

CHAPTER 6: Work and Energy

Answers to Questions

1. Some types of physical labor, particularly if it involves lifting objects, such as shoveling dirt or carrying shingles up to a roof, are “work” in the physics sense of the word. Or, pushing a lawn mower would be work corresponding to the physics definition. When we use the word “work” for employment, such as “go to work” or “school work”, there is often no sense of physical labor or of moving something through a distance by a force.
2. Since “centripetal” means “pointing to the center of curvature”, then a centripetal force will not do work on an object, because if an object is moving in a curved path, by definition the direction towards the center of curvature is always perpendicular to the direction of motion. For a force to do work, the force must have a component in the direction of displacement. So the centripetal force does no work.
3. The normal force can do work on an object if the normal force has a component in the direction of displacement of an object. If someone were to jump up in the air, then the floor pushing upward on the person (the normal force) would do positive work and increase the person’s kinetic energy. Likewise when they hit the floor coming back down, the force of the floor pushing upwards (the normal force) would do negative work and decrease the person’s kinetic energy.
4. The woman does work by moving the water with her hands and feet, because she must exert a force to move the water some distance. As she stops swimming and begins to float in the current, the current does work on her because she gains kinetic energy. Once she is floating the same speed as the water, her kinetic energy does not change, and so no net work is being done on her.
5. The kinetic force of friction opposes the relative motion between two objects. As in the example suggested, as the tablecloth is pulled from under the dishes, the relative motion is for the dishes to be left behind as the tablecloth is pulled, and so the kinetic friction opposes that and moves the dishes in the same direction as the tablecloth. This is a force that is in the direction of displacement, and so positive work is done. Also note that the cloth is moving faster than the dishes in this case, so that the friction is kinetic, not static.
6. While it is true that no work is being done on the wall by you, there is work being done inside your arm muscles. Exerting a force via a muscle causes small continual motions in your muscles, which is work, and which causes you to tire. An example of this is holding a heavy load at arm’s length. While at first you may hold the load steady, after a time your arm will begin to shake, which indicates the motion of muscles in your arm.
7. (a) In this case, the same force is applied to both springs. Spring 1 will stretch less, and so more work is done on spring 2.
(b) In this case, both springs are stretched the same distance. It takes more force to stretch spring 1, and so more work is done on spring 1.
8. At point C the block’s speed will be less than $2v_B$. The same amount of work was done on the block in going from A to B as from B to C since the force and the displacement are the same for each segment. Thus the change in kinetic energy will be the same for each segment. From A to B, the

block gained $\frac{1}{2}mv_B^2$ of kinetic energy. If the same amount is gained from B to C, then the total kinetic energy at C is $\frac{1}{2}mv_C^2 = 2\left(\frac{1}{2}mv_B^2\right)$ which results in $v_C = \sqrt{2}v_B$, or $v_C \approx 1.4v_B$

9. Your gravitational PE will change according to $\Delta PE = mg\Delta y$. If we choose some typical values of $m = 80 \text{ kg}$ and $\Delta y = 0.75 \text{ m}$, then $\Delta PE = (80 \text{ kg})(9.8 \text{ m/s}^2)(0.75 \text{ m}) = 590 \text{ J}$
10. Since each balloon has the same initial kinetic energy, and each balloon undergoes the same overall change in gravitational PE, each balloon will have the same kinetic energy at the ground, and so each one has the same speed at impact.
11. The two launches will result in the same largest angle. Applying conservation of energy between the launching point and the highest point, we have $E_1 = E_2 \rightarrow \frac{1}{2}mv^2 + mgh = mgh_{\text{max}}$. The direction of the launching velocity does not matter, and so the same maximum height (and hence maximum angle) will result from both launches. Also, for the first launch, the ball will rise to some maximum height and then come back to the launch point with the same speed as when launched. That then exactly duplicates the second launch.
12. The spring can leave the table if it is compressed enough. If the spring is compressed an amount x_0 , then the gain in elastic PE is $\frac{1}{2}kx_0^2$. As the spring is compressed, its center of mass is lowered by some amount. If the spring is uniform, then the center of mass is lowered by $x_0/2$, and the amount of decrease in gravitational PE is $\frac{1}{2}mgx_0$. If the gain in elastic PE is more than the loss in gravitational PE, so that $\frac{1}{2}kx_0^2 > \frac{1}{2}mgx_0$ or $x_0 > mg/k$, then the released spring should rise up off of the table, because there is more than enough elastic PE to restore the spring to its original position. That extra elastic energy will enable the spring to “jump” off the table – it can raise its center of mass to a higher point and thus rise up off the table. Where does that “extra” energy come from? From the work you did in compressing the spring.
13. If the instructor releases the ball without pushing it, the ball should return to exactly the same height (barring any dissipative forces) and just touch the instructor’s nose as it stops. But if the instructor pushes the ball, giving it extra kinetic energy and hence a larger total energy, the ball will then swing to a higher point before stopping, and hit the instructor in the face when it returns.
14. When water at the top of a waterfall falls to the pool below, initially the water’s gravitational PE is turned into kinetic energy. That kinetic energy then can do work on the pool water when it hits it, and so some of the pool water is given energy, which makes it splash upwards and outwards and creates outgoing water waves, which carry energy. Some of the energy will become heat, due to viscous friction between the falling water and the pool water. Some of the energy will become kinetic energy of air molecules, making sound waves that give the waterfall its “roar”.
15. Start the description with the child suspended in mid-air, at the top of a hop. All of the energy is gravitational PE at that point. Then, the child falls, and gains kinetic energy. When the child reaches the ground, most of the energy is kinetic. As the spring begins to compress, the kinetic energy is changed into elastic PE. The child also goes down a little bit further as the spring compresses, and so more gravitational PE is also changed into elastic PE. At the very bottom of a hop, the energy is all elastic PE. Then as the child rebounds, the elastic PE is turned into kinetic energy and gravitational PE. When the child reaches the top of the bounce, all of the elastic PE has

been changed into gravitational PE, because the child has a speed of 0 at the top. Then the cycle starts over again. Due to friction, the child must also add energy to the system by pushing down on the pogo stick while it is on the ground, getting a more forceful reaction from the ground.

16. As the skier goes down the hill, the gravitational PE is transformed mostly into kinetic energy, and small amount is transformed into heat energy due to the friction between the skis and the snow and air friction. As the skier strikes the snowdrift, the kinetic energy of the skier turns into kinetic energy of the snow (by making the snow move), and also into some heat from the friction in moving through the snowdrift.
17. (a) If there is no friction to dissipate any of the energy, then the gravitational PE that the child has at the top of the hill all turns into kinetic energy at the bottom of the hill. The same kinetic energy will be present regardless of the slope – the final speed is completely determined by the height. The time it takes to reach the bottom of the hill will be longer for a smaller slope.
 (b) If there is friction, then the longer the path is, the more work that friction will do, and so the slower the speed will be at the bottom. So for a steep hill, the sled will have a greater speed at the bottom than for a shallow hill.
18. Stepping on the log requires that the entire body mass be raised up the height of the log, requiring work (that is not recoverable) proportional to the entire body mass. Stepping over the log only requires the raising of the legs, making for a small mass being raised and thus less work. Also, when jumping down, energy is expended to stop the “fall” from the log. The potential energy that you had at the top of the log is lost when coming down from the log.
19. If we assume that all of the arrow’s kinetic energy is converted into work done against friction, then the following relationship exists:

$$W = \Delta KE = KE_f - KE_i \rightarrow F_{fr} d \cos 180^\circ = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \rightarrow -F_{fr}d = -\frac{1}{2}mv_0^2 \rightarrow$$

$$d = \frac{mv_0^2}{2F_{fr}}$$

Thus the distance is proportional to the square of the initial velocity. So if the initial velocity is doubled, the distance will be multiplied by a factor of 4. Thus the faster arrow penetrates 4 times further than the slower arrow.

20. (a) Consider that there is no friction to dissipate any energy. Start the pendulum at the top of a swing, and define the lowest point of the swing as the zero location for gravitational PE. The pendulum has maximum gravitational PE at the top of a swing. Then as it falls, the gravitational PE is changed to kinetic energy. At the bottom of the swing, the energy is all kinetic energy. Then the pendulum starts to rise, and kinetic energy is changed to gravitational PE. Since there is no dissipation, all of the original gravitational PE is converted to kinetic energy, and all of the kinetic energy is converted to gravitational PE. The pendulum rises to the same height on both sides of every swing, and reaches the same maximum speed at the bottom on every swing.
- (b) If there is friction to dissipate the energy, then on each downward swing, the pendulum will have less kinetic energy at the bottom than it had gravitational PE at the top. And then on each swing up, the pendulum will not rise as high as the previous swing, because energy is being lost to frictional dissipation any time the pendulum is moving. So each time it swings, it has a smaller maximum displacement. When a grandfather clock is wound up, a weight is elevated so that it has some PE. That weight then falls at the proper rate to put energy back in to the pendulum to replace the energy that was lost to dissipation.

21. The superball cannot rebound to a height greater than its original height when dropped. If it did, it would violate conservation of energy. When a ball collides with the floor, the KE of the ball is converted into elastic PE by deforming the ball, much like compressing a spring. Then as the ball springs back to its original shape, that elastic PE is converted to back to KE. But that process is “lossy” – not all of the elastic PE gets converted back to KE. Some of the PE is lost, primarily to friction. The superball rebounds higher than many other balls because it is less “lossy” in its rebound than many other materials.
22. The work done to lift the suitcase is equal to the change in PE of the suitcase, which is the weight of the suitcase times the change in height (the height of the table).
- Work does NOT depend on the path, as long as there are no non-conservative forces doing work.
 - Work does NOT depend on the time taken.
 - Work DOES depend on the height of the table – the higher the table, the more work it takes to lift the suitcase.
 - Work DOES depend on the weight of the suitcase – the more the suitcase weighs, the more work it takes to lift the suitcase.
23. The power needed to lift the suitcase is the work required to lift the suitcase, divided by the time that it takes.
- Since work does NOT depend on the path, the power will not depend on the path either, assuming the time is the same for all paths.
 - The power DOES depend on the time taken. The more time taken, the lower the power needed.
 - The power needed DOES depend on the height of the table. A higher table requires more work to lift the suitcase. If we assume that the time to lift the suitcase is the same in both cases, then to lift to the higher table takes more power.
 - The power DOES depend on the weight of the suitcase. A heavier suitcase requires more force to lift, and so requires more work. Thus the heavier the suitcase, the more power is needed to lift it (in the same amount of time).
24. The climber does the same amount of work whether climbing straight up or via a zigzag path, ignoring dissipative forces. But if a longer zigzag path is taken, it takes more time to do the work, and so the power output needed from the climber is less. That will make the climb easier. It is easier for the human body to generate a small amount of power for long periods of time rather than to generate a large power for a small period of time.
25. Assuming that there are no dissipative forces to consider, for every meter that the load is raised, two meters of rope must be pulled up. This is due to the rope passing over the bottom pulley. The work done by the person pulling must be equal to the work done on the piano. Since the force on the piano is twice that exerted by the person pulling, and since work is force times distance, the person must exert their smaller force over twice the distance that the larger pulley force moves the piano.

Solutions to Problems

1. The force and the displacement are both downwards, so the angle between them is 0° .

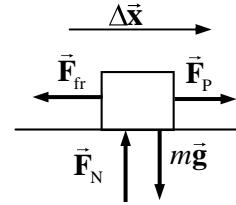
$$W_G = mgd \cos \theta = (265 \text{ kg})(9.80 \text{ m/s}^2)(2.80 \text{ m}) \cos 0^\circ = \boxed{7.27 \times 10^3 \text{ J}}$$

2. The minimum force required to lift the firefighter is equal to his weight. The force and the displacement are both upwards, so the angle between them is 0° .

$$W_{\text{climb}} = F_{\text{climb}} d \cos \theta = mgd \cos \theta = (65.0 \text{ kg})(9.80 \text{ m/s}^2)(20.0 \text{ m}) \cos 0^\circ = \boxed{1.27 \times 10^4 \text{ J}}$$

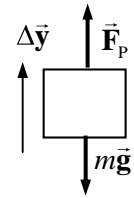
3. (a) See the free-body diagram for the crate as it is being pulled. Since the crate is not accelerating horizontally, $F_p = F_{\text{fr}} = 230 \text{ N}$. The work done to move it across the floor is the work done by the pulling force. The angle between the pulling force and the direction of motion is 0° .

$$W_p = F_p d \cos 0^\circ = (230 \text{ N})(4.0 \text{ m})(1) = \boxed{9.2 \times 10^2 \text{ J}}$$



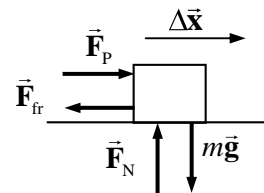
- (b) See the free-body diagram for the crate as it is being lifted. Since the crate is not accelerating vertically, the pulling force is the same magnitude as the weight. The angle between the pulling force and the direction of motion is 0° .

$$W_p = F_p d \cos 0^\circ = mgd = (1300 \text{ N})(4.0 \text{ m}) = \boxed{5.2 \times 10^3 \text{ J}}$$



4. Draw a free-body diagram for the crate as it is being pushed across the floor. Since it is not accelerating vertically, $F_N = mg$. Since it is not accelerating horizontally, $F_p = F_{\text{fr}} = \mu_k F_N = \mu_k mg$. The work done to move it across the floor is the work done by the pushing force. The angle between the pushing force and the direction of motion is 0° .

$$W_{\text{push}} = F_{\text{push}} d \cos 0^\circ = \mu_k mgd (1) = (0.50)(160 \text{ kg})(9.80 \text{ m/s}^2)(10.3 \text{ m}) = \boxed{8.1 \times 10^3 \text{ J}}$$



5. Since the acceleration of the box is constant, use Eq. 2-11b to find the distance moved. Assume that the box starts from rest.

$$\Delta x = x - x_0 = v_0 t + \frac{1}{2} a t^2 = 0 + \frac{1}{2} (2.0 \text{ m/s}^2) (7 \text{ s})^2 = 49 \text{ m}$$

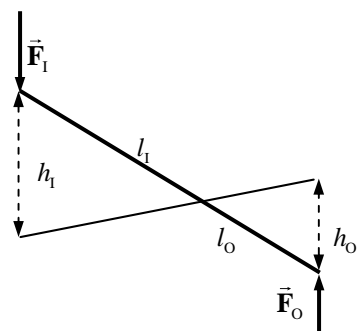
Then the work done in moving the crate is

$$W = F \Delta x \cos 0^\circ = ma \Delta x = (5 \text{ kg})(2.0 \text{ m/s}^2)(49 \text{ m}) = \boxed{4.9 \times 10^2 \text{ J}}$$

6. The first book is already in position, so no work is required to position it. The second book must be moved upwards by a distance d , by a force equal to its weight, mg . The force and the displacement are in the same direction, so the work is mgd . The third book will need to be moved a distance of $2d$ by the same size force, so the work is $2mgd$. This continues through all seven books, with each needing to be raised by an additional amount of d by a force of mg . The total work done is

$$W = mgd + 2mgd + 3mgd + 4mgd + 5mgd + 6mgd + 7mgd = 28mgd = 28(1.7 \text{ kg})(9.8 \text{ m/s}^2)(0.043 \text{ m}) = \boxed{2.0 \times 10^1 \text{ J}}$$

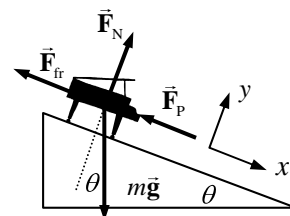
7. Consider the diagram shown. If we assume that the man pushes straight down on the end of the lever, then the work done by the man (the “input” work) is given by $W_1 = F_1 h_1$. The object moves a shorter distance, as seen from the diagram, and so $W_o = F_o h_o$. Equate the two amounts of work.



$$F_o h_o = F_1 h_1 \rightarrow \frac{F_o}{F_1} = \frac{h_1}{h_o}$$

But by similar triangles, we see that $\frac{h_1}{h_o} = \frac{l_1}{l_o}$, and so $\boxed{\frac{F_o}{F_1} = \frac{l_1}{l_o}}$.

8. The piano is moving with a constant velocity down the plane. \vec{F}_p is the force of the man pushing on the piano.



- (a) Write Newton’s 2nd law on each direction for the piano, with an acceleration of 0.

$$\sum F_y = F_N - mg \cos \theta = 0 \rightarrow F_N = mg \cos \theta$$

$$\sum F_x = mg \sin \theta - F_p - F_{fr} = 0 \rightarrow$$

$$F_p = mg \sin \theta - F_{fr} = mg (\sin \theta - \mu_k \cos \theta)$$

$$= (330 \text{ kg})(9.80 \text{ m/s}^2)(\sin 28^\circ - 0.40 \cos 28^\circ) = \boxed{3.8 \times 10^2 \text{ N}}$$

- (b) The work done by the man is the work done by \vec{F}_p . The angle between \vec{F}_p and the direction of motion is 180° .

$$W_p = F_p d \cos 180^\circ = -(380 \text{ N})(3.6 \text{ m}) = \boxed{-1.4 \times 10^3 \text{ J}}$$

- (c) The angle between \vec{F}_{fr} and the direction of motion is 180° .

$$W_{fr} = F_{fr} d \cos 180^\circ = -\mu_k mg d \cos \theta = -(0.40)(330 \text{ kg})(9.8 \text{ m/s}^2)(3.6 \text{ m}) \cos 28^\circ = \boxed{-4.1 \times 10^3 \text{ J}}$$

- (d) The angle between the force of gravity and the direction of motion is 62° . So the work done by gravity is

$$W_G = F_G d \cos 62^\circ = mg d \cos 62^\circ = (330 \text{ kg})(9.8 \text{ m/s}^2)(3.6 \text{ m}) \cos 62^\circ = \boxed{5.5 \times 10^3 \text{ J}}$$

- (e) Since the piano is unaccelerated, the net force on the piano is 0, and so the net work done on the piano is also 0. This can also be seen by adding the three work amounts calculated.

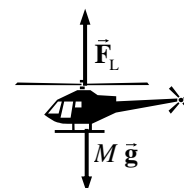
$$W_{\text{Net}} = W_p + W_{fr} + W_G = -1400 \text{ J} - 4100 \text{ J} + 5500 \text{ J} = \boxed{0 \text{ J}}$$

9. (a) Write Newton’s 2nd law for the vertical direction, with up as positive.

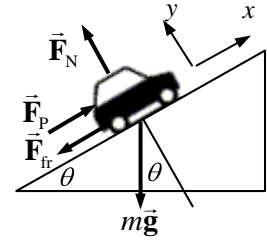
$$\sum F_y = F_L - Mg = Ma = M(0.10g) \rightarrow F_L = \boxed{1.10 Mg}$$

- (b) The lifting force and the displacement are in the same direction, so the work done by the lifting force in lifting the helicopter a vertical distance h is

$$W_L = F_L h \cos 0^\circ = \boxed{1.10 Mgh}$$



10. Draw a free-body diagram of the car on the incline. Include a frictional force, but ignore it in part (a) of the problem. The minimum work will occur when the car is moved at a constant velocity.
- (a) Write Newton's 2nd law in both the x and y directions, noting that the car is unaccelerated.



$$\sum F_y = F_N - mg \cos \theta = 0 \rightarrow F_N = mg \cos \theta$$

$$\sum F_x = F_p - mg \sin \theta = 0 \rightarrow F_p = mg \sin \theta$$

The work done by \vec{F}_p in moving the car a distance d along the plane (parallel to \vec{F}_p) is given by

$$W_p = F_p d \cos 0^\circ = mgd \sin \theta = (950 \text{ kg})(9.80 \text{ m/s}^2)(810 \text{ m}) \sin 9.0^\circ = \boxed{1.2 \times 10^6 \text{ J}}$$

- (b) Now include the frictional force, given by $F_{fr} = \mu_k F_N$. We still assume that the car is not accelerated. We again write Newton's 2nd law for each direction. The y -forces are unchanged by the addition of friction, and so we still have $F_N = mg \cos \theta$.

$$\sum F_x = F_p - F_{fr} - mg \sin \theta = 0 \rightarrow F_p = F_{fr} + mg \sin \theta = \mu_k mg \cos \theta + mg \sin \theta.$$

The work done by \vec{F}_p in moving the car a distance d along the plane (parallel to \vec{F}_p) is given by

$$W_p = F_p d \cos 0^\circ = mgd (\sin \theta + \mu_k \cos \theta)$$

$$= (950 \text{ kg})(9.80 \text{ m/s}^2)(810 \text{ m})(\sin 9.0^\circ + 0.25 \cos 9.0^\circ) = \boxed{3.0 \times 10^6 \text{ J}}$$

11. The work done is equal to the area under the graph. The area is roughly trapezoidal, and so the area of the region is found as follows.

$$W = \frac{1}{2}(F_{\parallel \max} + F_{\parallel \min})(d_B - d_A) = \frac{1}{2}(250 \text{ N} + 150 \text{ N})(35.0 \text{ m} - 10.0 \text{ m}) = \boxed{5.0 \times 10^3 \text{ J}}$$

12. The work done will be the area under the F_x vs. x graph.

- (a) From $x = 0.0$ to $x = 10.0$ m, the shape under the graph is trapezoidal. The area is

$$W_a = (400 \text{ N}) \frac{10 \text{ m} + 4 \text{ m}}{2} = \boxed{2.8 \times 10^3 \text{ J}}$$

- (b) From $x = 10.0$ m to $x = 15.0$ m, the force is in the opposite direction from the direction of motion, and so the work will be negative. Again, since the shape is trapezoidal, we find

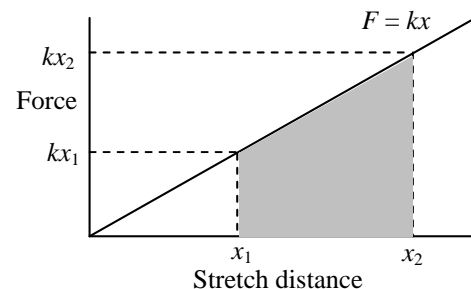
$$W_a = (-200 \text{ N}) \frac{5 \text{ m} + 2 \text{ m}}{2} = -700 \text{ J}.$$

Thus the total work from $x = 0.0$ to $x = 15.0$ m is $2800 \text{ J} - 700 \text{ J} = \boxed{2.1 \times 10^3 \text{ J}}$

13. The force exerted to stretch a spring is given by $F_{\text{stretch}} = kx$ (the opposite of the force exerted by the spring, which is given by $F = -kx$). A graph of F_{stretch} vs. x will be a straight line of slope k through the origin. The stretch from x_1 to x_2 , as shown on the graph, outlines a trapezoidal area. This area represents the work, and is calculated by

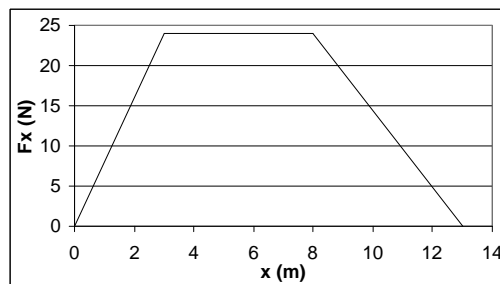
$$W = \frac{1}{2}(kx_1 + kx_2)(x_2 - x_1) = \frac{1}{2}k(x_1 + x_2)(x_2 - x_1)$$

$$= \frac{1}{2}(88 \text{ N/m})(0.096 \text{ m})(0.020 \text{ m}) = \boxed{8.4 \times 10^{-2} \text{ J}}.$$



14. See the graph of force vs. distance. The work done is the area under the graph. It can be found from the formula for a trapezoid.

$$W = \frac{1}{2}(13.0 \text{ m} + 5.0 \text{ m})(24.0 \text{ N}) = \boxed{216 \text{ J}}$$



15. Find the velocity from the kinetic energy, using Eq. 6-3.

$$KE = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2(KE)}{m}} = \sqrt{\frac{2(6.21 \times 10^{-21} \text{ J})}{5.31 \times 10^{-26}}} = \boxed{484 \text{ m/s}}$$

16. (a) Since $KE = \frac{1}{2}mv^2$, then $v = \sqrt{2(KE)/m}$ and so $v \propto \sqrt{KE}$. Thus if the kinetic energy is doubled, the speed will be multiplied by a factor of $\boxed{\sqrt{2}}$.

- (b) Since $KE = \frac{1}{2}mv^2$, then $KE \propto v^2$. Thus if the speed is doubled, the kinetic energy will be multiplied by a factor of $\boxed{4}$.

17. The work done on the electron is equal to the change in its kinetic energy.

$$W = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = 0 - \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(1.90 \times 10^6 \text{ m/s})^2 = \boxed{-1.64 \times 10^{-18} \text{ J}}$$

18. The work done on the car is equal to the change in its kinetic energy, and so

$$W = KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = 0 - \frac{1}{2}(1250 \text{ kg}) \left[(105 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \right]^2 = \boxed{-5.32 \times 10^5 \text{ J}}$$

19. The force exerted by the bow on the arrow is in the same direction as the displacement of the arrow. Thus $W = Fd \cos 0^\circ = Fd = (110 \text{ N})(0.78 \text{ m}) = 85.8 \text{ J}$. But that work changes the KE of the arrow, by the work-energy theorem. Thus

$$Fd = W = KE_2 - KE_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \rightarrow v_2 = \sqrt{\frac{2Fd}{m} + v_1^2} = \sqrt{\frac{2(85.8 \text{ J})}{0.088 \text{ kg}} + 0} = \boxed{44 \text{ m/s}}$$

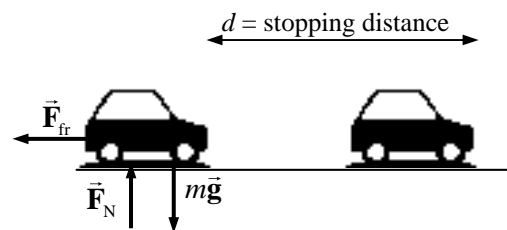
- 20.** The work done by the ball on the glove will be the opposite of the work done by the glove on the ball. The work done on the ball is equal to the change in the kinetic energy of the ball.

$$W_{\text{on ball}} = (KE_2 - KE_1)_{\text{ball}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = 0 - \frac{1}{2}(0.140 \text{ kg})(32 \text{ m/s})^2 = -72 \text{ J}$$

So $W_{\text{on glove}} = 72 \text{ J}$. But $W_{\text{on glove}} = F_{\text{on glove}} d \cos 0^\circ$, because the force on the glove is in the same direction as the motion of the glove.

$$72 \text{ J} = F_{\text{on glove}} (0.25 \text{ m}) \rightarrow F_{\text{on glove}} = \frac{72 \text{ J}}{0.25 \text{ m}} = \boxed{2.9 \times 10^2 \text{ N}}$$

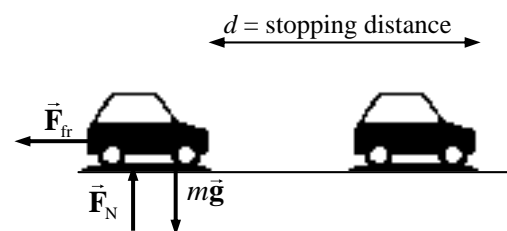
21. The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car. Assume the maximum possible frictional force, which results in the minimum braking distance. Thus $F_{\text{fr}} = \mu_s F_N$. The normal force is equal to the car's weight if it is on a level surface, and so $F_{\text{fr}} = \mu_s mg$. In the diagram, the car is traveling to the right.



$$W = \Delta KE \rightarrow F_{\text{fr}} d \cos 180^\circ = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \rightarrow -\mu_s mg d = -\frac{1}{2} m v_1^2 \rightarrow d = \frac{v_1^2}{2g\mu_s}$$

Since $d \propto v_1^2$, if v_1 increases by 50%, or is multiplied by 1.5, then d will be multiplied by a factor of $(1.5)^2$, or **2.25**.

- 22.** The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car, which is assumed to be static friction since the driver locked the brakes. Thus $F_{\text{fr}} = \mu_k F_N$. Since the car is on a level surface, the normal force is equal to the car's weight, and so $F_{\text{fr}} = \mu_k mg$ if it is on a level surface. See the diagram for the car. The car is traveling to the right.



$$W = \Delta KE \rightarrow F_{\text{fr}} d \cos 180^\circ = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \rightarrow -\mu_k mg d = 0 - \frac{1}{2} m v_1^2 \rightarrow$$

$$v_1 = \sqrt{2\mu_k g d} = \sqrt{2(0.42)(9.8 \text{ m/s}^2)(88 \text{ m})} = \boxed{27 \text{ m/s}}$$

The mass does not affect the problem, since both the change in kinetic energy and the work done by friction are proportional to the mass. The mass cancels out of the equation.

23. The original speed of the softball is $(95 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 26.39 \text{ m/s}$. The final speed is 90% of this, or 23.75 m/s. The work done by air friction causes a change in the kinetic energy of the ball, and thus the speed change. In calculating the work, notice that the force of friction is directed oppositely to the direction of motion of the ball.

$$W_{\text{fr}} = F_{\text{fr}} d \cos 180^\circ = KE_2 - KE_1 = \frac{1}{2} m (v_2^2 - v_1^2) \rightarrow$$

$$F_{\text{fr}} = \frac{m(v_2^2 - v_1^2)}{-2d} = \frac{m v_1^2 (0.9^2 - 1)}{-2d} = \frac{(0.25 \text{ kg})(26.39 \text{ m/s})^2 (0.9^2 - 1)}{-2(15 \text{ m})} = \boxed{1.1 \text{ N}}$$

24. If the rock has 80.0 J of work done to it, and it loses all 80.0 J by stopping, then the force of gravity must have done -80.0 J of work on the rock. The force is straight down, and the displacement is straight up, so the angle between the force and the displacement is 180° . The work done by the gravity force can be used to find the distance the rock rises.

$$W_G = F_G d \cos \theta = mgd \cos 180^\circ = -80.0 \text{ J}$$

$$d = \frac{W_G}{-mg} = \frac{-80.0 \text{ J}}{-(1.85 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{4.41 \text{ m}}$$

25. (a) From the free-body diagram for the load being lifted, write Newton's 2nd law for the vertical direction, with up being positive.

$$\sum F = F_T - mg = ma = 0.160mg \rightarrow$$

$$F_T = 1.16mg = 1.16(285 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{3.24 \times 10^3 \text{ N}}$$

- (b) The net work done on the load is found from the net force.

$$W_{\text{net}} = F_{\text{net}} d \cos 0^\circ = (0.160mg) d = 0.160(285 \text{ kg})(9.80 \text{ m/s}^2)(22.0 \text{ m})$$

$$= \boxed{9.83 \times 10^3 \text{ J}}$$

- (c) The work done by the cable on the load is

$$W_{\text{cable}} = F_T d \cos 0^\circ = (1.160mg) d = 1.16(285 \text{ kg})(9.80 \text{ m/s}^2)(22.0 \text{ m}) = \boxed{7.13 \times 10^4 \text{ J}}$$

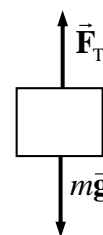
- (d) The work done by gravity on the load is

$$W_G = mgd \cos 180^\circ = -mgd = -(285 \text{ kg})(9.80 \text{ m/s}^2)(22.0 \text{ m}) = \boxed{-6.14 \times 10^4 \text{ J}}$$

- (e) Use the work-energy theory to find the final speed, with an initial speed of 0.

$$W_{\text{net}} = KE_2 - KE_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \rightarrow$$

$$v_2 = \sqrt{\frac{2W_{\text{net}}}{m} + v_1^2} = \sqrt{\frac{2(9.83 \times 10^3 \text{ J})}{285 \text{ kg}} + 0} = \boxed{8.31 \text{ m/s}}$$



26. The elastic PE is given by $PE_{\text{elastic}} = \frac{1}{2}kx^2$ where x is the distance of stretching or compressing of the spring from its natural length.

$$x = \sqrt{\frac{2PE_{\text{elastic}}}{k}} = \sqrt{\frac{2(25 \text{ J})}{440 \text{ N/m}}} = \boxed{0.34 \text{ m}}$$

27. Subtract the initial gravitational PE from the final gravitational PE.

$$\Delta PE_G = mgy_2 - mgy_1 = mg(y_2 - y_1) = (7.0 \text{ kg})(9.8 \text{ m/s}^2)(1.2 \text{ m}) = \boxed{82 \text{ J}}$$

28. Subtract the initial gravitational PE from the final gravitational PE.

$$\Delta PE_{\text{grav}} = mgy_2 - mgy_1 = mg(y_2 - y_1) = (64 \text{ kg})(9.8 \text{ m/s}^2)(4.0 \text{ m}) = \boxed{2.5 \times 10^3 \text{ J}}$$

29. Assume that all of the kinetic energy of the car becomes PE of the compressed spring.

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \rightarrow k = \frac{mv^2}{x^2} = \frac{(1200 \text{ kg}) \left[(65 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \right]^2}{(2.2 \text{ m})^2} = \boxed{8.1 \times 10^4 \text{ N/m}}$$

30. (a) Relative to the ground, the PE is given by

$$PE_G = mg(y_{\text{book}} - y_{\text{ground}}) = (2.10 \text{ kg})(9.80 \text{ m/s}^2)(2.20 \text{ m}) = \boxed{45.3 \text{ J}}$$

(b) Relative to the top of the person's head, the PE is given by

$$PE_G = mg(y_{\text{book}} - y_{\text{head}})h = (2.10 \text{ kg})(9.80 \text{ m/s}^2)(0.60 \text{ m}) = \boxed{12 \text{ J}}$$

(c) The work done by the person in lifting the book from the ground to the final height is the same as the answer to part (a), $\boxed{45.3 \text{ J}}$. In part (a), the PE is calculated relative to the starting location of the application of the force on the book. The work done by the person is not related to the answer to part (b).

31. (a) The change in PE is given by

$$\Delta PE_G = mg(y_2 - y_1) = (55 \text{ kg})(9.80 \text{ m/s}^2)(3300 \text{ m} - 1600 \text{ m}) = \boxed{9.2 \times 10^5 \text{ J}}$$

(b) The minimum work required by the hiker would equal the change in PE, which is $\boxed{9.2 \times 10^5 \text{ J}}$.

(c) $\boxed{\text{Yes}}$. The actual work may be more than this, because the climber almost certainly had to overcome some dissipative forces such as air friction. Also, as the person steps up and down, they do not get the full amount of work back from each up-down event. For example, there will be friction in their joints and muscles.

32. The spring will stretch enough to hold up the mass. The force exerted by the spring will be equal to the weight of the mass.

$$mg = k(\Delta x) \rightarrow \Delta x = \frac{mg}{k} = \frac{(2.5 \text{ kg})(9.80 \text{ m/s}^2)}{53 \text{ N/m}} = 0.46 \text{ m}$$

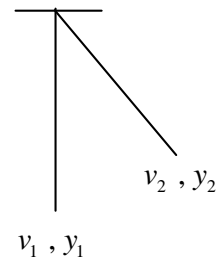
Thus the ruler reading will be $\boxed{46 \text{ cm} + 15 \text{ cm} = 61 \text{ cm}}$.

33. The only forces acting on Jane are gravity and the vine tension. The tension pulls in a centripetal direction, and so can do no work – the tension force is perpendicular at all times to her motion. So Jane's mechanical energy is conserved. Subscript 1 represents Jane at the point where she grabs the vine, and subscript 2 represents Jane at the highest point of her swing. The ground is the zero location for PE ($y = 0$). We have $v_1 = 5.3 \text{ m/s}$, $y_1 = 0$, and $v_2 = 0$ (top of swing). Solve for y_2 , the height of her swing.

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow \frac{1}{2}mv_1^2 + 0 = 0 + mgy_2 \rightarrow$$

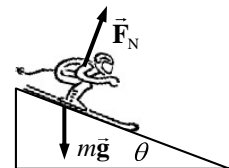
$$y_2 = \frac{v_1^2}{2g} = \frac{(5.3 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = \boxed{1.4 \text{ m}}$$

$\boxed{\text{No}}$, the length of the vine does not enter into the calculation, unless the vine is less than 0.7 m long. If that were the case, she could not rise 1.4 m high. Instead she would wrap the vine around the tree branch.



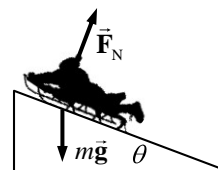
34. The forces on the skier are gravity and the normal force. The normal force is perpendicular to the direction of motion, and so does no work. Thus the skier's mechanical energy is conserved. Subscript 1 represents the skier at the top of the hill, and subscript 2 represents the skier at the bottom of the hill. The ground is the zero location for PE ($y = 0$). We have $v_1 = 0$, $y_1 = 185 \text{ m}$, and $y_2 = 0$ (bottom of the hill). Solve for v_2 , the speed at the bottom.

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow 0 + mgy_1 = \frac{1}{2}mv_2^2 + 0 \rightarrow$$



$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80 \text{ m/s}^2)(185 \text{ m})} = \boxed{60.2 \text{ m/s}} (\approx 135 \text{ mi/h})$$

35. The forces on the sled are gravity and the normal force. The normal force is perpendicular to the direction of motion, and so does no work. Thus the sled's mechanical energy is conserved. Subscript 1 represents the sled at the bottom of the hill, and subscript 2 represents the sled at the top of the hill. The ground is the zero location for PE ($y = 0$). We have $y_1 = 0$, $v_2 = 0$, and $y_2 = 1.35 \text{ m}$.



Solve for v_1 , the speed at the bottom.

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow \frac{1}{2}mv_1^2 + 0 = 0 + mgy_2 \rightarrow$$

$$v_1 = \sqrt{2gy_2} = \sqrt{2(9.80 \text{ m/s}^2)(1.35 \text{ m})} = \boxed{5.14 \text{ m/s}}$$

Notice that the angle is not used in the calculation.

36. We assume that all the forces on the jumper are conservative, so that the mechanical energy of the jumper is conserved. Subscript 1 represents the jumper at the bottom of the jump, and subscript 2 represents the jumper at the top of the jump. Call the ground the zero location for PE ($y = 0$). We have $y_1 = 0$, $v_2 = 0.70 \text{ m/s}$, and $y_2 = 2.10 \text{ m}$. Solve for v_1 , the speed at the bottom.

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow \frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow$$

$$v_1 = \sqrt{v_2^2 + 2gy_2} = \sqrt{(0.70 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(2.10 \text{ m})} = \boxed{6.45 \text{ m/s}}$$

37. (a) Since there are no dissipative forces present, the mechanical energy of the person – trampoline – Earth combination will be conserved. The level of the unstretched trampoline is the zero level for both the elastic and gravitational PE. Call up the positive direction. Subscript 1 represents the jumper at the top of the jump, and subscript 2 represents the jumper upon arriving at the trampoline. There is no elastic PE involved in this part of the problem. We have $v_1 = 5.0 \text{ m/s}$, $y_1 = 3.0 \text{ m}$, and $y_2 = 0$. Solve for v_2 , the speed upon arriving at the trampoline.

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + 0 \rightarrow$$

$$v_2 = \pm\sqrt{v_1^2 + 2gy_1} = \pm\sqrt{(5.0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(3.0 \text{ m})} = \pm 9.154 \text{ m/s} \approx \boxed{9.2 \text{ m/s}}$$

The speed is the absolute value of v_2 .

- (b) Now let subscript 3 represent the jumper at the maximum stretch of the trampoline. We have $v_2 = 9.154 \text{ m/s}$, $y_2 = 0$, $x_2 = 0$, $v_3 = 0$, and $x_3 = y_3$. There is no elastic energy at position 2, but there is elastic energy at position 3. Also, the gravitational PE at position 3 is negative, and so $y_3 < 0$. A quadratic relationship results from the conservation of energy condition.

$$E_2 = E_3 \rightarrow \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}kx_2^2 = \frac{1}{2}mv_3^2 + mgy_3 + \frac{1}{2}kx_3^2 \rightarrow$$

$$\frac{1}{2}mv_2^2 + 0 + 0 = 0 + mgy_3 + \frac{1}{2}ky_3^2 \rightarrow \frac{1}{2}ky_3^2 + mgy_3 - \frac{1}{2}mv_2^2 = 0 \rightarrow$$

$$y_3 = \frac{-mg \pm \sqrt{m^2 g^2 - 4(\frac{1}{2}k)(-\frac{1}{2}mv_2^2)}}{2(\frac{1}{2}k)} = \frac{-mg \pm \sqrt{m^2 g^2 + kmv_2^2}}{k}$$

$$= \frac{-(65 \text{ kg})(9.8 \text{ m/s}^2) \pm \sqrt{(65 \text{ kg})^2 (9.8 \text{ m/s}^2)^2 + (6.2 \times 10^4 \text{ N/m})(65 \text{ kg})(9.154 \text{ m/s})^2}}{(6.2 \times 10^4 \text{ N/m})}$$

$$= -0.307 \text{ m}, 0.286 \text{ m}$$

Since $y_3 < 0$, $y_3 = \boxed{-0.31 \text{ m}}$.

The first term under the quadratic is about 1000 times smaller than the second term, indicating that the problem could have been approximated by not even including gravitational PE for the final position. If that approximation would have been made, the result would have been found by taking the negative result from the following solution.

$$E_2 = E_3 \rightarrow \frac{1}{2}mv_2^2 = \frac{1}{2}ky_3^2 \rightarrow y_3 = v_2\sqrt{\frac{m}{k}} = (9.2 \text{ m/s})\sqrt{\frac{65 \text{ kg}}{6.2 \times 10^4 \text{ N/m}}} = \pm 0.30 \text{ m}$$

38. Use conservation of energy. Subscript 1 represents the projectile at the launch point, and subscript 2 represents the projectile as it reaches the ground. The ground is the zero location for PE ($y = 0$).

We have $v_1 = 185 \text{ m/s}$, $y_1 = 265 \text{ m}$, and $y_2 = 0$. Solve for v_2 .

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + 0 \rightarrow$$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(185 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(265 \text{ m})} = \boxed{199 \text{ m/s}}$$

Note that the angle of launch does not enter into the problem.

39. Use conservation of energy. The level of the ball on the uncompressed spring taken as the zero location for both gravitational PE ($y = 0$) and elastic PE ($x = 0$). Take up to be positive for both.

(a) Subscript 1 represents the ball at the launch point, and subscript 2 represents the ball at the location where it just leaves the spring, at the uncompressed length. We have $v_1 = 0$, $x_1 = y_1 = -0.150 \text{ m}$, and

$x_2 = y_2 = 0$. Solve for v_2 .

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}kx_2^2 \rightarrow$$

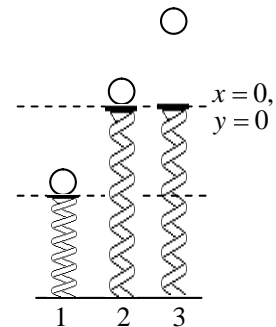
$$0 + mgy_1 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + 0 + 0 \rightarrow v_2 = \sqrt{\frac{kx_1^2 + 2mgy_1}{m}}$$

$$v_2 = \sqrt{\frac{(950 \text{ N/m})(0.150 \text{ m})^2 + 2(0.30 \text{ kg})(9.80 \text{ m/s}^2)(-0.150 \text{ m})}{(0.30 \text{ kg})}} = \boxed{8.3 \text{ m/s}}$$

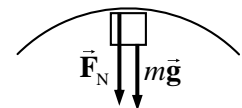
- (b) Subscript 3 represents the ball at its highest point. We have $v_1 = 0$, $x_1 = y_1 = -0.150 \text{ m}$, $v_3 = 0$, and $x_3 = 0$. Solve for y_3 .

$$E_1 = E_3 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_3^2 + mgy_3 + \frac{1}{2}kx_3^2 \rightarrow$$

$$0 + mgy_1 + \frac{1}{2}kx_1^2 = 0 + mgy_2 + 0 \rightarrow y_2 - y_1 = \frac{kx_1^2}{2mg} = \frac{(950 \text{ N/m})(0.150 \text{ m})^2}{2(0.30 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{3.6 \text{ m}}$$



40. Draw a free-body diagram for the block at the top of the curve. Since the block is moving in a circle, the net force is centripetal. Write Newton's 2nd law for the block, with down as positive. If the block is to be on the verge of falling off the track, then $F_N = 0$.



$$\sum F_R = F_N + mg = mv^2/r \rightarrow mg = mv_{\text{top}}^2/r \rightarrow v_{\text{top}} = \sqrt{gr}$$

Now use conservation of energy for the block. Since the track is frictionless, there are no non-conservative forces, and mechanical energy will be conserved. Subscript 1 represents the block at the release point, and subscript 2 represents the block at the top of the loop. The ground is the zero location for PE ($y = 0$). We have $v_1 = 0$, $y_1 = h$, $v_2 = \sqrt{gr}$, and $y_2 = 2r$. Solve for h .

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow 0 + mgh = \frac{1}{2}mgr + 2mgr \rightarrow h = \boxed{2.5r}$$

41. The block-spring combination is assumed to initially be at equilibrium, so the spring is neither stretched nor unstretched. At the release point, the speed of the mass is 0, and so the initial energy is all PE, given by $\frac{1}{2}kx_0^2$. That is the total energy of the system. Thus the energy of the system when the block is at a general location with some non-zero speed will still have this same total energy value. This is expressed by $E_{\text{total}} = \boxed{\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2}$.

42. Consider this diagram for the jumper's fall.

- (a) The mechanical energy of the jumper is conserved. Use y for the distance from the 0 of gravitational PE and x for the amount of bungee cord "stretch" from its unstretched length. Subscript 1 represents the jumper at the start of the fall, and subscript 2 represents the jumper at the lowest point of the fall. The bottom of the fall is the zero location for gravitational PE ($y = 0$), and the location where the bungee cord just starts to be stretched is the zero location for elastic PE ($x = 0$). We have $v_1 = 0$, $y_1 = 31$ m, $x_1 = 0$, $v_2 = 0$, $y_2 = 0$, and $x_2 = 19$ m. Apply conservation of energy.

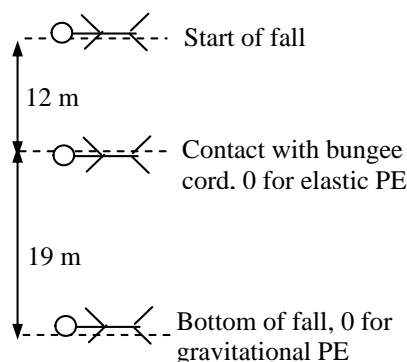
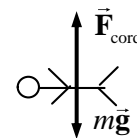
$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}kx_2^2 \rightarrow mgy_1 = \frac{1}{2}kx_2^2 \rightarrow$$

$$k = \frac{2mgy_1}{x_2^2} = \frac{2(62 \text{ kg})(9.8 \text{ m/s}^2)(31 \text{ m})}{(19 \text{ m})^2} = 104.4 \text{ N/m} \approx \boxed{1.0 \times 10^2 \text{ N/m}}$$

- (b) The maximum acceleration occurs at the location of the maximum force, which occurs when the bungee cord has its maximum stretch, at the bottom of the fall. Write Newton's 2nd law for the force on the jumper, with upward as positive.

$$F_{\text{net}} = F_{\text{cord}} - mg = kx_2 - mg = ma \rightarrow$$

$$a = \frac{kx_2}{m} - g = \frac{(104.4 \text{ N/m})(19 \text{ m})}{(62 \text{ kg})} - 9.8 \text{ m/s}^2 = 22.2 \text{ m/s}^2 \approx \boxed{22 \text{ m/s}^2}$$



43. Since there are no dissipative forces present, the mechanical energy of the roller coaster will be conserved. Subscript 1 represents the coaster at point 1, etc. The height of point 2 is the zero location for gravitational PE. We have $v_1 = 0$ and $y_1 = 35$ m.

$$\text{Point 2: } \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \quad ; \quad y_2 = 0 \rightarrow mgy_1 = \frac{1}{2}mv_2^2 \rightarrow$$

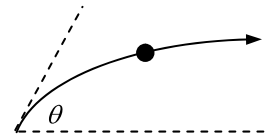
$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80 \text{ m/s}^2)(35 \text{ m})} = \boxed{26 \text{ m/s}}$$

Point 3: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_3^2 + mgy_3$; $y_3 = 28 \text{ m} \rightarrow mgy_1 = \frac{1}{2}mv_3^2 + mgy_3 \rightarrow$
 $v_3 = \sqrt{2g(y_1 - y_3)} = \sqrt{2(9.80 \text{ m/s}^2)(7 \text{ m})} = \boxed{12 \text{ m/s}}$

Point 4: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_4^2 + mgy_4$; $y_4 = 15 \text{ m} \rightarrow mgy_1 = \frac{1}{2}mv_4^2 + mgy_4 \rightarrow$
 $v_4 = \sqrt{2g(y_1 - y_4)} = \sqrt{2(9.80 \text{ m/s}^2)(20 \text{ m})} = \boxed{20 \text{ m/s}}$

44. (a) See the diagram for the thrown ball. The speed at the top of the path will be the horizontal component of the original velocity.

$$v_{\text{top}} = v_0 \cos \theta = (12 \text{ m/s}) \cos 33^\circ = \boxed{10 \text{ m/s}}$$



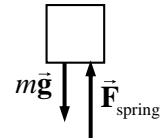
- (b) Since there are no dissipative forces in the problem, the mechanical energy of the ball is conserved. Subscript 1 represents the ball at the release point, and subscript 2 represents the ball at the top of the path. The ground is the zero location for PE ($y = 0$). We have $v_1 = 12 \text{ m/s}$, $y_1 = 0$, and $v_2 = v_1 \cos \theta$. Solve for y_2 .

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow \frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv_1^2 \cos^2 \theta + mgy_2 \rightarrow$$

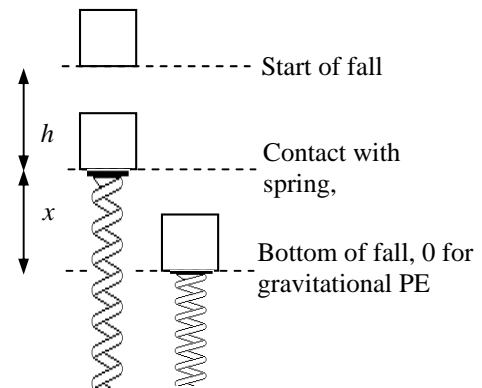
$$y_2 = \frac{v_1^2 (1 - \cos^2 \theta)}{2g} = \frac{(12 \text{ m/s})^2 (1 - \cos^2 33^\circ)}{2(9.8 \text{ m/s}^2)} = \boxed{2.2 \text{ m}}$$

45. The maximum acceleration of $5.0 g$ occurs where the force is at a maximum. The maximum force occurs at the bottom of the motion, where the spring is at its maximum compression. Write Newton's 2nd law for the elevator at the bottom of the motion, with up as the positive direction.

$$F_{\text{net}} = F_{\text{spring}} - Mg = Ma = 5.0Mg \rightarrow F_{\text{spring}} = 6.0Mg$$



Now consider the diagram for the elevator at various points in its motion. If there are no non-conservative forces, then mechanical energy is conserved. Subscript 1 represents the elevator at the start of its fall, and subscript 2 represents the elevator at the bottom of its fall. The bottom of the fall is the zero location for gravitational PE ($y = 0$). There is also a point at the top of the spring that we will define as the zero location for elastic PE ($x = 0$). We have $v_1 = 0$, $y_1 = x + h$, $x_1 = 0$, $v_2 = 0$, $y_2 = 0$, and $x_2 = x$. Apply conservation of energy.



$$E_1 = E_2 \rightarrow \frac{1}{2}Mv_1^2 + Mgy_1 + \frac{1}{2}kx_1^2 = \frac{1}{2}Mv_2^2 + Mgy_2 + \frac{1}{2}kx_2^2 \rightarrow$$

$$0 + Mg(x + h) + 0 = 0 + 0 + \frac{1}{2}kx_2^2 \rightarrow Mg(x + h) = \frac{1}{2}kx_2^2$$

$$F_{\text{spring}} = 6.0Mg = kx \rightarrow x = \frac{6.0Mg}{k} \rightarrow Mg\left(\frac{6Mg}{k} + h\right) = \frac{1}{2}k\left(\frac{6Mg}{k}\right)^2 \rightarrow \boxed{k = \frac{12Mg}{h}}$$

46. (a) The work done against gravity is the change in PE.

$$W_{\text{against gravity}} = \Delta PE = mg(y_2 - y_1) = (75 \text{ kg})(9.8 \text{ m/s}^2)(150 \text{ m}) = \boxed{1.1 \times 10^5 \text{ J}}$$

- (b) The work done by the force on the pedals in one revolution is equal to the tangential force times the circumference of the circular path of the pedals. That work is also equal to the energy change of the bicycle during that revolution. Note that a vertical rise on the incline is related to the distance along the incline by $\text{rise} = \text{distance}(\sin \theta)$.

$$W_{\text{pedal force}} = F_{\text{tan}} 2\pi r = \Delta PE_{1\text{ rev}} = mg(\Delta y)_{1\text{ rev}} = mgd_{1\text{ rev}} \sin \theta \rightarrow$$

$$F_{\text{tan}} = \frac{mgd_{1\text{ rev}} \sin \theta}{2\pi r} = \frac{(75 \text{ kg})(9.8 \text{ m/s}^2)(5.1 \text{ m}) \sin 7.8^\circ}{2\pi(0.18 \text{ m})} = \boxed{4.5 \times 10^2 \text{ N}}$$

47. Use conservation of energy, where all of the kinetic energy is transformed to thermal energy.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow \frac{1}{2}mv^2 = E_{\text{thermal}} = \frac{1}{2}(2)(7650 \text{ kg}) \left[(95 \text{ km/h}) \left(\frac{0.238 \text{ m/s}}{1 \text{ km/h}} \right) \right]^2 = \boxed{5.3 \times 10^6 \text{ J}}$$

48. Apply the conservation of energy to the child, considering work done by gravity and work changed into thermal energy. Subscript 1 represents the child at the top of the slide, and subscript 2 represents the child at the bottom of the slide. The ground is the zero location for PE ($y = 0$). We have $v_1 = 0$, $y_1 = 3.5 \text{ m}$, $v_2 = 2.2 \text{ m/s}$, and $y_2 = 0$. Solve for the work changed into thermal energy.

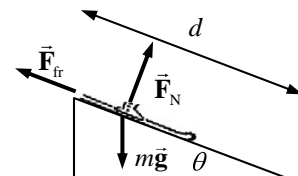
$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 + W_{\text{thermal}} \rightarrow$$

$$W_{\text{thermal}} = mgy_1 - \frac{1}{2}mv_2^2 = (21.7 \text{ kg})(9.8 \text{ m/s}^2)(3.5 \text{ m}) - \frac{1}{2}(21.7 \text{ kg})(2.2 \text{ m/s})^2 = \boxed{6.9 \times 10^2 \text{ J}}$$

49. (a) See the free-body diagram for the ski. Write Newton's 2nd law for forces perpendicular to the direction of motion, noting that there is no acceleration perpendicular to the plane.

$$\sum F_{\perp} = F_N - mg \cos \theta \rightarrow F_N = mg \cos \theta \rightarrow$$

$$F_{\text{fr}} = \mu_k F_N = \mu_k mg \cos \theta$$



Now use conservation of energy, including the non-conservative friction force. Subscript 1 represents the ski at the top of the slope, and subscript 2 represents the ski at the bottom of the slope. The location of the ski at the bottom of the incline is the zero location for gravitational PE ($y = 0$). We have $v_1 = 0$, $y_1 = d \sin \theta$, and $y_2 = 0$. Write the conservation of energy condition, and solve for the final speed. Note that $F_{\text{fr}} = \mu_k F_N = \mu_k mg \cos \theta$

$$W_{\text{NC}} = \Delta KE + \Delta PE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgy_2 - mgy_1 \rightarrow W_{\text{NC}} + E_1 = E_2$$

$$F_{\text{fr}} d \cos 180^\circ + \frac{1}{2}mv_2^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow -\mu_k mgd \cos \theta + mgd \sin \theta = \frac{1}{2}mv_2^2 \rightarrow$$

$$v_2 = \sqrt{2gd(\sin \theta - \mu_k \cos \theta)} = \sqrt{2(9.80 \text{ m/s}^2)(75 \text{ m})(\sin 22^\circ - 0.090 \cos 22^\circ)}$$

$$= 20.69 \text{ m/s} \approx \boxed{21 \text{ m/s}}$$

- (b) Now, on the level ground, $F_f = \mu_k mg$, and there is no change in PE. Let us again use conservation of energy, including the non-conservative friction force, to relate position 2 with position 3. Subscript 3 represents the ski at the end of the travel on the level, having traveled a distance d_3 on the level. We have $v_2 = 20.69 \text{ m/s}$, $y_2 = 0$, $v_3 = 0$, and $y_3 = 0$.

$$W_{\text{NC}} + E_2 = E_3 \rightarrow F_f d_3 \cos 180^\circ + \frac{1}{2} m v_2^2 + m g y_2 = \frac{1}{2} m v_3^2 + m g y_3 \rightarrow$$

$$-\mu_k m g d_3 + \frac{1}{2} m v_2^2 = 0 \rightarrow d_3 = \frac{v_2^2}{2 g \mu_k} = \frac{(20.69 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(0.090)} = 242.7 \text{ m} \approx \boxed{2.4 \times 10^2 \text{ m}}$$

50. (a) Apply energy conservation with no non-conservative work. Subscript 1 represents the ball as it is dropped, and subscript 2 represents the ball as it reaches the ground. The ground is the zero location for gravitational PE. We have $v_1 = 0$, $y_1 = 13.0 \text{ m}$, and $y_2 = 0$. Solve for v_2 .

$$E_1 = E_2 \rightarrow \frac{1}{2} m v_1^2 + m g y_1 = \frac{1}{2} m v_2^2 + m g y_2 \rightarrow m g y_1 = \frac{1}{2} m v_2^2 \rightarrow$$

$$v_2 = \sqrt{2 g y_1} = \sqrt{2(9.80 \text{ m/s}^2)(13.0 \text{ m})} = \boxed{16.0 \text{ m/s}}$$

- (b) Apply energy conservation, but with non-conservative work due to friction included. The work done by friction will be given by $W_{\text{NC}} = F_{\text{fr}} d \cos 180^\circ$, since the force of friction is in the opposite direction as the motion. The distance d over which the frictional force acts will be the 13.0 m distance of fall. With the same parameters as above, and $v_2 = 8.00 \text{ m/s}$, solve for the force of friction.

$$W_{\text{nc}} + E_1 = E_2 \rightarrow -F_{\text{fr}} d + \frac{1}{2} m v_1^2 + m g y_1 = \frac{1}{2} m v_2^2 + m g y_2 \rightarrow -F_{\text{fr}} d + m g y_1 = \frac{1}{2} m v_2^2 \rightarrow$$

$$F_{\text{fr}} = m \left(g \frac{y_1}{d} - \frac{v_2^2}{2d} \right) = (0.145 \text{ kg}) \left(9.80 \text{ m/s}^2 - \frac{(8.00 \text{ m/s})^2}{2(13.0 \text{ m})} \right) = \boxed{1.06 \text{ N}}$$

51. (a) Calculate the energy of the ball at the two maximum heights, and subtract to find the amount of energy “lost”. The energy at the two heights is all gravitational PE, since the ball has no KE at those maximum heights.

$$E_{\text{lost}} = E_{\text{initial}} - E_{\text{final}} = m g y_{\text{initial}} - m g y_{\text{final}}$$

$$\frac{E_{\text{lost}}}{E_{\text{initial}}} = \frac{m g y_{\text{initial}} - m g y_{\text{final}}}{m g y_{\text{initial}}} = \frac{y_{\text{initial}} - y_{\text{final}}}{y_{\text{initial}}} = \frac{2.0 \text{ m} - 1.5 \text{ m}}{2.0 \text{ m}} = 0.25 = \boxed{25\%}$$

- (b) Due to energy conservation, the KE of the ball just as it leaves the ground is equal to its final PE.

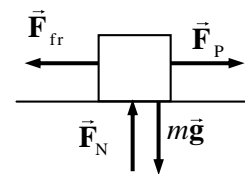
$$PE_{\text{final}} = KE_{\text{ground}} \rightarrow m g y_{\text{final}} = \frac{1}{2} m v_{\text{ground}}^2 \rightarrow$$

$$v_{\text{ground}} = \sqrt{2 g y_{\text{final}}} = \sqrt{2(9.8 \text{ m/s}^2)(1.5 \text{ m})} = \boxed{5.4 \text{ m/s}}$$

- (c) The energy “lost” was changed primarily into heat energy – the temperature of the ball and the ground would have increased slightly after the bounce. Some of the energy may have been changed into acoustic energy (sound waves). Some may have been lost due to non-elastic deformation of the ball or ground.

52. Since the crate moves along the floor, there is no change in gravitational PE, so use the work-energy theorem: $W_{\text{net}} = KE_2 - KE_1$. There are two forces doing work: F_p , the pulling force, and $F_{\text{fr}} = \mu_k F_N = \mu_k m g$, the frictional force. $KE_1 = 0$ since the crate starts from rest. Note that the two forces doing work do work over different distances.

$$W_p = F_p d_p \cos 0^\circ \quad W_{\text{fr}} = F_{\text{fr}} d_{\text{fr}} \cos 180^\circ = -\mu_k m g d_{\text{fr}}$$



$$\begin{aligned}
 W_{\text{net}} &= W_p + W_{\text{fr}} = KE_2 - KE_1 = \frac{1}{2}mv_2^2 - 0 \rightarrow \\
 v_2 &= \sqrt{\frac{2}{m}(W_p + W_{\text{fr}})} = \sqrt{\frac{2}{m}(F_p d_p - \mu_k mg d_{\text{fr}})} \\
 &= \sqrt{\frac{2}{(110 \text{ kg})}[(350 \text{ N})(30 \text{ m}) - (0.30)(110 \text{ kg})(9.8 \text{ m/s}^2)(15 \text{ m})]} = \boxed{10 \text{ m/s}}
 \end{aligned}$$

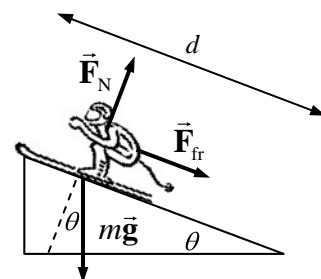
53. Since there is a non-conservative force, consider energy conservation with non-conservative work included. Subscript 1 represents the roller coaster at point 1, and subscript 2 represents the roller coaster at point 2. Point 2 is taken as the zero location for gravitational PE. We have $v_1 = 1.70 \text{ m/s}$, $y_1 = 35 \text{ m}$, and $y_2 = 0$. Solve for v_2 . The work done by the non-conservative friction force is given by $W_{\text{NC}} = F_{\text{fr}} d \cos 180^\circ = -0.20mgd$, since the force is one-fifth of mg , and the force is directed exactly opposite to the direction of motion.

$$\begin{aligned}
 W_{\text{NC}} + E_1 &= E_2 \rightarrow -0.2mgd + \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow \\
 v_2 &= \sqrt{-0.4gd + v_1^2 + 2gy_1} = \sqrt{-0.4(9.80 \text{ m/s}^2)(45.0 \text{ m}) + (1.70 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(35 \text{ m})} \\
 &= 22.64 \text{ m/s} \approx \boxed{23 \text{ m/s}}
 \end{aligned}$$

54. Consider the free-body diagram for the skier in the midst of the motion. Write Newton's 2nd law for the direction perpendicular to the plane, with an acceleration of 0.

$$\begin{aligned}
 \sum F_{\perp} &= F_N - mg \cos \theta = 0 \rightarrow F_N = mg \cos \theta \rightarrow \\
 F_{\text{fr}} &= \mu_k F_N = \mu_k mg \cos \theta
 \end{aligned}$$

Apply conservation of energy to the skier, including the non-conservative friction force. Subscript 1 represents the skier at the bottom of the slope, and subscript 2 represents the skier at the point furthest up the slope. The location of the skier at the bottom of the incline is the zero location for gravitational PE ($y = 0$). We have $v_1 = 12.0 \text{ m/s}$, $y_1 = 0$, $v_2 = 0$, and $y_2 = d \sin \theta$.



$$\begin{aligned}
 W_{\text{NC}} + E_1 &= E_2 \rightarrow F_{\text{fr}} d \cos 180^\circ + \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow \\
 -\mu_k mgd \cos \theta + \frac{1}{2}mv_1^2 + 0 &= 0 + mgd \sin \theta \rightarrow \\
 \mu_k &= \frac{\frac{1}{2}v_1^2 - gd \sin \theta}{gd \cos \theta} = \frac{v_1^2}{2gd \cos \theta} - \tan \theta = \frac{(12.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(12.2 \text{ m}) \cos 18.0^\circ} - \tan 18.0^\circ \\
 &= \boxed{0.308}
 \end{aligned}$$

55. Use conservation of energy, including the non-conservative frictional force. The block is on a level surface, so there is no gravitational PE change to consider. The frictional force is given by $F_{\text{fr}} = \mu_k F_N = \mu_k mg$, since the normal force is equal to the weight. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at the maximum stretched position. The location of the block when the spring is neither stretched nor compressed is the zero location for elastic PE ($x = 0$). Take right to be the positive direction. We have $v_1 = 0$, $x_1 = -0.050 \text{ m}$, $v_2 = 0$, and $x_2 = 0.023 \text{ m}$.

$$\begin{aligned}
 W_{\text{NC}} + E_1 &= E_2 \rightarrow F_{\text{fr}} \Delta x \cos 180^\circ + \frac{1}{2} m v_1^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} m v_2^2 + \frac{1}{2} k x_2^2 \rightarrow \\
 -\mu_k m g \Delta x + \frac{1}{2} k x_1^2 &= \frac{1}{2} k x_2^2 \rightarrow \\
 \mu_k &= \frac{k(x_1^2 - x_2^2)}{2mg\Delta x} = \frac{(180 \text{ N/m})[(0.050 \text{ m})^2 - (0.023 \text{ m})^2]}{2(0.620 \text{ kg})(9.80 \text{ m/s}^2)(0.073 \text{ m})} = \boxed{0.40}
 \end{aligned}$$

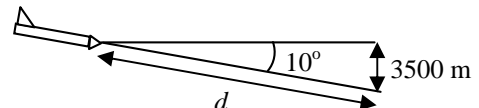
56. Use conservation of energy, including the non-conservative frictional force. The block is on a level surface, so there is no gravitational PE change to consider. Since the normal force is equal to the weight, the frictional force is $F_{\text{fr}} = \mu_k F_N = \mu_k m g$. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at the maximum stretched position. The location of the block when the spring is neither stretched nor compressed is the zero location for elastic PE ($x = 0$). Take right to be the positive direction. We have $v_1 = 0$, $x_1 = -0.18 \text{ m}$, and $v_2 = 0$. The value of the spring constant is found from the fact that a 22-N force compresses the spring 18 cm, and so $k = F/x = 22 \text{ N}/0.18 \text{ m} = 122.2 \text{ N/m}$. The value of x_2 must be positive.

$$\begin{aligned}
 W_{\text{NC}} + E_1 &= E_2 \rightarrow F_{\text{fr}} \Delta x \cos 180^\circ + \frac{1}{2} m v_1^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} m v_2^2 + \frac{1}{2} k x_2^2 \rightarrow \\
 -\mu_k m g (x_2 - x_1) + \frac{1}{2} k x_1^2 &= \frac{1}{2} k x_2^2 \rightarrow x_2^2 + \frac{2\mu_k m g}{k} x_2 - \left(\frac{2\mu_k m g}{k} x_1 + x_1^2 \right) = 0 \rightarrow \\
 x_2^2 + \frac{2(0.30)(0.28)(9.80)}{122.2} x_2 - \left(\frac{2(0.30)(0.28)(9.80)}{122.2} (-0.18) + (-0.18)^2 \right) &= 0 \rightarrow \\
 x_2^2 + 0.01347 x_2 - 0.02997 &= 0 \rightarrow x_2 = 0.1665 \text{ m}, -0.1800 \text{ m} \rightarrow x_2 = \boxed{0.17 \text{ m}}
 \end{aligned}$$

57. (a) If there is no air resistance, then conservation of mechanical energy can be used. Subscript 1 represents the glider when at launch, at subscript 2 represents the glider at landing. The landing location is the zero location for elastic PE ($x = 0$). We have $y_1 = 500 \text{ m}$, $y_2 = 0$, and

$$\begin{aligned}
 v_1 &= 500 \text{ km/h} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 138.9 \text{ m/s}. \text{ Solve for } v_2 \\
 E_1 &= E_2 \rightarrow \frac{1}{2} m v_1^2 + m g y_1 = \frac{1}{2} m v_2^2 + m g y_2 \rightarrow \\
 v_2 &= \sqrt{v_1^2 + 2g y_1} = \sqrt{(138.9 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(3500 \text{ m})} = 296 \text{ m/s} \left(\frac{3.6 \text{ km/h}}{1 \text{ m/s}} \right) \\
 &= 1067 \text{ km/h} \approx \boxed{1.1 \times 10^3 \text{ km/h}}
 \end{aligned}$$

- (b) Now include the work done by the non-conservative frictional force. Consider the diagram of the glider. Calculate the work done by friction.



$$W_{\text{NC}} = F_{\text{fr}} d \cos 180^\circ = -F_{\text{fr}} d = -F_{\text{fr}} \frac{3500 \text{ m}}{\sin 10^\circ}$$

Use the same subscript representations as above, with y_1 , v_1 , and y_2 as before, and

$$v_2 = 200 \text{ km/h} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 55.56 \text{ m/s}. \text{ Write the energy conservation equation and solve for the frictional force.}$$

$$\begin{aligned}
 W_{\text{NC}} + E_1 = E_2 &\rightarrow -F_{\text{fr}}d + \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow F_f = \frac{m(v_1^2 - v_2^2 + 2gy_1)}{2d} \\
 &= \frac{(980 \text{ kg}) \left[(138.9 \text{ m/s})^2 - (55.56 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(3500 \text{ m}) \right]}{2 \left(\frac{3500 \text{ m}}{\sin 10^\circ} \right)} \\
 &= 2062 \text{ N} \approx \boxed{2 \times 10^3 \text{ N}}
 \end{aligned}$$

58. The work necessary to lift the piano is the work done by an upward force, equal in magnitude to the weight of the piano. Thus $W = Fd \cos 0^\circ = mgh$. The average power output required to lift the piano is the work done divided by the time to lift the piano.

$$P = \frac{W}{t} = \frac{mgh}{t} \rightarrow t = \frac{mgh}{P} = \frac{(315 \text{ kg})(9.80 \text{ m/s}^2)(16.0 \text{ m})}{1750 \text{ W}} = \boxed{28.2 \text{ s}}$$

59. The 18 hp is the power generated by the engine in creating a force on the ground to propel the car forward. The relationship between the power and the force is given by $P = \frac{W}{t} = \frac{Fd}{t} = F \frac{d}{t} = Fv$.

Thus the force to propel the car forward is found by $F = P/v$. If the car has a constant velocity, then the total resistive force must be of the same magnitude as the engine force, so that the net force is zero. Thus the total resistive force is also found by $F = P/v$.

$$F = \frac{P}{v} = \frac{(18 \text{ hp})(746 \text{ W/1 hp})}{(88 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)} = \boxed{5.5 \times 10^2 \text{ N}}$$

60. The power is given by Eq. 6-16. The energy transformed is the change in kinetic energy of the car.

$$\begin{aligned}
 P &= \frac{\text{energy transformed}}{\text{time}} = \frac{\Delta KE}{t} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{t} = \frac{(1400 \text{ kg}) \left[(95 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \right]^2}{2(7.4 \text{ s})} \\
 &= \boxed{6.6 \times 10^4 \text{ W}} \approx 88 \text{ hp}
 \end{aligned}$$

$$61. (a) 1 \text{ hp} = (1 \text{ hp}) \left(\frac{550 \text{ ft}\cdot\text{lb/s}}{1 \text{ hp}} \right) \left(\frac{4.45 \text{ N}}{1 \text{ lb}} \right) \left(\frac{1 \text{ m}}{3.28 \text{ ft}} \right) = 746 \text{ N}\cdot\text{m/s} = \boxed{746 \text{ W}}$$

$$(b) 75 \text{ W} = (75 \text{ W}) \left(\frac{1 \text{ hp}}{746 \text{ W}} \right) = \boxed{0.10 \text{ hp}}$$

$$62. (a) 1 \text{ kW}\cdot\text{h} = 1 \text{ kW}\cdot\text{h} \left(\frac{1000 \text{ W}}{1 \text{ kW}} \right) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) \left(\frac{1 \text{ J/s}}{1 \text{ W}} \right) = \boxed{3.6 \times 10^6 \text{ J}}$$

$$(b) (520 \text{ W})(1 \text{ month}) = (520 \text{ W})(1 \text{ month}) \left(\frac{1 \text{ kW}}{1000 \text{ W}} \right) \left(\frac{30 \text{ d}}{1 \text{ month}} \right) \left(\frac{24 \text{ h}}{1 \text{ d}} \right) = 374 \text{ kW}\cdot\text{h}$$

$$\approx \boxed{370 \text{ kW}\cdot\text{h}}$$

$$(c) \quad 374 \text{ kW}\cdot\text{h} = 374 \text{ kW}\cdot\text{h} \left(\frac{3.6 \times 10^6 \text{ J}}{1 \text{ kW}\cdot\text{h}} \right) = \boxed{1.3 \times 10^9 \text{ J}}$$

$$(d) \quad (374 \text{ kW}\cdot\text{h}) \left(\frac{\$0.12}{1 \text{ kW}\cdot\text{h}} \right) = \$44.88 \approx \boxed{\$45}$$

Kilowatt-hours is a measure of energy, not power, and so **no**, the actual rate at which the energy is used does not figure into the bill. They could use the energy at a constant rate, or at a widely varying rate, and as long as the total used is 370 kilowatt-hours, the price would be \$45.

63. The energy transfer from the engine must replace the lost kinetic energy. From the two speeds, calculate the average rate of loss in kinetic energy while in neutral.

$$v_1 = 85 \text{ km/h} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 23.61 \text{ m/s} \quad v_2 = 65 \text{ km/h} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 18.06 \text{ m/s}$$

$$\Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}(1150 \text{ kg}) \left[(18.06 \text{ m/s})^2 - (23.61 \text{ m/s})^2 \right] = -1.330 \times 10^5 \text{ J}$$

$$P = \frac{W}{t} = \frac{1.330 \times 10^5 \text{ J}}{6.0 \text{ s}} = 2.216 \times 10^4 \text{ W}, \text{ or } (2.216 \times 10^4 \text{ W}) \frac{1 \text{ hp}}{746 \text{ W}} = 29.71 \text{ hp}$$

So $\boxed{2.2 \times 10^4 \text{ W}}$ or $\boxed{3.0 \times 10^1 \text{ hp}}$ is needed from the engine.

$$64. \text{ Since } P = \frac{W}{t}, \text{ we have } W = Pt = 3.0 \text{ hp} \left(\frac{746 \text{ W}}{1 \text{ hp}} \right) (1 \text{ hr}) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) = \boxed{8.1 \times 10^6 \text{ J}}$$

65. The work done in accelerating the shot put is given by its change in kinetic energy: The power is the energy change per unit time.

$$P = \frac{W}{t} = \frac{KE_2 - KE_1}{t} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{t} = \frac{\frac{1}{2}(7.3 \text{ kg}) \left[(14 \text{ m/s})^2 - 0 \right]}{1.5 \text{ s}} = 476.9 \text{ W} \approx \boxed{4.8 \times 10^2 \text{ W}}$$

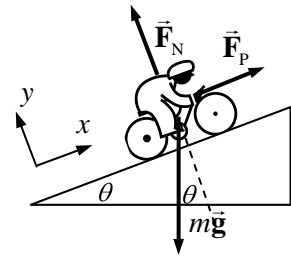
66. The force to lift the water is equal to its weight, and so the work to lift the water is equal to the weight times the distance. The power is the work done per unit time.

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{(18.0 \text{ kg})(9.80 \text{ m/s}^2)(3.60 \text{ m})}{60 \text{ sec}} = \boxed{10.6 \text{ W}}$$

67. The minimum force needed to lift the football player vertically is equal to his weight, mg . The distance over which that force would do work would be the change in height, $\Delta y = (140 \text{ m}) \sin 32^\circ = 74.2 \text{ m}$. So the work done in raising the player is $W = mg\Delta y$ and the power output required is the work done per unit time.

$$P = \frac{W}{t} = \frac{mg\Delta y}{t} = \frac{(95 \text{ kg})(9.80 \text{ m/s}^2)(74.2 \text{ m})}{66 \text{ sec}} = 1047 \text{ W} \approx \boxed{1.0 \times 10^3 \text{ W}}.$$

68. See the free-body diagram for the bicycle on the hill. Write Newton's 2nd law for the x direction, noting that the acceleration is 0. Solve for the magnitude of \vec{F}_p . The power output related to that force is given by Eq. 6-17, $P = F_p v$. Use that relationship to find the velocity.

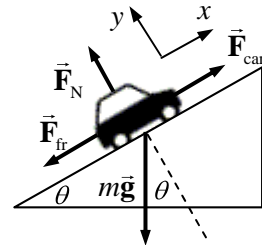


$$\sum F_x = F_p - mg \sin \theta = 0 \rightarrow F_p = mg \sin \theta$$

$$P = vF_p \rightarrow v = \frac{P}{F_p} = \frac{P}{mg \sin \theta} = \frac{(0.25 \text{ hp})(746 \text{ W/hp})}{(68 \text{ kg})(9.8 \text{ m/s}^2) \sin 6.0^\circ}$$

$$= \boxed{2.7 \text{ m/s}}$$

69. Consider the free-body diagram for the car. The car has a constant velocity, so the net force on the car is zero. F_{fr} is the friction force, and F_{car} is the force of the road pushing on the car. It is equal in magnitude to the force of the car pushing on the road, and so we can think of F_{car} as the force the car is able to generate by the engine. Write Newton's 2nd law in the x direction.



$$\sum F_x = F_{car} - F_{fr} - mg \sin \theta \rightarrow F_{car} = F_{fr} + mg \sin \theta$$

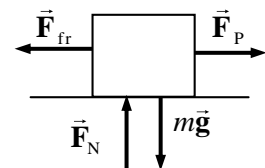
Use Eq. 6-17 to express the power output of the car, and then calculate the angle from that expression.

$$P = (F_{fr} + mg \sin \theta)v \rightarrow$$

$$\theta = \sin^{-1} \left[\frac{1}{mg} \left(\frac{P}{v} - F_{fr} \right) \right] = \sin^{-1} \left[\frac{1}{(1200 \text{ kg})(9.80 \text{ m/s}^2)} \left(\frac{120 \text{ hp} (746 \text{ W/1 hp})}{(75 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)} - 650 \text{ N} \right) \right]$$

$$= \boxed{18^\circ}$$

70. Draw a free-body diagram for the box being dragged along the floor. The box has a constant speed, so the acceleration is 0 in all directions. Write Newton's 2nd law for both the x (horizontal) and y (vertical) directions.



$$\sum F_y = F_N - mg = 0 \rightarrow F_N = mg$$

$$\sum F_x = F_p - F_{fr} = 0 \rightarrow F_p = F_{fr} = \mu_k F_N = \mu_k mg$$

The work done by F_p in moving the crate a distance Δx is given by $W = F_p \Delta x \cos 0^\circ = \mu_k mg \Delta x$.

The power required is the work done per unit time.

$$P = \frac{W}{t} = \frac{\mu_k mg \Delta x}{t} = \mu_k mg \frac{\Delta x}{t} = \mu_k mg v_x = (0.45)(310 \text{ kg})(9.80 \text{ m/s}^2)(1.20 \text{ m/s}) = 1641 \text{ W}$$

$$1641 \text{ W} \left(\frac{1 \text{ hp}}{746 \text{ W}} \right) = \boxed{2.2 \text{ hp}}$$

71. First, consider a free-body diagram for the cyclist going down hill. Write Newton's 2nd law for the x direction, with an acceleration of 0 since the cyclist has a constant speed.

$$\sum F_x = mg \sin \theta - F_{fr} = 0 \rightarrow F_{fr} = mg \sin \theta$$

Now consider the diagram for the cyclist going up the hill. Again, write Newton's 2nd law for the x direction, with an acceleration of 0.

$$\sum F_x = F_{fr} - F_p + mg \sin \theta = 0 \rightarrow F_p = F_{fr} + mg \sin \theta$$

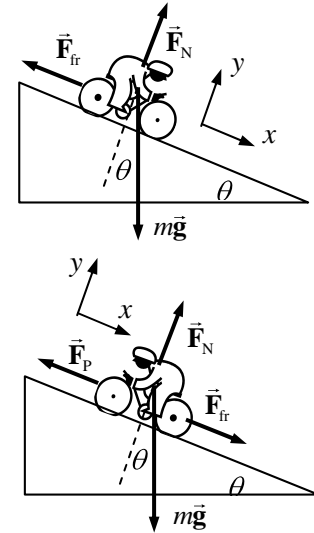
Assume that the friction force is the same when the speed is the same, so the friction force when going uphill is the same magnitude as when going downhill.

$$F_p = F_{fr} + mg \sin \theta = 2mg \sin \theta$$

The power output due to this force is given by Eq. 6-17.

$$P = F_p v = 2mgv \sin \theta = 2(75 \text{ kg})(9.8 \text{ m/s}^2)(5.0 \text{ m/s}) \sin 7.0^\circ$$

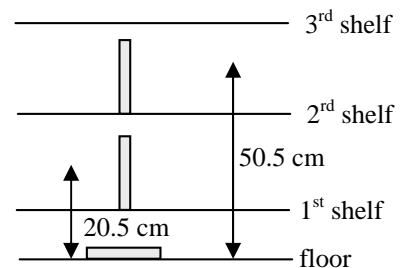
$$= \boxed{9.0 \times 10^2 \text{ W}}$$



72. The kinetic energy of the moving car is changed into the elastic PE of the bumper, before it deforms.

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \rightarrow k = \frac{mv^2}{x^2} = \frac{(1300 \text{ kg}) \left[(8 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \right]^2}{(0.015 \text{ m})^2} = \boxed{2.9 \times 10^7 \text{ N/m}}$$

73. The minimum work required to shelve a book is equal to the weight of the book times the vertical distance the book is moved – its increase in PE. See the diagram. Each book that is placed on the lowest shelf has its center of mass moved upwards by 20.5 cm. So the work done to move 25 books to the lowest shelf is $W_1 = 25mg(0.205 \text{ m})$. Each book that is placed on the second shelf has its center of mass moved upwards by 50.5 cm, so the work done to move 25 books to the second shelf is



$W_2 = 25mg(0.505 \text{ m})$. Similarly, $W_3 = 25mg(0.805 \text{ m})$, $W_4 = 25mg(1.105 \text{ m})$, and $W_5 = 25mg(1.405 \text{ m})$. The total work done is the sum of the five work expressions.

$$W = 25mg(0.205 \text{ m} + .505 \text{ m} + .805 \text{ m} + 1.105 \text{ m} + 1.405 \text{ m})$$

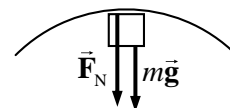
$$= 25(1.5 \text{ kg})(9.80 \text{ m/s}^2)(4.025 \text{ m}) = 1479 \text{ J} \approx \boxed{1.5 \times 10^3 \text{ J}}$$

74. Assume that there are no non-conservative forces doing work, so the mechanical energy of the jumper will be conserved. Subscript 1 represents the jumper at the launch point of the jump, and subscript 2 represents the jumper at the highest point. The starting height of the jump is the zero location for PE ($y = 0$). We have $y_1 = 0$, $y_2 = 1.1 \text{ m}$, and $v_2 = 6.5 \text{ m/s}$. Solve for v_1 .

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow$$

$$v_1 = \sqrt{v_2^2 + 2gy_2} = \sqrt{(6.5 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(1.1 \text{ m})} = \boxed{8.0 \text{ m/s}}$$

75. (a) Consider a free-body diagram for the block at the top of the curve. Since the block is moving in a circle, the net force is centripetal. Write Newton's 2nd law for the block, with down as vertical. If the block is to be on the verge of falling off the track, then $F_N = 0$.



$$\sum F_R = F_N + mg = mv_{\text{top}}^2/r \rightarrow mg = mv_{\text{top}}^2/r \rightarrow v_{\text{top}} = \sqrt{gr}$$

Now use conservation of energy for the block. Since the track is frictionless, there are no non-conservative forces, and mechanical energy will be conserved. Subscript 1 represents the block at the release point, and subscript 2 represents the block at the top of the loop. The ground is the zero location for PE ($y = 0$). We have $v_1 = 0$, $y_1 = h$, $v_2 = v_{\text{top}} = \sqrt{gr}$, and $y_2 = 2r$.

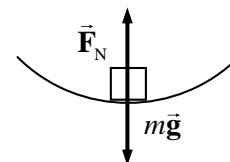
Solve for h .

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow mgh = \frac{1}{2}mgr + 2mgr \rightarrow h = \boxed{2.5r}$$

- (b) Now the release height is $2h = 5r$. Use conservation of energy again. Subscript 1 represents the block at the (new) release point, and subscript 2 represents the block at the bottom of the loop. We have $v_1 = 0$, $y_1 = 5r$, and $y_2 = 0$. Solve for v_2^2 .

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow v_2^2 = 10rg$$

Now consider the free-body diagram for the block at the bottom of the loop. The net force must be upward and radial. Write Newton's 2nd law for the vertical direction, with up as positive.



$$\sum F_R = F_N - mg = mv^2/r \rightarrow$$

$$F_N = mg + mv^2/r = mg + \frac{m10rg}{r} = \boxed{11mg}$$

- (c) Use conservation of energy again. Subscript 2 is as in part (b) above, and subscript 3 represents the block at the top of the loop. We have $y_2 = 0$, $v_2 = \sqrt{10rg}$, and $y_3 = 2r$. Solve for v_3^2 .

$$E_2 = E_3 \rightarrow \frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_3^2 + mgy_3 \rightarrow 5mrg + 0 = \frac{1}{2}mv_3^2 + 2mrg \rightarrow$$

$$v_3^2 = 6rg$$

Refer to the free-body diagram and analysis of part (a) to find the normal force.

$$\sum F_R = F_N + mg = mv_{\text{top}}^2/r \rightarrow F_N = mv^2/r - mg = \frac{6mrg}{r} - mg = \boxed{5mg}$$

- (d) When moving on the level, the normal force is the same as the weight, $F_N = \boxed{mg}$.

76. (a) Use conservation of energy, including the work done by the non-conservative force of the snow on the pilot. Subscript 1 represents the pilot at the top of the snowbank, and subscript 2 represents the pilot at the bottom of the crater. The bottom of the crater is the zero location for PE ($y = 0$). We have $v_1 = 35 \text{ m/s}$, $y_1 = 1.1 \text{ m}$, $v_2 = 0$, and $y_2 = 0$. Solve for the non-conservative work.

$$W_{\text{NC}} + E_1 = E_2 \rightarrow W_{\text{NC}} + \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow$$

$$W_{\text{NC}} = -\frac{1}{2}mv_1^2 - mgy_1 = -\frac{1}{2}(78 \text{ kg})(35 \text{ m/s})^2 - (78 \text{ kg})(9.8 \text{ m/s}^2)(1.1 \text{ m})$$

$$= -4.862 \times 10^4 \text{ J} \approx \boxed{-4.9 \times 10^4 \text{ J}}$$

- (b) The work done by the snowbank is done by an upward force, while the pilot moves down.

$$W_{\text{NC}} = F_{\text{snow}} d \cos 180^\circ = -F_{\text{snow}} d \rightarrow$$

$$F_{\text{snow}} = -\frac{W_{\text{NC}}}{d} = -\frac{-4.862 \times 10^4 \text{ J}}{1.1 \text{ m}} = 4.420 \times 10^4 \text{ N} \approx \boxed{4.4 \times 10^4 \text{ N}}$$

- (c) To find the work done by air friction, another non-conservative force, use energy conservation including the work done by the non-conservative force of air friction. Subscript 1 represents the pilot at the start of the descent, and subscript 3 represents the pilot at the top of the snowbank. The top of the snowbank is the zero location for PE ($y = 0$). We have $v_1 = 0 \text{ m/s}$, $y_1 = 370 \text{ m}$, $v_2 = 35 \text{ m/s}$, and $y_2 = 0$. Solve for the non-conservative work.

$$W_{\text{NC}} + E_1 = E_2 \rightarrow W_{\text{NC}} + \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow$$

$$\begin{aligned} W_{\text{NC}} &= \frac{1}{2}mv_2^2 - mgy_1 = \frac{1}{2}(78 \text{ kg})(35 \text{ m/s})^2 - (78 \text{ kg})(9.8 \text{ m/s}^2)(370 \text{ m}) \\ &= -2.351 \times 10^5 \text{ J} \approx \boxed{-2.4 \times 10^5 \text{ J}} \end{aligned}$$

77. (a) The tension in the cord is perpendicular to the path at all times, and so the tension in the cord does not do any work on the ball. Thus the mechanical energy of the ball is conserved. Subscript 1 represents the ball when it is horizontal, and subscript 2 represents the ball at the lowest point on its path. The lowest point on the path is the zero location for PE ($y = 0$). We have $v_1 = 0$, $y_1 = L$, and $y_2 = 0$. Solve for v_2 .

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow mgL = \frac{1}{2}mv_2^2 \rightarrow v_2 = \boxed{\sqrt{2gL}}$$

- (b) Use conservation of energy, to relate points 2 and 3. Point 2 is as described above. Subscript 3 represents the ball at the top of its circular path around the peg. The lowest point on the path is the zero location for PE ($y = 0$). We have $v_1 = \sqrt{2gL}$, $y_1 = 0$, and $y_2 = 2(L - h) = 2(L - 0.80L) = 0.40L$. Solve for v_2 .

$$\begin{aligned} E_2 = E_3 &\rightarrow \frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_3^2 + mgy_3 \rightarrow \frac{1}{2}m(2gL) = \frac{1}{2}mv_2^2 + mg(0.40L) \rightarrow \\ v_2 &= \boxed{\sqrt{1.2gL}} \end{aligned}$$

78. (a) The work done by the hiker against gravity is the change in gravitational PE.

$$W_G = mg\Delta y = (65 \text{ kg})(9.8 \text{ m/s}^2)(3700 \text{ m} - 2300 \text{ m}) = 8.918 \times 10^5 \text{ J} \approx \boxed{8.9 \times 10^5 \text{ J}}$$

- (b) The average power output is found by dividing the work by the time taken.

$$P_{\text{output}} = \frac{W_{\text{grav}}}{t} = \frac{8.918 \times 10^5 \text{ J}}{(5 \text{ h})(3600 \text{ s/1 h})} = 49.54 \text{ W} \approx \boxed{50 \text{ W}}$$

$$49.54 \text{ W} \left(\frac{1 \text{ hp}}{746 \text{ W}} \right) = \boxed{6.6 \times 10^{-2} \text{ hp}}$$

- (c) The output power is the efficiency times the input power.

$$P_{\text{output}} = 0.15P_{\text{input}} \rightarrow P_{\text{input}} = \frac{P_{\text{output}}}{0.15} = \frac{49.54 \text{ W}}{0.15} = \boxed{3.3 \times 10^2 \text{ W}} = \boxed{0.44 \text{ hp}}$$

79. (a) The work done by gravity as the elevator falls is the opposite of the change in gravitational PE.

$$\begin{aligned} W_G &= -\Delta PE = PE_1 - PE_2 = mg(y_1 - y_2) = (920 \text{ kg})(9.8 \text{ m/s}^2)(28 \text{ m}) \\ &= 2.524 \times 10^5 \text{ J} \approx \boxed{2.5 \times 10^5 \text{ J}} \end{aligned}$$

Gravity is the only force doing work on the elevator as it falls (ignoring friction), so this result is also the net work done on the elevator as it falls.

- (b) The net work done on the elevator is equal to its change in kinetic energy. The net work done just before striking the spring is the work done by gravity found above.

$$W_G = KE_2 - KE_1 \rightarrow mg(y_1 - y_2) = \frac{1}{2}mv_2^2 - 0 \rightarrow$$

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.8 \text{ m/s}^2)(28 \text{ m})} = 23.43 \text{ m/s} \approx \boxed{23 \text{ m/s}}$$

- (c) Use conservation of energy. Subscript 1 represents the elevator just before striking the spring, and subscript 2 represents the elevator at the bottom of its motion. The level of the elevator just before striking the spring is the zero location for both gravitational PE and elastic PE. We have $v_1 = 23.43 \text{ m/s}$, $y_1 = 0$, and $v_2 = 0$. We assume that $y_2 < 0$.

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}ky_1^2 = \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}ky_2^2 \rightarrow$$

$$y_2^2 + 2\frac{mg}{k}y_2 - \frac{m}{k}v_1^2 = 0 \rightarrow y_2 = \frac{-\frac{2mg}{k} \pm \sqrt{\frac{4m^2g^2}{k^2} + 4\frac{mv_1^2}{k}}}{2} = \frac{-mg \pm \sqrt{m^2g^2 + mkv_1^2}}{k}$$

We must choose the negative root so that y_2 is negative. Thus

$$y_2 = \frac{-(920 \text{ kg})(9.8 \text{ m/s}^2) - \sqrt{(920 \text{ kg})^2(9.8 \text{ m/s}^2)^2 + (920 \text{ kg})(2.2 \times 10^5 \text{ N/m})(23.43 \text{ m/s})^2}}{2.2 \times 10^5 \text{ N/m}}$$

$$= \boxed{-1.56 \text{ m}}$$

80. The force to lift a person is equal to the person's weight, so the work to lift a person up a vertical distance h is equal to mgh . The work needed to lift N people is $Nmgh$, and so the power needed is the total work divided by the total time. We assume the mass of the average person to be 70 kg,

$$P = \frac{W}{t} = \frac{Nmgh}{t} = \frac{47000(70 \text{ kg})(9.80 \text{ m/s}^2)(200 \text{ m})}{3600 \text{ s}} = 1.79 \times 10^6 \text{ W} \approx \boxed{2 \times 10^6 \text{ W}}$$

81. (a) Use conservation of mechanical energy, assuming there are no non-conservative forces. Subscript 1 represents the water at the top of the dam, and subscript 2 represents the water as it strikes the turbine blades. The level of the turbine blades is the zero location for PE ($y = 0$).

We have $v_1 = 0$, $y_1 = 80 \text{ m}$, and $y_2 = 0$. Solve for v_2 .

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow mgy_1 = \frac{1}{2}mv_2^2 \rightarrow$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.8 \text{ m/s}^2)(81 \text{ m})} = 39.84 \text{ m/s} \approx \boxed{4.0 \times 10^1 \text{ m/s}}$$

- (b) The energy of the water at the level of the turbine blades is all kinetic energy, and so is given by $\frac{1}{2}mv_2^2$. 58% of that energy gets transferred to the turbine blades. The rate of energy transfer to the turbine blades is the power developed by the water.

$$P = 0.58 \left(\frac{1}{2} \frac{m}{t} v_2^2 \right) = \frac{(0.58)(650 \text{ kg/s})(39.84 \text{ m/s})^2}{2} = \boxed{3.0 \times 10^5 \text{ W}}$$

82. Consider the free-body diagram for the coaster at the bottom of the loop. The net force must be an upward centripetal force.

$$\sum F_{\text{bottom}} = F_{N_{\text{bottom}}} - mg = m v_{\text{bottom}}^2 / R \rightarrow F_{N_{\text{bottom}}} = mg + m v_{\text{bottom}}^2 / R$$

Now consider the force diagram at the top of the loop. Again, the net force must be centripetal, and so must be downward.

$$\sum F_{\text{top}} = F_{N_{\text{top}}} + mg = m v_{\text{top}}^2 / R \rightarrow F_{N_{\text{top}}} = m v_{\text{top}}^2 / R - mg .$$

Assume that the speed at the top is large enough that $F_{N_{\text{top}}} > 0$, and so

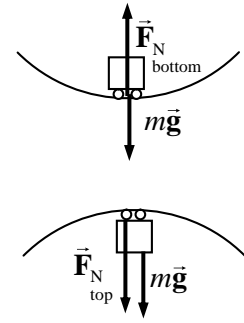
$v_{\text{top}} > \sqrt{Rg}$. Now apply the conservation of mechanical energy. Subscript 1 represents the coaster at the bottom of the loop, and subscript 2 represents the coaster at the top of the loop. The level of the bottom of the loop is the zero location for PE ($y = 0$). We have $y_1 = 0$ and $y_2 = 2R$.

$$E_1 = E_2 \rightarrow \frac{1}{2} m v_1^2 + m g y_1 = \frac{1}{2} m v_2^2 + m g y_2 \rightarrow v_{\text{bottom}}^2 = v_{\text{top}}^2 + 4gR .$$

The difference in apparent weights is the difference in the normal forces.

$$\begin{aligned} F_{N_{\text{bottom}}} - F_{N_{\text{top}}} &= \left(mg + m v_{\text{bottom}}^2 / R \right) - \left(m v_{\text{top}}^2 / R - mg \right) = 2mg + m \left(v_{\text{bottom}}^2 - v_{\text{top}}^2 \right) / R \\ &= 2mg + m(4gR) / R = \boxed{6mg} \end{aligned}$$

Notice that the result does not depend on either v or R .



83. (a) Assume that the energy of the candy bar is completely converted into a change of PE:

$$E_{\text{candy bar}} = \Delta PE = mg \Delta y \rightarrow \Delta y = \frac{E_{\text{candy bar}}}{mg} = \frac{1.1 \times 10^6 \text{ J}}{(82 \text{ kg})(9.8 \text{ m/s}^2)} = \boxed{1.4 \times 10^3 \text{ m}} .$$

- (b) If the person jumped to the ground, the same energy is all converted into kinetic energy.

$$E_{\text{candy bar}} = \frac{1}{2} m v^2 \rightarrow v = \sqrt{\frac{2E_{\text{candy bar}}}{m}} = \sqrt{\frac{2(1.1 \times 10^6 \text{ J})}{(82 \text{ kg})}} = \boxed{1.6 \times 10^2 \text{ m/s}}$$

84. Since there are no non-conservative forces, the mechanical energy of the projectile will be conserved. Subscript 1 represents the projectile at launch and subscript 2 represents the projectile as it strikes the ground. The ground is the zero location for PE ($y = 0$). We have $v_1 = 175 \text{ m/s}$, $y_1 = 165 \text{ m}$, and $y_2 = 0$. Solve for v_2 .

$$\begin{aligned} E_1 = E_2 \rightarrow \frac{1}{2} m v_1^2 + m g y_1 &= \frac{1}{2} m v_2^2 + m g y_2 \rightarrow \frac{1}{2} m v_1^2 + m g y_1 = \frac{1}{2} m v_2^2 \rightarrow \\ v_2 &= \sqrt{v_1^2 + 2g y_1} = \sqrt{(175 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(165 \text{ m})} = \boxed{184 \text{ m/s}} \end{aligned}$$

Notice that the launch angle does not enter the problem, and so does not influence the final speed.

85. The spring constant for the scale can be found from the 0.6 mm compression due to the 710 N force.

$$k = \frac{F}{x} = \frac{710 \text{ N}}{6.0 \times 10^{-4} \text{ m}} = 1.183 \times 10^6 \text{ N/m} .$$

Use conservation of energy for the jump. Subscript 1 represents the initial location, and subscript 2 represents the location at maximum compression of the scale spring. Assume that the location of the uncompressed scale spring is the 0 location for gravitational PE. We have $v_1 = v_2 = 0$ and $y_1 = 1.0 \text{ m}$. Solve for y_2 , which must be negative.

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}ky_2^2 \rightarrow$$

$$mgy_1 = mgy_2 + \frac{1}{2}ky_2^2 \rightarrow y_2^2 + 2\frac{mg}{k}y_2 - 2\frac{mg}{k}y_1 = y_2^2 + 1.200 \times 10^{-3}y_2 - 1.200 \times 10^{-3} = 0$$

$$y_2 = -3.52 \times 10^{-2} \text{ m}, 3.40 \times 10^{-2} \text{ m}$$

$$F_{\text{scale}} = k|x| = (1.183 \times 10^6 \text{ N/m})(3.52 \times 10^{-2} \text{ m}) = \boxed{4.2 \times 10^4 \text{ N}}$$

86. (a) Use conservation of energy for the swinging motion. Subscript 1 represents the student initially grabbing the rope, and subscript 2 represents the student at the top of the swing. The location where the student initially grabs the rope is the zero location for PE ($y = 0$).

We have $v_1 = 5.0 \text{ m/s}$, $y_1 = 0$, and $v_2 = 0$. Solve for y_2 .

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow$$

$$\frac{1}{2}mv_1^2 = mgy_2 \rightarrow y_2 = \frac{v_1^2}{2g} = h$$

Calculate the angle from the relationship in the diagram.

$$\cos \theta = \frac{L-h}{L} = 1 - \frac{h}{L} = 1 - \frac{v_1^2}{2gL} \rightarrow$$

$$\theta = \cos^{-1} \left(1 - \frac{v_1^2}{2gL} \right) = \cos^{-1} \left(1 - \frac{(5.0 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)(10 \text{ m})} \right) = \boxed{29^\circ}$$

- (b) At the release point, the speed is 0, and so there is no radial acceleration, since $a_R = v^2/r$. Thus the centripetal force must be 0. Use the free-body diagram to write Newton's 2nd law for the radial direction.

$$\sum F_R = F_T - mg \cos \theta = 0 \rightarrow$$

$$F_T = mg \cos \theta = (65 \text{ kg})(9.8 \text{ m/s}^2) \cos 29^\circ = \boxed{5.6 \times 10^2 \text{ N}}$$

- (c) Write Newton's 2nd law for the radial direction for any angle, and solve for the tension.

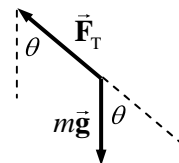
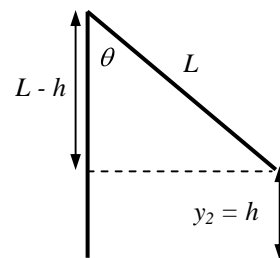
$$\sum F_R = F_T - mg \cos \theta = mv^2/r \rightarrow F_T = mg \cos \theta + mv^2/r$$

As the angle decreases, the tension increases, and as the speed increases, the tension increases. Both effects are greatest at the bottom of the swing, and so that is where the tension will be at its maximum.

$$F_{T \text{ max}} = mg \cos 0 + mv_1^2/r = (65 \text{ kg})(9.8 \text{ m/s}^2) + \frac{(65 \text{ kg})(5.0 \text{ m/s})^2}{10 \text{ m}} = \boxed{8.0 \times 10^2 \text{ N}}$$

87. The minimum vertical force needed to raise the athlete is equal to the athlete's weight. If the athlete moves upward a distance Δy , then the work done by the lifting force is $W = Fd \cos 0^\circ = mg\Delta y$, the change in PE. The power output needed to accomplish this work in a certain time t is the work divided by the time.

$$P = \frac{W}{t} = \frac{mg\Delta y}{t} = \frac{(72 \text{ kg})(9.8 \text{ m/s}^2)(5.0 \text{ m})}{9.0 \text{ s}} = \boxed{3.9 \times 10^2 \text{ W}}$$



88. The energy to be stored is the power multiplied by the time: $E = Pt$. The energy will be stored as the gravitational PE increase in the water: $E = \Delta PE = mg\Delta y = \rho Vg\Delta y$, where ρ is the density of the water, and V is the volume of the water.

$$Pt = \rho Vg\Delta y \rightarrow V = \frac{Pt}{\rho g\Delta y} = \frac{(120 \times 10^6 \text{ W})(3600 \text{ s})}{(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(520 \text{ m})} = \boxed{8.5 \times 10^4 \text{ m}^3}$$

89. If the original spring is stretched a distance x from equilibrium, then the potential energy stored is $PE_{\text{full}} = \frac{1}{2}kx^2$. Alternatively, think of the original spring as being made up of the two halves of the spring, connected from end to end. Each half of the spring has a spring constant k' , to be determined. As the spring is stretched a distance x , each half-spring is stretched a distance $x/2$. Each half-spring will have an amount of potential energy stored of $PE_{\text{half}} = \frac{1}{2}k'(x/2)^2$. The amount of energy in the two half-springs must equal the amount of energy in the full spring.

$$PE_{\text{full}} = 2PE_{\text{half}} \rightarrow \frac{1}{2}kx^2 = 2\left[\frac{1}{2}k'(x/2)^2\right] \rightarrow k' = \boxed{2k}$$

90. Consider the free-body diagram for the block. The block is moving up the plane.

$$(a) KE_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(6.0 \text{ kg})(2.2 \text{ m/s})^2 = 14.52 \text{ J} \approx \boxed{15 \text{ J}}$$

$$(b) W_p = F_p d \cos 37^\circ = (75 \text{ N})(8.0 \text{ m}) \cos 37^\circ = 479.2 \text{ J} \approx \boxed{4.8 \times 10^2 \text{ J}}$$

$$(c) W_{\text{fr}} = F_{\text{fr}} d \cos 180^\circ = -(25 \text{ N})(8.0 \text{ m}) = \boxed{-2.0 \times 10^2 \text{ J}}$$

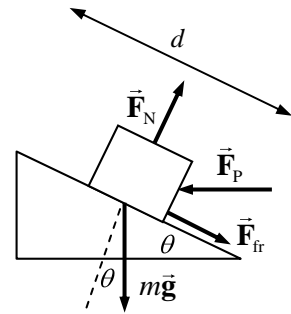
$$(d) W_G = mgd \cos 127^\circ = (6.0 \text{ kg})(9.8 \text{ m/s}^2)(8.0 \text{ m}) \cos 127^\circ \\ = -283.1 \text{ J} \approx \boxed{-2.8 \times 10^2 \text{ J}}$$

$$(e) W_N = F_N d \cos 90^\circ = \boxed{0 \text{ J}}$$

- (f) By the work-energy theorem,

$$W_{\text{total}} = KE_2 - KE_1 \rightarrow$$

$$KE_2 = W_{\text{total}} + KE_1 = W_p + W_{\text{fr}} + W_G + W_N + KE_1 = 10.62 \text{ J} \approx \boxed{11 \text{ J}}$$



91. The power output for either scenario is given by the change in kinetic energy, divided by the time required to change the kinetic energy. Subscripts of f and i are used for final and initial values of speed and kinetic energy. Subscript 1 represents the acceleration from 35 km/h to 55 km/h, and subscript 2 represents the acceleration from 55 km/h to 75 km/h.

$$P_1 = \frac{KE_{1f} - KE_{1i}}{t_1} = \frac{\frac{1}{2}m(v_{1f}^2 - v_{1i}^2)}{t_1} \quad P_2 = \frac{KE_{2f} - KE_{2i}}{t_2} = \frac{\frac{1}{2}m(v_{2f}^2 - v_{2i}^2)}{t_2}$$

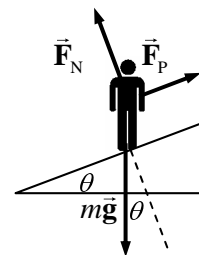
Equate the two expressions for power, and solve for t_2 .

$$\frac{\frac{1}{2}m(v_{1f}^2 - v_{1i}^2)}{t_1} = \frac{\frac{1}{2}m(v_{2f}^2 - v_{2i}^2)}{t_2} \rightarrow t_2 = t_1 \frac{(v_{2f}^2 - v_{2i}^2)}{(v_{1f}^2 - v_{1i}^2)}$$

Since the velocities are included as a ratio, any consistent set of units may be used for the velocities. Thus no conversion from km/h to some other units is needed.

$$t_2 = t_1 \frac{(v_{2f}^2 - v_{2i}^2)}{(v_{1f}^2 - v_{1i}^2)} = (3.2 \text{ s}) \frac{(75 \text{ km/h})^2 - (55 \text{ km/h})^2}{(55 \text{ km/h})^2 - (35 \text{ km/h})^2} = \boxed{4.6 \text{ s}}$$

92. See the free-body diagram for the patient on the treadmill. We assume that there are no dissipative forces. Since the patient has a constant velocity, the net force parallel to the plane must be 0. Write Newton's 2nd law for forces parallel to the plane, and then calculate the power output of force \vec{F}_p .



$$\sum F_{\text{parallel}} = F_p - mg \sin \theta = 0 \rightarrow F_p = mg \sin \theta$$

$$P = F_p v = mgv \sin \theta = (75 \text{ kg})(9.8 \text{ m/s}^2)(3.3 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \sin 15^\circ$$

$$= 174.4 \text{ W} \approx \boxed{170 \text{ W}}$$

This is about 2 to 3 times the wattage of typical household light bulbs (60–100 W).

93. (a) Assume that there are no non-conservative forces on the rock, and so its mechanical energy is conserved. Subscript 1 represents the rock as it leaves the volcano, and subscript 2 represents the rock at its highest point. The location as the rock leaves the volcano is the zero location for PE ($y = 0$). We have $y_1 = 0$, $y_2 = 500 \text{ m}$, and $v_2 = 0$. Solve for v_1 .

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow \frac{1}{2}mv_1^2 = mgy_2 \rightarrow$$

$$v_1 = \sqrt{2gy_2} = \sqrt{2(9.80 \text{ m/s}^2)(500 \text{ m})} = 98.99 \text{ m/s} \approx \boxed{1 \times 10^2 \text{ m/s}}$$

- (b) The power output is the energy transferred to the launched rocks per unit time. The launching energy of a single rock is $\frac{1}{2}mv_1^2$, and so the energy of 1000 rocks is $1000(\frac{1}{2}mv_1^2)$. Divide this energy by the time it takes to launch 1000 rocks to find the power output needed to launch the rocks.

$$P = \frac{1000(\frac{1}{2}mv_1^2)}{t} = \frac{500(500 \text{ kg})(98.99 \text{ m/s})^2}{60 \text{ sec}} = \boxed{4 \times 10^7 \text{ W}}$$

94. (a) The maximum power output from the falling water would occur if all of the potential energy available were converted into work to turn the wheel. The rate of potential energy delivery to the wheel from the falling water is the power available.

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{(95 \text{ kg})(9.8 \text{ m/s}^2)(2.0 \text{ m})}{1 \text{ sec}} = \boxed{1.9 \times 10^3 \text{ W}}$$

- (b) To find the speed of the water as it hits the wheel, use energy conservation with no non-conservative forces. Subscript 1 represents the water at the start of the descent, and subscript 2 represents the water as it hits the wheel at the bottom of the descent. The bottom of the descent is the zero location for PE ($y = 0$). We have $v_1 = 0$, $y_1 = 2.0 \text{ m}$, and $y_2 = 0$. Solve for v_2 .

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow mgy_1 = \frac{1}{2}mv_2^2 \rightarrow$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.8 \text{ m/s}^2)(2.0 \text{ m})} = \boxed{6.3 \text{ m/s}}$$