PUZZLER

In a moment the arresting cable will be pulled taut, and the 140-mi/h landing of this F/A-18 Hornet on the aircraft carrier USS *Nimitz* will be brought to a sudden conclusion. The pilot cuts power to the engine, and the plane is stopped in less than 2 s. If the cable had not been successfully engaged, the pilot would have had to take off quickly before reaching the end of the flight deck. Can the motion of the plane be described quantitatively in a way that is useful to ship and aircraft designers and to pilots learning to land on a "postage stamp?" *(Courtesy of the USS* Nimitz*/U.S. Navy)*

chapter 2

Motion in One Dimension

Chapter Outline

As a first step in studying classical mechanics, we describe motion in terms of space and time while ignoring the agents that caused that motion. This portion of classical mechanics is called *kinematics.* (The word *kinematics* has the same root as *cinema.* Can you see why?) In this chapter we consider only motion in one dimension. We first define displacement, velocity, and acceleration. Then, using these concepts, we study the motion of objects traveling in one dimension with a constant acceleration.

From everyday experience we recognize that motion represents a continuous change in the position of an object. In physics we are concerned with three types of motion: translational, rotational, and vibrational. A car moving down a highway is an example of translational motion, the Earth's spin on its axis is an example of rotational motion, and the back-and-forth movement of a pendulum is an example of vibrational motion. In this and the next few chapters, we are concerned only with translational motion. (Later in the book we shall discuss rotational and vibrational motions.)

In our study of translational motion, we describe the moving object as a *particle* regardless of its size. In general, **a particle is a point-like mass having infinitesimal size.** For example, if we wish to describe the motion of the Earth around the Sun, we can treat the Earth as a particle and obtain reasonably accurate data about its orbit. This approximation is justified because the radius of the Earth's orbit is large compared with the dimensions of the Earth and the Sun. As an example on a much smaller scale, it is possible to explain the pressure exerted by a gas on the walls of a container by treating the gas molecules as particles.

2.1 DISPLACEMENT, VELOCITY, AND SPEED

TABLE 2.1
Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

The motion of a particle is completely known if the particle's position in space is known at all times. Consider a car moving back and forth along the x axis, as shown in Figure 2.1a. When we begin collecting position data, the car is 30 m to the right of a road sign. (Let us assume that all data in this example are known to two significant figures. To convey this information, we should report the initial position as 3.0×10^1 m. We have written this value in this simpler form to make the discussion easier to follow.) We start our clock and once every 10 s note the car's location relative to the sign. As you can see from Table 2.1, the car is moving to the right (which we have defined as the positive direction) during the first 10 s of motion, from position Ⓐ to position Ⓑ. The position values now begin to decrease, however, because the car is backing up from position Ⓑ through position Ⓕ. In fact, at Ⓓ, 30 s after we start measuring, the car is alongside the sign we are using as our origin of coordinates. It continues moving to the left and is more than 50 m to the left of the sign when we stop recording information after our sixth data point. A graph of this information is presented in Figure 2.1b. Such a plot is called a *position–time graph.*

If a particle is moving, we can easily determine its change in position. **The displacement of a particle is defined as its change in position.** As it moves from an initial position x_i to a final position x_f, its displacement is given by $x_f - x_i$. We use the Greek letter delta (Δ) to denote the *change* in a quantity. Therefore, we write the displacement, or change in position, of the particle as

$$\Delta x \equiv x_f - x_i \tag{2.1}$$

From this definition we see that Δx is positive if x_f is greater than x_i and negative if x_f is less than x_i.

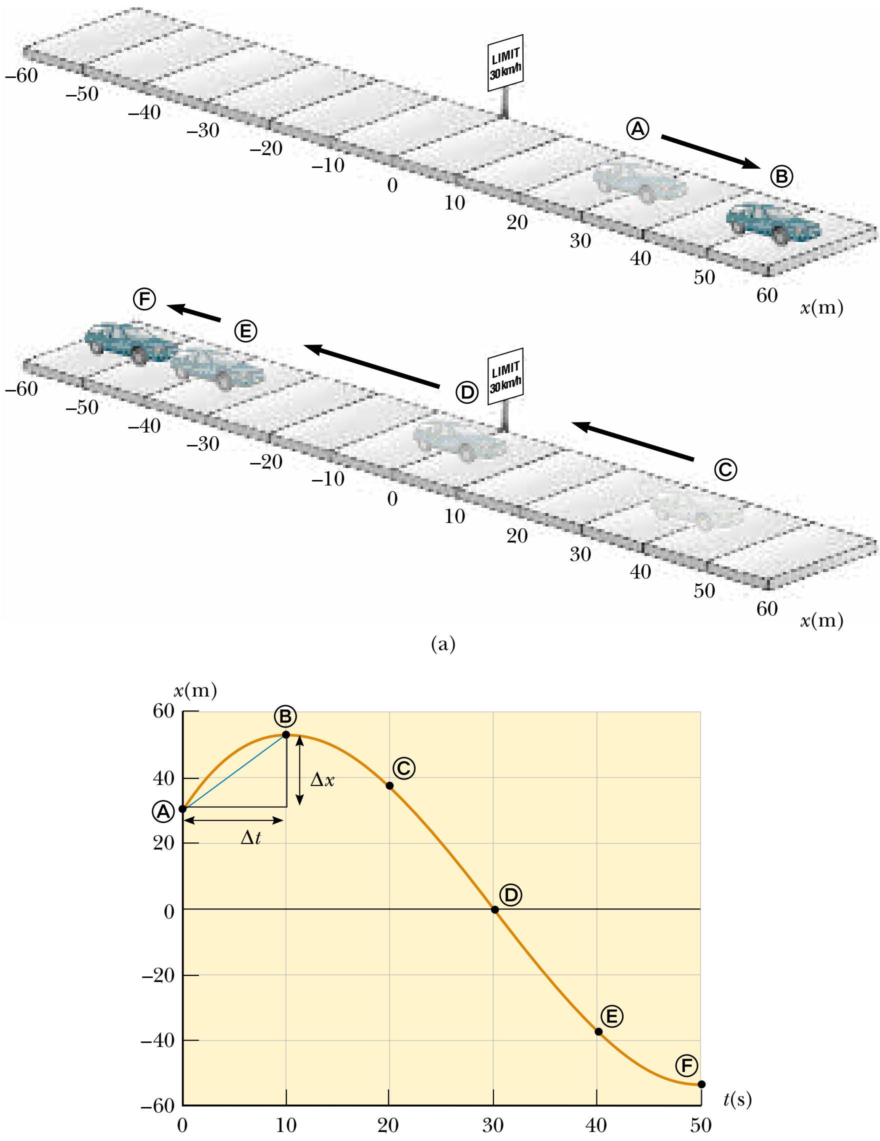


Figure 2.1 (a) A car moves back and forth along a straight line taken to be the *x* axis. Because we are interested only in the car's translational motion, we can treat it as a particle. (b) Position–time graph for the motion of the "particle."

A very easy mistake to make is not to recognize the difference between displacement and distance traveled (Fig. 2.2). A baseball player hitting a home run travels a distance of 360 ft in the trip around the bases. However, the player's displacement is zero because his final and initial positions are identical.

Displacement is an example of a vector quantity. Many other physical quantities, including velocity and acceleration, also are vectors. In general, **a vector is a physical quantity that requires the specification of both direction and magnitude.** By contrast, **a scalar is a quantity that has magnitude and no direction.** In this chapter, we use plus and minus signs to indicate vector direction. We can do this because the chapter deals with one-dimensional motion only; this means that any object we study can be moving only along a straight line. For example, for horizontal motion, let us arbitrarily specify to the right as being the positive direction. It follows that any object always moving to the right undergoes a

Figure 2.2 Bird's-eye view of a baseball diamond. A batter who hits a home run travels 360 ft as he rounds the bases, but his displacement for the round trip is zero. *(Mark C. Burnett/Photo Researchers, Inc.)*

positive displacement $+\Delta x$, and any object moving to the left undergoes a negative displacement $-\Delta x$. We shall treat vectors in greater detail in Chapter 3.

There is one very important point that has not yet been mentioned. Note that the graph in Figure 2.1b does not consist of just six data points but is actually a smooth curve. The graph contains information about the entire 50-s interval during which we watched the car move. It is much easier to see changes in position from the graph than from a verbal description or even a table of numbers. For example, it is clear that the car was covering more ground during the middle of the 50-s interval than at the end. Between positions Ⓒ and Ⓓ, the car traveled almost 40 m, but during the last 10 s, between positions Ⓔ and Ⓕ, it moved less than half that far. A common way of comparing these different motions is to divide the displacement Δx that occurs between two clock readings by the length of that particular time interval Δt. This turns out to be a very useful ratio, one that we shall use many times. For convenience, the ratio has been given a special name—*average velocity.* **The average velocity $\overline{v}_x$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurred:**

Average velocity

$$\overline{v}_x \equiv \frac{\Delta x}{\Delta t} \tag{2.2}$$

3.2

where the subscript x indicates motion along the x axis. From this definition we see that average velocity has dimensions of length divided by time (L/T)—meters per second in SI units.

Although the distance traveled for any motion is always positive, the average velocity of a particle moving in one dimension can be positive or negative, depending on the sign of the displacement. (The time interval Δt is always positive.) If the coordinate of the particle increases in time (that is, if $x_f > x_i$), then Δx is positive and $\overline{v}_x = \Delta x/\Delta t$ is positive. This case corresponds to motion in the positive x direction. If the coordinate decreases in time (that is, if $x_f < x_i$), then Δx is negative and hence $\overline{v}_x$ is negative. This case corresponds to motion in the negative x direction.

We can interpret average velocity geometrically by drawing a straight line between any two points on the position–time graph in Figure 2.1b. This line forms the hypotenuse of a right triangle of height Δx and base Δt. The slope of this line is the ratio $\Delta x/\Delta t$. For example, the line between positions Ⓐ and Ⓑ has a slope equal to the average velocity of the car between those two times, $(52\text{ m} - 30\text{ m})/(10\text{ s} - 0) = 2.2\text{ m/s}$.

In everyday usage, the terms *speed* and *velocity* are interchangeable. In physics, however, there is a clear distinction between these two quantities. Consider a marathon runner who runs more than 40 km, yet ends up at his starting point. His average velocity is zero! Nonetheless, we need to be able to quantify how fast he was running. A slightly different ratio accomplishes this for us. **The average speed of a particle, a scalar quantity, is defined as the total distance traveled divided by the total time it takes to travel that distance:**

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

Average speed

The SI unit of average speed is the same as the unit of average velocity: meters per second. However, unlike average velocity, average speed has no direction and hence carries no algebraic sign.

Knowledge of the average speed of a particle tells us nothing about the details of the trip. For example, suppose it takes you 8.0 h to travel 280 km in your car. The average speed for your trip is 35 km/h. However, you most likely traveled at various speeds during the trip, and the average speed of 35 km/h could result from an infinite number of possible speed values.

EXAMPLE 2.1 Calculating the Variables of Motion

Find the displacement, average velocity, and average speed of the car in Figure 2.1a between positions Ⓐ and Ⓕ.

Solution The units of displacement must be meters, and the numerical result should be of the same order of magnitude as the given position data (which means probably not 10 or 100 times bigger or smaller). From the position–time graph given in Figure 2.1b, note that $x_A = 30$ m at $t_A = 0$ s and that $x_F = -53$ m at $t_F = 50$ s. Using these values along with the definition of displacement, Equation 2.1, we find that

$$\Delta x = x_F - x_A = -53\text{ m} - 30\text{ m} = \boxed{-83\text{ m}}$$

This result means that the car ends up 83 m in the negative direction (to the left, in this case) from where it started. This number has the correct units and is of the same order of magnitude as the supplied data. A quick look at Figure 2.1a indicates that this is the correct answer.

It is difficult to estimate the average velocity without completing the calculation, but we expect the units to be meters per second. Because the car ends up to the left of where we started taking data, we know the average velocity must be negative. From Equation 2.2,

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_F - x_A}{t_F - t_A}$$

$$= \frac{-53\text{ m} - 30\text{ m}}{50\text{ s} - 0\text{ s}} = \frac{-83\text{ m}}{50\text{ s}} = \boxed{-1.7\text{ m/s}}$$

We find the car's average speed for this trip by adding the distances traveled and dividing by the total time:

$$\text{Average speed} = \frac{22\text{ m} + 52\text{ m} + 53\text{ m}}{50\text{ s}} = \boxed{2.5\text{ m/s}}$$

2.2 INSTANTANEOUS VELOCITY AND SPEED

Often we need to know the velocity of a particle at a particular instant in time, rather than over a finite time interval. For example, even though you might want to calculate your average velocity during a long automobile trip, you would be especially interested in knowing your velocity at the *instant* you noticed the police

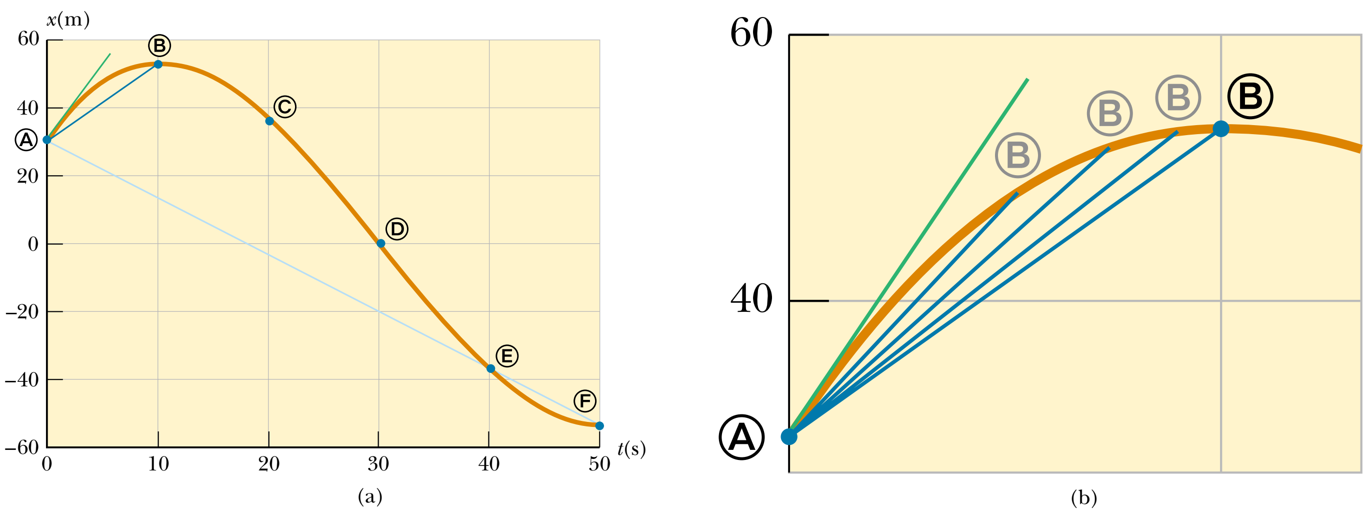


Figure 2.3 (a) Graph representing the motion of the car in Figure 2.1. (b) An enlargement of the upper left-hand corner of the graph shows how the blue line between positions Ⓐ and Ⓑ approaches the green tangent line as point Ⓑ gets closer to point Ⓐ.

car parked alongside the road in front of you. In other words, you would like to be able to specify your velocity just as precisely as you can specify your position by noting what is happening at a specific clock reading—that is, at some specific instant. It may not be immediately obvious how to do this. What does it mean to talk about how fast something is moving if we "freeze time" and talk only about an individual instant? This is a subtle point not thoroughly understood until the late 1600s. At that time, with the invention of calculus, scientists began to understand how to describe an object's motion at any moment in time.

To see how this is done, consider Figure 2.3a. We have already discussed the average velocity for the interval during which the car moved from position Ⓐ to position Ⓑ (given by the slope of the dark blue line) and for the interval during which it moved from Ⓐ to Ⓕ (represented by the slope of the light blue line). Which of these two lines do you think is a closer approximation of the initial velocity of the car? The car starts out by moving to the right, which we defined to be the positive direction. Therefore, being positive, the value of the average velocity during the Ⓐ to Ⓑ interval is probably closer to the initial value than is the value of the average velocity during the Ⓐ to Ⓕ interval, which we determined to be negative in Example 2.1. Now imagine that we start with the dark blue line and slide point Ⓑ to the left along the curve, toward point Ⓐ, as in Figure 2.3b. The line between the points becomes steeper and steeper, and as the two points get extremely close together, the line becomes a tangent line to the curve, indicated by the green line on the graph. The slope of this tangent line represents the velocity of the car at the moment we started taking data, at point Ⓐ. What we have done is determine the *instantaneous velocity* at that moment. In other words, the **instantaneous velocity v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero:**[1]

Definition of instantaneous velocity

3.3

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad \textbf{(2.3)}$$

[1] Note that the displacement Δx also approaches zero as Δt approaches zero. As Δx and Δt become smaller and smaller, the ratio $\Delta x/\Delta t$ approaches a value equal to the slope of the line tangent to the x-versus-t curve.

In calculus notation, this limit is called the *derivative* of x with respect to t, written dx/dt:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad \textbf{(2.4)}$$

The instantaneous velocity can be positive, negative, or zero. When the slope of the position–time graph is positive, such as at any time during the first 10 s in Figure 2.3, v_x is positive. After point Ⓑ, v_x is negative because the slope is negative. At the peak, the slope and the instantaneous velocity are zero.

From here on, we use the word *velocity* to designate instantaneous velocity. When it is *average velocity* we are interested in, we always use the adjective *average.*

The instantaneous speed of a particle is defined as the magnitude of its velocity. As with average speed, instantaneous speed has no direction associated with it and hence carries no algebraic sign. For example, if one particle has a velocity of $+25$ m/s along a given line and another particle has a velocity of -25 m/s along the same line, both have a speed[2] of 25 m/s.

EXAMPLE 2.2 Average and Instantaneous Velocity

A particle moves along the x axis. Its x coordinate varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds.[3] The position–time graph for this motion is shown in Figure 2.4. Note that the particle moves in the negative x direction for the first second of motion, is at rest at the moment $t = 1$ s, and moves in the positive x direction for $t > 1$ s. (a) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

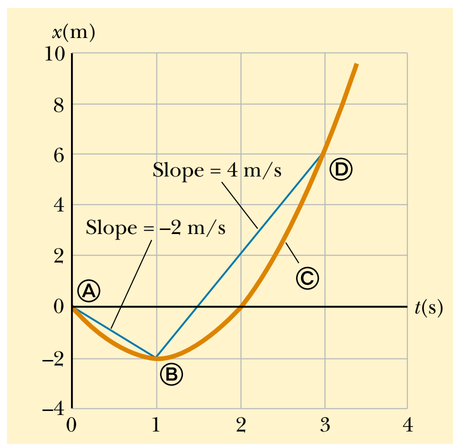


Figure 2.4 Position–time graph for a particle having an x coordinate that varies in time according to the expression $x = -4t + 2t^2$.

Solution During the first time interval, we have a negative slope and hence a negative velocity. Thus, we know that the displacement between Ⓐ and Ⓑ must be a negative number having units of meters. Similarly, we expect the displacement between Ⓑ and Ⓓ to be positive.

In the first time interval, we set $t_i = t_A = 0$ and $t_f = t_B = 1$ s. Using Equation 2.1, with $x = -4t + 2t^2$, we obtain for the first displacement

$$\begin{aligned}\Delta x_{A\to B} &= x_f - x_i = x_B - x_A \\ &= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] \\ &= \boxed{-2 \text{ m}}\end{aligned}$$

To calculate the displacement during the second time interval, we set $t_i = t_B = 1$ s and $t_f = t_D = 3$ s:

$$\begin{aligned}\Delta x_{B\to D} &= x_f - x_i = x_D - x_B \\ &= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] \\ &= \boxed{+8 \text{ m}}\end{aligned}$$

These displacements can also be read directly from the position–time graph.

[2] As with velocity, we drop the adjective for instantaneous speed: "Speed" means instantaneous speed.

[3] Simply to make it easier to read, we write the empirical equation as $x = -4t + 2t^2$ rather than as $x = (-4.00 \text{ m/s})t + (2.00 \text{ m/s}^2)t^{2.00}$. When an equation summarizes measurements, consider its coefficients to have as many significant digits as other data quoted in a problem. Consider its coefficients to have the units required for dimensional consistency. When we start our clocks at $t = 0$ s, we usually do not mean to limit the precision to a single digit. Consider any zero value in this book to have as many significant figures as you need.

(b) Calculate the average velocity during these two time intervals.

Solution In the first time interval, $\Delta t = t_f - t_i = t_B - t_A = 1$ s. Therefore, using Equation 2.2 and the displacement calculated in (a), we find that

$$\bar{v}_{x(A\to B)} = \frac{\Delta x_{A\to B}}{\Delta t} = \frac{-2\text{ m}}{1\text{ s}} = \boxed{-2\text{ m/s}}$$

In the second time interval, $\Delta t = 2$ s; therefore,

$$\bar{v}_{x(B\to D)} = \frac{\Delta x_{B\to D}}{\Delta t} = \frac{8\text{ m}}{2\text{ s}} = \boxed{+4\text{ m/s}}$$

These values agree with the slopes of the lines joining these points in Figure 2.4.

(c) Find the instantaneous velocity of the particle at $t = 2.5$ s.

Solution Certainly we can guess that this instantaneous velocity must be of the same order of magnitude as our previous results, that is, around 4 m/s. Examining the graph, we see that the slope of the tangent at position Ⓒ is greater than the slope of the blue line connecting points Ⓑ and Ⓓ. Thus, we expect the answer to be greater than 4 m/s. By measuring the slope of the position–time graph at $t = 2.5$ s, we find that

$$v_x = \boxed{+6\text{ m/s}}$$

2.3 ACCELERATION

In the last example, we worked with a situation in which the velocity of a particle changed while the particle was moving. This is an extremely common occurrence. (How constant is your velocity as you ride a city bus?) It is easy to quantify changes in velocity as a function of time in exactly the same way we quantify changes in position as a function of time. When the velocity of a particle changes with time, the particle is said to be *accelerating*. For example, the velocity of a car increases when you step on the gas and decreases when you apply the brakes. However, we need a better definition of acceleration than this.

Suppose a particle moving along the x axis has a velocity v_{xi} at time t_i and a velocity v_{xf} at time t_f, as in Figure 2.5a.

The average acceleration of the particle is defined as the *change* in velocity Δv_x divided by the time interval Δt during which that change occurred:

Average acceleration

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \tag{2.5}$$

As with velocity, when the motion being analyzed is one-dimensional, we can use positive and negative signs to indicate the direction of the acceleration. Because the dimensions of velocity are L/T and the dimension of time is T, accelera-

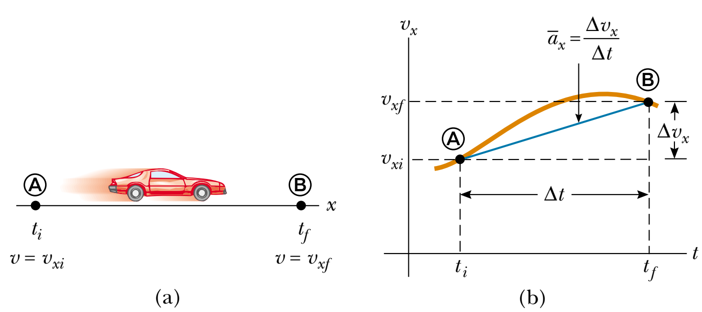


Figure 2.5 (a) A "particle" moving along the x axis from Ⓐ to Ⓑ has velocity v_{xi} at $t = t_i$ and velocity v_{xf} at $t = t_f$. (b) Velocity–time graph for the particle moving in a straight line. The slope of the blue straight line connecting Ⓐ and Ⓑ is the average acceleration in the time interval $\Delta t = t_f - t_i$.

tion has dimensions of length divided by time squared, or L/T^2. The SI unit of acceleration is meters per second squared (m/s^2). It might be easier to interpret these units if you think of them as meters per second per second. For example, suppose an object has an acceleration of 2 m/s^2. You should form a mental image of the object having a velocity that is along a straight line and is increasing by 2 m/s during every 1-s interval. If the object starts from rest, you should be able to picture it moving at a velocity of + 2 m/s after 1 s, at + 4 m/s after 2 s, and so on.

In some situations, the value of the average acceleration may be different over different time intervals. It is therefore useful to define the *instantaneous acceleration* as the limit of the average acceleration as Δt approaches zero. This concept is analogous to the definition of instantaneous velocity discussed in the previous section. If we imagine that point Ⓑ is brought closer and closer to point Ⓐ in Figure 2.5a and take the limit of $\Delta v_x/\Delta t$ as Δt approaches zero, we obtain the instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad \textbf{(2.6)}$$

Instantaneous acceleration

That is, **the instantaneous acceleration equals the derivative of the velocity with respect to time,** which by definition is the slope of the velocity–time graph (Fig. 2.5b). Thus, we see that just as the velocity of a moving particle is the slope of the particle's x-t graph, the acceleration of a particle is the slope of the particle's v_x-t graph. One can interpret the derivative of the velocity with respect to time as the time rate of change of velocity. If a_x is positive, then the acceleration is in the positive x direction; if a_x is negative, then the acceleration is in the negative x direction.

From now on we shall use the term *acceleration* to mean instantaneous acceleration. When we mean average acceleration, we shall always use the adjective *average.*

Because $v_x = dx/dt$, the acceleration can also be written

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \quad \textbf{(2.7)}$$

That is, in one-dimensional motion, the acceleration equals the *second derivative* of x with respect to time.

Figure 2.6 illustrates how an acceleration–time graph is related to a velocity–time graph. The acceleration at any time is the slope of the velocity–time graph at that time. Positive values of acceleration correspond to those points in Figure 2.6a where the velocity is increasing in the positive x direction. The acceler-

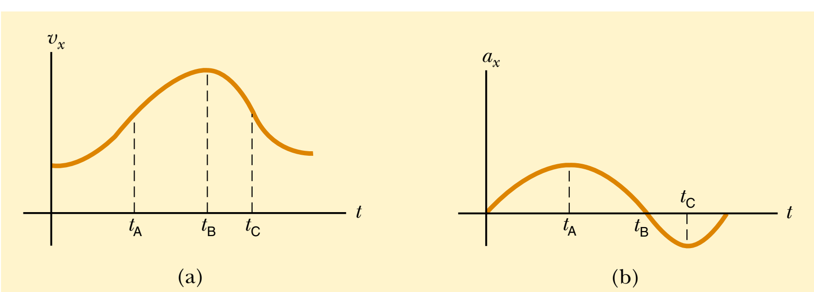


Figure 2.6 Instantaneous acceleration can be obtained from the v_x-t graph. (a) The velocity–time graph for some motion. (b) The acceleration–time graph for the same motion. The acceleration given by the a_x-t graph for any value of t equals the slope of the line tangent to the v_x-t graph at the same value of t.

ation reaches a maximum at time t_A, when the slope of the velocity–time graph is a maximum. The acceleration then goes to zero at time t_B, when the velocity is a maximum (that is, when the slope of the v_x-t graph is zero). The acceleration is negative when the velocity is decreasing in the positive x direction, and it reaches its most negative value at time t_C.

CONCEPTUAL EXAMPLE 2.3 Graphical Relationships Between x, v_x, and a_x

The position of an object moving along the x axis varies with time as in Figure 2.7a. Graph the velocity versus time and the acceleration versus time for the object.

Solution The velocity at any instant is the slope of the tangent to the x-t graph at that instant. Between $t = 0$ and $t = t_A$, the slope of the x-t graph increases uniformly, and so the velocity increases linearly, as shown in Figure 2.7b. Between t_A and t_B, the slope of the x-t graph is constant, and so the velocity remains constant. At t_D, the slope of the x-t graph is zero, so the velocity is zero at that instant. Between t_D and t_E, the slope of the x-t graph and thus the velocity are negative and decrease uniformly in this interval. In the interval t_E to t_F, the slope of the x-t graph is still negative, and at t_F it goes to zero. Finally, after t_F, the slope of the x-t graph is zero, meaning that the object is at rest for $t > t_F$.

The acceleration at any instant is the slope of the tangent to the v_x-t graph at that instant. The graph of acceleration versus time for this object is shown in Figure 2.7c. The acceleration is constant and positive between 0 and t_A, where the slope of the v_x-t graph is positive. It is zero between t_A and t_B and for $t > t_F$ because the slope of the v_x-t graph is zero at these times. It is negative between t_B and t_E because the slope of the v_x-t graph is negative during this interval.

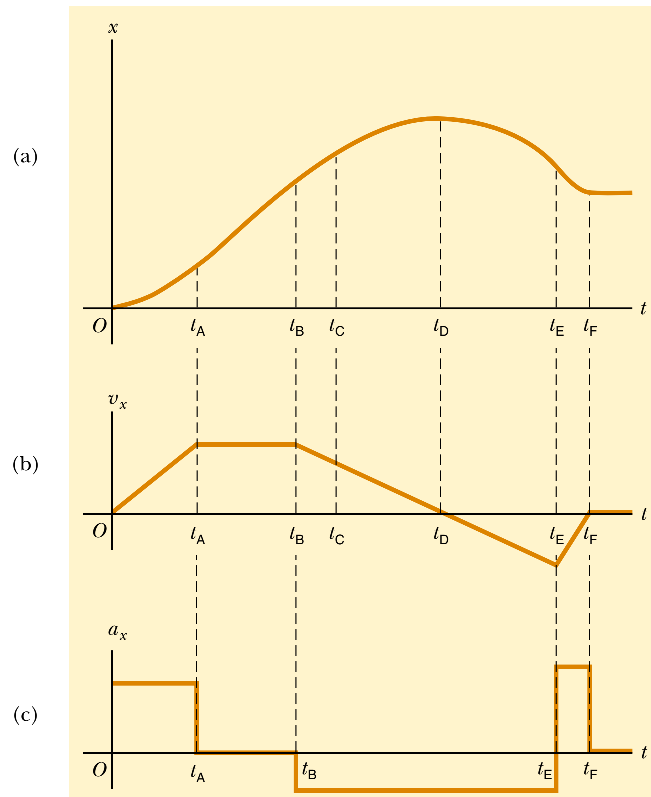


Figure 2.7 (a) Position–time graph for an object moving along the x axis. (b) The velocity–time graph for the object is obtained by measuring the slope of the position–time graph at each instant. (c) The acceleration–time graph for the object is obtained by measuring the slope of the velocity–time graph at each instant.

Quick Quiz 2.1

Make a velocity–time graph for the car in Figure 2.1a and use your graph to determine whether the car ever exceeds the speed limit posted on the road sign (30 km/h).

EXAMPLE 2.4 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds. (a) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

Solution Figure 2.8 is a v_x-t graph that was created from the velocity versus time expression given in the problem statement. Because the slope of the entire v_x-t curve is negative, we expect the acceleration to be negative.

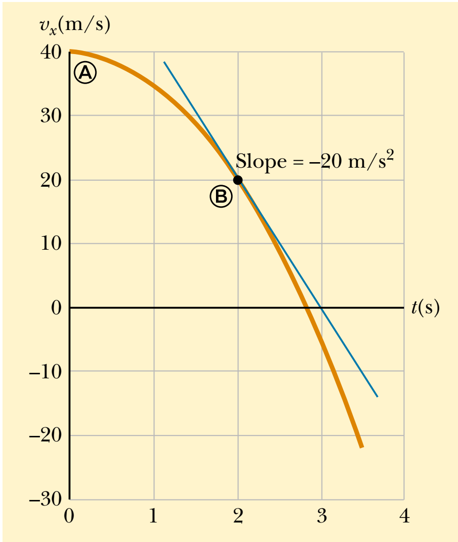


Figure 2.8 The velocity–time graph for a particle moving along the x axis according to the expression $v_x = (40 - 5t^2)$ m/s. The acceleration at $t = 2$ s is equal to the slope of the blue tangent line at that time.

We find the velocities at $t_i = t_A = 0$ and $t_f = t_B = 2.0$ s by substituting these values of t into the expression for the velocity:

$$v_{xA} = (40 - 5t_A^2)\text{ m/s} = [40 - 5(0)^2]\text{ m/s} = +40\text{ m/s}$$

$$v_{xB} = (40 - 5t_B^2)\text{ m/s} = [40 - 5(2.0)^2]\text{ m/s} = +20\text{ m/s}$$

Therefore, the average acceleration in the specified time interval $\Delta t = t_B - t_A = 2.0$ s is

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20 - 40)\text{ m/s}}{(2.0 - 0)\text{ s}}$$

$$= \boxed{-10\text{ m/s}^2}$$

The negative sign is consistent with our expectations—namely, that the average acceleration, which is represented by the slope of the line (not shown) joining the initial and final points on the velocity–time graph, is negative.

(b) Determine the acceleration at $t = 2.0$ s.

Solution The velocity at any time t is $v_{xi} = (40 - 5t^2)$ m/s, and the velocity at any later time $t + \Delta t$ is

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Therefore, the change in velocity over the time interval Δt is

$$\Delta v_x = v_{xf} - v_{xi} = [-10t\,\Delta t - 5(\Delta t)^2]\text{ m/s}$$

Dividing this expression by Δt and taking the limit of the result as Δt approaches zero gives the acceleration at *any* time t:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t\text{ m/s}^2$$

Therefore, at $t = 2.0$ s,

$$a_x = (-10)(2.0)\text{ m/s}^2 = \boxed{-20\text{ m/s}^2}$$

What we have done by comparing the average acceleration during the interval between Ⓐ and Ⓑ (-10 m/s²) with the instantaneous value at Ⓑ (-20 m/s²) is compare the slope of the line (not shown) joining Ⓐ and Ⓑ with the slope of the tangent at Ⓑ.

Note that the acceleration is not constant in this example. Situations involving constant acceleration are treated in Section 2.5.

So far we have evaluated the derivatives of a function by starting with the definition of the function and then taking the limit of a specific ratio. Those of you familiar with calculus should recognize that there are specific rules for taking derivatives. These rules, which are listed in Appendix B.6, enable us to evaluate derivatives quickly. For instance, one rule tells us that the derivative of any constant is zero. As another example, suppose x is proportional to some power of t, such as in the expression

$$x = At^n$$

where A and n are constants. (This is a very common functional form.) The derivative of x with respect to t is

$$\frac{dx}{dt} = nAt^{n-1}$$

Applying this rule to Example 2.4, in which $v_x = 40 - 5t^2$, we find that $a_x = dv_x/dt = -10t$.

2.4 MOTION DIAGRAMS

The concepts of velocity and acceleration are often confused with each other, but in fact they are quite different quantities. It is instructive to use motion diagrams to describe the velocity and acceleration while an object is in motion. In order not to confuse these two vector quantities, for which both magnitude and direction are important, we use red for velocity vectors and violet for acceleration vectors, as shown in Figure 2.9. The vectors are sketched at several instants during the motion of the object, and the time intervals between adjacent positions are assumed to be equal. This illustration represents three sets of strobe photographs of a car moving from left to right along a straight roadway. The time intervals between flashes are equal in each diagram.

In Figure 2.9a, the images of the car are equally spaced, showing us that the car moves the same distance in each time interval. Thus, the car moves with *constant positive velocity* and has *zero acceleration.*

In Figure 2.9b, the images become farther apart as time progresses. In this case, the velocity vector increases in time because the car's displacement between adjacent positions increases in time. The car is moving with a *positive velocity* and a *positive acceleration.*

In Figure 2.9c, we can tell that the car slows as it moves to the right because its displacement between adjacent images decreases with time. In this case, the car moves to the right with a constant negative acceleration. The velocity vector decreases in time and eventually reaches zero. From this diagram we see that the acceleration and velocity vectors are *not* in the same direction. The car is moving with a *positive velocity* but with a *negative acceleration.*

You should be able to construct motion diagrams for a car that moves initially to the left with a constant positive or negative acceleration.

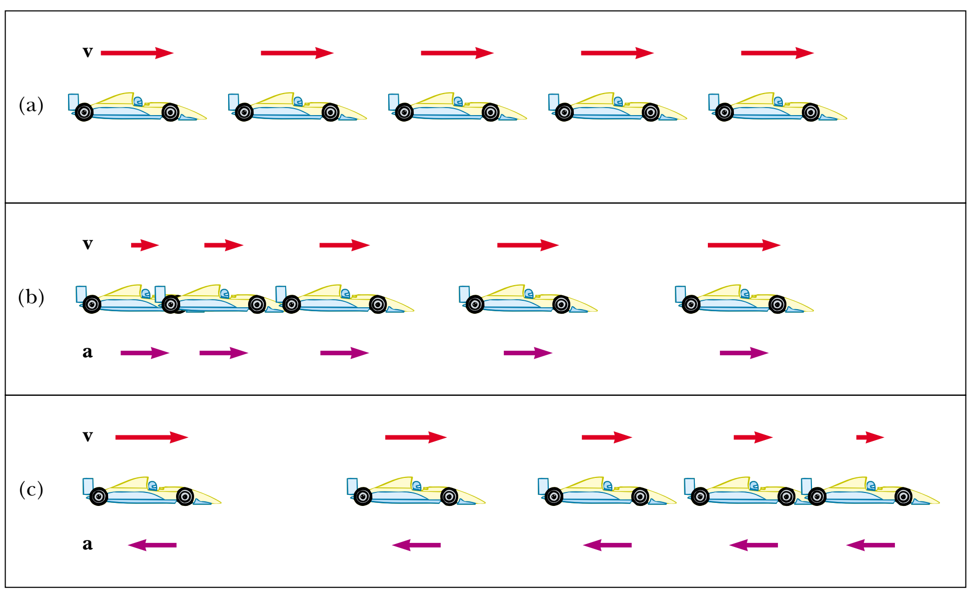


Figure 2.9 (a) Motion diagram for a car moving at constant velocity (zero acceleration). (b) Motion diagram for a car whose constant acceleration is in the direction of its velocity. The velocity vector at each instant is indicated by a red arrow, and the constant acceleration by a violet arrow. (c) Motion diagram for a car whose constant acceleration is in the direction *opposite* the velocity at each instant.

Quick Quiz 2.2

(a) If a car is traveling eastward, can its acceleration be westward? (b) If a car is slowing down, can its acceleration be positive?

2.5 ONE-DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze. However, a very common and simple type of one-dimensional motion is that in which the acceleration is constant. When this is the case, the average acceleration over any time interval equals the instantaneous acceleration at any instant within the interval, and the velocity changes at the same rate throughout the motion.

If we replace $\overline{a}_x$ by a_x in Equation 2.5 and take $t_i = 0$ and t_f to be any later time t, we find that

$$a_x = \frac{v_{xf} - v_{xi}}{t}$$

or

$$v_{xf} = v_{xi} + a_x t \qquad \text{(for constant } a_x\text{)} \qquad \textbf{(2.8)}$$

Velocity as a function of time

This powerful expression enables us to determine an object's velocity at *any* time t if we know the object's initial velocity and its (constant) acceleration. A velocity–time graph for this constant-acceleration motion is shown in Figure 2.10a. The graph is a straight line, the (constant) slope of which is the acceleration a_x; this is consistent with the fact that $a_x = dv_x/dt$ is a constant. Note that the slope is positive; this indicates a positive acceleration. If the acceleration were negative, then the slope of the line in Figure 2.10a would be negative.

When the acceleration is constant, the graph of acceleration versus time (Fig. 2.10b) is a straight line having a slope of zero.

Quick Quiz 2.3

Describe the meaning of each term in Equation 2.8.

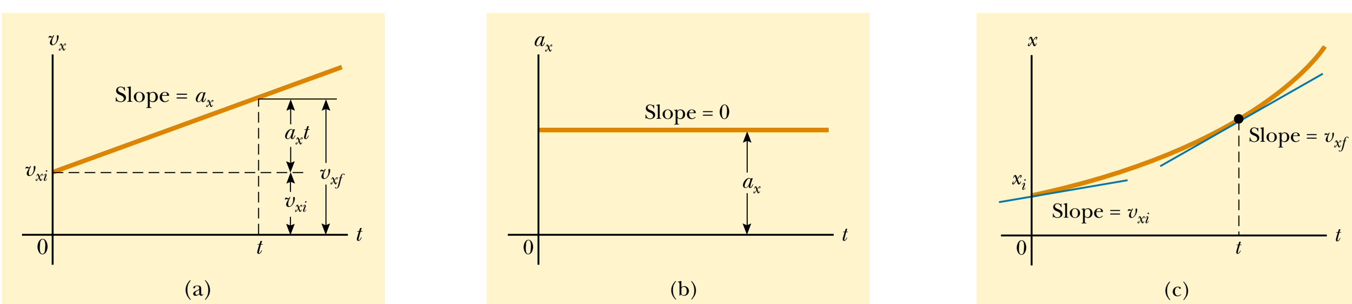


Figure 2.10 An object moving along the x axis with constant acceleration a_x. (a) The velocity–time graph. (b) The acceleration–time graph. (c) The position–time graph.

Because velocity at constant acceleration varies linearly in time according to Equation 2.8, we can express the average velocity in any time interval as the arithmetic mean of the initial velocity v_{xi} and the final velocity v_{xf}:

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} \qquad \text{(for constant } a_x\text{)} \tag{2.9}$$

Note that this expression for average velocity applies *only* in situations in which the acceleration is constant.

We can now use Equations 2.1, 2.2, and 2.9 to obtain the displacement of any object as a function of time. Recalling that Δx in Equation 2.2 represents $x_f - x_i$, and now using t in place of Δt (because we take $t_i = 0$), we can say

Displacement as a function of velocity and time

$$x_f - x_i = \bar{v}_x t = \tfrac{1}{2}(v_{xi} + v_{xf})\,t \qquad \text{(for constant } a_x\text{)} \tag{2.10}$$

We can obtain another useful expression for displacement at constant acceleration by substituting Equation 2.8 into Equation 2.10:

$$x_f - x_i = \tfrac{1}{2}(v_{xi} + v_{xi} + a_x t)\,t$$

$$x_f - x_i = v_{xi}t + \tfrac{1}{2}a_x t^2 \tag{2.11}$$

The position–time graph for motion at constant (positive) acceleration shown in Figure 2.10c is obtained from Equation 2.11. Note that the curve is a parabola. The slope of the tangent line to this curve at $t = t_i = 0$ equals the initial velocity v_{xi}, and the slope of the tangent line at any later time t equals the velocity at that time, v_{xf}.

We can check the validity of Equation 2.11 by moving the x_i term to the right-hand side of the equation and differentiating the equation with respect to time:

$$v_{xf} = \frac{dx_f}{dt} = \frac{d}{dt}\left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right) = v_{xi} + a_x t$$

Finally, we can obtain an expression for the final velocity that does not contain a time interval by substituting the value of t from Equation 2.8 into Equation 2.10:

$$x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = \frac{{v_{xf}}^2 - {v_{xi}}^2}{2a_x}$$

$${v_{xf}}^2 = {v_{xi}}^2 + 2a_x(x_f - x_i) \qquad \text{(for constant } a_x\text{)} \tag{2.12}$$

For motion at *zero* acceleration, we see from Equations 2.8 and 2.11 that

$$\left.\begin{aligned} v_{xf} &= v_{xi} = v_x \\ x_f - x_i &= v_x t \end{aligned}\right\} \quad \text{when } a_x = 0$$

That is, when acceleration is zero, velocity is constant and displacement changes linearly with time.

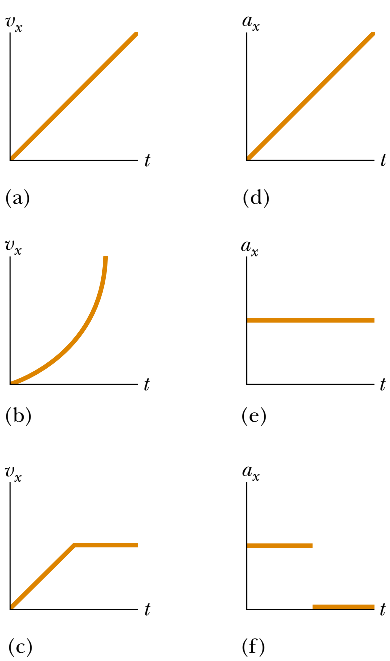


Figure 2.11 Parts (a), (b), and (c) are v_x-t graphs of objects in one-dimensional motion. The possible accelerations of each object as a function of time are shown in scrambled order in (d), (e), and (f).

Quick Quiz 2.4

In Figure 2.11, match each v_x-t graph with the a_x-t graph that best describes the motion.

Equations 2.8 through 2.12 are **kinematic expressions that may be used to solve any problem involving one-dimensional motion at constant accelera-**

TABLE 2.2 Kinematic Equations for Motion in a Straight Line Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t$	Displacement as a function of velocity and time
$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$	Displacement as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of displacement

Note: Motion is along the *x* axis.

tion. Keep in mind that these relationships were derived from the definitions of velocity and acceleration, together with some simple algebraic manipulations and the requirement that the acceleration be constant.

The four kinematic equations used most often are listed in Table 2.2 for convenience. The choice of which equation you use in a given situation depends on what you know beforehand. Sometimes it is necessary to use two of these equations to solve for two unknowns. For example, suppose initial velocity v_{xi} and acceleration a_x are given. You can then find (1) the velocity after an interval t has elapsed, using $v_{xf} = v_{xi} + a_x t$, and (2) the displacement after an interval t has elapsed, using $x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$. You should recognize that the quantities that vary during the motion are velocity, displacement, and time.

You will get a great deal of practice in the use of these equations by solving a number of exercises and problems. Many times you will discover that more than one method can be used to obtain a solution. Remember that these equations of kinematics *cannot* be used in a situation in which the acceleration varies with time. They can be used only when the acceleration is constant.

CONCEPTUAL EXAMPLE 2.5 The Velocity of Different Objects

Consider the following one-dimensional motions: (a) A ball thrown directly upward rises to a highest point and falls back into the thrower's hand. (b) A race car starts from rest and speeds up to 100 m/s. (c) A spacecraft drifts through space at constant velocity. Are there any points in the motion of these objects at which the instantaneous velocity is the same as the average velocity over the entire motion? If so, identify the point(s).

Solution (a) The average velocity for the thrown ball is zero because the ball returns to the starting point; thus its displacement is zero. (Remember that average velocity is defined as $\Delta x/\Delta t$.) There is one point at which the instantaneous velocity is zero—at the top of the motion.

(b) The car's average velocity cannot be evaluated unambiguously with the information given, but it must be some value between 0 and 100 m/s. Because the car will have every instantaneous velocity between 0 and 100 m/s at some time during the interval, there must be some instant at which the instantaneous velocity is equal to the average velocity.

(c) Because the spacecraft's instantaneous velocity is constant, its instantaneous velocity at *any* time and its average velocity over *any* time interval are the same.

EXAMPLE 2.6 Entering the Traffic Flow

(a) Estimate your average acceleration as you drive up the entrance ramp to an interstate highway.

Solution This problem involves more than our usual amount of estimating! We are trying to come up with a value of a_x, but that value is hard to guess directly. The other three variables involved in kinematics are position, velocity, and time. Velocity is probably the easiest one to approximate. Let us assume a final velocity of 100 km/h, so that you can merge with traffic. We multiply this value by 1 000 to convert kilometers to meters and then divide by 3 600 to convert hours to seconds. These two calculations together are roughly equivalent to dividing by 3. In fact, let us just say that the final velocity is $v_{xf} \approx 30$ m/s. (Remember, you can get away with this type of approximation and with dropping digits when performing mental calculations. If you were starting with British units, you could approximate 1 mi/h as roughly 0.5 m/s and continue from there.)

Now we assume that you started up the ramp at about one-third your final velocity, so that $v_{xi} \approx 10$ m/s. Finally, we assume that it takes about 10 s to get from v_{xi} to v_{xf}, basing this guess on our previous experience in automobiles. We can then find the acceleration, using Equation 2.8:

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{30 \text{ m/s} - 10 \text{ m/s}}{10 \text{ s}} = \boxed{2 \text{ m/s}^2}$$

Granted, we made many approximations along the way, but this type of mental effort can be surprisingly useful and often yields results that are not too different from those derived from careful measurements.

(b) How far did you go during the first half of the time interval during which you accelerated?

Solution We can calculate the distance traveled during the first 5 s from Equation 2.11:

$$x_f - x_i = v_{xi}t + \tfrac{1}{2}a_x t^2 \approx (10 \text{ m/s})(5 \text{ s}) + \tfrac{1}{2}(2 \text{ m/s}^2)(5 \text{ s})^2$$
$$= 50 \text{ m} + 25 \text{ m} = \boxed{75 \text{ m}}$$

This result indicates that if you had not accelerated, your initial velocity of 10 m/s would have resulted in a 50-m movement up the ramp during the first 5 s. The additional 25 m is the result of your increasing velocity during that interval.

Do not be afraid to attempt making educated guesses and doing some fairly drastic number rounding to simplify mental calculations. Physicists engage in this type of thought analysis all the time.

EXAMPLE 2.7 Carrier Landing

A jet lands on an aircraft carrier at 140 mi/h ($\approx$63 m/s). (a) What is its acceleration if it stops in 2.0 s?

Solution We define our x axis as the direction of motion of the jet. A careful reading of the problem reveals that in addition to being given the initial speed of 63 m/s, we also know that the final speed is zero. We also note that we are not given the displacement of the jet while it is slowing down. Equation 2.8 is the only equation in Table 2.2 that does not involve displacement, and so we use it to find the acceleration:

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63 \text{ m/s}}{2.0 \text{ s}} = \boxed{-31 \text{ m/s}^2}$$

(b) What is the displacement of the plane while it is stopping?

Solution We can now use any of the other three equations in Table 2.2 to solve for the displacement. Let us choose Equation 2.10:

$$x_f - x_i = \tfrac{1}{2}(v_{xi} + v_{xf})t = \tfrac{1}{2}(63 \text{ m/s} + 0)(2.0 \text{ s}) = \boxed{63 \text{ m}}$$

If the plane travels much farther than this, it might fall into the ocean. Although the idea of using arresting cables to enable planes to land safely on ships originated at about the time of the First World War, the cables are still a vital part of the operation of modern aircraft carriers.

EXAMPLE 2.8 Watch Out for the Speed Limit!

A car traveling at a constant speed of 45.0 m/s passes a trooper hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch it, accelerating at a constant rate of 3.00 m/s^2. How long does it take her to overtake the car?

Solution A careful reading lets us categorize this as a constant-acceleration problem. We know that after the 1-s delay in starting, it will take the trooper 15 additional seconds to accelerate up to 45.0 m/s. Of course, she then has to continue to pick up speed (at a rate of 3.00 m/s per second) to catch up to the car. While all this is going on, the car continues to move. We should therefore expect our result to be well over 15 s. A sketch (Fig. 2.12) helps clarify the sequence of events.

First, we write expressions for the position of each vehicle as a function of time. It is convenient to choose the position of the billboard as the origin and to set $t_{\text{B}} \equiv 0$ as the time the trooper begins moving. At that instant, the car has already traveled a distance of 45.0 m because it has traveled at a constant speed of $v_x = 45.0$ m/s for 1 s. Thus, the initial position of the speeding car is $x_{\text{B}} = 45.0$ m.

Because the car moves with constant speed, its accelera-

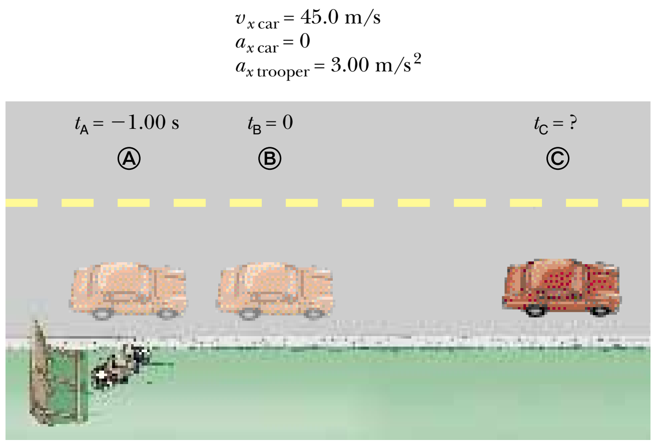


Figure 2.12 A speeding car passes a hidden police officer.

tion is zero, and applying Equation 2.11 (with $a_x = 0$) gives for the car's position at any time t:

$$x_{\text{car}} = x_B + v_{x\,\text{car}}t = 45.0 \text{ m} + (45.0 \text{ m/s})t$$

A quick check shows that at $t = 0$, this expression gives the car's correct initial position when the trooper begins to move: $x_{\text{car}} = x_B = 45.0$ m. Looking at limiting cases to see whether they yield expected values is a very useful way to make sure that you are obtaining reasonable results.

The trooper starts from rest at $t = 0$ and accelerates at 3.00 m/s^2 away from the origin. Hence, her position after any time interval t can be found from Equation 2.11:

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_xt^2$$

$$x_{\text{trooper}} = 0 + 0t + \tfrac{1}{2}a_xt^2 = \tfrac{1}{2}(3.00 \text{ m/s}^2)t^2$$

The trooper overtakes the car at the instant her position matches that of the car, which is position C:

$$x_{\text{trooper}} = x_{\text{car}}$$

$$\tfrac{1}{2}(3.00 \text{ m/s}^2)t^2 = 45.0 \text{ m} + (45.0 \text{ m/s})t$$

This gives the quadratic equation

$$1.50t^2 - 45.0t - 45.0 = 0$$

The positive solution of this equation is $t = \boxed{31.0 \text{ s}}$.

(For help in solving quadratic equations, see Appendix B.2.) Note that in this 31.0-s time interval, the trooper travels a distance of about 1440 m. [This distance can be calculated from the car's constant speed: (45.0 m/s)(31 + 1) s = 1 440 m.]

Exercise This problem can be solved graphically. On the same graph, plot position versus time for each vehicle, and from the intersection of the two curves determine the time at which the trooper overtakes the car.

2.6 FREELY FALLING OBJECTS

It is now well known that, in the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration under the influence of the Earth's gravity. It was not until about 1600 that this conclusion was accepted. Before that time, the teachings of the great philosopher Aristotle (384–322 B.C.) had held that heavier objects fall faster than lighter ones.

Astronaut David Scott released a hammer and a feather simultaneously, and they fell in unison to the lunar surface. *(Courtesy of NASA)*

It was the Italian Galileo Galilei (1564–1642) who originated our present-day ideas concerning falling objects. There is a legend that he demonstrated the law of falling objects by observing that two different weights dropped simultaneously from the Leaning Tower of Pisa hit the ground at approximately the same time. Although there is some doubt that he carried out this particular experiment, it is well established that Galileo performed many experiments on objects moving on inclined planes. In his experiments he rolled balls down a slight incline and measured the distances they covered in successive time intervals. The purpose of the incline was to reduce the acceleration; with the acceleration reduced, Galileo was able to make accurate measurements of the time intervals. By gradually increasing the slope of the incline, he was finally able to draw conclusions about freely falling objects because a freely falling ball is equivalent to a ball moving down a vertical incline.

QuickLab

Use a pencil to poke a hole in the bottom of a paper or polystyrene cup. Cover the hole with your finger and fill the cup with water. Hold the cup up in front of you and release it. Does water come out of the hole while the cup is falling? Why or why not?

You might want to try the following experiment. Simultaneously drop a coin and a crumpled-up piece of paper from the same height. If the effects of air resistance are negligible, both will have the same motion and will hit the floor at the same time. In the idealized case, in which air resistance is absent, such motion is referred to as *free fall.* If this same experiment could be conducted in a vacuum, in which air resistance is truly negligible, the paper and coin would fall with the same acceleration even when the paper is not crumpled. On August 2, 1971, such a demonstration was conducted on the Moon by astronaut David Scott. He simultaneously released a hammer and a feather, and in unison they fell to the lunar surface. This demonstration surely would have pleased Galileo!

Definition of free fall

When we use the expression *freely falling object,* we do not necessarily refer to an object dropped from rest. **A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all falling freely once they are released. Any freely falling object experiences an acceleration directed *downward*, regardless of its initial motion.**

We shall denote the magnitude of the *free-fall acceleration* by the symbol g. The value of g near the Earth's surface decreases with increasing altitude. Furthermore, slight variations in g occur with changes in latitude. It is common to define "up" as the $+y$ direction and to use y as the position variable in the kinematic equations. At the Earth's surface, the value of g is approximately 9.80 m/s^2. Unless stated otherwise, we shall use this value for g when performing calculations. For making quick estimates, use $g = 10\ \text{m/s}^2$.

Free-fall acceleration
$g = 9.80\ \text{m/s}^2$

If we neglect air resistance and assume that the free-fall acceleration does not vary with altitude over short vertical distances, then the motion of a freely falling object moving vertically is equivalent to motion in one dimension under constant acceleration. Therefore, the equations developed in Section 2.5 for objects moving with constant acceleration can be applied. The only modification that we need to make in these equations for freely falling objects is to note that the motion is in the vertical direction (the y direction) rather than in the horizontal (x) direction and that the acceleration is downward and has a magnitude of 9.80 m/s^2. Thus, we always take $a_y = -g = -9.80\ \text{m/s}^2$, where the minus sign means that the acceleration of a freely falling object is downward. In Chapter 14 we shall study how to deal with variations in g with altitude.

CONCEPTUAL EXAMPLE 2.9 The Daring Sky Divers

A sky diver jumps out of a hovering helicopter. A few seconds later, another sky diver jumps out, and they both fall along the same vertical line. Ignore air resistance, so that both sky divers fall with the same acceleration. Does the difference in their speeds stay the same throughout the fall? Does the vertical distance between them stay the same throughout the fall? If the two divers were connected by a long bungee cord, would the tension in the cord increase, lessen, or stay the same during the fall?

Solution At any given instant, the speeds of the divers are different because one had a head start. In any time interval Δt after this instant, however, the two divers increase their speeds by the same amount because they have the same acceleration. Thus, the difference in their speeds remains the same throughout the fall.

The first jumper always has a greater speed than the second. Thus, in a given time interval, the first diver covers a greater distance than the second. Thus, the separation distance between them increases.

Once the distance between the divers reaches the length of the bungee cord, the tension in the cord begins to increase. As the tension increases, the distance between the divers becomes greater and greater.

Example 2.10 Describing the Motion of a Tossed Ball

A ball is tossed straight up at 25 m/s. Estimate its velocity at 1-s intervals.

Solution Let us choose the upward direction to be positive. Regardless of whether the ball is moving upward or downward, its vertical velocity changes by approximately -10 m/s for every second it remains in the air. It starts out at 25 m/s. After 1 s has elapsed, it is still moving upward but at 15 m/s because its acceleration is downward (downward acceleration causes its velocity to decrease). After another second, its upward velocity has dropped to 5 m/s. Now comes the tricky part—after another half second, its velocity is zero. The ball has gone as high as it will go. After the last half of this 1-s interval, the ball is moving at -5 m/s. (The minus sign tells us that the ball is now moving in the negative direction, that is, *downward.* Its velocity has changed from $+5$ m/s to -5 m/s during that 1-s interval. The change in velocity is still $-5 - [+5] = -10$ m/s in that second.) It continues downward, and after another 1 s has elapsed, it is falling at a velocity of -15 m/s. Finally, after another 1 s, it has reached its original starting point and is moving downward at -25 m/s. If the ball had been tossed vertically off a cliff so that it could continue downward, its velocity would continue to change by about -10 m/s every second.

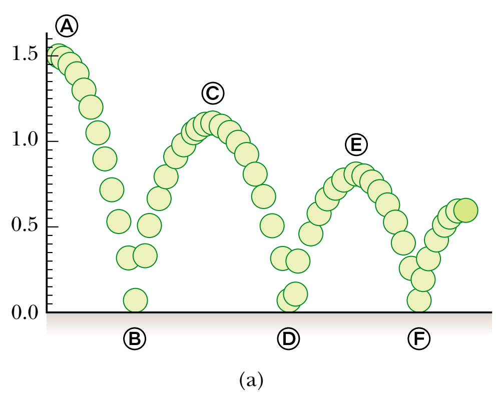
Conceptual Example 2.11 Follow the Bouncing Ball

A tennis ball is dropped from shoulder height (about 1.5 m) and bounces three times before it is caught. Sketch graphs of its position, velocity, and acceleration as functions of time, with the $+y$ direction defined as upward.

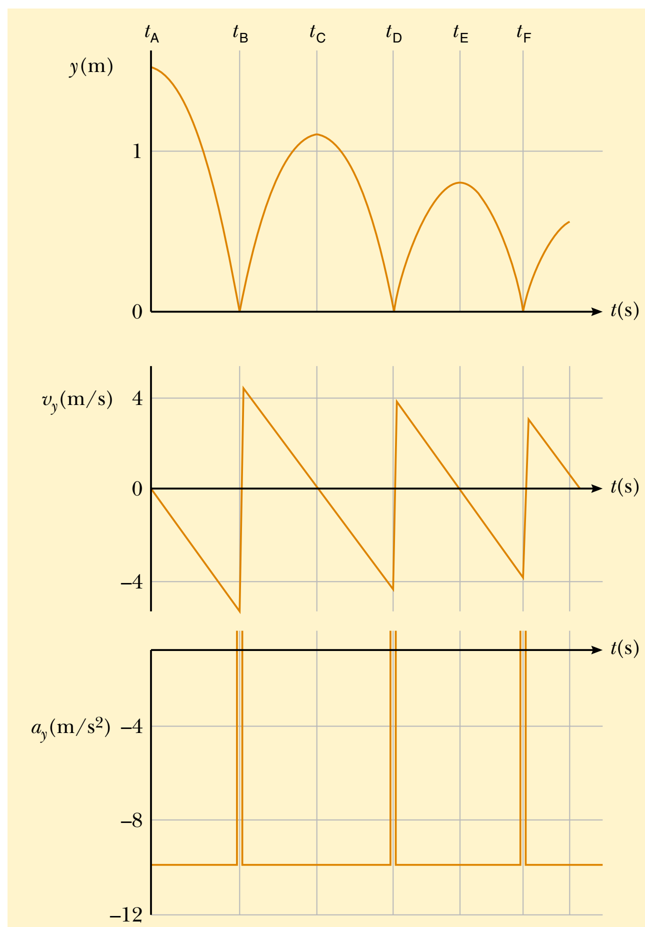
Solution For our sketch let us stretch things out horizontally so that we can see what is going on. (Even if the ball were moving horizontally, this motion would not affect its vertical motion.)

From Figure 2.13 we see that the ball is in contact with the floor at points Ⓑ, Ⓓ, and Ⓕ. Because the velocity of the ball changes from negative to positive three times during these bounces, the slope of the position–time graph must change in the same way. Note that the time interval between bounces decreases. Why is that?

During the rest of the ball's motion, the slope of the velocity–time graph should be -9.80 m/s^2. The acceleration–time graph is a horizontal line at these times because the acceleration does not change when the ball is in free fall. When the ball is in contact with the floor, the velocity changes substantially during a very short time interval, and so the acceleration must be quite great. This corresponds to the very steep upward lines on the velocity–time graph and to the spikes on the acceleration–time graph.



(a)



(b)

Figure 2.13 (a) A ball is dropped from a height of 1.5 m and bounces from the floor. (The horizontal motion is not considered here because it does not affect the vertical motion.) (b) Graphs of position, velocity, and acceleration versus time.

Quick Quiz 2.5

Which values represent the ball's velocity and acceleration at points Ⓐ, Ⓒ, and Ⓔ in Figure 2.13?

(a) $v_y = 0$, $a_y = 0$
(b) $v_y = 0$, $a_y = 9.80 \text{ m/s}^2$
(c) $v_y = 0$, $a_y = -9.80 \text{ m/s}^2$
(d) $v_y = -9.80 \text{ m/s}$, $a_y = 0$

Example 2.12 Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine (a) the time at which the stone reaches its maximum height, (b) the maximum height, (c) the time at which the stone returns to the height from which it was thrown, (d) the velocity of the stone at this instant, and (e) the velocity and position of the stone at $t = 5.00$ s.

Solution (a) As the stone travels from Ⓐ to Ⓑ, its velocity must change by 20 m/s because it stops at Ⓑ. Because gravity causes vertical velocities to change by about 10 m/s for every second of free fall, it should take the stone about 2 s to go from Ⓐ to Ⓑ in our drawing. (In a problem like this, a sketch definitely helps you organize your thoughts.) To calculate the time t_B at which the stone reaches maximum height, we use Equation 2.8, $v_{yB} = v_{yA} + a_y t$, noting that $v_{yB} = 0$ and setting the start of our clock readings at $t_A \equiv 0$:

$$20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)t = 0$$

$$t = t_B = \frac{20.0 \text{ m/s}}{9.80 \text{ m/s}^2} = \boxed{2.04 \text{ s}}$$

Our estimate was pretty close.

(b) Because the average velocity for this first interval is 10 m/s (the average of 20 m/s and 0 m/s) and because it travels for about 2 s, we expect the stone to travel about 20 m. By substituting our time interval into Equation 2.11, we can find the maximum height as measured from the position of the thrower, where we set $y_i = y_A = 0$:

$$y_{max} = y_B = v_{yA}\, t + \tfrac{1}{2}a_y t^2$$

$$y_B = (20.0 \text{ m/s})(2.04 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.04 \text{ s})^2$$

$$= \boxed{20.4 \text{ m}}$$

Our free-fall estimates are very accurate.

(c) There is no reason to believe that the stone's motion from Ⓑ to Ⓒ is anything other than the reverse of its motion

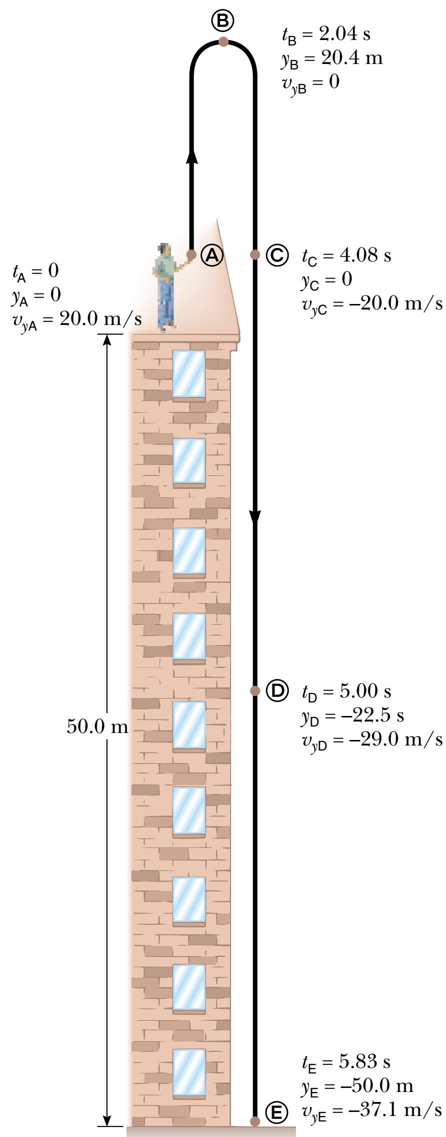


Figure 2.14 Position and velocity versus time for a freely falling stone thrown initially upward with a velocity $v_{yi} = 20.0$ m/s.

from Ⓐ to Ⓑ. Thus, the time needed for it to go from Ⓐ to Ⓒ should be twice the time needed for it to go from Ⓐ to Ⓑ. When the stone is back at the height from which it was thrown (position Ⓒ), the y coordinate is again zero. Using Equation 2.11, with $y_f = y_C = 0$ and $y_i = y_A = 0$, we obtain

$$y_C - y_A = v_{yA}\,t + \tfrac{1}{2}a_y t^2$$
$$0 = 20.0t - 4.90t^2$$

This is a quadratic equation and so has two solutions for $t = t_C$. The equation can be factored to give

$$t(20.0 - 4.90t) = 0$$

One solution is $t = 0$, corresponding to the time the stone starts its motion. The other solution is $\boxed{t = 4.08 \text{ s},}$ which is the solution we are after. Notice that it is double the value we calculated for t_B.

(d) Again, we expect everything at Ⓒ to be the same as it is at Ⓐ, except that the velocity is now in the opposite direction. The value for t found in (c) can be inserted into Equation 2.8 to give

$$v_{yC} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(4.08 \text{ s})$$
$$= \boxed{-20.0 \text{ m/s}}$$

The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but opposite in direction. This indicates that the motion is symmetric.

(e) For this part we consider what happens as the stone falls from position Ⓑ, where it had zero vertical velocity, to position Ⓓ. Because the elapsed time for this part of the motion is about 3 s, we estimate that the acceleration due to gravity will have changed the speed by about 30 m/s. We can calculate this from Equation 2.8, where we take $t = t_D - t_B$:

$$v_{yD} = v_{yB} + a_y t = 0 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.00 \text{ s} - 2.04 \text{ s})$$
$$= \boxed{-29.0 \text{ m/s}}$$

We could just as easily have made our calculation between positions Ⓐ and Ⓓ by making sure we use the correct time interval, $t = t_D - t_A = 5.00$ s:

$$v_{yD} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.00 \text{ s})$$
$$= -29.0 \text{ m/s}$$

To demonstrate the power of our kinematic equations, we can use Equation 2.11 to find the position of the stone at $t_D = 5.00$ s by considering the change in position between a different pair of positions, Ⓒ and Ⓓ. In this case, the time is $t_D - t_C$:

$$y_D = y_C + v_{yC}t + \tfrac{1}{2}a_y t^2$$
$$= 0 \text{ m} + (-20.0 \text{ m/s})(5.00 \text{ s} - 4.08 \text{ s})$$
$$+ \tfrac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s} - 4.08 \text{ s})^2$$
$$= \boxed{-22.5 \text{ m}}$$

Exercise Find (a) the velocity of the stone just before it hits the ground at Ⓔ and (b) the total time the stone is in the air.

Answer (a) -37.1 m/s (b) 5.83 s

Optional Section

2.7 KINEMATIC EQUATIONS DERIVED FROM CALCULUS

This is an optional section that assumes the reader is familiar with the techniques of integral calculus. If you have not yet studied integration in your calculus course, you should skip this section or cover it after you become familiar with integration.

The velocity of a particle moving in a straight line can be obtained if its position as a function of time is known. Mathematically, the velocity equals the derivative of the position coordinate with respect to time. It is also possible to find the displacement of a particle if its velocity is known as a function of time. In calculus, the procedure used to perform this task is referred to either as *integration* or as finding the *antiderivative*. Graphically, it is equivalent to finding the area under a curve.

Suppose the v_x-t graph for a particle moving along the x axis is as shown in Figure 2.15. Let us divide the time interval $t_f - t_i$ into many small intervals, each of duration Δt_n. From the definition of average velocity we see that the displacement during any small interval, such as the one shaded in Figure 2.15, is given by $\Delta x_n = \overline{v}_{xn}\,\Delta t_n$, where $\overline{v}_{xn}$ is the average velocity in that interval. Therefore, the displacement during this small interval is simply the area of the shaded rectangle.

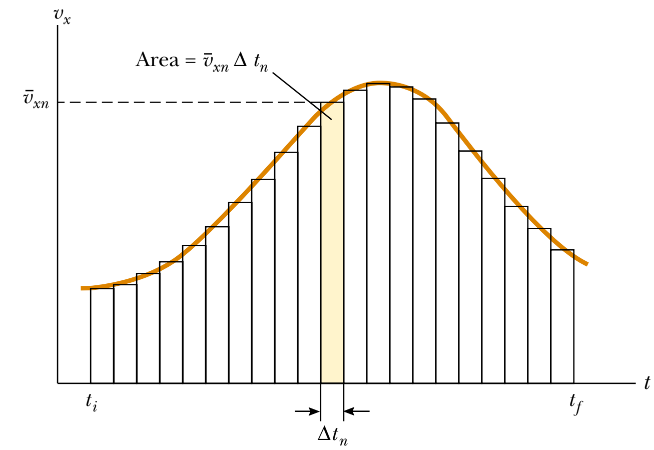


Figure 2.15 Velocity versus time for a particle moving along the x axis. The area of the shaded rectangle is equal to the displacement Δx in the time interval Δt_n, while the total area under the curve is the total displacement of the particle.

The total displacement for the interval $t_f - t_i$ is the sum of the areas of all the rectangles:

$$\Delta x = \sum_n \bar{v}_{xn}\,\Delta t_n$$

where the symbol Σ (upper case Greek sigma) signifies a sum over all terms. In this case, the sum is taken over all the rectangles from t_i to t_f. Now, as the intervals are made smaller and smaller, the number of terms in the sum increases and the sum approaches a value equal to the area under the velocity–time graph. Therefore, in the limit $n \to \infty$, or $\Delta t_n \to 0$, the displacement is

$$\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_{xn}\,\Delta t_n \qquad \textbf{(2.13)}$$

or

$$\text{Displacement} = \text{area under the } v_x\text{-}t \text{ graph}$$

Note that we have replaced the average velocity $\bar{v}_{xn}$ with the instantaneous velocity v_{xn} in the sum. As you can see from Figure 2.15, this approximation is clearly valid in the limit of very small intervals. We conclude that if we know the v_x-t graph for motion along a straight line, we can obtain the displacement during any time interval by measuring the area under the curve corresponding to that time interval.

The limit of the sum shown in Equation 2.13 is called a **definite integral** and is written

Definite integral

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn}\Delta t_n = \int_{t_i}^{t_f} v_x(t)\ dt \qquad \textbf{(2.14)}$$

where $v_x(t)$ denotes the velocity at any time t. If the explicit functional form of $v_x(t)$ is known and the limits are given, then the integral can be evaluated.

Sometimes the v_x-t graph for a moving particle has a shape much simpler than that shown in Figure 2.15. For example, suppose a particle moves at a constant ve-

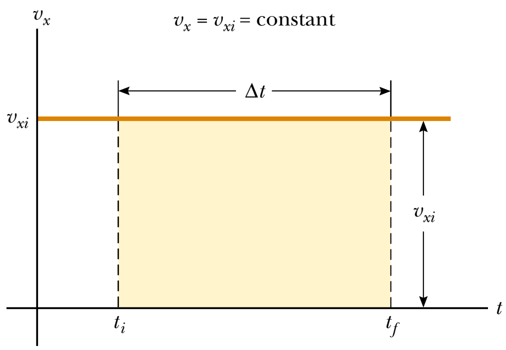


Figure 2.16 The velocity–time curve for a particle moving with constant velocity v_{xi}. The displacement of the particle during the time interval $t_f - t_i$ is equal to the area of the shaded rectangle.

locity v_{xi}. In this case, the v_x-t graph is a horizontal line, as shown in Figure 2.16, and its displacement during the time interval Δt is simply the area of the shaded rectangle:

$$\Delta x = v_{xi}\Delta t \qquad \text{(when } v_{xf} = v_{xi} = \text{constant)}$$

As another example, consider a particle moving with a velocity that is proportional to t, as shown in Figure 2.17. Taking $v_x = a_x t$, where a_x is the constant of proportionality (the acceleration), we find that the displacement of the particle during the time interval $t = 0$ to $t = t_{\text{A}}$ is equal to the area of the shaded triangle in Figure 2.17:

$$\Delta x = \tfrac{1}{2}(t_{\text{A}})(a_x t_{\text{A}}) = \tfrac{1}{2}a_x {t_{\text{A}}}^2$$

Kinematic Equations

We now use the defining equations for acceleration and velocity to derive two of our kinematic equations, Equations 2.8 and 2.11.

The defining equation for acceleration (Eq. 2.6),

$$a_x = \frac{dv_x}{dt}$$

may be written as $dv_x = a_x dt$ or, in terms of an integral (or antiderivative), as

$$v_x = \int a_x\, dt + C_1$$

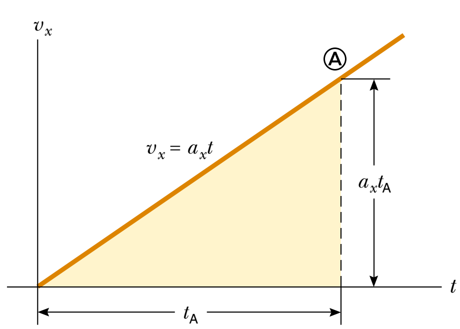


Figure 2.17 The velocity–time curve for a particle moving with a velocity that is proportional to the time.

where C_1 is a constant of integration. For the special case in which the acceleration is constant, the a_x can be removed from the integral to give

$$v_x = a_x \int dt + C_1 = a_x t + C_1 \tag{2.15}$$

The value of C_1 depends on the initial conditions of the motion. If we take $v_x = v_{xi}$ when $t = 0$ and substitute these values into the last equation, we have

$$v_{xi} = a_x(0) + C_1$$

$$C_1 = v_{xi}$$

Calling $v_x = v_{xf}$ the velocity after the time interval t has passed and substituting this and the value just found for C_1 into Equation 2.15, we obtain kinematic Equation 2.8:

$$v_{xf} = v_{xi} + a_x t \qquad \text{(for constant } a_x\text{)}$$

Now let us consider the defining equation for velocity (Eq. 2.4):

$$v_x = \frac{dx}{dt}$$

We can write this as $dx = v_x dt$ or in integral form as

$$x = \int v_x\, dt + C_2$$

where C_2 is another constant of integration. Because $v_x = v_{xf} = v_{xi} + a_x t$, this expression becomes

$$x = \int (v_{xi} + a_x t)\, dt + C_2$$

$$x = \int v_{xi}\, dt + a_x \int t\, dt + C_2$$

$$x = v_{xi} t + \tfrac{1}{2} a_x t^2 + C_2$$

To find C_2, we make use of the initial condition that $x = x_i$ when $t = 0$. This gives $C_2 = x_i$. Therefore, after substituting x_f for x, we have

$$x_f = x_i + v_{xi} t + \tfrac{1}{2} a_x t^2 \qquad \text{(for constant } a_x\text{)}$$

Once we move x_i to the left side of the equation, we have kinematic Equation 2.11. Recall that $x_f - x_i$ is equal to the displacement of the object, where x_i is its initial position.

Besides what you might expect to learn about physics concepts, a very valuable skill you should hope to take away from your physics course is the ability to solve complicated problems. The way physicists approach complex situations and break them down into manageable pieces is extremely useful. We have developed a memory aid to help you easily recall the steps required for successful problem solving. When working on problems, the secret is to keep your GOAL in mind!

GOAL Problem-Solving Steps

Gather information

The first thing to do when approaching a problem is to understand the situation. Carefully read the problem statement, looking for key phrases like "at rest" or "freely falls." What information is given? Exactly what is the question asking? Don't forget to gather information from your own experiences and common sense. What should a reasonable answer look like? You wouldn't expect to calculate the speed of an automobile to be 5×10^6 m/s. Do you know what units to expect? Are there any limiting cases you can consider? What happens when an angle approaches 0° or 90° or when a mass becomes huge or goes to zero? Also make sure you carefully study any drawings that accompany the problem.

Organize your approach

Once you have a really good idea of what the problem is about, you need to think about what to do next. Have you seen this type of question before? Being able to classify a problem can make it much easier to lay out a plan to solve it. You should almost always make a quick drawing of the situation. Label important events with circled letters. Indicate any known values, perhaps in a table or directly on your sketch.

Analyze the problem

Because you have already categorized the problem, it should not be too difficult to select relevant equations that apply to this type of situation. Use algebra (and calculus, if necessary) to solve for the unknown variable in terms of what is given. Substitute in the appropriate numbers, calculate the result, and round it to the proper number of significant figures.

Learn from your efforts

This is the most important part. Examine your numerical answer. Does it meet your expectations from the first step? What about the algebraic form of the result—before you plugged in numbers? Does it make sense? (Try looking at the variables in it to see whether the answer would change in a physically meaningful way if they were drastically increased or decreased or even became zero.) Think about how this problem compares with others you have done. How was it similar? In what critical ways did it differ? Why was this problem assigned? You should have learned something by doing it. Can you figure out what?

When solving complex problems, you may need to identify a series of subproblems and apply the GOAL process to each. For very simple problems, you probably don't need GOAL at all. But when you are looking at a problem and you don't know what to do next, remember what the letters in GOAL stand for and use that as a guide.

SUMMARY

After a particle moves along the x axis from some initial position x_i to some final position x_f, its **displacement** is

$$\Delta x \equiv x_f - x_i \tag{2.1}$$

The **average velocity** of a particle during some time interval is the displacement Δx divided by the time interval Δt during which that displacement occurred:

$$\bar{v}_x \equiv \frac{\Delta x}{\Delta t} \tag{2.2}$$

The **average speed** of a particle is equal to the ratio of the total distance it travels to the total time it takes to travel that distance.

The **instantaneous velocity** of a particle is defined as the limit of the ratio $\Delta x/\Delta t$ as Δt approaches zero. By definition, this limit equals the derivative of x with respect to t, or the time rate of change of the position:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{2.4}$$

The **instantaneous speed** of a particle is equal to the magnitude of its velocity.

The **average acceleration** of a particle is defined as the ratio of the change in its velocity Δv_x divided by the time interval Δt during which that change occurred:

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \tag{2.5}$$

The **instantaneous acceleration** is equal to the limit of the ratio $\Delta v_x/\Delta t$ as Δt approaches 0. By definition, this limit equals the derivative of v_x with respect to t, or the time rate of change of the velocity:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \tag{2.6}$$

The **equations of kinematics** for a particle moving along the x axis with uniform acceleration a_x (constant in magnitude and direction) are

$$v_{xf} = v_{xi} + a_x t \tag{2.8}$$

$$x_f - x_i = \bar{v}_x t = \tfrac{1}{2}(v_{xi} + v_{xf})t \tag{2.10}$$

$$x_f - x_i = v_{xi}t + \tfrac{1}{2}a_x t^2 \tag{2.11}$$

$$v_{xf}^{\,2} = v_{xi}^{\,2} + 2a_x(x_f - x_i) \tag{2.12}$$

You should be able to use these equations and the definitions in this chapter to analyze the motion of any object moving with constant acceleration.

An object falling freely in the presence of the Earth's gravity experiences a free-fall acceleration directed toward the center of the Earth. If air resistance is neglected, if the motion occurs near the surface of the Earth, and if the range of the motion is small compared with the Earth's radius, then the free-fall acceleration g is constant over the range of motion, where g is equal to 9.80 m/s^2.

Complicated problems are best approached in an organized manner. You should be able to recall and apply the steps of the GOAL strategy when you need them.

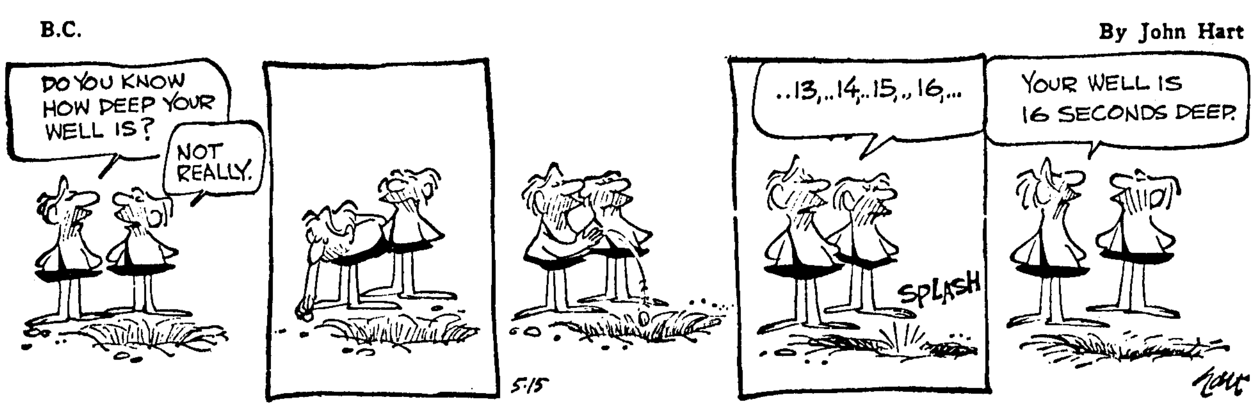
Questions

1. Average velocity and instantaneous velocity are generally different quantities. Can they ever be equal for a specific type of motion? Explain.
2. If the average velocity is nonzero for some time interval, does this mean that the instantaneous velocity is never zero during this interval? Explain.
3. If the average velocity equals zero for some time interval Δt and if $v_x(t)$ is a continuous function, show that the instantaneous velocity must go to zero at some time in this interval. (A sketch of x versus t might be useful in your proof.)
4. Is it possible to have a situation in which the velocity and acceleration have opposite signs? If so, sketch a velocity–time graph to prove your point.
5. If the velocity of a particle is nonzero, can its acceleration be zero? Explain.
6. If the velocity of a particle is zero, can its acceleration be nonzero? Explain.
7. Can an object having constant acceleration ever stop and stay stopped?
8. A stone is thrown vertically upward from the top of a building. Does the stone's displacement depend on the location of the origin of the coordinate system? Does the stone's velocity depend on the origin? (Assume that the coordinate system is stationary with respect to the building.) Explain.
9. A student at the top of a building of height h throws one ball upward with an initial speed v_{yi} and then throws a second ball downward with the same initial speed. How do the final speeds of the balls compare when they reach the ground?
10. Can the magnitude of the instantaneous velocity of an object ever be greater than the magnitude of its average velocity? Can it ever be less?
11. If the average velocity of an object is zero in some time interval, what can you say about the displacement of the object for that interval?
12. A rapidly growing plant doubles in height each week. At the end of the 25th day, the plant reaches the height of a building. At what time was the plant one-fourth the height of the building?
13. Two cars are moving in the same direction in parallel lanes along a highway. At some instant, the velocity of car A exceeds the velocity of car B. Does this mean that the acceleration of car A is greater than that of car B? Explain.
14. An apple is dropped from some height above the Earth's surface. Neglecting air resistance, how much does the apple's speed increase each second during its descent?
15. Consider the following combinations of signs and values for velocity and acceleration of a particle with respect to a one-dimensional x axis:

Velocity	Acceleration
a. Positive	Positive
b. Positive	Negative
c. Positive	Zero
d. Negative	Positive
e. Negative	Negative
f. Negative	Zero
g. Zero	Positive
h. Zero	Negative

Describe what the particle is doing in each case, and give a real-life example for an automobile on an east-west one-dimensional axis, with east considered to be the positive direction.

16. A pebble is dropped into a water well, and the splash is heard 16 s later, as illustrated in Figure Q2.16. Estimate the distance from the rim of the well to the water's surface.
17. Average velocity is an entirely contrived quantity, and other combinations of data may prove useful in other contexts. For example, the ratio $\Delta t/\Delta x$, called the "slowness" of a moving object, is used by geophysicists when discussing the motion of continental plates. Explain what this quantity means.



By permission of John Hart and Field Enterprises Inc.

Figure Q2.16

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
□ = paired numerical/symbolic problems

Section 2.1 Displacement, Velocity, and Speed

1. The position of a pinewood derby car was observed at various times; the results are summarized in the table below. Find the average velocity of the car for (a) the first second, (b) the last 3 s, and (c) the entire period of observation.

x (m)	0	2.3	9.2	20.7	36.8	57.5
t (s)	0	1.0	2.0	3.0	4.0	5.0

2. A motorist drives north for 35.0 min at 85.0 km/h and then stops for 15.0 min. He then continues north, traveling 130 km in 2.00 h. (a) What is his total displacement? (b) What is his average velocity?

3. The displacement versus time for a certain particle moving along the x axis is shown in Figure P2.3. Find the average velocity in the time intervals (a) 0 to 2 s, (b) 0 to 4 s, (c) 2 s to 4 s, (d) 4 s to 7 s, (e) 0 to 8 s.

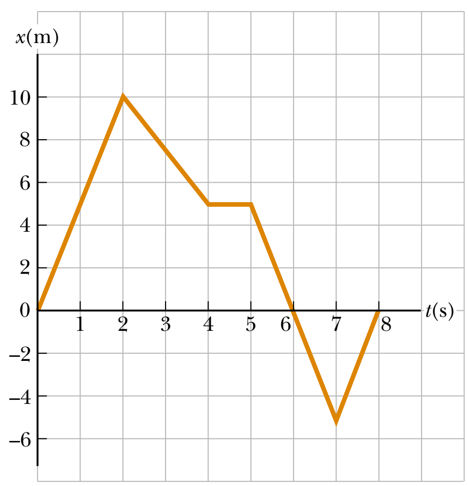


Figure P2.3 Problems 3 and 11.

4. A particle moves according to the equation $x = 10t^2$, where x is in meters and t is in seconds. (a) Find the average velocity for the time interval from 2.0 s to 3.0 s. (b) Find the average velocity for the time interval from 2.0 s to 2.1 s.

5. A person walks first at a constant speed of 5.00 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3.00 m/s. What are (a) her average speed over the entire trip and (b) her average velocity over the entire trip?

6. A person first walks at a constant speed v_1 along a straight line from A to B and then back along the line from B to A at a constant speed v_2. What are (a) her average speed over the entire trip and (b) her average velocity over the entire trip?

Section 2.2 Instantaneous Velocity and Speed

7. At $t = 1.00$ s, a particle moving with constant velocity is located at $x = -3.00$ m, and at $t = 6.00$ s the particle is located at $x = 5.00$ m. (a) From this information, plot the position as a function of time. (b) Determine the velocity of the particle from the slope of this graph.

8. The position of a particle moving along the x axis varies in time according to the expression $x = 3t^2$, where x is in meters and t is in seconds. Evaluate its position (a) at $t = 3.00$ s and (b) at $3.00 \text{ s} + \Delta t$. (c) Evaluate the limit of $\Delta x/\Delta t$ as Δt approaches zero to find the velocity at $t = 3.00$ s.

WEB 9. A position–time graph for a particle moving along the x axis is shown in Figure P2.9. (a) Find the average velocity in the time interval $t = 1.5$ s to $t = 4.0$ s. (b) Determine the instantaneous velocity at $t = 2.0$ s by measuring the slope of the tangent line shown in the graph. (c) At what value of t is the velocity zero?

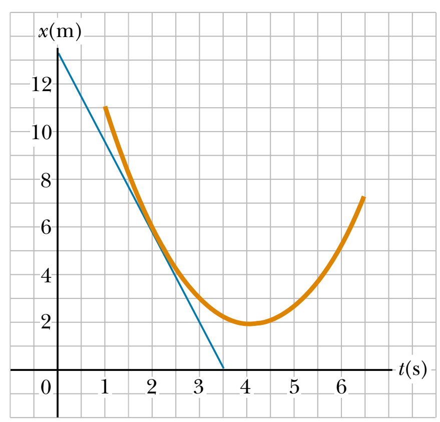


Figure P2.9

10. (a) Use the data in Problem 1 to construct a smooth graph of position versus time. (b) By constructing tangents to the $x(t)$ curve, find the instantaneous velocity of the car at several instants. (c) Plot the instantaneous velocity versus time and, from this, determine the average acceleration of the car. (d) What was the initial velocity of the car?

11. Find the instantaneous velocity of the particle described in Figure P2.3 at the following times: (a) $t = 1.0$ s, (b) $t = 3.0$ s, (c) $t = 4.5$ s, and (d) $t = 7.5$ s.

Section 2.3 Acceleration

12. A particle is moving with a velocity of 60.0 m/s in the positive x direction at $t = 0$. Between $t = 0$ and $t = 15.0$ s, the velocity decreases uniformly to zero. What was the acceleration during this 15.0-s interval? What is the significance of the sign of your answer?

13. A 50.0-g superball traveling at 25.0 m/s bounces off a brick wall and rebounds at 22.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval? (*Note:* 1 ms $= 10^{-3}$ s.)

14. A particle starts from rest and accelerates as shown in Figure P2.14. Determine: (a) the particle's speed at $t = 10$ s and at $t = 20$ s, and (b) the distance traveled in the first 20 s.

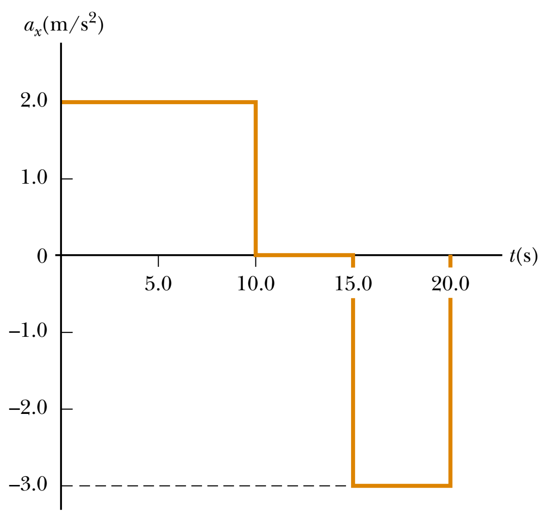


Figure P2.14

15. A velocity–time graph for an object moving along the x axis is shown in Figure P2.15. (a) Plot a graph of the acceleration versus time. (b) Determine the average acceleration of the object in the time intervals $t = 5.00$ s to $t = 15.0$ s and $t = 0$ to $t = 20.0$ s.

16. A student drives a moped along a straight road as described by the velocity–time graph in Figure P2.16. Sketch this graph in the middle of a sheet of graph paper. (a) Directly above your graph, sketch a graph of the position versus time, aligning the time coordinates of the two graphs. (b) Sketch a graph of the acceleration versus time directly below the v_x-t graph, again aligning the time coordinates. On each graph, show the

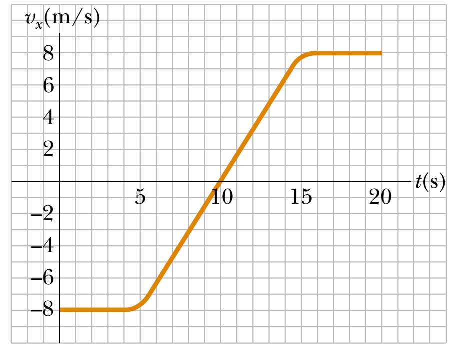


Figure P2.15

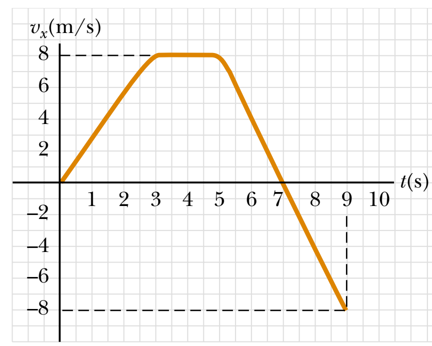


Figure P2.16

numerical values of x and a_x for all points of inflection. (c) What is the acceleration at $t = 6$ s? (d) Find the position (relative to the starting point) at $t = 6$ s. (e) What is the moped's final position at $t = 9$ s?

WEB $\boxed{17.}$ A particle moves along the x axis according to the equation $x = 2.00 + 3.00t - t^2$, where x is in meters and t is in seconds. At $t = 3.00$ s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.

18. An object moves along the x axis according to the equation $x = (3.00t^2 - 2.00t + 3.00)$ m. Determine (a) the average speed between $t = 2.00$ s and $t = 3.00$ s, (b) the instantaneous speed at $t = 2.00$ s and at $t = 3.00$ s, (c) the average acceleration between $t = 2.00$ s and $t = 3.00$ s, and (d) the instantaneous acceleration at $t = 2.00$ s and $t = 3.00$ s.

19. Figure P2.19 shows a graph of v_x versus t for the motion of a motorcyclist as he starts from rest and moves along the road in a straight line. (a) Find the average acceleration for the time interval $t = 0$ to $t = 6.00$ s. (b) Estimate the time at which the acceleration has its greatest positive value and the value of the acceleration at that instant. (c) When is the acceleration zero? (d) Estimate the maximum negative value of the acceleration and the time at which it occurs.

Figure P2.19

Section 2.4 Motion Diagrams

20. Draw motion diagrams for (a) an object moving to the right at constant speed, (b) an object moving to the right and speeding up at a constant rate, (c) an object moving to the right and slowing down at a constant rate, (d) an object moving to the left and speeding up at a constant rate, and (e) an object moving to the left and slowing down at a constant rate. (f) How would your drawings change if the changes in speed were not uniform; that is, if the speed were not changing at a constant rate?

Section 2.5 One-Dimensional Motion with Constant Acceleration

21. Jules Verne in 1865 proposed sending people to the Moon by firing a space capsule from a 220-m-long cannon with a final velocity of 10.97 km/s. What would have been the unrealistically large acceleration experienced by the space travelers during launch? Compare your answer with the free-fall acceleration, 9.80 m/s^2.

22. A certain automobile manufacturer claims that its super-deluxe sports car will accelerate from rest to a speed of 42.0 m/s in 8.00 s. Under the (improbable) assumption that the acceleration is constant, (a) determine the acceleration of the car. (b) Find the distance the car travels in the first 8.00 s. (c) What is the speed of the car 10.0 s after it begins its motion, assuming it continues to move with the same acceleration?

23. A truck covers 40.0 m in 8.50 s while smoothly slowing down to a final speed of 2.80 m/s. (a) Find its original speed. (b) Find its acceleration.

24. The minimum distance required to stop a car moving at 35.0 mi/h is 40.0 ft. What is the minimum stopping distance for the same car moving at 70.0 mi/h, assuming the same rate of acceleration?

WEB **25.** A body moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is -5.00 cm, what is the magnitude of its acceleration?

26. Figure P2.26 represents part of the performance data of a car owned by a proud physics student. (a) Calculate from the graph the total distance traveled. (b) What distance does the car travel between the times $t = 10$ s and $t = 40$ s? (c) Draw a graph of its acceleration versus time between $t = 0$ and $t = 50$ s. (d) Write an equation for x as a function of time for each phase of the motion, represented by (i) $0a$, (ii) ab, (iii) bc. (e) What is the average velocity of the car between $t = 0$ and $t = 50$ s?

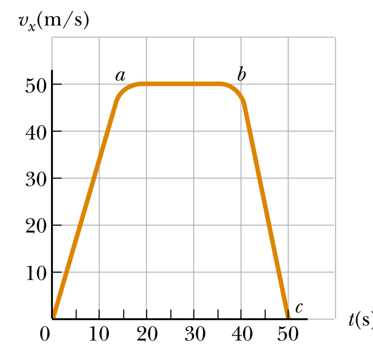


Figure P2.26

27. A particle moves along the x axis. Its position is given by the equation $x = 2.00 + 3.00t - 4.00t^2$ with x in meters and t in seconds. Determine (a) its position at the instant it changes direction and (b) its velocity when it returns to the position it had at $t = 0$.

28. The initial velocity of a body is 5.20 m/s. What is its velocity after 2.50 s (a) if it accelerates uniformly at 3.00 m/s^2 and (b) if it accelerates uniformly at -3.00 m/s^2?

29. A drag racer starts her car from rest and accelerates at 10.0 m/s^2 for the entire distance of 400 m ($\frac{1}{4}$ mi). (a) How long did it take the race car to travel this distance? (b) What is the speed of the race car at the end of the run?

30. A car is approaching a hill at 30.0 m/s when its engine suddenly fails, just at the bottom of the hill. The car moves with a constant acceleration of -2.00 m/s^2 while coasting up the hill. (a) Write equations for the position along the slope and for the velocity as functions of time, taking $x = 0$ at the bottom of the hill, where $v_i = 30.0$ m/s. (b) Determine the maximum distance the car travels up the hill.

31. A jet plane lands with a speed of 100 m/s and can accelerate at a maximum rate of -5.00 m/s^2 as it comes to rest. (a) From the instant the plane touches the runway, what is the minimum time it needs before it can come to rest? (b) Can this plane land at a small tropical island airport where the runway is 0.800 km long?

32. The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with an acceleration of -5.60 m/s^2 for 4.20 s, making straight skid marks 62.4 m long ending at the tree. With what speed does the car then strike the tree?

33. *Help! One of our equations is missing!* We describe constant-acceleration motion with the variables and parameters v_{xi}, v_{xf}, a_x, t, and $x_f - x_i$. Of the equations in Table 2.2, the first does not involve $x_f - x_i$. The second does not contain a_x, the third omits v_{xf}, and the last

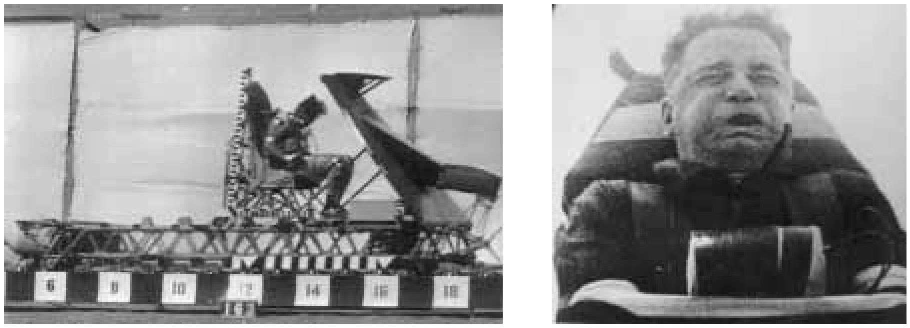


Figure P2.37 *(Left)* Col. John Stapp on rocket sled. *(Courtesy of the U.S. Air Force)* *(Right)* Col. Stapp's face is contorted by the stress of rapid negative acceleration. *(Photri, Inc.)*

leaves out t. So to complete the set there should be an equation *not* involving v_{xi}. Derive it from the others. Use it to solve Problem 32 in one step.

34. An indestructible bullet 2.00 cm long is fired straight through a board that is 10.0 cm thick. The bullet strikes the board with a speed of 420 m/s and emerges with a speed of 280 m/s. (a) What is the average acceleration of the bullet as it passes through the board? (b) What is the total time that the bullet is in contact with the board? (c) What thickness of board (calculated to 0.1 cm) would it take to stop the bullet, assuming the bullet's acceleration through all parts of the board is the same?

35. A truck on a straight road starts from rest, accelerating at 2.00 m/s^2 until it reaches a speed of 20.0 m/s. Then the truck travels for 20.0 s at constant speed until the brakes are applied, stopping the truck in a uniform manner in an additional 5.00 s. (a) How long is the truck in motion? (b) What is the average velocity of the truck for the motion described?

36. A train is traveling down a straight track at 20.0 m/s when the engineer applies the brakes. This results in an acceleration of $-$ 1.00 m/s^2 as long as the train is in motion. How far does the train move during a 40.0-s time interval starting at the instant the brakes are applied?

$\boxed{\textbf{37.}}$ For many years the world's land speed record was held by Colonel John P. Stapp, USAF (Fig. P2.37). On March 19, 1954, he rode a rocket-propelled sled that moved down the track at 632 mi/h. He and the sled were safely brought to rest in 1.40 s. Determine (a) the negative acceleration he experienced and (b) the distance he traveled during this negative acceleration.

38. An electron in a cathode-ray tube (CRT) accelerates uniformly from 2.00×10^4 m/s to 6.00×10^6 m/s over 1.50 cm. (a) How long does the electron take to travel this 1.50 cm? (b) What is its acceleration?

39. A ball starts from rest and accelerates at 0.500 m/s^2 while moving down an inclined plane 9.00 m long. When it reaches the bottom, the ball rolls up another plane, where, after moving 15.0 m, it comes to rest. (a) What is the speed of the ball at the bottom of the first plane? (b) How long does it take to roll down the first plane? (c) What is the acceleration along the second plane? (d) What is the ball's speed 8.00 m along the second plane?

40. Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. Sue applies her brakes but can accelerate only at -2.00 m/s^2 because the road is wet. Will there be a collision? If so, determine how far into the tunnel and at what time the collision occurs. If not, determine the distance of closest approach between Sue's car and the van.

Section 2.6 Freely Falling Objects

Note: In all problems in this section, ignore the effects of air resistance.

41. A golf ball is released from rest from the top of a very tall building. Calculate (a) the position and (b) the velocity of the ball after 1.00 s, 2.00 s, and 3.00 s.

42. *Every morning at seven o'clock*
There's twenty terriers drilling on the rock.
The boss comes around and he says, "Keep still
And bear down heavy on the cast-iron drill
And drill, ye terriers, drill." And drill, ye terriers, drill.
It's work all day for sugar in your tea . . .
And drill, ye terriers, drill.

One day a premature blast went off
And a mile in the air went big Jim Goff. And drill . . .

Then when next payday came around
Jim Goff a dollar short was found.
When he asked what for, came this reply:
"You were docked for the time you were up in the sky." And drill . . .

—American folksong

What was Goff's hourly wage? State the assumptions you make in computing it.

WEB **43.** A student throws a set of keys vertically upward to her sorority sister, who is in a window 4.00 m above. The keys are caught 1.50 s later by the sister's outstretched hand. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?

44. A ball is thrown directly downward with an initial speed of 8.00 m/s from a height of 30.0 m. How many seconds later does the ball strike the ground?

45. Emily challenges her friend David to catch a dollar bill as follows: She holds the bill vertically, as in Figure P2.45, with the center of the bill between David's index finger and thumb. David must catch the bill after Emily releases it without moving his hand downward. If his reaction time is 0.2 s, will he succeed? Explain your reasoning.

Figure P2.45 *(George Semple)*

46. A ball is dropped from rest from a height h above the ground. Another ball is thrown vertically upward from the ground at the instant the first ball is released. Determine the speed of the second ball if the two balls are to meet at a height $h/2$ above the ground.

47. A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3.00 s for the ball to reach its maximum height. Find (a) its initial velocity and (b) the maximum height it reaches.

48. A woman is reported to have fallen 144 ft from the 17th floor of a building, landing on a metal ventilator box, which she crushed to a depth of 18.0 in. She suffered only minor injuries. Calculate (a) the speed of the woman just before she collided with the ventilator box, (b) her average acceleration while in contact with the box, and (c) the time it took to crush the box.

WEB **49.** A daring ranch hand sitting on a tree limb wishes to drop vertically onto a horse galloping under the tree. The speed of the horse is 10.0 m/s, and the distance from the limb to the saddle is 3.00 m. (a) What must be the horizontal distance between the saddle and limb when the ranch hand makes his move? (b) How long is he in the air?

50. A ball thrown vertically upward is caught by the thrower after 20.0 s. Find (a) the initial velocity of the ball and (b) the maximum height it reaches.

51. A ball is thrown vertically upward from the ground with an initial speed of 15.0 m/s. (a) How long does it take the ball to reach its maximum altitude? (b) What is its maximum altitude? (c) Determine the velocity and acceleration of the ball at $t = 2.00$ s.

52. The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

(Optional)

2.7 Kinematic Equations Derived from Calculus

53. Automotive engineers refer to the time rate of change of acceleration as the "jerk." If an object moves in one dimension such that its jerk J is constant, (a) determine expressions for its acceleration a_x, velocity v_x, and position x, given that its initial acceleration, speed, and position are a_{xi}, v_{xi}, and x_i, respectively. (b) Show that $a_x^2 = a_{xi}^2 + 2J(v_x - v_{xi})$.

54. The speed of a bullet as it travels down the barrel of a rifle toward the opening is given by the expression $v = (-5.0 \times 10^7)t^2 + (3.0 \times 10^5)t$, where v is in meters per second and t is in seconds. The acceleration of the bullet just as it leaves the barrel is zero. (a) Determine the acceleration and position of the bullet as a function of time when the bullet is in the barrel. (b) Determine the length of time the bullet is accelerated. (c) Find the speed at which the bullet leaves the barrel. (d) What is the length of the barrel?

55. The acceleration of a marble in a certain fluid is proportional to the speed of the marble squared and is given (in SI units) by $a = -3.00v^2$ for $v > 0$. If the marble enters this fluid with a speed of 1.50 m/s, how long will it take before the marble's speed is reduced to half of its initial value?

ADDITIONAL PROBLEMS

56. A motorist is traveling at 18.0 m/s when he sees a deer in the road 38.0 m ahead. (a) If the maximum negative acceleration of the vehicle is -4.50 m/s^2, what is the maximum reaction time Δt of the motorist that will allow him to avoid hitting the deer? (b) If his reaction time is actually 0.300 s, how fast will he be traveling when he hits the deer?

57. Another scheme to catch the roadrunner has failed. A safe falls from rest from the top of a 25.0-m-high cliff toward Wile E. Coyote, who is standing at the base. Wile first notices the safe after it has fallen 15.0 m. How long does he have to get out of the way?

58. A dog's hair has been cut and is now getting longer by 1.04 mm each day. With winter coming on, this rate of hair growth is steadily increasing by 0.132 mm/day every week. By how much will the dog's hair grow during five weeks?

59. A test rocket is fired vertically upward from a well. A catapult gives it an initial velocity of 80.0 m/s at ground level. Subsequently, its engines fire and it accelerates upward at 4.00 m/s^2 until it reaches an altitude of 1000 m. At that point its engines fail, and the rocket goes into free fall, with an acceleration of -9.80 m/s^2. (a) How long is the rocket in motion above the ground? (b) What is its maximum altitude? (c) What is its velocity just before it collides with the Earth? (*Hint:* Consider the motion while the engine is operating separate from the free-fall motion.)

60. A motorist drives along a straight road at a constant speed of 15.0 m/s. Just as she passes a parked motorcycle police officer, the officer starts to accelerate at 2.00 m/s^2 to overtake her. Assuming the officer maintains this acceleration, (a) determine the time it takes the police officer to reach the motorist. Also find (b) the speed and (c) the total displacement of the officer as he overtakes the motorist.

61. In Figure 2.10a, the area under the velocity–time curve between the vertical axis and time t (vertical dashed line) represents the displacement. As shown, this area consists of a rectangle and a triangle. Compute their areas and compare the sum of the two areas with the expression on the righthand side of Equation 2.11.

62. A commuter train travels between two downtown stations. Because the stations are only 1.00 km apart, the train never reaches its maximum possible cruising speed. The engineer minimizes the time t between the two stations by accelerating at a rate $a_1 = 0.100$ m/s^2 for a time t_1 and then by braking with acceleration $a_2 = -0.500$ m/s^2 for a time t_2. Find the minimum time of travel t and the time t_1.

63. In a 100-m race, Maggie and Judy cross the finish line in a dead heat, both taking 10.2 s. Accelerating uniformly, Maggie took 2.00 s and Judy 3.00 s to attain maximum speed, which they maintained for the rest of the race. (a) What was the acceleration of each sprinter? (b) What were their respective maximum speeds? (c) Which sprinter was ahead at the 6.00-s mark, and by how much?

64. A hard rubber ball, released at chest height, falls to the pavement and bounces back to nearly the same height. When it is in contact with the pavement, the lower side of the ball is temporarily flattened. Suppose the maximum depth of the dent is on the order of 1 cm. Compute an order-of-magnitude estimate for the maximum acceleration of the ball while it is in contact with the pavement. State your assumptions, the quantities you estimate, and the values you estimate for them.

65. A teenager has a car that speeds up at 3.00 m/s^2 and slows down at -4.50 m/s^2. On a trip to the store, he accelerates from rest to 12.0 m/s, drives at a constant speed for 5.00 s, and then comes to a momentary stop at an intersection. He then accelerates to 18.0 m/s, drives at a constant speed for 20.0 s, slows down for 2.67 s, continues for 4.00 s at this speed, and then comes to a stop. (a) How long does the trip take? (b) How far has he traveled? (c) What is his average speed for the trip? (d) How long would it take to walk to the store and back if he walks at 1.50 m/s?

66. A rock is dropped from rest into a well. (a) If the sound of the splash is heard 2.40 s later, how far below the top of the well is the surface of the water? The speed of sound in air (at the ambient temperature) is 336 m/s. (b) If the travel time for the sound is neglected, what percentage error is introduced when the depth of the well is calculated?

67. An inquisitive physics student and mountain climber climbs a 50.0-m cliff that overhangs a calm pool of water. He throws two stones vertically downward, 1.00 s apart, and observes that they cause a single splash. The first stone has an initial speed of 2.00 m/s. (a) How long after release of the first stone do the two stones hit the water? (b) What was the initial velocity of the second stone? (c) What is the velocity of each stone at the instant the two hit the water?

68. A car and train move together along parallel paths at 25.0 m/s, with the car adjacent to the rear of the train. Then, because of a red light, the car undergoes a uniform acceleration of -2.50 m/s^2 and comes to rest. It remains at rest for 45.0 s and then accelerates back to a speed of 25.0 m/s at a rate of 2.50 m/s^2. How far behind the rear of the train is the car when it reaches the speed of 25.0 m/s, assuming that the speed of the train has remained 25.0 m/s?

69. Kathy Kool buys a sports car that can accelerate at the rate of 4.90 m/s^2. She decides to test the car by racing with another speedster, Stan Speedy. Both start from rest, but experienced Stan leaves the starting line 1.00 s before Kathy. If Stan moves with a constant acceleration of 3.50 m/s^2 and Kathy maintains an acceleration of 4.90 m/s^2, find (a) the time it takes Kathy to overtake Stan, (b) the distance she travels before she catches up with him, and (c) the speeds of both cars at the instant she overtakes him.

70. To protect his food from hungry bears, a boy scout raises his food pack with a rope that is thrown over a tree limb at height h above his hands. He walks away from the vertical rope with constant velocity v_{boy}, holding the free end of the rope in his hands (Fig. P2.70).

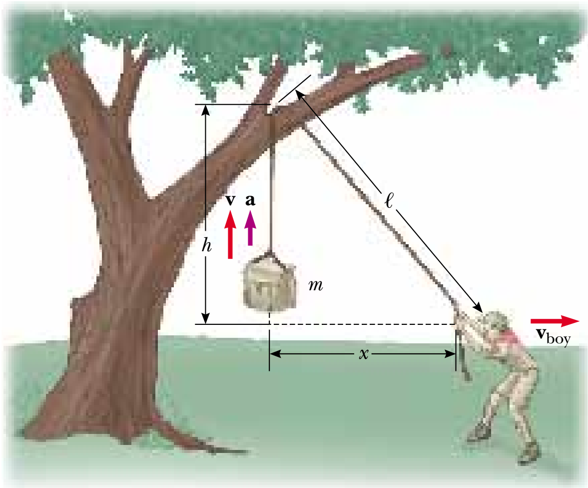


Figure P2.70

(a) Show that the speed v of the food pack is $x(x^2 + h^2)^{-1/2}\, v_{\text{boy}}$, where x is the distance he has walked away from the vertical rope. (b) Show that the acceleration a of the food pack is $h^2(x^2 + h^2)^{-3/2}\, v_{\text{boy}}{}^2$. (c) What values do the acceleration and velocity have shortly after he leaves the point under the pack ($x = 0$)? (d) What values do the pack's velocity and acceleration approach as the distance x continues to increase?

71. In Problem 70, let the height h equal 6.00 m and the speed v_{boy} equal 2.00 m/s. Assume that the food pack starts from rest. (a) Tabulate and graph the speed–time graph. (b) Tabulate and graph the acceleration–time graph. (Let the range of time be from 0 to 5.00 s and the time intervals be 0.500 s.)

72. Astronauts on a distant planet toss a rock into the air. With the aid of a camera that takes pictures at a steady rate, they record the height of the rock as a function of time as given in Table P2.72. (a) Find the average velocity of the rock in the time interval between each measurement and the next. (b) Using these average velocities to approximate instantaneous velocities at the midpoints of the time intervals, make a graph of velocity as a function of time. Does the rock move with constant acceleration? If so, plot a straight line of best fit on the graph and calculate its slope to find the acceleration.

TABLE P2.72 Height of a Rock versus Time

Time (s)	Height (m)	Time (s)	Height (m)
0.00	5.00	2.75	7.62
0.25	5.75	3.00	7.25
0.50	6.40	3.25	6.77
0.75	6.94	3.50	6.20
1.00	7.38	3.75	5.52
1.25	7.72	4.00	4.73
1.50	7.96	4.25	3.85
1.75	8.10	4.50	2.86
2.00	8.13	4.75	1.77
2.25	8.07	5.00	0.58
2.50	7.90		

73. Two objects, A and B, are connected by a rigid rod that has a length L. The objects slide along perpendicular guide rails, as shown in Figure P2.73. If A slides to the left with a constant speed v, find the speed of B when $\alpha = 60.0°$.

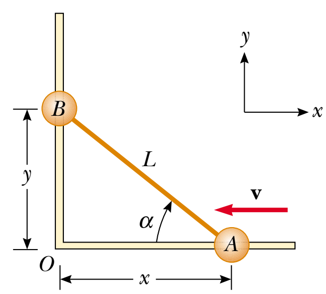


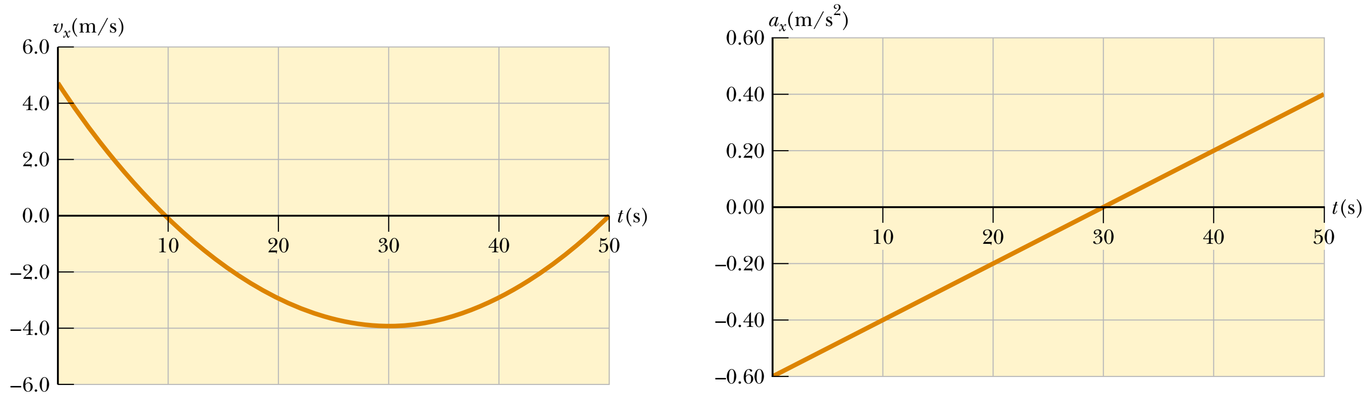
Figure P2.73

Answers to Quick Quizzes

2.1 Your graph should look something like the one in (a). This v_x-t graph shows that the maximum speed is about 5.0 m/s, which is 18 km/h (= 11 mi/h), and so the driver was not speeding. Can you derive the acceleration–time graph from the velocity–time graph? It should look something like the one in (b).

2.2 (a) Yes. This occurs when the car is slowing down, so that the direction of its acceleration is opposite the direction of its motion. (b) Yes. If the motion is in the direction chosen as negative, a positive acceleration causes a decrease in speed.

2.3 The left side represents the final velocity of an object. The first term on the right side is the velocity that the object had initially when we started watching it. The second term is the change in that initial velocity that is caused by the acceleration. If this second term is positive, then the initial velocity has increased ($v_{xf} > v_{xi}$). If this term is negative, then the initial velocity has decreased ($v_{xf} < v_{xi}$).



(a) (b)

2.4 Graph (a) has a constant slope, indicating a constant acceleration; this is represented by graph (e).

Graph (b) represents a speed that is increasing constantly but not at a uniform rate. Thus, the acceleration must be increasing, and the graph that best indicates this is (d).

Graph (c) depicts a velocity that first increases at a constant rate, indicating constant acceleration. Then the velocity stops increasing and becomes constant, indicating zero acceleration. The best match to this situation is graph (f).

2.5 (c). As can be seen from Figure 2.13b, the ball is at rest for an infinitesimally short time at these three points. Nonetheless, gravity continues to act even though the ball is instantaneously not moving.