CHAPTER 9: Static Equilibrium; Elasticity and Fracture

Answers to Questions

1. If the object has a net force on it of zero, then its center of mass does not accelerate. But since it is not in equilibrium, it must have a net torque, and therefore have an angular acceleration. Some examples are:
   a) A compact disk in a player as it comes up to speed, after just being put in the player.
   b) A hard drive on a computer when the computer is first turned on.
   c) A window fan immediately after the power to it has been shut off.

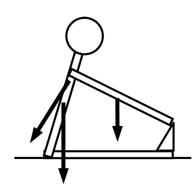
2. The bungee jumper is not in equilibrium, because the net force on the jumper is not zero. If the jumper were at rest and the net force were zero, then the jumper would stay at rest by Newton's 1st law. The jumper has a net upward force when at the bottom of the dive, and that is why the jumper is then pulled back upwards.

3. If the fingers are not the same distance from the CG, the finger closer to the CG will support a larger fraction of the weight of the meter stick so that the net torque on the stick is zero. That larger vertical force means there will be more friction between the stick and that closer finger, and thus the finger further from the CG will be easier to move. The more distant finger will slide easier, and therefore move in closer to the CG. That finger, when it becomes the one closest to the CG, will then have more friction and will "stick". The other finger will then slide. You then repeat the process. Whichever finger is farther from the CG will slide closer to it, until the two fingers eventually meet at the CG.

4. Like almost any beam balance, the movable weights are connected to the fulcrum point by relatively long lever arms, while the platform on which you stand is connected to the fulcrum point by a very short lever arm. The scale "balances" when the torque provided by your weight (large mass, small lever arm) is equal to that provided by the sliding weights (small mass, large lever arm).

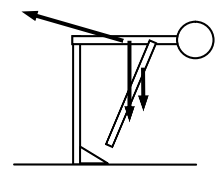
5. (*a*) If we assume that the pivot point of rotation is the lower left corner of the wall in the picture, then the gravity force acting through the CM provides the torque to keep the wall upright. Note that the gravity force would have a relatively small lever arm (about half the width of the wall) and so the sideways force would not have to be particularly large to start to move the wall.
   (*b*) With the horizontal extension, there are factors that make the wall less likely to overturn.
   - The mass of the second wall is larger, and so the torque caused by gravity (helping to keep the wall upright) will be larger for the second wall.
   - The center of gravity of the second wall is further to the right of the pivot point and so gravity exerts a larger torque to counteract the torque due to $\vec{\mathbf{F}}$.
   - The weight of the ground above the new part of the wall provides a large clockwise torque that helps to counteract the torque due to $\vec{\mathbf{F}}$.

6. For rotating the upper half body, the pivot point is near the waist and hips. In that position, the arms have a relatively small torque, even when extended, due to their smaller mass, and the more massive trunk–head combination has a very short lever arm, and so also has a relatively small torque. Thus the force of gravity on the upper body causes relatively little torque about the hips tending to rotate you forward, and so the back muscles need to produce little torque to keep you from rotating forward. The force on the upper half body due to the back muscles is small, and so the

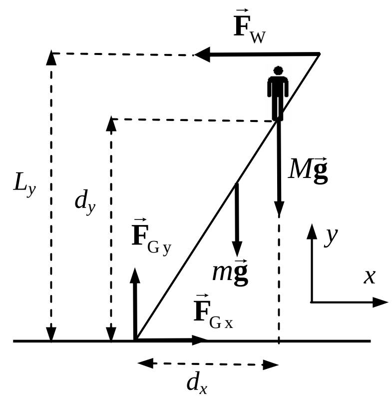


(partially rightward) force at the base of the spinal column, to keep the spine in equilibrium, will be small.

When standing and bending over, the lever arm for the upper body is much larger than while sitting, and so causes a much larger torque. The CM of the arms is also further from the support point, and so causes more torque. The back muscles, assumed to act at the center of the back, do not have a very long lever arm. Thus the back muscles will have to exert a large force to cause a counter-torque that keeps you from falling over. And accordingly, there will have to be a large force (mostly to the right in the picture) at the base of the spine to keep the spine in equilibrium.



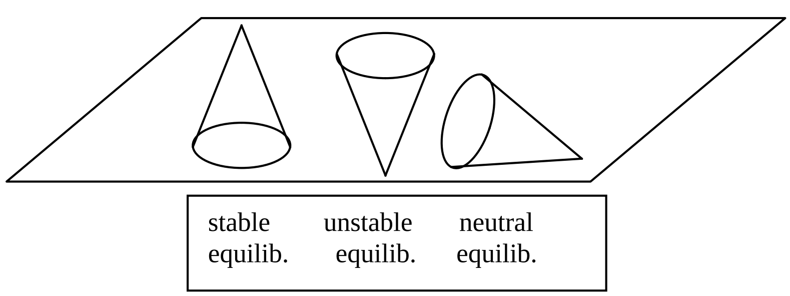
7. When the person stands near the top, the ladder is more likely to slip. In the accompanying diagram, the force of the person pushing down on the ladder $(M\mathbf{g})$ causes a clockwise torque about the contact point with the ground, with lever arm d_x. The only force causing a counterclockwise torque about that same point is the reaction force of the wall on the ladder, $\vec{\mathbf{F}}_W$. While the ladder is in equilibrium, $\vec{\mathbf{F}}_W$ will be the same magnitude as the frictional force at the ground, $\vec{\mathbf{F}}_{Gx}$. Since $\vec{\mathbf{F}}_{Gx}$ has a maximum value, $\vec{\mathbf{F}}_W$ will have the same maximum value, and so $\vec{\mathbf{F}}_W$ will have a maximum counterclockwise torque that it can exert. As the person climbs the ladder, their lever arm gets longer and so the torque due to their weight gets larger. Eventually, if the torque caused by the person is larger than the maximum torque caused by $\vec{\mathbf{F}}_W$, the ladder will start to slip – it will not stay in equilibrium.



8. The mass of the meter stick is equal to that of the rock. For purposes of calculating torques, the meter stick can be treated as if all of its mass were at the 50 cm mark. Thus the CM of the meter stick is the same distance from the pivot point as the rock, and so their masses must be the same in order to exert the same torque.

9. If the sum of the forces on an object are not zero, then the CM of the object will accelerate in the direction of the net force. If the sum of the torques on the object are zero, then the object has no angular acceleration. Some examples are:
   a) A satellite in a circular orbit around the Earth.
   b) A block sliding down an inclined plane.
   c) An object that is in projectile motion but not rotating
   d) The startup motion of an elevator, changing from rest to having a non-zero velocity.

10.



11. Configuration (b) is more likely to be stable. In configuration (a), the CG of the bottom brick is at the edge of the table, and the CG of the top brick is to the right of the edge of the table. Thus the CG of the two-brick system is not above the base of support, and so gravity will exert a torque to roll the bricks clockwise off the table. Another way to see this is that more than 50% of the brick mass is not above the base of support – 50% of the bottom brick and 75% of the top brick are to the right of the edge of the table. It is not in stable, neutral, or unstable equilibrium.

    In configuration (b), exactly half of the mass (75% of the top brick and 25% of the bottom brick) is over the edge of the table. Thus the CG of the pair is at the edge of the table – it is in unstable equilibrium.

12. When walking, you must keep your CG over your feet. If you have a heavy load in your arms, your CG is shifted forward, and so you must lean backwards to realign your CG over your feet.

13. When you rise on your tiptoes, your CM shifts forward. Since you are already standing with your nose and abdomen against the door, your CM cannot shift forward. Thus gravity exerts a torque on you and you are unable to stay on your tiptoes – you will return to being flat-footed on the floor.

14. When you start to stand up from a normal sitting position, your CM is not over your point of support (your feet), and so gravity will exert a torque about your feet that rotates you back down into the chair. You must lean forward in order that your CM be over your feet so that you can stand up.

15. In the midst of doing a sit-up, the abdomen muscles provide a torque to rotate you up away from the floor, while the force of gravity on your upper half-body is tending to pull you back down to the floor, providing the difficulty for doing sit-ups. The force of gravity on your lower half-body provides a torque that opposes the torque caused by the force of gravity on your upper half-body, making the sit-up a little easier. With the legs bent, the lever arm for the lower half-body is shorter, and so less counter-torque is available.

16. Position "A" is unstable equilibrium, position "B" is stable equilibrium, and position "C" is neutral equilibrium.

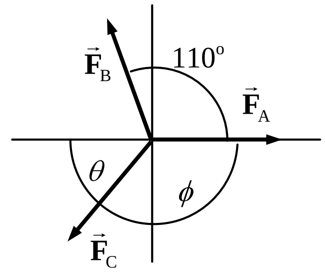
17. The Young's modulus for a bungee cord is much smaller than that for ordinary rope. From its behavior, we know that the bungee cord stretches relatively easily, compared to ordinary rope. From Eq. 9-4, we have $E = \dfrac{F/A}{\Delta L/L_o}$. The value of Young's modulus is inversely proportional to the change in length of a material under a tension. Since the change in length of a bungee cord is much larger than that of an ordinary rope if other conditions are identical (stressing force, unstretched length, cross-sectional area of rope or cord), it must have a smaller Young's modulus.

18. An object under shear stress has equal and opposite forces applied across it. One blade of the scissors pushes down on the cardboard while the other arm pushes up. These two forces cause the cardboard to shear between the two blades. Thus the name "shears" is justified.

19. The left support is under tension, since the force from the support pulls on the beam. Thus it would not be wise to use concrete or stone for that support. The right support is under compression, since it pushes on the beam. Concrete or stone would be acceptable for that support.

Solutions to Problems

1. If the tree is not accelerating, then the net force in all directions is 0.



$$\sum F_x = F_A + F_B \cos 110 + F_{Cx} = 0 \quad \rightarrow$$

$$F_{Cx} = -F_A - F_B \cos 110 = -310\text{ N} - (425\text{ N})\cos 110 = -164.6\text{ N}$$

$$\sum F_y = F_B \sin + F_{Cy} = 0 \quad \rightarrow$$

$$F_{Cy} = -F_B \sin 110 = -(425\text{ N})\sin 110 = -399.4\text{ N}$$

$$F_C = \sqrt{F_{Cx}^2 + F_{Cy}^2} = \sqrt{(-164.6\text{ N})^2 + (-399.4\text{ N})^2} = 432.0\text{ N} \approx \boxed{4.3\times10^2\text{ N}}$$

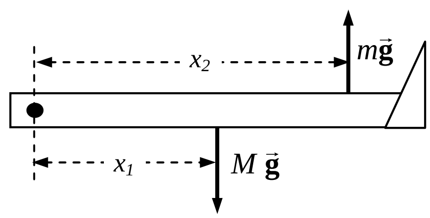
$$\theta = \tan^{-1}\frac{F_{Cy}}{F_{Cx}} = \tan^{-1}\frac{-399.4\text{ N}}{-164.6\text{ N}} = 67.6^\circ\ ,\ \phi = 180^\circ - 67.6^\circ = 112.4^\circ \approx \boxed{112^\circ}$$

And so $\vec{\mathbf{F}}_C$ is 430 N, at an angle of 112° clockwise from $\vec{\mathbf{F}}_A$.

2. The torque is the force times the lever arm.

$$\tau = Fr = (58\text{ kg})(9.8\text{ m/s}^2)(3.0\text{ m}) = \boxed{1.7\times10^3\text{ m}\bullet\text{N , clockwise}}$$

3. Because the mass m is stationary, the tension in the rope pulling up on the sling must be mg, and so the force of the sling on the leg must be mg, upward. Calculate torques about the hip joint, with counterclockwise torque taken as positive. See the free-body diagram for the leg. Note that the forces on the leg exerted by the hip joint are not drawn, because they do not exert a torque about the hip joint.

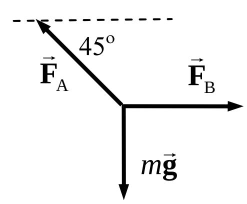


$$\sum \tau = mgx_2 - Mgx_1 = 0 \quad \rightarrow \quad m = M\frac{x_1}{x_2} = (15.0\text{ kg})\frac{(35.0\text{ cm})}{(80.5\text{ cm})} = \boxed{6.52\text{ kg}}$$

4. The torque is the force times the lever arm.

$$\tau = Fr \quad \rightarrow \quad r = \frac{\tau}{F} = \frac{1100\text{ m}\bullet\text{N}}{(58\text{ kg})(9.8\text{ m/s}^2)} = \boxed{1.9\text{ m}}$$

5. Write Newton's 2nd law for the junction, in both the x and y directions.



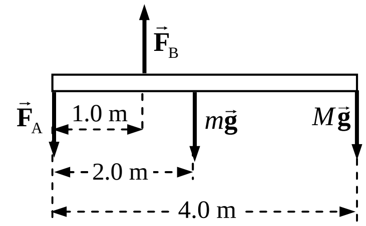
$$\sum F_x = F_B - F_A \cos 45^\circ = 0$$

From this, we see that $F_A > F_B$. Thus set $F_A = 1550\text{ N}$.

$$\sum F_y = F_A \sin 45^\circ - mg = 0$$

$$mg = F_A \sin 45^\circ = (1550\text{ N})\sin 45^\circ = \boxed{1.1\times10^3\text{ N}}$$

6. (*a*) Let $m = 0$. Calculate the net torque about the left end of the diving board, with counterclockwise torques positive. Since the board is in equilibrium, the net torque is zero.



$$\sum \tau = F_B(1.0\text{ m}) - Mg(4.0\text{ m}) = 0 \quad \rightarrow$$

$$F_B = 4Mg = 4(58\text{ kg})(9.80\text{ m/s}^2) = 2274\text{ N} \approx \boxed{2.3\times10^3\text{ N}}$$

Use Newton's 2nd law in the vertical direction to find F_A.

$$\sum F_y = F_B - Mg - F_A \quad \rightarrow$$

$$F_A = F_B - Mg = 4Mg - Mg = 3Mg = 3(58\text{ kg})(9.80\text{ m/s}^2) = 1705\text{ N} \approx \boxed{1.7\times10^3\text{ N}}$$

(*b*) Repeat the basic process, but with $m = 35$ kg. The weight of the board will add more clockwise torque.

$$\sum \tau = F_B(1.0\text{ m}) - mg(2.0\text{ m}) - Mg(4.0\text{ m}) = 0 \quad \rightarrow$$

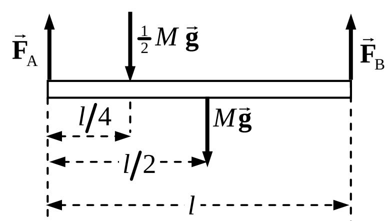
$$F_B = 4Mg + 2mg = [4(58\text{ kg}) + 2(35\text{ kg})](9.80\text{ m/s}^2) = 2960\text{ N} \approx \boxed{3.0\times10^3\text{ N}}$$

$$\sum F_y = F_B - Mg - mg - F_A \quad \rightarrow$$

$$F_A = F_B - Mg - mg = 4Mg + 2mg - Mg - mg = 3Mg + mg$$

$$= [3(58\text{ kg}) + 35\text{ kg}](9.80\text{ m/s}^2) = 2048\text{ N} \approx \boxed{2.0\times10^3\text{ N}}$$

7. The CG of each beam is at its center. Calculate torques about the left end of the beam, and take counterclockwise torques to be positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.



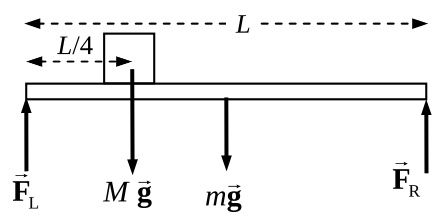
$$\sum \tau = F_B l - Mg(l/2) - \tfrac{1}{2}Mg(l/4) = 0 \quad \rightarrow$$

$$F_B = \tfrac{5}{8}Mg = \tfrac{5}{8}(940\text{ kg})(9.80\text{ m/s}^2) = 5758\text{ N} \approx \boxed{5.8\times10^3\text{ N}}$$

$$\sum F_y = F_A + F_B - Mg - \tfrac{1}{2}Mg = 0 \quad \rightarrow$$

$$F_A = \tfrac{3}{2}Mg - F_B = \tfrac{7}{8}Mg = \tfrac{7}{8}(940\text{ kg})(9.80\text{ m/s}^2) = 8061\text{ N} \approx \boxed{8.1\times10^3\text{ N}}$$

8. Let m be the mass of the beam, and M be the mass of the piano. Calculate torques about the left end of the beam, with counterclockwise torques positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.



$$\sum \tau = F_R L - mg(\tfrac{1}{2}L) - Mg(\tfrac{1}{4}L) = 0$$

$$F_R = (\tfrac{1}{2}m + \tfrac{1}{4}M)g = [\tfrac{1}{2}(140\text{ kg}) + \tfrac{1}{4}(320\text{ kg})](9.80\text{ m/s}^2) = 1.47\times10^3\text{ N}$$

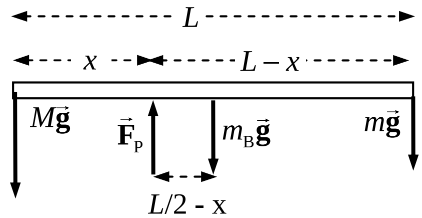
$$\sum F_y = F_L + F_R - mg - Mg = 0$$

$$F_L = (m+M)g - F_R = (460\text{ kg})(9.80\text{ m/s}^2) - 1.47\times10^3\text{ N} = 3.04\times10^3\text{ N}$$

The forces on the supports are equal in magnitude and opposite in direction to the above two results.

$$\boxed{F_R = 1.5\times10^3\text{ N down}} \qquad \boxed{F_L = 3.0\times10^3\text{ N down}}$$

9. The pivot should be placed so that the net torque on the board is zero. We calculate torques about the pivot point, with counterclockwise torques as positive. The upward force $\vec{\mathbf{F}}_P$ at the pivot point is shown, but it exerts no torque about the pivot point. The mass of the board is m_B, and the CG is at the middle of the board.



(*a*) Ignore the force $m_B g$.

$$\sum \tau = Mgx - mg(L-x) = 0 \quad \rightarrow$$

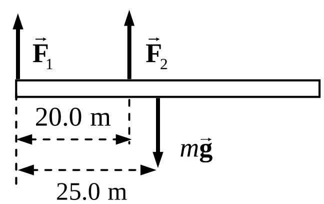
$$x = \frac{m}{m+M}L = \frac{(25\text{ kg})}{(25\text{ kg}+75\text{ kg})}(9.0\text{ m}) = 2.25\text{ m} \approx \boxed{2.3\text{ m from adult}}$$

(*b*) Include the force $m_B g$.

$$\sum \tau = Mgx - mg(L-x) - m_B g(L/2 - x) = 0$$

$$x = \frac{(m+m_B/2)}{(M+m+m_B)}L = \frac{(25\text{ kg}+7.5\text{ kg})}{(75\text{ kg}+25\text{ kg}+15\text{ kg})}(9.0\text{ m}) = 2.54\text{ m} \approx \boxed{2.5\text{ m from adult}}$$

10. Calculate torques about the left end of the beam, with counterclockwise torques positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.



$$\sum \tau = F_2(20.0\text{ m}) - mg(25.0\text{ m}) = 0 \quad \rightarrow$$

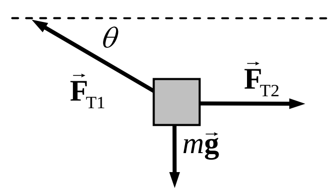
$$F_2 = \frac{25.0}{20.0}mg = (1.25)(1250\text{ kg})(9.80\text{ m/s}^2) = \boxed{1.53\times10^4\text{ N}}$$

$$\sum F_y = F_1 + F_2 - mg = 0$$

$$F_1 = mg - F_2 = mg - 1.25mg = -0.25mg = -(0.25)(1250\text{ kg})(9.80\text{ m/s}^2) = \boxed{-3.06\times10^3\text{ N}}$$

Notice that $\vec{\mathbf{F}}_1$ points down.

11. Using the free-body diagram, write Newton's second law for both the horizontal and vertical directions, with net forces of zero.



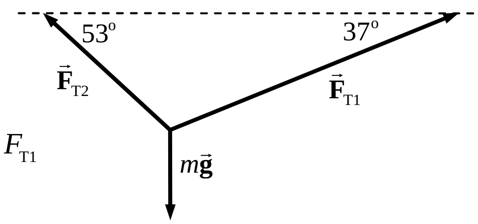
$$\sum F_x = F_{T2} - F_{T1}\cos\theta = 0 \quad \rightarrow \quad F_{T2} = F_{T1}\cos\theta$$

$$\sum F_y = F_{T1}\sin\theta - mg = 0 \quad \rightarrow \quad F_{T1} = \frac{mg}{\sin\theta}$$

$$F_{T2} = F_{T1}\cos\theta = \frac{mg}{\sin\theta}\cos\theta = \frac{mg}{\tan\theta} = \frac{(170\text{ kg})(9.80\text{ m/s}^2)}{\tan 33^\circ} = \boxed{2.6\times10^3\text{ N}}$$

$$F_{T1} = \frac{mg}{\sin\theta} = \frac{(170\text{ kg})(9.80\text{ m/s}^2)}{\sin 33^\circ} = \boxed{3.1\times10^3\text{ N}}$$

12. Draw a free-body diagram of the junction of the three wires. The tensions can be found from the conditions for force equilibrium.



$$\sum F_x = F_{T1}\cos 37^\circ - F_{T2}\cos 53^\circ = 0 \quad \rightarrow \quad F_{T2} = \frac{\cos 37^\circ}{\cos 53^\circ} F_{T1}$$

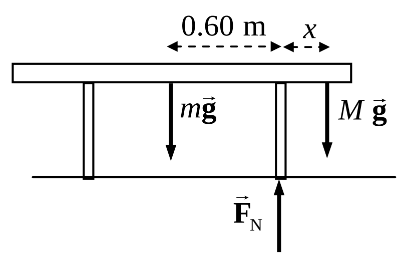
$$\sum F_y = F_{T1}\sin 37^\circ + F_{T2}\sin 53^\circ - mg = 0$$

$$F_{T1}\sin 37^\circ + \frac{\cos 37^\circ}{\cos 53^\circ} F_{T1}\sin 53^\circ - mg = 0 \quad \rightarrow$$

$$F_{T1} = \frac{(33\text{ kg})(9.8\text{ m/s}^2)}{\sin 37^\circ + \dfrac{\cos 37^\circ}{\cos 53^\circ}\sin 53^\circ} = 1.946\times10^2\text{ N} \approx \boxed{1.9\times10^2\text{ N}}$$

$$F_{T2} = \frac{\cos 37^\circ}{\cos 53^\circ} F_{T1} = \frac{\cos 37^\circ}{\cos 53^\circ}\left(1.946\times10^2\text{ N}\right) = 2.583\times10^2\text{ N} \approx \boxed{2.6\times10^2\text{ N}}$$

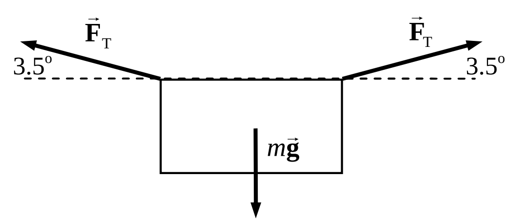
13. The table is symmetric, so the person can sit near either edge and the same distance will result. We assume that the person (mass M) is on the right side of the table, and that the table (mass m) is on the verge of tipping, so that the left leg is on the verge of lifting off the floor. There will then be no normal force between the left leg of the table and the floor. Calculate torques about the right leg of the table, so that the normal force between the table and the floor causes no torque. Counterclockwise torques are taken to be positive. The conditions of equilibrium for the table are used to find the person's location.



$$\sum \tau = mg(0.60\text{ m}) - Mgx = 0 \quad \rightarrow \quad x = (0.60\text{ m})\frac{m}{M} = (0.60\text{ m})\frac{20.0\text{ kg}}{66.0\text{ kg}} = 0.182\text{ m}$$

Thus the distance from the edge of the table is $0.50\text{ m} - 0.182\text{ m} = \boxed{0.32\text{ m}}$

14. Draw a force diagram for the sheet, and write Newton's second law for the vertical direction. Note that the tension is the same in both parts of the clothesline.

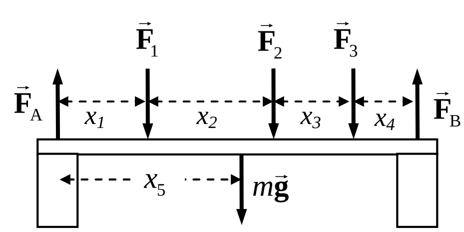


$$\sum F_y = F_T\sin 3.5^\circ + F_T\sin 3.5^\circ - mg = 0 \quad \rightarrow$$

$$F_T = \frac{mg}{2(\sin 3.5^\circ)} = \frac{(0.60\text{ kg})(9.80\text{ m/s}^2)}{2(\sin 3.5^\circ)} = \boxed{48\text{ N}}$$

The 48-N tension is much higher than the ~ 6-N weight of the sheet because of the angle. Only the vertical components of the tension are supporting the sheet, and since the angle is small, the tension has to be large to have a large enough vertical component.

15. The beam is in equilibrium, and so both the net torque and net force on it must be zero. From the free-body diagram, calculate the net torque about the center of the left support, with counterclockwise torques as positive. Calculate the net force, with upward as positive. Use those two equations to find F_A and F_B.



$$\sum \tau = F_B(x_1+x_2+x_3+x_4) - F_1x_1 - F_2(x_1+x_2) - F_3(x_1+x_2+x_3) - mgx_5$$

$$F_B = \frac{F_1x_1 + F_2(x_1+x_2) + F_3(x_1+x_2+x_3) + mgx_5}{(x_1+x_2+x_3+x_4)}$$

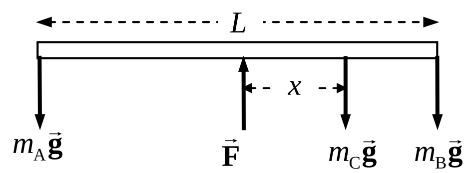
$$= \frac{(4300\text{ N})(2.0\text{ m}) + (3100\text{ N})(6.0\text{ m}) + (2200\text{ N})(9.0\text{ m}) + (250\text{ kg})(9.8\text{ m/s}^2)(5.0\text{ m})}{10.0\text{ m}}$$

$$= 5925\text{ N} \approx \boxed{5.9\times10^3\text{ N}}$$

$$\sum F = F_A + F_B - F_1 - F_2 - F_3 - mg = 0$$

$$F_A = F_1 + F_2 + F_3 + mg - F_B = 9600\text{ N} + (250\text{ kg})(9.8\text{ m/s}^2) - 5925\text{ N} = 6125\text{ N} \approx \boxed{6.1\times10^3\text{ N}}$$

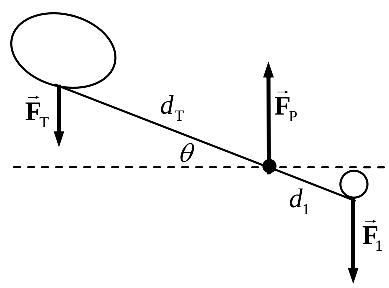
16. From the free-body diagram, the conditions of equilibrium are used to find the location of the girl (mass m_C). The 50-kg boy is represented by m_A, and the 35-kg girl by m_B. Calculate torques about the center of the see-saw, and take counterclockwise torques to be positive. The upward force of the fulcrum on the see-saw $(\vec{\mathbf{F}})$ causes no torque about the center.



$$\sum \tau = m_A g\left(\tfrac{1}{2}L\right) - m_C g x - m_B g\left(\tfrac{1}{2}L\right) = 0$$

$$x = \frac{(m_A - m_B)}{m_C}\left(\tfrac{1}{2}L\right) = \frac{(50\text{ kg} - 35\text{ kg})}{25\text{ kg}}\tfrac{1}{2}(3.6\text{ m}) = \boxed{1.1\text{ m}}$$

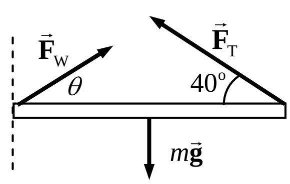
17. Since each half of the forceps is in equilibrium, the net torque on each half of the forceps is zero. Calculate torques with respect to an axis perpendicular to the plane of the forceps, through point P, counterclockwise being positive. Consider a force diagram for one half of the forceps. $\vec{\mathbf{F}}_1$ is the force on the half-forceps due to the plastic rod, and force $\vec{\mathbf{F}}_P$ is the force on the half-forceps from the pin joint. $\vec{\mathbf{F}}_P$ does not exert any torque about point P.



$$\sum \tau = F_T d_T \cos\theta - F_1 d_1 \cos\theta = 0 \;\rightarrow\; F_1 = F_T\frac{d_T}{d_1} = (11.0\text{ N})\frac{8.50\text{ cm}}{2.70\text{ cm}} = 34.6\text{ N}$$

The force that the forceps exerts on the rod is the opposite of $\vec{\mathbf{F}}_1$, and so is also $\boxed{34.6\text{ N}}$.

18. The beam is in equilibrium, and so the net force and net torque on the beam must be zero. From the free-body diagram for the beam, calculate the net torque (counterclockwise positive) about the wall support point to find $\vec{\mathbf{F}}_T$, and calculate the net force in both the *x* and *y* directions to find the components of $\vec{\mathbf{F}}_W$.



$$\sum \tau = F_T L \sin 40^\circ - mg\,L/2 = 0 \;\rightarrow$$

$$F_T = \frac{mg}{2\sin 40^\circ} = \frac{(27\text{ kg})(9.80\text{ m/s}^2)}{2\sin 40^\circ} = 205.8\text{ N} \approx \boxed{2.1\times10^2\text{ N}}$$

$$\sum F_x = F_{Wx} - F_T \cos 40^\circ = 0 \quad \rightarrow \quad F_{Wx} = F_T \cos 40^\circ = (205.8 \text{ N}) \cos 40^\circ = 157.7 \text{ N}$$

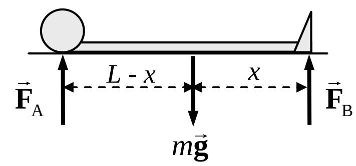
$$\sum F_y = F_{Wy} + F_T \sin 40^\circ - mg = 0 \quad \rightarrow$$

$$F_{Wy} = mg - F_T \sin 40^\circ = (27 \text{ kg})(9.80 \text{ m/s}^2) - (205.8 \text{ N}) \sin 40^\circ = 132.3 \text{ N}$$

$$F_W = \sqrt{F_{Wx}^2 + F_{Wx}^2} = \sqrt{(157.7 \text{ N})^2 + (132.3 \text{ N})^2} = 205.8 \text{ N} \approx \boxed{2.1 \times 10^2 \text{ N}}$$

$$\theta = \tan^{-1} \frac{F_{Wy}}{F_{Wx}} = \tan^{-1} \frac{132.3}{157.7} = \boxed{40^\circ}$$

19. The person is in equilibrium, and so both the net torque and net force must be zero. From the free-body diagram, calculate the net torque about the center of gravity, with counterclockwise torques as positive. Use that calculation to find the location of the center of gravity, a distance x from the feet.

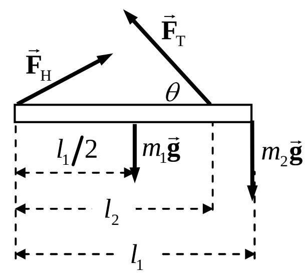


$$\sum \tau = F_B x - F_A (L - x) = 0$$

$$x = \frac{F_A}{F_A + F_B} L = \frac{m_A g}{m_A g + m_B g} L = \frac{m_A}{m_A + m_B} L = \frac{35.1 \text{ kg}}{31.6 \text{ kg} + 35.1 \text{ kg}} (1.72 \text{ m}) = \boxed{9.05 \times 10^{-1} \text{ m}}$$

The center of gravity is about 90.5 cm from the feet.

20. The beam is in equilibrium. Use the conditions of equilibrium to calculate the tension in the wire and the forces at the hinge. Calculate torques about the hinge, and take counterclockwise torques to be positive.



$$\sum \tau = (F_T \sin \theta) l_2 - m_1 g\, l_1/2 - m_2 g l_1 = 0 \quad \rightarrow$$

$$F_T = \frac{\frac{1}{2} m_1 g l_1 + m_2 g l_1}{l_2 \sin \theta} = \frac{\frac{1}{2}(155 \text{ N})(1.70 \text{ m}) + (245 \text{ N})(1.70 \text{ m})}{(1.35 \text{ m})(\sin 35.0^\circ)}$$

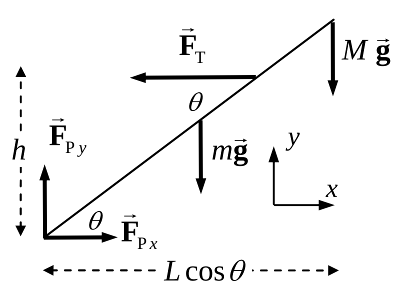
$$= 708.0 \text{ N} \approx \boxed{7.08 \times 10^2 \text{ N}}$$

$$\sum F_x = F_{Hx} - F_T \cos \theta = 0 \quad \rightarrow \quad F_{Hx} = F_T \cos \theta = (708 \text{ N}) \cos 35.0^\circ = 579.99 \text{ N} \approx \boxed{5.80 \times 10^2 \text{ N}}$$

$$\sum F_y = F_{Hy} + F_T \sin \theta - m_1 g - m_2 g = 0 \quad \rightarrow$$

$$F_{Hy} = m_1 g + m_2 g - F_T \sin \theta = 155 \text{ N} + 245 \text{ N} - (708 \text{ N}) \sin 35.0^\circ = -6.092 \text{ N} \approx \boxed{-6 \text{ N (down)}}$$

21. (*a*) The pole is in equilibrium, and so the net torque on it must be zero. From the free-body diagram, calculate the net torque about the lower end of the pole, with counterclockwise torques as positive. Use that calculation to find the tension in the cable. The length of the pole is L.



$$\sum \tau = F_T h - mg (L/2) \cos \theta - MgL \cos \theta = 0$$

$$F_T = \frac{(m/2 + M) gL \cos \theta}{h}$$

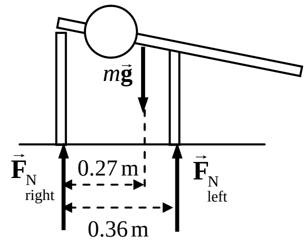
$$= \frac{(6.0 \text{ kg} + 21.5 \text{ kg})(9.80 \text{ m/s}^2)(7.50 \text{ m}) \cos 37^\circ}{3.80 \text{ m}} = 424.8 \text{ N} \approx \boxed{4.25 \times 10^2 \text{ N}}$$

(*b*) The net force on the pole is also zero since it is in equilibrium. Write Newton's 2nd law in both the x and y directions to solve for the forces at the pivot.

$$\sum F_x = F_{Px} - F_T = 0 \rightarrow F_{Px} = F_T = \boxed{4.25\times10^2\ \text{N}}$$

$$\sum F_y = F_{Py} - mg - Mg = 0 \rightarrow F_{Py} = (m+M)g = (33.5\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{3.28\times10^2\ \text{N}}$$

22. The man is in equilibrium, so the net force and the net torque on him are both zero. From the force diagram, write an expression for the net torque about a vertical axis through his right hand, with counterclockwise torques as positive. Also write an expression for the net force in the vertical direction.



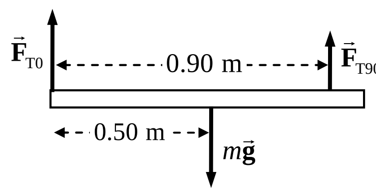
$$\sum \tau = F_{N\,\text{left}}(0.36\ \text{m}) - mg(0.27\ \text{m}) = 0 \rightarrow$$

$$F_{N\,\text{left}} = mg\frac{0.27}{0.36} = (72\ \text{kg})(9.80\ \text{m/s}^2)\frac{0.27}{0.36} = 529.2\ \text{N} \approx \boxed{5.3\times10^2\ \text{N}}$$

$$\sum F_y = F_{N\,\text{left}} + F_{N\,\text{right}} - mg = 0 \rightarrow$$

$$F_{N\,\text{right}} = mg - F_{N\,\text{left}} = (72\ \text{kg})(9.80\ \text{m/s}^2) - 529.2\ \text{N} = \boxed{1.8\times10^2\ \text{N}}$$

23. (*a*) The meter stick is in equilibrium, so the net torque and the net force are both zero. From the force diagram, write an expression for the net torque about the 90-cm mark, with counterclockwise torques as positive.



$$\sum \tau = mg(0.40\ \text{m}) - F_{T0}(0.90\ \text{m}) = 0 \rightarrow$$

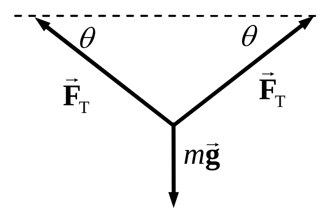
$$F_{T0} = mg\frac{0.40}{0.90} = (0.180\ \text{kg})(9.80\ \text{m/s}^2)\frac{0.40}{0.90} = \boxed{0.78\ \text{N}}$$

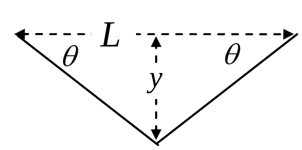
(*b*) Write Newton's 2nd law for the vertical direction with a net force of 0 to find the other tension.

$$\sum F_y = F_{T0} + F_{T90} - mg = 0 \rightarrow$$

$$F_{T90} = mg - F_{T0} = (0.180\ \text{kg})(9.80\ \text{m/s}^2) - 0.78\ \text{N} = \boxed{0.98\ \text{N}}$$

$\boxed{24.}$ Since the backpack is midway between the two trees, the angles in the diagram are equal. Write Newton's 2nd law for the vertical direction for the point at which the backpack is attached to the cord, with the weight of the backpack being the downward vertical force. The angle is determined by the distance between the trees and the amount of sag at the midpoint, as illustrated in the second diagram.





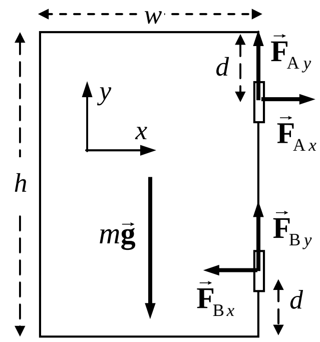
(*a*) $\theta = \tan^{-1}\dfrac{y}{L/2} = \tan^{-1}\dfrac{1.5\ \text{m}}{3.8\ \text{m}} = 21.5^\circ$

$$\sum F_y = 2F_T\sin\theta_1 - mg = 0 \rightarrow$$

$$F_T = \frac{mg}{2\sin\theta_1} = \frac{(19\ \text{kg})(9.80\ \text{m/s}^2)}{2\sin 21.5^\circ} = \boxed{2.5\times10^2\ \text{N}}$$

(*b*) $\theta = \tan^{-1}\dfrac{y}{L/2} = \tan^{-1}\dfrac{0.15\ \text{m}}{3.8\ \text{m}} = 2.26^\circ$ $\quad F_T = \dfrac{mg}{2\sin\theta_1} = \dfrac{(19\ \text{kg})(9.80\ \text{m/s}^2)}{2\sin 2.26^\circ} = \boxed{2.4\times10^3\ \text{N}}$

25. The forces on the door are due to gravity and the hinges. Since the door is in equilibrium, the net torque and net force must be zero. Write the three equations of equilibrium. Calculate torques about the bottom hinge, with counterclockwise torques as positive. From the statement of the problem, $F_{Ay} = F_{By} = \frac{1}{2}mg$.



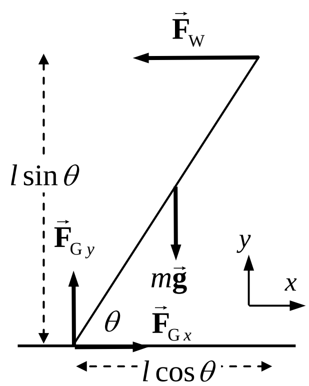
$$\sum \tau = mg\frac{w}{2} - F_{Ax}(h-2d) = 0$$

$$F_{Ax} = \frac{mgw}{2(h-2d)} = \frac{(13.0\text{ kg})(9.80\text{ m/s}^2)(1.30\text{ m})}{2(2.30\text{ m} - 0.80\text{ m})} = \boxed{55.2\text{ N}}$$

$$\sum F_x = F_{Ax} - F_{Bx} = 0 \rightarrow F_{Bx} = F_{Ax} = \boxed{55.2\text{ N}}$$

$$\sum F_y = F_{Ay} + F_{By} - mg = 0 \rightarrow F_{Ay} = F_{By} = \tfrac{1}{2}mg = \tfrac{1}{2}(13.0\text{ kg})(9.8\text{ m/s}^2) = \boxed{63.7\text{ N}}$$

26. Write the conditions of equilibrium for the ladder, with torques taken about the bottom of the ladder, and counterclockwise torques as positive.



$$\sum \tau = F_W l\sin\theta - mg\frac{l}{2}\cos\theta = 0 \rightarrow F_W = \tfrac{1}{2}\frac{mg}{\tan\theta}$$

$$\sum F_x = F_{Gx} - F_W = 0 \rightarrow F_{Gx} = F_W = \tfrac{1}{2}\frac{mg}{\tan\theta}$$

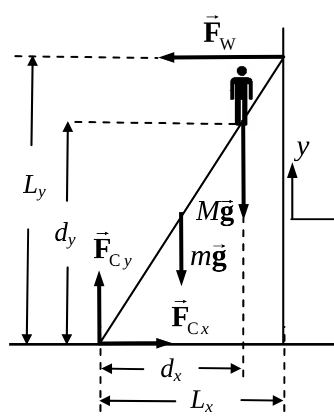
$$\sum F_y = F_{Gy} - mg = 0 \rightarrow F_{Gy} = mg$$

For the ladder to not slip, the force at the ground F_{Gx} must be less than or equal to the maximum force of static friction.

$$F_{Gx} \le \mu F_N = \mu F_{Gy} \rightarrow \tfrac{1}{2}\frac{mg}{\tan\theta} \le \mu mg \rightarrow \frac{1}{2\mu} \le \tan\theta \rightarrow \theta \ge \tan^{-1}(1/2\mu)$$

Thus the minimum angle is $\boxed{\theta_{min} = \tan^{-1}(1/2\mu)}$.

27. The ladder is in equilibrium, so the net torque and net force must be zero. By stating that the ladder is on the verge of slipping, the static frictional force at the ground, F_{Cx} is at its maximum value and so $F_{Cx} = \mu_s F_{Cy}$. Since the person is standing 70% of the way up the ladder, the height of the ladder is $L_y = d_y/0.7 = 2.8\text{ m}/0.7 = 4.0\text{ m}$. The width of the ladder is $L_x = d_x/0.7 = 2.1\text{ m}/0.7 = 3.0\text{ m}$. Torques are taken about the point of contact of the ladder with the ground, and counterclockwise torques are taken as positive. The three conditions of equilibrium are as follows.



$$\sum F_x = F_{Cx} - F_W = 0 \rightarrow F_{Cx} = F_W$$

$$\sum F_y = F_{Gy} - Mg - mg = 0 \quad \rightarrow$$
$$F_{Gy} = (M+m)g = (67.0\text{ kg})(9.80\,\text{m/s}^2) = 656.6\,\text{N}$$
$$\sum \tau = F_W L_y - mg\left(\tfrac{1}{2}L_x\right) - Mgd_x = 0$$

Solving the torque equation gives

$$F_W = \frac{\frac{1}{2}mL_x + Md_x}{L_y}g = \frac{\frac{1}{2}(12.0\text{ kg})(3.0\text{ m}) + (55.0\text{ kg})(2.1\text{ m})}{4.0\text{ m}}(9.80\,\text{m/s}^2) = 327.1\text{ N}.$$

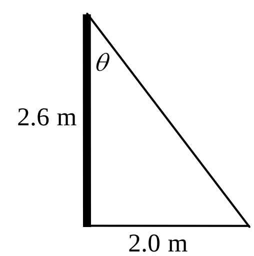
The coefficient of friction then is found to be

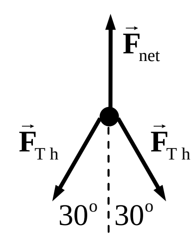
$$\mu_s = \frac{F_{Gx}}{F_{Gy}} = \frac{327.1\text{ N}}{656.6\text{ N}} = \boxed{0.50}.$$

28. If the lamp is just at the point of tipping, then the normal force will be acting at the edge of the base, 12 cm from the lamp stand pole. We assume the lamp is in equilibrium and just on the verge of tipping, and is being pushed sideways at a constant speed. Take torques about the center of the base, with counterclockwise torques positive. Also write Newton's 2nd law for both the vertical and horizontal directions.

$$\sum F_y = F_N - mg = 0 \quad \rightarrow \quad F_N = mg \qquad \sum F_x = F_P - F_{fr} = 0 \quad \rightarrow \quad F_P = F_{fr} = \mu F_N = \mu mg$$
$$\sum \tau = F_N(0.12\text{ m}) - F_P x = 0 \quad \rightarrow \quad x = \frac{F_N}{F_P}(0.12\text{ m}) = \frac{mg}{\mu mg}(0.12\text{ m}) = \frac{0.12\text{ m}}{0.20} = \boxed{0.60\text{ m}}$$

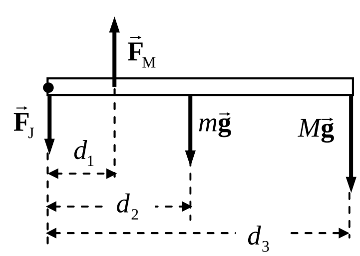
29. First consider the triangle made by the pole and one of the wires (first diagram). It has a vertical leg of 2.6 m, and a horizontal leg of 2.0 m. The angle that the tension (along the wire) makes with the vertical is $\theta = \tan^{-1}\frac{2.0}{2.6} = 37.6^\circ$. The part of the tension that is parallel to the ground is therefore $F_{Th} = F_T \sin\theta$. Now consider a top view of the pole, showing only force parallel to the ground (second diagram). The horizontal parts of the tension lie as the sides of an equilateral triangle, and so each make a 30° angle with the tension force of the net. Write the equilibrium equation for the forces along the direction of the tension in the net.





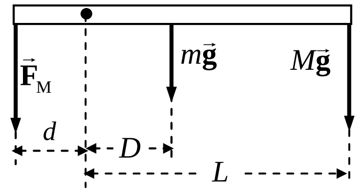
$$\sum F = F_{net} - 2F_{Th}\cos 30^\circ = 0 \quad \rightarrow$$
$$F_{net} = 2F_T \sin\theta\cos 30^\circ = 2(95\text{ N})\sin 37.6^\circ \cos 30^\circ = \boxed{1.0\times 10^2\text{ N}}$$

30. The arm is in equilibrium. Take torques about the elbow joint (the dot in the free-body diagram), so that the force at the elbow joint does not enter the calculation. Counterclockwise torques are positive. The mass of the lower arm is $m = 2.0$ kg, and the mass of the load is M. It is given that $F_M = 450$ N.



$$\sum \tau = F_M d_1 - mgd_2 - Mgd_3 = 0 \quad \rightarrow$$
$$M = \frac{F_M d_1 - mgd_2}{gd_3} = \frac{(450\text{ N})(0.060\text{ m}) - (2.0\text{ kg})(9.80\,\text{m/s}^2)(0.15\text{ m})}{(9.80\,\text{m/s}^2)(0.35\text{ m})} = \boxed{7.0\text{ kg}}$$

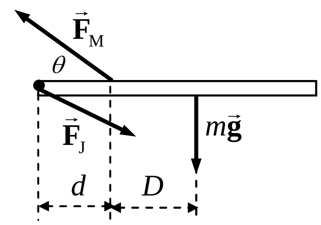
31. Calculate the torques about the elbow joint (the dot in the free body diagram). The arm is in equilibrium. Counterclockwise torques are positive.



$$\sum \tau = F_M d - mgD - MgL = 0$$

$$F_M = \frac{mD + ML}{d} g = \frac{(2.8\text{ kg})(0.12\text{ m}) + (7.3\text{ kg})(0.300\text{ m})}{0.025\text{ m}}\left(9.8\,\text{m/s}^2\right) = \boxed{9.9\times10^2\text{ N}}$$

32. (*a*) Calculate the torques about the elbow joint (the dot in the free-body diagram). The arm is in equilibrium. Take counterclockwise torques as positive.



$$\sum \tau = (F_M \sin\theta) d - mgD = 0 \quad \rightarrow$$

$$F_M = \frac{mgD}{d\sin\theta} = \frac{(3.3\text{ kg})\left(9.80\,\text{m/s}^2\right)(0.24\text{ m})}{(0.12\text{ m})\sin 15^\circ} = \boxed{2.5\times10^2\text{ N}}$$

(*b*) To find the components of F_J, write Newton's 2nd law for both the *x* and *y* directions. Then combine them to find the magnitude.

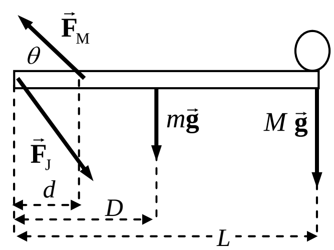
$$\sum F_x = F_{Jx} - F_M\cos\theta = 0 \quad \rightarrow \quad F_{Jx} = F_M\cos\theta = (250\text{ N})\cos 15^\circ = 241\text{ N}$$

$$\sum F_y = F_M\sin\theta - mg - F_{Jy} = 0 \quad \rightarrow$$

$$F_{Jy} = F_M\sin\theta - mg = (250\text{ N})\sin 15^\circ - (3.3\text{ kg})\left(9.80\,\text{m/s}^2\right) = 32\text{ N}$$

$$\mathrm{F}_J = \sqrt{F_{Jx}^2 + F_{Jy}^2} = \sqrt{(241\text{ N})^2 + (32\text{ N})^2} = 243.5\text{ N} \approx \boxed{2.4\times10^2\text{ N}}$$

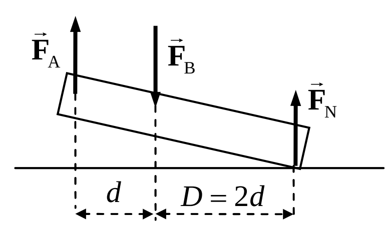
33. Calculate the torques about the shoulder joint, which is at the left end of the free-body diagram of the arm. Since the arm is in equilibrium, the sum of the torques will be zero. Take counterclockwise torques to be positive. The force due to the shoulder joint is drawn, but it does not exert any torque about the shoulder joint.



$$\sum \tau = F_m d\sin\theta - mgD - MgL = 0$$

$$F_m = \frac{mD + ML}{d\sin\theta} g = \frac{(3.3\text{ kg})(0.24\text{ cm}) + (15\text{ kg})(0.52\text{ m})}{(0.12\text{ m})\sin 15^\circ}\left(9.8\,\text{m/s}^2\right) = \boxed{2.7\times10^3\text{ N}}$$

34. There will be a normal force upwards at the ball of the foot, equal to the person's weight $(F_N = mg)$. Calculate torques about a point on the floor directly below the leg bone (and so in line with the leg bone force, $\vec{\mathbf{F}}_B$). Since the foot is in equilibrium, the sum of the torques will be zero. Take counterclockwise torques as positive.



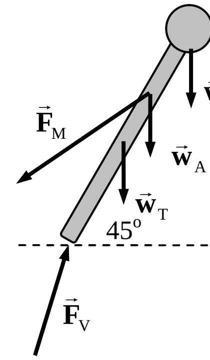
$$\sum \tau = F_N(2d) - F_A d = 0 \quad \rightarrow$$

$$F_A = 2F_N = 2mg = 2(72\text{ kg})\left(9.80\,\text{m/s}^2\right) = \boxed{1.4\times10^3\text{ N}}$$

The net force in the *y* direction must be zero. Use that to find F_B.

$$\sum F_y = F_N + F_A - F_B = 0 \quad \rightarrow \quad F_B = F_N + F_A = 2mg + mg = 3mg = \boxed{2.1\times10^3\text{ N}}$$

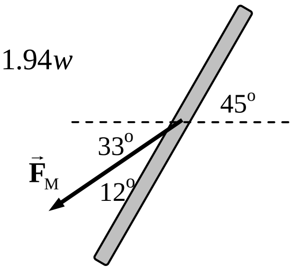
35. Figures 9-14 (b) and (c) are redrawn here with the person 45° from the horizontal, instead of the original 30°. The distances are all the same as in the original problem. We still assume that the back muscles pull at a 12° angle to the spine. The 18° angle from the original problem becomes 33°. Torques are taken about the same point at the base of the spine, with counterclockwise torques as positive.



$$\sum \tau = (0.48\text{ m})F_M \sin 12^\circ - (0.72\text{ m})(w_H)\sin 45^\circ - (0.48\text{ m})(w_A)\sin 45^\circ - (0.36\text{ m})(w_T)\sin 45^\circ = 0$$

As in the original problem, $w_H = 0.07w$, $w_A = 0.12w$, $w_T = 0.46w$. With this, the torque equation gives the following result.

$$F_M = \frac{[(0.72\text{ m})(0.07) + (0.48\text{ m})(0.12) + (0.36\text{ m})(0.46)]}{(0.48\text{ m})\sin 12^\circ} w \sin 45^\circ = 1.94w$$



Take the sum of the forces in the vertical direction, set equal to zero.

$$\sum F_y = F_{Vy} - F_M \sin 33^\circ - 0.07w - 0.12w - 0.46w = 0 \quad\to\quad F_{Vy} = 1.71w$$

Take the sum of the forces in the horizontal direction, set equal to zero.

$$\sum F_x = F_{Vx} - F_M \cos 33^\circ = 0 \quad\to\quad F_{Vy} = 1.63w$$

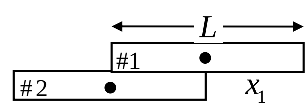
The final result is

$$F_V = \sqrt{F_{Vx}^2 + F_{Vy}^2} = \boxed{2.4w}$$

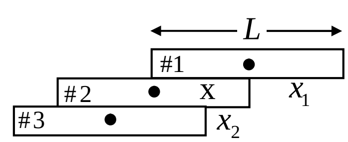
This compares to 2.5*w* for the more bent position.

36. From Section 9-4: "An object whose CG is above its base of support will be stable if a vertical line projected downward from the CG falls within the base of support." For the tower, the base of support is a circle of radius 3.5 m. If the top is 4.5 m off center, then the CG will be 2.25 m off center, and a vertical line downward from the CG will be 2.25 m from the center of the base. Thus the tower is in $\boxed{\text{stable equilibrium}}$. To be unstable, the CG has to be more than 3.5 m off center, and thus the top must be more than 7.0 m off center. Thus the top will have to lean $\boxed{2.5\text{ m}}$ further to reach the verge of instability.

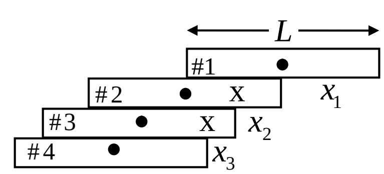
37. (*a*) The maximum distance for brick #1 to remain on brick #2 will be reached when the CM of brick #1 is directly over the edge of brick #2. Thus brick #1 will overhang brick #2 by $x_1 = L/2$.



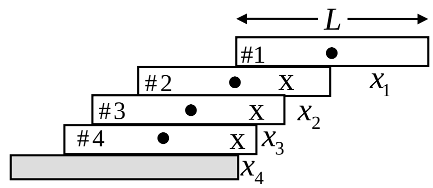
The maximum distance for the top two bricks to remain on brick #3 will be reached when the center of mass of the top two bricks is directly over the edge of brick #3. The CM of the top two bricks is (obviously) at the point labeled x on brick #2, a distance of $L/4$ from the right edge of brick #2. Thus $x_2 = L/4$.



The maximum distance for the top three bricks to remain on brick #4 will be reached when the center of mass of the top three bricks is directly over the edge of brick #4. The CM of the top three bricks is at the point labeled x on brick #3, and is found relative to the center of brick # 3 by



$$\text{CM} = \frac{m(0) + 2m(L/2)}{3m} = L/3 \text{, or } L/6 \text{ from the right edge of brick \#3. Thus } x_3 = L/6 .$$



The maximum distance for the four bricks to remain on a tabletop will be reached when the center of mass of the four bricks is directly over the edge of the table. The CM of all four bricks is at the point labeled x on brick #4, and is found relative to the center of brick #4 by

$$\text{CM} = \frac{m(0) + 3m(L/2)}{4m} = 3L/8 \text{, or } L/8 \text{ from the right}$$

edge of brick #4. Thus $x_4 = L/8$.

(*b*) From the last diagram, the distance from the edge of the tabletop to the right edge of brick #1 is

$$x_4 + x_3 + x_2 + x_1 = (L/8) + (L/6) + (L/4) + (L/2) = 25L/24 > L$$

Since this distance is greater than *L*, the answer is $\boxed{\text{yes}}$, the first brick is completely beyond the edge of the table.

(*c*) From the work in part (a), we see that the general formula for the total distance spanned by *n* bricks is

$$x_1 + x_2 + x_3 + \cdots x_n = (L/2) + (L/4) + (L/6) + \cdots + (L/2n) = \boxed{\sum_{i=1}^{n} \frac{L}{2i}}$$

(*d*) The arch is to span 1.0 m, so the span from one side will be 0.50 m. Thus we must solve $\sum_{i=1}^{n} \frac{0.30\,\text{m}}{2i} \geq 0.50\,\text{m}$. Evaluation of this expression for various values of *n* shows that 15 bricks will span a distance of 0.498 m, and that 16 bricks will span a distance of 0.507 m. Thus it takes 16 bricks for each half-span, plus 1 brick on top and 1 brick as the base on each side (as in Fig. 9-67(b)), for a total of $\boxed{\text{35 bricks}}$.

38. The amount of stretch can be found using the elastic modulus in Eq. 9-4.

$$\Delta L = \frac{1}{E}\frac{F}{A}L_0 = \frac{1}{5\times10^9\ \text{N/m}^2}\frac{275\ \text{N}}{\pi\left(5.00\times10^{-4}\right)^2}(0.300\ \text{m}) = \boxed{2.10\times10^{-2}\,\text{m}}$$

$\boxed{39.}$ (*a*) $\text{Stress} = \frac{F}{A} = \frac{mg}{A} = \frac{(25000\ \text{kg})(9.8\,\text{m/s}^2)}{1.2\text{m}^2} = 2.042\times10^5\ \text{N/m}^2 \approx \boxed{2.0\times10^5\ \text{N/m}^2}$

(*b*) $\text{Strain} = \frac{\text{Stress}}{\text{Young's Modulus}} = \frac{2.042\times10^5\ \text{N/m}^2}{50\times10^9\ \text{N/m}^2} = \boxed{4.1\times10^{-6}}$

40. The change in length is found from the strain.

$$\text{Strain} = \frac{\Delta L}{L_0} \quad\rightarrow\quad \Delta L = L_0(\text{Strain}) = (9.6\ \text{m})\left(4.1\times10^{-6}\right) = \boxed{3.9\times10^{-5}\,\text{m}}$$

41. (*a*) $\text{Stress} = \frac{F}{A} = \frac{mg}{A} = \frac{(2100\ \text{kg})(9.8\,\text{m/s}^2)}{0.15\text{m}^2} = 1.372\times10^5\ \text{N/m}^2 \approx \boxed{1.4\times10^5\ \text{N/m}^2}$

(*b*) $\text{Strain} = \frac{\text{Stress}}{\text{Young's Modulus}} = \frac{1.372\times10^5\ \text{N/m}^2}{200\times10^9\ \text{N/m}^2} = 6.86\times10^{-7} \approx \boxed{6.9\times10^{-7}}$

(*c*) $\Delta L = (\text{Strain})(L_o) = (6.86\times10^{-7})(9.50\text{ m}) = \boxed{6.5\times10^{-6}\text{m}}$

42. The change in volume is given by Eq. 9-7. We assume the original pressure is atmospheric pressure, $1.0\times10^5\text{ N/m}^2$.

$$\Delta V = -V_0\frac{\Delta P}{B} = -(1000\,\text{cm}^3)\frac{(2.6\times10^6\text{ N/m}^2 - 1.0\times10^5\text{ N/m}^2)}{1.0\times10^9\text{ N/m}^2} = -2.5\,\text{cm}^3$$

$$V = V_0 + \Delta V = 1000\,\text{cm}^3 - 2.5\,\text{cm}^3 = \boxed{997\,\text{cm}^3}$$

43. The Young's Modulus is the stress divided by the strain.

$$\text{Young's Modulus} = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L_o} = \frac{(13.4\text{ N})\Big/\left[\pi\left(\frac{1}{2}\times8.5\times10^{-3}\text{m}\right)^2\right]}{(3.7\times10^{-3}\text{m})/(15\times10^{-2}\text{m})} = \boxed{9.6\times10^6\text{ N/m}^2}$$

44. The relationship between pressure change and volume change is given by Eq. 9-7.

$$\Delta V = -V_0\frac{\Delta P}{B} \quad\rightarrow\quad \Delta P = -\frac{\Delta V}{V_0}B = -(0.10\times10^{-2})(90\times10^9\text{ N/m}^2) = \boxed{9.0\times10^7\text{ N/m}^2}$$

$$\frac{\Delta P}{P_{\text{atm}}} = \frac{9.0\times10^7\text{ N/m}^2}{1.0\times10^5\text{ N/m}^2} = \boxed{9.0\times10^2}$$

45. The percentage change in volume is found by multiplying the relative change in volume by 100. The change in pressure is 199 times atmospheric pressure, since it increases from atmospheric pressure to 200 times atmospheric pressure.

$$100\frac{\Delta V}{V_o} = -100\frac{\Delta P}{B} = -100\frac{199(1.0\times10^5\text{ N/m}^2)}{90\times10^9\text{ N/m}^2} = \boxed{-2\times10^{-2}\%}$$

The negative sign indicates that the interior space got smaller.

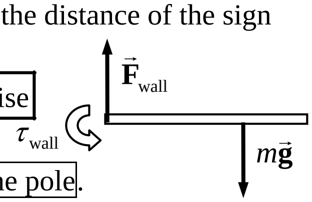
46. Elastic potential energy is given by $PE_{\text{elastic}} = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}F\Delta x$. The force is found from Eq. 9-4, using ΔL as Δx.

$$PE_{\text{elastic}} = \frac{1}{2}F\Delta x = \frac{1}{2}\left(\frac{EA}{L_0}\Delta L\right)\Delta L = \frac{1}{2}\frac{(2.0\times10^6\text{ N/m}^2)(0.50\times10^{-4}\text{m}^2)}{(3.0\times10^{-3}\text{m})}(1.0\times10^{-3}\text{m})^2$$

$$= \boxed{1.7\times10^{-2}\text{J}}$$

47. (*a*) The torque due to the sign is the product of the weight of the sign and the distance of the sign from the wall.

$$\tau = mgd = (5.1\text{ kg})(9.80\text{ m/s}^2)(2.2\text{ m}) = \boxed{1.1\times10^2\text{m}\bullet\text{N , clockwise}}$$



(*b*) Since the wall is the only other object that can put force on the pole (ignoring the weight of the pole), then $\boxed{\text{the wall must put a torque on the pole}}$. The torque due to the sign is clockwise, so the torque due to the wall must be counterclockwise. See the diagram. Also note that the wall must put an upward force on the pole as well, so that the net force on the pole will be zero.

(*c*) The torque on the rod can be considered as the wall pulling horizontally to the left on the top left corner of the rod and pushing horizontally to the right at the bottom left corner of the rod. The reaction forces to these put a shear on the wall at the point of contact. Also, since the wall is pulling upwards on the rod, the rod is pulling down on the wall at the top surface of contact, causing tension. Likewise the rod is pushing down on the wall at the bottom surface of contact, causing compression. Thus $\boxed{\text{all three are present}}$.

48. Set the compressive strength of the bone equal to the stress of the bone.

$$\text{Compressive Strength} = \frac{F_{\max}}{A} \rightarrow F_{\max} = \left(170\times10^{6}\ \text{N/m}^2\right)\left(3.0\times10^{-4}\,\text{m}^2\right) = \boxed{5.1\times10^{4}\,\text{N}}$$

49. (*a*) The maximum tension can be found from the ultimate tensile strength of the material.

$$\text{Tensile Strength} = \frac{F_{\max}}{A} \rightarrow$$

$$F_{\max} = \left(\text{Tensile Strength}\right)A = \left(500\times10^{6}\ \text{N/m}^2\right)\pi\left(5.00\times10^{-4}\,\text{m}\right)^2 = \boxed{393\ \text{N}}$$

(*b*) To prevent breakage, $\boxed{\text{thicker strings}}$ should be used, which will increase the cross-sectional area of the strings, and thus increase the maximum force. Breakage occurs because when the strings are hit by the ball, they stretch, increasing the tension. The strings are reasonably tight in the normal racket configuration, so when the tension is increased by a particularly hard hit, the tension may exceed the maximum force.

50. (*a*) Compare the stress on the bone to the compressive strength to see if the bone breaks.

$$\text{Stress} = \frac{F}{A} = \frac{3.6\times10^{4}\,\text{N}}{3.6\times10^{-4}\,\text{m}^2} = 1.0\times10^{8}\ \text{N/m}^2 < \text{Compressive Strength of bone}$$

$\boxed{\text{The bone will not break.}}$

(*b*) The change in length is calculated from Eq. 9-4.

$$\Delta L = \frac{L_0}{E}\frac{F}{A} = \left(\frac{0.22\ \text{m}}{15\times10^{9}\ \text{N/m}^2}\right)\left(1.0\times10^{8}\ \text{N/m}^2\right) = \boxed{1.5\times10^{-3}\,\text{m}}$$

51. (*a*) The area can be found from the ultimate tensile strength of the material.

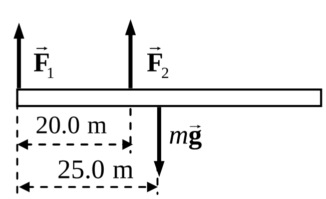
$$\frac{\text{Tensile Strength}}{\text{Safety Factor}} = \frac{F}{A}$$

$$A = F\left(\frac{\text{Safety Factor}}{\text{Tensile Strength}}\right) = \left(320\ \text{kg}\right)\left(9.8\,\text{m/s}^2\right)\frac{7.0}{500\times10^{6}\ \text{N/m}^2} = \boxed{4.4\times10^{-5}\,\text{m}^2}$$

(*b*) The change in length can be found from the stress-strain relationship, equation (9-5).

$$\frac{F}{A} = E\frac{\Delta L}{L_0} \rightarrow \Delta L = \frac{L_0 F}{AE} = \frac{\left(7.5\ \text{m}\right)\left(320\ \text{kg}\right)\left(9.8\,\text{m/s}^2\right)}{\left(4.4\times10^{-5}\,\text{m}^2\right)\left(200\times10^{9}\ \text{N/m}^2\right)} = \boxed{2.7\times10^{-3}\,\text{m}}$$

52. For each support, to find the minimum cross-sectional area with a safety factor means that $\frac{F}{A} = \frac{\text{Strength}}{\text{Safety Factor}}$, where either the tensile or compressive strength is used, as appropriate for each force. To find the force on each support, use the conditions of equilibrium for the beam. Take torques about the left end of the beam, calling counterclockwise



torques positive, and also sum the vertical forces, taking upward forces as positive.

$$\sum \tau = F_2(20.0\text{ m}) - mg(25.0\text{ m}) = 0 \quad \rightarrow \quad F_2 = \tfrac{25.0}{20.0} mg = 1.25mg$$

$$\sum F_y = F_1 + F_2 - mg = 0 \quad \rightarrow \quad F_1 = mg - F_2 = mg - 1.25mg = -0.25mg$$

Notice that the forces on the supports are the opposite of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. So the force on support # 1 is directed upwards, which means that support # 1 is in tension. The force on support # 2 is directed downwards, so support # 2 is in compression.

$$\frac{F_1}{A_1} = \frac{\text{Tensile Strength}}{8.5} \quad \rightarrow$$

$$A_1 = 8.5\frac{(0.25mg)}{\text{Tensile Strength}} = 8.5\frac{(0.25)(2.6\times10^3\text{ kg})(9.80\text{ m/s}^2)}{40\times10^6\text{ N/m}^2} = \boxed{1.4\times10^{-3}\text{ m}^2}$$

$$\frac{F_2}{A_2} = \frac{\text{Compressive Strength}}{8.5} \quad \rightarrow$$

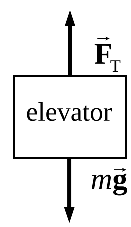
$$A_1 = 8.5\frac{(1.25mg)}{\text{Compressive Strength}} = 8.5\frac{(1.25)(2.6\times10^3\text{ kg})(9.80\text{ m/s}^2)}{35\times10^6\text{ N/m}^2} = \boxed{7.7\times10^{-3}\text{ m}^2}$$

53. The diameter can be found from the ultimate shear strength of the material.

$$\frac{\text{Shear Strength}}{\text{Safety Factor}} = \frac{F}{A} = \frac{F}{\pi(d/2)^2}$$

$$d = \sqrt{\frac{4F}{\pi}\left(\frac{\text{Safety Factor}}{\text{Shear Strength}}\right)} = \sqrt{\frac{4(3200\text{ N})}{\pi}\frac{6.0}{170\times10^6\text{ N/m}^2}} = \boxed{1.2\times10^{-2}\text{ m}}$$

54. See the free-body diagram. The largest tension will occur when the elevator has an upward acceleration. Use that with the maximum tension to calculate the diameter of the bolt, *d*. Write Newton's second law for the elevator to find the tension.



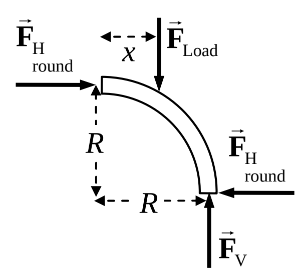
$$\sum F_y = F_T - mg = ma \quad \rightarrow$$

$$F_T = mg + ma = m(g+a) = m\left(9.8\text{ m/s}^2 + 1.2\text{ m/s}^2\right) = 11.0m$$

$$\frac{F_T}{A} = \frac{F_T}{\pi(d/2)^2} = \tfrac{1}{7}(\text{Tensile strength}) \quad \rightarrow$$

$$d = \sqrt{\frac{28F_T}{\pi(\text{Tensile strength})}} = \sqrt{\frac{28(11.0)(3.1\times10^3\text{ kg})}{\pi\left(500\times10^6\text{ N/m}^2\right)}} = \boxed{2.5\times10^{-2}\text{ m}}$$

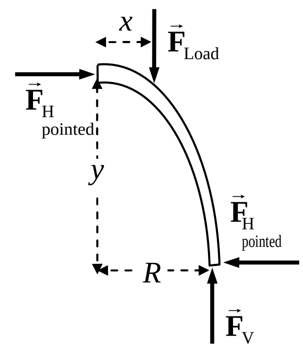
55. Draw free-body diagrams similar to Figures 9-31(a) and 9-31(b) for the forces on the right half of a round arch and a pointed arch. The load force is placed at the same horizontal position on each arch. For each half-arch, take torques about the lower right hand corner, with counterclockwise as positive.



For the round arch:

$$\sum \tau = F_{\text{Load}}(R-x) - F_{\substack{H\\ \text{round}}} R = 0 \quad \rightarrow \quad F_{\substack{H\\ \text{round}}} = F_{\text{Load}}\frac{R-x}{R}$$

For the pointed arch:



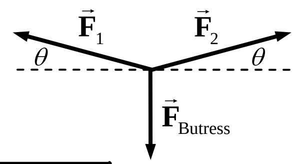
$$\sum\tau = F_{Load}\left(R-x\right) - F_{\underset{\text{pointed}}{H}}\, y = 0 \;\rightarrow\; F_{\underset{\text{pointed}}{H}} = F_{Load}\frac{R-x}{y}$$

Solve for y, given that $F_{\underset{\text{pointed}}{H}} = \frac{1}{3}F_{\underset{\text{round}}{H}}$.

$$F_{\underset{\text{pointed}}{H}} = \frac{1}{3}F_{\underset{\text{round}}{H}} \;\rightarrow\; F_{Load}\frac{R-x}{y} = \frac{1}{3}F_{Load}\frac{R-x}{R} \;\rightarrow$$

$$y = 3R = 3\left(\tfrac{1}{2}8.0\text{ m}\right) = \boxed{12\text{ m}}$$

56. Write Newton's 2nd law for the horizontal direction.



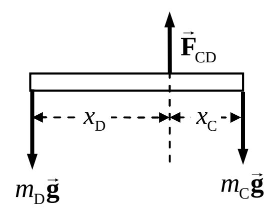
$$\sum F_x = F_2\cos\theta - F_1\cos\theta = 0 \;\rightarrow\; F_2 = F_1$$

Thus the two forces are the same size. Now write Newton's 2nd law for the vertical direction.

$$\sum F_y = F_1\sin\theta + F_1\sin\theta - F_{butress} = 0 \;\rightarrow\; F_1 = \frac{F_{butress}}{2\sin\theta} = \frac{4.3\times10^5\text{ N}}{2\left(\sin 5^\circ\right)} = \boxed{2.5\times10^6\text{ N}}$$

57. Each crossbar in the mobile is in equilibrium, and so the net torque about the suspension point for each crossbar must be 0. Counterclockwise torques will be taken as positive. The suspension point is used so that the tension in the suspension string need not be known initially. The net vertical force must also be 0.

The bottom bar:

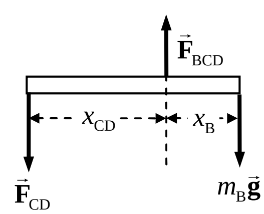


$$\sum\tau = m_D g x_D - m_C g x_C = 0 \;\rightarrow$$

$$m_C = m_D\frac{x_D}{x_C} = m_D\frac{17.50\text{ cm}}{5.00\text{ cm}} = 3.50 m_D$$

$$\sum F_y = F_{CD} - m_C g - m_D g = 0 \;\rightarrow\; F_{CD} = \left(m_C + m_D\right)g = 4.50 m_D g$$

The middle bar:



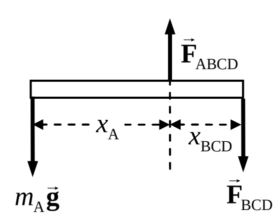
$$\sum\tau = F_{CD}x_{CD} - m_B g x_B = 0 \;\rightarrow\; F_{CD} = m_B g\frac{x_B}{x_{CD}} \;\rightarrow\; 4.50 m_D g = m_B g\frac{x_B}{x_{CD}}$$

$$m_D = \frac{m_B}{4.50}\frac{x_B}{x_{CD}} = \frac{\left(0.885\text{ kg}\right)\left(5.00\text{ cm}\right)}{\left(4.50\right)\left(15.00\text{ cm}\right)} = 0.06555 \approx \boxed{6.56\times10^{-2}\text{kg}}$$

$$m_C = 3.50 m_D = \left(3.50\right)\left(0.06555\text{kg}\right) = \boxed{2.29\times10^{-1}\text{kg}}$$

$$\sum F_y = F_{BCD} - F_{CD} - m_B g = 0 \;\rightarrow\; F_{BCD} = F_{CD} + m_B g = \left(4.50 m_D + m_B\right)g$$

The top bar:

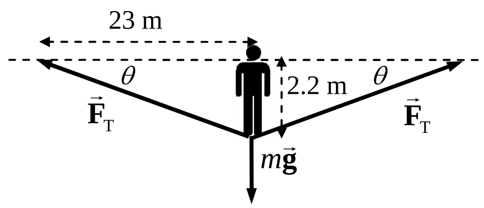


$$\sum\tau = m_A g x_A - F_{BCD}x_{BCD} = 0 \;\rightarrow$$

$$m_A = \frac{\left(4.50 m_D + m_B\right)g x_{BCD}}{g x_A} = \left(4.50 m_D + m_B\right)\frac{x_{BCD}}{x_A}$$

$$= \left[\left(4.50\right)\left(0.06555\text{ kg}\right) + 0.885\text{ kg}\right]\frac{7.50\text{ cm}}{30.00\text{ cm}} = \boxed{2.94\times10^{-1}\text{kg}}$$

58. From the free-body diagram (not to scale), write the force equilibrium condition for the vertical direction.

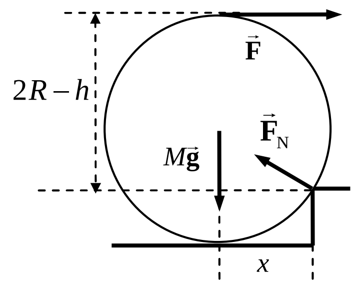


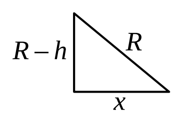
$$\sum F_y = 2T\sin\theta - mg = 0$$

$$T = \frac{mg}{2\sin\theta} = \frac{(60.0\text{ kg})(9.80\,\text{m/s}^2)}{2\left(\dfrac{2.2\text{ m}}{23\text{ m}}\right)} = \boxed{3.1\times10^3\,\text{N}}$$

It is not possible to increase the tension so that there is no sag. There must always be a vertical component of the tension to balance the gravity force. The larger the tension gets, the smaller the sag angle will be, however.

59. (*a*) If the wheel is just lifted off the lowest level, then the only forces on the wheel are the horizontal pull, its weight, and the contact force $\vec{\mathbf{F}}_N$ at the corner. Take torques about the corner point, for the wheel just barely off the ground, being held in equilibrium. The contact force at the corner exerts no torque and so does not enter the calculation. The pulling force has a lever arm of $R + R - h = 2R - h$, and gravity has a lever arm of x, found from the triangle shown.



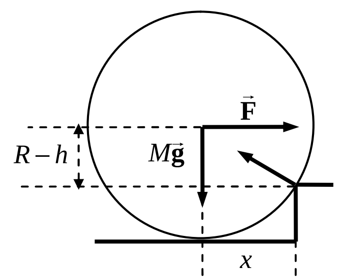


$$x = \sqrt{R^2 - (R-h)^2} = \sqrt{h(2R-h)}$$

$$\sum\tau = Mgx - F(2R-h) = 0 \quad \rightarrow$$

$$F = \frac{Mgx}{2R-h} = Mg\frac{\sqrt{h(2R-h)}}{2R-h} = \boxed{Mg\sqrt{\frac{h}{2R-h}}}$$

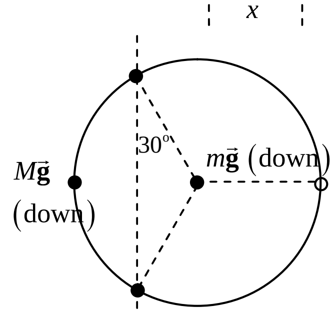
. (*b*) The only difference is that now the pulling force has a lever arm of $R - h$.

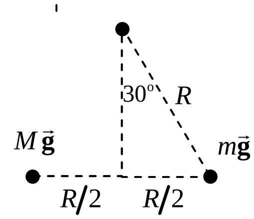


$$\sum\tau = Mgx - F(R-h) = 0 \quad \rightarrow$$

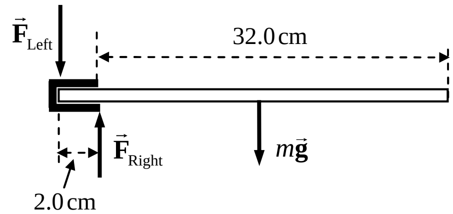
$$F = \frac{Mgx}{R-h} = \boxed{Mg\frac{\sqrt{h(2R-h)}}{R-h}}$$

60. The mass is to be placed symmetrically between two legs of the table. When enough mass is added, the table will rise up off of the third leg, and then the normal force on the table will all be on just two legs. Since the table legs are equally spaced, the angle marked in the diagram is 30°. Take torques about a line connecting the two legs that remain on the floor, so that the normal forces cause no torque. It is seen from the second diagram (a portion of the first diagram but enlarged) that the two forces are equidistant from the line joining the two legs on the floor. Since the lever arms are equal, then the torques will be equal if the forces are equal. Thus, to be in equilibrium, the two forces must be the same. If the force on the edge of the table is any bigger than the weight of the table, it will tip. Thus $\boxed{M > 25\text{ kg}}$ will cause the table to tip.





61. (*a*) The weight of the shelf exerts a downward force and a clockwise torque about the point where the shelf touches the wall. Thus there must be an upward force and a counterclockwise torque exerted by the slot for the shelf to be in equilibrium. Since any force exerted by the slot will have a short lever arm relative to the point where the shelf touches the wall, the upward force must be larger than the gravity force. Accordingly, there then must be a downward force exerted by the slot at its left edge, exerting no torque, but balancing the vertical forces.



(*b*) Calculate the values of the three forces by first taking torques about the left end of the shelf, with the net torque being zero, and then sum the vertical forces, with the sum being zero.

$$\sum \tau = F_{\text{Right}}\left(2.0\times10^{-2}\,\text{m}\right) - mg\left(17.0\times10^{-2}\,\text{m}\right) = 0 \quad \rightarrow$$

$$F_{\text{Right}} = \left(5.0\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)\left(\frac{17.0\times10^{-2}\,\text{m}}{2.0\times10^{-2}\,\text{m}}\right) = 416.5\,\text{N} \approx \boxed{4.2\times10^2\,\text{N}}$$

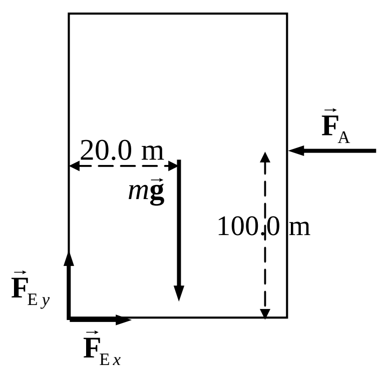
$$\sum F_y = F_{\text{Right}} - F_{\text{Left}} - mg \quad \rightarrow$$

$$F_{\text{Left}} = F_{\text{Right}} - mg = 416.5\text{ N} - \left(5.0\text{ kg}\right)\left(9.80\,\text{m/s}^2\right) = \boxed{3.7\times10^2\,\text{N}}$$

(*c*) The torque exerted by the support about the left end of the rod is

$$\tau = F_{\text{Right}}\left(2.0\times10^{-2}\,\text{m}\right) = \left(416.5\text{ N}\right)\left(2.0\times10^{-2}\,\text{m}\right) = \boxed{8.3\,\text{m}\bullet\text{N}}$$

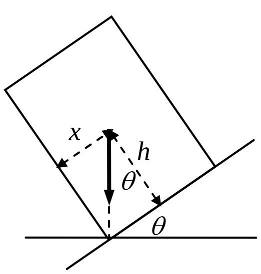
62. Assume that the building has just begun to tip, so that it is essentially vertical, but that all of the force on the building due to contact with the Earth is at the lower left corner, as shown in the figure. Take torques about that corner, with counterclockwise torques as positive.



$$\begin{aligned}\sum \tau &= F_{\text{A}}\left(100.0\text{ m}\right) - mg\left(20.0\text{ m}\right)\\ &= \left(950\,\text{N/m}^2\right)\left(200.0\text{ m}\right)\left(70.0\text{ m}\right)\left(100.0\text{ m}\right)\\ &\quad - \left(1.8\times10^7\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(20.0\text{ m}\right)\\ &= -2.2\times10^9\,\text{m}\bullet\text{N}\end{aligned}$$

Since this is a negative torque, the building will tend to rotate clockwise, which means it will rotate back down to the ground. Thus $\boxed{\text{the building will not topple}}$.

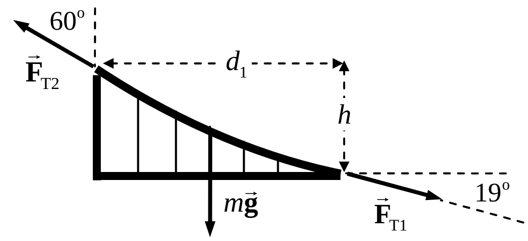
63. The truck will not tip as long as a vertical line down from the CG is between the wheels. When that vertical line is at the wheel, it is in unstable equilibrium and will tip if the road is inclined any more. See the diagram for the truck at the tipping angle, showing the truck's weight vector.



$$\tan\theta = \frac{x}{h} \quad \rightarrow \quad \theta = \tan^{-1}\frac{x}{h} = \tan^{-1}\frac{1.2\text{ m}}{2.2\text{ m}} = \boxed{29^\circ}$$

64. Draw a force diagram for the cable that is supporting the right-hand section. The forces will be the tension at the left end, $\vec{\mathbf{F}}_{\text{T2}}$, the tension at the right end, $\vec{\mathbf{F}}_{\text{T1}}$, and the weight of the section, $m\vec{\mathbf{g}}$. The weight acts at the midpoint of the horizontal span of the cable. The system is in equilibrium. Write Newton's 2nd law in both the x and y directions to find the tensions.

$$\sum F_x = F_{T1}\cos 19^\circ - F_{T2}\sin 60^\circ = 0 \quad \rightarrow$$



$$F_{T2} = F_{T1}\frac{\cos 19^\circ}{\sin 60^\circ}$$

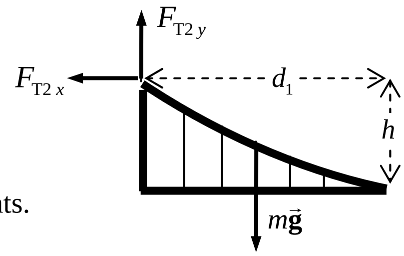
$$\sum F_y = F_{T2}\cos 60^\circ - F_{T1}\sin 19^\circ - mg = 0 \quad \rightarrow$$

$$F_{T1} = \frac{F_{T2}\cos 60^\circ - mg}{\sin 19^\circ} = \frac{F_{T1}\dfrac{\cos 19^\circ}{\sin 60^\circ}\cos 60^\circ - mg}{\sin 19^\circ} \quad \rightarrow$$

$$F_{T1} = mg\frac{\sin 60^\circ}{\left(\cos 19^\circ \cos 60^\circ - \sin 19^\circ \sin 60^\circ\right)} = 4.539\,mg \approx \boxed{4.5\,mg}$$

$$F_{T2} = F_{T1}\frac{\cos 19^\circ}{\sin 60^\circ} = 4.539\frac{\cos 19^\circ}{\sin 60^\circ}mg = 4.956\,mg \approx \boxed{5.0\,mg}$$

To find the height of the tower, take torques about the point where the roadway meets the ground, at the right side of the roadway. Note that then $\vec{F}_{T1}$ will exert no torque. Take counterclockwise torques as positive. For purposes of calculating the torque due to $\vec{F}_{T2}$, split it into x and y components.



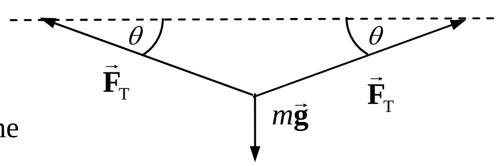
$$\sum \tau = mg\left(\tfrac{1}{2}d_1\right) + F_{T2\,x}h - F_{T2\,y}d_1 = 0 \quad \rightarrow$$

$$h = \frac{\left(F_{T2\,y} - \frac{1}{2}mg\right)}{F_{T2\,x}}d_1 = \frac{\left(F_{T2}\cos 60^\circ - \frac{1}{2}mg\right)}{F_{T2}\sin 60^\circ}d_1 = \frac{\left(4.956\,mg\cos 60^\circ - 0.50\,mg\right)}{4.956\,mg\sin 60^\circ}\left(343\text{ m}\right)$$

$$= \boxed{158\text{ m}}$$

65. The radius of the wire can be determined from the relationship between stress and strain, expressed by equation (9-5).

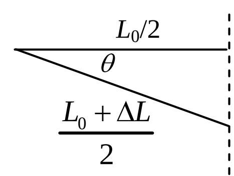
$$\frac{F}{A} = E\frac{\Delta L}{L_0} \quad \rightarrow \quad A = \frac{FL_0}{E\Delta L} = \pi r^2 \quad \rightarrow \quad r = \sqrt{\frac{1}{\pi}\frac{F}{E}\frac{L_0}{\Delta L}}$$



Use the free-body diagram for the point of connection of the mass to the wire to determine the tension force in the wire.

$$\sum F_y = 2F_T\sin\theta - mg = 0 \quad \rightarrow \quad F_T = \frac{mg}{2\sin\theta} = \frac{\left(25\text{ kg}\right)\left(9.8\,\text{m/s}^2\right)}{2\sin 12^\circ} = 589.2\text{ N}$$

The fractional change in the length of the wire can be found from the geometry of the problem.

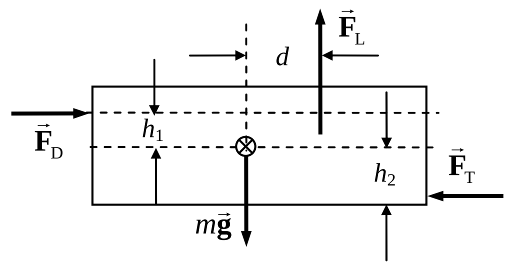


$$\cos\theta = \frac{L_0/2}{\dfrac{L_0 + \Delta L}{2}} \quad \rightarrow \quad \frac{\Delta L}{L_0} = \frac{1}{\cos\theta} - 1 = \frac{1}{\cos 12^\circ} - 1 = 2.234\times10^{-2}$$

Thus the radius is

$$r = \sqrt{\frac{1}{\pi}\frac{F_T}{E}\frac{L_0}{\Delta L}} = \sqrt{\frac{1}{\pi}\frac{589.2\text{ N}}{70\times10^9\,\text{N/m}^2}\frac{1}{\left(2.234\times10^{-2}\right)}} = \boxed{3.5\times10^{-4}\,\text{m}}$$

66. The airplane is in equilibrium, and so the net force in each direction and the net torque are all equal to zero. First write Newton's 2nd law for both the horizontal and vertical directions, to find the values of the forces.



$$\sum F_x = F_D - F_T = 0 \quad \rightarrow \quad F_D = F_T = \boxed{5.0\times 10^5\,\text{N}}$$

$$\sum F_y = F_L - mg = 0$$

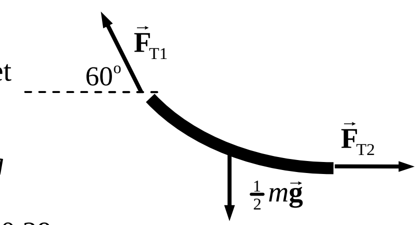
$$F_L = mg = \left(6.7\times 10^4\,\text{kg}\right)\left(9.8\,\text{m/s}^2\right) = \boxed{6.6\times 10^5\,\text{N}}$$

Calculate the torques about the CM, calling counterclockwise torques positive.

$$\sum \tau = F_L d - F_D h_1 - F_T h_2 = 0$$

$$h_1 = \frac{F_L d - F_T h_2}{F_D} = \frac{\left(6.6\times 10^5\,\text{N}\right)(3.2\text{ m}) - \left(5.0\times 10^5\,\text{N}\right)(1.6\text{ m})}{\left(5.0\times 10^5\,\text{N}\right)} = \boxed{2.6\text{ m}}$$

67. Draw a free-body diagram for half of the cable. Write Newton's 2nd law for both the vertical and horizontal directions, with the net force equal to 0 in each direction.



$$\sum F_y = F_{T1}\sin 60^\circ - \tfrac{1}{2}mg = 0 \quad \rightarrow \quad F_{T1} = \tfrac{1}{2}\frac{mg}{\sin 60^\circ} = 0.58mg$$

$$\sum F_x = F_{T2} - F_{T1}\cos 60^\circ = 0 \quad \rightarrow \quad F_{T2} = 0.58mg\left(\cos 60^\circ\right) = 0.29mg$$

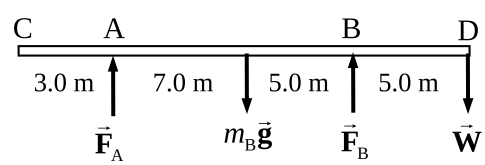
So the results are:

(*a*) $F_{T2} = \boxed{0.29mg}$

(*b*) $F_{T1} = \boxed{0.58mg}$

(*c*) The direction of the tension force is tangent to the cable at all points on the cable. Thus the direction of the tension force is $\boxed{\text{horizontal at the lowest point}}$, and is $\boxed{60^\circ\text{above the horizontal at the attachment point}}$.

68. (*a*) For the extreme case of the beam being ready to tip, there would be no normal force at point A from the support. Use the free-body diagram to write the equation of rotational equilibrium under that condition to find the weight of the person, with $F_A = 0$. Take torques about the location of support B, and call counterclockwise torques positive. $\vec{\mathbf{W}}$ is the weight of the person.



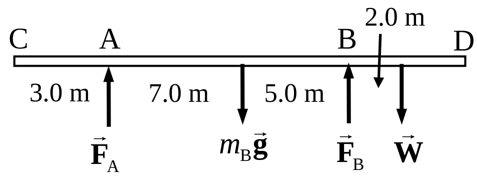
$$\sum \tau = m_B g\,(5.0\text{ m}) - W\,(5.0\text{ m}) = 0 \quad \rightarrow$$

$$W = m_B g = \boxed{550\text{ N}}$$

(*b*) With the person standing at point D, we have already assumed that $\boxed{F_A = 0}$. The net force in the vertical direction must also be zero.

$$\sum F_y = F_A + F_B - m_B g - W = 0 \quad \rightarrow \quad F_B = m_B g + W = 550\text{ N} + 550\text{ N} = \boxed{1100\text{ N}}$$

(*c*) Now the person moves to a different spot, so the free-body diagram changes as shown. Again use the net torque about support B and then use the net vertical force.



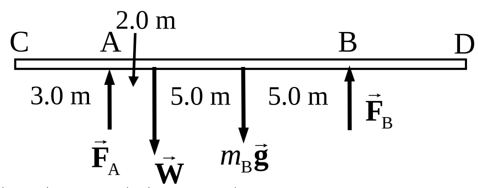
$$\sum \tau = m_B g\left(5.0\text{ m}\right) - W\left(2.0\text{ m}\right) - F_A\left(12.0\text{ m}\right) = 0$$

$$F_A = \frac{m_B g\left(5.0\text{ m}\right) - W\left(2.0\text{ m}\right)}{12.0\text{ m}} = \frac{\left(550\text{ N}\right)\left(3.0\text{ m}\right)}{12.0\text{ m}}$$

$$= \boxed{140\text{ N}}$$

$$\sum F_y = F_A + F_B - m_B g - W = 0 \quad \rightarrow \quad F_B = m_B g + W - F_A = 1100\text{ N} - 140\text{ N} = \boxed{960\text{ N}}$$

(*d*) Again the person moves to a different spot, so the free-body diagram changes again as shown. Again use the net torque about support B and then use the net vertical force.

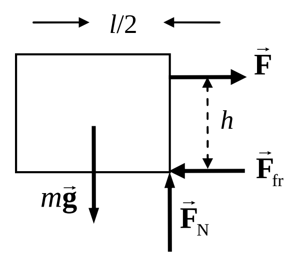


$$\sum \tau = m_B g\left(5.0\text{ m}\right) + W\left(10.0\text{ m}\right) - F_A\left(12.0\text{ m}\right) = 0$$

$$F_A = \frac{m_B g\left(5.0\text{ m}\right) + W\left(10.0\text{ m}\right)}{12.0\text{ m}} = \frac{\left(550\text{ N}\right)\left(5.0\text{ m}\right) + \left(550\text{ N}\right)\left(10.0\text{ m}\right)}{12.0\text{ m}} = \boxed{690\text{ N}}$$

$$\sum F_y = F_A + F_B - m_B g - W = 0 \quad \rightarrow \quad F_B = m_B g + W - F_A = 1100\text{ N} - 690\text{ N} = \boxed{410\text{ N}}$$

69. If the block is on the verge of tipping, the normal force will be acting at the lower right corner of the block, as shown in the free-body diagram. The block will begin to rotate when the torque caused by the pulling force is larger than the torque caused by gravity. For the block to be able to slide, the pulling force must be as large as the maximum static frictional force. Write the equations of equilibrium for forces in the *x* and *y* directions and for torque with the conditions as stated above.



$$\sum F_y = F_N - mg = 0 \quad \rightarrow \quad F_N = mg$$

$$\sum F_x = F - F_{fr} = 0 \quad \rightarrow \quad F = F_{fr} = \mu_s F_N = \mu_s mg$$

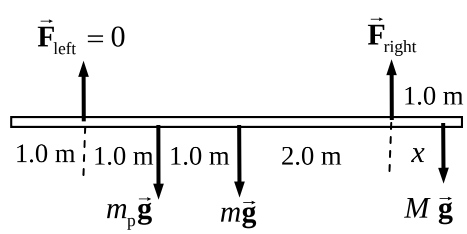
$$\sum \tau = mg\frac{l}{2} - Fh = 0 \quad \rightarrow \quad \frac{mgl}{2} = Fh = \mu_s mgh$$

Solve for the coefficient of friction in this limiting case, to find $\mu_s = \dfrac{l}{2h}$.

(*a*) If $\boxed{\mu_s < l/2h}$, then sliding will happen before tipping.

(*b*) If $\boxed{\mu_s > l/2h}$, then tipping will happen before sliding.

70. The limiting condition for the safety of the painter is the tension in the ropes. The ropes can only exert an upward tension on the scaffold. The tension will be least in the rope that is farther from the painter. The mass of the pail is m_p, the mass of the scaffold is m, and the mass of the painter is M.



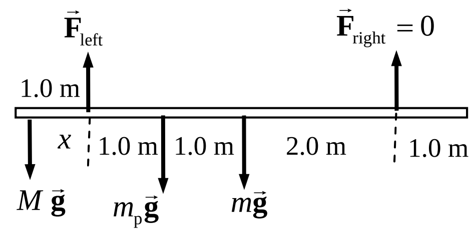
Find the distance to the right that the painter can walk before the tension in the left rope becomes zero. Take torques about the point where the right-side tension is attached to the scaffold, so that its value need not be known. Take counterclockwise torques as positive.

$$\sum \tau = mg(2.0\,\text{m}) + m_p g(3.0\,\text{m}) - Mgx = 0 \quad \rightarrow$$

$$x = \frac{m(2.0\,\text{m}) + m_p(3.0\,\text{m})}{M} = \frac{(25\text{ kg})(2.0\text{ m}) + (4.0\text{ kg})(3.0\text{ m})}{60.0\text{ kg}} = 1.03\text{ m}$$

Since the maximum value for x is 1.0 m, the painter can walk to the right edge of the scaffold safely.

Now find the distance to the left that the painter can walk before the tension in the right rope becomes zero. Take torques about the point where the left-side tension is attached to the scaffold, so that its value need not be known. Take counterclockwise torques as positive.

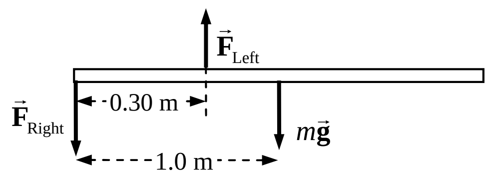


$$\sum \tau = Mgx - m_p g(1.0\,\text{m}) - mg(2.0\text{ m}) = 0 \quad \rightarrow$$

$$x = \frac{m(2.0\,\text{m}) + m_p(1.0\,\text{m})}{M} = \frac{(25\text{ kg})(2.0\text{ m}) + (4.0\text{ kg})(1.0\text{ m})}{60.0\text{ kg}} = 0.90\text{ m}$$

Thus the $\boxed{\text{left end}}$ is dangerous, and he can get within $\boxed{0.10\text{ m}}$ of the left end safely.

71. (*a*) The pole will exert a downward force and a clockwise torque about the woman's right hand. Thus there must be an upward force exerted by the left hand to cause a counterclockwise torque for the pole to have a net torque of zero. The force exerted by the right hand is then of such a magnitude and direction for the net vertical force on the pole to be zero.



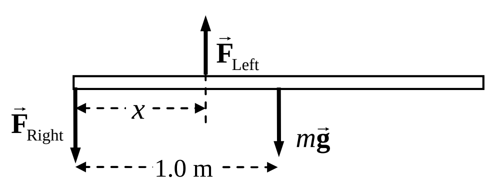
$$\sum \tau = F_{Left}(0.30\text{ m}) - mg(1.0\text{ m}) = 0 \quad \rightarrow$$

$$F_{Left} = mg\left(\frac{1.0\text{ m}}{0.30\text{ m}}\right) = \frac{(10.0\text{ kg})(9.80\,\text{m/s}^2)}{0.30} = 326.7\text{ N} \approx \boxed{3.3\times10^2\text{ N, upward}}$$

$$\sum F_y = F_{Left} - F_{Right} - mg = 0 \quad \rightarrow$$

$$F_{Right} = F_{Left} - mg = 326.7\text{ N} - (10.0\text{ kg})(9.80\,\text{m/s}^2) = 228.7\text{ N} \approx \boxed{2.3\times10^2\text{ N, downward}}$$

(*b*) We see that the force due to the left hand is larger than the force due to the right hand, since both the right hand and gravity are downward. Set the left hand force equal to 150 N and calculate the location of the left hand, by setting the net torque equal to zero.



$$\sum \tau = F_{Left}x - mg(1.0\text{ m}) = 0 \quad \rightarrow$$

$$x = \frac{mg}{F_{Left}}(1.0\text{ m}) = \frac{98.0\text{ N}}{150\text{ N}}(1.0\text{ m}) = \boxed{0.65\text{ m}}$$

As a check, calculate the force due to the right hand.

$$F_{Right} = F_{Left} - mg = 150\text{ N} - 98.0\text{ N} = 52\text{ N} \quad \text{OK}$$

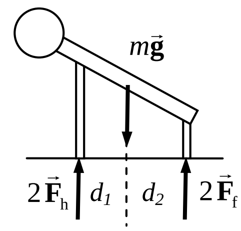
(*c*) Follow the same procedure, setting the left hand force equal to 85 N.

$$\sum \tau = F_{\text{Left}} x - mg(1.0\text{ m}) = 0 \;\rightarrow\; x = \frac{mg}{F_{\text{Left}}}(1.0\text{ m}) = \frac{98.0\text{ N}}{85\text{ N}}(1.0\text{ m}) = 1.153\text{ m} \approx \boxed{1.2\text{ m}}$$

$$F_{\text{Right}} = F_{\text{Left}} - mg = 85\text{ N} - 98.0\text{ N} = -13\text{ N} \quad \text{OK}$$

Note that now the force due to the right hand must be pulling upwards.

$\boxed{72.}$ The man is in equilibrium, so the net force and the net torque on him must be zero. We represent the force on the hands as twice the force on one hand, and the force on the feet as twice the force on one foot. Take torques about the point where his hands touch the ground, with counterclockwise as positive.



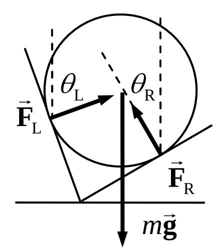
$$\sum \tau = 2F_f(d_1 + d_2) - mgd_1 = 0$$

$$F_f = \frac{mgd_1}{2(d_1+d_2)} = \frac{(75\text{ kg})(9.8\text{ m/s}^2)(0.40\text{ m})}{2(1.35\text{ m})} = 109\text{ N} \approx \boxed{1.1\times10^2\text{ N}}$$

$$\sum F_y = 2F_h + 2F_f - mg = 0$$

$$F_h = \tfrac{1}{2}mg - F_f = \tfrac{1}{2}(75\text{ kg})(9.8\text{ m/s}^2) - 109\text{ N} = 259\text{ N} \approx \boxed{2.6\times10^2\text{ N}}$$

73. The force on the sphere from each plane will be normal to the sphere, and so perpendicular to the plane at the point of contact. Use Newton's 2nd law in both the horizontal and vertical directions to determine the magnitudes of the forces.



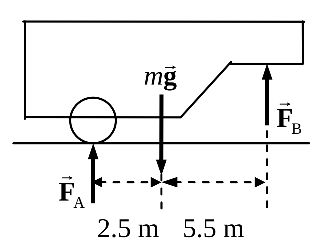
$$\sum F_x = F_L \sin\theta_L - F_R \sin\theta_R = 0 \;\rightarrow\; F_R = F_L \frac{\sin\theta_L}{\sin\theta_R} = F_L \frac{\sin 70^\circ}{\sin 30^\circ}$$

$$\sum F_y = F_L \cos\theta_L + F_R \cos\theta_R - mg = 0 \;\rightarrow\; F_L\left(\cos 70^\circ + \frac{\sin 70^\circ}{\sin 30^\circ}\cos 30^\circ\right) = mg$$

$$F_L = \frac{mg}{\left(\cos 70^\circ - \frac{\sin 70^\circ}{\sin 30^\circ}\cos 30^\circ\right)} = \frac{(20\text{ kg})(9.80\text{ m/s}^2)}{\left(\cos 70^\circ - \frac{\sin 70^\circ}{\sin 30^\circ}\cos 30^\circ\right)} = 99.51\text{ N} \approx \boxed{1.0\times10^2\text{ N}}$$

$$F_R = F_L \frac{\sin 70^\circ}{\sin 30^\circ} = (99.51\text{ N})\frac{\sin 70^\circ}{\sin 30^\circ} = 187.0\text{ N} \approx \boxed{1.9\times10^2\text{ N}}$$

74. To find the normal force exerted on the road by the trailer tires, take the torques about point B, with counterclockwise torques as positive.



$$\sum \tau = mg(5.5\text{ m}) - F_A(8.0\text{ m}) = 0 \;\rightarrow$$

$$F_A = mg\left(\frac{5.5\text{ m}}{8.0\text{ m}}\right) = (2200\text{ kg})(9.80\text{ m/s}^2)\left(\frac{5.5\text{ m}}{8.0\text{ m}}\right) = 14{,}823\text{ N}$$

$$\approx \boxed{1.5\times10^4\text{ N}}$$

The net force in the vertical direction must be zero.

$$\sum F_y = F_B + F_A - mg = 0 \;\rightarrow\; F_B = mg - F_A = (2200\text{ kg})(9.80\text{ m/s}^2) - 14{,}823\text{ N} = \boxed{6.7\times10^3\text{ N}}$$

75. Assume a constant acceleration as the person is brought to rest, with up as the positive direction. Use Eq. 2-11c to find the acceleration. From the acceleration, find the average force of the snow on the person, and compare the force per area to the strength of body tissue.

$$v^2 = v_0^2 - 2a\left(x - x_0\right) \rightarrow a = \frac{v^2 - v_0^2}{2\left(x - x_0\right)} = \frac{0 - \left(60\,\text{m/s}\right)^2}{2\left(-1.0\text{ m}\right)} = 1800\,\text{m/s}^2$$

$$\frac{F}{A} = \frac{ma}{A} = \frac{\left(75\text{ kg}\right)\left(1800\,\text{m/s}^2\right)}{0.30\text{m}^2} = 4.5\times10^5\text{ N/m}^2 < \text{ Tissue strength} = 5\times10^5\text{ N/m}^2$$

Since the average force on the person is less than the strength of body tissue, the person may escape serious injury. Certain parts of the body, such as the legs if landing feet first, may get more than the average force, though, and so still sustain injury.

76. The mass can be calculated from the equation for the relationship between stress and strain. The force causing the strain is the weight of the mass suspended from the wire.

$$\frac{\Delta L}{L_0} = \frac{1}{E}\frac{F}{A} = \frac{mg}{EA} \rightarrow m = \frac{EA}{g}\frac{\Delta L}{L_0} = \left(200\times10^9\text{ N/m}^2\right)\frac{\pi\left(1.0\times10^{-3}\,\text{m}\right)^2}{\left(9.80\,\text{m/s}^2\right)}\frac{0.030}{100} = \boxed{19\text{ kg}}$$

77. (*a*) From Example 7-6, the total force of the ground on one leg for a "stiff-legged landing" is $2.1\times10^5\,\text{N}/2 = 1.05\times10^5\,\text{N}$. The stress in the tibia bone is the force divided by the cross-sectional area of the bone.

$$\text{Stress} = \frac{F}{A} = \frac{1.05\times10^5\,\text{N}}{3.0\times10^{-4}\,\text{m}^2} = \boxed{3.5\times10^8\text{ N/m}^2}$$

(*b*) Since the stress from above is greater than the compressive strength of bone $\left(1.7\times10^8\text{ N/m}^2\right)$, the $\boxed{\text{bone will break}}$.

(*c*) From Example 7-6, the total force of the ground on one leg for a "bent-legged landing" is $4.9\times10^3\,\text{N}/2 = 2.45\times10^3\,\text{N}$. The stress in the tibia bone is the force divided by the cross-sectional area of the bone.

$$\text{Stress} = \frac{F}{A} = \frac{2.45\times10^3\,\text{N}}{3.0\times10^{-4}\,\text{m}^2} = \boxed{8.2\times10^6\text{ N/m}^2}$$

Since the stress from above is less than the compressive strength of bone $\left(1.7\times10^8\text{ N/m}^2\right)$, the $\boxed{\text{bone will not break}}$.

78. The number of supports can be found from the compressive strength of the wood. Since the wood will be oriented longitudinally, the stress will be parallel to the grain.

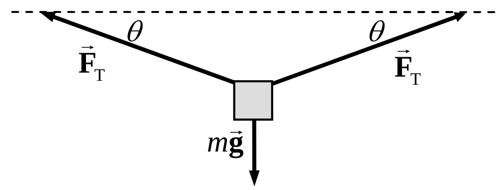
$$\frac{\text{Compressive Strength}}{\text{Safety Factor}} = \frac{\text{Load force on supports}}{\text{Area of supports}} = \frac{\text{Weight of roof}}{\left(\#\text{ supports}\right)\left(\text{area per support}\right)}$$

$$\left(\#\text{ supports}\right) = \frac{\text{Weight of roof}}{\left(\text{area per support}\right)}\frac{\text{Safety Factor}}{\text{Compressive Strength}}$$

$$= \frac{\left(1.26\times10^4\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(0.040\text{ m}\right)\left(0.090\text{ m}\right)}\frac{12}{\left(35\times10^6\text{ N/m}^2\right)} = 12\text{ supports}$$

Since there are to be 12 supports, there will be $\boxed{\text{6 supports on each side}}$. That means there will be 5 gaps on each side between the supports, and so $\text{Spacing} = \dfrac{10.0\text{ m}}{5\text{ gaps}} = \boxed{2.0\,\text{m/gap}}$.

79. The tension in the string when it breaks is found from the ultimate strength of nylon under tension, from Table 9-2.



$$\frac{F_T}{A} = \text{Tensile Strength} \quad\rightarrow$$

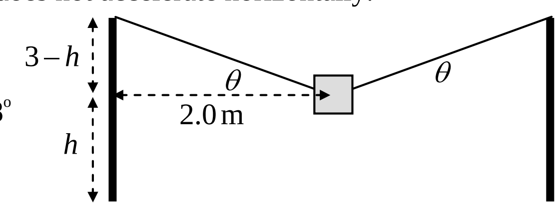
$$F_T = A(\text{Tensile Strength})$$

$$= \pi\left(5.00\times10^{-4}\,\text{m}\right)^2\left(500\times10^6\ \text{N/m}^2\right) = 392.7\ \text{N}$$

From the force diagram for the box, we calculate the angle of the rope relative to the horizontal from Newton's 2nd law in the vertical direction. Note that since the tension is the same throughout the string, the angles must be the same so that the object does not accelerate horizontally.

$$\sum F_y = 2F_T\sin\theta - mg = 0 \quad\rightarrow$$

$$\theta = \sin^{-1}\frac{mg}{2F_T} = \sin^{-1}\frac{(25\text{ kg})\left(9.80\,\text{m/s}^2\right)}{2(392.7\text{ N})} = 18.18^\circ$$



To find the height above the ground, consider the second diagram.

$$\tan\theta = \frac{3.00\,\text{m} - h}{2.00\,\text{m}} \quad\rightarrow\quad h = 3.00\,\text{m} - 2.00\,\text{m}(\tan\theta) = 3.00\,\text{m} - 2.00\,\text{m}\left(\tan 18.18^\circ\right) = \boxed{2.34\text{ m}}$$

80. The maximum compressive force in a column will occur at the bottom. The bottom layer supports the entire weight of the column, and so the compressive force on that layer is mg. For the column to be on the verge of buckling, the weight divided by the area of the column will be the compressive strength of the material. The mass of the column is its volume (area x height) times its density.

$$\frac{mg}{A} = \text{Compressive Strength} = \frac{hA\rho g}{A} \quad\rightarrow\quad h = \frac{\text{Compressive Strength}}{\rho g}$$

Note that the area of the column cancels out of the expression, and so the height does not depend on the cross-sectional area of the column.

(*a*) $$h_{\text{steel}} = \frac{\text{Compressive Strength}}{\rho g} = \frac{500\times10^6\ \text{N/m}^2}{\left(7.8\times10^3\ \text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{6.5\times10^3\,\text{m}}$$

(*b*) $$h_{\text{granite}} = \frac{\text{Compressive Strength}}{\rho g} = \frac{170\times10^6\ \text{N/m}^2}{\left(2.7\times10^3\ \text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{6.4\times10^3\,\text{m}}$$