CHAPTER 13: Temperature and Kinetic Theory

Answers to Questions

1. Because the atomic mass of aluminum is smaller than that of iron, an atom of aluminum has less mass than an atom of iron. Thus 1 kg of aluminum will have more atoms than 1 kg of iron.

2. Properties of materials that can be exploited for the making of a thermometer include:
    i) Volume of a liquid (mercury thermometer)
    ii) Electrical resistance
    iii) Color (frequency) of emitted light from a heated object
    iv) Volume of a gas
    v) Expansion of a metal (bimetallic strip)

3. 1 C^o is larger than 1 F^o. There are 100 C^o between the freezing and boiling temperatures of water, while there are 180 F^o between the same two temperatures.

4. The following conclusions can be drawn:
    A and B are at the same temperature
    B and C are not at the same temperature
    A and C are not at the same temperature

5. When heated, the aluminum expands more than the iron, because the expansion coefficient of aluminum is larger than that of iron. Thus the aluminum will be on the outside of the curve.

6. To be precise, L_0 is to be the initial length of the object. In practice, however, since the value of the coefficient of expansion is so small, there will be little difference in the calculation of ΔL caused by using either the initial or final length, unless the temperature change is quite large.

7. The coefficient of expansion is derived from a ratio of lengths: $\alpha = \frac{\Delta L}{L_0} \frac{1}{\Delta T}$. The length units cancel, and so the coefficient does not depend on the specific length unit used in its determination, as long as the same units are used for both ΔL and L_0.

8. The device controls the furnace by the expansion and contraction of the bimetallic strip. As the temperature increases, the strip coils more, and as the temperature decrease, the strip coils less. As the strip changes shape, it will move the liquid mercury switch. In the diagram, if the switch were tilted more to the right, the mercury would move and make contact between the heater wires, turning on the heater. By adjusting the temperature setting lever, the tilt of the mercury switch is changed, and a different amount of temperature change is needed to tilt the switch to the on (or off) position.

9. The steam pipe can have a large temperature change as the steam enters or leaves the pipe. If the pipe is fixed at both ends and the temperature changes significantly, there will be large thermal stresses which might break joints. The "U" in the pipe allows for expansion and contraction which is not possible at the fixed ends. This is similar to the joints placed in concrete roadway surfaces to allow expansion and contraction.

10. The lead floats in the mercury because $\rho_{Hg} > \rho_{Pb}$. As the substances are heated, the density of both substances will decrease due to volume expansion (see problem 17 for the derivation of this result). The density of the mercury decreases more upon heating than the density of the lead, because $\beta_{Hg} > \beta_{Pb}$. The net effect is that the densities get closer together, and so relatively more mercury will have to be displaced to hold up the lead, and the lead will float lower in the mercury.

11. The glass is the first to warm due to the hot water, and so the glass will initially expand a small amount. As the glass initially expands, the mercury level will decrease. As thermal equilibrium is reached, the mercury will expand more than the glass expands, since mercury has a larger coefficient of expansion than water, and the mercury level will rise to indicate the higher temperature.

12. If one part is heated or cooled more than another part, there will be more expansion or contraction of one part of the glass compared to an adjacent part. This causes internal stress forces which may exceed the maximum strength of the glass.

13. When Pyrex glass is heated or cooled, it will expand or contract much less than ordinary glass due to its small coefficient of linear expansion. The smaller changes in dimensions result in lower internal stresses than would be present in ordinary glass. Thus there is less of a chance of breaking the Pyrex by heating or cooling it.

14. On a hot day, the pendulum will be slightly longer than at 20°C, due to thermal expansion. Since the period of a pendulum is proportional to the square root of its length, the period will be slightly longer on the hot day, meaning that the pendulum takes more time for one oscillation. Thus the clock will run slow.

15. The soda is mostly water. As water cools below 4°C it expands. There is more expansion of the soda as it cools below 4°C and freezes than there is available room in the can, and so the freezing soda pushes against the can surfaces hard enough to push them outward. Evidently the top and bottom of the can are the weakest parts.

16. When a small mass object collides with a stationary massive object, the speed of the small mass is not changed. But when a small mass object collides with a massive object moving in the opposite direction, the speed of the small object increases. For example, a tennis ball of a given speed that is struck with a racket will rebound with a greater speed than a tennis ball of the same speed bouncing off a wall. So as the gas molecules collide with the piston that is moving toward them, their speed increases. The microscopic increase in molecular speed is manifested macroscopically as a higher temperature.

   In a similar fashion, when the molecules collide with a piston that is moving away from them, they rebound with a reduced speed compared to their initial speed. This lower speed is manifested macroscopically as a lower temperature.

17. The buoyant force on the aluminum sphere is the weight of the water displaced by the sphere, which is the volume of the sphere times the density of water times *g*. As the substances are heated, the volume of the sphere increases and the density of the water decreases. Since the volume expansion coefficient of the water is almost three times larger than that of the aluminum, the fractional decrease in the water density is larger than the fractional increase in the aluminum volume. Thus the product of the volume of the sphere times the density of water decreases, and the buoyant force gets smaller.

18. Charles's law states that the volume of a fixed mass of gas increases proportionately to the absolute temperature, when the pressure is held constant. As the temperature increases, the molecules have more kinetic energy, and the average force exerted by a gas molecule colliding with the boundaries of the container is proportional to the kinetic energy. Thus the force exerted during the collisions increases. The pressure is the force per unit area, and so for the pressure to remain constant, the surface area of the boundaries must increase, which means the volume of the container must increase.

19. Gay-Lussac's law states that at constant volume, the absolute pressure of a gas is proportional to the absolute temperature. Kinetic molecular theory has a result that the average force exerted by gas particles as they collide with the container boundaries is proportional to the kinetic energy, assuming a fixed container size. For the pressure to increase, the force on the walls must increase, which means the kinetic energy must therefore increase. But the kinetic energy of the particles is proportional to the absolute temperature, and so for the pressure to increase, the temperature must also increase.

20. Since an N_2 molecule has less mass than an O_2 molecule, at the same temperature (and thus the same kinetic energy), N_2 molecules will have a larger speed on average than O_2 molecules. If we consider "launching" molecules of both types from the Earth's surface, the faster-moving N_2 will rise higher before stopping and falling back to Earth. Thus there will be proportionally more N_2 molecules at higher altitudes than at lower altitudes.

21. Because the escape velocity is much smaller for the Moon than for the Earth, most gas molecules even at low temperatures have a speed great enough to escape the Moon's gravity. Thus the atmosphere has "evaporated" over the long time of the Moon's existence.

22. Since the alcohol evaporates more quickly, the alcohol molecules escape "easier" than the water molecules. One explanation could be that the intermolecular forces (bonds) for alcohol are smaller than those for water. Another explanation could be that the alcohol molecules are moving more rapidly than the water molecules, indicating that alcohol molecules are less massive than water molecules. However, the simplest alcohol (CH_3OH) has a molecular mass higher than that of water, so mass is probably not the explanation.

23. On a hot humid day, there is little evaporation from a human due to perspiration, because the air is already saturated with water vapor. Since perspiration is a major cooling mechanism, when it is restricted, humans will feel more uncomfortable. On a hot dry day, water molecules more easily evaporate into the air (taking their kinetic energy with them) and the body is cooled.

24. Liquids boil when their saturated vapor pressure equals the external pressure. For water, from Table 13-3, the saturated vapor pressure of water at 20°C is about 0.023 atm. So if the external pressure is lowered to that level (about 2.3% of normal air pressure), the water will boil.

25. On a day when the relative humidity is high, the percentage of air molecules that are water (as opposed to N_2 or O_2 molecules) is increased. Since the molecular mass of water is less than that of either N_2 or O_2, the average mass of air molecules in a given volume will decrease, and thus the density will be lower for the humid air.

26. The water in the radiator of an overheated automobile engine is under pressure. Similar to a pressure cooker, that high pressure keeps the water in the liquid state even though the water is quite hot – hotter than 100°C. When the cap is opened, the pressure is suddenly lowered, and the superheated water boils quickly and violently. That hot steam can cause severe burns if it contacts the skin.

Also, the violent bursting forth of steam propels some of the overheated water out of the radiator as well, which can spray onto the person opening the cap and again cause serious burns.

27. Exhaled air contains water vapor, at a relatively high percentage. Since the air inside the lungs is quite warm, the partial pressure of water in the lungs can be high without saturating the air in the lungs, and condensation does not occur. But in the cold winter air, the air can hold very little water without condensation. Thus as the warm, water-laden exhaled air cools, the partial pressure of water vapor exceeds the saturated vapor pressure in the cold air, and some of the water will condense. The white cloud seen is due to the condensed water vapor.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted temperatures as correct to the number of digits shown, especially where other values might indicate that. For example, in problem 17, values of 25°C and –40°C are used. We took both of those values to have 2 significant figures in calculating the temperature change.

1. The number of atoms is found by dividing the mass of the substance by the mass of a single atom. Take the atomic mass of carbon to be 63.

$$N_{Cu} = \frac{3.4 \times 10^{-3}\,\text{kg}}{63\left(1.66 \times 10^{-27}\right)\text{kg/atom}} = \boxed{3.3 \times 10^{22}\,\text{atoms of Cu}}$$

2. The number of atoms in a pure substance can be found by dividing the mass of the substance by the mass of a single atom. Take the atomic mass of gold to be 197, and silver to be 108.

$$\frac{N_{Au}}{N_{Ag}} = \frac{\dfrac{2.65 \times 10^{-2}\,\text{kg}}{\left(197\right)\left(1.66 \times 10^{-27}\ \text{kg/atom}\right)}}{\dfrac{2.65 \times 10^{-2}\,\text{kg}}{\left(108\right)\left(1.66 \times 10^{-27}\ \text{kg/atom}\right)}} = \frac{108}{197} = 0.548 \quad \rightarrow \quad \boxed{N_{Au} = 0.548 N_{Ag}}$$

Because a gold atom is heavier than a silver atom, there are fewer gold atoms in the given mass.

3. (a) $T\left(^\circ\text{C}\right) = \tfrac{5}{9}\left[T\left(^\circ\text{F}\right) - 32\right] = \tfrac{5}{9}\left[68 - 32\right] = \boxed{20^\circ\text{C}}$

    (b) $T\left(^\circ\text{F}\right) = \tfrac{9}{5}T\left(^\circ\text{C}\right) + 32 = \tfrac{9}{5}\left(1800\right) + 32 = \boxed{3300^\circ\text{F}}$

4. High: $T\left(^\circ\text{C}\right) = \tfrac{5}{9}\left[T\left(^\circ\text{F}\right) - 32\right] = \tfrac{5}{9}\left[136 - 32\right] = \boxed{57.8^\circ\text{C}}$

    Low: $T\left(^\circ\text{C}\right) = \tfrac{5}{9}\left[T\left(^\circ\text{F}\right) - 32\right] = \tfrac{5}{9}\left[-129 - 32\right] = \boxed{-89.4^\circ\text{C}}$

5. (a) $T\left(^\circ\text{F}\right) = \tfrac{9}{5}T\left(^\circ\text{C}\right) + 32 = \tfrac{9}{5}\left(-15\right) + 32 = \boxed{5^\circ\text{F}}$

    (b) $T\left(^\circ\text{C}\right) = \tfrac{5}{9}\left[T\left(^\circ\text{F}\right) - 32\right] = \tfrac{5}{9}\left[-15 - 32\right] = \boxed{-26^\circ\text{C}}$

6. Assume that the temperature and the length are linearly related. The change in temperature per unit length change is as follows.

$$\frac{\Delta T}{\Delta L} = \frac{100.0^{\circ}C - 0.0^{\circ}C}{22.85 \text{ cm} - 11.82 \text{ cm}} = 9.066 \, C^{\circ}/\text{cm}$$

Then the temperature corresponding to length L is $T(L) = 0.0^{\circ}C + (L - 11.82 \text{ cm})(9.066 \, C^{\circ}/\text{cm})$.

(*a*) $T(16.70 \text{ cm}) = 0.0^{\circ}C + (16.70 \text{ cm} - 11.82 \text{ cm})(9.066 \, C^{\circ}/\text{cm}) = \boxed{44.2^{\circ}C}$

(*b*) $T(20.50 \text{ cm}) = 0.0^{\circ}C + (20.50 \text{ cm} - 11.82 \text{ cm})(9.066 \, C^{\circ}/\text{cm}) = \boxed{78.7^{\circ}C}$

7. When the concrete cools in the winter, it will contract, and there will be no danger of buckling. Thus the low temperature in the winter is not a factor in the design of the highway. But when the concrete warms in the summer, it will expand. A crack must be left between the slabs equal to the increase in length of the concrete as it heats from $20^{\circ}C$ to $50^{\circ}C$.

$$\Delta L = \alpha L_0 \Delta T = (12 \times 10^{-6}/C^{\circ})(12 \text{ m})(50^{\circ}C - 20^{\circ}C) = \boxed{4.3 \times 10^{-3} \text{ m}}$$

8. The increase in length of the table is given by Equation 13-1a.

$$\Delta L = \alpha L_0 \Delta T = (0.2 \times 10^{-6}/C^{\circ})(2.0 \text{ m})(5.0C^{\circ}) = \boxed{2 \times 10^{-6} \text{ m}}$$

For steel, $\Delta L = \alpha L_0 \Delta T = (12 \times 10^{-6}/C^{\circ})(2.0 \text{ m})(5.0C^{\circ}) = \boxed{1.2 \times 10^{-4} \text{ m}}$.

The change for Super Invar is approximately only 2% of the change for steel.

9. Take the 300 m height to be the height in January. Then the increase in the height of the tower is given by Equation 13-1a.

$$\Delta L = \alpha L_0 \Delta T = (12 \times 10^{-6}/C^{\circ})(300 \text{ m})(25^{\circ}C - 2^{\circ}C) = \boxed{8 \times 10^{-2} \text{ m}}$$

$\boxed{10.}$ The rivet must be cooled so that its diameter becomes the same as the diameter of the hole.

$$\Delta L = \alpha L_0 \Delta T \quad \rightarrow \quad L - L_0 = \alpha L_0 (T - T_0)$$

$$T = T_0 + \frac{L - L_0}{\alpha L_0} = 20^{\circ}C + \frac{1.869 \text{ cm} - 1.871 \text{ cm}}{(12 \times 10^{-6}/C^{\circ})(1.871 \text{ cm})} = \boxed{-69^{\circ}C}$$

11. The density at $4^{\circ}C$ is $\rho = \dfrac{M}{V} = \dfrac{1.00 \times 10^3 \text{ kg}}{1.00 \text{ m}^3}$. When the water is warmed, the mass will stay the same, but the volume will increase according to Equation 13-2.

$$\Delta V = \beta V_0 \Delta T = (210 \times 10^{-6}/C^{\circ})(1.00 \text{ m}^3)(94^{\circ}C - 4^{\circ}C) = 1.89 \times 10^{-2} \text{ m}^3$$

The density at the higher temperature is $\rho = \dfrac{M}{V} = \dfrac{1.00 \times 10^3 \text{ kg}}{1.00 \text{ m}^3 + 1.89 \times 10^{-2} \text{ m}^3} = \boxed{981 \text{ kg}/\text{m}^3}$

12. The change in volume of the quartz is given by the volume expansion formula, Equation 13-2.

$$\Delta V = \beta V_0 \Delta T = (1 \times 10^{-6}/C^{\circ})\left(\tfrac{4}{3}\pi\left(\frac{8.75 \text{ cm}}{2}\right)^3\right)(200^{\circ}C - 30^{\circ}C) = \boxed{6.0 \times 10^{-2} \text{ cm}^3}$$

13. The amount of water that can be added to the container is the final volume of the container minus the final volume of the water. Also note that the original volumes of the water and the container are the same. We assume that the density of water is constant over the temperature change involved.

$$V_{\text{added}} = (V_0 + \Delta V)_{\text{container}} - (V_0 + \Delta V)_{\text{H}_2\text{O}} = \Delta V_{\text{container}} - \Delta V_{\text{H}_2\text{O}} = (\beta_{\text{container}} - \beta_{\text{H}_2\text{O}})V_0\Delta T$$

$$= (27 \times 10^{-6}/\text{C}^\circ - 210 \times 10^{-6}/\text{C}^\circ)(350.0\ \text{mL})(-80.0\ \text{C}^\circ) = \boxed{5.12\ \text{mL}}$$

14. (*a*) The amount of water lost is the final volume of the water minus the final volume of the container. Also note that the original volumes of the water and the container are the same.

$$V_{\text{lost}} = (V_0 + \Delta V)_{\text{H}_2\text{O}} - (V_0 + \Delta V)_{\text{container}} = \Delta V_{\text{H}_2\text{O}} - \Delta V_{\text{container}} = \beta_{\text{H}_2\text{O}}V_0\Delta T - \beta_{\text{container}}V_0\Delta T$$

$$\beta_{\text{container}} = \beta_{\text{H}_2\text{O}} - \frac{V_{\text{lost}}}{V_0\Delta T} = 210 \times 10^{-6}/\text{C}^\circ - \frac{(0.35\text{g})\left(\dfrac{1\ \text{mL}}{0.98324\ \text{g}}\right)}{(55.50\ \text{mL})(60^\circ\text{C} - 20^\circ\text{C})} = \boxed{5.0 \times 10^{-5}/\text{C}^\circ}$$

(*b*) From Table 13-1, the most likely material is $\boxed{\text{copper}}$.

15. (*a*) The sum of the original diameter plus the expansion must be the same for both the plug and the ring.

$$(L_0 + \Delta L)_{\text{iron}} = (L_0 + \Delta L)_{\text{brass}} \quad \rightarrow \quad L_{\text{iron}} + \alpha_{\text{iron}}L_{\text{iron}}\Delta T = L_{\text{brass}} + \alpha_{\text{brass}}L_{\text{brass}}\Delta T$$

$$\Delta T = \frac{L_{\text{brass}} - L_{\text{iron}}}{\alpha_{\text{iron}}L_{\text{iron}} - \alpha_{\text{brass}}L_{\text{brass}}} = \frac{8.753\ \text{cm} - 8.743\ \text{cm}}{(12 \times 10^{-6}/\text{C}^\circ)(8.743\ \text{cm}) - (19 \times 10^{-6}/\text{C}^\circ)(8.753\ \text{cm})}$$

$$= -163\text{C}^\circ = T_{\text{final}} - T_{\text{initial}} = T_{\text{final}} - 20^\circ\text{C} \quad \rightarrow \quad T_{\text{final}} = -143^\circ\text{C} \approx \boxed{-140^\circ\text{C}}$$

(*b*) Simply switch the initial values in the above calculation.

$$\Delta T = \frac{L_{\text{brass}} - L_{\text{iron}}}{\alpha_{\text{iron}}L_{\text{iron}} - \alpha_{\text{brass}}L_{\text{brass}}} = \frac{8.743\ \text{cm} - 8.753\ \text{cm}}{(12 \times 10^{-6}/\text{C}^\circ)(8.753\ \text{cm}) - (19 \times 10^{-6}/\text{C}^\circ)(8.743\ \text{cm})} =$$

$$= 164\text{C}^\circ = T_{\text{final}} - T_{\text{initial}} = T_{\text{final}} - 20^\circ\text{C} \quad \rightarrow \quad T_{\text{final}} = 184^\circ\text{C} \approx \boxed{180^\circ\text{C}}$$

16. We model the vessel as having a constant cross-sectional area A. Then a volume V_0 of fluid will occupy a length L_0 of the tube, given that $V_0 = AL_0$. Likewise $V = AL$.

$$\Delta V = V - V_0 = AL - AL_0 = A\Delta L \text{ and } \Delta V = \beta V_0\Delta T = \beta AL_0\Delta T.$$

Equate the two expressions for ΔV, and get $A\Delta L = \beta AL_0\Delta T \quad \rightarrow \quad \Delta L = \beta L_0\Delta T$. But $\Delta L = \alpha L_0\Delta T$, so we see that under the conditions of the problem, $\boxed{\alpha = \beta}$

17. (*a*) When a substance changes temperature, its volume will change by an amount given by Equation 13-2. This causes the density to change.

$$\Delta\rho = \rho - \rho_0 = \frac{M}{V} - \frac{M}{V_0} = \frac{M}{V_0 + \Delta V} - \frac{M}{V_0} = \frac{M}{V_0 + \beta V_0\Delta T} - \frac{M}{V_0} = \frac{M}{V_0}\left(\frac{1}{1 + \beta\Delta T} - 1\right)$$

$$= \rho_0\left(\frac{1}{1 + \beta\Delta T} - \frac{1 + \beta\Delta T}{1 + \beta\Delta T}\right) = \rho_0\left(\frac{-\beta\Delta T}{1 + \beta\Delta T}\right)$$

If we assume that $\beta\Delta T \ll 1$, then the denominator is approximately 1, so $\boxed{\Delta\rho = -\rho_0\beta\Delta T}$.

(*b*) The fractional change in density is

$$\frac{\Delta\rho}{\rho_0} = \frac{-\rho_0\beta\Delta T}{\rho_0} = -\beta\Delta T = -\left(87\times10^{-6}/{}^{\circ}\mathrm{C}\right)\left(-40^{\circ}\mathrm{C}-25^{\circ}\mathrm{C}\right) = 5.7\times10^{-3}$$

This is a $\boxed{0.57\% \text{ increase}}$.

18. Assume that each dimension of the plate changes according to Equation 13-1a.

$$\Delta A = A - A_0 = \left(l+\Delta l\right)\left(w+\Delta w\right)-lw = lw+l\Delta w+w\Delta l+\Delta l\Delta w-lw = l\Delta w+w\Delta l+\Delta l\Delta w$$

Neglect the very small quantity $\Delta l\Delta w$.

$$\Delta A = l\Delta w+w\Delta l = l\left(\alpha w\Delta T\right)+w\left(\alpha l\Delta T\right) = \boxed{2\alpha l w\Delta T}$$

19. Consider a cubic solid, where the original length of each dimension of the cube is l .

$$\Delta V = V - V_0 = \left(l+\Delta l\right)^3 - l^3 = l^3 + 3l^2\Delta l + 3l\left(\Delta l\right)^2 + \left(\Delta l\right)^3 - l^3 = 3l^2\Delta l + 3l\left(\Delta l\right)^2 + \left(\Delta l\right)^3$$

Neglect the very small quantities involving Δl squared or cubed.

$$\Delta V = 3l^2\Delta l = 3l^2\alpha l\Delta T = 3\alpha l^3\Delta T = 3\alpha V_0\Delta T$$

But Equation 13-2 states $\Delta V = \beta V_0\Delta T$. Equate the two statements for ΔV .

$$\Delta V = 3\alpha V_0\Delta T = \beta V_0\Delta T \quad\rightarrow\quad \boxed{\beta = 3\alpha}$$

20. The pendulum has a period of $\tau_0 = 2\pi\sqrt{L_0/g}$ at $17^{\circ}\mathrm{C}$, and a period of $\tau = 2\pi\sqrt{L/g}$ at $25^{\circ}\mathrm{C}$. Notice that $\tau > \tau_0$ since $L > L_0$. With every swing of the clock, the clock face will indicate that a time τ_0 has passed, but the actual amount of time that has passed is τ . Thus the clock face is "losing time" by an amount of $\Delta\tau = \tau - \tau_0$ every swing. The fractional loss is given by $\dfrac{\Delta\tau}{\tau_0}$, and the length at the higher temperature is given by

$$\frac{\Delta\tau}{\tau_0} = \frac{\tau-\tau_0}{\tau_0} = \frac{2\pi\sqrt{L/g}-2\pi\sqrt{L_0/g}}{2\pi\sqrt{L_0/g}} = \frac{\sqrt{L}-\sqrt{L_0}}{\sqrt{L_0}} = \frac{\sqrt{L_0+\Delta L}-\sqrt{L_0}}{\sqrt{L_0}} = \frac{\sqrt{L_0+\alpha L_0\Delta T}-\sqrt{L_0}}{\sqrt{L_0}}$$

$$= \sqrt{1+\alpha\Delta T}-1 = \sqrt{1+\left(19\times10^{-6}/\mathrm{C}^{\circ}\right)\left(8\mathrm{C}^{\circ}\right)}-1 = 7.60\times10^{-5}$$

Thus the amount of time lost in any time period τ_0 is $\Delta\tau = \left(7.60\times10^{-5}\right)\tau_0$. For one year, we have the following.

$$\Delta\tau = \left(7.60\times10^{-5}\right)\left(3.16\times10^{7}\,\mathrm{s}\right) = 2402\,\mathrm{s} \approx \boxed{40\ \text{min}}$$

$\boxed{21.}$ (*a*) Consider the adjacent diagrams. The mercury expands due to the heat, as does the bulb volume. The volume of filled glass is equal to the volume of mercury at both temperatures. The value δL is the amount the thread of mercury moves. The additional length of the mercury column in the tube multiplied by the tube cross sectional area will be equal to the expansion of the volume of mercury, minus the expansion of the volume of the glass bulb. Since the tube volume is so much smaller than the bulb volume we can ignore any changes in the tube dimensions and in the mercury initially in the tube volume.

Bulb volume V_0^{bulb}

Tube radius r_0

L_0

$L_0 + \delta L$
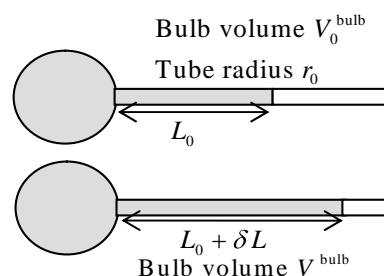
Bulb volume V^{bulb}

Original volume for glass bulb and Hg in bulb: V_0^{bulb}

Change in glass bulb volume: $\Delta V_{\text{glass}} = V_0^{\text{bulb}} \beta_{\text{glass}} \Delta T$

Change in Hg volume in glass bulb: $\Delta V_{\text{Hg}} = V_0^{\text{bulb}} \beta_{\text{Hg}} \Delta T$

Now find the additional volume of Hg, and use that to find the change in length of Hg in the tube.

$$(\delta L)\pi r_0^2 = \Delta V_{\text{Hg}} - \Delta V_{\text{glass}} = V_0^{\text{bulb}} \beta_{\text{Hg}} \Delta T - V_0^{\text{bulb}} \beta_{\text{glass}} \Delta T \quad \rightarrow$$

$$\delta L = \frac{V_0^{\text{bulb}}}{\pi r_0^2} \Delta T \left(\beta_{\text{Hg}} - \beta_{\text{glass}} \right) = \frac{V_0^{\text{bulb}}}{\pi (d_0/2)^2} \Delta T \left(\beta_{\text{Hg}} - \beta_{\text{glass}} \right) = \frac{4V_0^{\text{bulb}}}{\pi d_0^2} \Delta T \left(\beta_{\text{Hg}} - \beta_{\text{glass}} \right)$$

$$= \frac{4(0.255\,\text{cm}^3)}{\pi (1.40 \times 10^{-2}\,\text{cm})^2} (33.0^\circ\text{C} - 11.5^\circ\text{C}) \left[(180-9) \times 10^{-6} / \text{C}^\circ \right] = \boxed{6.1\,\text{cm}}$$

(b) The formula is quoted above: $\delta L = \dfrac{4V_0^{\text{bulb}}}{\pi d_0^2} \Delta T \left(\beta_{\text{Hg}} - \beta_{\text{glass}} \right)$.

22. The change in radius with heating does not cause a torque on the rotating wheel, and so the wheel's angular momentum does not change. Also recall that for a cylindrical wheel rotating about its axis, the moment of inertia is $I = \frac{1}{2}mr^2$.

$$L_0 = L_{\text{final}} \quad \rightarrow \quad I_0 \omega_0 = I_{\text{final}} \omega_{\text{final}} \quad \rightarrow \quad \omega_{\text{final}} = \frac{I_0 \omega_0}{I_{\text{final}}} = \frac{\frac{1}{2}mr_0^2 \omega_0}{\frac{1}{2}mr^2} = \frac{r_0^2 \omega_0}{r^2}$$

$$\frac{\Delta \omega}{\omega} = \frac{\omega_{\text{final}} - \omega_0}{\omega_0} = \frac{\dfrac{r_0^2 \omega_0}{r^2} - \omega_0}{\omega_0} = \frac{r_0^2}{r^2} - 1 = \frac{r_0^2}{\left(r_0 + \Delta r\right)^2} - 1 = \frac{r_0^2}{\left(r_0 + \alpha r_0 \Delta T\right)^2} - 1 = \frac{r_0^2}{\left(r_0 + \alpha r_0 \Delta T\right)^2} - 1$$

$$= \frac{1}{\left(1 + \alpha \Delta T\right)^2} - 1 = \frac{1 - \left(1 + 2\alpha\Delta T + \left(\alpha\Delta T\right)^2\right)}{\left(1 + \alpha\Delta T\right)^2} = \frac{-2\alpha\Delta T - \left(\alpha\Delta T\right)^2}{\left(1 + \alpha\Delta T\right)^2} = -\alpha\Delta T \frac{2 + \alpha\Delta T}{\left(1 + \alpha\Delta T\right)^2}$$

Now assume that $\alpha\Delta T \ll 1$, and so $\dfrac{\Delta \omega}{\omega} = -\alpha\Delta T \dfrac{2 + \alpha\Delta T}{\left(1 + \alpha\Delta T\right)^2} \approx -2\alpha\Delta T$. Evaluate at the given values.

$$-2\alpha\Delta T = -2\left(25 \times 10^{-6} / \text{C}^\circ\right)\left(55\,\text{C}^\circ\right) = \boxed{-2.8 \times 10^{-3}}$$

23. The thermal stress must compensate for the thermal expansion. E is Young's modulus for the aluminum.

$$\text{Stress} = F/A = \alpha E \Delta T = \left(25 \times 10^{-6} / \text{C}^\circ\right)\left(70 \times 10^9\,\text{N}/\text{m}^2\right)\left(35^\circ\text{C} - 15^\circ\text{C}\right) = \boxed{3.5 \times 10^7\,\text{N}/\text{m}^2}$$

24. (a) Since the beam cannot shrink while cooling, the tensile stress must compensate in order to keep the length constant.

$$\text{Stress} = F/A = \alpha E \Delta T = \left(12 \times 10^{-6} / \text{C}^\circ\right)\left(200 \times 10^9\,\text{N}/\text{m}^2\right)\left(60\,\text{C}^\circ\right) = \boxed{1.4 \times 10^8\,\text{N}/\text{m}^2}$$

(b) The ultimate tensile strength of steel (from chapter 9) is $5 \times 10^8\,\text{N}/\text{m}^2$, and so $\boxed{\text{the ultimate strength is not exceeded}}$. There would only be a safety factor of about 3.5.

(*c*)   For concrete, repeat the calculation with the expansion coefficient and elastic modulus for concrete.

$$\text{Stress} = F/A = \alpha E \Delta T = \left(12 \times 10^{-6}/\text{C}^\circ\right)\left(20 \times 10^9 \text{ N}/\text{m}^2\right)\left(60\,\text{C}^\circ\right) = \boxed{1.4 \times 10^7 \text{ N}/\text{m}^2}$$

The ultimate tensile strength of concrete is $2 \times 10^6 \text{ N}/\text{m}^2$, and so $\boxed{\text{the concrete will fracture}}$.

25.   (*a*)   Calculate the change in temperature needed to increase the diameter of the iron band so that it fits over the barrel. Assume that the barrel does not change in dimensions.

$$\Delta L = \alpha L_0 \Delta T \quad \rightarrow \quad L - L_0 = \alpha L_0 \left(T - T_0\right)$$

$$T = T_0 + \frac{L - L_0}{\alpha L_0} = 20^\circ\text{C} + \frac{134.122 \text{ cm} - 134.110 \text{ cm}}{\left(12 \times 10^{-6}/\text{C}^\circ\right)\left(134.110 \text{ cm}\right)} = 27.457^\circ\text{C} \approx \boxed{27^\circ\text{C}}$$

(*b*)   Since the band cannot shrink while cooling, the thermal stress must compensate in order to keep the length at a constant 132.122 cm. E is Young's modulus for the material.

$$\text{Stress} = F/A = \alpha E \Delta T \quad \rightarrow \quad F = AE \frac{\Delta L}{L_0} = AE\alpha\Delta T$$

$$= \left(7.4 \times 10^{-2}\,\text{m}\right)\left(6.5 \times 10^{-3}\,\text{m}\right)\left(100 \times 10^9 \text{ N}/\text{m}^2\right)\left(12 \times 10^{-6}/\text{C}^\circ\right)\left(7.457\text{C}^\circ\right) = \boxed{4.3 \times 10^3 \text{ N}}$$

26.   Use the relationships $T(\text{K}) = T(^\circ\text{C}) + 273.15$ and $T(\text{K}) = \frac{5}{9}\left[T(^\circ\text{F}) - 32\right] + 273.15$.

(*a*) $T(\text{K}) = T(^\circ\text{C}) + 273.15 = 86 + 273.15 = \boxed{359 \text{ K}}$

(*b*) $T(\text{K}) = \frac{5}{9}\left[T(^\circ\text{F}) - 32\right] + 273.15 = \frac{5}{9}\left[78 - 32\right] + 273.15 = \boxed{299 \text{ K}}$

(*c*) $T(\text{K}) = T(^\circ\text{C}) + 273.15 = -100 + 273.15 = \boxed{173 \text{ K}}$

(*d*) $T(\text{K}) = T(^\circ\text{C}) + 273.15 = 5500 + 273.15 = \boxed{5773 \text{ K}}$

(*e*) $T(\text{K}) = \frac{5}{9}\left[T(^\circ\text{F}) - 32\right] + 273.15 = \frac{5}{9}\left[-459 - 32\right] + 273.15 = \boxed{0.37 \text{ K}}$

27.   Use the relationship that $T(\text{K}) = \frac{5}{9}\left[T(^\circ\text{F}) - 32\right] + 273.15$.

$$T(\text{K}) = \frac{5}{9}\left[T(^\circ\text{F}) - 32\right] + 273.15 \quad \rightarrow$$

$$T(^\circ\text{F}) = \frac{9}{5}\left[T(\text{K}) - 273.15\right] + 32 = \frac{9}{5}\left[0 - 273.15\right] + 32 = \boxed{-459.67^\circ\text{F}}$$

28.   Use the relationship that $T(\text{K}) = T(^\circ\text{C}) + 273.15$.

(*a*) $T(\text{K}) = T(^\circ\text{C}) + 273.15 = 4270 \text{ K} \approx \boxed{4300 \text{ K}}$; $T(\text{K}) = T(^\circ\text{C}) + 273.15 = \boxed{15 \times 10^6 \text{ K}}$

(*b*)   % error $= \dfrac{\Delta T}{T(\text{K})} \times 100 = \dfrac{273}{T(\text{K})} \times 100$

$4000^\circ\text{C}:\ \dfrac{273}{4000} \times 100 \approx \boxed{7\%}$ $15 \times 10^6 \ ^\circ\text{C}:\ \dfrac{273}{15 \times 10^6} \times 100 \approx \boxed{2 \times 10^{-3}\%}$

29. Assume the gas is ideal. Since the amount of gas is constant, the value of $\dfrac{PV}{T}$ is constant.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \rightarrow \quad V_2 = V_1 \frac{P_1}{P_2} \frac{T_2}{T_1} = \left(3.00\,\text{m}^3\right)\left(\frac{1.00\,\text{atm}}{3.20\,\text{atm}}\right)\frac{\left(273+38\right)\text{K}}{273\,\text{K}} = \boxed{1.07\,\text{m}^3}$$

30. Assume the air is an ideal gas. Since the amount of air is constant, the value of $\dfrac{PV}{T}$ is constant.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \rightarrow \quad T_2 = T_1 \frac{P_2}{P_1} \frac{V_2}{V_1} = \left(293\,\text{K}\right)\left(\frac{40\,\text{atm}}{1\,\text{atm}}\right)\left(\frac{1}{9}\right) = 1300\,\text{K} = 1030^\circ\text{C} \approx \boxed{1000^\circ\text{C}}$$

31. Assume the oxygen is an ideal gas. From Example 13-10, the volume of one mole of oxygen gas at STP is $22.4 \times 10^{-3}\,\text{m}^3$. The mass of one mole of oxygen, with a molecular mass of 32.0 u, is 32.0 grams. Use these values to calculate the density of the oxygen gas.

$$\rho = \frac{M}{V} = \frac{32.0 \times 10^{-3}\,\text{kg}}{22.4 \times 10^{-3}\,\text{m}^3} = \boxed{1.43\,\text{kg}/\text{m}^3}$$

32. Assume that the nitrogen and carbon dioxide are ideal gases, and that the volume and temperature are constant for the two gases. From the ideal gas law, the value of $\dfrac{P}{n} = \dfrac{RT}{V}$ is constant. Also note that concerning the ideal gas law, the identity of the gas is unimportant, as long as the number of moles is considered.

$$\frac{P_1}{n_1} = \frac{P_2}{n_2} \quad \rightarrow \quad P_2 = P_1 \frac{n_2}{n_1} = \left(3.65\,\text{atm}\right)\left(\frac{\dfrac{21.6\,\text{kg CO}_2}{44 \times 10^{-3}\,\text{kg CO}_2/\text{mol}}}{\dfrac{21.6\,\text{kg N}_2}{28 \times 10^{-3}\,\text{kg N}_2/\text{mol}}}\right) = \left(3.65\,\text{atm}\right)\left(\frac{28}{44}\right) = \boxed{2.32\,\text{atm}}$$

33. (*a*) Assume the nitrogen is an ideal gas. The number of moles of nitrogen is found from the atomic weight, and then the ideal gas law is used to calculate the volume of the gas.

$$n = \left(18.5\,\text{kg}\right)\frac{1\,\text{mole N}_2}{28.0 \times 10^{-3}\,\text{kg}} = 660.71\,\text{mol}$$

$$PV = nRT \quad \rightarrow \quad V = \frac{nRT}{P} = \frac{\left(660.71\,\text{mol}\right)\left(8.315\,\text{J}/\text{mol}\cdot\text{K}\right)\left(273\,\text{K}\right)}{1.013 \times 10^5\,\text{Pa}} = 14.806\,\text{m}^3$$

$$\approx \boxed{14.8\,\text{m}^3}$$

     (*b*) Hold the volume and temperature constant, and again use the ideal gas law.

$$n = \left(18.5\,\text{kg} + 15.0\,\text{kg}\right)\frac{1\,\text{mole N}_2}{28.0 \times 10^{-3}\,\text{kg}} = 1196\,\text{mol}$$

$$PV = nRT \quad \rightarrow$$

$$P = \frac{nRT}{V} = \frac{\left(1196\,\text{mol}\right)\left(8.315\,\text{J}/\text{mol}\cdot\text{K}\right)\left(273\,\text{K}\right)}{14.806\,\text{m}^3} = \boxed{1.83 \times 10^5\,\text{Pa} = 1.82\,\text{atm}}$$

34. (*a*) Assume that the helium is an ideal gas, and then use the ideal gas law to calculate the volume. Absolute pressure must be used, even though gauge pressure is given.

$$PV = nRT \quad \rightarrow \quad V = \frac{nRT}{P} = \frac{(18.75 \text{ mol})(8.315 \text{ J/mol·K})(283 \text{ K})}{(1.350 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})} = \boxed{0.323 \text{ m}^3}$$

(*b*) Since the amount of gas is not changed, the value of PV/T is constant.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \rightarrow \quad T_2 = T_1 \frac{P_2}{P_1} \frac{V_2}{V_1} = (283 \text{ K}) \left(\frac{2.00 \text{ atm}}{1.350 \text{ atm}} \right) \left(\frac{1}{2} \right) = 210 \text{ K} = \boxed{-63^\circ \text{C}}$$

35. Assume the argon is an ideal gas. The number of moles of argon is found from the atomic weight, and then the ideal gas law is used to find the pressure.

$$n = (105.0 \text{ kg}) \frac{1 \text{ mole Ar}}{39.95 \times 10^{-3} \text{ kg}} = 2628 \text{ mol}$$

$$PV = nRT \quad \rightarrow \quad P = \frac{nRT}{V} = \frac{(2628 \text{ mol})(8.315 \text{ J/mol·k})(385 \text{ K})}{(35.0 \text{ L})(1.00 \times 10^{-3} \text{ m}^3/\text{L})} = \boxed{2.40 \times 10^8 \text{ Pa}}$$

36. Assume that the oxygen and helium are ideal gases, and that the volume and temperature are constant for the two gases. From the ideal gas law, the value of $\dfrac{P}{n} = \dfrac{RT}{V}$ is constant. Also note that concerning the ideal gas law, the identity of the gas is unimportant, as long as the number of moles is considered. Finally, gauge pressure must be changed to absolute pressure.

$$\frac{P_1}{n_1} = \frac{P_2}{n_2} \quad \rightarrow \quad n_2 = n_1 \frac{P_2}{P_1} = (26.0 \text{ kg O}_2) \left(\frac{1 \text{ mole O}_2}{32 \times 10^{-3} \text{ kg}} \right) \frac{(8.00 \text{ atm})}{(9.70 \text{ atm})} = 6.70 \times 10^2 \text{ moles}$$

$$(6.70 \times 10^2 \text{ moles}) \left(\frac{4.0 \times 10^{-3} \text{ kg}}{1 \text{ mole He}} \right) = \boxed{2.68 \text{ kg He}}$$

37. There are three forces to consider: the buoyant force upwards (which is the weight of the cold air displaced by the volume of the balloon), the downward weight of the hot air in the balloon, and the downward weight of the passengers and equipment. For the balloon to rise at constant speed, the buoyant force must equal the two weights.

$$F_{\text{buoyant}} = m_{\text{hot}} g + 2700 \text{ N} \quad \rightarrow \quad V \rho_{\text{cold}} g = V \rho_{\text{hot}} g + 2700 \text{ N}$$

The ideal gas law can be written in terms of the gas density ρ and the molecular mass M as follows.

$$PV = nRT = \frac{m}{M} RT \quad \rightarrow \quad \frac{PM}{R} = \frac{m}{V} T = \rho T$$

The gas inside and outside the balloon is air, and so M is the same for inside and outside. Also, since the balloon is open to the atmosphere, the pressure in the balloon is the same as the pressure outside the balloon. Thus the ideal gas law reduces to $\rho T = \text{constant} = (\rho T)_{\text{cold}} = (\rho T)_{\text{hot}}$.

$$V \rho_{\text{cold}} g = V \rho_{\text{hot}} g + 2700 \text{ N} = V \rho_{\text{cold}} \frac{T_{\text{cold}}}{T_{\text{hot}}} g + 2700 \text{ N} \quad \rightarrow$$

$$T_{\text{hot}} = \frac{V \rho_{\text{cold}} T_{\text{cold}} g}{(V \rho_{\text{cold}} g - 2700 \text{ N})} = \frac{(1800 \text{ m}^3)(1.29 \text{ kg/m}^3)(273 \text{ K})(9.80 \text{ m/s}^2)}{\left[(1800 \text{ m}^3)(1.29 \text{ kg/m}^3)(9.80 \text{ m/s}^2) - 2700 \text{ N} \right]}$$

$$= 309.8 \text{ K} \approx \boxed{37^\circ \text{C}}$$

One factor limiting the maximum altitude would be that as the balloon rises, the density of the air decreases, and thus the temperature required gets higher. Eventually the air would be too hot and the balloon fabric might be damaged.

38. Assume that the air is an ideal gas. The pressure and volume are held constant. From the ideal gas law, the value of $\dfrac{PV}{R} = nT$ is held constant.

$$n_1 T_1 = n_2 T_2 \quad \rightarrow \quad \frac{n_2}{n_1} = \frac{T_1}{T_2} = \frac{(273+15)\,\text{K}}{(273+38)\,\text{K}} = \frac{288}{311} = 0.926$$

Thus $1 - 0.926 = 0.074 = \boxed{7.4\,\%}$ must be removed.

39. Assume the oxygen is an ideal gas. Since the amount of gas is constant, the value of PV/T is constant.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \rightarrow \quad P_2 = P_1 \frac{V_1}{V_2}\frac{T_2}{T_1} = (2.45\ \text{atm})\left(\frac{61.5\ \text{L}}{48.8\ \text{L}}\right)\frac{(273+50.0)\,\text{K}}{(273+18.0)\,\text{K}} = \boxed{3.43\ \text{atm}}$$

40. Assume the helium is an ideal gas. Since the amount of gas is constant, the value of PV/T is constant. We assume that since the outside air pressure decreases by 30%, the air pressure inside the balloon will also decrease 30%.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \rightarrow \quad \frac{V_2}{V_1} = \frac{P_1}{P_2}\frac{T_2}{T_1} = \left(\frac{1.0}{0.70}\right)\frac{(273+5.0)\,\text{K}}{(273+20.0)\,\text{K}} = \boxed{1.4\ \text{times the original volume}}$$

41. At STP, 1 mole of ideal gas occupies 22.4 L.

$$\frac{1\ \text{mole}}{22.4\ \text{L}}\left(\frac{6.02\times10^{23}\,\text{molecules}}{\text{mole}}\right)\left(\frac{1\ \text{L}}{10^{-3}\,\text{m}^3}\right) = \boxed{2.69\times10^{25}\ \text{molecules}/\text{m}^3}$$

42. We assume that the water is at 4°C so that its density is $1000\ \text{kg}/\text{m}^3$.

$$1.000\ \text{L}\left(\frac{10^{-3}\,\text{m}^3}{1\ \text{L}}\right)\left(\frac{1000\ \text{kg}}{1\ \text{m}^3}\right)\left(\frac{1\ \text{mol}}{(15.9994 + 2\times1.00794)\times10^{-3}\,\text{kg}}\right) = \boxed{55.51\ \text{mol}}$$

$$55.51\ \text{mol}\left(\frac{6.02\times10^{23}\,\text{molecules}}{1\ \text{mol}}\right) = \boxed{3.34\times10^{25}\ \text{molecules}}$$

43. (*a*) Since the average depth of the oceans is very small compared to the radius of the Earth, the ocean's volume can be calculated as that of a spherical shell with surface area $4\pi R_{\text{Earth}}^2$ and a thickness Δy . Then use the density of sea water to find the mass, and the molecular weight of water to find the number of moles.

$$\text{Volume} = 0.75\left(4\pi R_{\text{Earth}}^2\right)\Delta y = 0.75(4\pi)\left(6.38\times10^6\,\text{m}\right)^2\left(3\times10^3\,\text{m}\right) = 1.15\times10^{18}\,\text{m}^3$$

$$1.15\times10^{18}\,\text{m}^3\left(\frac{1025\ \text{kg}}{\text{m}^3}\right)\left(\frac{1\ \text{mol}}{18\times10^{-3}\,\text{kg}}\right) = 6.55\times10^{22}\ \text{moles} \approx \boxed{7\times10^{22}\ \text{moles}}$$

(b) $6.55 \times 10^{22} \text{ moles} \left(\dfrac{6.02 \times 10^{23} \text{ molecules}}{1 \text{ mol}} \right) \approx \boxed{4 \times 10^{46} \text{ molecules}}$

44. The net force on each side of the box will be the pressure difference between the inside and outside of the box, times the area of a side of the box. The outside pressure is 1 atmosphere. The ideal gas law is used to find the pressure inside the box, assuming that the mass of gas and the volume are constant.

$$\frac{P}{T} = \frac{nR}{V} = \text{constant} \quad \rightarrow \quad \frac{P_2}{T_2} = \frac{P_1}{T_1} \quad \rightarrow \quad P_2 = P_1 \frac{T_2}{T_1} = (1.00 \text{ atm}) \frac{(273 + 180)\,\text{K}}{(273 + 20)\,\text{K}} = 1.55 \text{ atm}$$

The area of a side of the box is given by

$$\text{Area} = L^2 = \left[\left(\text{Volume of box} \right)^{1/3} \right]^2 = \left(5.1 \times 10^{-2} \text{ m}^2 \right)^{2/3} = 1.4 \times 10^{-1} \text{ m}^2$$

The net force on a side of the box is the pressure difference times the area.

$$F = \left(\Delta \text{Pressure} \right) \left(\text{Area} \right) = \left(0.55 \text{ atm} \right) \left(1.01 \times 10^5 \text{ Pa} \right) \left(1.4 \times 10^{-1} \text{ m}^2 \right) = \boxed{7.6 \times 10^3 \text{ N}}$$

45. We assume that the last breath Galileo took has been spread uniformly throughout the atmosphere since his death. Calculate the number of molecules in Galileo's last breath, and divide it by the volume of the atmosphere, to get "Galileo molecules/m³". Multiply that factor times the size of a breath to find the number of Galileo molecules in one of our breaths.

$$PV = NkT \quad \rightarrow \quad N = \frac{PV}{kT} = \frac{\left(1.01 \times 10^5 \text{ Pa} \right) \left(2.0 \times 10^{-3} \text{ m}^3 \right)}{\left(1.38 \times 10^{-23} \text{ J/K} \right) \left(300 \text{ K} \right)} = 4.9 \times 10^{22} \text{ molecules}$$

$$\text{Atmospheric volume} = 4\pi R_{\text{Earth}}^2 h = 4\pi \left(6.38 \times 10^6 \text{ m} \right)^2 \left(1.0 \times 10^4 \text{ m} \right) = 5.1 \times 10^{18} \text{ m}^3$$

$$\frac{\text{Galileo molecules}}{\text{m}^3} = \frac{4.9 \times 10^{22} \text{ molecules}}{5.8 \times 10^{18} \text{ m}^3} = 9.6 \times 10^3 \text{ molecules} / \text{m}^3$$

$$\frac{\text{\# Galileo molecules}}{\text{breath}} = 9.6 \times 10^3 \frac{\text{molecules}}{\text{m}^3} \left(\frac{2.0 \times 10^{-3} \text{ m}^3}{1 \text{ breath}} \right) = \boxed{19 \frac{\text{molecules}}{\text{breath}}}$$

46. (a) The average translational kinetic energy of a gas molecule is $\frac{3}{2} kT$.

$$KE_{\text{avg}} = \tfrac{3}{2} kT = \tfrac{3}{2} \left(1.38 \times 10^{-23} \text{ J/K} \right) \left(273 \text{ K} \right) = \boxed{5.65 \times 10^{-21} \text{ J}}$$

(b) The total translational kinetic energy is the average kinetic energy per molecule, times the number of molecules.

$$KE_{\text{total}} = N \left(KE_{\text{avg}} \right) = \left(2.0 \text{ mol} \right) \left(\frac{6.02 \times 10^{23} \text{ molecules}}{1} \right) \tfrac{3}{2} \left(1.38 \times 10^{-23} \text{ J/K} \right) \left(293 \text{ K} \right)$$

$$= \boxed{7.3 \times 10^3 \text{ J}}$$

$\boxed{47.}$ The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$. Helium has an atomic mass of 4.0.

$$v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3 \left(1.38 \times 10^{-23} \text{ J/K} \right) \left(6000 \text{ K} \right)}{4.0 \left(1.66 \times 10^{-27} \text{ kg} \right)}} = 6116 \text{ m/s} \approx \boxed{6 \times 10^3 \text{ m/s}}$$

48. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \frac{\sqrt{3kT_2/m}}{\sqrt{3kT_1/m}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{373 \text{ K}}{273 \text{ K}}} = \boxed{1.17}$$

49. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$. Since the rms speed is proportional to the square root of the absolute temperature, to double the rms speed without changing the mass, the absolute temperature must be multiplied by a factor of 4.

$$T_{\text{fast}} = 4T_{\text{slow}} = 4(273 + 20) \text{ K} = 1172 \text{ K} = \boxed{899^\circ \text{C}}$$

50. The rms speed is the square root of the mean (average) of the squares of the speeds.

$$v_{\text{rms}} = \sqrt{\frac{6^2 + 2^2 + 4^2 + 6^2 + 0^2 + 4^2 + 1^2 + 8^2 + 5^2 + 3^2 + 7^2 + 8^2}{12}} = \sqrt{\frac{320}{12}} = \boxed{5.2 \text{ km/s}}$$

The average speed is $v_{\text{avg}} = \dfrac{6+2+4+6+0+4+1+8+5+3+7+8}{12} = \dfrac{54}{12} = 4.5 \text{ km/s}$.

51. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = 1.010 = \frac{\sqrt{3kT_2/m}}{\sqrt{3kT_1/m}} = \sqrt{\frac{T_2}{T_1}} \quad \rightarrow$$

$$T_2 = T_1 (1.010)^2 = (293.2 \text{ K})(1.010)^2 = 299.1 \text{ K} = \boxed{25.9^\circ \text{C}}$$

52. From the ideal gas law, $PV = nRT$, if the volume and amount of gas are held constant, the temperature is proportional to the pressure, $PV = nRT \;\rightarrow\; P = \dfrac{nR}{V}T = (\text{constant})T$. Thus the temperature will be doubled . Since the rms speed is proportional to the square root of the temperature, $v_{\text{rms}} = \sqrt{3kT/m} = (\text{constant})\sqrt{T}$, the rms speed will be multiplied by a factor of $\sqrt{2} = \boxed{1.41}$.

53. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$. The temperature can be found from the ideal gas law, $PV = NkT \;\rightarrow\; kT = PV/N$. The mass of the gas is the mass of a molecule times the number of molecules: $M = Nm$, and the density of the gas is the mass per unit volume, $\rho = \dfrac{M}{V}$.

Combining these relationships gives the following.

$$v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3PV}{Nm}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}}$$

54. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \frac{\sqrt{3kT/m_2}}{\sqrt{3kT/m_1}} \quad \rightarrow \quad \boxed{\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \sqrt{\frac{m_1}{m_2}}}$$

55. The temperature of the nitrogen gas is found from the ideal gas law, and then the rms speed is found from the temperature.

$$PV = nRT \quad \rightarrow \quad T = \frac{PV}{nR} = \frac{(2.1\,\text{atm})(1.013 \times 10^5\,\text{Pa/atm})(8.5\,\text{m}^3)}{(1300\,\text{mol})(8.315\,\text{J/mol}\cdot\text{K})} = 167.3\,\text{K}$$

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23}\,\text{J/K})(167.3\,\text{K})}{28(1.66 \times 10^{-27}\,\text{kg})}} = 386.0\,\text{m/s} \approx \boxed{3.9 \times 10^2\,\text{m/s}}$$

56. (a) The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23}\,\text{J/K})(273\,\text{K})}{32(1.66 \times 10^{-27}\,\text{kg})}} = \boxed{461\,\text{m/s}}$$

(b) Assuming that the particle has no preferred direction, then we have the following:

$$v_{\text{rms}}^2 = v_x^2 + v_y^2 + v_z^2 = 3v_x^2 \quad \rightarrow \quad v_x = v_{\text{rms}}/\sqrt{3} .$$

The time for one crossing of the room is then given by $t = d/v_x = \sqrt{3}d/v_{\text{rms}}$, and so the time for a round trip is $2\sqrt{3}d/v_{\text{rms}}$. Thus the number of back and forth round trips per second is the reciprocal of this time, $\dfrac{v_{\text{rms}}}{2\sqrt{3}d}$.

$$\text{\# round trips per sec} = \frac{v_{\text{rms}}}{2\sqrt{3}d} = \frac{461\,\text{m/s}}{2\sqrt{3}(7.0\,\text{m})} = \boxed{19\,\text{round trips per sec}}$$

57. Assume that nitrogen is an ideal gas, and that each molecule occupies the same cubical volume of L^3 . Find the volume per molecule from the ideal gas law, and then the side length of that cubical molecular volume will be an estimate of the average distance between molecules.

$$PV = NkT \quad \rightarrow \quad \frac{V}{N} = \frac{kT}{P} = \frac{(1.38 \times 10^{-23}\,\text{J/K})(273\,\text{K})}{1.01 \times 10^5\,\text{Pa}} = 3.73 \times 10^{-26}\,\text{m}^3/\text{molecule}$$

$$L = \left(\frac{V}{N}\right)^{1/3} = (3.73 \times 10^{-26}\,\text{m}^3)^{1/3} = \boxed{3.34 \times 10^{-9}\,\text{m}}$$

58. It is stated in the text that the relationship $v_{\text{rms}} = \sqrt{3kT/m}$ is applicable to molecules within living cells at body temperature (37°C).

(a) For the amino acid: $\quad v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\dfrac{3(1.38 \times 10^{-23}\,\text{J/K})(310\,\text{K})}{89(1.66 \times 10^{-27}\,\text{kg})}} = \boxed{2.9 \times 10^2\,\text{m/s}}$.

(b) For the protein: $\quad v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\dfrac{3(1.38 \times 10^{-23}\,\text{J/K})(310\,\text{K})}{(50,000)(1.66 \times 10^{-27}\,\text{kg})}} = \boxed{12\,\text{m/s}}$

59. The pressure can be stated in terms of the ideal gas law, $P = NkT/V$. Substitute for the temperature from the expression for the rms speed, $v_{rms} = \sqrt{3kT/m} \;\rightarrow\; T = mv_{rms}^2/3k$. The mass of the gas is the mass of a molecule times the number of molecules: $M = Nm$, and the density of the gas is the mass per unit volume, $\rho = M/V$. Combining these relationships gives the following.

$$P = \frac{NkT}{V} = \frac{Nk}{V}\frac{mv_{rms}^2}{3k} = \tfrac{1}{3}\frac{Nm}{V}v_{rms}^2 = \tfrac{1}{3}\frac{M}{V}v_{rms}^2 = \tfrac{1}{3}\rho v_{rms}^2 \;\rightarrow\; \boxed{P = \tfrac{1}{3}\rho v_{rms}^2}$$

60. The rms speed is given by Equation 13-9, $v_{rms} = \sqrt{3kT/m}$.

$$\frac{(v_{rms})_{^{235}UF_6}}{(v_{rms})_{^{235}UF_6}} = \frac{\sqrt{3kT/m_{^{235}UF_6}}}{\sqrt{3kT/m_{^{238}UF_6}}} = \sqrt{\frac{m_{^{238}UF_6}}{m_{^{235}UF_6}}} = \sqrt{\frac{238+6(19)}{235+6(19)}} = \sqrt{\frac{352}{349}} = \boxed{1.004}$$

61. (*a*) From Fig. 13-21, at atmospheric pressure, CO_2 can exist as solid or vapor.

(*b*) From Fig. 13-21, for CO_2 to exist as a liquid, $\boxed{5.11\,\text{atm} \le P \le 73\,\text{atm}}$ and $\boxed{-56.6^{\circ}\text{C} \le T \le 31^{\circ}\text{C}}$.

62. (*a*) From Fig. 13-20, water is vapor when the pressure is 0.01 atm and the temperature is 90°C.

(*b*) From Fig. 13-20, water is solid when the pressure is 0.01 atm and the temperature is -20°C.

63. From Table 13-3, if the temperature is 25°C, the saturated vapor pressure is 23.8 torr. If the relative humidity is 50%, then the partial pressure of water is half the saturated vapor pressure, or 11.9 torr. The dew point is the temperature at which the saturated vapor pressure is 11.9 torr, and from Table 13-3 that is between 10°C and 15°C. Since there is no entry for 11.9 torr, the temperature can be estimated by a linear interpolation. Between 10°C and 15°C, the temperature change per torr is as follows:

$$\frac{(15-10)\,\text{C}^{\circ}}{(12.8-9.21)\,\text{torr}} = 1.393\,\text{C}^{\circ}/\text{torr}.$$

Thus the temperature corresponding to 11.9 torr is

$$10^{\circ}\text{C} + \left[(11.9-9.21)\,\text{torr}\right]\left(1.393\,\text{C}^{\circ}/\text{torr}\right) = 13.7^{\circ}\text{C} \approx \boxed{14^{\circ}\text{C}}.$$

64. At the boiling temperature, the external air pressure equals the saturated vapor pressure. Thus from Table 13-3, the air pressure is $\boxed{526\,\text{torr}}$ or $\boxed{7.01\times10^4\,\text{Pa}}$ or $\boxed{0.692\,\text{atm}}$.

65. At the boiling temperature, the air pressure equals the saturated vapor pressure. The pressure of 0.72 atm is equal to $7.27\times10^4\,\text{Pa}$. From Table 13-3, the temperature is between 90°C and 100°C. Since there is no entry for $7.27\times10^4\,\text{Pa}$, the temperature can be estimated by a linear interpolation. Between 90°C and 100°C, the temperature change per Pa is as follows:

$$\frac{(100-90)\,\text{C}^{\circ}}{(10.1-7.01)\times10^4\,\text{Pa}} = 3.236\times10^{-4}\,\text{C}^{\circ}/\text{Pa}.$$

Thus the temperature corresponding to $7.27\times10^4\,\text{Pa}$ is

$$90^{\circ}\text{C} + \left[(7.27-7.01)\times10^4\,\text{Pa}\right]\left(3.236\times10^{-4}\,\text{C}^{\circ}/\text{Pa}\right) = 90.8^{\circ}\text{C} \approx \boxed{91^{\circ}\text{C}}.$$

66. The relative humidity of 40% with a partial pressure of 530 Pa of water gives a saturated vapor pressure of

$$0.40 P_{\text{saturated}} = 530 \text{ Pa} \quad \rightarrow \quad P_{\text{saturated}} = \frac{530 \text{ Pa}}{0.40} = 1325 \text{ Pa}$$

From Table 13-3, the temperature at which the saturated vapor pressure is 1325 Pa is between 10°C and 15°C. Since there is no entry for 1325 Pa, the temperature can be estimated by a linear interpolation. Between 10°C and 15°C, the temperature change per Pa is as follows:

$$\frac{(15-10) \text{C}^{\circ}}{(1.71-1.23) \times 10^3 \text{Pa}} = 1.042 \times 10^{-2} \text{ C}^{\circ}/\text{Pa} \, .$$

Thus the temperature corresponding to 1325 Pa is

$$10^{\circ}\text{C} + \left[(1325-1230) \text{Pa}\right]\left(1.042 \times 10^{-2} \text{ C}^{\circ}/\text{Pa}\right) = 10.99^{\circ}\text{C} \approx \boxed{11^{\circ}\text{C}} \, .$$

67. From Table 13-3, the saturated vapor pressure at 25°C is 3170 Pa. Since the relative humidity is 35%, the partial pressure of water is

$$P_{\text{water}} = 0.35 P_{\text{saturated}} = 0.35 (3170 \text{ Pa}) = \boxed{1.1 \times 10^3 \text{Pa}}$$

68. Since the water is boiling at 120°C, the saturated vapor pressure is the same as the pressure inside the pressure cooker. From Table 13-3, the pressure is $\boxed{1.99 \times 10^5 \text{Pa} = 1.97 \text{ atm}}$.

69. The total amount of water vapor that can be in the air can be found from the saturated vapor pressure in Table 13-3, using the ideal gas law. At 25°C, that pressure is $3.17 \times 10^3 \text{Pa}$.

$$PV = nRT \quad \rightarrow \quad n = \frac{PV}{RT} = \frac{\left(3.17 \times 10^3 \text{Pa}\right)\left(680 \text{ m}^3\right)}{(8.315 \text{ J}/\text{mol} \cdot \text{K})(273+25) \text{K}} = 8.70 \times 10^2 \text{ moles}$$

Since the relative humidity is only 80%, only 80% of the total possible water is in the air. Thus 20% of the total possible water can still evaporate into the air.

$$m_{\text{evaporate}} = 0.20 \left(8.70 \times 10^2 \text{ moles}\right) \left(\frac{18 \times 10^{-3} \text{kg}}{1 \text{ mole}}\right) = \boxed{3.1 \text{ kg}}$$

70. The air is saturated at the lower temperature, so the vapor pressure of water is 872 Pa. The ideal gas law gives the following result for the change in volume of the given mass of air.

$$PV = nRT \quad \rightarrow \quad \frac{P}{nR} = \frac{T}{V} = \text{constant} \quad \rightarrow \quad \frac{T_1}{V_1} = \frac{T_2}{V_2}$$

Thus the vapor pressure of a given mass of air that moves from outside to inside is as follows.

$$P_{\text{in}} = \frac{nRT_{\text{in}}}{V_{\text{in}}} = \frac{nRT_{\text{out}}}{V_{\text{out}}} = P_{\text{out}} = 872 \text{ Pa}$$

The saturated vapor pressure at the inside temperature is 3170 Pa, and so the relative humidity is as follows.

$$\text{rel. hum.} = \frac{872 \text{ Pa}}{3170 \text{ Pa}} = 0.275 = \boxed{27.5\%}$$

71. From Example 13-19, we have an expression for the time to diffuse a given distance. Divide the distance by the time to get the average speed.

$$t = \frac{\overline{C}}{\Delta C}\frac{(\Delta x)^2}{D} = \frac{\frac{1}{2}(1.00+0.40)\,\text{mol}/\text{m}^3}{(1.00-0.40)\,\text{mol}/\text{m}^3}\frac{(15\times10^{-6}\,\text{m})^2}{(95\times10^{-11}\,\text{m}^2/\text{s})} = 0.2763\text{ s} \approx \boxed{0.28\text{ s}}$$

$$v_{\text{diffuse}} = \frac{\Delta x}{t} = \frac{15\times10^{-6}\,\text{m}}{0.2763\text{ s}} = \boxed{5.4\times10^{-5}\,\text{m}/\text{s}}$$

The rms thermal speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3(1.38\times10^{-23}\,\text{J}/\text{K})(293\text{ K})}{75(1.66\times10^{-27}\,\text{kg})}} = \boxed{3.1\times10^2\,\text{m}/\text{s}}$$

$$\frac{v_{\text{diffuse}}}{v_{\text{rms}}} = \frac{5.4\times10^{-5}\,\text{m}/\text{s}}{3.1\times10^2\,\text{m}/\text{s}} = 1.7\times10^{-7}$$

The diffusion speed is several orders of magnitude smaller than the thermal speed.

72. (*a*) Use the ideal gas law to find the concentration of the oxygen. We assume that the air pressure is 1.00 atm, and so the pressure caused by the oxygen is 0.21 atm.

$PV = nRT$ →

$$\frac{n}{V} = \frac{P}{RT} = \frac{(0.21\,\text{atm})(1.013\times10^5\,\text{Pa}/\text{atm})}{(8.315\,\text{J}/\text{mol}\cdot\text{K})(293\text{ K})} = 8.732\,\text{mol}/\text{m}^3 \approx \boxed{8.7\,\text{mol}/\text{m}^3}$$

(*b*) Use Equation 13-10 to calculate the diffusion rate.

$$J = DA\frac{C_1 - C_2}{\Delta x} = (1\times10^{-5}\,\text{m}^2/\text{s})(2\times10^{-9}\,\text{m}^2)\left(\frac{8.732\,\text{mol}/\text{m}^3 - 4.366\,\text{mol}/\text{m}^3}{2\times10^{-3}\,\text{m}}\right)$$

$$= 4.366\times10^{-11}\,\text{mol}/\text{s} \approx \boxed{4\times10^{-11}\,\text{mol}/\text{s}}$$

(*c*) From Example 13-19, we have an expression for the time to diffuse a given distance.

$$t = \frac{\overline{C}}{\Delta C}\frac{(\Delta x)^2}{D} = \frac{\frac{1}{2}(8.732\,\text{mol}/\text{m}^3 + 4.366\,\text{mol}/\text{m}^3)}{(8.732\,\text{mol}/\text{m}^3 - 4.366\,\text{mol}/\text{m}^3)}\frac{(2\times10^{-3}\,\text{m})^2}{1\times10^{-5}\,\text{m}^2/\text{s}} = \boxed{0.6\text{ s}}$$

73. (*a*) At 34°C, the tape will expand from its calibration, and so will $\boxed{\text{read low}}$.

(*b*) $\dfrac{\Delta L}{L_0} = \alpha\Delta T = (12\times10^{-6}/{}^\circ\text{C})(34^\circ\text{C} - 20^\circ\text{C}) = 1.68\times10^{-4} \approx \boxed{1.7\times10^{-2}\,\%}$

74. Since the glass does not expand, the measuring cup will contain 300 mL of hot water. Find the volume of water after it cools.

$$\Delta V = V_0\beta\Delta T = (300\text{ mL})(210\times10^{-6}/\text{C}^\circ)(20^\circ\text{C} - 80^\circ\text{C}) = -3.78\text{mL} \approx \boxed{-3.8\text{ mL}}$$

The volume of cool water is 3.8 mL less than the desired volume of 300 mL.

75. Assume the helium is an ideal gas. The volume of the cylinder is constant, and we assume that the temperature of the gas is also constant in the cylinder. From the ideal gas law, $PV = nRT$, under these conditions the amount of gas is proportional to the absolute pressure.

$$PV = nRT \quad \rightarrow \quad \frac{P}{n} = \frac{RT}{V} = \text{constant} \quad \rightarrow \quad \frac{P_1}{n_1} = \frac{P_2}{n_2} \quad \rightarrow \quad \frac{n_2}{n_1} = \frac{P_2}{P_1} = \frac{5\,\text{atm}+1\,\text{atm}}{28\,\text{atm}+1\,\text{atm}} = \frac{6}{29}$$

Thus $\boxed{6/29 = 0.207 \approx 21\%}$ of the original gas remains in the cylinder.

76. Assume the air is an ideal gas, and that the pressure is 1.0 atm.

$PV = NkT \quad \rightarrow$

$$N = \frac{PV}{kT} = \frac{\left(1.013 \times 10^5 \,\text{Pa}\right)\left(6.5 \times 3.1 \times 2.5\right)\text{m}^3}{\left(1.38 \times 10^{-23} \,\text{J/K}\right)\left(273 + 22\right)\text{K}} = 1.253 \times 10^{27} \text{ molecules} \approx \boxed{1.3 \times 10^{27} \text{ molecules}}$$

$$1.253 \times 10^{27} \text{ molecules}\left(\frac{1 \text{ mole}}{6.02 \times 10^{23} \text{ molecules}}\right) = 2082 \text{ moles} \approx \boxed{2.1 \times 10^3 \text{ moles}}$$

77. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$. Hydrogen atoms have a mass of 1 atomic mass unit.

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38 \times 10^{-23} \,\text{J/K}\right)\left(2.7 \text{ K}\right)}{1\left(1.66 \times 10^{-27} \,\text{kg}\right)}} = \boxed{260 \,\text{m/s}}$$

The pressure is found from the ideal gas law, $PV = NkT$.

$$PV = NkT \quad \rightarrow \quad P = \frac{NkT}{V} = \frac{(1)\left(1.38 \times 10^{-23} \,\text{J/K}\right)\left(2.7 \text{ K}\right)}{1 \text{ cm}^3\left(\dfrac{1 \times 10^{-6} \,\text{m}^3}{1 \text{cm}^3}\right)} = 3.726 \times 10^{-17} \,\text{Pa}\left(\frac{1 \text{ atm}}{1.01 \times 10^5 \,\text{Pa}}\right)$$

$$= 3.689 \times 10^{-22} \,\text{atm} \approx \boxed{4 \times 10^{-22} \,\text{atm}}$$

78. Assume the gas is ideal at those low pressures, and use the ideal gas law.

$$PV = NkT \quad \rightarrow \quad \frac{N}{V} = \frac{P}{kT} = \frac{1 \times 10^{-12} \,\text{N/m}^2}{\left(1.38 \times 10^{-23} \,\text{J/K}\right)\left(273 \text{ K}\right)} = \left(3 \times 10^8 \,\frac{\text{molecules}}{\text{m}^3}\right)\left(\frac{10^{-6} \,\text{m}^3}{1 \text{ cm}^3}\right)$$

$$= \boxed{3 \times 10^2 \,\frac{\text{molecules}}{\text{cm}^3}}$$

79. Assume that the air in the lungs is an ideal gas, that the amount of gas is constant, and that the temperature is constant. The ideal gas law then says that the value of PV is constant. The pressure a distance h below the surface of the water is discussed in chapter 10, and is given by $P = P_0 + \rho g h$, where P_0 is atmospheric pressure and ρ is the density of the water.

$$\left(PV\right)_{\text{surface}} = \left(PV\right)_{\text{submerged}} \quad \rightarrow \quad V_{\text{surface}} = V_{\text{submerged}} \frac{P_{\text{submerged}}}{P_{\text{surface}}} = V_{\text{submerged}} \frac{P_{\text{atm}} + \rho g h}{P_{\text{atm}}}$$

$$= \left(5.5 \text{ L}\right)\frac{1.01 \times 10^5 \,\text{Pa} + \left(1.0 \times 10^3 \,\text{kg/m}^3\right)\left(9.8 \,\text{m/s}^2\right)\left(10 \,\text{m}\right)}{1.01 \times 10^5 \,\text{Pa}} = \boxed{11 \text{ L}}$$

This is obviously very dangerous, to have the lungs attempt to inflate to twice their volume. Thus it is $\boxed{\text{not advisable}}$ to quickly rise to the surface.

80. The temperature can be found from the rms speed by Equation 13-9, $v_{rms} = \sqrt{3kT/m}$.

$$v_{rms} = \sqrt{3kT/m} \;\rightarrow$$

$$T = \frac{mv_{rms}^2}{3k} = \frac{(28)(1.66\times10^{-27}\,\text{kg})\left[4\times10^4\,\text{km/h}\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{3(1.38\times10^{-23}\,\text{J/K})} = \boxed{1.4\times10^5\,\text{K}}$$

81. From the ideal gas law, if the volume and number of moles stay constant, then the ratio of pressure and temperature is constant.

$$PV = nRT \;\rightarrow\; \frac{P}{T} = \frac{nR}{V} = \text{constant} \;\rightarrow\; \frac{P_2}{T_2} = \frac{P_1}{T_1} \;\rightarrow$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{(273+360)\,\text{K}}{(273+110)\,\text{K}} = \frac{633\,\text{K}}{383\,\text{K}} = \boxed{1.65}$$

From the relationship $v_{rms} = \sqrt{3kT/m}$, if the mass per molecule stays constant, then the ratio of rms speed to the square root of the temperature is constant.

$$v_{rms} = \sqrt{3kT/m} \;\rightarrow\; \frac{v_{rms}}{\sqrt{T}} = \sqrt{\frac{3k}{m}} = \text{constant} \;\rightarrow\; \frac{(v_{rms})_2}{\sqrt{T_2}} = \frac{(v_{rms})_1}{\sqrt{T_1}} \;\rightarrow$$

$$\frac{(v_{rms})_2}{(v_{rms})_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}} = \sqrt{\frac{633\,\text{K}}{383\,\text{K}}} = \boxed{1.29}$$

82. To do this problem, the "molecular weight" of air is needed. If we approximate air as 70% N_2 (molecular weight 28) and 30% O_2 (molecular weight 32), then the average molecular weight is
$$0.70(28) + 0.30(32) = 29$$

(*a*) Treat the air as an ideal gas. Assume that the pressure is 1.00 atm.

$$PV = nRT \;\rightarrow\; n = \frac{PV}{RT} = \frac{(1.01\times10^5\,\text{Pa})(770\,\text{m}^3)}{(8.315\,\text{J/mol·k})(293\,\text{K})} = 3.192\times10^4\,\text{moles}$$

$$m = (3.192\times10^4\,\text{moles})(29\times10^{-3}\,\text{kg/mol}) = 925.7\,\text{kg} \approx \boxed{9.3\times10^2\,\text{kg}}$$

(*b*) Find the mass of air at the lower temperature, and then subtract the mass at the higher temperature.

$$n = \frac{PV}{RT} = \frac{(1.01\times10^5\,\text{Pa})(770\,\text{m}^3)}{(8.315\,\text{J/mol·k})(263\,\text{K})} = 3.556\times10^4\,\text{moles}$$

$$m = (3.556\times10^4\,\text{moles})(29\times10^{-3}\,\text{kg/mol}) = 1031.2\,\text{kg}$$

The mass entering the house is $1031.2\,\text{kg} - 925.7\,\text{kg} = 105.5\,\text{kg} \approx \boxed{1.1\times10^2\,\text{kg}}$.

83. Since the pressure is force per unit area, if the pressure is multiplied by the surface area of the Earth, the force of the air is found. If we assume that the force of the air is due to its weight, then the mass of the air can be found. The number of molecules can then be found using the molecular mass of air (calculated in problem 82) and Avogadro's number.

$$P = \frac{F}{A} \;\rightarrow\; F = PA \;\rightarrow\; Mg = P4\pi R_{Earth}^2 \;\rightarrow$$

$$M = \frac{4\pi R_{Earth}^2 P}{g} = \frac{4\pi \left(6.38 \times 10^6 \, \text{m}\right)^2 \left(1.01 \times 10^5 \, \text{Pa}\right)}{9.80 \, \text{m/s}^2} = 5.27 \times 10^{18} \, \text{kg}$$

$$N = 5.27 \times 10^{18} \, \text{kg} \left(\frac{1 \, \text{mole}}{29 \times 10^{-3} \, \text{kg}}\right) \left(\frac{6.02 \times 10^{23} \, \text{molecules}}{1 \, \text{mole}}\right) = \boxed{1.1 \times 10^{44} \, \text{molecules}}$$

84. The temperature of the nitrogen gas is found from the ideal gas law, and then the rms speed is found from the temperature.

$$PV = nRT \quad \rightarrow \quad T = \frac{PV}{nR} = \frac{\left(4.2 \, \text{atm}\right)\left(1.013 \times 10^5 \, \text{Pa/atm}\right)\left(7.6 \, \text{m}^3\right)}{\left(1800 \, \text{mol}\right)\left(8.315 \, \text{J/mol} \cdot \text{K}\right)} = 216 \, \text{K}$$

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38 \times 10^{-23} \, \text{J/K}\right)\left(216 \, \text{K}\right)}{28\left(1.66 \times 10^{-27} \, \text{kg}\right)}} = 438.7 \, \text{m/s} \approx \boxed{4.4 \times 10^2 \, \text{m/s}}$$

85. The amount of gas is the same under both sets of conditions, as is the temperature. Thus the ideal gas law gives $PV = nRT = \text{constant} \quad \rightarrow \quad \left(PV\right)_1 = \left(PV\right)_2$. Absolute pressure must be used instead of gauge pressure.

$$\left(PV\right)_{\substack{\text{in} \\ \text{cylinder}}} = \left(PV\right)_{\substack{\text{flowing} \\ \text{out}}} \quad \rightarrow$$

$$V_{\substack{\text{flowing} \\ \text{out}}} = \frac{\left(PV\right)_{\substack{\text{in} \\ \text{cylinder}}}}{P_{\substack{\text{flowing} \\ \text{out}}}} = \frac{\left[\left(1.38 \times 10^7 + 1.01 \times 10^5\right)\text{Pa}\right]\left(16 \, \text{L}\right)}{\left(1.01 \times 10^5 \, \text{Pa}\right)} = 2202 \, \text{L}$$

So there are 2202 L of oxygen to dispense, at a rate of 2.4 L/min.

$$t = \frac{2202 \, \text{L}}{2.4 \, \text{L/min}} = 917.5 \, \text{min} \approx \boxed{15 \, \text{hours}}$$

86. (*a*) The iron floats in the mercury because $\rho_{Hg} > \rho_{Fe}$. As the substances are heated, the density of both substances will decrease due to volume expansion (see problem 17 for a detailed discussion of this effect). The density of the mercury decreases more upon heating than the density of the iron, because $\beta_{Hg} > \beta_{Fe}$. The net effect is that the densities get closer together, and so relatively more mercury will have to be displaced to hold up the iron, and the iron will float $\boxed{\text{lower}}$ in the mercury.

(*b*) The fraction of the volume submerged is $V_{\substack{Hg \\ \text{displaced}}} \Big/ V_{Fe}$. Both volumes expand as heated. The subscript "displace" is dropped for convenience.

$$\text{fractional change} = \frac{V_{Hg}/V_{Fe} - V_{0\,Hg}/V_{0\,Fe}}{V_{0\,Hg}/V_{0\,Fe}} = \frac{\dfrac{V_{0\,Hg}\left(1 + \beta_{Hg}\Delta T\right)}{V_{0\,Fe}\left(1 + \beta_{Fe}\Delta T\right)} - V_{0\,Hg}/V_{0\,Fe}}{V_{0\,Hg}/V_{0\,Fe}} = \frac{\left(1 + \beta_{Hg}\Delta T\right)}{\left(1 + \beta_{Fe}\Delta T\right)} - 1$$

$$= \frac{1 + \left(180 \times 10^{-6}/\text{C}^\circ\right)\left(25 \, \text{C}^\circ\right)}{1 + \left(35 \times 10^{-6}\right)\left(25 \, \text{C}^\circ\right)} - 1 = \frac{1.0045}{1.000875} - 1 = 3.6 \times 10^{-3}$$

$$\% \text{ change} = \left(3.6 \times 10^{-3}\right)\left(100\right) = \boxed{0.36 \, \%}$$

87. (*a*) Assume that a mass M of gasoline with volume V_0 at 0°C is under consideration, and so its
density is $\rho_0 = M/V_0$. At a temperature of 32°C, the same mass has a
volume $V = V_0 \left(1 + \beta \Delta T\right)$.

$$\rho = \frac{M}{V} = \frac{M}{V_0 \left(1 + \beta \Delta T\right)} = \frac{\rho_0}{1 + \beta \Delta T} = \frac{0.68 \times 10^3 \text{ kg}/\text{m}^3}{1 + \left(950 \times 10^{-6}/\text{C}^\circ\right)\left(38\text{C}^\circ\right)} = 0.6563 \times 10^3 \text{ kg}/\text{m}^3$$

$$\approx \boxed{0.66 \times 10^3 \text{ kg}/\text{m}^3}$$

(*b*) Calculate the percentage change in the density.

$$\% \text{ change} = \frac{\left(0.6563 - 0.68\right) \times 10^3 \text{ kg}/\text{m}^3}{0.68 \times 10^3 \text{ kg}/\text{m}^3 V} \times 100 = \boxed{-3.5\%}$$

88. The original length of the steel band is $L_0 = 2\pi R_{\text{Earth}}$. At the higher temperature, the length of the
band is $L = L_0 + \Delta L = 2\pi R = 2\pi \left(R_{\text{Earth}} + \Delta R\right)$. The change in radius, ΔR, would be the height above
the Earth.

$$\Delta L = \alpha L_0 \Delta T = 2\pi \Delta R \quad \rightarrow$$

$$\Delta R = \frac{\alpha L_0 \Delta T}{2\pi} = \alpha R_{\text{Earth}} \Delta T = \left(12 \times 10^{-6}/\text{C}^\circ\right)\left(6.38 \times 10^6 \text{ m}\right)\left(45^\circ\text{C} - 25 \,^\circ\text{C}\right) = \boxed{1.5 \times 10^3 \text{ m}}$$

89. The gap will be the radius of the lid minus the radius of the jar. Also note that the original radii of
the lid and the jar are the same.

$$r_{\text{gap}} = \left(r_0 + \Delta r\right)_{\text{lid}} - \left(r_0 + \Delta r\right)_{\text{jar}} = \Delta r_{\text{lid}} - \Delta r_{\text{jar}} = \left(\alpha_{\text{brass}} - \alpha_{\text{glass}}\right) r_0 \Delta T$$

$$= \left(19 \times 10^{-6}/\text{C}^\circ - 9 \times 10^{-6}/\text{C}^\circ\right)\left(4.0 \text{ cm}\right)\left(40\text{C}^\circ\right) = \boxed{1.6 \times 10^{-3} \text{ cm}}$$

90. The change in length is to be restricted to $\Delta L < 1.0 \times 10^{-6} \text{ m}$.

$$\Delta L = \alpha L_0 \Delta T \leq 1.0 \times 10^{-6} \text{ m} \quad \rightarrow \quad \Delta T \leq \frac{1.0 \times 10^{-6} \text{ m}}{\left(9 \times 10^{-6}/\text{C}^\circ\right)\left(1.0 \text{ m}\right)} \leq 0.11\text{C}^\circ$$

Thus the temperature would have to be controlled to within $\boxed{\pm 0.11\text{C}^\circ}$

91. (*a*) Treat the air as an ideal gas. Since the amount and temperature of the air are the same in both
cases, the ideal gas law says $PV = nRT$ is a constant.

$$P_2 V_2 = P_1 V_1 \quad \rightarrow \quad V_2 = V_1 \frac{P_1}{P_2} = \left(11.3 \text{ L}\right)\frac{195 \text{ atm}}{1.00 \text{ atm}} = 2203.5 \text{ L} \approx \boxed{2.20 \times 10^3 \text{ L}}$$

(*b*) Before entering the water, the air coming out of the tank will be at 1.00 atm pressure, and so the
person will be able to breathe 2203.5 L of air.

$$t = 2203.5 \text{ L} \left(\frac{1 \text{ breath}}{2.0 \text{ L}}\right)\left(\frac{1 \text{ min}}{12 \text{ breaths}}\right) = \boxed{92 \text{ min}}$$

(*c*) When the person is underwater, the temperature and pressure will be different. Use the ideal
gas equation to relate the original tank conditions to the underwater breathing conditions. The
amount of gas will be constant, so $PV/T = nR$ will be constant. The pressure a distance h

below the surface of the water is discussed in chapter 10, and is given by $P = P_0 + \rho g h$, where P_0 is atmospheric pressure and ρ is the density of the sea water.

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1} \quad \rightarrow \quad V_2 = V_1 \frac{P_1}{P_2} \frac{T_2}{T_1}$$

$$V_2 = (11.3 \text{ L}) \left[\frac{195 \text{ atm} \left(1.01 \times 10^5 \text{ Pa/atm}\right)}{1.01 \times 10^5 \text{ Pa} + \left(1.025 \times 10^3 \text{ kg/m}^3\right)\left(9.8 \text{ m/s}^2\right)(20.0 \text{ m})} \right] \left(\frac{283 \text{ K}}{293 \text{ K}}\right)$$

$$= 7.120 \times 10^2 \text{ L} \qquad t = 7.120 \times 10^2 \text{ L}\left(\frac{1 \text{ breath}}{2.0 \text{ L}}\right)\left(\frac{1 \text{ min}}{12 \text{ breaths}}\right) = \boxed{3.0 \times 10^1 \text{ min}}$$

92. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$. Using the escape velocity as v_{rms}, solve for the temperature.

    (*a*) For oxygen molecules: $T = \dfrac{m v_{\text{rms}}^2}{3k} = \dfrac{32\left(1.66 \times 10^{-27} \text{ kg}\right)\left(1.12 \times 10^4 \text{ m/s}\right)^2}{3\left(1.38 \times 10^{-23} \text{ J/K}\right)} = 1.6 \times 10^5 \text{ K}$

    (*b*) For helium atoms: $T = \dfrac{m v_{\text{rms}}^2}{3k} = \dfrac{4\left(1.66 \times 10^{-27} \text{ kg}\right)\left(1.12 \times 10^4 \text{ m/s}\right)^2}{3\left(1.38 \times 10^{-23} \text{ J/K}\right)} = 2.0 \times 10^4 \text{ K}$

    (*c*) Because the "escape temperature" is so high for oxygen, very few oxygen molecules ever escape the atmosphere. But helium, with one-eighth the mass, can escape at a much lower temperature. While the temperature of the Earth is not close to $2.0 \times 10^4 \text{ K}$ today, during the Earth's formation its temperature was possibly much hotter – presumably hot enough that helium was able to escape the atmosphere.

93. Following the development of the kinetic molecular theory in section 13-10 of the textbook, the tennis balls hitting the trash can lid are similar to the particles colliding with the walls of a container causing pressure. Quoting from the text, "the average force averaged over many collisions will be equal to the force exerted during one collision divided by the time between collisions." That average force must be the weight of the trash can lid in order to suspend it.

$$F_{\text{avg}} = M_{\text{lid}} g = \frac{2 m_{\text{ball}} v_{\text{ball}}}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{2 m_{\text{ball}} v_{\text{ball}}}{M_{\text{lid}} g}$$

The above expression is "seconds per ball", so its reciprocal will be "balls per second".

$$\text{balls/s} = \frac{1}{\Delta t} = \frac{M_{\text{lid}} g}{2 m_{\text{ball}} v_{\text{ball}}} = \frac{(1.0 \text{ kg})\left(9.8 \text{ m/s}^2\right)}{2(0.060 \text{ kg})(12 \text{ m/s})} = \boxed{6.8 \text{ balls/s}}$$

94. The amount of gas and the temperature of the gas are constant. Then from the ideal gas law, the quantity $PV = nRT$ is constant. The pressure at the surface will be 1 atmosphere. The pressure a distance h below the surface of the water is discussed in chapter 10, and is given by $P = P_0 + \rho g h$, where P_0 is atmospheric pressure and ρ is the density of the water. The volume of a spherical bubble is given by $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$.

$$P_{\text{surface}} V_{\text{surface}} = P_{\text{depth}} V_{\text{depth}} \quad \rightarrow \quad P_{\text{surface}} \tfrac{1}{6}\pi d_{\text{surface}}^3 = P_{\text{depth}} \tfrac{1}{6}\pi d_{\text{depth}}^3 \quad \rightarrow$$

$$d_{surface} = d_{depth} \left(\frac{P_{depth}}{P_{surface}} \right)^{1/3} = (3.00 \text{ cm}) \left(\frac{1.01 \times 10^5 \text{ Pa} + (1.0 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(14.0 \text{ m})}{1.01 \times 10^5 \text{ Pa}} \right)^{1/3}$$

$$= \boxed{3.99 \text{ cm}}$$

95. (*a*) At a temperature of 30°C, the saturated vapor pressure, from Table 13-6, is $4.24 \times 10^3 \text{ Pa}$. If the relative humidity is 40%, then the water vapor pressure is 40% of the saturated vapor pressure.

$$0.40 (4.24 \times 10^3 \text{ Pa}) = \boxed{1.7 \times 10^3 \text{ Pa}}$$

  (*b*) At a temperature of 5°C, the saturated vapor pressure, from Table 13-6, is $8.72 \times 10^2 \text{ Pa}$. If the relative humidity is 80%, then the water vapor pressure is 80% of the saturated vapor pressure.

$$0.80 (8.72 \times 10^2 \text{ Pa}) = \boxed{7.0 \times 10^2 \text{ Pa}}$$

96. Assume that the water is an ideal gas, and that the temperature is constant. From Table 13-3, saturated vapor pressure at 90°C is $7.01 \times 10^4 \text{ Pa}$, and so to have a relative humidity of 10%, the vapor pressure will be $7.01 \times 10^3 \text{ Pa}$. Use the ideal gas law to calculate the amount of water.

$$PV = nRT \quad \rightarrow$$

$$n = \frac{PV}{RT} = \frac{(7.01 \times 10^3 \text{ Pa})(7.0 \text{ m}^3)}{(8.315 \text{ J/mol·K})(273 + 90) \text{ K}} = 16.26 \text{ moles} \left(\frac{18 \times 10^{-3} \text{ kg}}{1 \text{ mole}} \right) = \boxed{0.29 \text{ kg}}$$

97. The density is the mass divided by the volume. Let the original volume of the mass of iron be V_0, the original density $\rho_0 = M/V_0$. The volume of that same mass deep in the Earth is $V = V_0 + \Delta V$, and so the density deep in the Earth is $\rho = M/V = M/(V_0 + \Delta V)$. The change in volume is due to two effects: the increase in volume due to a higher temperature, $\Delta V_{temp} = \beta V_0 \Delta T$, and the decrease in volume due to a higher pressure, $\Delta V_{pressure} = -V_0 \Delta P/B$. So $\Delta V = \Delta V_{temp} + \Delta V_{pressure}$. The new density is then

$$\rho = M/V = \frac{M}{V_0 + \Delta V} = \frac{M}{V_0 + \Delta V_{temp} + \Delta V_{pressure}} = \frac{M}{V_0 + \beta V_0 \Delta T - V_0 \Delta P/B} = \frac{M}{V_0} \frac{1}{(1 + \beta \Delta T - \Delta P/B)}$$

$$= \frac{\rho_0}{(1 + \beta \Delta T - \Delta P/B)}$$

$$= \frac{\rho_0}{\left[1 + (35 \times 10^{-6}/\text{C}°)(2000\text{C}°) - (5000 \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})/(90 \times 10^9 \text{ N/m}^2) \right]}$$

$$= \frac{\rho_0}{[1 + 0.07 - .00561]} = 0.9395 \rho_0 \quad \rightarrow \quad \boxed{6\% \text{ decrease}}$$

98. (*a*) Assume the pressure and amount of gas are held constant, and so $P_0 V_0 = nRT_0$ and $P_0 V = nRT$. From these two expressions calculate the change in volume and relate it to the change in temperature.

$$V = V_0 + \Delta V \quad \rightarrow \quad \Delta V = V - V_0 = \frac{nRT}{P_0} - \frac{nRT_0}{P_0} = \frac{nR}{P_0}(T - T_0) = \frac{V_0}{T_0} \Delta T$$

But $\Delta V = \beta V_0 \Delta T$, and so $\Delta V = \dfrac{V_0}{T_0} \Delta T = \beta V_0 \Delta T \rightarrow \beta = \dfrac{1}{T_0}$

For $T_0 = 293$ K, $\beta = \dfrac{1}{T_0} = \dfrac{1}{293 \text{ K}} = \boxed{3.4 \times 10^{-3}/\text{K}}$, which agrees well with Table 13-1.

(*b*) Assume the temperature and amount of gas are held constant, and so $P_0 V_0 = nRT_0 = PV$. From these two expressions calculate change in volume and relate it to the change in pressure.

$V = V_0 + \Delta V \rightarrow$

$$\Delta V = V - V_0 = \dfrac{nRT_0}{P} - \dfrac{nRT_0}{P_0} = nRT_0 \left(\dfrac{1}{P} - \dfrac{1}{P_0} \right) = \dfrac{nRT_0}{P_0} \left(\dfrac{P_0 - P}{P} \right) = V_0 \dfrac{1}{P} (-\Delta P)$$

But from chapter 9, $\Delta V = -V_0 \dfrac{1}{B} \Delta P$ and so $\Delta V = V_0 \dfrac{1}{P} (-\Delta P) = -V_0 \dfrac{1}{B} \Delta P \rightarrow \boxed{B = P}$

99. Assume that the water vapor behaves like an ideal gas. At 20°C, the saturated vapor pressure is 2.33×10^3 Pa. Using the ideal gas law, find the number of moles of water in the air at both 95% and 30%. Subtract those mole amounts to find the amount of water that must be removed.

$PV = nRT \rightarrow n = \dfrac{PV}{RT} \rightarrow$

$$n_1 - n_2 = \dfrac{V}{RT}(P_1 - P_2) = \dfrac{(95 \text{ m}^2)(2.8 \text{ m})}{(8.315 \text{ J/mol} \cdot \text{k})(293 \text{ K})}(2.33 \times 10^3 \text{ Pa})(0.95 - 0.30) = 165 \text{ mol}$$

$165 \text{ mol} \left(\dfrac{18 \times 10^{-3} \text{ kg}}{1 \text{ mol}} \right) = \boxed{3.0 \text{ kg}}$