

CHAPTER 8: Rotational Motion

Answers to Questions

1. The odometer designed for 27-inch wheels increases its reading by the circumference of a 27-inch wheel (27π ") for every revolution of the wheel. If a 24-inch wheel is used, the odometer will still register (27π ") for every revolution, but only 24π " of linear distance will have been traveled. Thus the odometer will read a distance that is further than you actually traveled, by a factor of $27/24 = 1.125$. The odometer will read 12.5% too high.
2. If a disk rotates at constant angular velocity, a point on the rim has radial acceleration only – no tangential acceleration. If the disk's angular velocity increases uniformly, the point will have both radial and tangential acceleration. If the disk rotates at constant angular velocity, neither component of linear acceleration is changing – both radial and tangential acceleration are constant. If the disk rotates with a uniformly increasing angular velocity, then the radial acceleration is changing, but the tangential acceleration is a constant non-zero value.
3. A non-rigid body cannot be described by a single value of angular velocity. Since the body is non-rigid, the angular position of one part of the body changes with respect to other parts of the body. Consider the solar system as an example of a non-rigid body or system. Each planet orbits in basically the same direction around the Sun, but each planet has its own angular velocity which is different than that of the other planets.
4. Since the torque involves the product of force times lever arm, a small force can exert a greater torque than a larger force if the small force has a large enough lever arm.
5. If the lever arm is zero, then the force does not exert any torque and so cannot produce an angular acceleration. There will be no change in the angular state of motion. However, the force will add to the net force on the body and so will change the linear acceleration of the body. The body's linear state of motion will change.
6. When you do a sit-up, torque from your abdomen muscles must rotate the upper half of the body from a laying-down position to a sitting-up position. The larger the moment of inertia of the upper half of the body, the more torque is needed, and thus the harder the sit-up is to do. With the hands behind the head, the moment of inertia of the upper half of the body is larger than with the hands outstretched in front.
7. The tension force in the bicycle chain can be assumed to be the same at both the front and rear sprockets. The force is related to the torque at each sprocket by $F = \tau/R$, and so $\tau_R/R_R = \tau_F/R_F$. The torque at the rear sprocket is what actually accelerates the bicycle, and so $\tau_R = \tau_F R_R/R_F$.

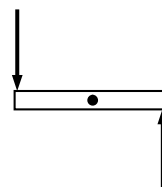
We see that, to achieve a given torque at the back sprocket, a larger front torque (due to pedaling) must be present when the rear sprocket is small. Thus it is harder to pedal with a small rear sprocket.

Likewise, to achieve a given torque at the back sprocket, a larger front torque (due to pedaling) must be present when the front sprocket is larger. Thus it is harder to pedal with a larger front sprocket.

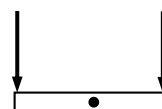
8. The legs have a lower moment of inertia when the leg mass is concentrated next to the body. That means the legs will require less torque to have a given angular acceleration, or, alternatively, a higher angular acceleration can be developed. Thus the animal can run fast.
9. The long beam increases the rotational inertia of the walker. If the walker gets off-center from the tightrope, gravity will exert a torque on the walker causing the walker to rotate with their feet as a pivot point. With a larger rotational inertia, the angular acceleration caused by that gravitational torque will be smaller, and the walker will therefore have more time to compensate.

The long size of the beam allows the walker to make relatively small shifts in their center of mass to bring them back to being centered on the tightrope. It is much easier for the walker to move a long, narrow object with the precision needed for small adjustments than a short, heavy object like a barbell.

10. Just because the net force on a system is zero, the net torque need not be zero. Consider a uniform object with two equal forces on it, and shown in the first diagram. The net force on the object is zero (it would not start to translate under the action of these forces), but there is a net counterclockwise torque about the center of the rod (it would start to rotate under the action of these forces).



Just because the net torque on a system is zero, the net force need not be zero. Consider an object with two equal forces on it, as shown in the second diagram. The net torque on the object is zero (it would not start to rotate under the action of these forces), but there is a net downward force on the rod (it would start to translate under the action of these forces).



11. Applying conservation of energy at the top and bottom of the incline, assuming that there is no work done by friction, gives $E_{\text{top}} = E_{\text{bottom}} \rightarrow Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. For a solid ball, $I = \frac{2}{5}MR^2$. If the ball rolls without slipping (no work done by friction) then $\omega = v/R$, and so

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\frac{2}{5}MR^2 v^2/R^2 \rightarrow v = \sqrt{10gh/7}.$$

This speed is independent of the angle of the incline, and so both balls will have the same speed at the bottom. The ball on the incline with the smaller angle will take more time to reach the bottom than the ball on the incline with the larger angle.

12. Applying conservation of energy at the top and bottom of the incline, and assuming that there is no work done by friction, gives $E_{\text{top}} = E_{\text{bottom}} \rightarrow Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. For a solid ball,

$I = \frac{2}{5}MR^2$. If the ball rolls without slipping (no work done by friction) then $\omega = v/R$, and so

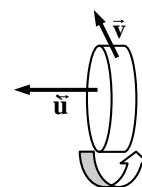
$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\frac{2}{5}MR^2 v^2/R^2 \rightarrow v = \sqrt{10gh/7}$$

This speed is independent of the mass and radius of the ball, and so both balls will have the same speed at the bottom. In fact, this is true for ANY height of fall, so the two balls will have identical instantaneous speeds all along their descent, and so both balls will take the same time to reach the bottom. The total kinetic energy is $KE = KE_{\text{trans}} + KE_{\text{rot}} = \frac{1}{2}Mv^2 + \frac{1}{2}\frac{2}{5}MR^2 v^2/R^2 = \frac{7}{10}Mv^2$, and so the ball with the larger mass has the greater total kinetic energy. Another way to consider this is that the initial potential energy of Mgh is all converted to kinetic energy. The larger mass has more potential energy to begin with (due to the larger mass), and so has more kinetic energy at the bottom.

13. Applying conservation of energy at the top and bottom of the incline, assuming that there is no work done by friction, gives $E_{\text{top}} = E_{\text{bottom}} \rightarrow Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. If the objects roll without slipping, then $\omega = v/R$, and so $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I(v/R)^2 \rightarrow v = \sqrt{\frac{2Mgh}{M + I/R^2}}$. For a solid ball, $I = \frac{2}{5}MR^2$, and for a cylinder, $I = \frac{1}{2}MR^2$. Thus $v_{\text{sphere}} = \sqrt{10gh/7}$ and $v_{\text{cyl}} = \sqrt{4gh/3}$. Since $v_{\text{sphere}} > v_{\text{cyl}}$, the sphere has the greater speed at the bottom. That is true for any amount of height change, and so the sphere is always moving faster than the cylinder after they start to move. Thus the sphere will reach the bottom first. Since both objects started with the same potential energy, both have the same total kinetic energy at the bottom. But since both objects have the same mass and the cylinder is moving slower, the cylinder has the smaller translational KE and thus the greater rotational KE.
14. Momentum and angular momentum are conserved for closed systems – systems in which there are no external forces or torques applied to the system. Probably no macroscopic systems on Earth are truly closed, and so external forces and torques (like those applied by air friction, for example) affect the systems over time.
15. If a large number of people went to the equator, the rotational inertia of the Earth would increase, since the people would be further from the axis of rotation. Angular momentum would be conserved in such an interaction, and so since the rotational inertia increased, the angular velocity would decrease – the Earth would “slow down” a small amount. The length of a day would therefore increase.
16. In order to do a somersault, the diver needs some initial angular momentum when she leaves the diving board, because angular momentum will be conserved during the free-fall motion of the dive. She cannot exert a torque on herself in isolation, and so if there is no angular momentum initially, there will be no rotation during the rest of the dive.
17. The moment of inertia will increase, because most the mass of the disk will be further from the axis of rotation than it was with the original axis position.
18. Your angular velocity will not change. Before you let go of the masses, your body has a certain angular momentum, which is the product of your moment of inertia and your angular velocity. No torques are put upon you by the act of dropping the masses, and so your angular momentum does not change. If you don't change your moment of inertia by changing the position of your body, then your angular velocity will not change. The masses, when dropped, will have a horizontal motion that is tangential to the circle in which they were moving before they were dropped. An object traveling horizontally at some distance from a vertical line (like your axis of rotation) has angular momentum relative to that vertical line. The masses keep the angular momentum that they had before being dropped.
19. The two spheres would have different rotational inertias. The sphere that is hollow will have a larger rotational inertia than the solid sphere. If the two spheres are allowed to roll down an incline without slipping, the sphere with the smaller moment of inertia (the solid one) will reach the bottom of the ramp first. See question number 13 for an explanation of why this happens.

20. Using the right hand rule, point the fingers in the direction of the Earth's rotation, from west to east. Then the thumb points north. Thus the Earth's angular velocity points along its axis of rotation, towards the North Star.

21. See the diagram. To the left is west, the direction of the angular velocity. The direction of the linear velocity of a point on the top of the wheel would be into the paper, which is north. If the angular acceleration is east, which is opposite the angular velocity, the wheel is slowing down – its angular speed is decreasing. The tangential linear acceleration of the point on top will be in the opposite direction to its linear velocity – it will point south.



22. The angular momentum of the turntable – person system will be conserved, since no external torques are being applied as the person walks to the center. As the person walks to the center, the overall moment of inertia of the system gets smaller, since the person is closer to the axis of rotation. Since the angular momentum is constant, the angular velocity must increase. So the turntable will begin to rotate faster as you walk to the center. This is similar to the spinning ice skater who pulls her arms in to increase her angular speed.

23. The shortstop, while in mid-air, cannot exert a torque on himself, and so his angular momentum will be conserved while in the air. If the upper half of his body rotates in a certain direction during the throwing motion, then to conserve angular momentum, the lower half of his body will rotate in the opposite direction.

24. Consider a helicopter in the air with the rotor spinning. To change the rotor's angular speed, a torque must be applied to the rotor. That torque has to come from the helicopter, and so by Newton's 3rd law, and equal and opposite torque will be applied by the rotor to the helicopter. Any change in rotor speed would therefore cause the body of the helicopter to spin in a direction opposite to the change in the rotor's angular velocity.

Some large helicopters have two rotor systems, spinning in opposite directions. That makes any change in the speed of the rotor pair require a net torque of 0, and so the helicopter body would not tend to spin. Smaller helicopters have a tail rotor which rotates in a vertical plane, causing a force on the tail of the helicopter in the opposite direction of the tendency of the tail to spin.

Solutions to Problems

- $(30^\circ)(2\pi \text{ rad}/360^\circ) = \boxed{\pi/6 \text{ rad}} = \boxed{0.52 \text{ rad}}$
 - $(57^\circ)(2\pi \text{ rad}/360^\circ) = \boxed{19\pi/60 \text{ rad}} = \boxed{0.99 \text{ rad}}$
 - $(90^\circ)(2\pi \text{ rad}/360^\circ) = \boxed{\pi/2 \text{ rad}} = \boxed{1.57 \text{ rad}}$
 - $(360^\circ)(2\pi \text{ rad}/360^\circ) = \boxed{2\pi \text{ rad}} = \boxed{6.28 \text{ rad}}$
 - $(420^\circ)(2\pi \text{ rad}/360^\circ) = \boxed{7\pi/3 \text{ rad}} = \boxed{7.33 \text{ rad}}$

2. The angle in radians is the diameter of the object divided by the distance to the object.

$$\Delta\theta_{\text{Sun}} = \frac{2R_{\text{Sun}}}{r_{\text{Earth-Sun}}} = \frac{2(6.96 \times 10^5 \text{ km})}{149.6 \times 10^6 \text{ km}} = \boxed{9.30 \times 10^{-3} \text{ rad}}$$

$$\Delta\theta_{\text{Moon}} = \frac{2R_{\text{Moon}}}{r_{\text{Earth-Moon}}} = \frac{2(1.74 \times 10^3 \text{ km})}{384 \times 10^3 \text{ km}} = \boxed{9.06 \times 10^{-3} \text{ rad}}$$

Since these angles are practically the same, solar eclipses occur.

3. We find the diameter of the spot from

$$\theta = \frac{\text{diameter}}{r_{\text{Earth-Moon}}} \rightarrow \text{diameter} = \theta r_{\text{Earth-Moon}} = (1.4 \times 10^{-5} \text{ rad})(3.8 \times 10^8 \text{ m}) = \boxed{5.3 \times 10^3 \text{ m}}$$

4. The initial angular velocity is $\omega_o = \left(6500 \frac{\text{rev}}{\text{min}}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 681 \text{ rad/s}$. Use the definition of angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 681 \text{ rad/s}}{3.0 \text{ s}} = -227 \text{ rad/s}^2 \approx \boxed{-2.3 \times 10^2 \text{ rad/s}^2}$$

5. The ball rolls $2\pi r = \pi d$ of linear distance with each revolution.

$$15.0 \text{ rev} \left(\frac{\pi d \text{ m}}{1 \text{ rev}}\right) = 3.5 \text{ m} \rightarrow d = \frac{3.5 \text{ m}}{15.0 \pi} = \boxed{7.4 \times 10^{-2} \text{ m}}$$

6. In each revolution, the wheel moves forward a distance equal to its circumference, πd .

$$\Delta x = N_{\text{rev}} (\pi d) \rightarrow N = \frac{\Delta x}{\pi d} = \frac{8000 \text{ m}}{\pi (0.68 \text{ m})} = \boxed{3.7 \times 10^3 \text{ rev}}$$

7. (a) $\omega = \left(\frac{2500 \text{ rev}}{1 \text{ min}}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 261.8 \text{ rad/sec} \sim \boxed{2.6 \times 10^2 \text{ rad/sec}}$

(b) $v = \omega r = (261.8 \text{ rad/sec})(0.175 \text{ m}) = \boxed{46 \text{ m/s}}$

$$a_{\text{R}} = \omega^2 r = (261.8 \text{ rad/sec})^2 (0.175 \text{ m}) = \boxed{1.2 \times 10^4 \text{ m/s}^2}$$

8. The angular speed of the merry-go-round is $2\pi \text{ rad}/4.0 \text{ s} = 1.57 \text{ rad/s}$

(a) $v = \omega r = (1.57 \text{ rad/sec})(1.2 \text{ m}) = \boxed{1.9 \text{ m/s}}$

- (b) The acceleration is radial. There is no tangential acceleration.

$$a_{\text{R}} = \omega^2 r = (1.57 \text{ rad/sec})^2 (1.2 \text{ m}) = \boxed{3.0 \text{ m/s}^2 \text{ towards the center}}$$

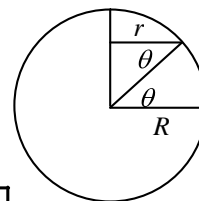
9. (a) The Earth makes one orbit around the Sun in one year.

$$\omega_{\text{orbit}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi \text{ rad}}{1 \text{ yr}}\right) \left(\frac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}}\right) = \boxed{1.99 \times 10^{-7} \text{ rad/s}}$$

- (b) The Earth makes one revolution about its axis in one day.

$$\omega_{\text{rotation}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi \text{ rad}}{1 \text{ d}}\right) \left(\frac{1 \text{ d}}{86,400 \text{ s}}\right) = \boxed{7.27 \times 10^{-5} \text{ rad/s}}$$

10. Each location will have the same angular velocity (1 revolution per day), but the radius of the circular path varies with the location. From the diagram, we see $r = R \cos \theta$, where R is the radius of the Earth, and r is the radius at latitude θ .



$$(a) \quad v = \omega r = \frac{2\pi}{T} r = \left(\frac{2\pi \text{ rad}}{1 \text{ d}} \right) \left(\frac{1 \text{ d}}{86400 \text{ s}} \right) (6.38 \times 10^6 \text{ m}) = \boxed{4.64 \times 10^2 \text{ m/s}}$$

$$(b) \quad v = \omega r = \frac{2\pi}{T} r = \left(\frac{2\pi \text{ rad}}{1 \text{ d}} \right) \left(\frac{1 \text{ d}}{86400 \text{ s}} \right) (6.38 \times 10^6 \text{ m}) \cos 66.5^\circ = \boxed{1.85 \times 10^2 \text{ m/s}}$$

$$(c) \quad v = \omega r = \frac{2\pi}{T} r = \left(\frac{2\pi \text{ rad}}{1 \text{ d}} \right) \left(\frac{1 \text{ d}}{86400 \text{ s}} \right) (6.38 \times 10^6 \text{ m}) \cos 45.0^\circ = \boxed{3.28 \times 10^2 \text{ m/s}}$$

11. The centripetal acceleration is given by $a = \omega^2 r$. Solve for the angular velocity.

$$\omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{(100,000)(9.8 \text{ m/s}^2)}{0.070 \text{ m}}} = 3741 \frac{\text{rad}}{\text{s}} \left(\frac{1 \text{ rev}}{2\pi \text{ rad}} \right) \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = \boxed{3.6 \times 10^4 \text{ rpm}}$$

12. Convert the rpm values to angular velocities.

$$\omega_0 = \left(130 \frac{\text{rev}}{\text{min}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ sec}} \right) = 13.6 \text{ rad/s}$$

$$\omega = \left(280 \frac{\text{rev}}{\text{min}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ sec}} \right) = 29.3 \text{ rad/s}$$

- (a) The angular acceleration is found from Eq. 8-9a.

$$\omega = \omega_0 + \alpha t \rightarrow \alpha = \frac{\omega - \omega_0}{t} = \frac{29.3 \text{ rad/s} - 13.6 \text{ rad/s}}{4.0 \text{ s}} = 3.93 \text{ rad/s}^2 \approx \boxed{3.9 \text{ rad/s}^2}$$

- (b) To find the components of the acceleration, the instantaneous angular velocity is needed.

$$\omega = \omega_0 + \alpha t = 13.6 \text{ rad/s} + (3.93 \text{ rad/s}^2)(2.0 \text{ s}) = 21.5 \text{ rad/s}$$

The instantaneous radial acceleration is given by $a_R = \omega^2 r$.

$$a_R = \omega^2 r = (21.5 \text{ rad/s})^2 (0.35 \text{ m}) = \boxed{1.6 \times 10^2 \text{ m/s}^2}$$

The tangential acceleration is given by $a_{\text{tan}} = \alpha r$.

$$a_{\text{tan}} = \alpha r = (3.93 \text{ rad/s}^2)(0.35 \text{ m}) = \boxed{1.4 \text{ m/s}^2}$$

13. The tangential speed of the turntable must be equal to the tangential speed of the roller, if there is no slippage.

$$v_1 = v_2 \rightarrow \omega_1 R_1 = \omega_2 R_2 \rightarrow \boxed{\omega_1 / \omega_2 = R_2 / R_1}$$

14. (a) The angular rotation can be found from Eq. 8-3a. The initial angular frequency is 0 and the final angular frequency is 1 rpm.

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{\left(1 \frac{\text{rev}}{\text{min}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1.0 \text{ min}}{60 \text{ s}} \right) - 0}{720 \text{ s}} = 1.454 \times 10^{-4} \text{ rad/s}^2 \approx \boxed{1.5 \times 10^{-4} \text{ rad/s}^2}$$

- (b) After 5.0 min (300 s), the angular speed is as follows.

$$\omega = \omega_0 + \alpha t = 0 + (1.454 \times 10^{-4} \text{ rad/s}^2)(300 \text{ s}) = 4.363 \times 10^{-2} \text{ rad/s}$$

Find the components of the acceleration of a point on the outer skin from the angular speed and the radius.

$$a_{\text{tan}} = \alpha R = (1.454 \times 10^{-4} \text{ rad/s}^2)(4.25 \text{ m}) = \boxed{6.2 \times 10^{-4} \text{ m/s}^2}$$

$$a_{\text{rad}} = \omega^2 R = (4.363 \times 10^{-2} \text{ rad/s})^2 (4.25 \text{ m}) = \boxed{8.1 \times 10^{-3} \text{ m/s}^2}$$

15. The angular displacement can be found from the following uniform angular acceleration relationship.

$$\theta = \frac{1}{2}(\omega_o + \omega)t = \frac{1}{2}(0 + 15000 \text{ rev/min})(220 \text{ s})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{2.8 \times 10^4 \text{ rev}}$$

16. (a) For constant angular acceleration:

$$\begin{aligned} \alpha &= \frac{\omega - \omega_o}{t} = \frac{1200 \text{ rev/min} - 4500 \text{ rev/min}}{2.5 \text{ s}} = \frac{-3300 \text{ rev/min}}{2.5 \text{ s}} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \\ &= \boxed{-1.4 \times 10^2 \text{ rad/s}^2} \end{aligned}$$

- (b) For the angular displacement, given constant angular acceleration:

$$\theta = \frac{1}{2}(\omega_o + \omega)t = \frac{1}{2}(4500 \text{ rev/min} + 1200 \text{ rev/min})(2.5 \text{ s})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{1.2 \times 10^2 \text{ rev}}$$

17. (a) The angular acceleration can be found from $\theta = \omega_o t + \frac{1}{2}\alpha t^2$ with $\omega_o = 0$.

$$\alpha = \frac{2\theta}{t^2} = \frac{2(20 \text{ rev})}{(1.0 \text{ min})^2} = \boxed{4.0 \times 10^1 \text{ rev/min}^2}$$

- (b) The final angular speed can be found from $\theta = \frac{1}{2}(\omega_o + \omega)t$, with $\omega_o = 0$.

$$\omega = \frac{2\theta}{t} - \omega_o = \frac{2(20 \text{ rev})}{1.0 \text{ min}} = \boxed{4.0 \times 10^1 \text{ rpm}}$$

18. Use Eq. 8-9d combined with Eq. 8-2a.

$$\bar{\omega} = \frac{\omega + \omega_o}{2} = \frac{240 \text{ rpm} + 360 \text{ rpm}}{2} = 300 \text{ rpm}$$

$$\theta = \bar{\omega}t = \left(300 \frac{\text{rev}}{\text{min}}\right) \left(\frac{1 \text{ min}}{60 \text{ sec}}\right) (6.5 \text{ s}) = 32.5 \text{ rev}$$

Each revolution corresponds to a circumference of travel distance.

$$32.5 \text{ rev} \left[\frac{\pi(0.33 \text{ m})}{1 \text{ rev}} \right] = \boxed{34 \text{ m}}$$

19. (a) The angular acceleration can be found from $\omega^2 = \omega_o^2 + 2\alpha\theta$.

$$\alpha = \frac{\omega^2 - \omega_o^2}{2\theta} = \frac{0 - (850 \text{ rev/min})^2}{2(1500 \text{ rev})} = \left(-241 \frac{\text{rev}}{\text{min}^2}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right)^2 = \boxed{-0.42 \frac{\text{rad}}{\text{s}^2}}$$

- (b) The time to come to a stop can be found from $\theta = \frac{1}{2}(\omega_o + \omega)t$.

$$t = \frac{2\theta}{\omega_o + \omega} = \frac{2(1500 \text{ rev})}{850 \text{ rev/min}} \left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{210 \text{ s}}$$

20. Since there is no slipping between the wheels, the tangential component of the linear acceleration of each wheel must be the same.

$$(a) \quad a_{\text{tan small}} = a_{\text{tan large}} \rightarrow \alpha_{\text{small}} r_{\text{small}} = \alpha_{\text{large}} r_{\text{large}} \rightarrow$$

$$\alpha_{\text{large}} = \alpha_{\text{small}} \frac{r_{\text{small}}}{r_{\text{large}}} = (7.2 \text{ rad/s}^2) \left(\frac{2.0 \text{ cm}}{25.0 \text{ cm}} \right) = 0.576 \text{ rad/s}^2 \approx \boxed{0.58 \text{ rad/s}^2}$$

- (b) Assume the pottery wheel starts from rest. Convert the speed to an angular speed, and then use Eq. 8-9a.

$$\omega = \left(65 \frac{\text{rev}}{\text{min}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 6.81 \text{ rad/s}$$

$$\omega = \omega_0 + \alpha t \rightarrow t = \frac{\omega - \omega_0}{\alpha} = \frac{6.81 \text{ rad/s}}{0.576 \text{ rad/s}^2} = \boxed{12 \text{ s}}$$

21. (a) The angular acceleration can be found from $\omega^2 = \omega_0^2 + 2\alpha\theta$, with the angular velocities being found from $\omega = v/r$.

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{(v^2 - v_0^2)}{2r^2\theta} = \frac{[(45 \text{ km/h})^2 - (95 \text{ km/h})^2] \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)^2}{2(0.40 \text{ m})^2 (65 \text{ rev}) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right)}$$

$$= -4.133 \text{ rad/s}^2 \approx \boxed{-4.1 \text{ rad/s}^2}$$

- (b) The time to stop can be found from $\omega = \omega_0 + \alpha t$, with a final angular velocity of 0.

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{v - v_0}{r\alpha} = \frac{-(45 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)}{(0.40 \text{ m})(-4.133 \text{ rad/s}^2)} = \boxed{7.6 \text{ s}}$$

22. (a) The maximum torque will be exerted by the force of her weight, pushing tangential to the circle in which the pedal moves.

$$\tau = r_{\perp} F = r_{\perp} mg = (0.17 \text{ m})(55 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{92 \text{ m}\cdot\text{N}}$$

- (b) She could exert more torque by pushing down harder with her legs, raising her center of mass. She could also pull upwards on the handle bars as she pedals, which will increase the downward force of her legs.

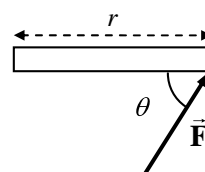
23. The torque is calculated by $\tau = rF \sin \theta$. See the diagram, from the top view.

- (a) For the first case, $\theta = 90^\circ$.

$$\tau = rF \sin \theta = (0.74 \text{ m})(55 \text{ N}) \sin 90^\circ = \boxed{41 \text{ m}\cdot\text{N}}$$

- (b) For the second case, $\theta = 45^\circ$.

$$\tau = rF \sin \theta = (0.74 \text{ m})(55 \text{ N}) \sin 45^\circ = \boxed{29 \text{ m}\cdot\text{N}}$$



24. Each force is oriented so that it is perpendicular to its lever arm. Call counterclockwise torques positive. The torque due to the three applied forces is given by

$$\tau_{\text{applied forces}} = (28 \text{ N})(0.24 \text{ m}) - (18 \text{ N})(0.24 \text{ m}) - (35 \text{ N})(0.12 \text{ m}) = -1.8 \text{ m}\cdot\text{N}.$$

Since this torque is clockwise, we assume the wheel is rotating clockwise, and so the frictional torque is counterclockwise. Thus the net torque is

$$\begin{aligned}\tau_{\text{net}} &= (28 \text{ N})(0.24 \text{ m}) - (18 \text{ N})(0.24 \text{ m}) - (35 \text{ N})(0.12 \text{ m}) + 0.40 \text{ m}\cdot\text{N} = -1.4 \text{ m}\cdot\text{N} \\ &= \boxed{1.4 \text{ m}\cdot\text{N}, \text{ clockwise}}\end{aligned}$$

25. There is a counterclockwise torque due to the force of gravity on the left block, and a clockwise torque due to the force of gravity on the right block. Call clockwise the positive direction.

$$\sum \tau = mgL_2 - mgL_1 = \boxed{mg(L_2 - L_1), \text{ clockwise}}$$

26. (a) The force required to produce the torque can be found from $\tau = rF \sin \theta$. The force is applied perpendicularly to the wrench, so $\theta = 90^\circ$. Thus

$$F = \frac{\tau}{r} = \frac{88 \text{ m}\cdot\text{N}}{0.28 \text{ m}} = \boxed{3.1 \times 10^2 \text{ N}}$$

- (b) The net torque still must be $88 \text{ m}\cdot\text{N}$. This is produced by 6 forces, one at each of the 6 points. Those forces are also perpendicular to the lever arm, and so

$$\tau_{\text{net}} = (6F_{\text{point}})r_{\text{point}} \rightarrow F_{\text{point}} = \frac{\tau}{6r} = \frac{88 \text{ m}\cdot\text{N}}{6(0.0075 \text{ m})} = \boxed{2.0 \times 10^3 \text{ N}}$$

27. For a sphere rotating about an axis through its center, the moment of inertia is given by

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(10.8 \text{ kg})(0.648 \text{ m})^2 = \boxed{1.81 \text{ kg}\cdot\text{m}^2}.$$

28. Since all of the significant mass is located at the same distance from the axis of rotation, the moment of inertia is given by

$$I = MR^2 = (1.25 \text{ kg})\left(\frac{0.667}{2} \text{ m}\right)^2 = \boxed{0.139 \text{ kg}\cdot\text{m}^2}.$$

The hub mass can be ignored because its distance from the axis of rotation is very small, and so it has a very small rotational inertia.

29. (a) The small ball can be treated as a particle for calculating its moment of inertia.

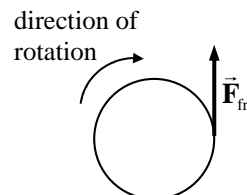
$$I = MR^2 = (0.650 \text{ kg})(1.2 \text{ m})^2 = \boxed{0.94 \text{ kg}\cdot\text{m}^2}$$

- (b) To keep a constant angular velocity, the net torque must be zero, and so the torque needed is the same magnitude as the torque caused by friction.

$$\sum \tau = \tau_{\text{applied}} - \tau_{\text{fr}} = 0 \rightarrow \tau_{\text{applied}} = \tau_{\text{fr}} = F_{\text{fr}}r = (0.020 \text{ N})(1.2 \text{ m}) = \boxed{2.4 \times 10^{-2} \text{ m}\cdot\text{N}}$$

30. (a) The torque exerted by the frictional force is $\tau = rF_{\text{fr}} \sin \theta$. The frictional force is assumed to be tangential to the clay, and so the angle is $\theta = 90^\circ$.

$$\tau_{\text{total}} = rF_{\text{fr}} \sin \theta = (6.0 \times 10^{-2} \text{ m})(1.5 \text{ N}) \sin 90^\circ = \boxed{9.0 \times 10^{-2} \text{ m}\cdot\text{N}}$$



- (b) The time to stop is found from $\omega = \omega_o + \alpha t$, with a final angular velocity of 0. The angular acceleration can be found from $\tau_{\text{total}} = I\alpha$. The net torque (and angular acceleration) is negative since the object is slowing.

$$t = \frac{\omega - \omega_o}{\alpha} = \frac{\omega - \omega_o}{\tau/I} = \frac{0 - (1.6 \text{ rev/s})(2\pi \text{ rad/rev})}{(-9.0 \times 10^{-2} \text{ m}\cdot\text{N}) / (0.11 \text{ kg}\cdot\text{m}^2)} = \boxed{12 \text{ s}}$$

31. (a) To calculate the moment of inertia about the y-axis (vertical), use

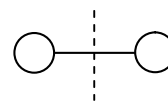
$$I = \sum M_i R_{ix}^2 = m(0.50 \text{ m})^2 + M(0.50 \text{ m})^2 + m(1.00 \text{ m})^2 + M(1.00 \text{ m})^2 \\ = (m + M) \left[(0.50 \text{ m})^2 + (1.00 \text{ m})^2 \right] = (4.9 \text{ kg}) \left[(0.50 \text{ m})^2 + (1.00 \text{ m})^2 \right] = \boxed{6.1 \text{ kg}\cdot\text{m}^2}$$

- (b) To calculate the moment of inertia about the x-axis (horizontal), use

$$I = \sum M_i R_{iy}^2 = (2m + 2M)(0.25 \text{ m})^2 = \boxed{0.61 \text{ kg}\cdot\text{m}^2}.$$

- (c) Because of the larger I value, it is harder to accelerate the array about the **vertical axis**.

- 32 The oxygen molecule has a “dumbbell” geometry, rotating about the dashed line, as shown in the diagram. If the total mass is M , then each atom has a mass of $M/2$. If the distance between them is d , then the distance from the axis of rotation to each atom is $d/2$. Treat each atom as a particle for calculating the moment of inertia.



$$I = (M/2)(d/2)^2 + (M/2)(d/2)^2 = 2(M/2)(d/2)^2 = \frac{1}{4}Md^2 \rightarrow$$

$$d = \sqrt{4I/M} = \sqrt{4(1.9 \times 10^{-46} \text{ kg}\cdot\text{m}^2) / (5.3 \times 10^{-26} \text{ kg})} = \boxed{1.2 \times 10^{-10} \text{ m}}$$

33. The firing force of the rockets will create a net torque, but no net force. Since each rocket fires tangentially, each force has a lever arm equal to the radius of the satellite, and each force is perpendicular to the lever arm. Thus $\tau_{\text{net}} = 4FR$. This torque will cause an angular acceleration according to $\tau = I\alpha$, where $I = \frac{1}{2}MR^2$ for a cylinder. The angular acceleration can be found from the kinematics by $\alpha = \frac{\Delta\omega}{\Delta t}$. Equating the two expressions for the torque and substituting enables us to solve for the force.

$$4FR = I\alpha = \frac{1}{2}MR^2 \frac{\Delta\omega}{\Delta t}$$

$$F = \frac{MR\Delta\omega}{8\Delta t} = \frac{(3600 \text{ kg})(4.0 \text{ m})(32 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{8(5.0 \text{ min})(60 \text{ s/min})} \\ = 20.11 \text{ N} \approx \boxed{2.0 \times 10^1 \text{ N}}$$

34. (a) The moment of inertia of a cylinder is found in Figure 8-21.

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(0.580 \text{ kg})(8.50 \times 10^{-2} \text{ m})^2 = 2.0953 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \approx \boxed{2.10 \times 10^{-3} \text{ kg}\cdot\text{m}^2}$$

- (b) The wheel slows down “on its own” from 1500 rpm to rest in 55.0s. This is used to calculate the frictional torque.

$$\tau_{\text{fr}} = I\alpha_{\text{fr}} = I \frac{\Delta\omega}{\Delta t} = (2.0953 \times 10^{-3} \text{ kg}\cdot\text{m}^2) \frac{(0 - 1500 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{55.0 \text{ s}} \\ = -5.984 \times 10^{-3} \text{ m}\cdot\text{N}$$

The net torque causing the angular acceleration is the applied torque plus the (negative) frictional torque.

$$\begin{aligned}\sum \tau &= \tau_{\text{applied}} + \tau_{\text{fr}} = I\alpha \rightarrow \\ \tau_{\text{applied}} &= I\alpha - \tau_{\text{fr}} = I \frac{\Delta\omega}{\Delta t} - \tau_{\text{fr}} \\ &= (2.0953 \times 10^{-3} \text{ kg}\cdot\text{m}^2) \frac{(1500 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{5.00 \text{ s}} + (5.984 \times 10^{-3} \text{ m}\cdot\text{N}) \\ &= \boxed{7.2 \times 10^{-2} \text{ m}\cdot\text{N}}\end{aligned}$$

35. The torque can be calculated from $\tau = I\alpha$. The rotational inertia of a rod about its end is given by $I = \frac{1}{3}ML^2$.

$$\tau = I\alpha = \frac{1}{3}ML^2 \frac{\Delta\omega}{\Delta t} = \frac{1}{3}(2.2 \text{ kg})(0.95 \text{ m})^2 \frac{(3.0 \text{ rev/s})(2\pi \text{ rad/rev})}{0.20 \text{ s}} = \boxed{62 \text{ m}\cdot\text{N}}$$

36. The torque needed is the moment of inertia of the system (merry-go-round and children) times the angular acceleration of the system. Let the subscript “mgr” represent the merry-go-round.

$$\begin{aligned}\tau &= I\alpha = (I_{\text{mgr}} + I_{\text{children}}) \frac{\Delta\omega}{\Delta t} = \left(\frac{1}{2}M_{\text{mgr}}R^2 + 2m_{\text{child}}R^2\right) \frac{\omega - \omega_0}{t} \\ &= \left[\frac{1}{2}(760 \text{ kg}) + 2(25 \text{ kg})\right](2.5 \text{ m})^2 \frac{(15 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{10.0 \text{ s}} \\ &= 422.15 \text{ m}\cdot\text{N} \approx \boxed{4.2 \times 10^2 \text{ m}\cdot\text{N}}\end{aligned}$$

The force needed is calculated from the torque and the radius. Assume that the force is all directed perpendicularly to the radius.

$$\tau = F_{\perp}R \sin \theta \rightarrow F_{\perp} = \tau/R = 4.2215 \times 10^2 \text{ m}\cdot\text{N}/2.5 \text{ m} = \boxed{1.7 \times 10^2 \text{ N}}$$

37. The torque on the rotor will cause an angular acceleration given by $\alpha = \tau/I$. The torque and angular acceleration will have the opposite sign of the initial angular velocity because the rotor is being brought to rest. The rotational inertia is that of a solid cylinder. Substitute the expressions for angular acceleration and rotational inertia into the equation $\omega^2 = \omega_0^2 + 2\alpha\theta$, and solve for the angular displacement.

$$\begin{aligned}\omega^2 &= \omega_0^2 + 2\alpha\theta \\ \theta &= \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{0 - \omega_0^2}{2(\tau/I)} = \frac{-\omega_0^2}{2(\tau/\frac{1}{2}MR^2)} = \frac{-MR^2\omega_0^2}{4\tau} \\ &= \frac{-(4.80 \text{ kg})(0.0710 \text{ m})^2 \left[\left(10,300 \frac{\text{rev}}{\text{min}}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \right]^2}{4(-1.20 \text{ N}\cdot\text{m})} = 5865 \text{ rad} \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) \\ &= \boxed{993 \text{ rev}}\end{aligned}$$

The time can be found from $\theta = \frac{1}{2}(\omega_0 + \omega)t$.

$$t = \frac{2\theta}{\omega_o + \omega} = \frac{2(993 \text{ rev})}{10,300 \text{ rev/min}} \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = \boxed{10.9 \text{ s}}$$

38. (a) The torque gives angular acceleration to the ball only, since the arm is considered massless. The angular acceleration of the ball is found from the given tangential acceleration.

$$\begin{aligned} \tau &= I\alpha = MR^2\alpha = MR^2 \frac{a_{\text{tan}}}{R} = MRa_{\text{tan}} = (3.6 \text{ kg})(0.31 \text{ m})(7.0 \text{ m/s}^2) \\ &= 7.812 \text{ m}\cdot\text{N} \approx \boxed{7.8 \text{ m}\cdot\text{N}} \end{aligned}$$

- (b) The triceps muscle must produce the torque required, but with a lever arm of only 2.5 cm, perpendicular to the triceps muscle force.

$$\tau = Fr_{\perp} \rightarrow F = \tau/r_{\perp} = 7.812 \text{ m}\cdot\text{N} / (2.5 \times 10^{-2} \text{ m}) = \boxed{3.1 \times 10^2 \text{ N}}$$

39. (a) The angular acceleration can be found from

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega}{t} = \frac{v/r}{t} = \frac{(10.0 \text{ m/s}) / (0.31 \text{ m})}{0.350 \text{ s}} = 92.17 \text{ rad/s}^2 \approx \boxed{92 \text{ rad/s}^2}$$

- (b) The force required can be found from the torque, since $\tau = Fr \sin \theta$. In this situation the force is perpendicular to the lever arm, and so $\theta = 90^\circ$. The torque is also given by $\tau = I\alpha$, where I is the moment of inertia of the arm-ball combination. Equate the two expressions for the torque, and solve for the force.

$$Fr \sin \theta = I\alpha$$

$$\begin{aligned} F &= \frac{I\alpha}{r \sin \theta} = \frac{m_{\text{ball}}d_{\text{ball}}^2 + \frac{1}{3}m_{\text{arm}}L_{\text{arm}}^2}{r \sin 90^\circ} \alpha \\ &= \frac{(1.00 \text{ kg})(0.31 \text{ m})^2 + \frac{1}{3}(3.70 \text{ kg})(0.31 \text{ m})^2}{(0.025 \text{ m})} (92.17 \text{ rad/s}^2) = \boxed{7.9 \times 10^2 \text{ N}} \end{aligned}$$

40. (a) The moment of inertia of a thin rod, rotating about its end, is given in Figure 8-21(g). There are three blades to add.

$$I_{\text{total}} = 3\left(\frac{1}{3}ML^2\right) = ML^2 = (160 \text{ kg})(3.75 \text{ m})^2 = 2250 \text{ kg}\cdot\text{m}^2 \approx \boxed{2.3 \times 10^2 \text{ kg}\cdot\text{m}^2}$$

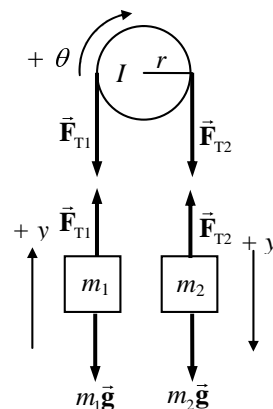
- (b) The torque required is the rotational inertia times the angular acceleration, assumed constant.

$$\tau = I_{\text{total}}\alpha = I_{\text{total}} \frac{\omega - \omega_0}{t} = (2250 \text{ kg}\cdot\text{m}^2) \frac{(5.0 \text{ rev/sec})(2\pi \text{ rad/rev})}{8.0 \text{ s}} = \boxed{8.8 \times 10^3 \text{ m}\cdot\text{N}}$$

41. We assume that $m_2 > m_1$, and so m_2 will accelerate down, m_1 will accelerate up, and the pulley will accelerate clockwise. Call the direction of acceleration the positive direction for each object. The masses will have the same acceleration since they are connected by a cord. The rim of the pulley will have that same acceleration since the cord is making it rotate, and so $\alpha_{\text{pulley}} = a/r$. From the free-body diagrams for each object, we have the following.

$$\sum F_{y1} = F_{T1} - m_1g = m_1a \rightarrow F_{T1} = m_1g + m_1a$$

$$\sum F_{y2} = m_2g - F_{T2} = m_2a \rightarrow F_{T2} = m_2g - m_2a$$



$$\sum \tau = F_{T2}r - F_{T1}r = I\alpha = I\frac{a}{r}$$

Substitute the expressions for the tensions into the torque equation, and solve for the acceleration.

$$F_{T2}r - F_{T1}r = I\frac{a}{r} \rightarrow (m_2g - m_2a)r - (m_1g + m_1a)r = I\frac{a}{r} \rightarrow a = \frac{(m_2 - m_1)}{(m_1 + m_2 + I/r^2)}g$$

If the moment of inertia is ignored, then from the torque equation we see that $F_{T2} = F_{T1}$, and the

acceleration will be $a_{I=0} = \frac{(m_2 - m_1)}{(m_1 + m_2)}g$. We see that the acceleration with the moment of inertia

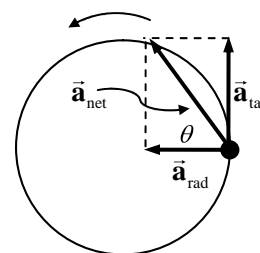
included will be smaller than if the moment of inertia is ignored.

42. A top view diagram of the hammer is shown, just at the instant of release, along with the acceleration vectors.

- (a) The angular acceleration is found from Eq. 8-9c.

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta \rightarrow$$

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\Delta\theta} = \frac{(v/r)^2 - 0}{2\Delta\theta} = \frac{[(28.0 \text{ m/s})/(1.20 \text{ m})]^2}{2(8\pi \text{ rad})} = 10.8 \text{ rad/s}^2$$



- (b) The tangential acceleration is found from the angular acceleration and the radius.

$$a_{\text{tan}} = \alpha r = (10.8 \text{ rad/s}^2)(1.20 \text{ m}) = 13.0 \text{ m/s}^2$$

- (c) The centripetal acceleration is found from the speed and the radius.

$$a_{\text{rad}} = v^2/r = (28.0 \text{ m/s})^2/(1.20 \text{ m}) = 653 \text{ m/s}^2$$

- (d) The net force is the mass times the net acceleration. It is in the same direction as the net acceleration, also.

$$F_{\text{net}} = ma_{\text{net}} = m\sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2} = (7.30 \text{ kg})\sqrt{(13.0 \text{ m/s}^2)^2 + (653 \text{ m/s}^2)^2} = 4.77 \times 10^3 \text{ N}$$

- (e) Find the angle from the two acceleration vectors.

$$\theta = \tan^{-1} \frac{a_{\text{tan}}}{a_{\text{rad}}} = \tan^{-1} \frac{13.0 \text{ m/s}^2}{653 \text{ m/s}^2} = 1.14^\circ$$

43. The energy required to bring the rotor up to speed from rest is equal to the final rotational KE of the rotor.

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(3.75 \times 10^{-2} \text{ kg}\cdot\text{m}^2) \left[8250 \frac{\text{rev}}{\text{min}} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \right]^2 = 1.40 \times 10^4 \text{ J}$$

44. Work can be expressed in rotational quantities as $W = \tau \Delta\theta$, and so power can be expressed in

rotational quantities as $P = \frac{W}{\Delta t} = \tau \frac{\Delta\theta}{\Delta t} = \tau\omega$.

$$P = \tau\omega = (280 \text{ m}\cdot\text{N}) \left(3800 \frac{\text{rev}}{\text{min}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 1.114 \times 10^5 \text{ W} \approx 1.1 \times 10^5 \text{ W}$$

$$1.114 \times 10^5 \text{ W} \left(\frac{1 \text{ hp}}{746 \text{ W}} \right) = 1.5 \times 10^2 \text{ hp}$$

45. The total kinetic energy is the sum of the translational and rotational kinetic energies. Since the ball is rolling without slipping, the angular velocity is given by $\omega = v/R$. The rotational inertia of a sphere about an axis through its center is $I = \frac{2}{5}mR^2$.

$$\begin{aligned} KE_{\text{total}} &= KE_{\text{trans}} + KE_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{v^2}{R^2} = \frac{7}{10}mv^2 \\ &= 0.7(7.3 \text{ kg})(3.3 \text{ m/s})^2 = \boxed{56 \text{ J}} \end{aligned}$$

46. (a) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$\begin{aligned} KE_{\text{daily}} &= \frac{1}{2}I\omega_{\text{daily}}^2 = \frac{1}{2}\left(\frac{2}{5}MR_{\text{Earth}}^2\right)\omega_{\text{daily}}^2 \\ &= \frac{1}{5}(6.0 \times 10^{24} \text{ kg})(6.4 \times 10^6 \text{ m})^2 \left[\left(\frac{2\pi \text{ rad}}{1 \text{ day}} \right) \left(\frac{1 \text{ day}}{86,400 \text{ s}} \right) \right]^2 = \boxed{2.6 \times 10^{29} \text{ J}} \end{aligned}$$

- (b) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$\begin{aligned} KE_{\text{yearly}} &= \frac{1}{2}I\omega_{\text{yearly}}^2 = \frac{1}{2}\left(MR_{\text{Sun-Earth}}^2\right)\omega_{\text{yearly}}^2 \\ &= \frac{1}{2}(6.0 \times 10^{24} \text{ kg})(1.5 \times 10^{11} \text{ m})^2 \left[\left(\frac{2\pi \text{ rad}}{365 \text{ day}} \right) \left(\frac{1 \text{ day}}{86,400 \text{ s}} \right) \right]^2 = \boxed{2.7 \times 10^{33} \text{ J}} \end{aligned}$$

Thus the total KE is $KE_{\text{daily}} + KE_{\text{yearly}} = 2.6 \times 10^{29} \text{ J} + 2.6 \times 10^{33} \text{ J} = \boxed{2.6 \times 10^{33} \text{ J}}$. The KE due to the daily motion is about 10,000 smaller than that due to the yearly motion.

47. The work required is the change in rotational kinetic energy. The initial angular velocity is 0.

$$W = \Delta KE_{\text{rot}} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_f^2 = \frac{1}{4}(1640 \text{ kg})(7.50 \text{ m})^2 \left(\frac{2\pi \text{ rad}}{8.00 \text{ s}} \right)^2 = \boxed{1.42 \times 10^4 \text{ J}}$$

48. Apply conservation of energy to the sphere, as done in Example 8-13.

- (a) The work of Example 8-13 is exactly applicable here. The symbol d is to represent the distance the sphere rolls along the plane. The sphere is rolling without slipping, so $v_{\text{CM}} = \omega R$.

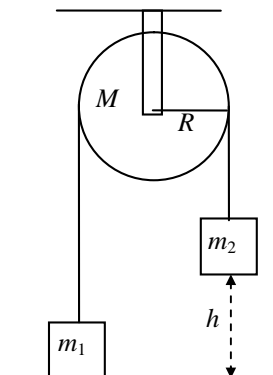
$$\begin{aligned} v_{\text{CM}} &= \sqrt{\frac{10}{7}gh} = \sqrt{\frac{10}{7}gd \sin \theta} = \sqrt{\frac{10}{7}(9.80 \text{ m/s}^2)(10.0 \text{ m})(\sin 30.0^\circ)} = 8.367 \\ &= \boxed{8.37 \text{ m/s}} \end{aligned}$$

$$\omega = v_{\text{CM}}/R = 8.367 \text{ m/s} / (2.00 \times 10^{-1} \text{ m}) = \boxed{41.8 \text{ rad/s}}$$

$$(b) \frac{KE_{\text{trans}}}{KE_{\text{rot}}} = \frac{\frac{1}{2}Mv_{\text{CM}}^2}{\frac{1}{2}I_{\text{CM}}\omega^2} = \frac{\frac{1}{2}Mv_{\text{CM}}^2}{\frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{v_{\text{CM}}^2}{R^2}} = \boxed{2.5}$$

- (c) Only the angular speed depends on the radius. None of the results depend on the mass.

49. The only force doing work in this system is gravity, so mechanical energy will be conserved. The initial state of the system is the configuration with m_1 on the ground and all objects at rest. The final state of the system has m_2 just reaching the ground, and all objects in motion. Call the zero level of gravitational potential energy to be the ground level. Both masses will have the same speed since they are connected by the rope. Assuming that the rope does not slip on the pulley, the angular speed of the pulley is related to the speed of the masses by $\omega = v/R$. All objects have an initial speed of 0.



$$E_i = E_f$$

$$\frac{1}{2}m_1v_i^2 + \frac{1}{2}m_2v_i^2 + \frac{1}{2}I\omega_i^2 + m_1gy_{1i} + m_2gy_{2i} = \frac{1}{2}m_1v_f^2 + \frac{1}{2}m_2v_f^2 + \frac{1}{2}I\omega_f^2 + m_1gy_{1f} + m_2gy_{2f}$$

$$m_2gh = \frac{1}{2}m_1v_f^2 + \frac{1}{2}m_2v_f^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f^2}{R^2}\right) + m_1gh$$

$$v_f = \sqrt{\frac{2(m_2 - m_1)gh}{(m_1 + m_2 + \frac{1}{2}M)}} = \sqrt{\frac{2(26.5 \text{ kg} - 18.0 \text{ kg})(9.80 \text{ m/s}^2)(3.00 \text{ m})}{(26.5 \text{ kg} + 18.0 \text{ kg} + (\frac{1}{2})7.50 \text{ kg})}} = \boxed{3.22 \text{ m/s}}$$

50. Since the lower end of the pole does not slip on the ground, the friction does no work, and so mechanical energy is conserved. The initial energy is the potential energy, treating all the mass as if it were at the CM. The final energy is rotational KE, for rotation about the point of contact with the ground. The linear velocity of the falling tip of the rod is its angular velocity divided by the length.

$$PE = KE \rightarrow mgh = \frac{1}{2}I\omega^2 \rightarrow mgL/2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\left(\frac{v_{\text{end}}}{L}\right)^2 \rightarrow$$

$$v_{\text{end}} = \sqrt{3gL} = \sqrt{3(9.80 \text{ m/s}^2)(2.30 \text{ m})} = \boxed{8.22 \text{ m/s}}$$

51. The angular momentum is given by Eq. 8-18.

$$L = I\omega = MR^2\omega = (0.210 \text{ kg})(1.10 \text{ m})^2(10.4 \text{ rad/s}) = \boxed{2.64 \text{ kg}\cdot\text{m}^2/\text{s}}$$

52. (a) The angular momentum is given by Eq. 8-18.

$$L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2}(2.8 \text{ kg})(0.18 \text{ m})^2 \left[\left(\frac{1500 \text{ rev}}{1 \text{ min}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \right]$$

$$= \boxed{7.1 \text{ kg}\cdot\text{m}^2/\text{s}}$$

- (b) The torque required is the change in angular momentum per unit time. The final angular momentum is zero.

$$\tau = \frac{L - L_0}{\Delta t} = \frac{0 - 7.1 \text{ kg}\cdot\text{m}^2/\text{s}}{6.0 \text{ s}} = \boxed{-1.2 \text{ m}\cdot\text{N}}$$

The negative sign indicates that the torque is used to oppose the initial angular momentum.

53. (a) Consider the person and platform a system for angular momentum analysis. Since the force and torque to raise and/or lower the arms is internal to the system, the raising or lowering of the arms will cause no change in the total angular momentum of the system. However, the rotational inertia increases when the arms are raised. Since angular momentum is conserved, an increase in rotational inertia must be accompanied by a decrease in angular velocity.

$$(b) \quad L_i = L_f \rightarrow I_i \omega_i = I_f \omega_f \rightarrow I_f = I_i \frac{\omega_i}{\omega_f} = I_i \frac{1.30 \text{ rev/s}}{0.80 \text{ rev/s}} = 1.625 I_i \approx 1.6 I_i$$

The rotational inertia has increased by a factor of $\boxed{1.6}$.

54. There is no net torque on the diver because the only external force (gravity) passes through the center of mass of the diver. Thus the angular momentum of the diver is conserved. Subscript 1 refers to the tuck position, and subscript 2 refers to the straight position.

$$L_1 = L_2 \rightarrow I_1 \omega_1 = I_2 \omega_2 \rightarrow \omega_2 = \omega_1 \frac{I_1}{I_2} = \left(\frac{2 \text{ rev}}{1.5 \text{ sec}} \right) \left(\frac{1}{3.5} \right) = \boxed{0.38 \text{ rev/s}}$$

55. The skater's angular momentum is constant, since no external torques are applied to her.

$$L_i = L_f \rightarrow I_i \omega_i = I_f \omega_f \rightarrow I_f = I_i \frac{\omega_i}{\omega_f} = (4.6 \text{ kg}\cdot\text{m}^2) \frac{0.50 \text{ rev/s}}{3.0 \text{ rev/s}} = \boxed{0.77 \text{ kg}\cdot\text{m}^2}$$

She accomplishes this by starting with her arms extended (initial angular velocity) and then $\boxed{\text{pulling her arms in to the center of her body}}$ (final angular velocity).

56. Because there is no external torque applied to the wheel-clay system, the angular momentum will be conserved. We assume that the clay is thrown with no angular momentum so that its initial angular momentum is 0. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the clay and the wheel. Subscript 1 represents before the clay is thrown, and subscript 2 represents after the clay is thrown.

$$\begin{aligned} L_1 &= L_2 \rightarrow I_1 \omega_1 = I_2 \omega_2 \rightarrow \\ \omega_2 &= \omega_1 \frac{I_1}{I_2} = \frac{I_{\text{wheel}}}{I_{\text{wheel}} + I_{\text{clay}}} = \omega_1 \left(\frac{\frac{1}{2} M_{\text{wheel}} R_{\text{wheel}}^2}{\frac{1}{2} M_{\text{wheel}} R_{\text{wheel}}^2 + \frac{1}{2} M_{\text{clay}} R_{\text{clay}}^2} \right) \\ &= (1.5 \text{ rev/s}) \left[\frac{(5.0 \text{ kg})(0.20 \text{ m})^2}{(5.0 \text{ kg})(0.20 \text{ m})^2 + (3.1 \text{ kg})(8.0 \times 10^{-2} \text{ m})^2} \right] = 1.36 \text{ rev/s} \approx \boxed{1.4 \text{ rev/s}} \end{aligned}$$

$$\boxed{57.} (a) \quad L = I \omega = \frac{1}{2} M R^2 \omega = \frac{1}{2} (55 \text{ kg})(0.15 \text{ m})^2 \left(3.5 \frac{\text{rev}}{\text{s}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = \boxed{14 \text{ kg}\cdot\text{m}^2/\text{s}}$$

- (b) If the rotational inertia does not change, then the change in angular momentum is strictly due to a change in angular velocity.

$$\tau = \frac{\Delta L}{\Delta t} = \frac{0 - 14 \text{ kg}\cdot\text{m}^2/\text{s}}{5.0 \text{ s}} = \boxed{-2.7 \text{ m}\cdot\text{N}}$$

The negative sign indicates that the torque is in the opposite direction as the initial angular momentum.

58. (a) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$\begin{aligned} L_{\text{daily}} &= I \omega_{\text{daily}} = \left(\frac{2}{5} M R_{\text{Earth}}^2 \right) \omega_{\text{daily}} \\ &= \frac{2}{5} (6.0 \times 10^{24} \text{ kg})(6.4 \times 10^6 \text{ m})^2 \left[\left(\frac{2\pi \text{ rad}}{1 \text{ day}} \right) \left(\frac{1 \text{ day}}{86,400 \text{ s}} \right) \right] = \boxed{7.1 \times 10^{33} \text{ kg}\cdot\text{m}^2/\text{s}} \end{aligned}$$

- (b) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$L_{\text{daily}} = I\omega_{\text{daily}} = \left(MR_{\text{Sun-Earth}}^2 \right) \omega_{\text{daily}}$$

$$= (6.0 \times 10^{24} \text{ kg})(1.5 \times 10^{11} \text{ m})^2 \left[\left(\frac{2\pi \text{ rad}}{365 \text{ day}} \right) \left(\frac{1 \text{ day}}{86,400 \text{ s}} \right) \right] = \boxed{2.7 \times 10^{40} \text{ kg}\cdot\text{m}^2/\text{s}}$$

59. Since there are no external torques on the system, the angular momentum of the 2-disk system is conserved. The two disks have the same final angular velocity.

$$L_i = L_f \rightarrow I\omega + I(0) = 2I\omega_f \rightarrow \boxed{\omega_f = \frac{1}{2}\omega}$$

60. The angular momentum of the disk – rod combination will be conserved because there are no external torques on the combination. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the disk and the rod. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The rod has no initial angular momentum.

$$L_1 = L_2 \rightarrow I_1\omega_1 = I_2\omega_2 \rightarrow$$

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \omega_1 \frac{I_{\text{disk}}}{I_{\text{disk}} + I_{\text{rod}}} = \omega_1 \left[\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + \frac{1}{12}M(2R)^2} \right] = (2.4 \text{ rev/s}) \left(\frac{3}{5} \right) = \boxed{1.4 \text{ rev/s}}$$

61. Since the person is walking radially, no torques will be exerted on the person-platform system, and so angular momentum will be conserved. The person will be treated as a point mass. Since the person is initially at the center, they have no initial rotational inertia.

(a) $L_i = L_f \rightarrow I_{\text{platform}}\omega_i = (I_{\text{platform}} + I_{\text{person}})\omega_f$

$$\omega_f = \frac{I_{\text{platform}}}{I_{\text{platform}} + mR^2} \omega_i = \frac{920 \text{ kg}\cdot\text{m}^2}{920 \text{ kg}\cdot\text{m}^2 + (75 \text{ kg})(3.0 \text{ m})^2} (2.0 \text{ rad/s}) = 1.154 \text{ rad/s} \approx \boxed{1.2 \text{ rad/s}}$$

(b) $KE_i = \frac{1}{2}I_{\text{platform}}\omega_i^2 = \frac{1}{2}(920 \text{ kg}\cdot\text{m}^2)(2.0 \text{ rad/s})^2 = \boxed{1.8 \times 10^3 \text{ J}}$

$$KE_f = \frac{1}{2}(I_{\text{platform}} + I_{\text{person}})\omega_f^2 = \frac{1}{2}(I_{\text{platform}} + m_{\text{person}}r_{\text{person}}^2)\omega_f^2$$

$$= \frac{1}{2}[920 \text{ kg}\cdot\text{m}^2 + (75 \text{ kg})(3.0 \text{ m})^2](1.154 \text{ rad/s})^2 = 1062 \text{ J} \approx \boxed{1.1 \times 10^3 \text{ J}}$$

62. The angular momentum of the merry-go-round and people combination will be conserved because there are no external torques on the combination. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the merry-go-round and the people. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The people have no initial angular momentum.

$$L_1 = L_2 \rightarrow I_1\omega_1 = I_2\omega_2 \rightarrow$$

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \omega_1 \frac{I_{\text{m-g-r}}}{I_{\text{m-g-r}} + I_{\text{people}}} = \omega_1 \left[\frac{I_{\text{m-g-r}}}{I_{\text{m-g-r}} + 4M_{\text{person}}R^2} \right]$$

$$= (0.80 \text{ rad/s}) \left[\frac{1760 \text{ kg}\cdot\text{m}^2}{1760 \text{ kg}\cdot\text{m}^2 + 4(65 \text{ kg})(2.1 \text{ m})^2} \right] = \boxed{0.48 \text{ rad/s}}$$

If the people jump off the merry-go-round radially, then they exert no torque on the merry-go-round, and thus cannot change the angular momentum of the merry-go-round. The merry-go-round would continue to rotate at $\boxed{0.80 \text{ rad/s}}$.

63. Since the lost mass carries away no angular momentum, the angular momentum of the remaining mass will be the same as the initial angular momentum.

$$L_i = L_f \rightarrow I_i \omega_i = I_f \omega_f \rightarrow \frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{\frac{2}{5} M_i R_i^2}{\frac{2}{5} M_f R_f^2} = \frac{M_i R_i^2}{(0.5 M_i) (0.01 R_f)^2} = 2.0 \times 10^4$$

$$\omega_f = 2.0 \times 10^4 \omega_i = 2.0 \times 10^4 \left(\frac{2\pi \text{ rad}}{30 \text{ day}} \right) \left(\frac{1 \text{ d}}{86400 \text{ s}} \right) = 4.848 \times 10^{-2} \text{ rad/s} \approx \boxed{5 \times 10^{-2} \text{ rad/s}}$$

The period would be a factor of 20,000 smaller, which would make it about 130 seconds. The ratio of angular kinetic energies of the spinning mass would be

$$\frac{KE_f}{KE_i} = \frac{\frac{1}{2} I_f \omega_f^2}{\frac{1}{2} I_i \omega_i^2} = \frac{\frac{1}{2} \left[\frac{2}{5} (0.5 M_i) (0.01 R_i)^2 \right] (2.0 \times 10^4 \omega_i)^2}{\frac{1}{2} \left(\frac{2}{5} M_i R_i^2 \right) \omega_i^2} = 2.0 \times 10^4 \rightarrow \boxed{KE_f = 2 \times 10^4 KE_i}$$

64. For our crude estimate, we model the hurricane as a rigid cylinder of air. Since the “cylinder” is rigid, each part of it has the same angular velocity. The mass of the air is the product of the density of air times the volume of the air cylinder.

$$M = \rho V = \rho \pi R^2 h = (1.3 \text{ kg/m}^3) \pi (1.00 \times 10^5 \text{ m})^2 (4.0 \times 10^3 \text{ m}) = 1.634 \times 10^{14} \text{ kg}$$

$$(a) \quad KE = \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{1}{2} MR^2 \right) (v_{\text{edge}}/R)^2 = \frac{1}{4} M v_{\text{edge}}^2$$

$$= \frac{1}{4} (1.634 \times 10^{14} \text{ kg}) \left[(120 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \right]^2 = 4.539 \times 10^{16} \text{ J} \approx \boxed{5 \times 10^{16} \text{ J}}$$

$$(b) \quad L = I \omega = \left(\frac{1}{2} MR^2 \right) (v_{\text{edge}}/R) = \frac{1}{2} MR v_{\text{edge}}$$

$$= \frac{1}{2} (1.634 \times 10^{14} \text{ kg}) (1.00 \times 10^5 \text{ m}) \left[(120 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \right] = 2.723 \times 10^{20} \text{ kg} \cdot \text{m}^2/\text{s}$$

$$\approx \boxed{3 \times 10^{20} \text{ kg} \cdot \text{m}^2/\text{s}}$$

65. Angular momentum will be conserved in the Earth – asteroid system, since all forces and torques are internal to the system. The initial angular velocity of the satellite, just before collision, can be found from $\omega_{\text{asteroid}} = v_{\text{asteroid}}/R_{\text{Earth}}$. Assuming the asteroid becomes imbedded in the Earth at the surface, the Earth and the asteroid will have the same angular velocity after the collision. We model the Earth as a uniform sphere, and the asteroid as a point mass.

$$L_i = L_f \rightarrow I_{\text{Earth}} \omega_{\text{Earth}} + I_{\text{asteroid}} \omega_{\text{asteroid}} = (I_{\text{Earth}} + I_{\text{asteroid}}) \omega_f$$

The moment of inertia of the satellite can be ignored relative to that of the Earth on the right side of the above equation, and so the percent change in Earth’s angular velocity is found as follows.

$$I_{\text{Earth}} \omega_{\text{Earth}} + I_{\text{asteroid}} \omega_{\text{asteroid}} = I_{\text{Earth}} \omega_f \rightarrow \frac{(\omega_f - \omega_{\text{Earth}})}{\omega_{\text{Earth}}} = \frac{I_{\text{asteroid}} \omega_{\text{asteroid}}}{I_{\text{Earth}} \omega_{\text{Earth}}}$$

$$\begin{aligned} \% \text{ change} &= \frac{(\omega_f - \omega_{\text{Earth}})}{\omega_{\text{Earth}}} (100) = \frac{m_{\text{asteroid}} R_{\text{Earth}}^2 \frac{v_{\text{asteroid}}}{R_{\text{Earth}}}}{\frac{2}{5} M_{\text{Earth}} R_{\text{Earth}}^2 \omega_{\text{Earth}}} = \frac{m_{\text{asteroid}} v_{\text{asteroid}}}{\frac{2}{5} M_{\text{Earth}} \omega_{\text{Earth}} R_{\text{Earth}}} (100) \\ &= \frac{(1.0 \times 10^5 \text{ kg})(3.0 \times 10^4 \text{ m/s})}{(0.4)(5.97 \times 10^{24} \text{ kg}) \left(\frac{2\pi \text{ rad}}{86400 \text{ s}} \right) (6.38 \times 10^6 \text{ m})} (100) = \boxed{2.7 \times 10^{-16} \%} \end{aligned}$$

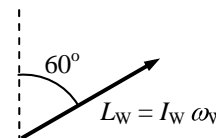
66. When the person and the platform rotate, they do so about the vertical axis. Initially there is no angular momentum pointing along the vertical axis, and so any change that the person – wheel – platform undergoes must result in no net angular momentum along the vertical axis.

- (a) If the wheel is moved so that its angular momentum points upwards, then the person and platform must get an equal but opposite angular momentum, which will point downwards. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

$$L_i = L_f \rightarrow 0 = I_w \omega_w + I_p \omega_p \rightarrow \boxed{\omega_p = -\frac{I_w}{I_p} \omega_w}$$

The negative sign means that the platform is rotating in the opposite direction of the wheel. If the wheel is spinning counterclockwise when viewed from above, the platform is spinning clockwise.

- (b) If the wheel is pointing at a 60° angle to the vertical, then the component of its angular momentum that is along the vertical direction is $I_w \omega_w \cos 60^\circ$. See the diagram. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.



$$L_i = L_f \rightarrow 0 = I_w \omega_w \cos 60^\circ + I_p \omega_p \rightarrow \boxed{\omega_p = -\frac{I_w}{2I_p} \omega_w}$$

Again, the negative sign means that the platform is rotating in the opposite direction of the wheel.

- (c) If the wheel is moved so that its angular momentum points downwards, then the person and platform must get an equal but opposite angular momentum, which will point upwards. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

$$L_i = L_f \rightarrow 0 = -I_w \omega_w + I_p \omega_p \rightarrow \boxed{\omega_p = \omega_w I_w / I_p}$$

The platform is rotating in the same direction as the wheel. If the wheel is spinning counterclockwise when viewed from above, the platform is also spinning counterclockwise.

- (d) Since the total angular momentum is 0, if the wheel is stopped from rotating, the platform will also stop. Thus $\boxed{\omega_p = 0}$.

67. The angular momentum of the person – turntable system will be conserved. Call the direction of the person's motion the positive rotation direction. Relative to the ground, the person's speed will be $v + v_T$, where v is the person's speed relative to the turntable, and v_T is the speed of the rim of the turntable with respect to the ground. The turntable's angular speed is $\omega_T = v_T / R$, and the person's

angular speed relative to the ground is $\omega_p = \frac{v + v_T}{R} = \frac{v}{R} + \omega_T$. The person is treated as a point particle for calculation of the moment of inertia.

$$L_i = L_f \rightarrow 0 = I_T \omega_T + I_P \omega_p = I_T \omega_T + mR^2 \left(\omega_T + \frac{v}{R} \right) \rightarrow$$

$$\omega_T = -\frac{mRv}{I_T + mR^2} = -\frac{(55 \text{ kg})(3.25 \text{ m})(3.8 \text{ m/s})}{1700 \text{ kg}\cdot\text{m}^2 + (55 \text{ kg})(3.25 \text{ m})^2} = \boxed{-0.30 \text{ rad/s}}$$

68. Since the spool rolls without slipping, each point on the edge of the spool moves with a speed of $v = r\omega = v_{\text{CM}}$ relative to the center of the spool, where v_{CM} is the speed of the center of the spool relative to the ground. Since the spool is moving to the right relative to the ground, and the top of the spool is moving to the right relative to the center of the spool, the top of the spool is moving with a speed of $2v_{\text{CM}}$ relative to the ground. This is the speed of the rope, assuming it is unrolling without slipping and is at the outer edge of the spool. The speed of the rope is the same as the speed of the person, since the person is holding the rope. So the person is walking with a speed of twice that of the center of the spool. Thus if the person moves forward a distance L , in the same time the center of the spool, traveling with half the speed, moves forward a distance $\boxed{L/2}$. The rope, to stay connected both to the person and to the spool, must therefore unwind by an amount $\boxed{L/2}$ also.

69. The spin angular momentum of the Moon can be calculated by $L_{\text{spin}} = I_{\text{spin}} \omega_{\text{spin}} = \frac{2}{5} MR_{\text{Moon}}^2 \omega_{\text{spin}}$. The orbital angular momentum can be calculated by $L_{\text{orbit}} = I_{\text{orbit}} \omega_{\text{orbit}} = MR_{\text{orbit}}^2 \omega_{\text{orbit}}$. Because the same side of the Moon always faces the Earth, $\omega_{\text{spin}} = \omega_{\text{orbit}}$.

$$\frac{L_{\text{spin}}}{L_{\text{orbit}}} = \frac{\frac{2}{5} MR_{\text{Moon}}^2 \omega_{\text{spin}}}{MR_{\text{orbit}}^2 \omega_{\text{orbit}}} = \frac{2}{5} \left(\frac{R_{\text{Moon}}}{R_{\text{orbit}}} \right)^2 = 0.4 \left(\frac{1.74 \times 10^6 \text{ m}}{3.84 \times 10^8 \text{ m}} \right)^2 = \boxed{8.21 \times 10^{-6}}$$

70. As discussed in section 8-3, from the reference frame of the axle of the wheel, the points on the wheel are all moving with the same speed of $v = r\omega$, where v is the speed of the axle of the wheel relative to the ground. The top of the tire has a velocity of v to the right relative to the axle, so it has a velocity of $2v$ to the right relative to the ground.

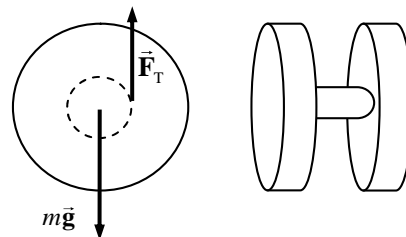
$$\vec{v}_{\text{top rel ground}} = \vec{v}_{\text{top rel center}} + \vec{v}_{\text{center rel ground}} = (v \text{ to the right}) + (v \text{ to the right}) = 2v \text{ to the right}$$

$$v_{\text{top rel ground}} = 2v = 2(v_0 + at) = 2at = 2(1.00 \text{ m/s}^2)(3.0 \text{ s}) = \boxed{6.0 \text{ m/s}}$$

71. The torque is found from $\tau = I\alpha$. The angular acceleration can be found from $\omega = \omega_o + \alpha t$, with an initial angular velocity of 0. The rotational inertia is that of a cylinder.

$$\tau = I\alpha = \frac{1}{2} MR^2 \left(\frac{\omega - \omega_o}{t} \right) = 0.5(1.4 \text{ kg})(0.20 \text{ m})^2 \frac{(1800 \text{ rev/s})(2\pi \text{ rad/rev})}{6.0 \text{ s}} = \boxed{53 \text{ m}\cdot\text{N}}$$

72. (a) There are two forces on the yo-yo: gravity and the string tension. If we assume that the top of the string is held in a fixed position, then the tension does no work, and so mechanical energy is conserved. The initial gravitational PE is converted into rotational and translational KE. Since the yo-yo rolls without slipping at the point of contact of the string, the velocity of the CM is simply related to the angular velocity of the yo-yo: $v_{\text{CM}} = r\omega$, where r is the



radius of the inner hub. Let m be the mass of the inner hub, and M and R be the mass and radius of each outer disk. Calculate the rotational inertia of the yo-yo about its CM, and then use conservation of energy to find the linear speed of the CM. We take the 0 of gravitational PE to be at the bottom of its fall.

$$I_{\text{CM}} = \frac{1}{2}mr^2 + 2\left(\frac{1}{2}MR^2\right) = \frac{1}{2}mr^2 + MR^2$$

$$= \frac{1}{2}(5.0 \times 10^{-3} \text{ kg})(5.0 \times 10^{-3} \text{ m})^2 + (5.0 \times 10^{-2} \text{ kg})(3.75 \times 10^{-2} \text{ m})^2 = 7.038 \times 10^{-5} \text{ kg}\cdot\text{m}^2$$

$$m_{\text{total}} = m + 2M = 5.0 \times 10^{-3} \text{ kg} + 2(5.0 \times 10^{-2} \text{ kg}) = 0.105 \text{ kg}$$

$$PE_i = KE_f \rightarrow$$

$$m_{\text{total}}gh = \frac{1}{2}m_{\text{total}}v_{\text{CM}}^2 + \frac{1}{2}I_{\text{CM}}\omega^2 = \frac{1}{2}m_{\text{total}}v_{\text{CM}}^2 + \frac{1}{2}\frac{I_{\text{CM}}}{r^2}v_{\text{CM}}^2 = \left(\frac{1}{2}m_{\text{total}} + \frac{1}{2}\frac{I_{\text{CM}}}{r^2}\right)v_{\text{CM}}^2 \rightarrow$$

$$v_{\text{CM}} = \sqrt{\frac{m_{\text{total}}gh}{\frac{1}{2}\left(m_{\text{total}} + \frac{I_{\text{CM}}}{r^2}\right)}} = \sqrt{\frac{(0.105 \text{ kg})(9.80 \text{ m/s}^2)(1.0 \text{ m})}{\frac{1}{2}\left[(0.105 \text{ kg}) + \frac{(7.038 \times 10^{-5} \text{ kg}\cdot\text{m}^2)}{(5.0 \times 10^{-3} \text{ m})^2}\right]}} = 0.8395 = \boxed{0.84 \text{ m/s}}$$

- (b) Calculate the ratio $KE_{\text{rot}}/KE_{\text{tot}}$.

$$\frac{KE_{\text{rot}}}{KE_{\text{tot}}} = \frac{KE_{\text{rot}}}{PE_{\text{tot}}} = \frac{\frac{1}{2}I_{\text{CM}}\omega^2}{m_{\text{total}}gh} = \frac{\frac{1}{2}\frac{I_{\text{CM}}}{r^2}v_{\text{CM}}^2}{m_{\text{total}}gh} = \frac{I_{\text{CM}}v_{\text{CM}}^2}{2r^2m_{\text{total}}gh}$$

$$= \frac{(7.038 \times 10^{-5} \text{ kg}\cdot\text{m}^2)(0.8395 \text{ m/s})^2}{2(5.0 \times 10^{-3} \text{ m})^2(0.105 \text{ kg})(9.8 \text{ m/s}^2)(1.0 \text{ m})} = 0.96 = \boxed{96\%}$$

73. (a) The linear speed of the chain must be the same as it passes over both sprockets. The linear speed is related to the angular speed by $v = \omega R$, and so

$$\omega_R R_R = \omega_F R_F.$$

If the spacing of the teeth on the sprockets is a distance d , then the number of teeth on a sprocket times the spacing distance must give the circumference of the sprocket.

$$Nd = 2\pi R \text{ and so } R = \frac{Nd}{2\pi}. \text{ Thus } \omega_R \frac{N_R d}{2\pi} = \omega_F \frac{N_F d}{2\pi} \rightarrow \boxed{\frac{\omega_R}{\omega_F} = \frac{N_F}{N_R}}$$

(b) $\boxed{\omega_R/\omega_F = 52/13 = 4.0}$

(c) $\boxed{\omega_R/\omega_F = 42/28 = 1.5}$

74. Since the lost mass carries away no angular momentum, the angular momentum of the remaining mass will be the same as the initial angular momentum.

$$L_i = L_f \rightarrow I_i \omega_i = I_f \omega_f$$

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{\frac{2}{5} M_i R_i^2}{\frac{2}{5} M_f R_f^2} = \frac{(8.0 M_{\text{Sun}})(6.96 \times 10^8 \text{ m})^2}{(0.25)(8.0 M_{\text{Sun}})(1.1 \times 10^4 \text{ m})^2} = 1.601 \times 10^{10}$$

$$\omega_f = 1.601 \times 10^{10} \omega_i = 1.601 \times 10^{10} \left(\frac{1 \text{ rev}}{12 \text{ day}} \right) = 1.334 \times 10^9 \text{ rev/day}$$

$$\approx \boxed{1.3 \times 10^9 \text{ rev/day} = 1.5 \times 10^4 \text{ rev/s}}$$

75. (a) The initial energy of the flywheel is used for two purposes – to give the car translational kinetic energy 20 times, and to replace the energy lost due to friction, from air resistance and from braking. The statement of the problem leads us to ignore any gravitational potential energy changes.

$$W_{\text{fr}} = KE_{\text{final}} - KE_{\text{initial}} \rightarrow F_{\text{fr}} \Delta x \cos 180^\circ = \frac{1}{2} M_{\text{car}} v_{\text{car}}^2 - KE_{\text{flywheel}}$$

$$KE_{\text{flywheel}} = F_{\text{fr}} \Delta x + \frac{1}{2} M_{\text{car}} v_{\text{car}}^2$$

$$= (450 \text{ N})(3.5 \times 10^5 \text{ m}) + (20) \frac{1}{2} (1400 \text{ kg}) \left[(95 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \right]^2$$

$$= 1.672 \times 10^8 \text{ J} \approx \boxed{1.7 \times 10^8 \text{ J}}$$

(b) $KE_{\text{flywheel}} = \frac{1}{2} I \omega^2$

$$\omega = \sqrt{\frac{2 KE}{I}} = \sqrt{\frac{2 KE}{\frac{1}{2} M_{\text{flywheel}} R_{\text{flywheel}}^2}} = \sqrt{\frac{2(1.672 \times 10^8 \text{ J})}{\frac{1}{2} (240 \text{ kg})(0.75 \text{ m})^2}} = \boxed{2.2 \times 10^3 \text{ rad/s}}$$

- (c) To find the time, use the relationship that $\text{Power} = \frac{\text{Work}}{t}$, where the work done by the motor will be equal to the kinetic energy of the flywheel.

$$P = \frac{W}{t} \rightarrow t = \frac{W}{P} = \frac{(1.672 \times 10^8 \text{ J})}{(150 \text{ hp})(746 \text{ W/hp})} = 1.494 \times 10^3 \text{ s} \approx \boxed{25 \text{ min}}$$

76. The mass of a hydrogen atom is 1.01 atomic mass units. The atomic mass unit is $1.66 \times 10^{-27} \text{ kg}$. Since the axis passes through the oxygen atom, it will have no rotational inertia.

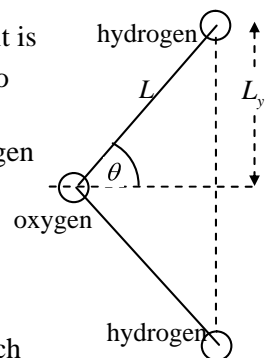
- (a) If the axis is perpendicular to the plane of the molecule, then each hydrogen atom is a distance L from the axis of rotation.

$$I_{\text{perp}} = 2m_H L^2 = 2(1.01)(1.66 \times 10^{-27} \text{ kg})(0.96 \times 10^{-9} \text{ m})^2$$

$$= \boxed{3.1 \times 10^{-45} \text{ kg}\cdot\text{m}^2}$$

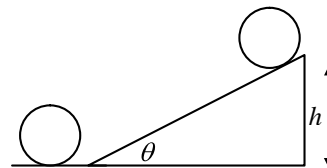
- (b) If the axis is in the plane of the molecule, bisecting the H-O-H bonds, each hydrogen atom is a distance of $L_y = L \sin \theta = (9.6 \times 10^{-10} \text{ m}) \sin 52^\circ$

$$= 7.564 \times 10^{-10} \text{ m}. \text{ Thus the moment of inertia is}$$



$$I_{\text{plane}} = 2m_H L_y^2 = 2(1.01)(1.66 \times 10^{-27} \text{ kg})(7.564 \times 10^{-10} \text{ m})^2 = \boxed{1.9 \times 10^{-45} \text{ kg} \cdot \text{m}^2}$$

77. (a) Assuming that there are no dissipative forces doing work, conservation of energy may be used to find the final height h of the hoop. Take the bottom of the incline to be the zero level of gravitational potential energy. We assume that the hoop is rolling without sliding, so that $\omega = v/R$. Relate the conditions at the bottom of the incline to the conditions at the top by conservation of energy. The hoop has both translational and rotational kinetic energy at the bottom, and the rotational inertia of the hoop is given by $I = mR^2$.



$$E_{\text{bottom}} = E_{\text{top}} \rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh \rightarrow \frac{1}{2}mv^2 + \frac{1}{2}mR^2 \frac{v^2}{R^2} = mgh \rightarrow$$

$$h = \frac{v^2}{g} = \frac{(3.3 \text{ m/s})^2}{9.8 \text{ m/s}^2} = 1.111 \text{ m}$$

The distance along the plane is given by $d = \frac{h}{\sin \theta} = \frac{1.111 \text{ m}}{\sin 15^\circ} = 4.293 \text{ m} \approx \boxed{4.3 \text{ m}}$

- (b) The time can be found from the constant accelerated linear motion. Use the relationship

$$\Delta x = \frac{1}{2}(v + v_o)t \rightarrow t = \frac{2\Delta x}{v + v_o} = \frac{2(4.293 \text{ m})}{0 + 3.3 \text{ m/s}} = 2.602 \text{ s}.$$

This is the time to go up the plane. The time to come back down the plane is the same, and so the total time is $\boxed{5.2 \text{ s}}$.

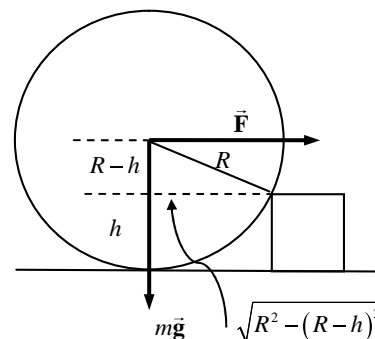
78. (a) The force of gravity acting through the CM will cause a clockwise torque which produces an angular acceleration. At the moment of release, the force of gravity is perpendicular to the lever arm from the hinge to the CM.

$$\tau = I\alpha \rightarrow \alpha = \frac{\tau_{\text{gravity}}}{I_{\text{rod about end}}} = \frac{Mg L/2}{\frac{1}{3}ML^2} = \boxed{\frac{3g}{2L}}$$

- (b) At the end of the rod, there is a tangential acceleration equal to the angular acceleration times the distance from the hinge. There is no radial acceleration since at the moment of release, the speed of the end of the rod is 0. Thus the tangential acceleration is the entire linear acceleration.

$$a_{\text{linear}} = a_{\text{tan}} = \alpha L = \boxed{\frac{3}{2}g}$$

79. The wheel is rolling about the point of contact with the step, and so all torques are to be taken about that point. As soon as the wheel is off the floor, there will be only two forces that can exert torques on the wheel – the pulling force and the force of gravity. There will not be a normal force of contact between the wheel and the floor once the wheel is off the floor, and any force on the wheel from the point of the step cannot exert a torque about that very point. Calculate the net torque on the wheel, with clockwise torques positive. The minimum force occurs when the net torque is 0.



$$\sum \tau = F(R-h) - mg\sqrt{R^2 - (R-h)^2} = 0$$

$$F = \frac{Mg\sqrt{R^2 - (R-h)^2}}{R-h} = \boxed{\frac{Mg\sqrt{2Rh - h^2}}{R-h}}$$

80. (a) In order not to fall over, the net torque on the cyclist about an axis through the CM and parallel to the ground must be zero. Consider the free-body diagram shown. Sum torques about the CM, with counterclockwise as positive, and set the sum equal to zero.

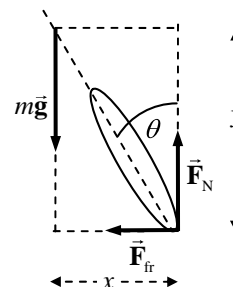
$$\sum \tau = F_N x - F_{fr} y = 0 \rightarrow \frac{F_{fr}}{F_N} = \frac{x}{y} = \tan \theta$$

- (b) The cyclist is not accelerating vertically, so $F_N = mg$. The cyclist is accelerating horizontally, because he is traveling in a circle. Thus the frictional force must be supplying the centripetal force, so $F_{fr} = mv^2/r$.

$$\tan \theta = \frac{F_{fr}}{F_N} = \frac{mv^2/r}{mg} = \frac{v^2}{rg} \rightarrow \theta = \tan^{-1} \frac{v^2}{rg} = \tan^{-1} \frac{(4.2 \text{ m/s})^2}{(6.4 \text{ m})(9.8 \text{ m/s}^2)} = 15.71^\circ \approx \boxed{16^\circ}$$

- (c) From $F_{fr} = mv^2/r$, the smallest turning radius results in the maximum force. The maximum static frictional force is $F_{fr} = \mu F_N$. Use this to calculate the radius.

$$mv^2/r_{\min} = \mu F_N = \mu mg \rightarrow r_{\min} = \frac{v^2}{\mu g} = \frac{(4.2 \text{ m/s})^2}{(0.70)(9.8 \text{ m/s}^2)} = \boxed{2.6 \text{ m}}$$

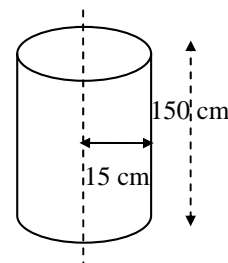


81. Assume that the angular acceleration is uniform. Then the torque required to whirl the rock is the moment of inertia of the rock (treated as a particle) times the angular acceleration.

$$\tau = I\alpha = (mr^2) \left(\frac{\omega - \omega_0}{t} \right) = \frac{(0.50 \text{ kg})(1.5 \text{ m})^2}{5.0 \text{ s}} \left[\left(120 \frac{\text{rev}}{\text{min}} \right) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \right] = \boxed{2.8 \text{ m}\cdot\text{N}}$$

That torque comes from the arm swinging the sling, and so comes from the arm muscles.

82. Assume a mass of 50 kg, corresponding to a weight of about 110 lb. From Table 7-1, we find that the total arm and hand mass is about 12.5% of the total mass, and so the rest of the body is about 87.5% of the total mass. Model the skater as a cylinder of mass 44 kg, and model each arm as a thin rod of mass 3 kg. Estimate the body as 150 cm tall with a radius of 15 cm. Estimate the arm dimension as 50 cm long.



With the arms held tightly, we approximate that the arms are part of the body cylinder. A sketch of the skater in this configuration is then as shown in the first diagram. In this configuration, the rotational inertia is

$$I_{\text{in}} = I_{\text{cylinder}} = \frac{1}{2} M_{\text{total}} R_{\text{body}}^2$$

With the arms extended, the configuration changes to the second diagram. In this configuration, the rotational inertia is

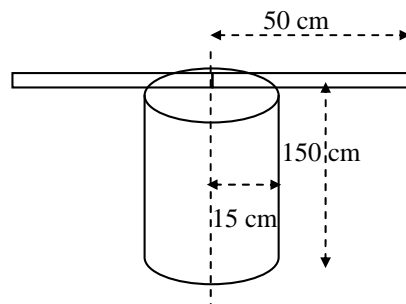
$$I_{\text{out}} = I_{\text{body}} + I_{\text{arms}} = \frac{1}{2} M_{\text{body}} R_{\text{body}}^2 + 2 \frac{1}{3} M_{\text{arm}} L_{\text{arm}}^2$$

The forces and torques involved in changing the configuration of the skater are internal to the skater, and so the skater's angular momentum is conserved during a configuration change.

Thus

$$L_{\text{in}} = L_{\text{out}} \rightarrow I_{\text{in}} \omega_{\text{in}} = I_{\text{out}} \omega_{\text{out}} \rightarrow$$

$$\begin{aligned} \frac{\omega_{\text{out}}}{\omega_{\text{in}}} &= \frac{I_{\text{in}}}{I_{\text{out}}} = \frac{\frac{1}{2} M_{\text{total}} R_{\text{body}}^2}{\frac{1}{2} M_{\text{body}} R_{\text{body}}^2 + 2 \frac{1}{3} M_{\text{arm}} L_{\text{arm}}^2} = \frac{\frac{1}{2} (50 \text{ kg}) (0.15 \text{ m})^2}{\frac{1}{2} (44 \text{ kg}) (0.15 \text{ m})^2 + 2 \frac{1}{3} (3 \text{ kg}) (0.50 \text{ m})^2} \\ &= 0.575 \approx \boxed{0.6} \end{aligned}$$



83. (a) The angular momentum of M_A will be

$$L_A = I_A \omega_1 = \frac{1}{2} M_A R^2 \omega_1 = \frac{1}{2} (6.0 \text{ kg}) (0.60 \text{ m})^2 (7.2 \text{ rad/s}) = \boxed{7.8 \text{ kg}\cdot\text{m}^2/\text{s}}$$

- (b) The torque required to accelerate M_A will be

$$\tau = \frac{\Delta L}{\Delta t} = \frac{7.8 \text{ kg}\cdot\text{m}^2/\text{s} - 0}{2.0 \text{ s}} = \boxed{3.9 \text{ m}\cdot\text{N}}$$

- (c) Since there are no torques external to the two plates, the angular momentum of the two plates will be conserved. Since the two plates stick together, they will have a common final angular velocity. This is a totally inelastic collision.

$$L_i = L_f \rightarrow I_A \omega_1 = (I_A + I_B) \omega_2 \rightarrow$$

$$\begin{aligned} \omega_2 &= \frac{I_A}{I_A + I_B} \omega_1 = \frac{\frac{1}{2} M_A R^2}{\frac{1}{2} M_A R^2 + \frac{1}{2} M_B R^2} \omega_1 = \frac{M_A}{M_A + M_B} \omega_1 = \left(\frac{6.0 \text{ kg}}{15.0 \text{ kg}} \right) (7.2 \text{ rad/s}) \\ &= \boxed{2.9 \text{ rad/s}} \end{aligned}$$

84. Since frictional losses can be ignored, energy will be conserved for the marble. Define the 0 position of PE to be the bottom of the track, so that the bottom of the ball is initially a height h above the 0 position of PE. Since $r \ll R$, the marble's CM is very close to the surface of the track. While the marble is on the loop, we then approximate that it will be moving in a circle of radius R . When the marble is at the top of the loop, we approximate that its CM is a distance of $2R$ above the 0 position of PE. For the marble to just be on the verge of leaving the track means the normal force between the marble and the track is zero, and so the centripetal force at the top must be equal to the gravitational force on the marble.

$$\frac{mv_{\text{top of loop}}^2}{R} = mg \rightarrow v_{\text{top of loop}}^2 = gR$$

We assume that the marble is rolling without slipping, and so $\omega = v/r$, and that the marble is released from rest. Use energy conservation to relate the release point to the point at the top of the loop. Note that the marble has both translational and rotational kinetic energy.

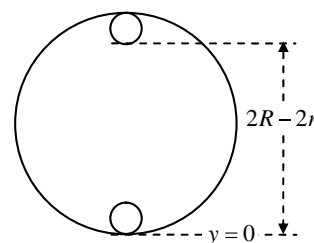
$$E_{\text{release}} = E_{\text{top of loop}} \rightarrow KE_{\text{release}} + PE_{\text{release}} = KE_{\text{top of loop}} + PE_{\text{top of loop}}$$

$$0 + mgh = \frac{1}{2}mv_{\text{top of loop}}^2 + \frac{1}{2}I\omega_{\text{top of loop}}^2 + mg2R = \frac{1}{2}mv_{\text{top of loop}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{v_{\text{top of loop}}^2}{r^2} + 2mgR$$

$$mgh = \frac{7}{10}mv_{\text{top of loop}}^2 + 2mgR = \frac{7}{10}mgR + 2mgR = 2.7mgR \rightarrow \boxed{h = 2.7R}$$

85. Since frictional losses can be ignored, energy will be conserved for the marble. Define the 0 position of PE to be bottom of the track, so that the bottom of the ball is initially a height h above the 0 position of PE. Since we are not to assume that $r \ll R$, then while the marble is on the loop portion of the track, it is moving in a circle of radius $R - r$, and when at the top of the loop, the bottom of the marble is a height of $2(R - r)$ above the 0 position of PE (see the diagram). For the marble to just be on the verge of leaving the track means the normal force between the marble and the track is zero, and so the centripetal force at the top must be equal to the gravitational force on the marble.

$$\frac{mv_{\text{top of loop}}^2}{R - r} = mg \rightarrow v_{\text{top of loop}}^2 = g(R - r)$$



We assume that the marble is rolling without slipping and so $\omega = v/r$, and that the marble is released from rest. Use energy conservation to relate the release point to the point at the top of the loop. Note that the marble has both translational and rotational kinetic energy.

$$E_{\text{release}} = E_{\text{top of loop}} \rightarrow KE_{\text{release}} + PE_{\text{release}} = KE_{\text{top of loop}} + PE_{\text{top of loop}}$$

$$0 + mgh = \frac{1}{2}mv_{\text{top of loop}}^2 + \frac{1}{2}I\omega_{\text{top of loop}}^2 + mg2(R - r) = \frac{1}{2}mv_{\text{top of loop}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{v_{\text{top of loop}}^2}{r^2} + 2mg(R - r)$$

$$mgh = \frac{7}{10}mv_{\text{top of loop}}^2 + 2mg(R - r) = \frac{7}{10}mg(R - r) + 2mg(R - r) = 2.7mg(R - r)$$

$$\boxed{h = 2.7(R - r)}$$

86. (a) The angular acceleration can be found from $\omega^2 = \omega_o^2 + 2\alpha\theta$, with the angular velocities being given by $\omega = v/r$.

$$\alpha = \frac{\omega^2 - \omega_o^2}{2\theta} = \frac{(v^2 - v_o^2)}{2r^2\theta} = \frac{\left[(60.0 \text{ km/h})^2 - (90.0 \text{ km/h})^2 \right] \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)^2}{2(0.45 \text{ m})^2 (85 \text{ rev})(2\pi \text{ rad/rev})}$$

$$= -1.6053 \text{ rad/s}^2 \approx \boxed{-1.61 \text{ rad/s}^2}$$

- (b) The time to stop can be found from $\omega = \omega_o + \alpha t$, with a final angular velocity of 0.

$$t = \frac{\omega - \omega_o}{\alpha} = \frac{v - v_o}{r\alpha} = \frac{0 - (60.0 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)}{(0.45 \text{ m})(-1.6053 \text{ rad/s}^2)} = \boxed{23 \text{ s}}$$