CHAPTER 7: Linear Momentum

Answers to Questions

1. For momentum to be conserved, the system under analysis must be "closed" – not have any forces on it from outside the system. A coasting car has air friction and road friction on it, for example, which are "outside" forces and thus reduce the momentum of the car. If the ground and the air were considered part of the system, and their velocities analyzed, then the momentum of the entire system would be conserved, but not necessarily the momentum of any single component, like the car.

2. Consider this problem as a very light object hitting and sticking to a very heavy object. The large object – small object combination (Earth + jumper) would have some momentum after the collision, but due to the very large mass of the Earth, the velocity of the combination is so small that it is not measurable. Thus the jumper lands on the Earth, and nothing more happens.

3. When you release an inflated but untied balloon at rest, the gas inside the balloon (at high pressure) rushes out the open end of the balloon. That escaping gas and the balloon form a closed system, and so the momentum of the system is conserved. The balloon and remaining gas acquires a momentum equal and opposite to the momentum of the escaping gas, and so move in the opposite direction to the escaping gas.

4. If the rich man would have faced away from the shore and thrown the bag of coins directly away from the shore, he would have acquired a velocity towards the shore by conservation of momentum. Since the ice is frictionless, he would slide all the way to the shore.

5. When a rocket expels gas in a given direction, it puts a force on that gas. The momentum of the gas-rocket system stays constant, and so if the gas is pushed to the left, the rocket will be pushed to the right due to Newton's 3rd law. So the rocket must carry some kind of material to be ejected (usually exhaust from some kind of engine) to change direction.

6. The air bag greatly increases the amount of time over which the stopping force acts on the driver. If a hard object like a steering wheel or windshield is what stops the driver, then a large force is exerted over a very short time. If a soft object like an air bag stops the driver, then a much smaller force is exerted over a much longer time. For instance, if the air bag is able to increase the time of stopping by a factor of 10, then the average force on the person will be decreased by a factor of 10. This greatly reduces the possibility of serious injury or death.

7. "Crumple zones" are similar to air bags in that they increase the time of interaction during a collision, and therefore lower the average force required for the change in momentum that the car undergoes in the collision.

8. From Eq. 7-7 for a 1-D elastic collision, $v_A - v_B = v'_B - v'_A$. Let "A" represent the bat, and let "B" represent the ball. The positive direction will be the (assumed horizontal) direction that the bat is moving when the ball is hit. We assume the batter can swing the bat with equal strength in either case, so that v_A is the same in both pitching situations. Because the bat is so much heavier than the ball, we assume that $v'_A \approx v_A$ – the speed of the bat doesn't change significantly during the collision. Then the velocity of the baseball after being hit is $v'_B = v'_A + v_A - v_B \approx 2v_A - v_B$. If $v_B = 0$, the ball tossed up into the air by the batter, then $v'_B \approx 2v_A$ – the ball moves away with twice the speed of the

bat. But if $v_B < 0$, the pitched ball situation, we see that the magnitude of $v'_B > 2v_A$, and so the ball moves away with greater speed. If, for example, the pitching speed of the ball was about twice the speed at which the batter could swing the bat, then we would have $v'_B \approx 4v_A$. Thus the ball has greater speed after being struck, and thus it is easier to hit a home run. This is similar to the "gravitational slingshot" effect discussed in problem 85.

9. The impulse is the product of the force and the time duration that the force is applied. So the impulse from a small force applied over a long time can be larger than the impulse applied by a large force over a small time.

10. The momentum of an object can be expressed in terms of its kinetic energy, as follows.

$$p = mv = \sqrt{m^2v^2} = \sqrt{m\left(mv^2\right)} = \sqrt{2m\left(\tfrac{1}{2}mv^2\right)} = \sqrt{2mKE}\ .$$

Thus if two objects have the same kinetic energy, then the one with more mass has the greater momentum.

11. Consider two objects, each with the same magnitude of momentum, moving in opposite directions. They have a total momentum of 0. If they collide and have a totally inelastic collision, in which they stick together, then their final common speed must be 0 so that momentum is conserved. But since they are not moving after the collision, they have no kinetic energy, and so all of their kinetic energy has been lost.

12. The turbine blades should be designed so that the water rebounds. If the water rebounds, that means that a larger momentum change for the water has occurred than if it just came to a stop. And if there is a larger momentum change for the water, there will also be a larger momentum change for the blades, making them spin faster.

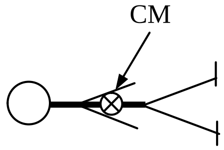
13. (*a*) The downward component of the momentum is unchanged. The horizontal component of momentum changes from rightward to leftward. Thus the change in momentum is to the left in the picture.
(*b*) Since the force on the wall is opposite that on the ball, the force on the wall is to the right.

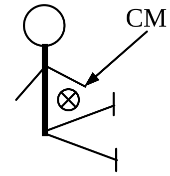
14. (*a*) The momentum of the ball is not conserved during any part of the process, because there is an external force acting on the ball at all times – the force of gravity. And there is an upward force on the ball during the collision. So considering the ball as the system, there are always external forces on it, and so its momentum is not conserved.
(*b*) With this definition of the system, all of the forces are internal, and so the momentum of the Earth-ball system is conserved during the entire process.
(*c*) The answer here is the same as for part (*b*).

15. In order to maintain balance, your CM must be located directly above your feet. If you have a heavy load in your arms, your CM will be out in front of your body and not above your feet. So you lean backwards to get your CM directly above your feet. Otherwise, you would fall over forwards.

16. The 1-m length of pipe is uniform – it has the same density throughout, and so its CM is at its geometric center, which is its midpoint. The arm and leg are not uniform – they are more dense where there is muscle, primarily in the parts that are closest to the body. Thus the CM of the arm or leg is closer the body than the geometric center. The CM is located closer to the more massive part of the arm or leg.

17. When you are lying flat on the floor, your CM is inside of the volume of your body. When you sit up on the floor with your legs extended, your CM is outside of the volume of your body.





18. The engine does not directly accelerate the car. The engine puts a force on the driving wheels, making them rotate. The wheels then push backwards on the roadway as they spin. The Newton's 3rd law reaction to this force is the forward-pushing of the roadway on the wheels, which accelerates the car. So it is the (external) road surface that accelerates the car.

19. The motion of the center of mass of the rocket will follow the original parabolic path, both before and after explosion. Each individual piece of the rocket will follow a separate path after the explosion, but since the explosion was internal to the system (consisting of the rocket), the center of mass of all the exploded pieces will follow the original path.

Solutions to Problems

1. $p = mv = (0.028\text{ kg})(8.4\text{ m/s}) = \boxed{0.24\text{ kg}\bullet\text{m/s}}$

2. From Newton's second law, $\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t$. For a constant mass object, $\Delta\vec{\mathbf{p}} = m\Delta\vec{\mathbf{v}}$. Equate the two expressions for $\Delta\vec{\mathbf{p}}$.

$$\vec{\mathbf{F}}\Delta t = m\Delta\vec{\mathbf{v}} \quad \rightarrow \quad \Delta\vec{\mathbf{v}} = \frac{\vec{\mathbf{F}}\Delta t}{m}.$$

If the skier moves to the right, then the speed will decrease, because the friction force is to the left.

$$\Delta v = -\frac{F\Delta t}{m} = -\frac{(25\text{ N})(20\text{ s})}{65\text{ kg}} = \boxed{-7.7\text{ m/s}}$$

The skier loses 7.7 m/s of speed.

3. Choose the direction from the batter to the pitcher to be the positive direction. Calculate the average force from the change in momentum of the ball.

$$\Delta p = F\Delta t = m\Delta v \quad \rightarrow$$

$$F = m\frac{\Delta v}{\Delta t} = (0.145\text{ kg})\left(\frac{52.0\text{ m/s} - -39.0\text{ m/s}}{3.00\times10^{-3}\text{ s}}\right) = \boxed{4.40\times10^{3}\text{ N, towards the pitcher}}$$

4. The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let "A" represent the boat and child together, and let "B" represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad (m_A + m_B)v = m_A v'_A + m_B v'_B \quad \rightarrow$$

$$v'_A = -\frac{m_B v'_B}{m_A} = -\frac{(6.40\text{ kg})(10.0\text{ m/s})}{(26.0\text{ kg} + 45.0\text{ kg})} = \boxed{-0.901\text{ m/s}}$$

The boat and child move in the opposite direction as the thrown package.

5. The force on the gas can be found from its change in momentum.

$$F = \frac{\Delta p}{\Delta t} = \frac{v\Delta m}{\Delta t} = v\frac{\Delta m}{\Delta t} = \left(4.0\times10^4\ \text{m/s}\right)\left(1500\ \text{kg/s}\right) = 6.0\times10^7\ \text{N downward}$$

The force on the rocket is the Newton's 3rd law pair (equal and opposite) to the force on the gas, and so is $\boxed{6.0\times10^7\ \text{N upward}}$.

6. The tackle will be analyzed as a one-dimensional momentum conserving situation. Let "A" represent the halfback, and "B" represent the tackling cornerback.

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad m_A v_A + m_B v_B = \left(m_A + m_B\right)v' \quad\rightarrow$$

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{\left(95\ \text{kg}\right)\left(4.1\,\text{m/s}\right) + \left(85\ \text{kg}\right)\left(5.5\,\text{m/s}\right)}{\left(95\ \text{kg}\right) + \left(85\ \text{kg}\right)} = \boxed{4.8\,\text{m/s}}$$

7. Consider the horizontal motion of the objects. The momentum in the horizontal direction will be conserved. Let "A" represent the car, and "B" represent the load. The positive direction is the direction of the original motion of the car.

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad m_A v_A + m_B v_B = \left(m_A + m_B\right)v' \quad\rightarrow$$

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{\left(12{,}600\ \text{kg}\right)\left(18.0\,\text{m/s}\right) + 0}{\left(12{,}600\ \text{kg}\right) + \left(5350\ \text{kg}\right)} = \boxed{12.6\,\text{m/s}}$$

8. Consider the motion in one dimension, with the positive direction being the direction of motion of the first car. Let "A" represent the first car, and "B" represent the second car. Momentum will be conserved in the collision. Note that $v_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad m_A v_A + m_B v_B = \left(m_A + m_B\right)v' \quad\rightarrow$$

$$m_B = \frac{m_A\left(v_A - v'\right)}{v'} = \frac{\left(9300\ \text{kg}\right)\left(15.0\,\text{m/s} - 6.0\,\text{m/s}\right)}{6.0\,\text{m/s}} = \boxed{1.4\times10^4\,\text{kg}}$$

9. The force stopping the wind is exerted by the person, so the force on the person would be equal in magnitude and opposite in direction to the force stopping the wind. Calculate the force from Eq. 7-2, in magnitude only.

$$\frac{m_{\text{wind}}}{\Delta t} = \frac{40\,\text{kg/s}}{\text{m}^2}\left(1.50\ \text{m}\right)\left(0.50\ \text{m}\right) = 30\,\text{kg/s} \qquad \Delta v_{\text{wind}} = 100\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 27.8\,\text{m/s}$$

$$F_{\substack{\text{on}\\\text{person}}} = F_{\substack{\text{on}\\\text{wind}}} = \frac{\Delta p_{\text{wind}}}{\Delta t} = \frac{m_{\text{wind}}\Delta v_{\text{wind}}}{\Delta t} = \frac{m_{\text{wind}}}{\Delta t}\Delta v_{\text{wind}} = \left(30\,\text{kg/s}\right)\left(27.8\,\text{m/s}\right)$$

$$= 833\,\text{N} \approx \boxed{8\times10^2\,\text{N}}$$

The typical maximum frictional force is $F_{\text{fr}} = \mu_s mg = \left(1.0\right)\left(70\ \text{kg}\right)\left(9.8\,\text{m/s}^2\right) = 690\ \text{N}$, and so we see that $\boxed{F_{\substack{\text{on}\\\text{person}}} > F_{\text{fr}}}$ – the wind is literally strong enough to blow a person off his feet.

10. Momentum will be conserved in the horizontal direction. Let "A" represent the car, and "B" represent the snow. For the horizontal motion, $v_B = 0$ and $v'_B = v'_A$. Momentum conservation gives the following.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A = \left(m_A + m_B\right) v'_A$$

$$v'_A = \frac{m_A v_A}{m_A + m_B} = \frac{\left(3800 \text{ kg}\right)\left(8.60 \text{ m/s}\right)}{3800 \text{ kg} + \left(\frac{3.50 \text{ kg}}{\text{min}}\right)\left(90.0 \text{ min}\right)} = 7.94 \text{ m/s} \approx \boxed{7.9 \text{ m/s}}$$

11. Consider the motion in one dimension, with the positive direction being the direction of motion of the original nucleus. Let "A" represent the alpha particle, with a mass of 4 u, and "B" represent the new nucleus, with a mass of 218 u. Momentum conservation gives the following.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad \left(m_A + m_B\right) v = m_A v'_A + m_B v'_B \quad \rightarrow$$

$$v'_A = \frac{\left(m_A + m_B\right) v - m_B v'_B}{m_A} = \frac{\left(222 \text{ u}\right)\left(420 \text{ m/s}\right) - \left(218 \text{ u}\right)\left(350 \text{ m/s}\right)}{4.0 \text{ u}} = \boxed{4.2 \times 10^3 \text{ m/s}}$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.

12. Consider the motion in one dimension with the positive direction being the direction of motion of the bullet. Let "A" represent the bullet, and "B" represent the block. Since there is no net force outside of the block-bullet system (like frictions with the table), the momentum of the block and bullet combination is conserved. Note that $v_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow$$

$$v'_B = \frac{m_A\left(v_A - v'_A\right)}{m_B} = \frac{\left(0.023 \text{ kg}\right)\left(230 \text{ m/s} - 170 \text{ m/s}\right)}{2.0 \text{ kg}} = \boxed{0.69 \text{ m/s}}$$

13. (*a*) Consider the motion in one dimension with the positive direction being the direction of motion before the separation. Let "A" represent the upper stage (that moves away faster) and "B" represent the lower stage. It is given that $m_A = m_B$, $v_A = v_B = v$, and $v'_B = v'_A - v_{rel}$. Momentum conservation gives the following.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad \left(m_A + m_B\right) v = m_A v'_A + m_B v'_B = m_A v'_A + m_B\left(v'_A - v_{rel}\right) \quad \rightarrow$$

$$v'_A = \frac{\left(m_A + m_B\right) v + m_B v_{rel}}{\left(m_A + m_B\right)} = \frac{\left(975 \text{ kg}\right)\left(5800 \text{ m/s}\right) + \frac{1}{2}\left(975 \text{ kg}\right)\left(2200 \text{ m/s}\right)}{\left(975 \text{ kg}\right)}$$

$$= \boxed{6.9 \times 10^3 \text{ m/s} \text{ , away from Earth}}$$

$$v'_B = v'_A - v_{rel} = 6.9 \times 10^3 \text{ m/s} - 2.20 \times 10^3 \text{ m/s} = \boxed{4.7 \times 10^3 \text{ m/s} \text{ , away from Earth}}$$

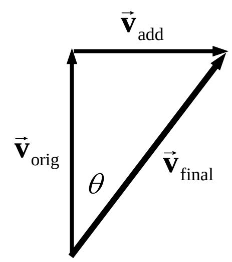
(*b*) The change in KE had to be supplied by the explosion.

$$\Delta KE = KE_f - KE_i = \left(\tfrac{1}{2} m_A v'^2_A + \tfrac{1}{2} m_B v'^2_B\right) - \tfrac{1}{2}\left(m_A + m_B\right) v^2$$

$$= \tfrac{1}{2}\left(487.5 \text{ kg}\right)\left[\left(6900 \text{ m/s}\right)^2 + \left(4700 \text{ m/s}\right)^2\right] - \tfrac{1}{2}\left(975 \text{ kg}\right)\left(5800 \text{ m/s}\right)^2$$

$$= \boxed{5.9 \times 10^8 \text{ J}}$$

14. To alter the course by 35.0°, a velocity perpendicular to the original velocity must be added. Call the direction of the added velocity, $\vec{\mathbf{v}}_{\text{add}}$, the positive direction. From the diagram, we see that $v_{\text{add}} = v_{\text{orig}} \tan\theta$. The momentum in the perpendicular direction will be conserved, considering that the gases are given perpendicular momentum in the opposite direction of $\vec{\mathbf{v}}_{\text{add}}$. The gas is expelled in the opposite direction to $\vec{\mathbf{v}}_{\text{add}}$, and so a negative value is used for $v_{\perp\text{ gas}}$.



$$p_{\perp\atop\text{before}} = p_{\perp\atop\text{after}} \rightarrow 0 = m_{\text{gas}} v_{\perp\text{ gas}} + \left(m_{\text{rocket}} - m_{\text{gas}}\right) v_{\text{add}} \rightarrow$$

$$m_{\text{gas}} = \frac{m_{\text{rocket}} v_{\text{add}}}{\left(v_{\text{add}} - v_{\perp\text{ gas}}\right)} = \frac{(3180\text{ kg})(115\text{ m/s})\tan 35.0^\circ}{\left[(115\text{ m/s})\tan 35^\circ - -1750\text{ m/s}\right]} = \boxed{1.40\times10^2\text{ kg}}$$

15. (*a*) The impulse is the change in momentum. The direction of travel of the struck ball is the positive direction.

$$\Delta p = m\Delta v = \left(4.5\times10^{-2}\text{ kg}\right)(45\text{ m/s} - 0) = \boxed{2.0\text{ kg}\bullet\text{m/s}}$$

(*b*) The average force is the impulse divided by the interaction time.

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{2.0\text{ kg}\bullet\text{m/s}}{3.5\times10^{-3}\text{ s}} = \boxed{5.8\times10^2\text{ N}}$$

16. (*a*) The impulse given to the nail is the opposite of the impulse given to the hammer. This is the change in momentum. Call the direction of the initial velocity of the hammer the positive direction.

$$\Delta p_{\text{nail}} = -\Delta p_{\text{hammer}} = mv_i - mv_f = (12\text{ kg})(8.5\text{ m/s}) - 0 = \boxed{1.0\times10^2\text{ kg}\bullet\text{m/s}}$$

(*b*) The average force is the impulse divided by the time of contact.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{1.0\times10^2\text{ kg}\bullet\text{m/s}}{8.0\times10^{-3}\text{ s}} = \boxed{1.3\times10^4\text{ N}}$$

17. The impulse given the ball is the change in the ball's momentum. From the symmetry of the problem, the vertical momentum of the ball does not change, and so there is no vertical impulse. Call the direction AWAY from the wall the positive direction for momentum perpendicular to the wall.

$$\Delta p_\perp = mv_{\perp\atop\text{final}} - mv_{\perp\atop\text{initial}} = m\left(v\sin 45^\circ - -v\sin 45^\circ\right) = 2mv\sin 45^\circ$$

$$= 2\left(6.0\times10^{-2}\text{ km}\right)(25\text{ m/s})\sin 45^\circ = \boxed{2.1\text{kg}\bullet\text{m/s}\text{ , to the left}}$$

18. (*a*) The average force on the car is the impulse (change in momentum) divided by the time of interaction. The positive direction is the direction of the car's initial velocity.

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = (1500\text{ kg})\left(\frac{0 - 50\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)}{0.15\text{ s}}\right) = -1.389\times10^5\text{ N} \approx \boxed{-1.4\times10^5\text{ N}}$$

(*b*) The deceleration is found from Newton's 2nd law.

$$\bar{F} = m\bar{a} \quad \rightarrow \quad \bar{a} = \frac{\bar{F}}{m} = \frac{\left(-1.389\times10^{5}\,\text{N}\right)}{1500\,\text{kg}} = \boxed{-93\,\text{m/s}^2}$$

19. Call east the positive direction.

(*a*) $p_{\substack{\text{original}\\\text{fullback}}} = mv_{\substack{\text{original}\\\text{fullback}}} = (95\text{ kg})(4.0\,\text{m/s}) = \boxed{3.8\times10^{2}\text{ kg}\bullet\text{m/s}}$

(*b*) The impulse on the fullback is the change in the fullback's momentum.

$$\Delta p_{\text{fullback}} = m\left(v_{\substack{\text{final}\\\text{fullback}}} - v_{\substack{\text{final}\\\text{fullback}}}\right) = (95\text{ kg})(0-4.0\,\text{m/s}) = \boxed{-3.8\times10^{2}\text{ kg}\bullet\text{m/s}}$$

(*c*) The impulse on the tackler is the opposite of the impulse on the fullback, so $\boxed{3.8\times10^{2}\text{ kg}\bullet\text{m/s}}$

(*d*) The average force on the tackler is the impulse on the tackler divided by the time of interaction.

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{3.8\times10^{2}\text{ kg}\bullet\text{m/s}}{0.75\text{ s}} = \boxed{5.1\times10^{2}\,\text{N}}$$

20. (*a*) The impulse given the ball is the area under the *F* vs. *t* graph. Approximate the area as a triangle of "height" 250 N, and "width" 0.01 sec.

$$\Delta p = \tfrac{1}{2}(250\text{ N})(0.01\text{ s}) = \boxed{1.25\text{N}\bullet\text{s}}$$

(*b*) The velocity can be found from the change in momentum. Call the positive direction the direction of the ball's travel after being served.

$$\Delta p = m\Delta v = m\left(v_f - v_i\right) \quad \rightarrow \quad v_f = v_i + \frac{\Delta p}{m} = 0 + \frac{1.25\text{N}\bullet\text{s}}{6.0\times10^{-2}\,\text{kg}} = \boxed{21\,\text{m/s}}$$

21. Find the velocity upon reaching the ground from energy conservation. Assume that all of the initial potential energy at the maximum height h_{max} is converted into kinetic energy. Take down to be the positive direction, so the velocity at the ground is positive.

$$mgh_{\text{max}} = \tfrac{1}{2}mv_{\text{ground}}^2 \quad \rightarrow \quad v_{\text{ground}} = \sqrt{2gh_{\text{max}}}$$

When contacting the ground, the impulse on the person causes a change in momentum. That relationship is used to find the time of the stopping interaction. The force of the ground acting on the person is negative since it acts in the upward direction.

$$\bar{F}\Delta t = m\left(0 - v_{\text{ground}}\right) \quad \rightarrow \quad \Delta t = -\frac{mv_{\text{ground}}}{\bar{F}}$$

We assume that the stopping force is so large that we call it the total force on the person – we ignore gravity for the stopping motion. The average acceleration of the person during stopping $\left(\bar{a} = \bar{F}/m\right)$ is used with Eq. 2-11b to find the displacement during stopping, h_{stop}.

$$y - y_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad h_{\text{stop}} = v_{\text{ground}}\left(-\frac{mv_{\text{ground}}}{\bar{F}}\right) + \tfrac{1}{2}\left(\frac{\bar{F}}{m}\right)\left(-\frac{mv_{\text{ground}}}{\bar{F}}\right)^2 \quad \rightarrow$$

$$h_{\text{stop}} = -\frac{mv_{\text{ground}}^2}{\bar{F}} + \tfrac{1}{2}\frac{mv_{\text{ground}}^2}{\bar{F}} = -\tfrac{1}{2}\frac{mv_{\text{ground}}^2}{\bar{F}} = -\frac{gh_{\text{max}}m}{\bar{F}} \quad \rightarrow \quad h_{\text{max}} = -\frac{\bar{F}h_{\text{stop}}}{mg}$$

We assume that the person lands with both feet striking the ground simultaneously, so the stopping force is divided between both legs. Thus the critical average stopping force is twice the breaking strength of a single leg.

$$h_{max} = -\frac{\bar{F} h_{stop}}{mg} = \frac{2F_{break} h_{stop}}{mg} = \frac{2\left(170\times10^{6}\ \mathrm{N/m^2}\right)\left(2.5\times10^{-4}\ \mathrm{m^2}\right)\left(0.60\ \mathrm{m}\right)}{\left(75\ \mathrm{kg}\right)\left(9.8\ \mathrm{m/s^2}\right)} = \boxed{69\ \mathrm{m}}$$

22. Let A represent the 0.440-kg ball, and B represent the 0.220-kg ball. We have $v_A = 3.30\ \mathrm{m/s}$ and $v_B = 0$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -\left(v'_A - v'_B\right) \rightarrow v'_B = v_A + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow m_A v_A = m_A v'_A + m_B\left(v_A + v'_A\right) \rightarrow$$

$$v'_A = \frac{\left(m_A - m_B\right)}{\left(m_A + m_B\right)} v_A = \frac{0.220\ \mathrm{kg}}{0.660\ \mathrm{kg}}\left(3.30\ \mathrm{m/s}\right) = \boxed{1.10\ \mathrm{m/s}\ (\mathrm{east})}$$

$$v'_B = v_A + v'_A = 3.30\ \mathrm{m/s} + 1.10\ \mathrm{m/s} = \boxed{4.40\ \mathrm{m/s}\ (\mathrm{east})}$$

23. Let A represent the 0.450-kg puck, and let B represent the 0.900-kg puck. The initial direction of puck A is the positive direction. We have $v_A = 3.00\ \mathrm{m/s}$ and $v_B = 0$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -\left(v'_A - v'_B\right) \rightarrow v'_B = v_A + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow m_A v_A = m_A v'_A + m_B\left(v_A + v'_A\right) \rightarrow$$

$$v'_A = \frac{\left(m_A - m_B\right)}{\left(m_A + m_B\right)} v_A = \frac{-0.450\ \mathrm{kg}}{1.350\ \mathrm{kg}}\left(3.00\ \mathrm{m/s}\right) = -1.00\ \mathrm{m/s} = \boxed{1.00\ \mathrm{m/s}\ (\mathrm{west})}$$

$$v'_B = v_A + v'_A = 3.00\ \mathrm{m/s} - 1.00\ \mathrm{m/s} = \boxed{2.00\ \mathrm{m/s}\ (\mathrm{east})}$$

24. Let A represent the ball moving at 2.00 m/s, and call that direction the positive direction. Let B represent the ball moving at 3.00 m/s in the opposite direction. So $v_A = 2.00\ \mathrm{m/s}$ and $v_B = -3.00\ \mathrm{m/s}$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -\left(v'_A - v'_B\right) \rightarrow v'_B = 5.00\ \mathrm{m/s} + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision, noting that $m_A = m_B$.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow v_A + v_B = v'_A + v'_B \rightarrow$$

$$-1.00\ \mathrm{m/s} = v'_A + \left(v'_A + 5.00\ \mathrm{m/s}\right) \rightarrow 2v'_A = -6.00\ \mathrm{m/s} \rightarrow v'_A = \boxed{-3.00\ \mathrm{m/s}}$$

$$v'_B = 5.00\ \mathrm{m/s} + v'_A = \boxed{2.00\ \mathrm{m/s}}$$

The two balls have exchanged velocities. This will always be true for 1-D elastic collisions of objects of equal mass.

25. Let A represent the 0.060-kg tennis ball, and let B represent the 0.090-kg ball. The initial direction of the balls is the positive direction. We have $v_A = 2.50\,\text{m/s}$ and $v_B = 1.15\,\text{m/s}$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = 1.35\,\text{m/s} + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow m_A v_A + m_B v_B = m_A v'_A + m_B(1.35\,\text{m/s} + v'_A) \rightarrow$$

$$v'_A = \frac{m_A v_A + m_B(v_B - 1.35\,\text{m/s})}{m_A + m_B} = \frac{(0.060\text{ kg})(2.50\,\text{m/s}) + (0.090\text{ kg})(1.15\,\text{m/s} - 1.35\,\text{m/s})}{0.150\text{ kg}}$$

$$= \boxed{0.88\,\text{m/s}}$$

$$v'_B = 1.35\,\text{m/s} + v'_A = \boxed{2.23\,\text{m/s}}$$

Both balls move in the direction of the tennis ball's initial motion.

26. Let A represent the moving softball, and let B represent the ball initially at rest. The initial direction of the softball is the positive direction. We have $v_A = 8.5\,\text{m/s}$, $v_B = 0$, and $v'_A = -3.7\,\text{m/s}$.

(*a*) Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = v_A - v_B + v'_A = 8.5\,\text{m/s} - 0 - 3.7\,\text{m/s} = \boxed{4.8\,\text{m/s}}$$

(*b*) Use momentum conservation to solve for the mass of the target ball.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow$$

$$m_B = m_A \frac{(v_A - v'_A)}{(v'_B - v_B)} = (0.220\text{ kg})\frac{(8.5\,\text{m/s} - -3.7\,\text{m/s})}{4.8\,\text{m/s}} = \boxed{0.56\text{ kg}}$$

27. Let the original direction of the cars be the positive direction. We have $v_A = 4.50\,\text{m/s}$ and $v_B = 3.70\,\text{m/s}$

(*a*) Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = v_A - v_B + v'_A = 0.80\,\text{m/s} + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow m_A v_A + m_B v_B = m_A v'_A + m_B(0.80\,\text{m/s} + v'_A) \rightarrow$$

$$v'_A = \frac{m_A v_A + m_B(v_B - 0.80\,\text{m/s})}{m_A + m_B} = \frac{(450\text{ kg})(4.50\,\text{m/s}) + (550\text{ kg})(2.90\,\text{m/s})}{1000\text{ kg}} = \boxed{3.62\,\text{m/s}}$$

$$v'_B = 0.80\,\text{m/s} + v'_A = \boxed{4.42\,\text{m/s}}$$

(*b*) Calculate $\Delta p = p' - p$ for each car.

$$\Delta p_A = m_A v'_A - m_A v_A = (450\text{ kg})(3.62\,\text{m/s} - 4.50\,\text{m/s}) = -3.96\times10^2\text{ kg}\bullet\text{m/s}$$

$$\approx \boxed{-4.0\times10^2\text{ kg}\bullet\text{m/s}}$$

$$\Delta p_B = m_B v'_B - m_B v_B = (550\text{ kg})(4.42\,\text{m/s} - 3.70\,\text{m/s}) = 3.96\times10^2\text{ kg}\bullet\text{m/s}$$

$$\approx \boxed{4.0\times10^2\text{ kg}\bullet\text{m/s}}$$

The two changes are equal and opposite because momentum was conserved.

28. (*a*) Momentum will be conserved in one dimension. Call the direction of the first ball the positive direction. Let A represent the first ball, and B represent the second ball. We have $v_B = 0$ and $v'_B = \frac{1}{2}v_A$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_A = -\tfrac{1}{2}v_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow m_A v_A = -\tfrac{1}{2}m_A v_A + m_B \tfrac{1}{2}v_A \rightarrow$$

$$m_B = 3m_A = 3(0.280 \text{ kg}) = \boxed{0.840 \text{ kg}}$$

(*b*) The fraction of the kinetic energy given to the second ball is as follows.

$$\frac{KE'_B}{KE_A} = \frac{\frac{1}{2}m_B v'^2_B}{\frac{1}{2}m_A v_A^2} = \frac{3m_A\left(\frac{1}{2}v_A\right)^2}{m_A v_A^2} = \boxed{0.75}$$

29. Let A represent the cube of mass *M*, and B represent the cube of mass *m*. Find the speed of A immediately before the collision, v_A, by using energy conservation.

$$Mgh = \tfrac{1}{2}Mv_A^2 \rightarrow v_A = \sqrt{2gh} = \sqrt{2(9.8\,\text{m/s}^2)(0.30\text{ m})} = 2.42\,\text{m/s}$$

Use Eq. 7-7 for elastic collisions to obtain a relationship between the velocities in the collision. We have $v_B = 0$ and $M = 2m$.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = v_A + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow m_A v_A = m_A v'_A + m_B(v_A + v'_A) \rightarrow$$

$$2mv_A = 2mv'_A + m(v_A + v'_A) \rightarrow v'_A = \frac{v_A}{3} = \frac{\sqrt{2gh}}{3} = \frac{\sqrt{2(9.8\,\text{m/s}^2)(0.30\text{ m})}}{3} = 0.808\,\text{m/s}$$

$$v'_B = v_A + v'_A = \tfrac{4}{3}v_A = 3.23\,\text{m/s}$$

Each mass is moving horizontally initially after the collision, and so each has a vertical velocity of 0 as they start to fall. Use constant acceleration Eq. 2-11b with down as positive and the table top as the vertical origin to find the time of fall.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \rightarrow H = 0 + 0 + \tfrac{1}{2}gt^2 \rightarrow t = \sqrt{2H/g}$$

Each cube then travels a horiztonal distance found by $\Delta x = v_x \Delta t$.

$$\Delta x_m = v'_A \Delta t = \frac{\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} = \tfrac{2}{3}\sqrt{hH} = \tfrac{2}{3}\sqrt{(0.30\text{ m})(0.90\text{ m})} = 0.3464\,\text{m} \approx \boxed{0.35\text{ m}}$$

$$\Delta x_M = v'_B \Delta t = \frac{4\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} = \tfrac{8}{3}\sqrt{hH} = \tfrac{8}{3}\sqrt{(0.30\text{ m})(0.90\text{ m})} = 1.386\,\text{m} \approx \boxed{1.4\text{ m}}$$

30. (*a*) Use Eq. 7-7, along with $v_B = 0$, to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = v_A + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B = m_A v'_A + m_B(v_A + v'_A) = m_A v'_A + m_B v_A + m_B v'_A \rightarrow$$

$$m_A v_A - m_B v_A = m_A v'_A + m_B v'_A \rightarrow (m_A - m_B)v_A = (m_A + m_B)v'_A \rightarrow v'_A = \frac{(m_A - m_B)}{(m_A + m_B)}v_A$$

Substitute this result into the result of Eq. 7-7.

$$v'_B = v_A + v'_A = v_A + \frac{(m_A - m_B)}{(m_A + m_B)} v_A = v_A \frac{(m_A + m_B)}{(m_A + m_B)} + \frac{(m_A - m_B)}{(m_A + m_B)} v_A = v_A \frac{2m_A}{(m_A + m_B)}$$

(*b*) If $m_A \ll m_B$, then approximate $m_A = 0$

$$v'_A = \frac{(m_A - m_B)}{(m_A + m_B)} v_A = \frac{(-m_B)}{(+m_B)} v_A = -v_A \qquad v'_B = \frac{2m_A v_A}{(m_A + m_B)} = 0$$

The result is $\boxed{v'_A = -v_A\ ;\ v'_B = 0}$. An example of this is a ball bouncing off of the floor. The massive floor has no speed after the collision, and the velocity of the ball is reversed (if dissipative forces are not present).

(*c*) If $m_A \gg m_B$, then approximate $m_B = 0$.

$$v'_A = \frac{(m_A - m_B)}{(m_A + m_B)} v_A = \frac{(m_A)}{(m_A)} v_A = v_A \qquad v'_B = \frac{2m_A v_A}{(m_A + m_B)} = \frac{2m_A v_A}{(m_A)} = 2v_A$$

The result is $\boxed{v'_A = v_A\ ;\ v'_B = 2v_A}$. An example of this would be a golf club hitting a golf ball. The speed of the club immediately after the collision is essentially the same as its speed before the collision, and the golf ball takes off with twice the speed of the club.

(*d*) If $m_A = m_B$, then set $m_A = m_B = m$.

$$v'_A = \frac{(m - m)}{(m + m)} v_A = 0 \qquad v'_B = \frac{2m v_A}{(m + m)} = \frac{2m v_A}{2m} = v_A$$

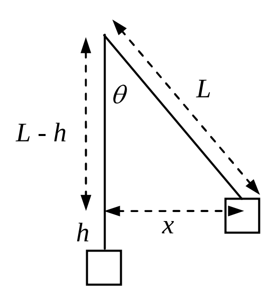
The result is $\boxed{v'_A = 0\ ;\ v'_B = v_A}$. An example of this is one billiard ball making a head-on collision with another. The first ball stops, and the second ball takes off with the same speed that the first one had.

31. From the analysis in Example 7-10, the initial projectile speed is given by $v = \frac{m+M}{m}\sqrt{2gh}$.

Compare the two speeds with the same masses.

$$\frac{v_2}{v_1} = \frac{\frac{m+M}{m}\sqrt{2gh_2}}{\frac{m+M}{m}\sqrt{2gh_1}} = \frac{\sqrt{h_2}}{\sqrt{h_1}} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{5.2}{2.6}} = \sqrt{2} \quad \rightarrow \quad \boxed{v_2 = \sqrt{2}v_1}$$

32. From the analysis in the Example 7-10, we know that



$$v = \frac{m+M}{m}\sqrt{2gh} \quad \rightarrow$$

$$h = \frac{1}{2g}\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2(9.8\,\mathrm{m/s^2})}\left(\frac{(0.028\ \mathrm{kg})(230\,\mathrm{m/s})}{0.028\ \mathrm{kg} + 3.6\mathrm{kg}}\right)^2$$

$$= 0.1607\,\mathrm{m} \approx \boxed{0.16\ \mathrm{m}}$$

From the diagram we see that

$$L^2 = (L-h)^2 + x^2$$

$$x = \sqrt{L^2 - (L-h)^2} = \sqrt{(2.8\ \mathrm{m})^2 - (2.8\ \mathrm{m} - 0.1607\ \mathrm{m})^2} = \boxed{0.94\ \mathrm{m}}$$

33. (*a*) In example 7-10, $KE_i = \frac{1}{2}mv^2$ and $KE_f = \frac{1}{2}(m+M)v'^2$. The speeds are related by

$$v' = \frac{m}{m+M}v .$$

$$\frac{\Delta KE}{KE_i} = \frac{KE_f - KE_i}{KE_i} = \frac{\frac{1}{2}(m+M)v'^2 - \frac{1}{2}mv^2}{\frac{1}{2}mv^2} = \frac{(m+M)\left(\frac{m}{m+M}v\right)^2 - mv^2}{mv^2}$$

$$= \frac{\frac{m^2v^2}{m+M} - mv^2}{mv^2} = \frac{m}{m+M} - 1 = \boxed{\frac{-M}{m+M}}$$

(*b*) For the given values, $\frac{-M}{m+M} = \frac{-380\text{ g}}{394\text{ g}} = -0.96$. Thus 96% of the energy is lost.

34. Use conservation of momentum in one dimension, since the particles will separate and travel in opposite directions. Call the direction of the heavier particle's motion the positive direction. Let A represent the heavier particle, and B represent the lighter particle. We have $m_A = 1.5m_B$, and $v_A = v_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad\to\quad 0 = m_A v'_A + m_B v'_B \quad\to\quad v'_A = -\frac{m_B v'_B}{m_A} = -\frac{2}{3}v'_B$$

The negative sign indicates direction.

Since there was no mechanical energy before the explosion, the kinetic energy of the particles after the explosion must equal the energy added.

$$E_{\text{added}} = KE'_A + KE'_B = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B = \frac{1}{2}(1.5m_B)\left(\frac{2}{3}v'_B\right)^2 + \frac{1}{2}m_B v'^2_B = \frac{5}{3}\left(\frac{1}{2}m_B v'^2_B\right) = \frac{5}{3}KE'_B$$

$$KE'_B = \frac{3}{5}E_{\text{added}} = \frac{3}{5}(7500\text{ J}) = 4500\text{ J} \qquad KE'_A = E_{\text{added}} - KE'_B = 7500\text{ J} - 4500\text{ J} = 3000\text{ J}$$

Thus $\boxed{KE'_A = 3.0\times10^3\text{ J} \quad KE'_B = 4.5\times10^3\text{ J}}$

35 Use conservation of momentum in one dimension. Call the direction of the sports car's velocity the positive *x* direction. Let A represent the sports car, and B represent the SUV. We have $v_B = 0$ and $v'_A = v'_B$. Solve for v_A.

$$p_{\text{initial}} = p_{\text{final}} \quad\to\quad m_A v_A + 0 = (m_A + m_B)v'_A \quad\to\quad v_A = \frac{m_A + m_B}{m_A}v'_A$$

The kinetic energy that the cars have immediately after the collision is lost due to negative work done by friction. The work done by friction can also be calculated using the definition of work. We assume the cars are on a level surface, so that the normal force is equal to the weight. The distance the cars slide forward is Δx. Equate the two expressions for the work done by friction, solve for v'_A, and use that to find v_A.

$$W_{\text{fr}} = (KE_{\text{final}} - KE_{\text{initial}})_{\substack{\text{after}\\ \text{collision}}} = 0 - \frac{1}{2}(m_A + m_B)v'^2_A$$

$$W_{\text{fr}} = F_{\text{fr}}\Delta x\cos 180^\circ = -\mu_k(m_A + m_B)g\Delta x$$

$$-\frac{1}{2}(m_A + m_B)v'^2_A = -\mu_k(m_A + m_B)g\Delta x \quad\to\quad v'_A = \sqrt{2\mu_k g\Delta x}$$

$$v_A = \frac{m_A + m_B}{m_A} v'_A = \frac{m_A + m_B}{m_A}\sqrt{2\mu_k g \Delta x} = \frac{920\,\text{kg} + 2300\,\text{kg}}{920\,\text{kg}}\sqrt{2(0.80)(9.8\,\text{m/s}^2)(2.8\text{ m})}$$
$$= 23.191\,\text{m/s} \approx \boxed{23\,\text{m/s}}$$

36. Consider conservation of energy during the rising and falling of the ball, between contacts with the floor. The gravitational potential energy at the top of a path will be equal to the kinetic energy at the start and the end of each rising-falling cycle. Thus $mgh = \frac{1}{2}mv^2$ for any particular bounce cycle. Thus for an interaction with the floor, the ratio of the energies before and after the bounce is $\frac{KE_{\text{after}}}{KE_{\text{before}}} = \frac{mgh'}{mgh} = \frac{1.20\text{ m}}{1.50\text{ m}} = 0.80$. We assume that each bounce will further reduce the energy to 80% of its pre-bounce amount. The number of bounces to lose 90% of the energy can be expressed as follows.

$$(0.8)^n = 0.1 \;\rightarrow\; n = \frac{\log 0.1}{\log 0.8} = 10.3$$

Thus after $\boxed{\text{11 bounces}}$, more than 90% of the energy is lost.

As an alternate method, after each bounce, 80% of the available energy is left. So after 1 bounce, 80% of the original energy is left. After the second bounce, only 80% of 80%, or 64% of the available energy is left. After the third bounce, 51 %. After the fourth bounce, 41%. After the fifth bounce, 33 %. After the sixth bounce, 26%. After the seventh bounce, 21%. After the eight bounce, 17%. After the ninth bounce, 13%. After the tenth bounce, 11%. After the eleventh bounce, 9% is left. So again, it takes 11 bounces.

37. (*a*) For a perfectly elastic collision, Eq. 7-7 says $v_A - v_B = -(v'_A - v'_B)$. Substitute that into the coefficient of restitution definition.

$$e = \frac{v'_A - v'_B}{v_B - v_A} = -\frac{(v_A - v_B)}{v_B - v_A} = 1.$$

For a completely inelastic collision, $v'_A = v'_B$. Substitute that into the coefficient of restitution definition.

$$e = \frac{v'_A - v'_B}{v_B - v_A} = 0.$$

(*b*) Let A represent the falling object, and B represent the heavy steel plate. The speeds of the steel plate are $v_B = 0$ and $v'_B = 0$. Thus $e = -v'_A / v_A$. Consider energy conservation during the falling or rising path. The potential energy of body A at height *h* is transformed into kinetic energy just before it collides with the plate. Choose down to be the positive direction.

$$mgh = \tfrac{1}{2}mv_A^2 \;\rightarrow\; v_A = \sqrt{2gh}$$

The kinetic energy of body A immediately after the collision is transformed into potential energy as it rises. Also, since it is moving upwards, it has a negative velocity.

$$mgh' = \tfrac{1}{2}mv_A'^2 \;\rightarrow\; v'_A = -\sqrt{2gh'}$$

Substitute the expressions for the velocities into the definition of the coefficient of restitution.

$$e = -v'_A / v_A = -\frac{-\sqrt{2gh'}}{\sqrt{2gh}} \;\rightarrow\; \boxed{e = \sqrt{h'/h}}$$

38. Let A represent the more massive piece, and B the less massive piece. Thus $m_A = 3m_B$. In the explosion, momentum is conserved. We have $v_A = v_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \to \quad 0 = m_A v'_A + m_B v'_B = 3m_B v'_A + m_B v'_B \quad \to \quad v'_A = -\tfrac{1}{3} v'_B$$

For each block, the kinetic energy gained during the explosion is lost to negative work done by friction on the block.

$$W_{\text{fr}} = KE_f - KE_i = -\tfrac{1}{2} m v^2$$

But work is also calculated in terms of the force doing the work and the distance traveled.

$$W_{\text{fr}} = F_{\text{fr}} \Delta x \cos 180^\circ = -\mu_k F_N \Delta x = -\mu_k m g \Delta x$$

Equate the two work expressions, solve for the distance traveled, and find the ratio of distances.

$$-\tfrac{1}{2} m v^2 = -\mu_k m g \Delta x \quad \to \quad \Delta x = \frac{v^2}{g\mu_k} \qquad \frac{(\Delta x)_A}{(\Delta x)_B} = \frac{\dfrac{v_A'^2}{g\mu_k}}{\dfrac{v_B'^2}{g\mu_k}} = \frac{v_A'^2}{v_B'^2} = \frac{\left(-\tfrac{1}{3} v'_B\right)^2}{v_B'^2} = \frac{1}{9}$$

And so $\boxed{(\Delta x)_{\text{heavy}} / (\Delta x)_{\text{light}} = 1/9}$

39. In each case, use momentum conservation. Let A represent the 15.0-kg object, and let B represent the 10.0-kg object. We have $v_A = 5.5\,\text{m/s}$ and $v_B = -4.0\,\text{m/s}$.

(*a*) In this case, $v'_A = v'_B$.

$$m_A v_A + m_B v_B = (m_A + m_B) v'_A \quad \to$$

$$v'_B = v'_A = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(15.0\text{ kg})(5.5\,\text{m/s}) + (10.0\text{ kg})(-4.0\,\text{m/s})}{25.0\text{ kg}} = \boxed{1.7\,\text{m/s}}$$

(*b*) In this case, use Eq. 7-7 to find a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \to \quad v'_B = v_A - v_B + v'_A$$

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B = m_A v'_A + m_B (v_A + v_B + v'_A) \quad \to$$

$$v'_A = \frac{(m_A - m_B) v_A + 2 m_B v_B}{m_A + m_B} = \frac{(5.0\text{ kg})(5.5\,\text{m/s}) + 2(10.0\text{ kg})(-4.0\,\text{m/s})}{25.0\text{ kg}} = \boxed{-2.1\,\text{m/s}}$$

$$v'_B = v_A - v_B + v'_A = 5.5\,\text{m/s} - (-4.0\,\text{m/s}) - 2.1\,\text{m/s} = \boxed{7.4\,\text{m/s}}$$

(*c*) In this case, $v'_A = 0$.

$$m_A v_A + m_B v_B = m_B v'_B \quad \to$$

$$v'_B = \frac{m_A v_A + m_B v_B}{m_B} = \frac{(15.0\text{ kg})(5.5\,\text{m/s}) + (10.0\text{ kg})(-4.0\,\text{m/s})}{10.0\text{ kg}} = \boxed{4.3\,\text{m/s}}$$

To check for "reasonableness", first note the final directions of motion. A has stopped, and B has gone in the opposite direction. This is reasonable. Secondly, calculate the change in kinetic energy.

$$\Delta KE = \tfrac{1}{2} m_B v_B'^2 - \left(\tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2\right)$$

$$= \tfrac{1}{2}(10.0\text{ kg}) - \left[\tfrac{1}{2}(15.0\text{ kg})(5.5\,\text{m/s})^2 + \tfrac{1}{2}(10.0\text{ kg})(-4.0\,\text{m/s})^2\right] = -220\text{ J}$$

Since the system has lost kinetic energy and the directions are possible, this interaction is $\boxed{\text{"reasonable"}}$.

(*d*) In this case, $v'_B = 0$.

$$m_A v_A + m_B v_B = m_A v'_A \quad \rightarrow$$

$$v'_A = \frac{m_A v_A + m_B v_B}{m_A} = \frac{(15.0\text{ kg})(5.5\text{ m/s}) + (10.0\text{ kg})(-4.0\text{ m/s})}{15.0\text{ kg}} = \boxed{2.8\text{ m/s}}$$

This answer is not reasonable because it has A moving in its original direction while B has stopped. Thus A has somehow passed through B. If B has stopped, A should have rebounded in the negative direction.

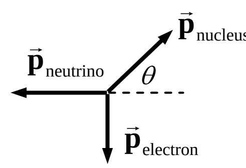
(*e*) In this case, $v'_A = -4.0\text{ m/s}$.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow$$

$$v'_B = \frac{(15.0\text{ kg})(5.5\text{ m/s} - -4.0\text{ m/s}) + (10.0\text{ kg})(-4.0\text{ m/s})}{10.0\text{ kg}} = \boxed{10.3\text{ m/s}}$$

The directions are reasonable, in that each object rebounds. However, the speed of both objects is larger than its speed in the perfectly elastic case (b). Thus the system has gained kinetic energy, and unless there is some other source adding energy, this is not reasonable.

40. Use this diagram for the momenta after the decay. Since there was no momentum before the decay, the three momenta shown must add to 0 in both the *x* and *y* directions.



$$(p_{nucleus})_x = p_{neutrino} \qquad (p_{nucleus})_y = p_{electron}$$

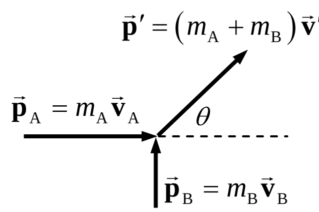
$$p_{nucleus} = \sqrt{(p_{nucleus})_x^2 + (p_{nucleus})_y^2} = \sqrt{(p_{neutrino})^2 + (p_{electron})^2}$$

$$= \sqrt{(5.40\times10^{-23}\text{ kg}\bullet\text{m/s})^2 + (9.30\times10^{-23}\text{ kg}\bullet\text{m/s})^2} = \boxed{1.08\times10^{-22}\text{ kg}\bullet\text{m/s}}$$

$$\theta = \tan^{-1}\frac{(p_{nucleus})_y}{(p_{nucleus})_x} = \tan^{-1}\frac{(p_{electron})}{(p_{neutrino})} = \tan^{-1}\frac{(9.30\times10^{-23}\text{ kg}\bullet\text{m/s})}{(5.40\times10^{-23}\text{ kg}\bullet\text{m/s})} = 59.9^\circ$$

The second nucleus' momentum is 150° from the momentum of the electron.

41. Consider the diagram for the momenta of the eagles. Momentum will be conserved in both the *x* and *y* directions.



$$p'_x = (m_A + m_B)v'_x = m_A v_A \quad \rightarrow \quad v'_x = \frac{m_A v_A}{m_A + m_B}$$

$$p'_y = (m_A + m_B)v'_y = m_B v_B \quad \rightarrow \quad v'_y = \frac{m_B v_B}{m_A + m_B}$$

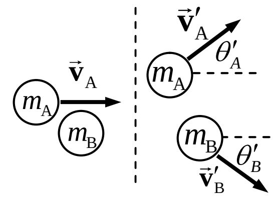
$$v' = \sqrt{v_x'^2 + v_y'^2} = \sqrt{\left(\frac{m_A v_A}{m_A + m_B}\right)^2 + \left(\frac{m_B v_B}{m_A + m_B}\right)^2} = \frac{\sqrt{(m_A v_A)^2 + (m_B v_B)^2}}{m_A + m_B}$$

$$= \frac{\sqrt{(4.3\text{ kg})^2(7.8\text{ m/s})^2 + (5.6\text{ kg})^2(10.2\text{ m/s})^2}}{4.3\text{ kg} + 5.6\text{ kg}} = \boxed{6.7\text{ m/s}}$$

$$\theta = \tan^{-1}\frac{v'_y}{v'_x} = \tan^{-1}\frac{\dfrac{m_B v_B}{m_A + m_B}}{\dfrac{m_A v_A}{m_A + m_B}} = \tan^{-1}\frac{m_B v_B}{m_A v_A} = \tan^{-1}\frac{(5.6\text{ kg})(10.2\text{ m/s})}{(4.3\text{ kg})(7.8\text{ m/s})} = \boxed{60^\circ \text{ rel. to eagle A}}$$

42. (*a*) $p_x: \quad m_A v_A = m_A v'_A \cos\theta'_A + m_B v'_B \cos\theta'_B$

$p_y: \quad 0 = m_A v'_A \sin\theta'_A - m_B v'_B \sin\theta'_B$



(*b*) Solve the *x* equation for $\cos\theta'_B$ and the *y* equation for $\sin\theta'_B$, and then find the angle from the tangent function.

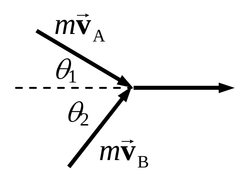
$$\tan\theta'_B = \frac{\sin\theta'_B}{\cos\theta'_B} = \frac{\dfrac{m_A v'_A \sin\theta'_A}{m_B v'_B}}{\dfrac{m_A\left(v_A - v'_A\cos\theta'_A\right)}{m_B v'_B}} = \frac{v'_A \sin\theta'_A}{\left(v_A - v'_A\cos\theta'_A\right)}$$

$$\theta'_B = \tan^{-1}\frac{v'_A\sin\theta'_A}{v_A - v'_A\cos\theta'_A} = \tan^{-1}\frac{(1.10\text{ m/s})\sin 30.0^\circ}{1.80\text{ m/s} - (1.10\text{ m/s})\cos 30.0^\circ} = \boxed{33.0^\circ}$$

With the value of the angle, solve the *y* equation for the velocity.

$$v'_B = \frac{m_A v'_A \sin\theta'_A}{m_B \sin\theta'_B} = \frac{(0.400\text{ kg})(1.10\text{ m/s})\sin 30.0^\circ}{(0.500\text{ kg})\sin 33.0^\circ} = \boxed{0.808\text{ m/s}}$$

43. Call the final direction of the joined objects the positive *x* axis. A diagram of the collision is shown. Momentum will be conserved in both the *x* and *y* directions. Note that $v_A = v_B = v$ and $v' = v/3$.



$p_y: \quad -mv\sin\theta_1 + mv\sin\theta_2 = 0 \quad\to\quad \sin\theta_1 = \sin\theta_2 \quad\to\quad \theta_1 = \theta_2$

$p_x: \quad mv\cos\theta_1 + mv\cos\theta_2 = (2m)(v/3) \quad\to\quad \cos\theta_1 + \cos\theta_2 = \frac{2}{3}$

$\cos\theta_1 + \cos\theta_2 = 2\cos\theta_1 = \frac{2}{3} \quad\to\quad \theta_1 = \cos^{-1}\frac{1}{3} = 70.5^\circ = \theta_2$

$\theta_1 + \theta_2 = \boxed{141^\circ}$

44. Write momentum conservation in the *x* and *y* directions, and KE conservation. Note that both masses are the same. We allow $\vec{\mathbf{v}}_A$ to have both *x* and *y* components.

$p_x: \quad mv_B = mv'_{Ax} \quad\to\quad v_B = v'_{Ax}$

$p_y: \quad mv_A = mv'_{Ay} + mv'_B \quad\to\quad v_A = v'_{Ay} + v'_B$

$KE: \quad \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 = \frac{1}{2}mv'^2_A + \frac{1}{2}mv'^2_B \quad\to\quad v_A^2 + v_B^2 = v'^2_A + v'^2_B$

Substitute the results from the momentum equations into the KE equation.

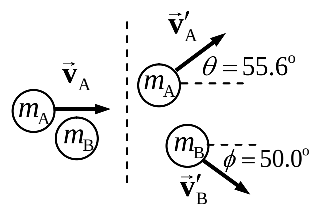
$\left(v'_{Ay} + v'_B\right)^2 + \left(v'_{Ax}\right)^2 = v'^2_A + v'^2_B \quad\to\quad v'^2_{Ay} + 2v'^2_{Ay}v'_B + v'^2_B + v'^2_{Ay} = v'^2_A + v'^2_B \quad\to$

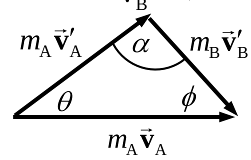
$v'^2_A + 2v'^2_{Ay}v'_B + v'^2_B = v'^2_A + v'^2_B \quad\to\quad 2v'^2_{Ay}v'_B = 0 \quad\to\quad v'_{Ay} = 0 \text{ or } v'_B = 0$

Since we are given that $v'_B \neq 0$, we must have $v'_{Ay} = 0$. This means that the final direction of A is the $\boxed{x\text{ direction}}$. Put this result into the momentum equations to find the final speeds.

$v'_A = v'_{Ax} = v_B = \boxed{3.7\text{ m/s}} \qquad v'_B = v_A = \boxed{2.0\text{ m/s}}$

45. Let A represent the incoming neon atom, and B represent the target atom. A momentum diagram of the collision looks like the first figure. The figure can be re-drawn as a triangle, the second figure, since $m_A \vec{\mathbf{v}}_A = m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B$. Write the law of sines for this triangle, relating each final momentum magnitude to the initial momentum magnitude.





$$\frac{m_A v'_A}{m_A v_A} = \frac{\sin\phi}{\sin\alpha} \quad \rightarrow \quad v'_A = v_A \frac{\sin\phi}{\sin\alpha}$$

$$\frac{m_B v'_B}{m_A v_A} = \frac{\sin\theta}{\sin\alpha} \quad \rightarrow \quad v'_B = v_A \frac{m_A}{m_B} \frac{\sin\theta}{\sin\alpha}$$

The collision is elastic, so write the KE conservation equation, and substitute the results from above. Also note that $\alpha = 180.0 - 55.6^o - 50.0^o = 74.4^o$

$$\tfrac{1}{2} m_A v_A^2 = \tfrac{1}{2} m_A v_A'^2 + \tfrac{1}{2} m_B v_B'^2 \quad \rightarrow \quad m_A v_A^2 = m_A \left(v_A \frac{\sin\phi}{\sin\alpha} \right)^2 + m_B \left(v_A \frac{m_A}{m_B} \frac{\sin\theta}{\sin\alpha} \right)^2 \quad \rightarrow$$

$$m_B = \frac{m_A \sin^2\theta}{\sin^2\alpha - \sin^2\phi} = \frac{(20.0\text{ u})\sin^2 55.6^o}{\sin^2 74.4 - \sin^2 50.0^o} = \boxed{39.9\text{ u}}$$

46. Use Eq. 7-9a, extended to three particles.

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(1.00\text{ kg})(0) + (1.50\text{ kg})(0.50\text{ m}) + (1.10\text{ kg})(0.75\text{ m})}{1.00\text{ kg} + 1.50\text{ kg} + 1.10\text{ kg}}$$

$$= \boxed{0.44\text{ m}}$$

47. Choose the carbon atom as the origin of coordinates.

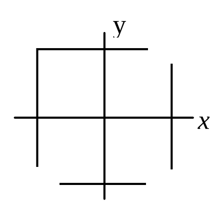
$$x_{CM} = \frac{m_C x_C + m_O x_O}{m_C + m_O} = \frac{(12\text{ u})(0) + (16\text{ u})(1.13\times10^{-10}\text{ m})}{12\text{ u} + 16\text{ u}} = \boxed{6.5\times10^{-11}\text{ m}} \text{ from the C atom.}$$

[48.] Find the CM relative to the front of the car.

$$x_{CM} = \frac{m_{car} x_{car} + m_{front} x_{front} + m_{back} x_{back}}{m_{car} + m_{front} + m_{back}}$$

$$= \frac{(1050\text{ kg})(2.50\text{ m}) + 2(70.0\text{ kg})(2.80\text{ m}) + 3(70.0\text{ kg})(3.90\text{ m})}{1050\text{ kg} + 2(70.0\text{ kg}) + 3(70.0\text{ kg})} = \boxed{2.74\text{ m}}$$

49. Consider this diagram of the cars on the raft. Notice that the origin of coordinates is located at the CM of the raft. Reference all distances to that location.



$$x_{CM} = \frac{(1200\text{ kg})(9\text{ m}) + (1200\text{ kg})(9\text{ m}) + (1200\text{ kg})(-9\text{ m})}{3(1200\text{ kg}) + 6800\text{ kg}} = \boxed{1.04\text{ m}}$$

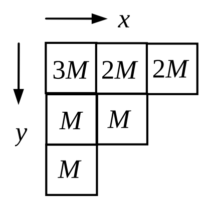
$$y_{CM} = \frac{(1200\text{ kg})(9\text{ m}) + (1200\text{ kg})(-9\text{ m}) + (1200\text{ kg})(-9\text{ m})}{3(1200\text{ kg}) + 6800\text{ kg}} = \boxed{-1.04\text{ m}}$$

50. By the symmetry of the problem, since the centers of the cubes are along a straight line, the vertical CM coordinate will be 0, and the depth CM coordinate will be 0. The only CM coordinate to calculate is the one along the straight line joining the centers. The mass of each cube will be the volume times the density, and so $m_1 = \rho(l_0)^3$, $m_2 = \rho(2l_0)^3$, $m_3 = \rho(3l_0)^3$. Measuring from the left edge of the smallest block, the locations of the CM's of the individual cubes are $x_1 = \frac{1}{2}l_0$, $x_2 = 2l_0$, $x_3 = 4.5l_0$. Use Eq. 7-9a to calculate the CM of the system.

$$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{\rho l_0^3\left(\frac{1}{2}l_0\right) + 8\rho l_0^3\left(2l_0\right) + 27\rho l_0^3\left(4.5l_0\right)}{\rho l_0^3 + 8\rho l_0^3 + 27\rho l_0^3}$$

$$= \boxed{3.8\,l_0 \text{ from the left edge of the smallest cube}}$$

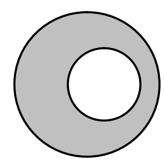
51. Let each crate have a mass M. A top view of the pallet is shown, with the total mass of each stack listed. Take the origin to be the back left corner of the pallet.



$$x_{CM} = \frac{(5M)(l/2) + (3M)(3l/2) + (2M)(5l/2)}{10M} = \boxed{1.2l}$$

$$y_{CM} = \frac{(7M)(l/2) + (2M)(3l/2) + (1M)(5l/2)}{10M} = \boxed{0.9l}$$

52. Consider the following. We start with a full circle of radius 2*R*, with its CM at the origin. Then we draw a circle of radius *R*, with its CM at the coordinates $(0.80R, 0)$. The full circle can now be labeled as a "gray" part and a "white" part. The *y* coordinate of the CM of the entire circle, the CM of the gray part, and the CM of the white part are all at $y = 0$ by the symmetry of the system. The *x* coordinate of the entire circle is at $x_{\text{CM}} = 0$, and can be calculated by $x_{\text{CM}} = \frac{m_{\text{gray}}x_{\text{gray}} + m_{\text{white}}x_{\text{white}}}{m_{\text{total}}}$. Rearrange this equation.



$$x_{\text{CM}} = \frac{m_{\text{gray}}x_{\text{gray}} + m_{\text{white}}x_{\text{white}}}{m_{\text{total}}} \rightarrow$$

$$x_{\text{gray}} = \frac{m_{\text{total}}x_{\text{CM}} - m_{\text{white}}x_{\text{white}}}{m_{\text{gray}}} = \frac{m_{\text{total}}x_{\text{CM}} - m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}} = \frac{-m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}}$$

This is functionally the same as treating the white part of the figure as a hole of negative mass. The mass of each part can be found by multiplying the area of the part times the uniform density of the plate.

$$x_{\text{gray}} = \frac{-m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}} = \frac{-\rho\pi R^2(0.80R)}{\rho\pi(2R)^2 - \rho\pi R^2} = \frac{-0.80R}{3} = \boxed{-0.27R}$$

53. Take the upper leg, lower leg, and foot all together. Note that Table 7-1 gives the relative mass of BOTH legs and feet, so a factor of 1/2 is needed. Assume a person of mass 70 kg.

$$(70\text{ kg})\frac{(21.5 + 9.6 + 3.4)}{100}\frac{1}{2} = \boxed{12\text{ kg}}.$$

54. With the shoulder as the origin of coordinates for measuring the center of mass, we have the following relative locations from Table 7-1 for the arm components, as percentages of the height. Down is positive.

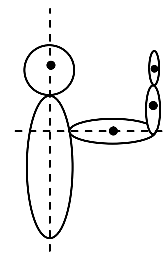
$$x_{\substack{\text{upper}\\\text{arm}}} = 81.2 - 7.7 = 9.5 \qquad x_{\substack{\text{lower}\\\text{arm}}} = 81.2 - 55.3 = 25.9 \qquad x_{\text{hand}} = 81.2 - 43.1 = 38.1$$

To find the CM, we can also use relative mass percentages. Since the expression includes the total mass in the denominator, there is no need to divide all masses by 2 to find single component masses. Simply use the relative mass percentages given in the table.

$$x_{\text{CM}} = \frac{x_{\substack{\text{upper}\\\text{arm}}} m_{\substack{\text{upper}\\\text{arm}}} + x_{\substack{\text{lower}\\\text{arm}}} m_{\substack{\text{lower}\\\text{arm}}} + x_{\text{hand}} m_{\text{hand}}}{m_{\substack{\text{upper}\\\text{arm}}} + m_{\substack{\text{lower}\\\text{arm}}} + m_{\text{hand}}} = \frac{(9.5)(6.6) + (25.9)(4.2) + (38.1)(1.7)}{6.6 + 4.2 + 1.7}$$

$$= \boxed{19\% \text{ of the person's height along the line from the shoulder to the hand}}$$

55. Take the shoulder to be the origin of coordinates. We assume that the arm is held with the upper arm parallel to the floor and the lower arm and hand extended upward. Measure *x* horizontally from the shoulder, and *y* vertically. Since the expression includes the total mass in the denominator, there is no need to divide all masses by 2 to find single component masses. Simply use the relative mass percentages given in the table.



$$x_{\text{CM}} = \frac{x_{\substack{\text{upper}\\\text{arm}}} m_{\substack{\text{upper}\\\text{arm}}} + x_{\substack{\text{lower}\\\text{arm}}} m_{\substack{\text{lower}\\\text{arm}}} + x_{\text{hand}} m_{\text{hand}}}{m_{\substack{\text{upper}\\\text{arm}}} + m_{\substack{\text{lower}\\\text{arm}}} + m_{\text{hand}}}$$

$$= \frac{(81.2 - 71.7)(6.6) + (81.2 - 62.2)(4.2 + 1.7)}{6.6 + 4.2 + 1.7} = 14.0$$

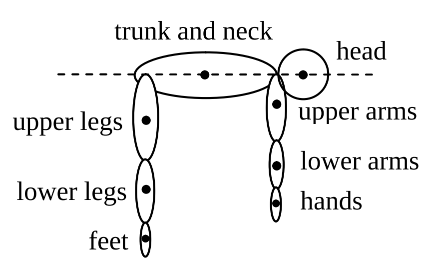
$$y_{\text{CM}} = \frac{y_{\substack{\text{upper}\\\text{arm}}} m_{\substack{\text{upper}\\\text{arm}}} + y_{\substack{\text{lower}\\\text{arm}}} m_{\substack{\text{lower}\\\text{arm}}} + y_{\text{hand}} m_{\text{hand}}}{m_{\substack{\text{upper}\\\text{arm}}} + m_{\substack{\text{lower}\\\text{arm}}} + m_{\text{hand}}}$$

$$= \frac{(0)(6.6) + (62.2 - 55.3)(4.2) + (62.2 - 43.1)(1.7)}{6.6 + 4.2 + 1.7} = 4.92$$

Convert the distance percentages to actual distance using the person's height.

$$x_{\text{CM}} = (14.0\%)(155\text{ cm}) = \boxed{21.7\text{ cm}} \qquad y_{\text{CM}} = (4.92\%)(155\text{ cm}) = \boxed{7.6\text{ cm}}$$

56. See the diagram of the person. The head, trunk, and neck are all lined up so that their CM's are on the torso's median line. Call down the positive *y* direction. The *y* distances of the CM of each body part from the median line, in terms of percentage of full height, are shown below, followed by the percentage each body part is of the full body mass.



On median line:	head (h):	0	;	6.9% body mass
	Trunk & neck (t n):	0	;	46.1% body mass
From shoulder hinge point:	upper arms (u a):	81.2 – 71.7 = 9.5	;	6.6% body mass
	lower arms (l a):	81.2 – 55.3 = 25.9	;	4.2% body mass
	hands (ha):	81.2 – 43.1 = 38.1	;	1.7% body mass

From hip hinge point:	upper legs (u l):	52.1 – 42.5 = 9.6 ;	21.5% body mass
	lower legs (l l):	52.1 – 18.2 = 33.9 ;	9.6% body mass
	feet (f):	52.1 – 1.8 = 50.3 ;	3.4% body mass

Using this data, calculate the vertical location of the CM.

$$y_{CM} = \frac{y_{\mathrm{h}} m_{\mathrm{h}} + y_{\mathrm{t\,n}} m_{\mathrm{t\,n}} + y_{\mathrm{u\,a}} m_{\mathrm{u\,a}} + y_{\mathrm{l\,a}} m_{\mathrm{l\,a}} + y_{\mathrm{ha}} m_{\mathrm{ha}} + y_{\mathrm{u\,l}} m_{\mathrm{u\,l}} + y_{\mathrm{l\,l}} m_{\mathrm{l\,l}} + y_{\mathrm{f}} m_{\mathrm{f}}}{m_{\substack{\mathrm{full}\\\mathrm{body}}}}$$

$$= \frac{0 + 0 + (9.5)(6.6) + (25.9)(4.2) + (38.1)(1.7) + (9.6)(21.5) + (33.9)(9.6) + (50.3)(3.4)}{100}$$

$$= 9.4$$

Thus the center of mass is $\boxed{\text{9.4\% of the full body height below the torso's median line}}$. For a person of height 1.7 m, this is about 16 cm. That is most likely slightly $\boxed{\text{outside the body}}$.

57. (*a*) Find the CM relative to the center of the Earth.

$$x_{CM} = \frac{m_E x_E + m_M x_M}{m_E + m_M} = \frac{(5.98\times10^{24}\,\mathrm{kg})(0) + (7.35\times10^{22}\,\mathrm{kg})(3.84\times10^{8}\,\mathrm{m})}{5.98\times10^{24}\,\mathrm{kg} + 7.35\times10^{22}\,\mathrm{kg}}$$

$$= \boxed{4.66\times10^{6}\,\mathrm{m\ from\ the\ center\ of\ the\ Earth}}$$

This is actually inside the volume of the Earth, since $R_E = 6.38\times10^{6}\,\mathrm{m}$

(*b*) It is this Earth – Moon CM location that actually traces out the orbit as discussed in chapter 5. The Earth and Moon will orbit about this location in (approximately) circular orbits. The motion of the Moon, for example, around the Sun would then be a sum of two motions: i) the motion of the Moon about the Earth – Moon CM; and ii) the motion of the Earth – Moon CM about the Sun. To an external observer, the Moon's motion would appear to be a small radius, higher frequency circular motion (motion about the Earth – Moon CM) combined with a large radius, lower frequency circular motion (motion about the Sun).

58. (*a*) Measure all distances from the original position of the woman.

$$x_{\mathrm{CM}} = \frac{m_{\mathrm{W}} x_{\mathrm{W}} + m_{\mathrm{M}} x_{\mathrm{M}}}{m_{\mathrm{W}} + m_{\mathrm{M}}} = \frac{(55\ \mathrm{kg})(0) + (80\ \mathrm{kg})(10.0\ \mathrm{m})}{135\ \mathrm{kg}} = \boxed{5.9\ \mathrm{m\ from\ the\ woman}}$$

(*b*) Since there is no force external to the man-woman system, the CM will not move, relative to the original position of the woman. The woman's distance will no longer be 0, and the man's distance has changed to 7.5 m.

$$x_{\mathrm{CM}} = \frac{m_{\mathrm{W}} x_{\mathrm{W}} + m_{\mathrm{M}} x_{\mathrm{M}}}{m_{\mathrm{W}} + m_{\mathrm{M}}} = \frac{(55\ \mathrm{kg}) x_{\mathrm{W}} + (80\ \mathrm{kg})(7.5\ \mathrm{m})}{135\ \mathrm{kg}} = 5.9\ \mathrm{m} \rightarrow$$

$$x_{\mathrm{W}} = \frac{(5.9\ \mathrm{m})(135\ \mathrm{kg}) - (80\ \mathrm{kg})(7.5\ \mathrm{m})}{55\ \mathrm{kg}} = 3.6\ \mathrm{m}$$

$$x_{\mathrm{M}} - x_{\mathrm{W}} = 7.5\ \mathrm{m} - 3.6\ \mathrm{m} = \boxed{3.9\ \mathrm{m}}$$

(*c*) When the man collides with the woman, he will be at the original location of the center of mass.

$$x_{\substack{\mathrm{M}\\\mathrm{final}}} - x_{\substack{\mathrm{M}\\\mathrm{initial}}} = 5.9\,\mathrm{m} - 10.0\ \mathrm{m} = -4.1\ \mathrm{m}$$

He has moved $\boxed{4.1\ \mathrm{m}}$ from his original position.

59. The point that will follow a parabolic trajectory is the center of mass. Find the CM relative to the bottom of the mallet. Each part of the hammer (handle and head) can be treated as a point mass located at the CM of the respective piece. So the CM of the handle is 12.0 cm from the bottom of the handle, and the CM of the head is 28.0 cm from the bottom of the handle.

$$x_{CM} = \frac{m_{handle}x_{handle} + m_{head}x_{head}}{m_{handle} + m_{head}} = \frac{(0.500 \text{ kg})(24.0 \text{ cm}) + (2.00 \text{ kg})(28.0 \text{ cm})}{2.50 \text{ kg}} = \boxed{24.8 \text{ cm}}$$

Note that this is inside the head of the mallet.

60. The CM of the system will follow the same path regardless of the way the mass splits, and so will still be $2d$ from the launch point when the parts land. Assume that the explosion is designed so that m_I still is stopped in midair and falls straight down.

(*a*) $$x_{CM} = \frac{m_I x_I + m_{II} x_{II}}{m_I + m_{II}} \rightarrow 2d = \frac{m_I d + 3m_I x_{II}}{4m_I} = \frac{d + 3x_{II}}{4} \rightarrow x_{II} = \boxed{\tfrac{7}{3}d}$$

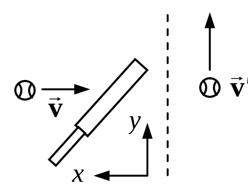
(*b*) $$x_{CM} = \frac{m_I x_I + m_{II} x_{II}}{m_I + m_{II}} \rightarrow 2d = \frac{3m_{II} d + m_{II} x_{II}}{4m_{II}} = \frac{3d + x_{II}}{4} \rightarrow x_{II} = \boxed{5d}$$

61. Call the origin of coordinates the CM of the balloon, gondola, and person at rest. Since the CM is at rest, the total momentum of the system relative to the ground is 0. The man climbing the rope cannot change the total momentum of the system, and so the CM must stay at rest. Call the upward direction positive. Then the velocity of the man with respect to the balloon is $-v$. Call the velocity of the balloon with respect to the ground v_{BG}. Then the velocity of the man with respect to the ground is $v_{MG} = -v + v_{BG}$. Apply Eq. 7-10.

$$0 = mv_{MG} + Mv_{BG} = m(-v + v_{BG}) + Mv_{BG} \rightarrow \boxed{v_{BG} = v\frac{m}{m+M} \text{, upward}}$$

If the passenger stops, the balloon also stops, and the CM of the system remains at rest.

62. To find the average force, divide the change in momentum by the time over which the momentum changes. Choose the *x* direction to be the opposite of the baseball s incoming direction. The velocity with which the ball is moving after hitting the bat can be found from conservation of energy, and knowing the height the ball rises.



$$\left(KE_{initial} = PE_{final}\right)_{\substack{after\\collision}} \rightarrow \tfrac{1}{2}mv'^2 = mg\Delta y \rightarrow$$

$$v' = \sqrt{2g\Delta y} = \sqrt{2(9.80 \text{ m/s}^2)(55.6 \text{ m})} = 33.0 \text{ m/s}$$

The average force can be calculated from the change in momentum and the time of contact.

$$\bar{F}_x = \frac{\Delta p_x}{\Delta t} = \frac{m(v'_x - v_x)}{\Delta t} = \frac{(0.145 \text{ kg})(0 - -35.0 \text{ m/s})}{1.4\times10^{-3}\text{s}} = 3.6\times10^3 \text{ N}$$

$$\bar{F}_y = \frac{\Delta p_y}{\Delta t} = \frac{m(v'_y - v_y)}{\Delta t} = \frac{(0.145 \text{ kg})(33.0 \text{ m/s} - 0)}{1.4\times10^{-3}\text{s}} = 3.4\times10^3 \text{ N}$$

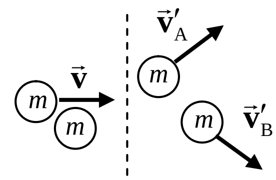
$$\bar{F} = \sqrt{\bar{F}_x^2 + \bar{F}_y^2} = \boxed{5.0\times10^3 \text{ N}} \qquad \theta = \tan^{-1}\frac{\bar{F}_y}{\bar{F}_x} = \boxed{43^\circ}$$

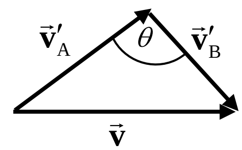
63. Momentum will be conserved in two dimensions. The fuel was ejected in the *y* direction as seen from the ground, and so the fuel had no *x*-component of velocity.

$$p_x: \quad m_{\text{rocket}} v_0 = \left(m_{\text{rocket}} - m_{\text{fuel}}\right) v_x' + m_{\text{fuel}} 0 = \tfrac{2}{3} m_{\text{rocket}} v_x' \quad \rightarrow \quad \boxed{v_x' = \tfrac{3}{2} v_0}$$

$$p_y: \quad 0 = m_{\text{fuel}} v_{\text{fuel}} + \left(m_{\text{rocket}} - m_{\text{fuel}}\right) v_y' = \tfrac{1}{3} m_{\text{rocket}} \left(2v_0\right) + \tfrac{2}{3} m_{\text{rocket}} v_y' \quad \rightarrow \quad \boxed{v_y' = -v_0}$$

64. In an elastic collision between two objects of equal mass, with the target object initially stationary, the angle between the final velocities of the objects is 90°. Here is a proof of that fact. Momentum conservation as a vector relationship says $m\vec{v} = m\vec{v}_A' + m\vec{v}_B' \rightarrow \vec{v} = \vec{v}_A' + \vec{v}_B'$. Kinetic energy conservation says $\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_A'^2 + \tfrac{1}{2}mv_B'^2 \rightarrow v^2 = v_A'^2 + v_B'^2$. The vector equation resulting from momentum conservation can be illustrated by the second diagram. Apply the law of cosines to that triangle of vectors, and then equate the two expressions for v^2.

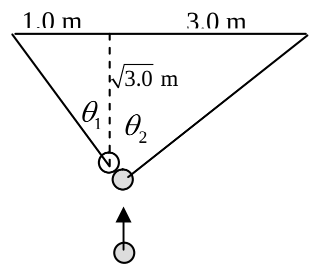




$$v^2 = v_A'^2 + v_B'^2 - 2v_A' v_B' \cos\theta$$

Equating the two expressions for v^2 gives

$$v_A'^2 + v_B'^2 - 2v_A' v_B' \cos\theta = v_A'^2 + v_B'^2 \quad \rightarrow \quad \cos\theta = 0 \quad \rightarrow \quad \theta = 90^\circ$$



For this specific circumstance, see the third diagram. We assume that the target ball is hit "correctly" so that it goes in the pocket. Find θ_1 from the geometry of the "left' triangle: $\theta_1 = \tan^{-1}\dfrac{1.0}{\sqrt{3.0}} = 30^\circ$. Find θ_2 from the geometry of the "right" triangle: $\theta_2 = \tan^{-1}\dfrac{3.0}{\sqrt{3.0}} = 60^\circ$. Since the balls will separate at a 90° angle, if the target ball goes in the pocket, this does appear to be a $\boxed{\text{good possibility of a scratch shot}}$.

65. (*a*) The momentum of the astronaut – space capsule combination will be conserved since the only forces are "internal" to that system. Let A represent the astronaut and B represent the space capsule, and let the direction the astronaut moves be the positive direction. Due to the choice of reference frame, $v_A = v_B = 0$. We also have $v_A' = 2.50\,\text{m/s}$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A + m_B v_B = 0 = m_A v_A' + m_B v_B' \quad \rightarrow$$

$$v_B' = -v_A' \frac{m_A}{m_B} = -\left(2.50\,\text{m/s}\right)\frac{140\ \text{kg}}{1800\ \text{kg}} = \boxed{-0.194\,\text{m/s}}$$

The negative sign indicates that the space capsule is moving in the opposite direction to the astronaut.

(*b*) The average force on the astronaut is the astronaut's change in momentum, divided by the time of interaction.

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{m\left(v_A' - v_A\right)}{\Delta t} = \frac{\left(140\ \text{kg}\right)\left(2.50\,\text{m/s} - 0\right)}{0.40\ \text{s}} = \boxed{8.8 \times 10^2\,\text{N}}$$

66. Since the only forces on the astronauts are internal to the 2-astronaut system, their CM will not change. Call the CM location the origin of coordinates. That is also the original location of the two astronauts.

$$x_{\text{CM}} = \frac{m_{\text{A}} x_{\text{A}} + m_{\text{B}} x_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} \rightarrow 0 = \frac{(60\text{ kg})(12\text{ m}) + (80\text{ kg})x_{\text{B}}}{140\text{ kg}} \rightarrow x = -9\text{ m}$$

Their distance apart is $x_{\text{A}} - x_{\text{B}} = 12\text{ m} - (-9\text{m}) = \boxed{21\text{ m}}$.

67. Let A represent the incoming ball, and B represent the target ball. We have $v_{\text{B}} = 0$ and $v'_{\text{A}} = -\frac{1}{4}v_{\text{A}}$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_{\text{A}} - v_{\text{B}} = -(v'_{\text{A}} - v'_{\text{B}}) \rightarrow v'_{\text{B}} = v_{\text{A}} + v'_{\text{A}} = \tfrac{3}{4}v_{\text{A}}$$

Substitute this relationship into the momentum conservation equation for the collision.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_{\text{A}}v_{\text{A}} = m_{\text{A}}v'_{\text{A}} + m_{\text{B}}v'_{\text{B}} = m_{\text{A}}\left(-\tfrac{1}{4}v_{\text{A}}\right) + m_{\text{B}}\left(\tfrac{3}{4}v_{\text{A}}\right) \rightarrow \boxed{m_{\text{B}} = \tfrac{5}{3}m_{\text{A}}}$$

68. We assume that all motion is along a single direction. The distance of sliding can be related to the change in the kinetic energy of a car, as follows.

$$W_{\text{fr}} = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) \qquad W_{\text{fr}} = F_{\text{fr}}\Delta x\cos 180^\circ\theta = -\mu_k F_{\text{N}}\Delta x = -\mu_k mg\Delta x \rightarrow$$

$$-\mu_k g\Delta x = \tfrac{1}{2}\left(v_f^2 - v_i^2\right)$$

For post-collision sliding, $v_f = 0$ and v_i is the speed immediately after the collision, v'. Use this relationship to find the speed of each car immediately after the collision.

$$\text{Car A:} \quad -\mu_k g\Delta x'_{\text{A}} = -\tfrac{1}{2}v'^2_{\text{A}} \rightarrow v'_{\text{A}} = \sqrt{2\mu_k g\Delta x'_{\text{A}}} = \sqrt{2(0.60)(9.8\text{ m/s}^2)(18\text{ m})} = 14.55\text{ m/s}$$

$$\text{Car B:} \quad -\mu_k g\Delta x'_{\text{B}} = -\tfrac{1}{2}v'^2_{\text{B}} \rightarrow v'_{\text{B}} = \sqrt{2\mu_k g\Delta x'_{\text{B}}} = \sqrt{2(0.60)(9.8\text{ m/s}^2)(30\text{ m})} = 18.78\text{ m/s}$$

During the collision, momentum is conserved in one dimension. Note that $v_{\text{B}} = 0$.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_{\text{A}}v_{\text{A}} = m_{\text{A}}v'_{\text{A}} + m_{\text{B}}v'_{\text{B}}$$

$$v_A = \frac{m_{\text{A}}v'_{\text{A}} + m_{\text{B}}v'_{\text{B}}}{m_{\text{A}}} = \frac{(1900\text{ kg})(14.55\text{ m/s}) + (1100\text{ kg})(18.78\text{ m/s})}{1900\text{ kg}} = 25.42\text{ m/s}$$

For pre-collision sliding, again apply the friction – energy relationship, with $v_f = v_{\text{A}}$ and v_i is the speed when the brakes were first applied.

$$-\mu_k g\Delta x_{\text{A}} = \tfrac{1}{2}\left(v_{\text{A}}^2 - v_i^2\right) \rightarrow v_i = \sqrt{v_A^2 + 2\mu_k g\Delta x_A} = \sqrt{(25.42\text{ m/s})^2 + 2(0.60)(9.8\text{ m/s}^2)(15\text{ m})}$$

$$= 28.68\text{ m/s}\left(\frac{1\text{ mi/h}}{0.447\text{ m/s}}\right) = \boxed{64\text{ mi/h}}$$

69. Because all of the collisions are perfectly elastic, no energy is lost in the collisions. With each collision, the horizontal velocity is constant, and the vertical velocity reverses direction. So, after each collision, the ball rises again to the same height from which it dropped. Thus, after five bounces, the bounce height will be $\boxed{4.00\text{ m}}$, the same as the starting height.

70. This is a ballistic "pendulum" of sorts, similar to Example 7-10 in the textbook. There is no difference in the fact that the block and bullet are moving vertically instead of horizontally. The collision is still totally inelastic and conserves momentum, and the energy is still conserved in the

rising of the block and embedded bullet after the collision. So we simply quote the equation from that example.

$$v = \frac{m+M}{m}\sqrt{2gh} \quad \rightarrow$$

$$h = \frac{1}{2g}\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2\left(9.80\,\text{m/s}^2\right)}\left(\frac{\left(0.0290\,\text{kg}\right)\left(510\,\text{m/s}\right)}{0.0290\,\text{kg}+1.40\,\text{kg}}\right)^2 = \boxed{5.47\text{ m}}$$

71. This is a ballistic "pendulum" of sorts, similar to Example 7-10 in the textbook. Momentum is conserved in the totally inelastic collision, and so $mv = \left(m+M\right)v'$. The kinetic energy present immediately after the collision is lost due to negative work being done by friction.

$$W_{\text{fr}} = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right)_{\substack{\text{after}\\ \text{collision}}} \qquad W_{\text{fr}} = F_{\text{fr}}\Delta x\cos 180^{\circ}\theta = -\mu_k F_{\text{N}}\Delta x = -\mu_k mg\Delta x \quad \rightarrow$$

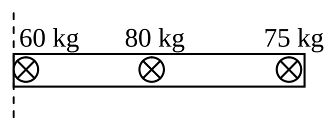
$$-\mu_k g\Delta x = \tfrac{1}{2}\left(v_f^2 - v_i^2\right) = -\tfrac{1}{2}v'^2 \quad \rightarrow \quad v' = \sqrt{2\mu_k g\Delta x}$$

Use this expression for v' in the momentum equation in order to solve for v.

$$mv = \left(m+M\right)v' = \left(m+M\right)\sqrt{2\mu_k g\Delta x} \quad \rightarrow$$

$$v = \left(\frac{m+M}{m}\right)\sqrt{2\mu_k g\Delta x} = \left(\frac{0.025\text{ kg}+1.35\text{ kg}}{0.025\text{ kg}}\right)\sqrt{2\left(0.25\right)\left(9.8\,\text{m/s}^2\right)\left(9.5\text{ m}\right)} = \boxed{3.8\times10^2\text{ m/s}}$$

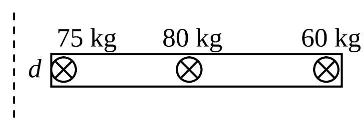
72. Calculate the CM relative to the 60-kg person's seat, at one end of the boat. See the first diagram. Don't forget to include the boat's mass.



$$x_{\text{CM}} = \frac{m_{\text{A}}x_{\text{A}} + m_{\text{B}}x_{\text{B}} + m_{\text{C}}x_{\text{C}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}$$

$$= \frac{\left(60\text{ kg}\right)\left(0\right) + \left(80\text{ kg}\right)\left(1.6\text{ m}\right) + \left(75\text{ kg}\right)\left(3.2\text{ m}\right)}{215\text{ kg}} = 1.712\text{ m}$$

Now, when the passengers exchange positions, the boat will move some distance "*d*" as shown, but the CM will not move. We measure the location of the CM from the same place as before, but now the boat has moved relative to that origin.



$$x_{\text{CM}} = \frac{m_{\text{A}}x_{\text{A}} + m_{\text{B}}x_{\text{B}} + m_{\text{C}}x_{\text{C}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}$$

$$1.712\text{ m} = \frac{\left(75\text{ kg}\right)\left(d\right) + \left(80\text{ kg}\right)\left(1.6\text{ m}+d\right) + \left(60\text{ kg}\right)\left(3.2\text{ m}+d\right)}{215\text{ kg}} = \frac{215d\text{ kg}\bullet\text{m} + 320\text{ kg}\bullet\text{m}}{215\text{ kg}}$$

$$d = 0.224\text{ m}$$

Thus the boat will move $\boxed{\text{0.22 m towards the initial position of the 75 kg person}}$.

73. (*a*) The meteor striking and coming to rest in the Earth is a totally inelastic collision. Let A represent the Earth and B represent the meteor. Use the frame of reference in which the Earth is at rest before the collision, and so $v_{\text{A}} = 0$. Write momentum conservation for the collision.

$$m_{\text{B}}v_{\text{B}} = \left(m_{\text{A}} + m_{\text{B}}\right)v' \quad \rightarrow$$

$$v' = v_B \frac{m_B}{m_A + m_B} = \left(1.5\times10^4\ \text{m/s}\right)\frac{1.0\times10^8\,\text{kg}}{6.0\times10^{24}\,\text{kg} + 1.0\times10^8\,\text{kg}} = \boxed{2.5\times10^{-13}\ \text{m/s}}$$

(*b*) The fraction of the meteor's KE transferred to the Earth is the final KE of the Earth divided by the initial KE of the meteor.

$$\frac{KE_{\substack{\text{final}\\\text{Earth}}}}{KE_{\substack{\text{initial}\\\text{meteor}}}} = \frac{\frac{1}{2}m_A v'^2}{\frac{1}{2}m_B v_B^2} = \frac{\frac{1}{2}\left(6.0\times10^{24}\,\text{kg}\right)\left(2.5\times10^{-13}\ \text{m/s}\right)^2}{\frac{1}{2}\left(1.0\times10^8\,\text{kg}\right)\left(1.5\times10^4\ \text{m/s}\right)^2} = \boxed{1.7\times10^{-17}}$$

(*c*) The Earth's change in KE can be calculated directly.

$$\Delta KE_{\text{Earth}} = KE_{\substack{\text{final}\\\text{Earth}}} - KE_{\substack{\text{initial}\\\text{Earth}}} = \tfrac{1}{2}m_A v'^2 - 0 = \tfrac{1}{2}\left(6.0\times10^{24}\,\text{kg}\right)\left(2.5\times10^{-13}\ \text{m/s}\right) = \boxed{0.19\ \text{J}}$$

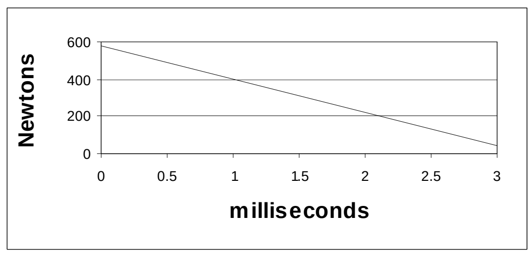
74. Momentum will be conserved in one dimension in the explosion. Let A represent the fragment with the larger KE.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_A v'_A + m_B v'_B \rightarrow v'_B = -\frac{m_A v'_A}{m_B}$$

$$KE_A = 2KE_B \rightarrow \tfrac{1}{2}m_A v_A'^2 = 2\left(\tfrac{1}{2}m_B v_B'^2\right) = m_B\left(-\frac{m_A v'_A}{m_B}\right)^2 \rightarrow \frac{m_A}{m_B} = \boxed{\frac{1}{2}}.$$

The fragment with the larger KE energy has half the mass of the other fragment.

75. (*a*) The force is linear, with a maximum force of 580 N at 0 seconds, and a minimum force of 40 N at 3 milliseconds.



(*b*) The impulse given the bullet is the "area" under the F vs. t graph. The area is trapezoidal.

$$\text{Impulse} = \left(\frac{580\ \text{N} + 40\ \text{N}}{2}\right)\left(3.0\times10^{-3}\text{s}\right) = \boxed{0.93\text{N}\bullet\text{s}}$$

(*c*) The impulse given the bullet is the change in momentum of the bullet. The starting speed of the bullet is 0.

$$\text{Impulse} = \Delta p = m\left(v - v_0\right) \rightarrow m = \frac{\text{Impulse}}{v} = \frac{0.93\,\text{N}\bullet\text{s}}{220\,\text{m/s}} = \boxed{4.2\times10^{-3}\text{kg}}$$

76. For the swinging balls, their velocity at the bottom of the swing and the height to which they rise are related by conservation of energy. If the zero of gravitational potential energy is taken to be the lowest point of the swing, then the kinetic energy at the low point is equal to the potential energy at the highest point of the swing, where the speed is zero. Thus we have $\frac{1}{2}mv_{\text{bottom}}^2 = mgh$ for any swinging ball, and so the relationship between speed and height is $v_{\text{bottom}}^2 = 2gh$.

(*a*) Calculate the speed of the lighter ball at the bottom of its swing.

$$v_A = \sqrt{2gh_A} = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(0.30\ \text{m} - 0.30\ \text{m}\cos 60^\circ\right)} = 1.715\,\text{m/s} \approx \boxed{1.7\,\text{m/s}}$$

(*b*) Assume that the collision is elastic, and use the results of problem 30. Take the direction that ball A is moving just before the collision as the positive direction.

$$v'_A = \frac{(m_A - m_B)}{(m_A + m_B)} v_A = \frac{(0.040 \text{ kg} - 0.060 \text{ kg})}{(0.040 \text{ kg} + 0.060 \text{ kg})}(1.715\,\text{m/s}) = -0.343\,\text{m/s} \approx \boxed{-0.34\,\text{m/s}}$$

$$v'_B = \frac{2m_A}{(m_A + m_B)} v_A = \frac{2(0.040 \text{ kg})}{(0.040 \text{ kg} + 0.060 \text{ kg})}(1.715\,\text{m/s}) = 1.372\,\text{m/s} \approx \boxed{1.4\,\text{m/s}}$$

Notice that ball A has rebounded backwards.

(*c*) After each collision, use the conservation of energy relationship again.

$$h'_A = \frac{v'^2_A}{2g} = \frac{(-0.343\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = \boxed{6.0\times10^{-3}\,\text{m}} \qquad h'_B = \frac{v'^2_B}{2g} = \frac{(1.372\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = \boxed{9.6\times10^{-2}\,\text{m}}$$

77. Use conservation of momentum in one dimension, since the particles will separate and travel in opposite directions. Let A represent the alpha particle, and B represent the smaller nucleus. Call the direction of the alpha particle's motion the positive direction. We have $m_B = 57m_A$, $v_A = v_B = 0$, and $v'_A = 3.8\times10^5$ m/s.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_A v'_A + m_B v'_B \rightarrow$$

$$v'_B = -\frac{m_A v'_A}{m_B} = -\frac{v'_A}{57} = -\frac{3.8\times10^5 \text{ m/s}}{57} = -6.7\times10^3 \text{ m/s} \;,\; |v'_B| = \boxed{6.7\times10^3 \text{ m/s}}$$

The negative sign indicates that the nucleus is moving in the opposite direction of the alpha particle.

78. The original horizontal distance can be found from the range formula from Example 3-8.

$$R = v_0^2 \sin 2\theta_0 / g = (25\,\text{m/s})^2 (\sin 60^\circ)/(9.8\,\text{m/s}^2) = 55.2 \text{ m}$$

The height at which the objects collide can be found from Eq. 2-11c for the vertical motion, with $v_y = 0$ at the top of the path. Take up to be positive.

$$v_y^2 = v_{y0}^2 + 2a(y - y_0) \rightarrow (y - y_0) = \frac{v_y^2 - v_{y0}^2}{2a} = \frac{0 - [(25\,\text{m/s})\sin 30^\circ]^2}{2(-9.8\,\text{m/s}^2)} = 7.97 \text{ m}$$

Let *m* represent the bullet and *M* the skeet. When the objects collide, the skeet is moving horizontally at $v_0 \cos\theta = (25\,\text{m/s})\cos 30^\circ = 21.65\,\text{m/s} = v_x$, and the bullet is moving vertically at $v_y = 200\,\text{m/s}$. Write momentum conservation in both directions to find the velocities after the totally inelastic collision.

$$p_x: \quad Mv_x = (M+m)v'_x \rightarrow v'_x = \frac{Mv_x}{M+m} = \frac{(0.25 \text{ kg})(21.65\,\text{m/s})}{(0.25+0.015)\,\text{kg}} = 20.42\,\text{m/s}$$

$$p_y: \quad mv_y = (M+m)v'_y \rightarrow v'_y = \frac{mv_y}{M+m} = \frac{(0.015 \text{ kg})(200\,\text{m/s})}{(0.25+0.015)\,\text{kg}} = 11.32\,\text{m/s}$$

(*a*) The speed v'_y can be used as the starting vertical speed in Eq. 2-11c to find the height that the skeet-bullet combination rises above the point of collision.

$$v_y^2 = v_{y0}^2 + 2a(y - y_0)_{\text{extra}} \rightarrow (y - y_0)_{\text{extra}} = \frac{v_y^2 - v_{y0}^2}{2a} = \frac{0 - (11.32\,\text{m/s})^2}{2(-9.8\,\text{m/s}^2)} = \boxed{6.5 \text{ m}}$$

(*b*) From Eq. 2-11b applied to the vertical motion after the collision, we can find the time for the skeet-bullet combination to reach the ground.

$$y = y_0 + v'_y t + \tfrac{1}{2}at^2 \quad \rightarrow \quad 0 = 7.97\text{ m} + (11.32\,\text{m/s})t + \tfrac{1}{2}(-9.8\,\text{m/s}^2)t^2 \quad \rightarrow$$

$$4.9t^2 - 11.32t - 7.97 = 0 \quad \rightarrow \quad t = 2.88\text{ s},\ -0.565\text{ s}$$

The positive time root is used to find the horizontal distance traveled by the combination after the collision.

$$x_{\text{after}} = v'_x t = (20.42\,\text{m/s})(2.88\text{ s}) = 58.7\text{ m}$$

If the collision would not have happened, the skeet would have gone $\tfrac{1}{2}R$ horizontally.

$$\Delta x = x_{\text{after}} - \tfrac{1}{2}R = 58.7\text{ m} - \tfrac{1}{2}(55.2\text{ m}) = 31.1\text{ m} \approx \boxed{31\text{ m}}$$

79. (*a*) Use conservation of energy to find the speed of mass *m* before the collision. The potential energy at the starting point is all transformed into kinetic energy just before the collision.

$$mgh_{\text{A}} = \tfrac{1}{2}mv_{\text{A}}^2 \quad \rightarrow \quad v_{\text{A}} = \sqrt{2gh_{\text{A}}} = \sqrt{2(9.80\,\text{m/s}^2)(3.60\text{ m})} = 8.40\,\text{m/s}$$

Use Eq. 7-7 to obtain a relationship between the velocities, noting that $v_B = 0$.

$$v_{\text{A}} - v_{\text{B}} = v'_{\text{B}} - v'_{\text{A}} \quad \rightarrow \quad v'_{\text{B}} = v'_{\text{A}} + v_{\text{A}}$$

Apply momentum conservation for the collision, and substitute the result from Eq. 7-7.

$$mv_{\text{A}} = mv'_{\text{A}} + Mv'_{\text{B}} = mv'_{\text{A}} + M\left(v_{\text{A}} + v'_{\text{A}}\right) \quad \rightarrow$$

$$v'_{\text{A}} = \frac{m-M}{m+M}v_{\text{A}} = \left(\frac{2.20\text{ kg} - 7.00\text{ kg}}{9.20\text{ kg}}\right)(8.4\,\text{m/s}) = -4.38\,\text{m/s} \approx \boxed{-4.4\,\text{m/s}}$$

$$v'_{\text{B}} = v'_{\text{A}} + v_{\text{A}} = -4.4\,\text{m/s} + 8.4\,\text{m/s} = \boxed{4.0\,\text{m/s}}$$

(*b*) Again use energy conservation to find the height to which mass *m* rises after the collision. The kinetic energy of *m* immediately after the collision is all transformed into potential energy. Use the angle of the plane to change the final height into a distance along the incline.

$$\tfrac{1}{2}mv'^2_{\text{A}} = mgh'_{\text{A}} \quad \rightarrow \quad h'_{\text{A}} = \frac{v'^2_{\text{A}}}{2g}$$

$$d'_{\text{A}} = \frac{h'_{\text{A}}}{\sin 30} = \frac{v'^2_{\text{A}}}{2g\sin 30} = \frac{(-4.38\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)g\sin 30} = 1.96\text{ m} \approx \boxed{2.0\text{ m}}$$

80. Let A represent mass *m*, and B represent mass *M*. Use Eq. 7-7 to obtain a relationship between the velocities, noting that $v_B = 0$.

$$v_{\text{A}} - v_{\text{B}} = v'_{\text{B}} - v'_{\text{A}} \quad \rightarrow \quad v'_{\text{A}} = v'_{\text{B}} - v_{\text{A}}.$$

After the collision, v'_{A} will be negative since *m* is moving in the negative direction. For there to be a second collision, then after *m* moves up the ramp and comes back down, with a positive velocity at the bottom of the incline of $-v'_{\text{A}}$, the speed of *m* must be greater than the speed of *M* so that *m* can catch *M*. Thus $-v'_{\text{A}} > v'_{\text{B}}$, or $v'_{\text{A}} < -v'_{\text{B}}$. Substitute the result from Eq. 7-7 into the inequality.

$$v'_{\text{B}} - v_{\text{A}} < -v'_{\text{B}} \quad \rightarrow \quad v'_{\text{B}} < \tfrac{1}{2}v_{\text{A}}.$$

Now write momentum conservation for the original collision, and substitute the result from Eq. 7-7.

$$mv_{\text{A}} = mv'_{\text{A}} + Mv'_{\text{B}} = m\left(v'_{\text{B}} - v_{\text{A}}\right) + Mv'_{\text{B}} \quad \rightarrow \quad v'_{\text{B}} = \frac{2m}{m+M}v_{\text{A}}$$

Finally, combine the above result with the inequality from above.

$$\frac{2m}{m+M}v_A < \tfrac{1}{2}v_A \quad \rightarrow \quad 4m < m+M \quad \rightarrow \quad \boxed{m < \tfrac{1}{3}M = 2.33\,\text{kg}}$$

81. The interaction between the planet and the spacecraft is elastic, because the force of gravity is conservative. Thus kinetic energy is conserved in the interaction. Consider the problem a 1-dimensional collision, with A representing the spacecraft and B representing Saturn. Because the mass of Saturn is so much bigger than the mass of the spacecraft, Saturn's speed is not changed appreciably during the interaction. Use Eq. 7-7, with $v_A = 10.4\,\text{km/s}$ and $v_B = v_B' = -9.6\,\text{km/s}$.

$$v_A - v_B = -v_A' + v_B' \quad \rightarrow \quad v_A' = 2v_B - v_A = 2(-9.6\,\text{km/s}) - 10.4\,\text{km/s} = \boxed{-29.6\,\text{km/s}}$$

Thus there is almost a threefold increase in the spacecraft's speed.