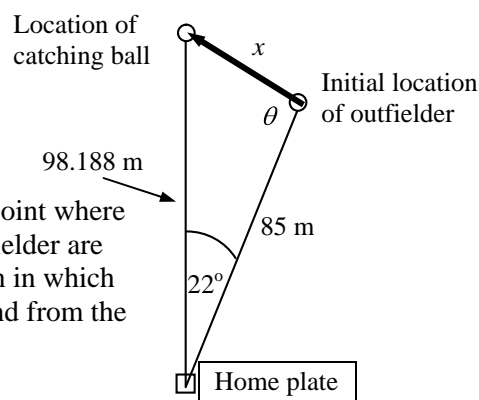
74. Choose the origin to be the point at the top of the building from which the ball is shot, and call upwards the positive y direction. The initial velocity is $v_0 = 18\,\text{m/s}$ at an angle of $\theta_0 = 42°$. The acceleration due to gravity is $a_y = -g$.

(a) $v_x = v_0 \cos\theta_0 = (18\,\text{m/s})\cos 42° = 13.38 \approx \boxed{13\,\text{m/s}}$

$v_{y0} = v_0 \sin\theta_0 = (18\,\text{m/s})\sin 42° = 12.04 \approx \boxed{12\,\text{m/s}}$

(*b*) Since the horizontal velocity is known and the horizontal distance is known, the time of flight can be found from the constant velocity equation for horizontal motion.

$$\Delta x = v_x t \quad \rightarrow \quad t = \frac{\Delta x}{v_x} = \frac{55 \text{ m}}{13.38 \text{ m/s}} = 4.111 \text{ s}$$

With that time of flight, calculate the vertical position of the ball using Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = \left(12.04 \text{ m/s}\right)\left(4.111 \text{ s}\right) + \tfrac{1}{2}\left(-9.80 \text{ m/s}^2\right)\left(4.111 \text{ s}\right)^2$$

$$= -33.3 = \boxed{-33 \text{ m}}$$

So the ball will strike 33 m below the top of the building.

75. When shooting the gun vertically, half the time of flight is spent moving upwards. Thus the upwards flight takes two seconds. Choose upward as the positive *y* direction. Since at the top of the flight, the vertical velocity is zero, find the launching velocity from Eq. 2-11a.

$$v_y = v_{y0} + at \quad \rightarrow \quad v_{y0} = v_y - at = 0 = \left(9.8 \text{ m/s}^2\right)\left(2.0 \text{ s}\right) = 19.6 \text{ m/s}$$

Using this initial velocity and an angle of 45° in the range formula will give the maximum range for the gun.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{\left(19.6 \text{ m/s}\right)^2 \sin\left(2 \times 45°\right)}{9.80 \text{ m/s}^2} = \boxed{39 \text{ m}}$$