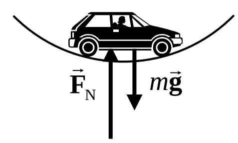
CHAPTER 5: Circular Motion; Gravitation

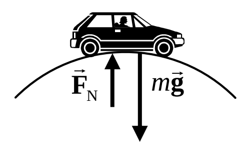
Answers to Questions

1. The problem with the statement is that there is nothing to cause an outward force, and so the water removed from the clothes is not thrown outward. Rather, the spinning drum pushes INWARD on the clothes and water. But where there are holes in the drum, the drum can't push on the water, and so the water is not pushed in. Instead, the water moves tangentially to the rotation, out the holes, in a straight line, and so the water is separated from the clothes.

2. The centripetal acceleration for an object moving in circular motion is inversely proportional to the radius of the curve, given a constant speed $\left(a = v^2/r\right)$. So for a gentle curve (which means a large radius), the acceleration is smaller, while for a sharp curve (which means a small radius), the acceleration is larger.

3. The force that the car exerts on the road is the Newton's 3rd law reaction to the normal force of the road on the car, and so we can answer this question in terms of the normal force. The car exerts the greatest force on the road at the dip between two hills. There the normal force from the road has to both support the weight AND provide a centripetal upward force to make the car move in an upward curved path. The car exerts the least force on the road at the top of a hill. We have all felt the "floating upward" sensation as we have driven over the crest of a hill. In that case, there must be a net downward centripetal force to cause the circular motion, and so the normal force from the road does not completely support the weight.





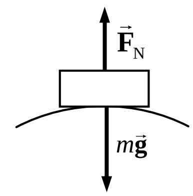
4. There are at least three distinct major forces on the child. The force of gravity is acting downward on the child. There is a normal force from the seat of the horse acting upward on the child. There must be friction between the seat of the horse and the child as well, or the child could not be accelerated by the horse. It is that friction that provides the centripetal acceleration. There may be smaller forces as well, such as a reaction force on the child's hands if the child is holding on to part of the horse. Any force that has a radially inward component will contribute to the centripetal acceleration.

5. For the water to remain in the bucket, there must be a centripetal force forcing the water to move in a circle along with the bucket. That centripetal force gets larger with the tangential velocity of the water, since $F_R = mv^2/r$. The centripetal force at the top of the motion comes from a combination of the downward force of gravity and the downward normal force of the bucket on the water. If the bucket is moving faster than some minimum speed, the water will stay in the bucket. If the bucket is moving too slow, there is insufficient force to keep the water moving in the circular path, and it spills out.

6. The three major "accelerators" are the accelerator pedal, the brake pedal, and the steering wheel. The accelerator pedal (or gas pedal) can be used to increase speed (by depressing the pedal) or to decrease speed in combination with friction (by releasing the pedal). The brake pedal can be used to decrease speed by depressing it. The steering wheel is used to change direction, which also is an acceleration. There are some other controls which could also be considered accelerators. The parking brake can be used to decrease speed by depressing it. The gear shift lever can be used to decrease speed by downshifting. If the car has a manual transmission, then the clutch can be used to

decrease speed by depressing it (friction will slow the car). Finally, shutting the car off can be used to decrease its speed. Any change in speed or direction means that an object is accelerating.

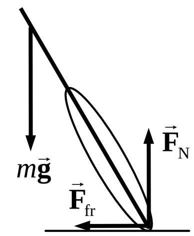
7. When the child is on a level surface, the normal force between his chest and the sled is equal to the child's weight, and thus he has no vertical acceleration. When he goes over the hill, the normal force on him will be reduced. Since the child is moving on a curved path, there must be a net centripetal force towards the center of the path, and so the normal force does not completely support the weight. Write Newton's 2nd law for the radial direction, with inward as positive.



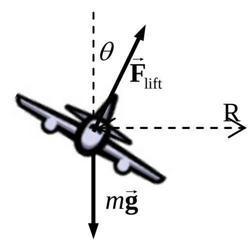
$$\sum F_R = mg - F_N = mv^2/r \quad \rightarrow \quad F_N = mg - mv^2/r$$

We see that the normal force is reduced from *mg* by the centripetal force.

8. When a bicycle rider leans inward, the bike tire pushes down on the ground at an angle. The road surface then pushes back on the tire both vertically (to provide the normal force which counteracts gravity) and horizontally toward the center of the curve (to provide the centripetal frictional force, enabling them to turn).

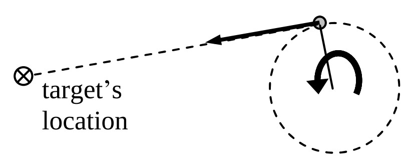


9. Airplanes bank when they turn because in order to turn, there must be a force that will be exerted towards the center of a circle. By tilting the wings, the lift force on the wings has a non-vertical component which points toward the center of the curve, providing the centripetal force. The banking angle can be computed from the free-body diagram. The sum of vertical forces must be zero for the plane to execute a level turn, and so $F_{lift}\cos\theta = mg$. The horizontal component of the lifting force must provide the centripetal force to move the airplane in a circle.



$$F_{lift}\sin\theta = mv^2/r \quad \rightarrow \quad \frac{mg}{\cos\theta}\sin\theta = mv^2/r \quad \rightarrow \quad \tan\theta = \frac{v^2}{Rg}$$

10. She should let go of the string when the ball is at a position where the tangent line to the circle at the ball's location, when extended, passes through the target's position. That tangent line indicates the direction of the velocity at that instant, and if the centripetal force is removed, then the ball will follow that line horizontally. See the top-view diagram.



11. The apple does exert a gravitational force on the Earth. By Newton's 3rd law, the force on the Earth due to the apple is the same magnitude as the force on the apple due to the Earth – the weight of the apple. The force is also independent of the state of motion of the apple. So for both a hanging apple and a falling apple, the force on the Earth due to the apple is equal to the weight of the apple.

12. The gravitational force on the Moon is given by $G\dfrac{M_{Earth}M_{Moon}}{R^2}$, where *R* is the radius of the Moon's orbit. This is a radial force, and so can be expressed as $M_{Moon}\,v_{Moon}^2/R$. This can be changed using the relationship $v_{Moon} = 2\pi R/T$, where *T* is the orbital period of the Moon, to $4\pi^2 M_{Moon}\,R/T^2$. If we equate these two expressions for the force, we get the following:

$G\frac{M_{\text{Earth}}M_{\text{Moon}}}{R^2} = 4\pi^2 M_{\text{Moon}} R/T^2 \quad \rightarrow \quad \frac{R^3}{T^2} = \frac{GM_{\text{Earth}}}{4\pi^2}$. Thus the mass of the Earth determines the ratio R^3/T^2. If the mass of the Earth were doubled, then the ratio R^3/T^2 would double, and so $R'^3/T'^2 = R^3/T^2$, where the primes indicated the "after doubling" conditions. For example, the radius might stay the same, and the period decrease by a factor of $\sqrt{2}$, which means the speed increased by a factor of $\sqrt{2}$. Or the period might stay the same, and the radius increase by a factor of $2^{1/3}$, which means the speed increased by the same factor of $2^{1/3}$. Or if both R and T were to double, keeping the speed constant, then R^3/T^2 would double. There are an infinite number of other combinations that would also satisfy the doubling of R^3/T^2.

13. The gravitational pull is the same in each case, by Newton's 3rd law. The magnitude of that pull is given by $F = G\frac{M_{\text{Earth}}M_{\text{Moon}}}{r^2_{\text{Earth-Moon}}}$. To find the acceleration of each body, the gravitational pulling force is divided by the mass of the body. Since the Moon has the smaller mass, it will have the larger acceleration.

14. The <u>difference</u> in force on the two sides of the Earth from the gravitational pull of either the Sun or the Moon is the primary cause of the tides. That difference in force comes about from the fact that the two sides of the Earth are a different distance away from the pulling body. Relative to the Sun, the difference in distance (Earth diameter) of the two sides from the Sun, relative to the average distance to the Sun, is given by $2R_{\text{Earth}}/R_{\substack{\text{Earth}\\\text{to Sun}}} = 8.5\times10^{-5}$. The corresponding relationship between the Earth and the Moon is $2R_{\text{Earth}}/R_{\substack{\text{Earth}\\\text{to Moon}}} = 3.3\times10^{-2}$. Since the relative change in distance is much greater for the Earth-Moon combination, we see that the Moon is the primary cause of the Earth's tides.

15. An object weighs more at the poles, due to two effects which complement (not oppose) each other. First of all, the Earth is slightly flattened at the poles and expanded at the equator, relative to a perfect sphere. Thus the mass at the poles is slightly closer to the center, and so experiences a slightly larger gravitational force. Secondly, objects at the equator have a centripetal acceleration due to the rotation of the Earth that objects at the poles do not have. To provide that centripetal acceleration, the apparent weight (the radially outward normal force of the Earth on an object) is slightly less than the gravitational pull inward. So the two effects both make the weight of an object at the equator less than that at the poles.

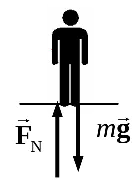
16. The Moon is not pulled away from the Earth because both the Moon and the Earth are experiencing the same radial acceleration due to the Sun. They both have the same period around the Sun because they are both, on average, the same distance from the Sun, and so they travel around the Sun together.

17. The centripetal acceleration of Mars is smaller than that of Earth. The acceleration of each planet can be found by dividing the gravitational force on each planet by the planet's mass. The resulting acceleration is inversely proportional to the square of the distance of the planet from the Sun. Since Mars is further from the Sun than the Earth is, the acceleration of Mars will be smaller. Also see the equation below.

$$F_{\text{on planet}} = G\frac{M_{\text{sun}}M_{\text{planet}}}{r^2_{\text{Sun to planet}}} \qquad a_{\text{planet}} = \frac{F_{\text{on planet}}}{M_{\text{planet}}} = G\frac{M_{\text{sun}}}{r^2_{\text{Sun to planet}}}$$

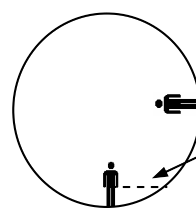
18. In order to orbit, a satellite must reach an orbital speed relative to the center of the Earth. Since the satellite is already moving eastward when launched (due to the rotation speed at the surface of the Earth), it requires less additional speed to launch it east to obtain the final orbital speed.

19. The apparent weight (the normal force) would be largest when the elevator is accelerating upward. From the free-body diagram, with up as positive, we have $F_N - mg = ma \;\rightarrow\; F_N = m(g+a)$. With a positive acceleration, the normal force is greater than your weight. The apparent weight would be the least when in free fall, because there the apparent weight is zero, since $a = -g$. When the elevator is moving with constant speed, your apparent weight would be the same as it is on the ground, since $a = 0$ and so $F_N = mg$.



20. A satellite remains in orbit due to the combination of gravitational force on the satellite directed towards the center of the orbit and the tangential speed of the satellite. First, the proper tangential speed had to be established by some other force than the gravitational force. Then, if the satellite has the proper combination of speed and radius such that the force required for circular motion is equal to the force of gravity on the satellite, then the satellite will maintain circular motion.

21. The passengers, as seen in the diagram, are standing on the floor.



(*a*) If a passenger held an object beside their waist and then released it, the object would move in a straight line, tangential to the circle in which the passenger's waist was moving when the object was released. In the figure, we see that the released object would hit the rotating shell, and so fall to the floor, but behind the person. The passenger might try to explain such motion by inventing some kind of "retarding" force on dropped objects, when really there is no such force.

(*b*) The floor exerts a centripetal force on the feet, pushing them towards the center. This force has the same direction ("upwards", away from the floor) that a passenger would experience on Earth, and so it seems to the passenger that gravity must be pulling them "down". Actually, the passengers are pushing down on the floor, because the floor is pushing up on them.

(*c*) The "normal" way of playing catch, for example, would have to change. Since the artificial gravity is not uniform, passengers would have to re-learn how to throw something across the room to each other. There would not be projectile motion as we experience it on Earth. Also, if the cylinder were small, there might be a noticeable difference in the acceleration of our head vs. our feet. Since the head is closer to the center of the circle than the feet, and both the head and the feet have the same period of rotation, the centripetal acceleration $\left(a_R = 4\pi^2 r / T^2\right)$ is smaller for the head. This might cause dizziness or a light-headed feeling.

22. When the runner has both feet off the ground, the only force on the runner is gravity – there is no normal force from the ground on the runner. This lack of normal force is interpreted as "free fall" and "apparent weightlessness".

23. By Kepler's 2nd law, the Earth moves faster around the Sun when it is nearest the Sun. Kepler's 2nd law says that an imaginary line drawn from the Sun to the Earth sweeps out equal areas in equal

times. So when the Earth is close to the Sun, it must move faster to sweep out a given area than when the Earth is far from the Sun. Thus the Earth is closer to the Sun in January.

24. Let the mass of Pluto be M, the mass of the moon be m, the radius of the moon's orbit be R, and the period of the moon's orbit be T. Then Newton's second law for the moon orbiting Pluto will be $F = \frac{GmM}{R^2}$. If that moon's orbit is a circle, then the form of the force must be centripetal, and so $F = mv^2/R$. Equate these two expressions for the force on the moon, and substitute the relationship for a circular orbit that $v = 2\pi R/T$.

$$\frac{GmM}{R^2} = \frac{mv^2}{R} = \frac{4\pi^2 mR}{T^2} \quad \rightarrow \quad M = \frac{4\pi^2 R^3}{GT^2}.$$

Thus a value for the mass of Pluto can be calculated knowing the period and radius of the moon's orbit.

Solutions to Problems

1. (*a*) Find the centripetal acceleration from Eq. 5-1.

$$a_R = v^2/r = (1.25\,\text{m/s})^2/1.10\text{ m} = \boxed{1.42\,\text{m/s}^2}$$

(*b*) The net horizontal force is causing the centripetal motion, and so will be the centripetal force.

$$F_R = ma_R = (25.0\text{ kg})(1.42\,\text{m/s}^2) = \boxed{35.5\text{ N}}$$

2. Find the centripetal acceleration from Eq. 5-1.

$$a_R = v^2/r = \frac{(525\,\text{m/s})^2}{6.00\times10^3\,\text{m}} = (45.94\,\text{m/s}^2)\left(\frac{1\,g}{9.80\,\text{m/s}^2}\right) = \boxed{4.69\,g\text{'s}}$$

3. The centripetal acceleration is $a_R = v^2/R_{\substack{\text{Earth}\\\text{orbit}}} = \frac{\left(2\pi R_{\substack{\text{Earth}\\\text{orbit}}}/T\right)^2}{R_{\substack{\text{Earth}\\\text{orbit}}}} = \frac{4\pi^2 R_{\substack{\text{Earth}\\\text{orbit}}}}{T^2}$. The force (from Newton's 2nd law) is $F_R = m_{\text{Earth}} a_R$. The period is one year, converted into seconds.

$$a_R = \frac{4\pi^2 R_{\substack{\text{Earth}\\\text{orbit}}}}{T^2} = \frac{4\pi^2(1.50\times10^{11}\,\text{m})}{(3.15\times10^7\text{ sec})^2} = \boxed{5.97\times10^{-3}\text{ m/s}^2}$$

$$F_R = ma = (5.97\times10^{24}\,\text{kg})(5.97\times10^{-3}\text{ m/s}^2) = \boxed{3.56\times10^{22}\text{ N}}$$

The Sun exerts this force on the Earth. It is a gravitational force.

4. The speed can be found from the centripetal force and centripetal acceleration.

$$F_R = ma_R = mv^2/r \quad \rightarrow \quad v = \sqrt{\frac{F_R r}{m}} = \sqrt{\frac{(210\text{ N})(0.90\text{ m})}{2.0\text{ kg}}} = \boxed{9.7\,\text{m/s}}$$

5. The orbit radius will be the sum of the Earth's radius plus the 400 km orbit height. The orbital period is about 90 minutes. Find the centripetal acceleration from these data.

$$r = 6380\,\text{km} + 400\,\text{km} = 6780\,\text{km} = 6.78\times10^6\,\text{m} \qquad T = 90\text{ min}\left(\frac{60\text{ sec}}{1\text{ min}}\right) = 5400\text{ sec}$$

$$a_R = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2\left(6.78\times10^6\,\text{m}\right)}{\left(5400\text{ sec}\right)^2} = \left(9.18\,\text{m/s}^2\right)\left(\frac{1\,g}{9.80\,\text{m/s}^2}\right) = 0.937 \approx \boxed{0.9\,g\text{'s}}$$

Notice how close this is to g, because the shuttle is not very far above the surface of the Earth, relative to the radius of the Earth.

6. To find the period, the rotational speed (in rev/min) is reciprocated to have min/rev, and then converted to sec/rev. Use the period to find the speed, and then the centripetal acceleration.

$$T = \left(\frac{1\text{ min}}{45\text{ rev}}\right)\left(\frac{60\text{ sec}}{1\text{ min}}\right) = 1.333\frac{\text{sec}}{\text{rev}} \qquad r = 0.16\text{ m} \qquad v = \frac{2\pi r}{T} = \frac{2\pi\left(0.16\text{ m}\right)}{1.333\text{ sec}} = 0.754\,\text{m/s}$$

$$a_R = v^2/r = \frac{\left(0.754\,\text{m/s}\right)^2}{0.16\text{ m}} = \boxed{3.6\,\text{m/s}^2}$$

7. See the free-body diagram in the textbook. Since the object is moving in a circle with a constant speed, the net force on the object at any point must point to the center of the circle.

(*a*) Take positive to be downward. Write Newton's 2nd law in the downward direction.

$$\sum F_R = mg + F_{T1} = ma_R = mv^2/r \quad \rightarrow$$

$$F_{T1} = m\left(v^2/r - g\right) = \left(0.300\text{ kg}\right)\left(\frac{\left(4.00\,\text{m/s}\right)^2}{0.720\text{ m}} - 9.80\,\text{m/s}^2\right) = \boxed{3.73\text{ N}}$$

This is a downward force, as expected.

(*b*) Take positive to be upward. Write Newton's 2nd law in the upward direction.

$$\sum F_R = F_{T2} - mg = ma = mv^2/r \quad \rightarrow$$

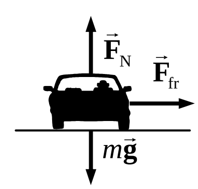
$$F_{T1} = m\left(v^2/r + g\right) = \left(0.300\text{ kg}\right)\left(\frac{\left(4.00\,\text{m/s}\right)^2}{0.720\text{ m}} + 9.80\,\text{m/s}^2\right) = \boxed{9.61\text{ N}}$$

This is an upward force, as expected.

8. The centripetal force that the tension provides is given by $F_R = mv^2/r$. Solve that for the speed.

$$v = \sqrt{\frac{F_R r}{\text{m}}} = \sqrt{\frac{\left(75\text{ N}\right)\left(1.3\text{ m}\right)}{0.45\,\text{kg}}} = \boxed{15\,\text{m/s}}$$

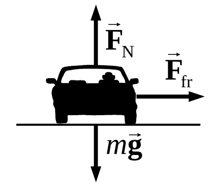
9. A free-body diagram for the car at one instant of time is shown. In the diagram, the car is coming out of the paper at the reader, and the center of the circular path is to the right of the car, in the plane of the paper. If the car has its maximum speed, it would be on the verge of slipping, and the force of static friction would be at its maximum value. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that the force of friction is the force causing the circular motion.



$$F_R = F_{fr} \quad\rightarrow\quad mv^2/r = \mu_s F_N = \mu_s mg \quad\rightarrow\quad v = \sqrt{\mu_s rg} = \sqrt{\left(0.80\right)\left(77\text{ m}\right)\left(9.8\,\text{m/s}^2\right)} = \boxed{25\,\text{m/s}}$$

Notice that the result is independent of the car's mass.

10. In the free-body diagram, the car is coming out of the paper at the reader, and the center of the circular path is to the right of the car, in the plane of the paper. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that the force of friction is the force causing the circular motion. If the car has its maximum speed, it would be on the verge of slipping, and the force of static friction would be at its maximum value.



$$F_R = F_{fr} \rightarrow mv^2/r = \mu_s F_N = \mu_s mg \rightarrow \mu_s = \frac{v^2}{rg} = \frac{\left[(95\,\text{km/hr})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/hr}}\right)\right]^2}{(85\text{ m})(9.8\,\text{m/s}^2)} = \boxed{0.84}$$

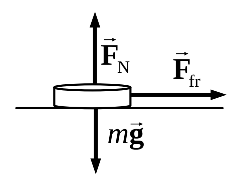
Notice that the result is independent of the car's mass.

11. Since the motion is all in a horizontal circle, gravity has no influence on the analysis. Set the general expression for centripetal force equal to the stated force in the problem.

$$F_R = mv^2/r = 7.85W = 7.85mg \rightarrow v = \sqrt{7.85rg} = \sqrt{7.85(12.0\text{m})(9.8\,\text{m/s}^2)} = \boxed{30.4\,\text{m/s}}$$

$$(30.4\,\text{m/s})\left(\frac{1\text{ rev}}{2\pi(12.0\text{ m})}\right) = \boxed{0.403\,\text{rev/s}}$$

12. The force of static friction is causing the circular motion – it is the centripetal force. The coin slides off when the static frictional force is not large enough to move the coin in a circle. The maximum static frictional force is the coefficient of static friction times the normal force, and the normal force is equal to the weight of the coin as seen in the free-body diagram, since there is no vertical acceleration. In the free-body diagram, the coin is coming out of the paper and the center of the circle is to the right of the coin, in the plane of the paper.

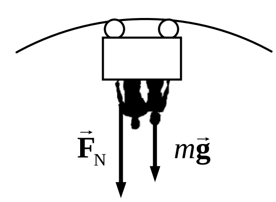


The rotational speed must be changed into a linear speed.

$$v = \left(36\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi(0.11\text{m})}{1\text{ rev}}\right) = 0.4147\,\text{m/s}$$

$$F_R = F_{fr} \rightarrow mv^2/r = \mu_s F_N = \mu_s mg \rightarrow \mu_s = \frac{v^2}{rg} = \frac{(0.4147\,\text{m/s})^2}{(0.11\text{ m})(9.8\,\text{m/s}^2)} = \boxed{0.16}$$

13. At the top of a circle, a free-body diagram for the passengers would be as shown, assuming the passengers are upside down. Then the car's normal force would be pushing DOWN on the passengers, as shown in the diagram. We assume no safety devices are present. Choose the positive direction to be down, and write Newton's 2nd law for the passengers.

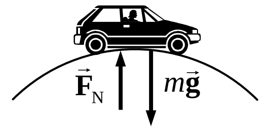


$$\sum F = F_N + mg = ma = mv^2/r \rightarrow F_N = m(v^2/r - g)$$

We see from this expression that for a high speed, the normal force is positive, meaning the passengers are in contact with the car. But as the speed decreases, the normal force also decreases. If the normal force becomes 0, the passengers are no longer in contact with the car – they are in free fall. The limiting condition is

$$v_{min}^2/r - g = 0 \rightarrow v_{min} = \sqrt{rg} = \sqrt{(9.8\,\text{m/s}^2)(7.4\text{ m})} = \boxed{8.5\,\text{m/s}}$$

14. (*a*) A free-body diagram of the car at the instant it is on the top of the hill is shown. Since the car is moving in a circular path, there must be a net centripetal force downward. Write Newton's 2nd law for the car, with down as the positive direction.



$$\sum F_R = mg - F_N = ma = mv^2/r \quad \rightarrow$$

$$F_N = m\left(g - v^2/r\right) = (950 \text{ kg})\left(9.8 \text{ m/s}^2 - \frac{(22 \text{ m/s})^2}{95 \text{ m}}\right) = \boxed{4.5\times10^3 \text{ N}}$$

(*b*) The free-body diagram for the passengers would be the same as the one for the car, leading to the same equation for the normal force on the passengers.

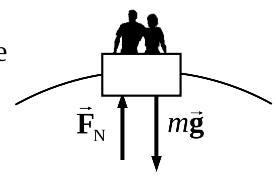
$$F_N = m\left(g - v^2/r\right) = (72 \text{ kg})\left(9.8 \text{ m/s}^2 - \frac{(22 \text{ m/s})^2}{95 \text{ m}}\right) = \boxed{3.4\times10^2 \text{ N}}$$

Notice that this is significantly less than the 700-N weight of the passenger. Thus the passenger will feel "light" as they drive over the hill.

(*c*) For the normal force to be zero, we see that we must have

$$F_N = m\left(g - v^2/r\right) = 0 \quad \rightarrow \quad g = v^2/r \quad \rightarrow \quad v = \sqrt{gr} = \sqrt{(9.8 \text{ m/s}^2)(95 \text{ m})} = \boxed{31 \text{ m/s}}.$$

15. The free-body diagram for passengers at the top of a Ferris wheel is as shown. F_N is the normal force of the seat pushing up on the passenger. The sum of the forces on the passenger is producing the centripetal motion, and so must be a centripetal force. Call the downward direction positive. Newton's 2nd law for the passenger is:



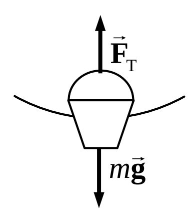
$$\sum F_R = mg - F_N = ma = mv^2/r$$

Since the passenger is to feel "weightless", they must lose contact with their seat, and so the normal force will be 0.

$$mg = mv^2/r \quad \rightarrow \quad v = \sqrt{gr} = \sqrt{(9.8 \text{ m/s}^2)(7.5 \text{ m})} = 8.6 \text{ m/s}$$

$$\left(8.6 \frac{\text{m}}{\text{s}}\right)\left(\frac{1 \text{ rev}}{2\pi(7.5 \text{ m})}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{11 \text{ rpm}}$$

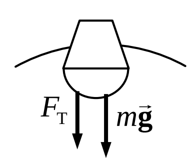
16. (*a*) At the bottom of the motion, a free-body diagram of the bucket would be as shown. Since the bucket is moving in a circle, there must be a net force on it towards the center of the circle, and a centripetal acceleration. Write Newton's 2nd law for the bucket, with up as the positive direction.



$$\sum F_R = F_T - mg = ma = mv^2/r \quad \rightarrow$$

$$v = \sqrt{\frac{r(F_T - mg)}{m}} = \sqrt{\frac{(1.10 \text{ m})\left[25.0 \text{ N} - (2.00 \text{ kg})(9.80 \text{ m/s}^2)\right]}{2.00 \text{ kg}}} = 1.723 \approx \boxed{1.7 \text{ m/s}}$$

(*b*) A free-body diagram of the bucket at the top of the motion is shown. Since the bucket is moving in a circle, there must be a net force on it towards the center of the circle, and a centripetal acceleration. Write Newton's 2nd law for the bucket, with down as the positive direction.



$$\sum F_R = F_T + mg = ma = mv^2/r \quad \rightarrow \quad v = \sqrt{\frac{r(F_T + mg)}{m}}$$

If the tension is to be zero, then

$$v=\sqrt{\frac{r(0+mg)}{m}}=\sqrt{rg}=\sqrt{(1.10\text{ m})(9.80\text{ m/s}^2)}=\boxed{3.28\text{ m/s}}$$

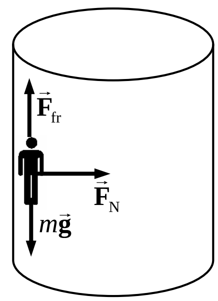
The bucket must move faster than 3.28 m/s in order for the rope not to go slack.

17. The centripetal acceleration of a rotating object is given by $a_R = v^2/r$. Thus

$$v=\sqrt{a_R r}=\sqrt{(1.15\times10^5 g)r}=\sqrt{(1.15\times10^5)(9.80\text{ m/s}^2)(9.00\times10^{-2}\text{ m})}=3.18\times10^2\text{ m/s}.$$

$$(3.18\times10^2\text{ m/s})\left(\frac{1\text{ rev}}{2\pi(9.00\times10^{-2}\text{m})}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)=\boxed{3.38\times10^4\text{ rpm}}.$$

18. Consider the free-body diagram for a person in the "Rotor-ride". $\vec{\mathbf{F}}_N$ is the normal force of contact between the rider and the wall, and $\vec{\mathbf{F}}_{fr}$ is the static frictional force between the back of the rider and the wall. Write Newton's 2nd law for the vertical forces, noting that there is no vertical acceleration.



$$\sum F_y = F_{fr} - mg = 0 \quad\rightarrow\quad F_{fr} = mg$$

If we assume that the static friction force is a maximum, then

$$F_{fr} = \mu_s F_N = mg \quad\rightarrow\quad F_N = mg/\mu_s .$$

But the normal force must be the force causing the centripetal motion – it is the only force pointing to the center of rotation. Thus $F_R = F_N = mv^2/r$. Using $v = 2\pi r/T$, we have $F_N = \frac{4\pi^2 mr}{T^2}$. Equate the two expressions for the normal force and solve for the coefficient of friction. Note that since there are 0.5 rev per sec, the period is 2.0 sec.

$$F_N = \frac{4\pi^2 mr}{T^2} = \frac{mg}{\mu_s} \quad\rightarrow\quad \mu_s = \frac{gT^2}{4\pi^2 r} = \frac{(9.8\text{ m/s}^2)(2\text{ s})^2}{4\pi^2(4.6\text{ m})} = \boxed{0.22}.$$

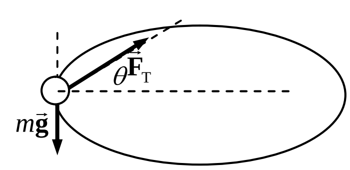
Any larger value of the coefficient of friction would mean that the normal force could be smaller to achieve the same frictional force, and so the period could be longer or the cylinder smaller.

There is no force pushing outward on the riders. Rather, the wall pushes against the riders, so by Newton's 3rd law the riders push against the wall. This gives the sensation of being pressed into the wall.

19. Since mass *m* is dangling, the tension in the cord must be equal to the weight of mass *m*, and so $F_T = mg$. That same tension is in the other end of the cord, maintaining the circular motion of mass *M*, and so $F_T = F_R = Ma_R = Mv^2/r$. Equate the two expressions for the tension and solve for the velocity.

$$Mv^2/r = mg \quad\rightarrow\quad v = \boxed{\sqrt{mgR/M}}.$$

20. A free-body diagram for the ball is shown. The tension in the suspending cord must not only hold the ball up, but also provide the centripetal force needed to make the ball move in a circle. Write Newton's 2nd law for the vertical direction, noting that the ball is not accelerating vertically.



$$\sum F_y = F_T \sin\theta - mg = 0 \quad \rightarrow \quad F_T = \frac{mg}{\sin\theta}$$

The force moving the ball in a circle is the horizontal portion of the tension. Write Newton's 2nd law for that radial motion.

$$\sum F_R = F_T \cos\theta = ma_R = mv^2/r$$

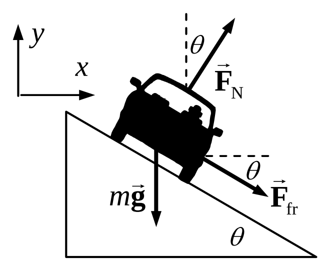
Substitute the expression for the tension from the first equation into the second equation, and solve for the angle. Also substitute in the fact that for a rotating object, $v = 2\pi r/T$. Finally we recognize that if the string is of length L, then the radius of the circle is $r = L\cos\theta$.

$$F_T \cos\theta = \frac{mg}{\sin\theta}\cos\theta = \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2} = \frac{4\pi^2 mL\cos\theta}{T^2} \quad \rightarrow$$

$$\sin\theta = \frac{gT^2}{4\pi^2 L} \quad \rightarrow \quad \theta = \sin^{-1}\frac{gT^2}{4\pi^2 L} = \sin^{-1}\frac{(9.80\,\text{m/s}^2)(0.500\text{ s})^2}{4\pi^2(0.600\text{ m})} = \boxed{5.94^\circ}$$

The tension is then given by $F_T = \dfrac{mg}{\sin\theta} = \dfrac{(0.150\text{ kg})(9.80\,\text{m/s}^2)}{\sin 5.94^\circ} = \boxed{14.2\text{ N}}$

21. Since the curve is designed for 75 km/h, traveling at a higher speed with the same radius means that more centripetal force will be required. That extra centripetal force will be supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. Note that from Example 5-7 in the textbook, the no-friction banking angle is given by



$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{\left[(75\,\text{km/h})\left(\frac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{(88\text{ m})(9.8\,\text{m/s}^2)} = 26.7^\circ$$

Write Newton's 2nd law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{fr} = \mu_s F_N$. Solve each equation for the normal force.

$$\sum F_y = F_N\cos\theta - mg - F_{fr}\sin\theta = 0 \quad \rightarrow \quad F_N\cos\theta - \mu_s F_N\sin\theta = mg \quad \rightarrow$$

$$F_N = \frac{mg}{(\cos\theta - \mu_s\sin\theta)}$$

$$\sum F_x = F_N\sin\theta + F_{fr}\cos\theta = F_R = mv^2/r \quad \rightarrow \quad F_N\sin\theta + \mu_s F_N\cos\theta = mv^2/r \quad \rightarrow$$

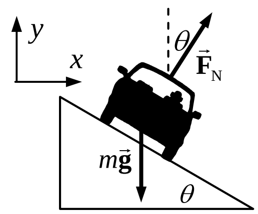
$$F_N = \frac{mv^2/r}{(\sin\theta + \mu_s\cos\theta)}$$

Equate the two expressions for F_N, and solve for the coefficient of friction. The speed of rounding the curve is given by $v = (95\,\text{km/h})\left(\dfrac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right) = 26.39\,\text{m/s}$.

$$\frac{mg}{(\cos\theta - \mu_s\sin\theta)} = \frac{mv^2/r}{(\sin\theta + \mu_s\cos\theta)} \quad \rightarrow$$

$$\mu_s = \frac{\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)}{\left(g\cos\theta + \frac{v^2}{r}\sin\theta\right)} = \frac{\left(\frac{v^2}{r} - g\tan\theta\right)}{\left(g + \frac{v^2}{r}\tan\theta\right)} = \frac{\left(\frac{(26.39\,\text{m/s})^2}{88\text{ m}} - (9.8\,\text{m/s}^2)\tan 26.7^\circ\right)}{\left(9.8\,\text{m/s}^2 + \frac{(26.39\,\text{m/s})^2}{88\text{ m}}\tan 26.7^\circ\right)} = \boxed{0.22}$$

22. The car moves in a horizontal circle, and so there must be a net horizontal centripetal force. The car is not accelerating vertically. Write Newton's 2nd law for both the *x* and *y* directions.



$$\sum F_y = F_N\cos\theta - mg = 0 \quad\rightarrow\quad F_N = \frac{mg}{\cos\theta}$$

$$\sum F_x = \sum F_R = F_N\sin\theta = ma_x$$

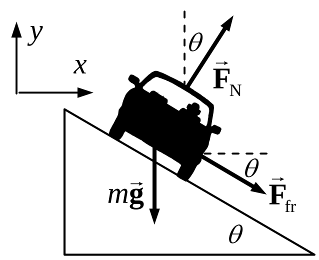
The amount of centripetal force needed for the car to round the curve is

$$F_R = mv^2/r = (1200\text{ kg})\frac{\left[(95\,\text{km/h})\left(\frac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{67\text{ m}} = 1.247\times10^4\,\text{N}.$$

The actual horizontal force available from the normal force is

$$F_N\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = (1200\text{ kg})(9.80\,\text{m/s}^2)\tan 12^\circ = 2.500\times10^3\,\text{N}.$$

Thus more force is necessary for the car to round the curve than can be supplied by the normal force. That extra force will have to have a horizontal component to the right in order to provide the extra centripetal force. Accordingly, we add a frictional force pointed down the plane. That corresponds to the car not being able to make the curve without friction.



Again write Newton's 2nd law for both directions, and again the *y* acceleration is zero.

$$\sum F_y = F_N\cos\theta - mg - F_{fr}\sin\theta = 0 \quad\rightarrow\quad F_N = \frac{mg + F_{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x = F_N\sin\theta + F_{fr}\cos\theta = mv^2/r$$

Substitute the expression for the normal force from the *y* equation into the *x* equation, and solve for the friction force.

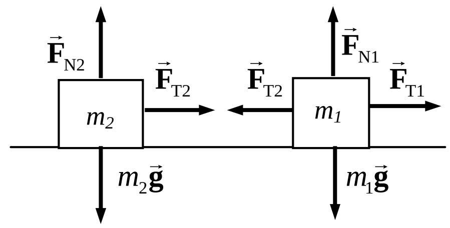
$$\frac{mg + F_{fr}\sin\theta}{\cos\theta}\sin\theta + F_{fr}\cos\theta = mv^2/r \quad\rightarrow\quad (mg + F_{fr}\sin\theta)\sin\theta + F_{fr}\cos^2\theta = m\frac{v^2}{r}\cos\theta$$

$$F_{fr} = m\frac{v^2}{r}\cos\theta - mg\sin\theta = (1.247\times10^4\,\text{N})\cos 12^\circ - (1200\text{ kg})(9.80\,\text{m/s}^2)\sin 12^\circ$$

$$= 9.752\times10^3\,\text{N}$$

So a frictional force of $\boxed{9.8\times10^3\,\text{N down the plane}}$ is needed to provide the necessary centripetal force to round the curve at the specified speed.

23. If the masses are in line and both have the same frequency, then they will always stay in line. Consider a free-body diagram for both masses, from a side view, at the instant that they are to the left of the post. Note that the same tension that pulls inward on mass 2 pulls outward on mass 1, by Newton's 3rd law. Also notice that since there is no vertical acceleration, the normal force on each mass is equal to its weight. Write Newton's 2nd law for the horizontal direction for both masses, noting that they are in uniform circular motion.



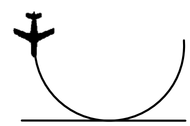
$$\sum F_{1R} = F_{T1} - F_{T2} = m_1 a_1 = m_1 v_1^2/r_1 \qquad \sum F_{2R} = F_{T2} = m_2 a_2 = m_2 v_2^2/r_2$$

The speeds can be expressed in terms of the frequency as follows: $v = \left(f \frac{\text{rev}}{\text{sec}} \right)\left(\frac{2\pi r}{1\ \text{rev}} \right) = 2\pi r f$.

$$F_{T2} = m_2 v_2^2/r_2 = m_2 (2\pi r_2 f)^2/r_2 = \boxed{4\pi^2 m_2 r_2 f^2}$$

$$F_{T1} = F_{T2} + m_1 v_1^2/r_1 = 4\pi m_2 r_2 f^2 + m_1 (2\pi r_1 f)^2/r_1 = \boxed{4\pi^2 f^2 (m_1 r_1 + m_2 r_2)}$$

24. The fact that the pilot can withstand 9.0 g's without blacking out, along with the speed of the aircraft, will determine the radius of the circle that he must fly as he pulls out of the dive. To just avoid crashing into the sea, he must begin to form that circle (pull out of the dive) at a height equal to the radius of that circle.



$$a_R = v^2/r = 9.0g \quad \to \quad r = \frac{v^2}{9.0g} = \frac{(310\ \text{m/s})^2}{9.0(9.80\ \text{m/s}^2)} = \boxed{1.1\times10^3\ \text{m}}$$

25. From example 5.8, we are given that the track radius is 500 m, and the tangential acceleration is 3.2 m/s^2. Thus the tangential force is

$$F_{\tan} = m a_{\tan} = (1100\ \text{kg})(3.2\ \text{m/s}^2) = \boxed{3.5\times10^3\ \text{N}}.$$

The centripetal force is given by

$$F_R = m v^2/r = (1100\ \text{kg})(15\ \text{m/s})^2/(500\ \text{m}) = \boxed{5.0\times10^2\ \text{N}}.$$

26. The car has constant tangential acceleration, which is the acceleration that causes the speed to change. Thus use constant acceleration equations to calculate the tangential acceleration. The initial speed is 0, the final speed is $320\ \text{km/h}\left(\frac{1.0\ \text{m/s}}{3.6\ \text{km/h}} \right) = 88.89\ \text{m/s}$, and the distance traveled is one half of a circular arc of radius 220 m, so $\Delta x_{\tan} = 220\pi\ \text{m}$. Find the tangential acceleration using Eq. 2-11c.

$$v_{\tan}^2 - v_{0\ \tan}^2 = 2a_{\tan}\Delta x_{\tan} \quad \to \quad a_{\tan} = \frac{v_{\tan}^2 - v_{0\ \tan}^2}{2\Delta x_{\tan}} = \frac{(88.89\ \text{m/s})^2}{2(220\pi\ \text{m})} = 5.72\ \text{m/s}^2$$

With this tangential acceleration, we can find the speed that the car has halfway through the turn, using Eq. 2-11c, and then calculate the radial acceleration.

$$v_{\tan}^2 - v_{0\ \tan}^2 = 2a_{\tan}\Delta x_{\tan} \quad \to \quad v_{\tan} = \sqrt{v_{0\ \tan}^2 + 2a_{\tan}\Delta x_{\tan}} = \sqrt{2(5.72\ \text{m/s}^2)(110\pi\ \text{m})} = 62.9\ \text{m/s}$$

$$a_R = v^2/r = \frac{(62.9\ \text{m/s})^2}{220\ \text{m}} = 18.0\ \text{m/s}^2$$

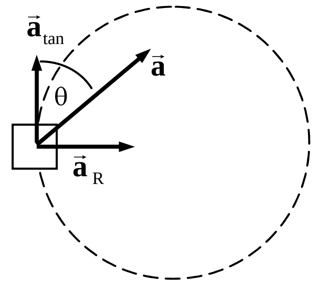
The total acceleration is given by the Pythagorean combination of the tangential and centripetal accelerations. $a_{\text{total}} = \sqrt{a_{\text{R}}^2 + a_{\tan}^2}$. If static friction is to provide the total acceleration, then $F_{\text{fr}} = ma_{\text{total}} = m\sqrt{a_{\text{R}}^2 + a_{\tan}^2}$. We assume that the car is on the verge of slipping, and is on a level surface, and so the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_{\text{N}} = \mu_s mg$. If we equate these two expressions for the frictional force, we can solve for the coefficient of static friction.

$$F_{\text{fr}} = ma_{\text{total}} = m\sqrt{a_{\text{R}}^2 + a_{\tan}^2} = \mu_s mg \quad \rightarrow$$

$$\mu_s = \frac{\sqrt{a_{\text{R}}^2 + a_{\tan}^2}}{g} = \frac{\sqrt{\left(18.0\,\text{m/s}^2\right)^2 + \left(5.72\,\text{m/s}^2\right)^2}}{9.80\,\text{m/s}^2} = 1.92 \approx \boxed{1.9}$$

This is an exceptionally large coefficient of friction, and so the curve had better be banked.

27. We show a top view of the particle in circular motion, traveling clockwise. Because the particle is in circular motion, there must be a radially-inward component of the acceleration.



(*a*) $a_{\text{R}} = a\sin\theta = v^2/r \quad \rightarrow$

$$v = \sqrt{ar\sin\theta} = \sqrt{\left(1.05\,\text{m/s}^2\right)\left(2.90\ \text{m}\right)\sin 32.0^{\circ}} = \boxed{1.27\,\text{m/s}}$$

(*b*) The particle's speed change comes from the tangential acceleration, which is given by $a_{\tan} = a\cos\theta$. If the tangential acceleration is constant, then using Eq. 2-11a,

$$v_{\tan} - v_{0\ \tan} = a_{\tan}t \quad \rightarrow$$

$$v_{\tan} = v_{0\ \tan} + a_{\tan}t = 1.27\,\text{m/s} + \left(1.05\,\text{m/s}^2\right)\left(\cos 32.0^{\circ}\right)\left(2.00\ \text{s}\right) = \boxed{3.05\,\text{m/s}}$$

28. The spacecraft is three times as far from the Earth's center as when at the surface of the Earth. Therefore, since the force as gravity decreases as the square of the distance, the force of gravity on the spacecraft will be one-ninth of its weight at the Earth's surface.

$$F_G = \tfrac{1}{9}mg_{\substack{\text{Earth's}\\ \text{surface}}} = \frac{\left(1350\ \text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{9} = \boxed{1.47\times10^3\,\text{N}}$$

This could also have been found using Newton's law of Universal Gravitation.

29. (*a*) Mass is independent of location and so the mass of the ball is $\boxed{21.0\ \text{kg}}$ on both the Earth and the planet.

(*b*) The weight is found by $W = mg$.

$$W_{\text{Earth}} = mg_{\text{Earth}} = \left(21.0\ \text{kg}\right)\left(9.80\,\text{m/s}^2\right) = \boxed{206\ \text{N}}$$

$$W_{\text{Planet}} = mg_{\text{Planet}} = \left(21.0\ \text{kg}\right)\left(12.0\,\text{m/s}^2\right) = \boxed{252\ \text{N}}.$$

30. The force of gravity on an object at the surface of a planet is given by Newton's law of Universal Gravitation, using the mass and radius of the planet. If that is the only force on an object, then the acceleration of a freely-falling object is acceleration due to gravity.

$$F_G = G\frac{M_{\text{Moon}}m}{r_{\text{Moon}}^2} = mg_{\text{Moon}} \quad \rightarrow$$

$$g_{\text{Moon}} = G\frac{M_{\text{Moon}}}{r_{\text{Moon}}^2} = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(7.35\times10^{22}\,\text{kg}\right)}{\left(1.74\times10^{6}\,\text{m}\right)^2} = \boxed{1.62\,\text{m}/\text{s}^2}$$

31. The acceleration due to gravity at any location on or above the surface of a planet is given by $g_{\text{planet}} = G\,M_{\text{Planet}}/r^2$, where r is the distance from the center of the planet to the location in question.

$$g_{\text{planet}} = G\frac{M_{\text{Planet}}}{r^2} = G\frac{M_{\text{Earth}}}{\left(1.5R_{\text{Earth}}\right)^2} = \frac{1}{1.5^2}G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2} = \frac{1}{1.5^2}g_{\text{Earth}} = \frac{9.8\,\text{m}/\text{s}^2}{1.5^2} = \boxed{4.4\,\text{m}/\text{s}^2}$$

32. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{\text{planet}} = G\,M_{\text{Planet}}/r^2$, where r is the distance from the center of the planet to the location in question.

$$g_{\text{planet}} = G\frac{M_{\text{Planet}}}{r^2} = G\frac{1.66M_{\text{Earth}}}{R_{\text{Earth}}^2} = 1.66\left(G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2}\right) = 1.66g_{\text{Earth}} = 1.66\left(9.80\,\text{m}/\text{s}^2\right) = \boxed{16.3\,\text{m}/\text{s}^2}$$

33. Assume that the two objects can be treated as point masses, with $m_1 = m$ and $m_2 = 4\ \text{kg} - m$. The gravitational force between the two masses is given by

$$F = G\frac{m_1m_2}{r^2} = G\frac{m(4-m)}{r^2} = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{4m-m^2}{(0.25\ \text{m})^2} = 2.5\times10^{-10}\,\text{N}.$$

This can be rearranged into a quadratic form of $m^2 - 4m + 0.234 = 0$. Use the quadratic formula to solve for *m*, resulting in two values which are the two masses.

$\boxed{m_1 = 3.9\ \text{kg},\ m_2 = 0.1\ \text{kg}}$.

34. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{\text{planet}} = G\,M_{\text{Planet}}/r^2$, where r is the distance from the center of the planet to the location in question.

For this problem, $M_{\text{Planet}} = M_{\text{Earth}} = 5.97\times10^{24}\,\text{kg}$

(*a*) $r = R_{\text{Earth}} + 3200\ \text{m} = 6.38\times10^6\,\text{m} + 3200\ \text{m}$

$$g = G\frac{M_{\text{Earth}}}{r^2} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\,\text{kg}\right)}{\left(6.38\times10^6\,\text{m} + 3200\ \text{m}\right)^2} = \boxed{9.77\,\text{m}/\text{s}^2}$$

(*b*) $r = R_{\text{Earth}} + 3200\ \text{km} = 6.38\times10^6\,\text{m} + 3.20\times10^6\ \text{m} = 9.58\times10^6\,\text{m}$

$$g = G\frac{M_{\text{Earth}}}{r^2} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\,\text{kg}\right)}{\left(9.58\times10^6\,\text{m}\right)^2} = \boxed{4.34\,\text{m}/\text{s}^2}$$

35. In general, the acceleration due to gravity of the Earth is given by $g = G\,M_{\text{Earth}}/r^2$, where r is the distance from the center of the Earth to the location in question. So for the location in question,

$$g = \tfrac{1}{10}g_{\text{surface}} \quad\rightarrow\quad G\frac{M_{\text{Earth}}}{r^2} = \tfrac{1}{10}G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2} \quad\rightarrow\quad r^2 = 10\,R_{\text{Earth}}^2$$

$$r = \sqrt{10}R_{\text{Earth}} = \sqrt{10}\left(6.38\times10^6\,\text{m}\right) = \boxed{2.02\times10^7\,\text{m}}$$

36. The acceleration due to gravity at any location at or above the surface of a star is given by $g_{\text{star}} = G\,M_{\text{star}} / r^2$, where r is the distance from the center of the star to the location in question.

$$g_{\text{star}} = G\frac{M_{\text{star}}}{r^2} = G\frac{5M_{\text{Sun}}}{r^2} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{5\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1\times10^4\,\text{m}\right)^2} = \boxed{7\times10^{12}\ \text{m}/\text{s}^2}$$

37. The acceleration due to gravity at any location at or above the surface of a star is given by $g_{\text{star}} = G\,M_{\text{star}} / r^2$, where r is the distance from the center of the star to the location in question.

$$g_{\text{star}} = G\frac{M_{\text{sun}}}{R^2_{\text{Moon}}} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1.74\times10^6\ \text{m}\right)^2} = \boxed{4.38\times10^7\ \text{m}/\text{s}^2}$$

38. The distance from the Earth's center is

$$r = R_{\text{Earth}} + 250\ \text{km} = 6.38\times10^6\,\text{m} + 2.5\times10^5\,\text{m} = 6.63\times10^6\,\text{m}.$$

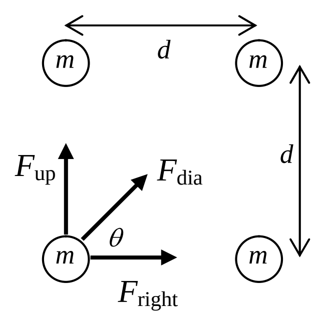
Calculate the acceleration due to gravity at that location.

$$g = G\frac{M_{\text{Earth}}}{r^2} = G\frac{M_{\text{Earth}}}{r^2} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{5.97\times10^{24}\,\text{kg}}{\left(6.63\times10^6\,\text{m}\right)^2} = 9.059\,\text{m}/\text{s}^2$$

$$= 9.059\,\text{m}/\text{s}^2\left(\frac{1\text{"}g\text{"}}{9.80\,\text{m}/\text{s}^2}\right) = \boxed{0.924g\text{'s}}$$

This is only about a 7.5% reduction from the value of g at the surface of the Earth.

39. Calculate the force on the sphere in the lower left corner, using the free-body diagram shown. From the symmetry of the problem, the net forces in the x and y directions will be the same. Note $\theta = 45^\circ$



$$F_x = F_{\text{right}} + F_{\text{dia}}\cos\theta = G\frac{m^2}{d^2} + G\frac{m^2}{\left(\sqrt{2}d\right)^2}\frac{1}{\sqrt{2}} = G\frac{m^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)$$

and so $F_y = F_x = G\dfrac{m^2}{d^2}\left(1+\dfrac{1}{2\sqrt{2}}\right)$. The net force can be found by the Pythagorean combination of the two component forces. Due to the symmetry of the arrangement, the net force will be along the diagonal of the square.

$$F = \sqrt{F_x^2+F_y^2} = \sqrt{2F_x^2} = F_x\sqrt{2} = G\frac{m^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)\sqrt{2} = G\frac{m^2}{d^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$= \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(9.5\ \text{kg}\right)^2}{\left(0.60\ \text{m}\right)^2}\left(\sqrt{2}+\frac{1}{2}\right) = \boxed{3.2\times10^{-8}\,\text{N at }45^\circ}$$

The force points towards the center of the square.

40. We are to calculate the force on Earth, so we need the distance of each planet from Earth.

$$r_{\substack{\text{Earth}\\\text{Venus}}} = \left(150-108\right)\times10^6\ \text{km} = 4.2\times10^{10}\,\text{m} \qquad r_{\substack{\text{Earth}\\\text{Jupiter}}} = \left(778-150\right)\times10^6\ \text{km} = 6.28\times10^{11}\,\text{m}$$

$$r_{\substack{\text{Earth}\\\text{Saturn}}} = \left(1430-150\right)\times10^6\ \text{km} = 1.28\times10^{12}\,\text{m}$$

Jupiter and Saturn will exert a rightward force, while Venus will exert a leftward force. Take the right direction as positive.

$$F_{\substack{\text{Earth-}\\\text{planets}}} = G\frac{M_{\text{Earth}}M_{\text{Jupiter}}}{r^2_{\substack{\text{Earth}\\\text{Jupiter}}}} + G\frac{M_{\text{Earth}}M_{\text{Saturn}}}{r^2_{\substack{\text{Earth}\\\text{Saturn}}}} - G\frac{M_{\text{Earth}}M_{\text{Venus}}}{r^2_{\substack{\text{Earth}\\\text{Venus}}}}$$

$$= GM^2_{\text{Earth}}\left(\frac{318}{\left(6.28\times10^{11}\,\text{m}\right)^2} + \frac{95.1}{\left(1.28\times10^{12}\,\text{m}\right)^2} - \frac{0.815}{\left(4.2\times10^{10}\,\text{m}\right)^2}\right)$$

$$= \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\,\text{kg}\right)^2\left(4.02\times10^{-22}\,\text{m}^{-2}\right) = \boxed{9.56\times10^{17}\,\text{N}}$$

The force of the Sun on the Earth is as follows.

$$F_{\substack{\text{Earth-}\\\text{Sun}}} = G\frac{M_{\text{Earth}}M_{\text{Sun}}}{r^2_{\substack{\text{Earth}\\\text{Sun}}}} = \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\,\text{kg}\right)\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1.50\times10^{11}\,\text{m}\right)^2} = 3.52\times10^{22}\,\text{N}$$

And so the ratio is $F_{\substack{\text{Earth-}\\\text{planets}}} \big/ F_{\substack{\text{Earth-}\\\text{Sun}}} = 9.56\times10^{17}\,\text{N}/3.52\times10^{22}\,\text{N} = \boxed{2.71\times10^{-5}}$, which is 27 millionths.

41. The expression for the acceleration due to gravity at the surface of a body is $g_{\text{body}} = G\frac{M_{\text{body}}}{R^2_{\text{body}}}$, where R_{body} is the radius of the body. For Mars, $g_{\text{Mars}} = 0.38g_{\text{Earth}}$. Thus

$$G\frac{M_{\text{Mars}}}{R^2_{\text{Mars}}} = 0.38\,G\frac{M_{\text{Earth}}}{R^2_{\text{Earth}}} \quad\rightarrow$$

$$M_{\text{Mars}} = 0.38M_{\text{Earth}}\left(\frac{R_{\text{Mars}}}{\text{R}_{\text{Earth}}}\right)^2 = 0.38\left(5.97\times10^{24}\,\text{kg}\right)\left(\frac{3400\text{ km}}{6380\text{ km}}\right)^2 = \boxed{6.4\times10^{23}\,\text{kg}}$$

42. The speed of an object in an orbit of radius *r* around the Sun is given by $v = \sqrt{G\,M_{\text{Sun}}/r}$, and is also given by $v = 2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for M_{Sun}, using data for the Earth.

$$\sqrt{G\frac{M_{\text{Sun}}}{r}} = \frac{2\pi r}{T} \quad\rightarrow\quad M_{\text{Sun}} = \frac{4\pi^2r^3}{GT^2} = \frac{4\pi^2\left(1.50\times10^{11}\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(3.15\times10^{7}\,\text{sec}\right)^2} = \boxed{2.01\times10^{30}\text{ kg}}$$

This is the same result obtained in Example 5-16 using Kepler's third law.

43. The speed of a satellite in a circular orbit around a body is given by $v = \sqrt{G\,M_{\text{body}}/r}$, where *r* is the distance from the satellite to the center of the body. So for this satellite,

$$v = \sqrt{G\frac{M_{\text{body}}}{r}} = \sqrt{G\frac{M_{\text{Earth}}}{R_{\text{Earth}} + 3.6\times10^6\,\text{m}}} = \sqrt{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\,\text{kg}\right)}{\left(9.98\times10^{6}\,\text{m}\right)}}$$

$$= \boxed{6.32\times10^3\,\text{m/s}}$$

44. The shuttle must be moving at "orbit speed" in order for the satellite to remain in the orbit when released. The speed of a satellite in circular orbit around the Earth is given by

$$v = \sqrt{G\frac{M_{\text{Earth}}}{r}} = \sqrt{G\frac{M_{\text{Earth}}}{(R_{\text{Earth}} + 650\text{ km})}} = \sqrt{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\text{kg}\right)}{\left(6.38\times10^{6}\text{m} + 6.5\times10^{5}\text{m}\right)}}$$

$$= \boxed{7.53\times10^{3}\text{ m/s}}$$

45. The centripetal acceleration will simulate gravity. Thus $v^2/r = 0.60g \rightarrow v = \sqrt{0.60gr}$. Also for a rotating object, the speed is given by $v = 2\pi r/T$. Equate the two expressions for the speed and solve for the period.

$$v = \sqrt{0.60gr} = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{\sqrt{0.60gr}} = \frac{2\pi(16\text{ m})}{\sqrt{(0.60)(9.8\text{ m/s}^2)(16\text{ m})}} = \boxed{10\text{ sec}}$$

46. The speed of an object in an orbit of radius r around the Earth is given by $v = \sqrt{GM_{\text{Earth}}/r}$, and is also given by $v = 2\pi r/T$, where T is the period of the object in orbit. Equate the two expressions for the speed and solve for T. Also, for a "near-Earth" orbit, $r = R_{\text{Earth}}$.

$$\sqrt{G\frac{M_{\text{Earth}}}{r}} = \frac{2\pi r}{T} \rightarrow T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Earth}}}}$$

$$T = 2\pi\sqrt{\frac{R_{\text{Earth}}^3}{GM_{\text{Earth}}}} = 2\pi\sqrt{\frac{\left(6.38\times10^{6}\text{m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{m}\right)}} = \boxed{5070\text{ s} \sim 84\text{ min}}$$

$\boxed{\text{No}}$, the result does not depend on the mass of the satellite.

47. At the top of Mt. Everest (elevation 8848 meters), the distance of the orbit from the center of the Earth would be $r = R_{\text{Earth}} + 8848\text{ m} = 6.38\times10^{6}\text{m} + 8848\text{ m}$. The orbit speed is given by

$$v = \sqrt{G\frac{M_{\text{Earth}}}{r}} = \sqrt{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.98\times10^{24}\text{m}\right)}{\left(6.38\times10^{6}\text{m} + 8848\text{ m}\right)}} = \boxed{7.90\times10^{3}\text{ m/s}}.$$

A comment – a launch would have some initial orbit speed from the fact that the Earth is rotating to the east. That is why most space launches are to the east.

48. The speed of an object in an orbit of radius r around the Moon is given by $v = \sqrt{GM_{\text{Moon}}/r}$, and is also given by $v = 2\pi r/T$, where T is the period of the object in orbit. Equate the two expressions for the speed and solve for T.

$$\sqrt{GM_{\text{Moon}}/r} = 2\pi r/T \rightarrow$$

$$T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Moon}}}} = 2\pi\sqrt{\frac{\left(R_{\text{Moon}} + 100\text{ km}\right)^3}{GM_{\text{Moon}}}} = 2\pi\sqrt{\frac{\left(1.74\times10^{6}\text{m} + 1.0\times10^{5}\text{m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\text{kg}\right)}}$$

$$= \boxed{7.08\times10^{3}\text{s}\left(\sim 2\text{ h}\right)}$$

49. The speed of an object in an orbit of radius *r* around a planet is given by $v = \sqrt{G M_{\text{planet}}/r}$, and is also given by $v = 2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for *T*.

$$\sqrt{G\frac{M_{\text{Planet}}}{r}} = \frac{2\pi r}{T} \quad \rightarrow \quad T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Planet}}}}$$

For this problem, the inner orbit is at $r_{\text{inner}} = 7.3\times10^{7}\,\text{m}$, and the outer orbit is at $r_{\text{inner}} = 1.7\times10^{8}\,\text{m}$. Use these values to calculate the periods.

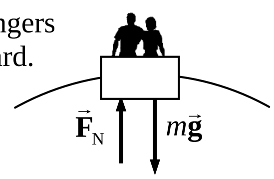
$$T_{\text{inner}} = 2\pi\sqrt{\frac{\left(7.3\times10^{7}\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.7\times10^{26}\,\text{kg}\right)}} = \boxed{2.0\times10^{4}\,\text{s}}$$

$$T_{\text{outer}} = 2\pi\sqrt{\frac{\left(1.7\times10^{8}\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.7\times10^{26}\,\text{kg}\right)}} = \boxed{7.1\times10^{4}\,\text{s}}$$

Saturn's rotation period (day) is 10 hr 39 min which is about $3.8\times10^{4}\,\text{sec}$. Thus the inner ring will appear to move across the sky "faster" than the Sun (about twice per Saturn day), while the outer ring will appear to move across the sky "slower" than the Sun (about once every two Saturn days).

50. The apparent weight is the normal force on the passenger. For a person at rest, the normal force is equal to the actual weight. If there is acceleration in the vertical direction, either up or down, then the normal force (and hence the apparent weight) will be different than the actual weight. The speed of the Ferris wheel is $v = 2\pi r/T = 2\pi\left(12.0\text{ m}\right)/15.5\text{ s} = 4.86\,\text{m/s}$.

(*a*) At the top, consider the free-body diagram shown. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's 2nd law with downward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.

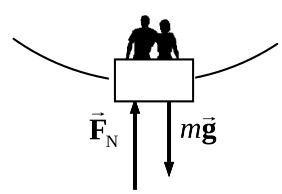


$$\sum F = F_{\text{R}} = mg - F_{\text{N}} = ma = mv^2/r \quad \rightarrow \quad F_{\text{N}} = mg - mv^2/r$$

The ratio of apparent weight to real weight is given by

$$\frac{mg - mv^2/r}{mg} = \frac{g - v^2/r}{g} = 1 - \frac{v^2}{rg} = 1 - \frac{\left(4.86\,\text{m/s}\right)^2}{\left(12.0\text{ m}\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{0.799}$$

(*b*) At the bottom, consider the free-body diagram shown. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's 2nd law with upward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.

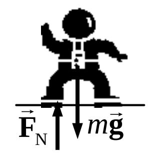


$$\sum F = F_{\text{R}} = F_{\text{N}} - mg = ma = mv^2/r \quad \rightarrow \quad F_{\text{N}} = mg + mv^2/r$$

The ratio of apparent weight to real weight is given by

$$\frac{mg + mv^2/r}{mg} = 1 + \frac{v^2}{rg} = \boxed{1.201}$$

51. Consider the free-body diagram for the astronaut in the space vehicle. The Moon is below the astronaut in the figure. We assume that the astronaut is touching the inside of the space vehicle, or in a seat, or strapped in somehow, and so a force will be exerted on the astronaut by the spacecraft. That force has been labeled $\vec{\mathbf{F}}_N$. The magnitude of that force is the apparent weight of the astronaut. Take down as the positive direction.



(*a*) If the spacecraft is moving with a constant velocity, then the acceleration of the astronaut must be 0, and so the net force on the astronaut is 0.

$$\sum F = mg - F_N = 0 \quad \rightarrow$$

$$F_N = mg = G\frac{mM_{\text{Moon}}}{r^2} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{(75\ \text{kg})\left(7.4\times10^{22}\ \text{kg}\right)}{\left(4.2\times10^6\ \text{m}\right)^2} = 21\ \text{N}$$

Since the value here is positive, the normal force points in the original direction as shown on the free-body diagram. The astronaut will be pushed "upward" by the floor or the seat. Thus the astronaut will perceive that he has a "weight" of $\boxed{21\ \text{N, towards the Moon}}$.

(*b*) Now the astronaut has an acceleration towards the Moon. Write Newton's 2nd law for the astronaut, with down as the positive direction.

$$\sum F = mg - F_N = ma \quad \rightarrow \quad F_N = mg - ma = 21\ \text{N} - (75\ \text{kg})\left(2.9\ \text{m/s}^2\right) = -2.0\times10^2\ \text{N}$$

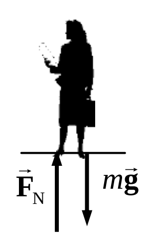
Because of the negative value, the normal force points in the opposite direction from what is shown on the free-body diagram – it is pointing towards the Moon. So perhaps the astronaut is pinned against the "ceiling" of the spacecraft, or safety belts are pulling down on the astronaut. The astronaut will perceive being "pushed downwards", and so has an upward apparent weight of $\boxed{2.0\times10^2\ \text{N, away from the Moon}}$.

52. Consider the motion of one of the stars. The gravitational force on the star is given by $F = G\dfrac{mm}{d^2}$, where d is the distance separating the two stars. But since the star is moving in a circle of radius $d/2$, the force on the star can be expressed as $F_R = mv^2/r = m\dfrac{v^2}{d/2}$. Equate these two force expressions, and use $v = 2\pi r/T = d\pi/T$.

$$G\frac{mm}{d^2} = m\frac{v^2}{d/2} = m\frac{(d\pi/T)^2}{d/2} \quad \rightarrow$$

$$m = \frac{2\pi^2 d^3}{GT^2} = \frac{2\pi^2\left(3.6\times10^{11}\ \text{m}\right)^3}{\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left[(5.7\ \text{y})\left(3.15\times10^7\ \text{sec/y}\right)\right]^2} = \boxed{4.3\times10^{29}\ \text{kg}}$$

53. Consider a free-body diagram for the woman in the elevator. $\vec{\mathbf{F}}_N$ is the force the spring scale exerts. Write Newton's 2nd law for the vertical direction, with up as positive.



$$\sum F = F_N - mg = ma \quad \rightarrow \quad F_N = m(g + a)$$

(*a*, *b*) For constant speed motion in a straight line, the acceleration is 0, and so

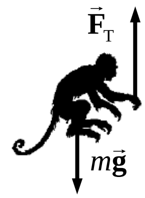
$$F_N = mg = (55\ \text{kg})\left(9.8\ \text{m/s}^2\right) = \boxed{5.4\times10^2\ \text{N}}$$

(*c*) Here $a = +0.33g$ and so $F_N = 1.33mg = 1.33(55\ \text{kg})\left(9.8\ \text{m/s}^2\right) = \boxed{7.2\times10^2\ \text{N}}$

(*d*) Here $a = -0.33g$ and so $F_N = 0.67\,mg = 0.67(55\text{ kg})(9.8\,\text{m/s}^2) = \boxed{3.6\times10^2\text{ N}}$

(*e*) Here $a = -g$ and so $F_N = \boxed{0\text{ N}}$

54. Draw a free-body diagram of the monkey. Then write Newton's 2nd law for the vertical direction, with up as positive.



$$\sum F = F_T - mg = ma \quad\rightarrow\quad a = \frac{F_T - mg}{m}$$

For the maximum tension of 220 N,

$$a = \frac{220\text{ N} - (17.0\text{ kg})(9.80\,\text{m/s}^2)}{(17.0\text{ kg})} = 3.1\,\text{m/s}^2$$

Thus the elevator must have an $\boxed{\text{upward acceleration greater than } a = 3.1\,\text{m/s}^2}$ for the cord to break. Any downward acceleration would result in a tension less than the monkey's weight.

55. (*a*) The speed of an object in near-surface orbit around a planet is given by $v = \sqrt{GM/R}$, where M is the planet mass and R is the planet radius. The speed is also given by $v = 2\pi R/T$, where T is the period of the object in orbit. Equate the two expressions for the speed.

$$\sqrt{G\frac{M}{R}} = \frac{2\pi R}{T} \quad\rightarrow\quad G\frac{M}{R} = \frac{4\pi^2R^2}{T^2} \quad\rightarrow\quad \frac{M}{R^3} = \frac{4\pi^2}{GT^2}$$

The density of a uniform spherical planet is given by $\rho = \dfrac{M}{\text{Volume}} = \dfrac{M}{\frac{4}{3}\pi R^3}$. Thus

$$\rho = \frac{3M}{4\pi R^3} = \frac{3}{4\pi}\frac{4\pi^2}{GT^2} = \frac{3\pi}{GT^2}$$

(*b*) For Earth,

$$\rho = \frac{3\pi}{GT^2} = \frac{3\pi}{(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2)[(85\text{ min})(60\text{ s/min})]^2} = \boxed{5.4\times10^3\text{ kg/m}^3}$$

56. Use Kepler's 3rd law for objects orbiting the Earth. The following are given.

$$T_2 = \text{period of Moon} = (27.4\text{ day})\left(\frac{86,400\text{ s}}{1\text{ day}}\right) = 2.367\times10^6\text{ sec}$$

$$r_2 = \text{radius of Moon's orbit} = 3.84\times10^8\text{ m}$$

$$r_1 = \text{radius of near-Earth orbit} = R_{\text{Earth}} = 6.38\times10^6\text{ m}$$

$$(T_1/T_2)^2 = (r_1/r_2)^3 \quad\rightarrow$$

$$T_1 = T_2(r_1/r_2)^{3/2} = (2.367\times10^6\text{ sec})\left(\frac{6.38\times10^6\text{ m}}{3.84\times10^8\text{ m}}\right)^{3/2} = \boxed{5.07\times10^3\text{sec}}(\sim 84.5\text{ min})$$

57. Use Kepler's 3rd law for objects orbiting the Sun.

$$\left(\frac{r_{\text{Icarus}}}{r_{\text{Earth}}}\right)^3 = \left(\frac{T_{\text{Icarus}}}{T_{\text{Earth}}}\right)^2 \quad\rightarrow\quad r_{\text{Icarus}} = r_{\text{Earth}}\left(\frac{T_{\text{Icarus}}}{T_{\text{Earth}}}\right)^{2/3} = (1.50\times10^{11}\text{m})\left(\frac{410\text{ d}}{365\text{ d}}\right)^{2/3} = \boxed{1.62\times10^{11}\text{m}}$$

58. Use Kepler's 3rd law for objects orbiting the Sun.

$$\left(T_{\text{Neptune}}/T_{\text{Earth}}\right)^2 = \left(r_{\text{Neptune}}/r_{\text{Earth}}\right)^3 \rightarrow$$

$$T_{\text{Neptune}} = T_{\text{Earth}}\left(\frac{r_{\text{Neptune}}}{r_{\text{Earth}}}\right)^{3/2} = (1\text{ year})\left(\frac{4.5\times10^9\text{ km}}{1.5\times10^8\text{ km}}\right)^{3/2} = \boxed{1.6\times10^2\text{ years}}$$

59. Use Kepler's 3rd law to relate the orbits of Earth and Halley's comet around the Sun.

$$\left(r_{\text{Halley}}/r_{\text{Earth}}\right)^3 = \left(T_{\text{Halley}}/T_{\text{Earth}}\right)^2 \rightarrow$$

$$r_{\text{Halley}} = r_{\text{Earth}}\left(T_{\text{Halley}}/T_{\text{Earth}}\right)^{2/3} = \left(150\times10^6\text{ km}\right)\left(76\text{ y}/1\text{ y}\right)^{2/3} = \boxed{2690\times10^6\text{ km}}$$

This value is half the sum of the nearest and farthest distances of Halley's comet from the Sun. Since the nearest distance is very close to the Sun, we will approximate that nearest distance as 0. Then the farthest distance is twice the value above, or $5380\times10^6\text{ km}$. This distance approaches the mean orbit distance of Pluto, which is $5900\times10^6\text{ km}$. It is still in the Solar System, nearest to Pluto's orbit.

60. There are two expressions for the velocity of an object in circular motion around a mass M: $v = \sqrt{GM/r}$ and $v = 2\pi r/T$. Equate the two expressions and solve for T.

$$\sqrt{GM/r} = 2\pi r/T \rightarrow$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{\left(\left(3\times10^4\text{ ly}\right)\frac{\left(3\times10^8\text{ m/s}\right)\left(3.16\times10^7\text{ sec}\right)}{1\text{ ly}}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(4\times10^{41}\text{ kg}\right)}} = 5.8\times10^{15}\text{ s} = 1.8\times10^8\text{ y}$$

$$\approx \boxed{2\times10^8\text{ y}}$$

61. (*a*) The relationship between satellite period T, mean satellite distance r, and planet mass M can be derived from the two expressions for satellite speed: $v = \sqrt{GM/r}$ and $v = 2\pi r/T$. Equate the two expressions and solve for M.

$$\sqrt{GM/r} = 2\pi r/T \rightarrow M = \frac{4\pi^2 r^3}{GT^2}$$

Substitute the values for Io to get the mass of Jupiter.

$$M_{\text{Jupiter-Io}} = \frac{4\pi^2\left(4.22\times10^8\text{ m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(1.77\text{d}\times\frac{24\text{ h}}{1\text{ d}}\times\frac{3600\text{ s}}{1\text{ h}}\right)^2} = \boxed{1.90\times10^{27}\text{ kg}}.$$

(*b*) For the other moons:

$$M_{\text{Jupiter-Europa}} = \frac{4\pi^2\left(6.71\times10^8\text{ m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(3.55\times24\times3600\text{ s}\right)^2} = \boxed{1.90\times10^{27}\text{ kg}}$$

$$M_{\text{Jupiter-Ganymede}} = \frac{4\pi^2\left(1.07\times10^9\text{ m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.16\times24\times3600\text{ s}\right)^2} = \boxed{1.89\times10^{27}\text{ kg}}$$

$$M_{\substack{\text{Jupiter-}\\\text{Callisto}}} = \frac{4\pi^2\left(1.883\times10^9\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(16.7\times24\times3600\,\text{s}\right)^2} = \boxed{1.90\times10^{27}\,\text{kg}}$$

$\boxed{\text{Yes}}$, the results are consistent – only about 0.5% difference between them.

62. Knowing the period of the Moon and the distance to the Moon, we can calculate the speed of the Moon by $v = 2\pi r/T$. But the speed can also be calculated for any Earth satellite by $v=\sqrt{G\,M_{\text{Earth}}/r}$. Equate the two expressions for the speed, and solve for the mass of the Earth.

$$\sqrt{G\,M_{\text{Earth}}/r} = 2\pi r/T \quad\rightarrow$$

$$M_{\text{Earth}} = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2\left(3.84\times10^8\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left[\left(27.4\,\text{d}\right)\left(86,400\,\text{s/d}\right)\right]^2} = \boxed{5.98\times10^{24}\,\text{kg}}$$

63. Use Kepler's 3rd law to find the radius of each moon of Jupiter, using Io's data for r_2 and T_2.

$$\left(r_1/r_2\right)^3 = \left(T_1/T_2\right)^2 \quad\rightarrow\quad r_1 = r_2\left(T_1/T_2\right)^{2/3}$$

$$r_{\text{Europa}} = r_{\text{Io}}\left(T_{\text{Europa}}/T_{\text{Io}}\right)^{2/3} = \left(422\times10^3\,\text{km}\right)\left(3.55\,\text{d}/1.77\,\text{d}\right)^{2/3} = \boxed{671\times10^3\,\text{km}}$$

$$r_{\text{Ganymede}} = \left(422\times10^3\,\text{km}\right)\left(7.16\,\text{d}/1.77\,\text{d}\right)^{2/3} = \boxed{1070\times10^3\,\text{km}}$$

$$r_{\text{Callisto}} = \left(422\times10^3\,\text{km}\right)\left(16.7\,\text{d}/1.77\,\text{d}\right)^{2/3} = \boxed{1880\times10^3\,\text{km}}$$

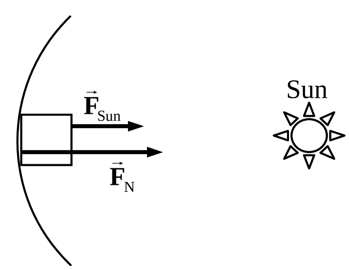
The agreement with the data in the table is excellent.

64. (*a*) Use Kepler's 3rd law to relate the Earth and the hypothetical planet in their orbits around the Sun.

$$\left(T_{\text{planet}}/T_{\text{Earth}}\right)^2 = \left(r_{\text{planet}}/r_{\text{Earth}}\right)^3 \quad\rightarrow\quad T_{\text{planet}} = T_{\text{Earth}}\left(r_{\text{planet}}/r_{\text{Earth}}\right)^{3/2} = \left(1\,\text{y}\right)\left(3/1\right)^{3/2} \approx \boxed{5\,\text{y}}$$

(*b*) No mass data can be calculated from this relationship, because the relationship is mass-independent. Any object at the orbit radius of 3 times the Earth's orbit radius would have a period of 5.2 years, regardless of its mass.

65. If the ring is to produce an apparent gravity equivalent to that of Earth, then the normal force of the ring on objects must be given by $F_{\text{N}} = mg$. The Sun will also exert a force on objects on the ring. See the free-body diagram.



Write Newton's 2nd law for the object, with the fact that the acceleration is centripetal.

$$\sum F = F_{\text{R}} = F_{\text{Sun}} + F_{\text{N}} = m v^2/r$$

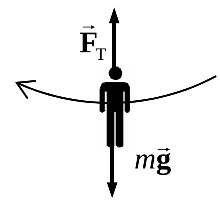
Substitute in the relationships that $v = 2\pi r/T$, $F_{\text{N}} = mg$, and $F_{\text{Sun}} = G\dfrac{M_{\text{Sun}}m}{r^2}$, and solve for the period of the rotation.

$$F_{Sun} + F_N = m v^2/r \quad\rightarrow\quad G\frac{M_{\text{Sun}}m}{r^2} + mg = \frac{4\pi^2 mr}{T^2} \quad\rightarrow\quad G\frac{M_{\text{Sun}}}{r^2} + g = \frac{4\pi^2 r}{T^2}$$

$$T=\sqrt{\frac{4\pi^2 r}{G\frac{M_{\text{Sun}}}{r^2}+g}}=\sqrt{\frac{4\pi^2\left(1.50\times10^{11}\,\text{m}\right)}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1.50\times10^{11}\,\text{m}\right)^2}+9.8\,\text{m/s}^2}}=7.8\times10^5\,\text{s}=\boxed{9.0\text{ d}}$$

The force of the Sun is only about 1/1600 the size of the normal force. The force of the Sun could have been ignored in the calculation with no effect in the result as given above.

66. A free-body diagram of Tarzan at the bottom of his swing is shown. The upward tension force is created by his pulling down on the vine. Write Newton's 2nd law in the vertical direction. Since he is moving in a circle, his acceleration will be centripetal, and points upward when he is at the bottom.



$$\sum F=F_{\text{T}}-mg=ma=mv^2/r\quad\rightarrow\quad v=\sqrt{\frac{\left(F_{\text{T}}-mg\right)r}{m}}$$

The maximum speed will be obtained with the maximum tension.

$$v_{\text{max}}=\sqrt{\frac{\left(\vec{\mathbf{F}}_{\text{T max}}-mg\right)r}{m}}=\sqrt{\frac{\left(1400\text{ N}-\left(80\text{ kg}\right)\left(9.8\,\text{m/s}^2\right)\right)5.5\text{ m}}{80\text{ kg}}}=\boxed{6.5\,\text{m/s}}$$

67. The acceleration due to the Earth's gravity at a location at or above the surface is given by $g=GM_{\text{Earth}}/r^2$, where r is the distance from the center of the Earth to the location in question. Find the location where $g=\frac{1}{2}g_{\text{surface}}$.

$$\frac{GM_{\text{Earth}}}{r^2}=\frac{1}{2}\frac{GM_{\text{Earth}}}{R_{\text{Earth}}^2}\quad\rightarrow\quad r^2=2R_{\text{Earth}}^2\quad\rightarrow\quad r=\sqrt{2}R_{\text{Earth}}$$

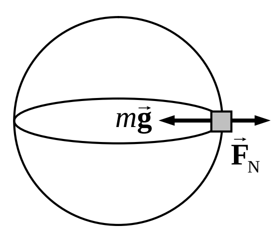
The distance above the Earth's surface is

$$r-R_{\text{Earth}}=\left(\sqrt{2}-1\right)R_{\text{Earth}}=\left(\sqrt{2}-1\right)\left(6.38\times10^6\,\text{m}\right)=\boxed{2.64\times10^6\,\text{m}}.$$

68. The radius of either skater's motion is 0.80 m, and the period is 2.5 sec. Thus their speed is given by $v=2\pi r/T=\frac{2\pi\left(0.80\text{ m}\right)}{2.5\text{ s}}=2.0\,\text{m/s}$. Since each skater is moving in a circle, the net radial force on each one is given by Eq. 5-3.

$$F_{\text{R}}=mv^2/r=\frac{\left(60.0\text{ kg}\right)\left(2.0\,\text{m/s}\right)^2}{0.80\text{ m}}=\boxed{3.0\times10^2\,\text{N}}.$$

69. Consider this free-body diagram for an object at the equator. Since the object is moving in a circular path, there must be a net force on the object, pointing towards the center of the Earth, producing a centripetal acceleration. Write Newton's 2nd law, with the inward direction positive.



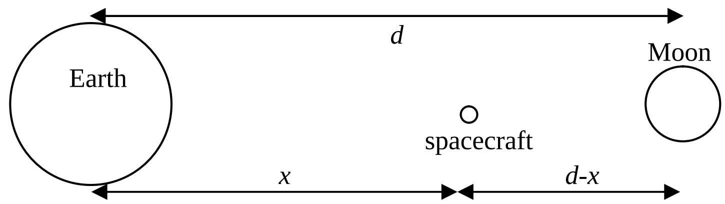
$$\sum F_{\text{R}}=mg-F_{\text{N}}=mv^2/R_{\text{Earth}}\quad\rightarrow\quad F_N=m\left(g-v^2/R_{\text{Earth}}\right)$$

We see that the normal force, which without the rotation would be the "expected" value of *mg*, has now been reduced. This effect can be described as saying that the acceleration due to gravity has been reduced, by an amount equal to v^2/R_{Earth}. To calculate this "change" in *g*, use $v=2\pi R_{\text{Earth}}/T$ to get the following.

$$\Delta g = -v^2/R_{\text{Earth}} = -\frac{4\pi^2 R_{\text{Earth}}}{T^2} = -\frac{4\pi^2\left(6.38\times10^6\,\text{m}\right)}{\left[(1\text{ d})(86,400\text{ s}/1\text{ d})\right]^2} = -0.03374\,\text{m/s}^2$$

This is a reduction of $-0.03374\,\text{m/s}^2 \times \dfrac{1\,g}{9.80\,\text{m/s}^2} = \boxed{-3.44\times10^{-3}\,g}$.

70. For the forces to balance means that the gravitational force on the spacecraft due to the Earth must be the same as that due to the Moon. Write the gravitational forces on the spacecraft, equate them, and solve for the distance *x*.



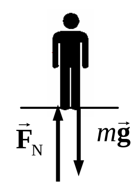
$$F_{\substack{\text{Earth-}\\\text{spacecraft}}} = G\frac{M_{\text{Earth}} m_{\text{spacecraft}}}{x^2} \quad ; \quad F_{\substack{\text{Moon}\\\text{spacecraft}}} = G\frac{M_{\text{Moon}} m_{\text{spacecraft}}}{(d-x)^2}$$

$$G\frac{M_{\text{Earth}} m_{\text{spacecraft}}}{x^2} = G\frac{M_{\text{Moon}} m_{\text{spacecraft}}}{(d-x)^2} \rightarrow \frac{x^2}{M_{\text{Earth}}} = \frac{(d-x)^2}{M_{\text{Moon}}} \rightarrow \frac{x}{\sqrt{M_{\text{Earth}}}} = \frac{d-x}{\sqrt{M_{\text{Moon}}}}$$

$$x = d\frac{\sqrt{M_{\text{Earth}}}}{\left(\sqrt{M_{\text{Moon}}}+\sqrt{M_{\text{Earth}}}\right)} = \left(3.84\times10^8\,\text{m}\right)\frac{\sqrt{5.97\times10^{24}\,\text{kg}}}{\left(\sqrt{7.35\times10^{22}\,\text{kg}}+\sqrt{5.97\times10^{24}\,\text{kg}}\right)} = \boxed{3.46\times10^8\,\text{m}}$$

This is only about 22 Moon radii away from the Moon.

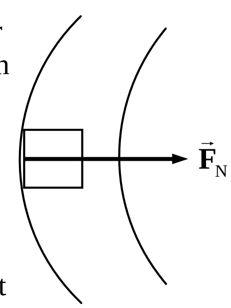
71. Consider a free-body diagram of yourself in the elevator. $\vec{\mathbf{F}}_{\text{N}}$ is the force of the scale pushing up on you, and reads the normal force. Since the scale reads 82 kg, if it were calibrated in Newtons, the normal force would be $F_{\text{N}} = (82\text{ kg})\left(9.8\,\text{m/s}^2\right) = 804\text{ N}$. Write Newton's 2nd law in the vertical direction, with upward as positive.



$$\sum F = F_{\text{N}} - mg = ma \rightarrow a = \frac{F_{\text{N}} - mg}{m} = \frac{804\text{ N} - (65\text{ kg})\left(9.8\,\text{m/s}^2\right)}{65\text{ kg}} = \boxed{2.6\ \text{m/s}^2\text{ upward}}$$

Since the acceleration is positive, the acceleration is upward.

72. To experience a gravity-type force, objects must be on the inside of the outer wall of the tube, so that there can be a centripetal force to move the objects in a circle. See the free-body diagram for an object on the inside of the outer wall, and a portion of the tube. The normal force of contact between the object and the wall must be maintaining the circular motion. Write Newton's 2nd law for the radial direction.



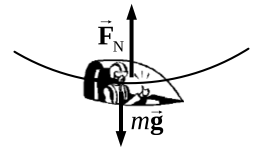
$$\sum F_{\text{R}} = F_{\text{N}} = ma = mv^2/r$$

If this is to have the same effect as Earth gravity, then we must also have that $F_{\text{N}} = mg$. Equate the two expressions for normal force and solve for the speed.

$$F_{\text{N}} = mv^2/r = mg \rightarrow v = \sqrt{gr} = \sqrt{\left(9.8\,\text{m/s}^2\right)(550\,\text{m})} = 73\,\text{m/s}$$

$$(73\,\text{m/s})\left(\frac{1\text{ rev}}{2\pi(550\text{ m})}\right)\left(\frac{86,400\text{ s}}{1\text{ d}}\right) = 1825\,\text{rev/d} \approx \boxed{1.8\times10^3\text{ rev/d}}$$

73. (*a*) See the free-body diagram for the pilot in the jet at the bottom of the loop. For $a_R = v^2/r = 6g$,



$$v^2/r = 6.0g \;\rightarrow\; r = \frac{v^2}{6.0g} = \frac{\left[(1300\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{6.0(9.8\,\text{m/s}^2)} = \boxed{2.2\times10^3\,\text{m}}$$

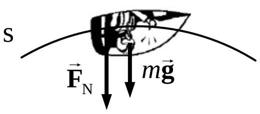
(*b*) The net force must be centripetal, to make the pilot go in a circle. Write Newton's 2nd law for the vertical direction, with up as positive. The normal force is the apparent weight.

$$\sum F_R = F_N - mg = m v^2/r$$

The centripetal acceleration is to be $v^2/r = 6.0g$.

$$F_N = mg + m v^2/r = 7mg = 7(78\text{ kg})(9.80\,\text{m/s}^2) = 5350\text{ N} = \boxed{5.4\times10^3\,\text{N}}$$

(*c*) See the free-body diagram for the pilot at the top of the loop. Notice that the normal force is down, because the pilot is upside down. Write Newton's 2nd law in the vertical direction, with down as positive.

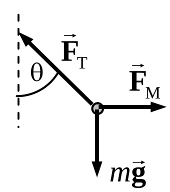


$$\sum F_R = F_N + mg = m v^2/r = 6mg \;\rightarrow\; F_N = 5mg = \boxed{3.8\times10^3\,\text{N}}$$

74. The force of gravity on an object at the surface is given by $F_{grav} = mg_P$. But by Newton's law of Universal Gravitation, the force of gravity on an object at the surface is given by $F_{grav} = G\frac{mM_{planet}}{r^2}$. Equate the expressions for the force of gravity and solve for the mass of the planet.

$$G\frac{mM_{planet}}{r^2} = mg_P \;\rightarrow\; \boxed{M_{planet} = \frac{r^2 g_P}{G}}.$$

75. (*a*) See the free-body diagram for the plumb bob. The attractive gravitational force on the plumb bob is $F_M = G\frac{mm_M}{D_M^2}$. Since the bob is not accelerating, the net force in any direction will be zero. Write the net force for both vertical and horizontal directions. Use $g = G\frac{M_{Earth}}{R_{Earth}^2}$



$$\sum F_{vertical} = F_T\cos\theta - mg = 0 \;\rightarrow\; F_T = \frac{mg}{\cos\theta}$$

$$\sum F_{horizontal} = F_M - F_T\sin\theta = 0 \;\rightarrow\; F_M = F_T\sin\theta = mg\tan\theta$$

$$G\frac{mm_M}{D_M^2} = mg\tan\theta \;\rightarrow\; \theta = \tan^{-1} G\frac{m_M}{gD_M^2} = \boxed{\tan^{-1}\frac{m_M R_{Earth}^2}{M_{Earth}D_M^2}}$$

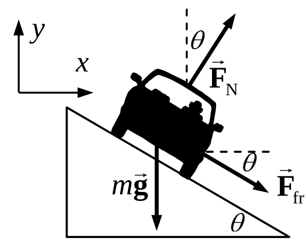
(*b*) We estimate the mass of Mt. Everest by taking its volume times its mass density. If we approximate Mt. Everest as a cone with the same size diameter as height, then its volume is $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2000\text{ m})^2(4000\text{ m}) = 1.7\times10^{10}\,\text{m}^3$. The density is $\rho = 3\times10^3\text{ kg/m}^3$. Find the mass by multiplying the volume times the density.

$$M = \rho V = (3\times10^3\text{ kg/m}^3)(1.7\times10^{10}\,\text{m}^3) = \boxed{5\times10^{13}\,\text{kg}}$$

(*c*) With $D = 5000$ m, use the relationship derived in part (a).

$$\theta = \tan^{-1}\frac{M_M R_{Earth}^2}{M_{Earth} D_M^2} = \tan^{-1}\frac{\left(5\times10^{13}\,\text{kg}\right)\left(6.38\times10^{6}\,\text{m}\right)^2}{\left(5.97\times10^{24}\,\text{kg}\right)\left(5000\,\text{m}\right)^2} = \boxed{8\times10^{-4}\,\text{degrees}}$$

76. Since the curve is designed for a speed of 95 km/h, traveling at that speed would mean no friction is needed to round the curve. From Example 5-7 in the textbook, the no-friction banking angle is given by



$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{\left[\left(95\,\text{km/h}\right)\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{\left(67\text{ m}\right)\left(9.8\,\text{m/s}^2\right)} = 46.68^\circ$$

Driving at a higher speed with the same radius means that more centripetal force will be required than is present by the normal force alone. That extra centripetal force will be supplied by a force of static friction, downward along the incline, as shown in the first free-body diagram for the car on the incline. Write Newton's 2nd law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{fr} = \mu_s F_N$.

$$\sum F_y = F_N\cos\theta - mg - F_{fr}\sin\theta = 0 \;\rightarrow\; F_N\cos\theta - \mu_s F_N\sin\theta = mg \;\rightarrow$$

$$F_N = \frac{mg}{\left(\cos\theta - \mu_s\sin\theta\right)}$$

$$\sum F_x = F_N\sin\theta + F_{fr}\cos\theta = mv^2/r \;\rightarrow\; F_N\sin\theta + \mu_s F_N\cos\theta = mv^2/r \;\rightarrow$$

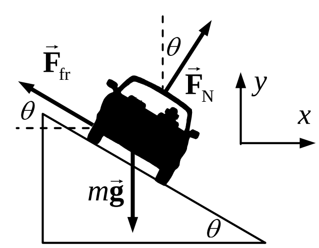
$$F_N = \frac{mv^2/r}{\left(\sin\theta + \mu_s\cos\theta\right)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/r}{\left(\sin\theta + \mu_s\cos\theta\right)} = \frac{mg}{\left(\cos\theta - \mu_s\sin\theta\right)} \;\rightarrow$$

$$v = \sqrt{rg\frac{\left(\sin\theta + \mu_s\cos\theta\right)}{\left(\cos\theta - \mu_s\sin\theta\right)}} = \sqrt{\left(67\text{ m}\right)\left(9.8\,\text{m/s}^2\right)\frac{\left(\sin 46.68^\circ + 0.30\cos 46.68^\circ\right)}{\left(\cos 46.68^\circ - 0.30\sin 46.68^\circ\right)}} = 36\,\text{m/s}$$

Now for the slowest possible speed. Driving at a slower speed with the same radius means that less centripetal force will be required than that supplied by the normal force. That decline in centripetal force will be supplied by a force of static friction, upward along the incline, as shown in the second free-body diagram for the car on the incline. Write Newton's 2nd law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{fr} = \mu_s F_N$.



$$\sum F_y = F_N\cos\theta - mg + F_{fr}\sin\theta = 0 \;\rightarrow$$

$$F_N\cos\theta + \mu_s F_N\sin\theta = mg \;\rightarrow\; F_N = \frac{mg}{\left(\cos\theta + \mu_s\sin\theta\right)}$$

$$\sum F_x = F_N \sin\theta - F_{fr}\cos\theta = mv^2/r \quad \rightarrow \quad F_N \sin\theta - \mu_s F_N \cos\theta = mv^2/r \quad \rightarrow$$

$$F_N = \frac{mv^2/r}{(\sin\theta - \mu_s \cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/r}{(\sin\theta - \mu_s\cos\theta)} = \frac{mg}{(\cos\theta + \mu_s \sin\theta)} \quad \rightarrow$$

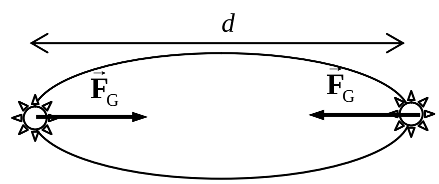
$$v = \sqrt{rg\frac{(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)}} = \sqrt{(67\text{ m})(9.8\text{ m/s}^2)\frac{(\sin 46.67^\circ - 0.30\cos 46.67^\circ)}{(\cos 46.67^\circ + 0.30\sin 46.67^\circ)}} = 19\text{ m/s}$$

Thus the range is $\boxed{19\text{ m/s} \le v \le 36\text{ m/s}}$, which is $\boxed{68\text{ km/h} \le v \le 130\text{ km/h}}$.

77. For an object to be apparently weightless would mean that the object would have a centripetal acceleration equal to g. This is really the same as asking what the orbital period would be for an object orbiting the Earth with an orbital radius equal to the Earth's radius. To calculate, use $g = a_C = v^2/R_{\text{Earth}}$, along with $v = 2\pi R_{\text{Earth}}/T$, and solve for T.

$$g = \frac{v^2}{R_{\text{Earth}}} = \frac{4\pi^2 R_{\text{Earth}}}{T^2} \quad \rightarrow \quad T = 2\pi\sqrt{\frac{R_{\text{Earth}}}{g}} = 2\pi\sqrt{\frac{6.38\times10^6\text{ m}}{9.80\text{ m/s}^2}} = \boxed{5.07\times10^3\text{ s}}(\sim 84.5\text{ min})$$

78. See the diagram for the two stars.



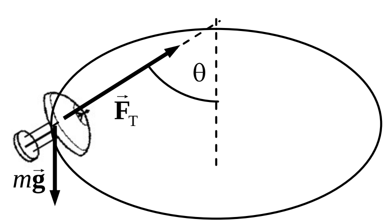
(*a*) The two stars don't crash into each other because of their circular motion. The force on them is centripetal, and maintains their circular motion. Another way to consider it is that the stars have a velocity, and the gravity force causes CHANGE in velocity, not actual velocity. If the stars were somehow brought to rest and then released under the influence of their mutual gravity, they would crash into each other.

(*b*) Set the gravity force on one of the stars equal to the centripetal force, using the relationship that $v = 2\pi r/T = \pi d/T$, and solve for the mass.

$$F_G = G\frac{M^2}{d^2} = F_R = M\frac{v^2}{d/2} = M\frac{2(\pi d/T)^2}{d} = \frac{2\pi^2 Md}{T^2} \quad \rightarrow \quad G\frac{M^2}{d^2} = \frac{2\pi^2 Md}{T^2} \quad \rightarrow$$

$$M = \frac{2\pi^2 d^3}{GT^2} = \frac{2\pi^2(8.0\times10^{10}\text{ m})^3}{(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2)\left(12.6\text{ y}\times\frac{3.15\times10^7\text{ s}}{1\text{ y}}\right)^2} = \boxed{9.6\times10^{26}\text{ kg}}$$

79. The lamp must have the same speed and acceleration as the train. The forces on the lamp as the train rounds the corner are shown in the free-body diagram included. The tension in the suspending cord must not only hold the lamp up, but also provide the centripetal force needed to make the lamp move in a circle. Write Newton's 2nd law for the vertical direction, noting that the lamp is not accelerating vertically.



$$\sum F_y = F_T \cos\theta - mg = 0 \quad \rightarrow \quad F_T = \frac{mg}{\cos\theta}$$

The force moving the lamp in a circle is the horizontal portion of the tension. Write Newton's 2nd law for that radial motion.

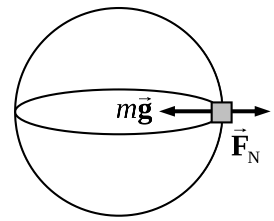
$$\sum F_R = F_T \sin\theta = ma_R = mv^2/r$$

Substitute the expression for the tension from the first equation into the second equation, and solve for the speed.

$$F_T \sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = mv^2/r \quad \rightarrow$$

$$v = \sqrt{rg\tan\theta} = \sqrt{(235\text{ m})(9.80\text{ m/s}^2)\tan 17.5^\circ} = \boxed{26.9\text{ m/s}}$$

80. For a body on the equator, the net motion is circular. Consider the free-body diagram as shown. F_N is the normal force, which is the apparent weight. The net force must point to the center of the circle for the object to be moving in a circular path at constant speed. Write Newton's 2nd law with the inward direction as positive.



$$\sum F_R = mg_{\text{Jupiter}} - F_N = mv^2/R_{\text{Jupiter}} \quad \rightarrow$$

$$F_N = m\left(g_{\text{Jupiter}} - v^2/R_{\text{Jupiter}}\right) = m\left(G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}} - \frac{v^2}{R_{\text{Jupiter}}}\right)$$

Use the fact that for a rotating object, $v = 2\pi r/T$.

$$F_N = m\left(G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}} - \frac{4\pi^2 R_{\text{Jupiter}}}{T^2_{\text{Jupiter}}}\right)$$

Thus the perceived acceleration of the object on the surface of Jupiter is

$$G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}} - \frac{4\pi^2 R_{\text{Jupiter}}}{T^2_{\text{Jupiter}}} = \left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(1.9\times10^{27}\text{ kg}\right)}{\left(7.1\times10^7\text{ m}\right)^2} - \frac{4\pi^2\left(7.1\times10^7\text{ m}\right)}{\left[(595\text{ min})\left(\frac{60\text{ s}}{1\text{ min}}\right)\right]^2}$$

$$= 22.94\text{ m/s}^2\left(\frac{1\,g}{9.8\text{ m/s}^2}\right) = \boxed{2.3\ g\text{'s}}$$

Thus you would not be crushed at all. You would certainly feel "heavy", but not at all crushed.

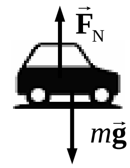
81. The speed of an orbiting object is given by $v = \sqrt{GM/r}$, where r is the radius of the orbit, and M is the mass around which the object is orbiting. Solve the equation for M.

$$v = \sqrt{GM/r} \quad \rightarrow \quad M = \frac{rv^2}{G} = \frac{\left(5.7\times10^{17}\text{ m}\right)\left(7.8\times10^5\text{ m/s}\right)^2}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)} = \boxed{5.2\times10^{39}\text{ kg}}$$

The number of solar masses is found by dividing the result by the solar mass.

$$\#\text{ solar masses} = \frac{M_{\text{galaxy}}}{M_{\text{Sun}}} = \frac{5.2\times10^{39}\text{ kg}}{2\times10^{30}\text{ kg}} = \boxed{2.6\times10^9\text{ solar masses}}$$

82. A generic free-body diagram for the car at any of the three locations is shown. Write Newton's 2nd law for the vertical direction, with downward positive.



$$\sum F = mg - F_N = ma$$

(*a*) At point B, the net force is 0, so the acceleration is 0, and so $F_N = mg$. At point A, the net force is positive, so $mg > F_N$, which is interpreted as a relatively small normal force. At point B, the net force is negative, so $mg < F_N$, which is interpreted as a relatively large normal force. And so $\boxed{F_{NC} > F_{NB} > F_{NA}}$.

(*b*) The driver "feels heavy" when the normal force is larger than his weight, so the driver will feel heavy at point C. The driver "feels light" when the normal force is smaller than his weight, so the driver will feel light at point A. $\boxed{\text{From feels heaviest to feels lightest, C, B, A.}}$

(*c*) In general, $mg - F_N = mv^2/r$ if the car is executing a part of the road that is curved up or down with a radius of *R*. If the normal force goes to 0, then the car loses contact with the road. This happens if $mg = mv^2/r \rightarrow \boxed{v_{max} = \sqrt{gR}}$. Any faster and the car will lose contact with the road.

83. (*a*) The speed of a satellite orbiting the Earth is given by $v = \sqrt{GM_{Earth}/r}$. For the GPS satellites, $r = R_{Earth} + (11,000)(1.852\text{ km}) = 2.68\times10^7\text{ m}$.

$$v = \sqrt{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\text{kg}\right)}{2.68\times10^7\text{ m}}} = \boxed{3.86\times10^3\text{ m/s}}$$

(*b*) The period can be found from the speed and the radius.

$$v = 2\pi r/T \rightarrow T = \frac{2\pi r}{v} = \frac{2\pi\left(2.68\times10^7\text{ m}\right)}{3.86\times10^3\text{ m/s}} = \boxed{4.36\times10^4\text{sec}} = 12.1\text{ hours}$$

84. (*a*) The speed of an object orbiting a mass *M* is given by $v = \sqrt{GM/r}$. The mass of the asteroid is found by multiplying the density times the volume. The period of an object moving in a circular path is given by $T = 2\pi r/v$. Combine these relationships to find the period.

$$M = \rho V = \left(2.3\times10^3\frac{\text{kg}}{\text{m}^3}\right)\left(40000\times6000\times6000\text{ m}^3\right) = 3.312\times10^{15}\text{kg}$$

$$T = \frac{2\pi r}{\sqrt{G\frac{M}{r}}} = \frac{2\pi\left(1.5\times10^4\text{ m}\right)}{\sqrt{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(3.312\times10^{15}\text{kg}\right)}{\left(1.5\times10^4\text{ m}\right)}}} = 2.456\times10^4\text{sec}$$

$$\approx \boxed{2\times10^4\text{sec}} \approx 7\text{ h}$$

(*b*) If the asteroid were a sphere, then the mass would be given by $M = \rho V = \frac{4}{3}\pi\rho r^3$. Solve this for the radius.

$$r = \left(\frac{3M}{4\pi\rho}\right)^{1/3} = \left(\frac{3\left(3.312\times10^{15}\text{kg}\right)}{4\pi\left(2.3\times10^3\frac{\text{kg}}{\text{m}^3}\right)}\right)^{1/3} = 7005\text{ m} \approx \boxed{7\times10^3\text{m}}$$

(*c*) The acceleration due to gravity is found from the mass and the radius.

$$g = GM/r^2 = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(3.312\times10^{15}\,\text{kg}\right)}{\left(7005\ \text{m}\right)^2} = 4.502\times10^{-3}\ \text{m/s}^2$$

$$\approx \boxed{5\times10^{-3}\ \text{m/s}^2}$$

85. The relationship between orbital speed and orbital radius for objects in orbit around the Earth is given by $v=\sqrt{GM_{\text{Earth}}/r}$. There are two orbital speeds involved – the one at the original radius, $v_0=\sqrt{GM_{\text{Earth}}/r_0}$, and the faster speed at the reduced radius, $v=\sqrt{GM_{\text{Earth}}/(r_0-\Delta r)}$.

(*a*) At the faster speed, 25,000 more meters will be traveled during the "catch-up" time, *t*. Note that $r_0 = 6.38\times10^6\,\text{m} + 4\times10^5\,\text{m} = 6.78\times10^6\,\text{m}$.

$$vt = v_0t + 2.5\times10^4\,\text{m} \;\rightarrow\; \left(\sqrt{G\frac{M_{\text{Earth}}}{r_0-\Delta r}}\right)t = \left(\sqrt{G\frac{M_{\text{Earth}}}{r_0}}\right)t + 2.5\times10^4\,\text{m} \;\rightarrow$$

$$t = \frac{2.5\times10^4\,\text{m}}{\sqrt{GM_{\text{Earth}}}\left(\frac{1}{\sqrt{r_0-\Delta r}}-\frac{1}{\sqrt{r_0}}\right)}$$

$$= \frac{2.5\times10^4\,\text{m}}{\sqrt{\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\,\text{kg}\right)}\left(\frac{1}{\sqrt{6.78\times10^6\,\text{m}-1\times10^3\,\text{m}}}-\frac{1}{\sqrt{6.78\times10^6\,\text{m}}}\right)}$$

$$= 4.42\times10^4\,\text{s} \approx \boxed{12\,\text{h}}$$

(*b*) Again, 25,000 more meters must be traveled at the faster speed in order to catch up to the satellite.

$$vt = v_0t + 2.5\times10^4\,\text{m} \;\rightarrow\; \left(\sqrt{G\frac{M_{\text{Earth}}}{r_0-\Delta r}}\right)t = \left(\sqrt{G\frac{M_{\text{Earth}}}{r_0}}\right)t + 2.5\times10^4\,\text{m} \;\rightarrow$$

$$\sqrt{\frac{1}{r_0-\Delta r}} = \frac{1}{\sqrt{r_0}} + \frac{2.5\times10^4\,\text{m}}{t\sqrt{GM_{\text{Earth}}}} \;\rightarrow\; \sqrt{r_0-\Delta r} = \left[\frac{1}{\sqrt{r_0}} + \frac{2.5\times10^4\,\text{m}}{t\sqrt{GM_{\text{Earth}}}}\right]^{-1} \;\rightarrow$$

$$\Delta r = r_0 + \left[\frac{1}{\sqrt{r_0}} + \frac{2.5\times10^4\,\text{m}}{t\sqrt{GM_{\text{Earth}}}}\right]^{-2} = 1755\ \text{m} \approx \boxed{1.8\times10^3\,\text{m}}$$

86. (*a*) Use Kepler's 3rd law to relate the orbits of the Earth and the comet around the Sun.

$$\left(\frac{r_{\text{comet}}}{r_{\text{Earth}}}\right)^3 = \left(\frac{T_{\text{comet}}}{T_{\text{Earth}}}\right)^2 \;\rightarrow\; r_{\text{comet}} = r_{\text{Earth}}\left(\frac{T_{\text{comet}}}{T_{\text{Earth}}}\right)^{2/3} = \left(1\ \text{AU}\right)\left(\frac{3000\ \text{y}}{1\ \text{y}}\right)^{2/3} = \boxed{208\ \text{AU}}$$

(*b*) The mean distance is the numeric average of the closest and farthest distances.

$$208\ \text{AU} = \frac{1\ \text{AU} + r_{\max}}{2} \;\rightarrow\; r_{\max} = \boxed{415\ \text{AU}}.$$

(*c*) Refer to figure 5-29, which illustrates Kepler's second law. If the time for each shaded region is made much shorter, then the area of each region can be approximated as a triangle. The area of each triangle is half the "base" (speed of comet multiplied by the amount of time) times the "height" (distance from Sun). So we have the following.

$$\text{Area}_{\text{min}} = \text{Area}_{\text{max}} \rightarrow \tfrac{1}{2}(v_{\text{min}}t)r_{\text{min}} = \tfrac{1}{2}(v_{\text{max}}t)r_{\text{max}} \rightarrow$$
$$v_{\text{min}}/v_{\text{max}} = r_{\text{max}}/r_{\text{min}} = \boxed{415/1}.$$

87. Let us assume that each person has a mass of 70 kg (a weight of ~ 150 lb). We shall assume that the people can be treated as point masses, and that their centers of mass are about 0.5 m apart. Finally, we assume that we can feel a gravitational force of about 1 N. The expression for the gravitational force becomes

$$F = G\frac{m_1 m_2}{r^2} \rightarrow G = \frac{Fr^2}{m_1 m_2} = \frac{(1\text{ N})(0.5\text{ m})^2}{(70\text{ kg})^2} = \boxed{5\times10^{-5}\text{ N}\bullet\text{m}^2/\text{kg}^2}.$$

This is roughly one million times larger than G actually is.

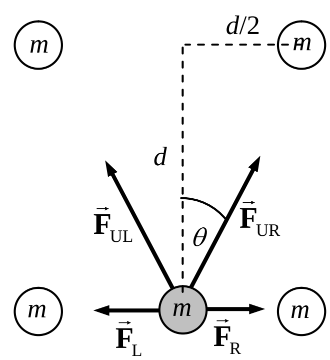
88. The speed of rotation of the Sun about the galactic center, under the assumptions made, is given by $v = \sqrt{G\frac{M_{\text{galaxy}}}{r_{\text{Sun orbit}}}}$ and so $M_{\text{galaxy}} = \frac{r_{\text{Sun orbit}}v^2}{G}$. Substitute in the relationship that $v = 2\pi r_{\text{Sun orbit}}/T$.

$$M_{\text{galaxy}} = \frac{4\pi^2(r_{\text{Sun orbit}})^3}{GT^2} = \frac{4\pi^2\left[(30,000)(9.5\times10^{15}\text{ m})\right]^3}{(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2)\left[(200\times10^6\text{ y})\left(\frac{3.15\times10^7\text{ s}}{1\text{ y}}\right)\right]^2}$$
$$= 3.452\times10^{41}\text{ kg} \approx \boxed{3\times10^{41}\text{ kg}}$$

The number of solar masses is found by dividing the result by the solar mass.

$$\#\text{ stars} = \frac{M_{\text{galaxy}}}{M_{\text{Sun}}} = \frac{3.452\times10^{41}\text{ kg}}{2.0\times10^{30}\text{ kg}} = 1.726\times10^{11} \approx \boxed{2\times10^{11}\text{ stars}}$$

89. See the free-body diagram. $\theta = \tan^{-1} 0.25/0.50 = 27^\circ$. Because of the symmetry of the problem, the forces $\vec{\mathbf{F}}_{\text{L}}$ and $\vec{\mathbf{F}}_{\text{R}}$ cancel each other. Likewise, the horizontal components of $\vec{\mathbf{F}}_{\text{UR}}$ and $\vec{\mathbf{F}}_{\text{UL}}$ cancel each other. Thus the only forces on the fifth mass will be the vertical components of $\vec{\mathbf{F}}_{\text{UR}}$ and $\vec{\mathbf{F}}_{\text{UL}}$. These components are also equal, and so only one needs to be calculated, and then doubled.



$$F_{\text{net}} = 2F_{UR}\cos\theta = 2G\frac{m^2}{d^2 + (d/2)^2}\cos\theta$$
$$= 2(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2)\frac{(1.0\text{ kg})^2}{\tfrac{5}{4}(0.50\text{ m})^2}\cos 27^\circ = \boxed{3.8\times10^{-10}\text{ N, upward}}$$

90. The gravitational force on the satellite is given by $F_{\text{grav}} = G\frac{M_{\text{Earth}}m}{r^2}$, where r is the distance of the satellite from the center of the Earth. Since the satellite is moving in circular motion, then the net force on the satellite can be written as $F_{\text{net}} = mv^2/r$. By substituting $v = 2\pi r/T$ for a circular orbit,

we have $F_{\text{net}} = \dfrac{4\pi^2 mr}{T^2}$. Then, since gravity is the only force on the satellite, the two expressions for force can be equated, and solved for the orbit radius.

$$G\frac{M_{\text{Earth}} m}{r^2} = \frac{4\pi^2 mr}{T^2} \quad \rightarrow$$

$$r = \left(\frac{GM_{\text{Earth}}T^2}{4\pi^2}\right)^{1/3} = \left[\frac{\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(6.0\times10^{24}\,\text{kg}\right)\left(6200\ \text{s}\right)^2}{4\pi^2}\right]^{1/3} = 7.304\times10^6\,\text{m}$$

(*a*) From this value the gravitational force on the satellite can be calculated.

$$F_{\text{grav}} = G\frac{M_{\text{Earth}} m}{r^2} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(6.0\times10^{24}\,\text{kg}\right)\left(5500\ \text{kg}\right)}{\left(7.304\times10^6\,\text{m}\right)^2} = 4.126\times10^4\,\text{N}$$

$$\approx \boxed{4.1\times10^4\,\text{N}}$$

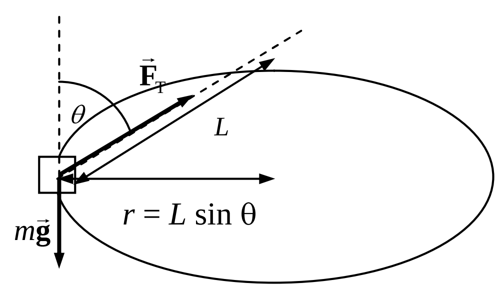
(*b*) The altitude of the satellite above the Earth's surface is given by

$$r - R_{\text{Earth}} = 7.304\times10^6\,\text{m} - 6.38\times10^6\,\text{m} = \boxed{9.2\times10^5\,\text{m}}.$$

91. The radial acceleration is given by $a_R = v^2/r$. Substitute in the speed of the tip of the sweep hand, given by $v = 2\pi r/T$, to get $a_R = \dfrac{4\pi^2 r}{T^2}$. For the tip of the sweep hand, r = 0.015 m, and T = 60 sec.

$$a_R = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2\left(0.015\ \text{m}\right)}{\left(60\ \text{s}\right)^2} = \boxed{1.6\times10^{-4}\ \text{m/s}^2}.$$

92. A free-body diagram for the sinker weight is shown. L is the length of the string actually swinging the sinker. The radius of the circle of motion is moving is $r = L\sin\theta$. Write Newton's 2nd law for the vertical direction, noting that the sinker is not accelerating vertically. Take up to be positive.



$$\sum F_y = F_T\cos\theta - mg = 0 \quad \rightarrow \quad F_T = \frac{mg}{\cos\theta}$$

The radial force is the horizontal portion of the tension. Write Newton's 2nd law for the radial motion.

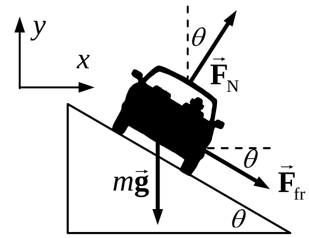
$$\sum F_R = F_T\sin\theta = ma_R = mv^2/r$$

Substitute the tension from the vertical equation, and the relationships $r = L\sin\theta$ and $v = 2\pi r/T$.

$$F_T\sin\theta = mv^2/r \quad \rightarrow \quad \frac{mg}{\cos\theta}\sin\theta = \frac{4\pi^2 mL\sin\theta}{T^2} \quad \rightarrow \quad \cos\theta = \frac{gT^2}{4\pi^2 L}$$

$$\theta = \cos^{-1}\frac{gT^2}{4\pi^2 L} = \cos^{-1}\frac{\left(9.8\,\text{m/s}^2\right)\left(0.50\ \text{s}\right)^2}{4\pi^2\left(0.25\ \text{m}\right)} = \boxed{76^\circ}$$

93. From Example 5-7 in the textbook, the no-friction banking angle is given by $\theta = \tan^{-1}\frac{v_0^2}{Rg}$. The centripetal force in this case is provided by a component of the normal force. Driving at a higher speed with the same radius requires more centripetal force than that provided by the normal force alone. The additional centripetal force is supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. The center of the circle of the car's motion is to the right of the car in the diagram. Write Newton's 2nd law in both the x and y directions. The car will have no acceleration in the y direction, and centripetal acceleration in the x direction. Assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{fr} = \mu_s F_N$.



$$\sum F_y = F_N\cos\theta - mg - F_{fr}\sin\theta = 0 \;\rightarrow\; F_N\cos\theta - \mu_s F_N\sin\theta = mg \;\rightarrow$$

$$F_N = \frac{mg}{(\cos\theta - \mu_s\sin\theta)}$$

$$\sum F_x = F_R = F_N\sin\theta + F_{fr}\cos\theta = mv^2/R \;\rightarrow\; F_N\sin\theta + \mu_s F_N\cos\theta = mv^2/R \;\rightarrow$$

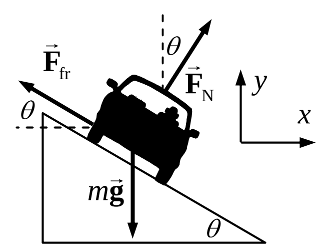
$$F_N = \frac{mv^2/R}{(\sin\theta + \mu_s\cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed, which is the maximum speed that the car can have.

$$\frac{mv^2/R}{(\sin\theta + \mu_s\cos\theta)} = \frac{mg}{(\cos\theta - \mu_s\sin\theta)} \;\rightarrow$$

$$v_{max} = \sqrt{Rg\frac{\sin\theta}{\cos\theta}\frac{(1+\mu_s/\tan\theta)}{(1-\mu_s\tan\theta)}} = v_0\sqrt{\frac{(1+Rg\mu_s/v_0^2)}{(1-\mu_s v_0^2/Rg)}}$$

Driving at a slower speed with the same radius requires less centripetal force than that provided by the normal force alone. The decrease in centripetal force is supplied by a force of static friction, upward along the incline. See the free-body diagram for the car on the incline. Write Newton's 2nd law in both the x and y directions. The car will have no acceleration in the y direction, and centripetal acceleration in the x direction. Assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{fr} = \mu_s F_N$.



$$\sum F_y = F_N\cos\theta - mg + F_{fr}\sin\theta = 0 \;\rightarrow$$

$$F_N\cos\theta + \mu_s F_N\sin\theta = mg \;\rightarrow\; F_N = \frac{mg}{(\cos\theta + \mu_s\sin\theta)}$$

$$\sum F_x = F_R = F_N\sin\theta - F_{fr}\cos\theta = mv^2/R \;\rightarrow\; F_N\sin\theta - \mu_s F_N\cos\theta = mv^2/R \;\rightarrow$$

$$F_N = \frac{mv^2/R}{(\sin\theta - \mu_s\cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/R}{(\sin\theta - \mu_s\cos\theta)} = \frac{mg}{(\cos\theta + \mu_s\sin\theta)} \quad \rightarrow$$

$$v_{\min} = \sqrt{Rg\frac{\sin\theta}{\cos\theta}\frac{(1-\mu_s/\tan\theta)}{(1+\mu_s\tan\theta)}} = v_0\sqrt{\frac{(1-\mu_s Rg/v_0^2)}{(1+\mu_s v_0^2/Rg)}}$$

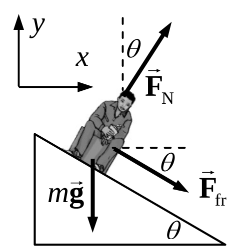
Thus $\boxed{v_{\min} = v_0\sqrt{\frac{(1-\mu_s Rg/v_0^2)}{(1+\mu_s v_0^2/Rg)}}}$ and $\boxed{v_{\max} = v_0\sqrt{\frac{(1+Rg\mu_s/v_0^2)}{(1-\mu_s v_0^2/Rg)}}}$.

94. The speed of the train is $(160\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 44.44\,\text{m/s}$

(*a*) If there is no tilt, then the friction force must supply the entire centripetal force on the passenger.

$$F_R = mv^2/R = \frac{(75\text{ kg})(44.44\,\text{m/s})^2}{(620\text{ m})} = 238.9\text{ N} \approx \boxed{2.4\times10^2\text{ N}}$$

(*b*) For the banked case, the normal force will contribute to the radial force needed. Write Newton's 2nd law for both the *x* and *y* directions. The *y* acceleration is zero, and the *x* acceleration is radial.



$$\sum F_y = F_N\cos\theta - mg - F_{fr}\sin\theta = 0 \quad \rightarrow \quad F_N = \frac{mg + F_{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x = F_N\sin\theta + F_{fr}\cos\theta = mv^2/r$$

Substitute the expression for the normal force from the *y* equation into the *x* equation, and solve for the friction force.

$$\frac{mg + F_{fr}\sin\theta}{\cos\theta}\sin\theta + F_{fr}\cos\theta = mv^2/r \quad \rightarrow$$

$$(mg + F_{fr}\sin\theta)\sin\theta + F_{fr}\cos^2\theta = m\frac{v^2}{r}\cos\theta \quad \rightarrow$$

$$F_{fr} = m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)$$

$$= (75\text{ kg})\left[\frac{(44.44\,\text{m/s})^2}{620\text{ m}}\cos 8.0^\circ - (9.80\,\text{m/s}^2)\sin 8.0^\circ\right] = 134\text{ N} \approx \boxed{1.3\times10^2\text{ N}}$$