PUZZLER

Chum salmon "climbing a ladder" in the McNeil River in Alaska. Why are fish ladders like this often built around dams? Do the ladders reduce the amount of work that the fish must do to get past the dam? *(Daniel J. Cox/Tony Stone Images)*

chapter

7 Work and Kinetic Energy

Chapter Outline

The concept of energy is one of the most important topics in science and engineering. In everyday life, we think of energy in terms of fuel for transportation and heating, electricity for lights and appliances, and foods for consumption. However, these ideas do not really define energy. They merely tell us that fuels are needed to do a job and that those fuels provide us with something we call *energy*.

In this chapter, we first introduce the concept of work. Work is done by a force acting on an object when the point of application of that force moves through some distance and the force has a component along the line of motion. Next, we define kinetic energy, which is energy an object possesses because of its motion. In general, we can think of *energy* as the capacity that an object has for performing work. We shall see that the concepts of work and kinetic energy can be applied to the dynamics of a mechanical system without resorting to Newton's laws. In a complex situation, in fact, the "energy approach" can often allow a much simpler analysis than the direct application of Newton's second law. However, it is important to note that the work–energy concepts are based on Newton's laws and therefore allow us to make predictions that are always in agreement with these laws.

This alternative method of describing motion is especially useful when the force acting on a particle varies with the position of the particle. In this case, the acceleration is not constant, and we cannot apply the kinematic equations developed in Chapter 2. Often, a particle in nature is subject to a force that varies with the position of the particle. Such forces include the gravitational force and the force exerted on an object attached to a spring. Although we could analyze situations like these by applying numerical methods such as those discussed in Section 6.5, utilizing the ideas of work and energy is often much simpler. We describe techniques for treating complicated systems with the help of an extremely important theorem called the *work–kinetic energy theorem,* which is the central topic of this chapter.

7.1 WORK DONE BY A CONSTANT FORCE

5.1 Almost all the terms we have used thus far—velocity, acceleration, force, and so on—convey nearly the same meaning in physics as they do in everyday life. Now, however, we encounter a term whose meaning in physics is distinctly different from its everyday meaning. That new term is *work*.

To understand what *work* means to the physicist, consider the situation illustrated in Figure 7.1. A force is applied to a chalkboard eraser, and the eraser slides along the tray. If we are interested in how effective the force is in moving the

(a) (b) (c)

Figure 7.1 An eraser being pushed along a chalkboard tray. *(Charles D. Winters)*

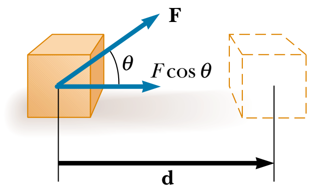


Figure 7.2 If an object undergoes a displacement **d** under the action of a constant force **F**, the work done by the force is $(F \cos \theta)\, d$.

eraser, we need to consider not only the magnitude of the force but also its direction. If we assume that the magnitude of the applied force is the same in all three photographs, it is clear that the push applied in Figure 7.1b does more to move the eraser than the push in Figure 7.1a. On the other hand, Figure 7.1c shows a situation in which the applied force does not move the eraser at all, regardless of how hard it is pushed. (Unless, of course, we apply a force so great that we break something.) So, in analyzing forces to determine the work they do, we must consider the vector nature of forces. We also need to know how far the eraser moves along the tray if we want to determine the work required to cause that motion. Moving the eraser 3 m requires more work than moving it 2 cm.

Let us examine the situation in Figure 7.2, where an object undergoes a displacement **d** along a straight line while acted on by a constant force **F** that makes an angle θ with **d**.

Work done by a constant force

The **work** W done on an object by an agent exerting a constant force on the object is the product of the component of the force in the direction of the displacement and the magnitude of the displacement:

$$W = Fd \cos \theta \tag{7.1}$$

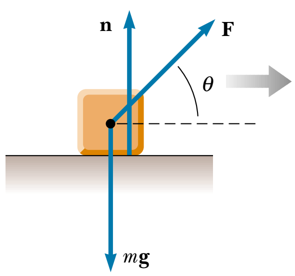


Figure 7.3 When an object is displaced on a frictionless, horizontal, surface, the normal force **n** and the force of gravity $m\mathbf{g}$ do no work on the object. In the situation shown here, **F** is the only force doing work on the object.

As an example of the distinction between this definition of work and our everyday understanding of the word, consider holding a heavy chair at arm's length for 3 min. At the end of this time interval, your tired arms may lead you to think that you have done a considerable amount of work on the chair. According to our definition, however, you have done no work on it whatsoever.[1] You exert a force to support the chair, but you do not move it. A force does no work on an object if the object does not move. This can be seen by noting that if $d = 0$, Equation 7.1 gives $W = 0$—the situation depicted in Figure 7.1c.

Also note from Equation 7.1 that the work done by a force on a moving object is zero when the force applied is perpendicular to the object's displacement. That is, if $\theta = 90°$, then $W = 0$ because $\cos 90° = 0$. For example, in Figure 7.3, the work done by the normal force on the object and the work done by the force of gravity on the object are both zero because both forces are perpendicular to the displacement and have zero components in the direction of **d**.

The sign of the work also depends on the direction of **F** relative to **d**. The work done by the applied force is positive when the vector associated with the component $F \cos \theta$ is in the same direction as the displacement. For example, when an object is lifted, the work done by the applied force is positive because the direction of that force is upward, that is, in the same direction as the displacement. When the vector associated with the component $F \cos \theta$ is in the direction opposite the displacement, W is negative. For example, as an object is lifted, the work done by the gravitational force on the object is negative. The factor $\cos \theta$ in the definition of W (Eq. 7.1) automatically takes care of the sign. It is important to note that **work is an energy transfer;** if energy is transferred *to* the system (object), W is positive; if energy is transferred *from* the system, W is negative.

5.3

[1] Actually, you do work while holding the chair at arm's length because your muscles are continuously contracting and relaxing; this means that they are exerting internal forces on your arm. Thus, work is being done by your body—but internally on itself rather than on the chair.

If an applied force **F** acts along the direction of the displacement, then $\theta = 0$ and $\cos 0 = 1$. In this case, Equation 7.1 gives

$$W = Fd$$

Work is a scalar quantity, and its units are force multiplied by length. Therefore, the SI unit of work is the **newton·meter** (N·m). This combination of units is used so frequently that it has been given a name of its own: the **joule** (J).

Quick Quiz 7.1

Can the component of a force that gives an object a centripetal acceleration do any work on the object? (One such force is that exerted by the Sun on the Earth that holds the Earth in a circular orbit around the Sun.)

In general, a particle may be moving with either a constant or a varying velocity under the influence of several forces. In these cases, because work is a scalar quantity, the total work done as the particle undergoes some displacement is the algebraic sum of the amounts of work done by all the forces.

Example 7.1 Mr. Clean

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50.0$ N at an angle of 30.0° with the horizontal (Fig. 7.4a). Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

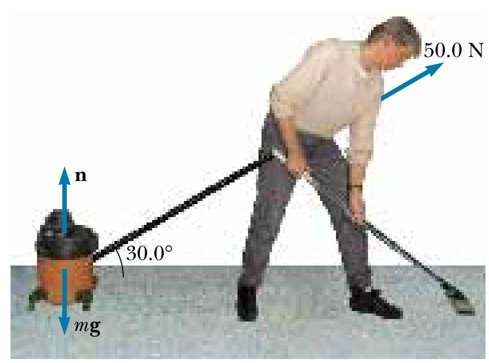
Solution Because they aid us in clarifying which forces are acting on the object being considered, drawings like Figure 7.4b are helpful when we are gathering information and organizing a solution. For our analysis, we use the definition of work (Eq. 7.1):

$$\begin{aligned} W &= (F\cos\theta)\,d \\ &= (50.0\text{ N})(\cos 30.0°)(3.00\text{ m}) = 130\text{ N·m} \\ &= \boxed{130\text{ J}} \end{aligned}$$

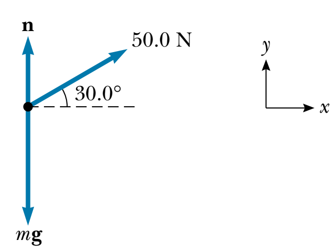
One thing we should learn from this problem is that the normal force **n**, the force of gravity $\mathbf{F}_g = m\mathbf{g}$, and the upward component of the applied force (50.0 N) (sin 30.0°) do no work on the vacuum cleaner because these forces are perpendicular to its displacement.

Exercise Find the work done by the man on the vacuum cleaner if he pulls it 3.0 m with a horizontal force of 32 N.

Answer 96 J.

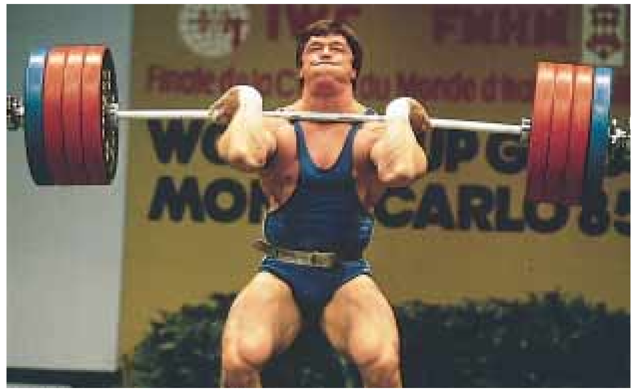


(a)



(b)

Figure 7.4 (a) A vacuum cleaner being pulled at an angle of 30.0° with the horizontal. (b) Free-body diagram of the forces acting on the vacuum cleaner.



The weightlifter does no work on the weights as he holds them on his shoulders. (If he could rest the bar on his shoulders and lock his knees, he would be able to support the weights for quite some time.) Did he do any work when he raised the weights to this height?

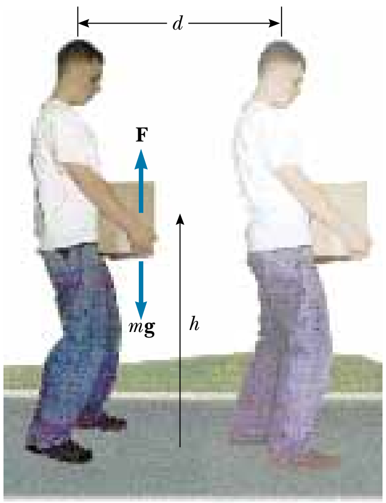


Figure 7.5 A person lifts a box of mass m a vertical distance h and then walks horizontally a distance d.

Quick Quiz 7.2

A person lifts a heavy box of mass m a vertical distance h and then walks horizontally a distance d while holding the box, as shown in Figure 7.5. Determine (a) the work he does on the box and (b) the work done on the box by the force of gravity.

7.2 THE SCALAR PRODUCT OF TWO VECTORS

2.6 Because of the way the force and displacement vectors are combined in Equation 7.1, it is helpful to use a convenient mathematical tool called the **scalar product.** This tool allows us to indicate how **F** and **d** interact in a way that depends on how close to parallel they happen to be. We write this scalar product **F·d.** (Because of the dot symbol, the scalar product is often called the **dot product.**) Thus, we can express Equation 7.1 as a scalar product:

Work expressed as a dot product

$$W = \mathbf{F} \cdot \mathbf{d} = Fd \cos \theta \qquad \textbf{(7.2)}$$

In other words, **F·d** (read "F dot d") is a shorthand notation for $Fd \cos \theta$.

Scalar product of any two vectors **A** and **B**

In general, the scalar product of any two vectors **A** and **B** is a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them:

$$\mathbf{A} \cdot \mathbf{B} \equiv AB \cos \theta \qquad \textbf{(7.3)}$$

This relationship is shown in Figure 7.6. Note that **A** and **B** need not have the same units.

In Figure 7.6, $B \cos \theta$ is the projection of **B** onto **A**. Therefore, Equation 7.3 says that $\mathbf{A} \cdot \mathbf{B}$ is the product of the magnitude of **A** and the projection of **B** onto **A**.[2]

From the right-hand side of Equation 7.3 we also see that the scalar product is **commutative.**[3] That is,

The order of the dot product can be reversed

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

Finally, the scalar product obeys the **distributive law of multiplication,** so that

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$$

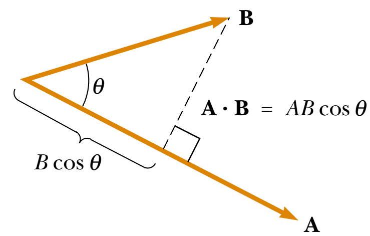


Figure 7.6 The scalar product $\mathbf{A} \cdot \mathbf{B}$ equals the magnitude of **A** multiplied by $B \cos \theta$, which is the projection of **B** onto **A**.

The dot product is simple to evaluate from Equation 7.3 when **A** is either perpendicular or parallel to **B**. If **A** is perpendicular to **B** ($\theta = 90°$), then $\mathbf{A} \cdot \mathbf{B} = 0$. (The equality $\mathbf{A} \cdot \mathbf{B} = 0$ also holds in the more trivial case when either **A** or **B** is zero.) If vector **A** is parallel to vector **B** and the two point in the same direction ($\theta = 0$), then $\mathbf{A} \cdot \mathbf{B} = AB$. If vector **A** is parallel to vector **B** but the two point in opposite directions ($\theta = 180°$), then $\mathbf{A} \cdot \mathbf{B} = -AB$. The scalar product is negative when $90° < \theta < 180°$.

The unit vectors **i**, **j**, and **k**, which were defined in Chapter 3, lie in the positive *x*, *y*, and *z* directions, respectively, of a right-handed coordinate system. Therefore, it follows from the definition of $\mathbf{A} \cdot \mathbf{B}$ that the scalar products of these unit vectors are

Dot products of unit vectors

$$\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1 \tag{7.4}$$

$$\mathbf{i} \cdot \mathbf{j} = \mathbf{i} \cdot \mathbf{k} = \mathbf{j} \cdot \mathbf{k} = 0 \tag{7.5}$$

Equations 3.18 and 3.19 state that two vectors **A** and **B** can be expressed in component vector form as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

Using the information given in Equations 7.4 and 7.5 shows that the scalar product of **A** and **B** reduces to

$$\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z \tag{7.6}$$

(Details of the derivation are left for you in Problem 7.10.) In the special case in which $\mathbf{A} = \mathbf{B}$, we see that

$$\mathbf{A} \cdot \mathbf{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$

Quick Quiz 7.3

If the dot product of two vectors is positive, must the vectors have positive rectangular components?

[2] This is equivalent to stating that $\mathbf{A} \cdot \mathbf{B}$ equals the product of the magnitude of **B** and the projection of **A** onto **B**.

[3] This may seem obvious, but in Chapter 11 you will see another way of combining vectors that proves useful in physics and is not commutative.

EXAMPLE 7.2 The Scalar Product

The vectors **A** and **B** are given by $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{B} = -\mathbf{i} + 2\mathbf{j}$. (a) Determine the scalar product $\mathbf{A} \cdot \mathbf{B}$.

Solution

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= (2\mathbf{i} + 3\mathbf{j}) \cdot (-\mathbf{i} + 2\mathbf{j}) \\ &= -2\mathbf{i}\cdot\mathbf{i} + 2\mathbf{i}\cdot 2\mathbf{j} - 3\mathbf{j}\cdot\mathbf{i} + 3\mathbf{j}\cdot 2\mathbf{j} \\ &= -2(1) + 4(0) - 3(0) + 6(1) \\ &= -2 + 6 = \boxed{4}\end{aligned}$$

where we have used the facts that $\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = 1$ and $\mathbf{i}\cdot\mathbf{j} = \mathbf{j}\cdot\mathbf{i} = 0$. The same result is obtained when we use Equation 7.6 directly, where $A_x = 2$, $A_y = 3$, $B_x = -1$, and $B_y = 2$.

(b) Find the angle θ between **A** and **B**.

Solution The magnitudes of **A** and **B** are

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

Using Equation 7.3 and the result from part (a) we find that

$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{8.06} = \boxed{60.2°}$$

EXAMPLE 7.3 Work Done by a Constant Force

A particle moving in the xy plane undergoes a displacement $\mathbf{d} = (2.0\mathbf{i} + 3.0\mathbf{j})$ m as a constant force $\mathbf{F} = (5.0\mathbf{i} + 2.0\mathbf{j})$ N acts on the particle. (a) Calculate the magnitude of the displacement and that of the force.

Solution

$$d = \sqrt{x^2 + y^2} = \sqrt{(2.0)^2 + (3.0)^2} = \boxed{3.6\ \text{m}}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = \boxed{5.4\ \text{N}}$$

(b) Calculate the work done by **F**.

Solution Substituting the expressions for **F** and **d** into Equations 7.4 and 7.5, we obtain

$$\begin{aligned}W = \mathbf{F}\cdot\mathbf{d} &= (5.0\mathbf{i} + 2.0\mathbf{j})\cdot(2.0\mathbf{i} + 3.0\mathbf{j})\ \text{N}\cdot\text{m} \\ &= 5.0\mathbf{i}\cdot 2.0\mathbf{i} + 5.0\mathbf{i}\cdot 3.0\mathbf{j} + 2.0\mathbf{j}\cdot 2.0\mathbf{i} + 2.0\mathbf{j}\cdot 3.0\mathbf{j} \\ &= 10 + 0 + 0 + 6 = 16\ \text{N}\cdot\text{m} = \boxed{16\ \text{J}}\end{aligned}$$

Exercise Calculate the angle between **F** and **d**.

Answer 35°.

7.3 WORK DONE BY A VARYING FORCE

5.2

Consider a particle being displaced along the x axis under the action of a varying force. The particle is displaced in the direction of increasing x from $x = x_i$ to $x = x_f$. In such a situation, we cannot use $W = (F\cos\theta)d$ to calculate the work done by the force because this relationship applies only when **F** is constant in magnitude and direction. However, if we imagine that the particle undergoes a very small displacement Δx, shown in Figure 7.7a, then the x component of the force F_x is approximately constant over this interval; for this small displacement, we can express the work done by the force as

$$\Delta W = F_x\,\Delta x$$

This is just the area of the shaded rectangle in Figure 7.7a. If we imagine that the F_x versus x curve is divided into a large number of such intervals, then the total work done for the displacement from x_i to x_f is approximately equal to the sum of a large number of such terms:

$$W \approx \sum_{x_i}^{x_f} F_x\,\Delta x$$

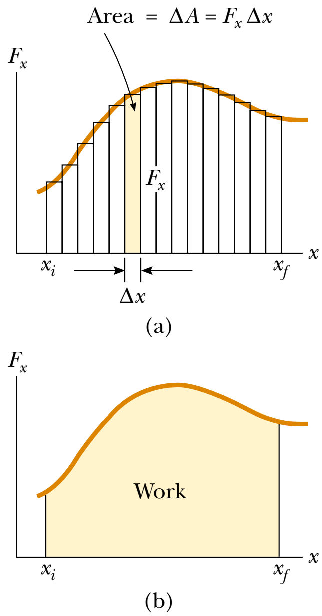


Figure 7.7 (a) The work done by the force component F_x for the small displacement Δx is $F_x\,\Delta x$, which equals the area of the shaded rectangle. The total work done for the displacement from x_i to x_f is approximately equal to the sum of the areas of all the rectangles. (b) The work done by the component F_x of the varying force as the particle moves from x_i to x_f is exactly equal to the area under this curve.

If the displacements are allowed to approach zero, then the number of terms in the sum increases without limit but the value of the sum approaches a definite value equal to the area bounded by the F_x curve and the x axis:

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x\,\Delta x = \int_{x_i}^{x_f} F_x\,dx$$

This definite integral is numerically equal to the area under the F_x-versus-x curve between x_i and x_f. Therefore, we can express the work done by F_x as the particle moves from x_i to x_f as

$$W = \int_{x_i}^{x_f} F_x\,dx \qquad \textbf{(7.7)}$$

Work done by a varying force

This equation reduces to Equation 7.1 when the component $F_x = F\cos\theta$ is constant.

If more than one force acts on a particle, the total work done is just the work done by the resultant force. If we express the resultant force in the x direction as ΣF_x, then the total work, or *net work*, done as the particle moves from x_i to x_f is

$$\sum W = W_{\text{net}} = \int_{x_i}^{x_f} \left(\sum F_x\right) dx \qquad \textbf{(7.8)}$$

EXAMPLE 7.4 Calculating Total Work Done from a Graph

A force acting on a particle varies with x, as shown in Figure 7.8. Calculate the work done by the force as the particle moves from $x = 0$ to $x = 6.0$ m.

Solution The work done by the force is equal to the area under the curve from $x_A = 0$ to $x_C = 6.0$ m. This area is equal to the area of the rectangular section from Ⓐ to Ⓑ plus

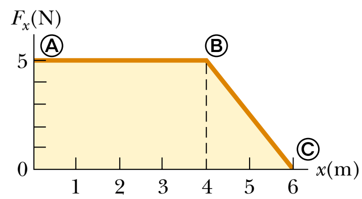


Figure 7.8 The force acting on a particle is constant for the first 4.0 m of motion and then decreases linearly with x from $x_B = 4.0$ m to $x_C = 6.0$ m. The net work done by this force is the area under the curve.

the area of the triangular section from Ⓑ to Ⓒ. The area of the rectangle is (4.0)(5.0) N·m = 20 J, and the area of the triangle is $\frac{1}{2}(2.0)(5.0)$ N·m = 5.0 J. Therefore, the total work done is 25 J.

EXAMPLE 7.5 Work Done by the Sun on a Probe

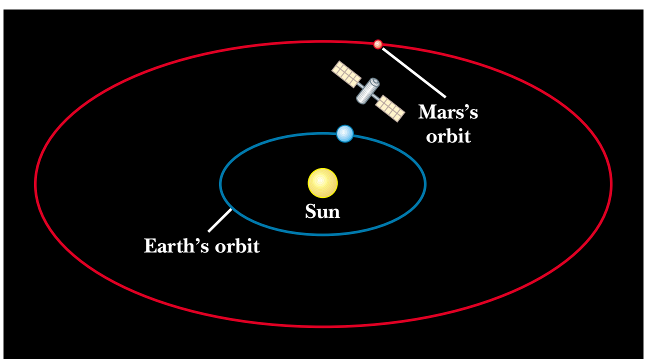
The interplanetary probe shown in Figure 7.9a is attracted to the Sun by a force of magnitude

$$F = -1.3 \times 10^{22}/x^2$$

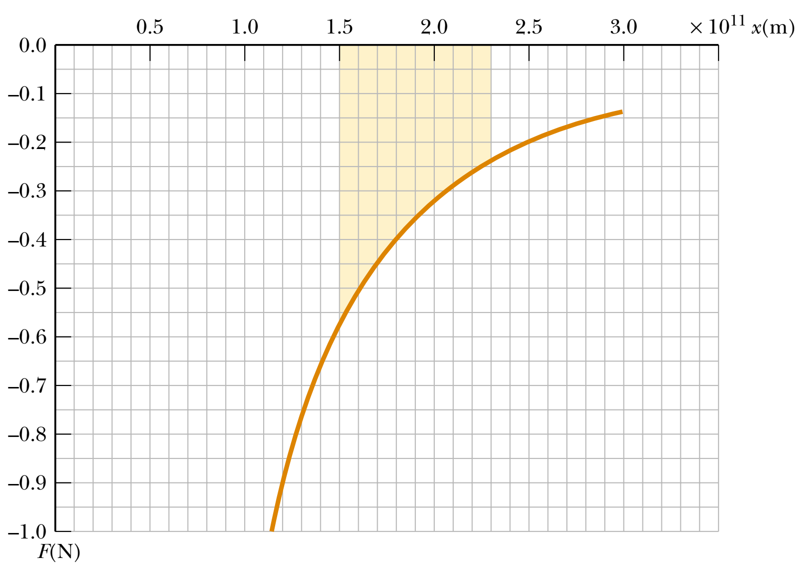
where x is the distance measured outward from the Sun to the probe. Graphically and analytically determine how much work is done by the Sun on the probe as the probe–Sun separation changes from 1.5×10^{11} m to 2.3×10^{11} m.

Graphical Solution The minus sign in the formula for the force indicates that the probe is attracted to the Sun. Because the probe is moving away from the Sun, we expect to calculate a negative value for the work done on it.

A spreadsheet or other numerical means can be used to generate a graph like that in Figure 7.9b. Each small square of the grid corresponds to an area $(0.05 \text{ N})(0.1 \times 10^{11} \text{ m}) = 5 \times 10^8$ N·m. The work done is equal to the shaded area in Figure 7.9b. Because there are approximately 60 squares shaded, the total area (which is negative because it is below the x axis) is about -3×10^{10} N·m. This is the work done by the Sun on the probe.



(a)



(b)

Figure 7.9 (a) An interplanetary probe moves from a position near the Earth's orbit radially outward from the Sun, ending up near the orbit of Mars. (b) Attractive force versus distance for the interplanetary probe.

Analytical Solution We can use Equation 7.7 to calculate a more precise value for the work done on the probe by the Sun. To solve this integral, we use the first formula of Table B.5 in Appendix B with $n = -2$:

$$W = \int_{1.5\times10^{11}}^{2.3\times10^{11}} \left(\frac{-1.3 \times 10^{22}}{x^2}\right) dx$$

$$= (-1.3 \times 10^{22}) \int_{1.5\times10^{11}}^{2.3\times10^{11}} x^{-2}\, dx$$

$$= (-1.3 \times 10^{22})(-x^{-1}) \Big|_{1.5\times10^{11}}^{2.3\times10^{11}}$$

$$= (-1.3 \times 10^{22}) \left(\frac{-1}{2.3 \times 10^{11}} - \frac{-1}{1.5 \times 10^{11}}\right)$$

$$= \boxed{-3.0 \times 10^{10}\ \text{J}}$$

Exercise Does it matter whether the path of the probe is not directed along a radial line away from the Sun?

Answer No; the value of W depends only on the initial and final positions, not on the path taken between these points.

Work Done by a Spring

5.3 A common physical system for which the force varies with position is shown in Figure 7.10. A block on a horizontal, frictionless surface is connected to a spring. If the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force of magnitude

$$F_s = -kx \quad \textbf{(7.9)}$$

Spring force

where x is the displacement of the block from its unstretched ($x = 0$) position and k is a positive constant called the **force constant** of the spring. In other words, the force required to stretch or compress a spring is proportional to the amount of stretch or compression x. This force law for springs, known as **Hooke's law,** is valid only in the limiting case of small displacements. The value of k is a measure of the *stiffness* of the spring. Stiff springs have large k values, and soft springs have small k values.

Quick Quiz 7.4

What are the units for k, the force constant in Hooke's law?

The negative sign in Equation 7.9 signifies that the force exerted by the spring is always directed *opposite* the displacement. When $x > 0$ as in Figure 7.10a, the spring force is directed to the left, in the negative x direction. When $x < 0$ as in Figure 7.10c, the spring force is directed to the right, in the positive x direction. When $x = 0$ as in Figure 7.10b, the spring is unstretched and $F_s = 0$. Because the spring force always acts toward the equilibrium position ($x = 0$), it sometimes is called a *restoring force.* If the spring is compressed until the block is at the point $-x_{max}$ and is then released, the block moves from $-x_{max}$ through zero to $+x_{max}$. If the spring is instead stretched until the block is at the point x_{max} and is then released, the block moves from $+x_{max}$ through zero to $-x_{max}$. It then reverses direction, returns to $+x_{max}$, and continues oscillating back and forth.

Suppose the block has been pushed to the left a distance x_{max} from equilibrium and is then released. Let us calculate the work W_s done by the spring force as the block moves from $x_i = -x_{max}$ to $x_f = 0$. Applying Equation 7.7 and assuming the block may be treated as a particle, we obtain

$$W_s = \int_{x_i}^{x_f} F_s\, dx = \int_{-x_{max}}^{0} (-kx)\, dx = \tfrac{1}{2}kx_{max}^2 \quad \textbf{(7.10)}$$

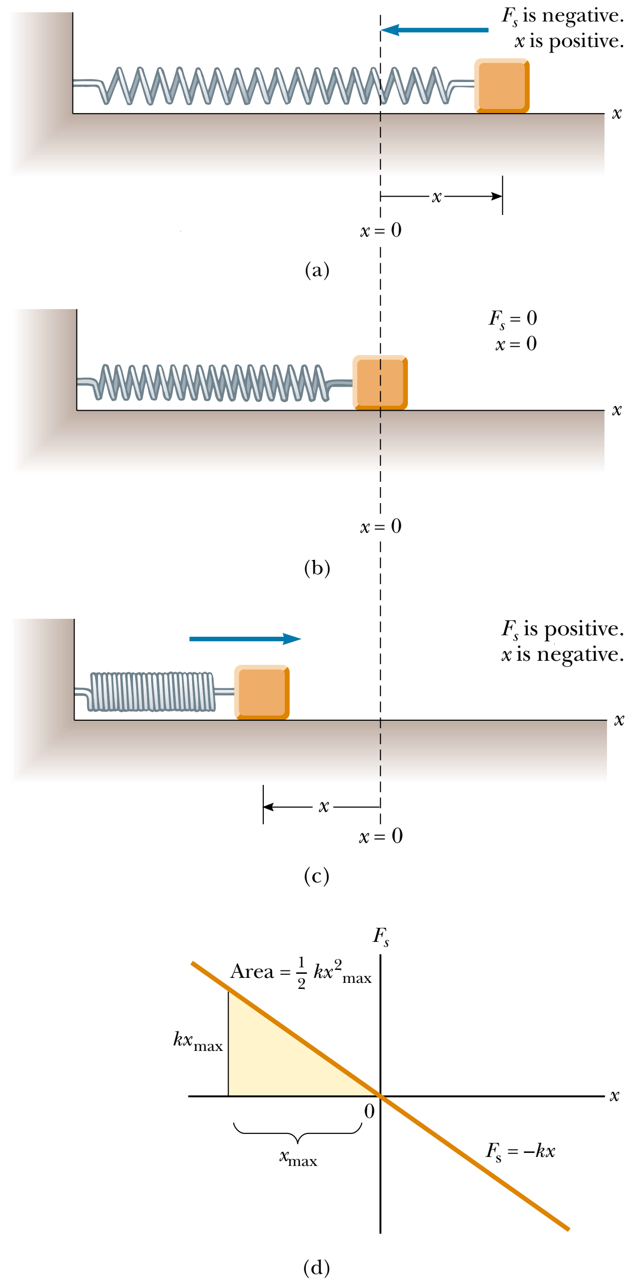


Figure 7.10 The force exerted by a spring on a block varies with the block's displacement x from the equilibrium position $x = 0$. (a) When x is positive (stretched spring), the spring force is directed to the left. (b) When x is zero (natural length of the spring), the spring force is zero. (c) When x is negative (compressed spring), the spring force is directed to the right. (d) Graph of F_s versus x for the block–spring system. The work done by the spring force as the block moves from $-x_{max}$ to 0 is the area of the shaded triangle, $\frac{1}{2}kx^2_{max}$.

where we have used the indefinite integral $\int x^n dx = x^{n+1}/(n+1)$ with $n = 1$. The work done by the spring force is positive because the force is in the same direction as the displacement (both are to the right). When we consider the work done by the spring force as the block moves from $x_i = 0$ to $x_f = x_{max}$, we find that

$W_s = -\frac{1}{2}kx_{\max}^2$ because for this part of the motion the displacement is to the right and the spring force is to the left. Therefore, the *net* work done by the spring force as the block moves from $x_i = -x_{\max}$ to $x_f = x_{\max}$ is *zero*.

Figure 7.10d is a plot of F_s versus x. The work calculated in Equation 7.10 is the area of the shaded triangle, corresponding to the displacement from $-x_{\max}$ to 0. Because the triangle has base $x_{\max}$ and height $kx_{\max}$, its area is $\frac{1}{2}kx_{\max}^2$, the work done by the spring as given by Equation 7.10.

If the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$, the work done by the spring force is

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \textbf{(7.11)}$$

Work done by a spring

For example, if the spring has a force constant of 80 N/m and is compressed 3.0 cm from equilibrium, the work done by the spring force as the block moves from $x_i = -3.0$ cm to its unstretched position $x_f = 0$ is 3.6×10^{-2} J. From Equation 7.11 we also see that the work done by the spring force is zero for any motion that ends where it began ($x_i = x_f$). We shall make use of this important result in Chapter 8, in which we describe the motion of this system in greater detail.

Equations 7.10 and 7.11 describe the work done by the spring on the block. Now let us consider the work done on the spring by an *external agent* that stretches the spring very slowly from $x_i = 0$ to $x_f = x_{\max}$, as in Figure 7.11. We can calculate this work by noting that at any value of the displacement, the *applied force* $\mathbf{F}_{\text{app}}$ is equal to and opposite the spring force $\mathbf{F}_s$, so that $F_{\text{app}} = -(-kx) = kx$. Therefore, the work done by this applied force (the external agent) is

$$W_{F_{\text{app}}} = \int_0^{x_{\max}} F_{\text{app}}\,dx = \int_0^{x_{\max}} kx\,dx = \tfrac{1}{2}kx_{\max}^2$$

This work is equal to the negative of the work done by the spring force for this displacement.

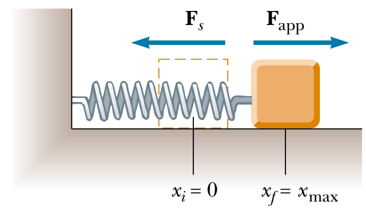


Figure 7.11 A block being pulled from $x_i = 0$ to $x_f = x_{\max}$ on a frictionless surface by a force $\mathbf{F}_{\text{app}}$. If the process is carried out very slowly, the applied force is equal to and opposite the spring force at all times.

EXAMPLE 7.6 Measuring *k* for a Spring

A common technique used to measure the force constant of a spring is described in Figure 7.12. The spring is hung vertically, and an object of mass m is attached to its lower end. Under the action of the "load" mg, the spring stretches a distance d from its equilibrium position. Because the spring force is upward (opposite the displacement), it must balance the downward force of gravity $m\mathbf{g}$ when the system is at rest. In this case, we can apply Hooke's law to give $|\mathbf{F}_s| = kd = mg$, or

$$k = \frac{mg}{d}$$

For example, if a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, then the force constant is

$$k = \frac{mg}{d} = \frac{(0.55\text{ kg})(9.80\text{ m/s}^2)}{2.0 \times 10^{-2}\text{ m}} = \boxed{2.7 \times 10^2\text{ N/m}}$$

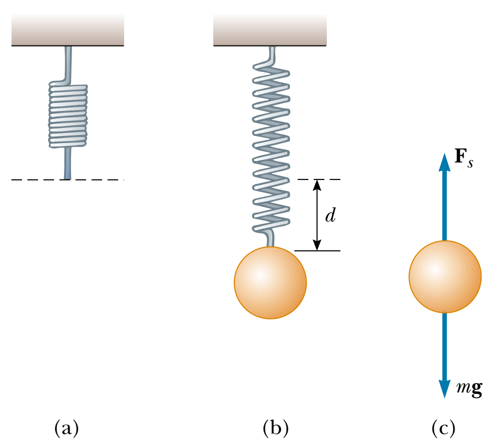


Figure 7.12 Determining the force constant k of a spring. The elongation d is caused by the attached object, which has a weight mg. Because the spring force balances the force of gravity, it follows that $k = mg/d$.

7.4 KINETIC ENERGY AND THE WORK–KINETIC ENERGY THEOREM

5.7

It can be difficult to use Newton's second law to solve motion problems involving complex forces. An alternative approach is to relate the speed of a moving particle to its displacement under the influence of some net force. If the work done by the net force on a particle can be calculated for a given displacement, then the change in the particle's speed can be easily evaluated.

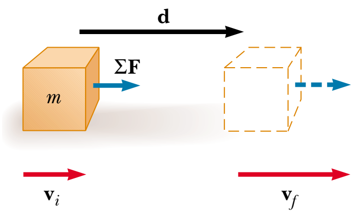


Figure 7.13 A particle undergoing a displacement **d** and a change in velocity under the action of a constant net force $\Sigma\mathbf{F}$.

Figure 7.13 shows a particle of mass m moving to the right under the action of a constant net force $\Sigma\mathbf{F}$. Because the force is constant, we know from Newton's second law that the particle moves with a constant acceleration **a**. If the particle is displaced a distance d, the net work done by the total force $\Sigma\mathbf{F}$ is

$$\sum W = \left(\sum F\right)d = (ma)\,d \qquad \textbf{(7.12)}$$

In Chapter 2 we found that the following relationships are valid when a particle undergoes constant acceleration:

$$d = \tfrac{1}{2}(v_i + v_f)\,t \qquad a = \frac{v_f - v_i}{t}$$

where v_i is the speed at $t = 0$ and v_f is the speed at time t. Substituting these expressions into Equation 7.12 gives

$$\sum W = m\left(\frac{v_f - v_i}{t}\right)\tfrac{1}{2}(v_i + v_f)\,t$$

$$\sum W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad \textbf{(7.13)}$$

The quantity $\frac{1}{2}mv^2$ represents the energy associated with the motion of the particle. This quantity is so important that it has been given a special name—**kinetic energy.** The net work done on a particle by a constant net force $\Sigma\mathbf{F}$ acting on it equals the change in kinetic energy of the particle.

In general, the kinetic energy K of a particle of mass m moving with a speed v is defined as

Kinetic energy is energy associated with the motion of a body

$$K \equiv \tfrac{1}{2}mv^2 \qquad \textbf{(7.14)}$$

TABLE 7.1 **Kinetic Energies for Various Objects**

Object	Mass (kg)	Speed (m/s)	Kinetic Energy (J)
Earth orbiting the Sun	5.98×10^{24}	2.98×10^4	2.65×10^{33}
Moon orbiting the Earth	7.35×10^{22}	1.02×10^3	3.82×10^{28}
Rocket moving at escape speed[a]	500	1.12×10^4	3.14×10^{10}
Automobile at 55 mi/h	2 000	25	6.3×10^5
Running athlete	70	10	3.5×10^3
Stone dropped from 10 m	1.0	14	9.8×10^1
Golf ball at terminal speed	0.046	44	4.5×10^1
Raindrop at terminal speed	3.5×10^{-5}	9.0	1.4×10^{-3}
Oxygen molecule in air	5.3×10^{-26}	500	6.6×10^{-21}

[a] *Escape speed* is the minimum speed an object must attain near the Earth's surface if it is to escape the Earth's gravitational force.

5.4 Kinetic energy is a scalar quantity and has the same units as work. For example, a 2.0-kg object moving with a speed of 4.0 m/s has a kinetic energy of 16 J. Table 7.1 lists the kinetic energies for various objects.

It is often convenient to write Equation 7.13 in the form

$$\sum W = K_f - K_i = \Delta K \tag{7.15}$$

Work–kinetic energy theorem

That is, $K_i + \Sigma W = K_f$.

Equation 7.15 is an important result known as the **work–kinetic energy theorem.** It is important to note that when we use this theorem, we must include *all* of the forces that do work on the particle in the calculation of the net work done. From this theorem, we see that the speed of a particle increases if the net work done on it is positive because the final kinetic energy is greater than the initial kinetic energy. The particle's speed decreases if the net work done is negative because the final kinetic energy is less than the initial kinetic energy.

The work–kinetic energy theorem as expressed by Equation 7.15 allows us to think of kinetic energy as the work a particle can do in coming to rest, or the amount of energy stored in the particle. For example, suppose a hammer (our particle) is on the verge of striking a nail, as shown in Figure 7.14. The moving hammer has kinetic energy and so can do work on the nail. The work done on the nail is equal to Fd, where F is the average force exerted on the nail by the hammer and d is the distance the nail is driven into the wall.[4]

Figure 7.14 The moving hammer has kinetic energy and thus can do work on the nail, driving it into the wall.

We derived the work–kinetic energy theorem under the assumption of a constant net force, but it also is valid when the force varies. To see this, suppose the net force acting on a particle in the x direction is ΣF_x. We can apply Newton's second law, $\Sigma F_x = ma_x$, and use Equation 7.8 to express the net work done as

$$\sum W = \int_{x_i}^{x_f}\left(\sum F_x\right)dx = \int_{x_i}^{x_f} ma_x\, dx$$

If the resultant force varies with x, the acceleration and speed also depend on x. Because we normally consider acceleration as a function of t, we now use the following chain rule to express a in a slightly different way:

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

Substituting this expression for a into the above equation for ΣW gives

$$\sum W = \int_{x_i}^{x_f} mv\frac{dv}{dx}\, dx = \int_{v_i}^{v_f} mv\, dv$$

$$\sum W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \tag{7.16}$$

The net work done on a particle equals the change in its kinetic energy

The limits of the integration were changed from x values to v values because the variable was changed from x to v. Thus, we conclude that the net work done on a particle by the net force acting on it is equal to the change in the kinetic energy of the particle. This is true whether or not the net force is constant.

[4] Note that because the nail and the hammer are *systems* of particles rather than single particles, part of the hammer's kinetic energy goes into warming the hammer and the nail upon impact. Also, as the nail moves into the wall in response to the impact, the large frictional force between the nail and the wood results in the continuous transformation of the kinetic energy of the nail into further temperature increases in the nail and the wood, as well as in deformation of the wall. Energy associated with temperature changes is called *internal energy* and will be studied in detail in Chapter 20.

Situations Involving Kinetic Friction

One way to include frictional forces in analyzing the motion of an object sliding on a *horizontal* surface is to describe the kinetic energy lost because of friction. Suppose a book moving on a horizontal surface is given an initial horizontal velocity $\mathbf{v}_i$ and slides a distance d before reaching a final velocity $\mathbf{v}_f$ as shown in Figure 7.15. The external force that causes the book to undergo an acceleration in the negative x direction is the force of kinetic friction $\mathbf{f}_k$ acting to the left, opposite the motion. The initial kinetic energy of the book is $\frac{1}{2}mv_i^2$, and its final kinetic energy is $\frac{1}{2}mv_f^2$. Applying Newton's second law to the book can show this. Because the only force acting on the book in the x direction is the friction force, Newton's second law gives $-f_k = ma_x$. Multiplying both sides of this expression by d and using Equation 2.12 in the form $v_{xf}^2 - v_{xi}^2 = 2a_xd$ for motion under constant acceleration give $-f_kd = (ma_x)d = \frac{1}{2}mv_{xf}^2 - \frac{1}{2}mv_{xi}^2$ or

Loss in kinetic energy due to friction

$$\Delta K_{\text{friction}} = -f_kd \tag{7.17a}$$

This result specifies that the amount by which the force of kinetic friction changes the kinetic energy of the book is equal to $-f_kd$. Part of this lost kinetic energy goes into warming up the book, and the rest goes into warming up the surface over which the book slides. In effect, the quantity $-f_kd$ is equal to the work done by kinetic friction on the book *plus* the work done by kinetic friction on the surface. (We shall study the relationship between temperature and energy in Part III of this text.) When friction—as well as other forces—acts on an object, the work–kinetic energy theorem reads

$$K_i + \sum W_{\text{other}} - f_kd = K_f \tag{7.17b}$$

Here, $\sum W_{\text{other}}$ represents the sum of the amounts of work done on the object by forces other than kinetic friction.

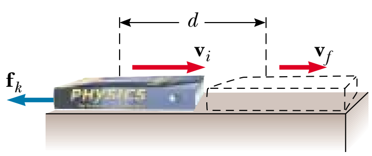


Figure 7.15 A book sliding to the right on a horizontal surface slows down in the presence of a force of kinetic friction acting to the left. The initial velocity of the book is $\mathbf{v}_i$, and its final velocity is $\mathbf{v}_f$. The normal force and the force of gravity are not included in the diagram because they are perpendicular to the direction of motion and therefore do not influence the book's velocity.

Quick Quiz 7.5

Can frictional forces ever *increase* an object's kinetic energy?

Example 7.7 A Block Pulled on a Frictionless Surface

A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.

Solution We have made a drawing of this situation in Figure 7.16a. We could apply the equations of kinematics to determine the answer, but let us use the energy approach for

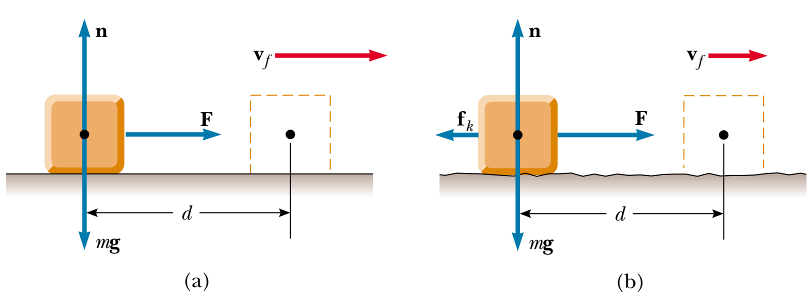


Figure 7.16 A block pulled to the right by a constant horizontal force. (a) Frictionless surface. (b) Rough surface.

practice. The normal force balances the force of gravity on the block, and neither of these vertically acting forces does work on the block because the displacement is horizontal. Because there is no friction, the net external force acting on the block is the 12-N force. The work done by this force is

$$W = Fd = (12\text{ N})(3.0\text{ m}) = 36\text{ N}\cdot\text{m} = 36\text{ J}$$

Using the work–kinetic energy theorem and noting that the initial kinetic energy is zero, we obtain

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - 0$$

$$v_f^2 = \frac{2W}{m} = \frac{2(36\text{ J})}{6.0\text{ kg}} = 12\text{ m}^2/\text{s}^2$$

$$v_f = \boxed{3.5\text{ m/s}}$$

Exercise Find the acceleration of the block and determine its final speed, using the kinematics equation $v_{xf}^2 = v_{xi}^2 + 2a_xd$.

Answer $a_x = 2.0\text{ m/s}^2$; $v_f = 3.5\text{ m/s}$.

EXAMPLE 7.8 A Block Pulled on a Rough Surface

Find the final speed of the block described in Example 7.7 if the surface is not frictionless but instead has a coefficient of kinetic friction of 0.15.

Solution The applied force does work just as in Example 7.7:

$$W = Fd = (12\text{ N})(3.0\text{ m}) = 36\text{ J}$$

In this case we must use Equation 7.17a to calculate the kinetic energy lost to friction $\Delta K_{\text{friction}}$. The magnitude of the frictional force is

$$f_k = \mu_k n = \mu_k mg = (0.15)(6.0\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$$

The change in kinetic energy due to friction is

$$\Delta K_{\text{friction}} = -f_kd = -(8.82\text{ N})(3.0\text{ m}) = -26.5\text{ J}$$

The final speed of the block follows from Equation 7.17b:

$$\tfrac{1}{2}mv_i^2 + \sum W_{\text{other}} - f_kd = \tfrac{1}{2}mv_f^2$$

$$0 + 36\text{ J} - 26.5\text{ J} = \tfrac{1}{2}(6.0\text{ kg})\,v_f^2$$

$$v_f^2 = 2(9.5\text{ J})/(6.0\text{ kg}) = 3.18\text{ m}^2/\text{s}^2$$

$$v_f = \boxed{1.8\text{ m/s}}$$

After sliding the 3-m distance on the rough surface, the block is moving at a speed of 1.8 m/s; in contrast, after covering the same distance on a frictionless surface (see Example 7.7), its speed was 3.5 m/s.

Exercise Find the acceleration of the block from Newton's second law and determine its final speed, using equations of kinematics.

Answer $a_x = 0.53\text{ m/s}^2$; $v_f = 1.8\text{ m/s}$.

CONCEPTUAL EXAMPLE 7.9 Does the Ramp Lessen the Work Required?

A man wishes to load a refrigerator onto a truck using a ramp, as shown in Figure 7.17. He claims that less work would be required to load the truck if the length L of the ramp were increased. Is his statement valid?

Solution No. Although less force is required with a longer ramp, that force must act over a greater distance if the same amount of work is to be done. Suppose the refrigerator is wheeled on a dolly up the ramp at constant speed. The

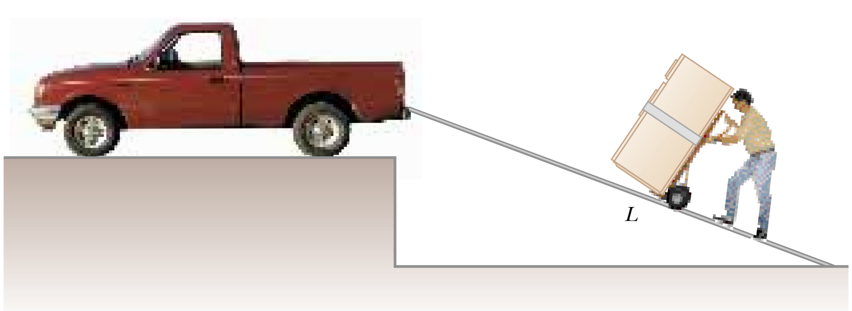


Figure 7.17 A refrigerator attached to a frictionless wheeled dolly is moved up a ramp at constant speed.

normal force exerted by the ramp on the refrigerator is directed 90° to the motion and so does no work on the refrigerator. Because $\Delta K = 0$, the work–kinetic energy theorem gives

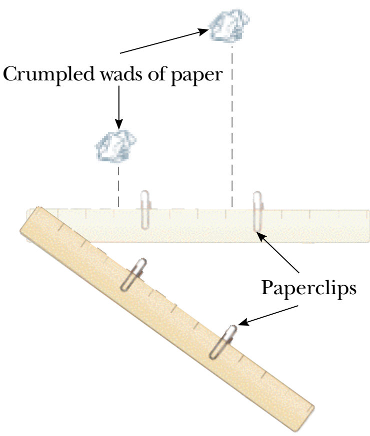
$$\sum W = W_{\text{by man}} + W_{\text{by gravity}} = 0$$

The work done by the force of gravity equals the weight of the refrigerator mg times the vertical height h through which it is displaced times cos 180°, or $W_{\text{by gravity}} = -mgh$. (The minus sign arises because the downward force of gravity is opposite the displacement.) Thus, the man must do work mgh on the refrigerator, regardless of the length of the ramp.

Consider the chum salmon attempting to swim upstream in the photograph at the beginning of this chapter. The "steps" of a fish ladder built around a dam do not change the total amount of work that must be done by the salmon as they leap through some vertical distance. However, the ladder allows the fish to perform that work in a series of smaller jumps, and the net effect is to raise the vertical position of the fish by the height of the dam.

QuickLab

Attach two paperclips to a ruler so that one of the clips is twice the distance from the end as the other. Place the ruler on a table with two small wads of paper against the clips, which act as stops. Sharply swing the ruler through a small angle, stopping it abruptly with your finger. The outer paper wad will have twice the speed of the inner paper wad as the two slide on the table away from the ruler. Compare how far the two wads slide. How does this relate to the results of Conceptual Example 7.10?



These cyclists are working hard and expending energy as they pedal uphill in Marin County, CA.

CONCEPTUAL EXAMPLE 7.10 Useful Physics for Safer Driving

A certain car traveling at an initial speed v slides a distance d to a halt after its brakes lock. Assuming that the car's initial speed is instead $2v$ at the moment the brakes lock, estimate the distance it slides.

Solution Let us assume that the force of kinetic friction between the car and the road surface is constant and the same for both speeds. The net force multiplied by the displacement of the car is equal to the initial kinetic energy of the car (because $K_f = 0$). If the speed is doubled, as it is in this example, the kinetic energy is quadrupled. For a given constant applied force (in this case, the frictional force), the distance traveled is four times as great when the initial speed is doubled, and so the estimated distance that the car slides is $4d$.

EXAMPLE 7.11 A Block–Spring System

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1.0×10^3 N/m, as shown in Figure 7.10. The spring is compressed 2.0 cm and is then released from rest. (a) Calculate the speed of the block as it passes through the equilibrium position $x = 0$ if the surface is frictionless.

Solution In this situation, the block starts with $v_i = 0$ at $x_i = -2.0$ cm, and we want to find v_f at $x_f = 0$. We use Equation 7.10 to find the work done by the spring with $x_{max} = x_i = -2.0\text{ cm} = -2.0 \times 10^{-2}$ m:

$$W_s = \tfrac{1}{2}kx_{max}^2 = \tfrac{1}{2}(1.0 \times 10^3\text{ N/m})(-2.0 \times 10^{-2}\text{ m})^2 = 0.20\text{ J}$$

Using the work–kinetic energy theorem with $v_i = 0$, we obtain the change in kinetic energy of the block due to the work done on it by the spring:

$$W_s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$0.20\text{ J} = \tfrac{1}{2}(1.6\text{ kg})v_f^2 - 0$$

$$v_f^2 = \frac{0.40\text{ J}}{1.6\text{ kg}} = 0.25\text{ m}^2/\text{s}^2$$

$$v_f = \boxed{0.50\text{ m/s}}$$

(b) Calculate the speed of the block as it passes through the equilibrium position if a constant frictional force of 4.0 N retards its motion from the moment it is released.

Solution Certainly, the answer has to be less than what we found in part (a) because the frictional force retards the motion. We use Equation 7.17 to calculate the kinetic energy lost because of friction and add this negative value to the kinetic energy found in the absence of friction. The kinetic energy lost due to friction is

$$\Delta K = -f_k d = -(4.0\text{ N})(2.0 \times 10^{-2}\text{ m}) = -0.080\text{ J}$$

In part (a), the final kinetic energy without this loss was found to be 0.20 J. Therefore, the final kinetic energy in the presence of friction is

$$K_f = 0.20\text{ J} - 0.080\text{ J} = 0.12\text{ J} = \tfrac{1}{2}mv_f^2$$

$$\tfrac{1}{2}(1.6\text{ kg})v_f^2 = 0.12\text{ J}$$

$$v_f^2 = \frac{0.24\text{ J}}{1.6\text{ kg}} = 0.15\text{ m}^2/\text{s}^2$$

$$v_f = \boxed{0.39\text{ m/s}}$$

As expected, this value is somewhat less than the 0.50 m/s we found in part (a). If the frictional force were greater, then the value we obtained as our answer would have been even smaller.

7.5 POWER

5.8 Imagine two identical models of an automobile: one with a base-priced four-cylinder engine; and the other with the highest-priced optional engine, a mighty eight-cylinder powerplant. Despite the differences in engines, the two cars have the same mass. Both cars climb a roadway up a hill, but the car with the optional engine takes much less time to reach the top. Both cars have done the same amount of work against gravity, but in different time periods. From a practical viewpoint, it is interesting to know not only the work done by the vehicles but also the *rate* at which it is done. In taking the ratio of the amount of work done to the time taken to do it, we have a way of quantifying this concept. The time rate of doing work is called **power.**

If an external force is applied to an object (which we assume acts as a particle), and if the work done by this force in the time interval Δt is W, then the **average power** expended during this interval is defined as

$$\overline{\mathcal{P}} \equiv \frac{W}{\Delta t}$$

Average power

The work done on the object contributes to the increase in the energy of the object. Therefore, a more general definition of power is the *time rate of energy transfer.* In a manner similar to how we approached the definition of velocity and acceleration, we can define the **instantaneous power** $\mathcal{P}$ as the limiting value of the average power as Δt approaches zero:

$$\mathcal{P} \equiv \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

where we have represented the increment of work done by dW. We find from Equation 7.2, letting the displacement be expressed as $d\mathbf{s}$, that $dW = \mathbf{F} \cdot d\mathbf{s}$. Therefore, the instantaneous power can be written

Instantaneous power

$$\mathcal{P} = \frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{s}}{dt} = \mathbf{F} \cdot \mathbf{v} \qquad \textbf{(7.18)}$$

where we use the fact that $\mathbf{v} = d\mathbf{s}/dt$.

The SI unit of power is joules per second (J/s), also called the *watt* (W) (after James Watt, the inventor of the steam engine):

The watt

$$1 \text{ W} = 1 \text{ J/s} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^3$$

The symbol W (not italic) for watt should not be confused with the symbol W (italic) for work.

A unit of power in the British engineering system is the horsepower (hp):

$$1 \text{ hp} = 746 \text{ W}$$

A unit of energy (or work) can now be defined in terms of the unit of power. One **kilowatt hour** (kWh) is the energy converted or consumed in 1 h at the constant rate of 1 kW = 1 000 J/s. The numerical value of 1 kWh is

The kilowatt hour is a unit of energy

$$1 \text{ kWh} = (10^3 \text{ W})(3\,600 \text{ s}) = 3.60 \times 10^6 \text{ J}$$

It is important to realize that a kilowatt hour is a unit of energy, not power. When you pay your electric bill, you pay the power company for the total electrical energy you used during the billing period. This energy is the power used multiplied by the time during which it was used. For example, a 300-W lightbulb run for 12 h would convert (0.300 kW)(12 h) = 3.6 kWh of electrical energy.

Quick Quiz 7.6

Suppose that an old truck and a sports car do the same amount of work as they climb a hill but that the truck takes much longer to accomplish this work. How would graphs of $\mathcal{P}$ versus t compare for the two vehicles?

EXAMPLE 7.12 Power Delivered by an Elevator Motor

An elevator car has a mass of 1 000 kg and is carrying passengers having a combined mass of 800 kg. A constant frictional force of 4 000 N retards its motion upward, as shown in Figure 7.18a. (a) What must be the minimum power delivered by the motor to lift the elevator car at a constant speed of 3.00 m/s?

Solution The motor must supply the force of magnitude T that pulls the elevator car upward. Reading that the speed is constant provides the hint that $a = 0$, and therefore we know from Newton's second law that $\Sigma F_y = 0$. We have drawn a free-body diagram in Figure 7.18b and have arbitrarily specified that the upward direction is positive. From Newton's second law we obtain

$$\sum F_y = T - f - Mg = 0$$

where M is the *total* mass of the system (car plus passengers), equal to 1 800 kg. Therefore,

$$\begin{aligned} T &= f + Mg \\ &= 4.00 \times 10^3 \text{ N} + (1.80 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 2.16 \times 10^4 \text{ N} \end{aligned}$$

Using Equation 7.18 and the fact that **T** is in the same direction as **v**, we find that

$$\mathcal{P} = \mathbf{T}\cdot\mathbf{v} = Tv$$

$$= (2.16 \times 10^4\ \text{N})(3.00\ \text{m/s}) = \boxed{6.48 \times 10^4\ \text{W}}$$

(b) What power must the motor deliver at the instant its speed is v if it is designed to provide an upward acceleration of $1.00\ \text{m/s}^2$?

Solution Now we expect to obtain a value greater than we did in part (a), where the speed was constant, because the motor must now perform the additional task of accelerating the car. The only change in the setup of the problem is that now $a > 0$. Applying Newton's second law to the car gives

$$\sum F_y = T - f - Mg = Ma$$

$$T = M(a + g) + f$$

$$= (1.80 \times 10^3\ \text{kg})(1.00 + 9.80)\text{m/s}^2 + 4.00 \times 10^3\ \text{N}$$

$$= 2.34 \times 10^4\ \text{N}$$

Therefore, using Equation 7.18, we obtain for the required power

$$\mathcal{P} = Tv = \boxed{(2.34 \times 10^4 v)\ \text{W}}$$

where v is the instantaneous speed of the car in meters per second. The power is less than that obtained in part (a) as long as the speed is less than $\mathcal{P}/T = 2.77$ m/s, but it is greater when the elevator's speed exceeds this value.

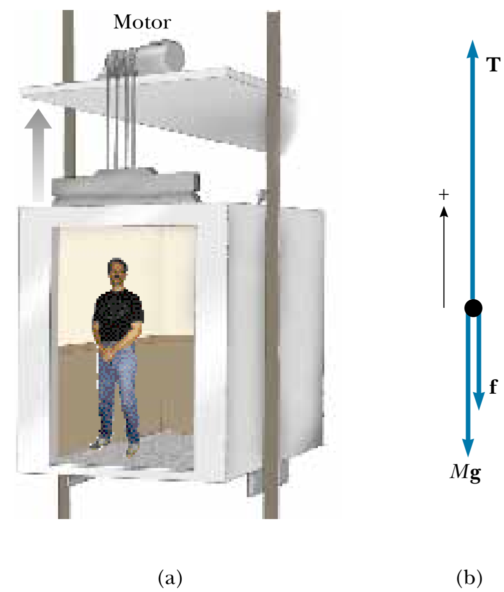


Figure 7.18 (a) The motor exerts an upward force **T** on the elevator car. The magnitude of this force is the tension T in the cable connecting the car and motor. The downward forces acting on the car are a frictional force **f** and the force of gravity $\mathbf{F}_g = M\mathbf{g}$. (b) The free-body diagram for the elevator car.

CONCEPTUAL EXAMPLE 7.13

In part (a) of the preceding example, the motor delivers power to lift the car, and yet the car moves at constant speed. A student analyzing this situation notes that the kinetic energy of the car does not change because its speed does not change. This student then reasons that, according to the work–kinetic energy theorem, $W = \Delta K = 0$. Knowing that $\mathcal{P} = W/t$, the student concludes that the power delivered by the motor also must be zero. How would you explain this apparent paradox?

Solution The work–kinetic energy theorem tells us that the *net* force acting on the system multiplied by the displacement is equal to the change in the kinetic energy of the system. In our elevator case, the net force is indeed zero (that is, $T - Mg - f = 0$), and so $W = (\Sigma F_y)d = 0$. However, the power from the motor is calculated not from the *net* force but rather from the force exerted by the motor acting in the direction of motion, which in this case is T and not zero.

Optional Section

7.6 ENERGY AND THE AUTOMOBILE

Automobiles powered by gasoline engines are very inefficient machines. Even under ideal conditions, less than 15% of the chemical energy in the fuel is used to power the vehicle. The situation is much worse under stop-and-go driving conditions in a city. In this section, we use the concepts of energy, power, and friction to analyze automobile fuel consumption.

Many mechanisms contribute to energy loss in an automobile. About 67% of the energy available from the fuel is lost in the engine. This energy ends up in the atmosphere, partly via the exhaust system and partly via the cooling system. (As we shall see in Chapter 22, the great energy loss from the exhaust and cooling systems is required by a fundamental law of thermodynamics.) Approximately 10% of the available energy is lost to friction in the transmission, drive shaft, wheel and axle bearings, and differential. Friction in other moving parts dissipates approximately 6% of the energy, and 4% of the energy is used to operate fuel and oil pumps and such accessories as power steering and air conditioning. This leaves a mere 13% of the available energy to propel the automobile! This energy is used mainly to balance the energy loss due to flexing of the tires and the friction caused by the air, which is more commonly referred to as *air resistance*.

Let us examine the power required to provide a force in the forward direction that balances the combination of the two frictional forces. The coefficient of rolling friction μ between the tires and the road is about 0.016. For a 1 450-kg car, the weight is 14 200 N and the force of rolling friction has a magnitude of $\mu n = \mu mg = 227$ N. As the speed of the car increases, a small reduction in the normal force occurs as a result of a decrease in atmospheric pressure as air flows over the top of the car. (This phenomenon is discussed in Chapter 15.) This reduction in the normal force causes a slight reduction in the force of rolling friction f_r with increasing speed, as the data in Table 7.2 indicate.

Now let us consider the effect of the resistive force that results from the movement of air past the car. For large objects, the resistive force f_a associated with air friction is proportional to the square of the speed (in meters per second; see Section 6.4) and is given by Equation 6.6:

$$f_a = \frac{1}{2} D\rho A v^2$$

where D is the drag coefficient, ρ is the density of air, and A is the cross-sectional area of the moving object. We can use this expression to calculate the f_a values in Table 7.2, using $D = 0.50$, $\rho = 1.293 \text{ kg/m}^3$, and $A \approx 2 \text{ m}^2$.

The magnitude of the total frictional force f_t is the sum of the rolling frictional force and the air resistive force:

$$f_t = f_r + f_a$$

At low speeds, road friction is the predominant resistive force, but at high speeds air drag predominates, as shown in Table 7.2. Road friction can be decreased by a reduction in tire flexing (for example, by an increase in the air pres-

TABLE 7.2 Frictional Forces and Power Requirements for a Typical Car[a]

v (m/s)	n (N)	f_r (N)	f_a (N)	f_t (N)	$\mathcal{P} = f_t v$ (kW)
0	14 200	227	0	227	0
8.9	14 100	226	51	277	2.5
17.8	13 900	222	204	426	7.6
26.8	13 600	218	465	683	18.3
35.9	13 200	211	830	1 041	37.3
44.8	12 600	202	1 293	1 495	67.0

[a] In this table, n is the normal force, f_r is road friction, f_a is air friction, f_t is total friction, and $\mathcal{P}$ is the power delivered to the wheels.

sure slightly above recommended values) and by the use of radial tires. Air drag can be reduced through the use of a smaller cross-sectional area and by streamlining the car. Although driving a car with the windows open increases air drag and thus results in a 3% decrease in mileage, driving with the windows closed and the air conditioner running results in a 12% decrease in mileage.

The total power needed to maintain a constant speed v is $f_t v$, and it is this power that must be delivered to the wheels. For example, from Table 7.2 we see that at $v = 26.8$ m/s (60 mi/h) the required power is

$$\mathcal{P} = f_t v = (683 \text{ N})\left(26.8 \frac{\text{m}}{\text{s}}\right) = 18.3 \text{ kW}$$

This power can be broken down into two parts: (1) the power $f_r v$ needed to compensate for road friction, and (2) the power $f_a v$ needed to compensate for air drag. At $v =$ 26.8 m/s, we obtain the values

$$\mathcal{P}_r = f_r v = (218 \text{ N})\left(26.8 \frac{\text{m}}{\text{s}}\right) = 5.84 \text{ kW}$$

$$\mathcal{P}_a = f_a v = (465 \text{ N})\left(26.8 \frac{\text{m}}{\text{s}}\right) = 12.5 \text{ kW}$$

Note that $\mathcal{P} = \mathcal{P}_r + \mathcal{P}_a$.

On the other hand, at $v = 44.8$ m/s (100 mi/h), $\mathcal{P}_r = 9.05$ kW, $\mathcal{P}_a = 57.9$ kW, and $\mathcal{P} = 67.0$ kW. This shows the importance of air drag at high speeds.

EXAMPLE 7.14 Gas Consumed by a Compact Car

A compact car has a mass of 800 kg, and its efficiency is rated at 18%. (That is, 18% of the available fuel energy is delivered to the wheels.) Find the amount of gasoline used to accelerate the car from rest to 27 m/s (60 mi/h). Use the fact that the energy equivalent of 1 gal of gasoline is 1.3×10^8 J.

Solution The energy required to accelerate the car from rest to a speed v is its final kinetic energy $\frac{1}{2}mv^2$:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(800 \text{ kg})(27 \text{ m/s})^2 = 2.9 \times 10^5 \text{ J}$$

If the engine were 100% efficient, each gallon of gasoline would supply 1.3×10^8 J of energy. Because the engine is only 18% efficient, each gallon delivers only $(0.18)(1.3 \times 10^8 \text{ J}) = 2.3 \times 10^7$ J. Hence, the number of gallons used to accelerate the car is

$$\text{Number of gallons} = \frac{2.9 \times 10^5 \text{ J}}{2.3 \times 10^7 \text{ J/gal}} = 0.013 \text{ gal}$$

At cruising speed, this much gasoline is sufficient to propel the car nearly 0.5 mi. This demonstrates the extreme energy requirements of stop-and-start driving.

EXAMPLE 7.15 Power Delivered to Wheels

Suppose the compact car in Example 7.14 gets 35 mi/gal at 60 mi/h. How much power is delivered to the wheels?

Solution By simply canceling units, we determine that the car consumes 60 mi/h ÷ 35 mi/gal = 1.7 gal/h. Using the fact that each gallon is equivalent to 1.3×10^8 J, we find that the total power used is

$$\mathcal{P} = \frac{(1.7 \text{ gal/h})(1.3 \times 10^8 \text{ J/gal})}{3.6 \times 10^3 \text{ s/h}}$$

$$= \frac{2.2 \times 10^8 \text{ J}}{3.6 \times 10^3 \text{ s}} = 62 \text{ kW}$$

Because 18% of the available power is used to propel the car, the power delivered to the wheels is (0.18)(62 kW) = 11 kW. This is 40% less than the 18.3-kW value obtained for the 1 450-kg car discussed in the text. Vehicle mass is clearly an important factor in power-loss mechanisms.

EXAMPLE 7.16 Car Accelerating Up a Hill

Consider a car of mass m that is accelerating up a hill, as shown in Figure 7.19. An automotive engineer has measured the magnitude of the total resistive force to be

$$f_t = (218 + 0.70v^2) \text{ N}$$

where v is the speed in meters per second. Determine the power the engine must deliver to the wheels as a function of speed.

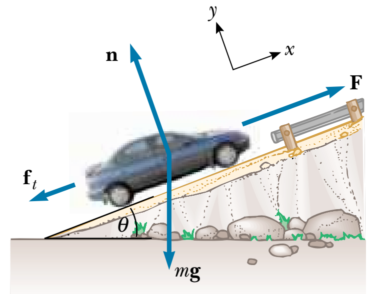


Figure 7.19

Solution The forces on the car are shown in Figure 7.19, in which **F** is the force of friction from the road that propels the car; the remaining forces have their usual meaning. Applying Newton's second law to the motion along the road surface, we find that

$$\sum F_x = F - f_t - mg\sin\theta = ma$$

$$F = ma + mg\sin\theta + f_t$$

$$= ma + mg\sin\theta + (218 + 0.70v^2)$$

Therefore, the power required to move the car forward is

$$\mathcal{P} = Fv = mva + mvg\sin\theta + 218v + 0.70v^3$$

The term mva represents the power that the engine must deliver to accelerate the car. If the car moves at constant speed, this term is zero and the total power requirement is reduced. The term $mvg \sin\theta$ is the power required to provide a force to balance a component of the force of gravity as the car moves up the incline. This term would be zero for motion on a horizontal surface. The term $218v$ is the power required to provide a force to balance road friction, and the term $0.70v^3$ is the power needed to do work on the air.

If we take $m = 1\,450$ kg, $v = 27$ m/s $(= 60$ mi/h$)$, $a = 1.0$ m/s^2, and $\theta = 10°$, then the various terms in $\mathcal{P}$ are calculated to be

$$mva = (1\,450 \text{ kg})(27 \text{ m/s})(1.0 \text{ m/s}^2) = 39 \text{ kW} = 52 \text{ hp}$$

$$mvg\sin\theta = (1\,450 \text{ kg})(27 \text{ m/s})(9.80 \text{ m/s}^2)(\sin 10°) = 67 \text{ kW} = 89 \text{ hp}$$

$$218v = 218(27 \text{ m/s}) = 5.9 \text{ kW} = 7.9 \text{ hp}$$

$$0.70v^3 = 0.70(27 \text{ m/s})^3 = 14 \text{ kW} = 19 \text{ hp}$$

Hence, the total power required is 126 kW, or 168 hp.

Note that the power requirements for traveling at constant speed on a horizontal surface are only 20 kW, or 27 hp (the sum of the last two terms). Furthermore, if the mass were halved (as in the case of a compact car), then the power required also is reduced by almost the same factor.

Optional Section

7.7 KINETIC ENERGY AT HIGH SPEEDS

The laws of Newtonian mechanics are valid only for describing the motion of particles moving at speeds that are small compared with the speed of light in a vacuum c $(= 3.00 \times 10^8$ m/s$)$. When speeds are comparable to c, the equations of Newtonian mechanics must be replaced by the more general equations predicted by the theory of relativity. One consequence of the theory of relativity is that the kinetic energy of a particle of mass m moving with a speed v is no longer given by $K = mv^2/2$. Instead, one must use the relativistic form of the kinetic energy:

Relativistic kinetic energy

$$K = mc^2\left(\frac{1}{\sqrt{1 - (v/c)^2}} - 1\right) \quad \textbf{(7.19)}$$

According to this expression, speeds greater than c are not allowed because, as v approaches c, K approaches ∞. This limitation is consistent with experimental observations on subatomic particles, which have shown that no particles travel at speeds greater than c. (In other words, c is the ultimate speed.) From this relativistic point of view, the work–kinetic energy theorem says that v can only approach c because it would take an infinite amount of work to attain the speed $v = c$.

All formulas in the theory of relativity must reduce to those in Newtonian mechanics at low particle speeds. It is instructive to show that this is the case for the kinetic energy relationship by analyzing Equation 7.19 when v is small compared with c. In this case, we expect K to reduce to the Newtonian expression. We can check this by using the binomial expansion (Appendix B.5) applied to the quantity $[1 - (v/c)^2]^{-1/2}$, with $v/c \ll 1$. If we let $x = (v/c)^2$, the expansion gives

$$\frac{1}{(1-x)^{1/2}} = 1 + \frac{x}{2} + \frac{3}{8}x^2 + \cdots$$

Making use of this expansion in Equation 7.19 gives

$$K = mc^2\left(1 + \frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots - 1\right)$$

$$= \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \cdots$$

$$= \frac{1}{2}mv^2 \qquad \text{for} \qquad \frac{v}{c} \ll 1$$

Thus, we see that the relativistic kinetic energy expression does indeed reduce to the Newtonian expression for speeds that are small compared with c. We shall return to the subject of relativity in Chapter 39.

SUMMARY

The work done by a constant force **F** acting on a particle is defined as the product of the component of the force in the direction of the particle's displacement and the magnitude of the displacement. Given a force **F** that makes an angle θ with the displacement vector **d** of a particle acted on by the force, you should be able to determine the work done by **F** using the equation

$$W \equiv Fd\cos\theta \tag{7.1}$$

The **scalar product** (dot product) of two vectors **A** and **B** is defined by the relationship

$$\mathbf{A}\cdot\mathbf{B} \equiv AB\cos\theta \tag{7.3}$$

where the result is a scalar quantity and θ is the angle between the two vectors. The scalar product obeys the commutative and distributive laws.

If a varying force does work on a particle as the particle moves along the x axis from x_i to x_f, you must use the expression

$$W \equiv \int_{x_i}^{x_f} F_x\,dx \tag{7.7}$$

where F_x is the component of force in the x direction. If several forces are acting on the particle, the net work done by all of the forces is the sum of the amounts of work done by all of the forces.

The **kinetic energy** of a particle of mass m moving with a speed v (where v is small compared with the speed of light) is

$$K \equiv \frac{1}{2}mv^2 \tag{7.14}$$

The **work–kinetic energy theorem** states that the net work done on a particle by external forces equals the change in kinetic energy of the particle:

$$\sum W = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \tag{7.16}$$

If a frictional force acts, then the work–kinetic energy theorem can be modified to give

$$K_i + \sum W_{\text{other}} - f_k d = K_f \tag{7.17b}$$

The **instantaneous power** $\mathcal{P}$ is defined as the time rate of energy transfer. If an agent applies a force **F** to an object moving with a velocity **v**, the power delivered by that agent is

$$\mathcal{P} \equiv \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v} \tag{7.18}$$

Questions

1. Consider a tug-of-war in which two teams pulling on a rope are evenly matched so that no motion takes place. Assume that the rope does not stretch. Is work done on the rope? On the pullers? On the ground? Is work done on anything?
2. For what values of θ is the scalar product (a) positive and (b) negative?
3. As the load on a spring hung vertically is increased, one would not expect the F_s-versus-x curve to always remain linear, as shown in Figure 7.10d. Explain qualitatively what you would expect for this curve as m is increased.
4. Can the kinetic energy of an object be negative? Explain.
5. (a) If the speed of a particle is doubled, what happens to its kinetic energy? (b) If the net work done on a particle is zero, what can be said about the speed?
6. In Example 7.16, does the required power increase or decrease as the force of friction is reduced?
7. An automobile sales representative claims that a "souped-up" 300-hp engine is a necessary option in a compact car (instead of a conventional 130-hp engine). Suppose you intend to drive the car within speed limits ($\leq$ 55 mi/h) and on flat terrain. How would you counter this sales pitch?
8. One bullet has twice the mass of another bullet. If both bullets are fired so that they have the same speed, which has the greater kinetic energy? What is the ratio of the kinetic energies of the two bullets?
9. When a punter kicks a football, is he doing any work on the ball while his toe is in contact with it? Is he doing any work on the ball after it loses contact with his toe? Are any forces doing work on the ball while it is in flight?
10. Discuss the work done by a pitcher throwing a baseball. What is the approximate distance through which the force acts as the ball is thrown?
11. Two sharpshooters fire 0.30-caliber rifles using identical shells. The barrel of rifle A is 2.00 cm longer than that of rifle B. Which rifle will have the higher muzzle speed? (*Hint:* The force of the expanding gases in the barrel accelerates the bullets.)
12. As a simple pendulum swings back and forth, the forces acting on the suspended mass are the force of gravity, the tension in the supporting cord, and air resistance. (a) Which of these forces, if any, does no work on the pendulum? (b) Which of these forces does negative work at all times during its motion? (c) Describe the work done by the force of gravity while the pendulum is swinging.
13. The kinetic energy of an object depends on the frame of reference in which its motion is measured. Give an example to illustrate this point.
14. An older model car accelerates from 0 to a speed v in 10 s. A newer, more powerful sports car accelerates from 0 to $2v$ in the same time period. What is the ratio of powers expended by the two cars? Consider the energy coming from the engines to appear only as kinetic energy of the cars.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
= paired numerical/symbolic problems

Section 7.1 Work Done by a Constant Force

1. A tugboat exerts a constant force of 5 000 N on a ship moving at constant speed through a harbor. How much work does the tugboat do on the ship in a distance of 3.00 km?

2. A shopper in a supermarket pushes a cart with a force of 35.0 N directed at an angle of 25.0° downward from the horizontal. Find the work done by the shopper as she moves down an aisle 50.0 m in length.

3. A raindrop ($m = 3.35 \times 10^{-5}$ kg) falls vertically at constant speed under the influence of gravity and air resistance. After the drop has fallen 100 m, what is the work done (a) by gravity and (b) by air resistance?

4. A sledge loaded with bricks has a total mass of 18.0 kg and is pulled at constant speed by a rope. The rope is inclined at 20.0° above the horizontal, and the sledge moves a distance of 20.0 m on a horizontal surface. The coefficient of kinetic friction between the sledge and the surface is 0.500. (a) What is the tension of the rope? (b) How much work is done on the sledge by the rope? (c) What is the energy lost due to friction?

5. A block of mass 2.50 kg is pushed 2.20 m along a frictionless horizontal table by a constant 16.0-N force directed 25.0° below the horizontal. Determine the work done by (a) the applied force, (b) the normal force exerted by the table, and (c) the force of gravity. (d) Determine the total work done on the block.

6. A 15.0-kg block is dragged over a rough, horizontal surface by a 70.0-N force acting at 20.0° above the horizontal. The block is displaced 5.00 m, and the coefficient of kinetic friction is 0.300. Find the work done by (a) the 70-N force, (b) the normal force, and (c) the force of gravity. (d) What is the energy loss due to friction? (e) Find the total change in the block's kinetic energy.

WEB **7.** Batman, whose mass is 80.0 kg, is holding onto the free end of a 12.0-m rope, the other end of which is fixed to a tree limb above. He is able to get the rope in motion as only Batman knows how, eventually getting it to swing enough so that he can reach a ledge when the rope makes a 60.0° angle with the vertical. How much work was done against the force of gravity in this maneuver?

Section 7.2 The Scalar Product of Two Vectors

In Problems 8 to 14, calculate all numerical answers to three significant figures.

8. Vector **A** has a magnitude of 5.00 units, and vector **B** has a magnitude of 9.00 units. The two vectors make an angle of 50.0° with each other. Find **A · B.**

9. Vector **A** extends from the origin to a point having polar coordinates (7, 70°), and vector **B** extends from the origin to a point having polar coordinates (4, 130°). Find **A · B.**

10. Given two arbitrary vectors **A** and **B**, show that **A·B** = $A_xB_x + A_yB_y + A_zB_z$. (*Hint:* Write **A** and **B** in unit vector form and use Equations 7.4 and 7.5.)

WEB **11.** A force **F** = (6**i** − 2**j**) N acts on a particle that undergoes a displacement **d** = (3**i** + **j**)m. Find (a) the work done by the force on the particle and (b) the angle between **F** and **d**.

12. For **A** = 3**i** + **j** − **k**, **B** = −**i** + 2**j** + 5**k**, and **C** = 2**j** − 3**k**, find **C·**(**A** − **B**).

13. Using the definition of the scalar product, find the angles between (a) **A** = 3**i** − 2**j** and **B** = 4**i** − 4**j**; (b) **A** = − 2**i** + 4**j** and **B** = 3**i** − 4**j** + 2**k**; (c) **A** = **i** − 2**j** + 2**k** and **B** = 3**j** + 4**k**.

14. Find the scalar product of the vectors in Figure P7.14.

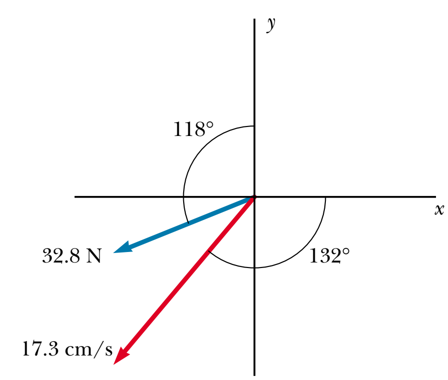


Figure P7.14

Section 7.3 Work Done by a Varying Force

15. The force acting on a particle varies as shown in Figure P7.15. Find the work done by the force as the particle moves (a) from $x = 0$ to $x = 8.00$ m, (b) from $x = 8.00$ m to $x = 10.0$ m, and (c) from $x = 0$ to $x = 10.0$ m.

16. The force acting on a particle is $F_x = (8x - 16)$ N, where x is in meters. (a) Make a plot of this force versus x from $x = 0$ to $x = 3.00$ m. (b) From your graph, find the net work done by this force as the particle moves from $x = 0$ to $x = 3.00$ m.

WEB **17.** A particle is subject to a force F_x that varies with position as in Figure P7.17. Find the work done by the force on the body as it moves (a) from $x = 0$ to $x = 5.00$ m,

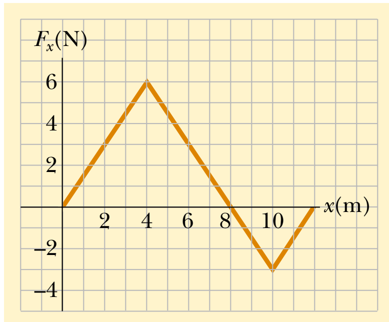


Figure P7.15

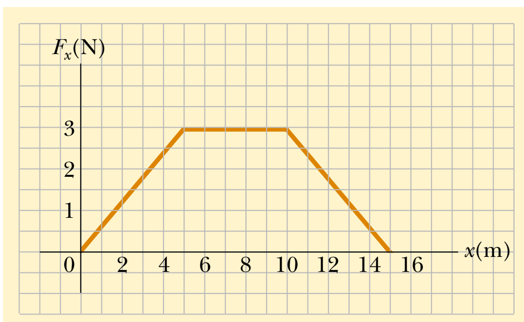


Figure P7.17 Problems 17 and 32.

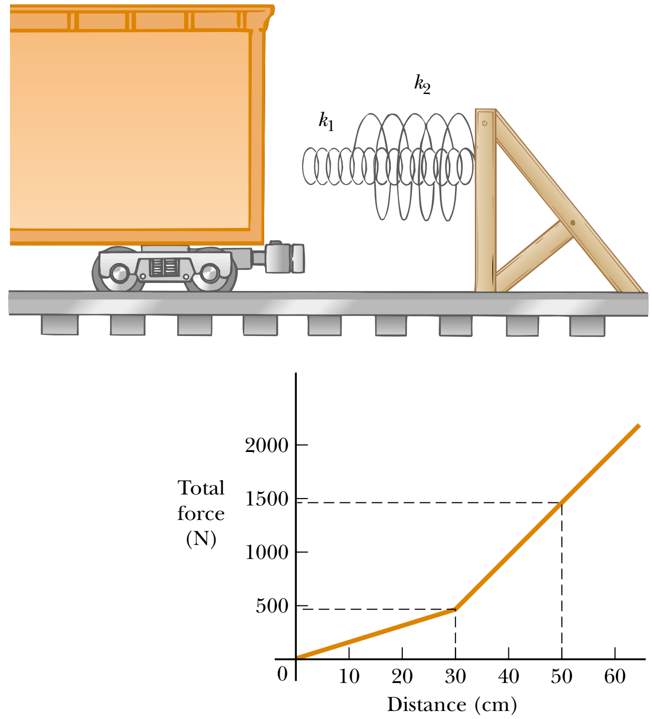


Figure P7.21

(b) from $x = 5.00$ m to $x = 10.0$ m, and (c) from $x = 10.0$ m to $x = 15.0$ m. (d) What is the total work done by the force over the distance $x = 0$ to $x = 15.0$ m?

18. A force $\mathbf{F} = (4x\mathbf{i} + 3y\mathbf{j})$ N acts on an object as it moves in the x direction from the origin to $x = 5.00$ m. Find the work $W = \int \mathbf{F} \cdot d\mathbf{r}$ done on the object by the force.

19. When a 4.00-kg mass is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.50 cm. If the 4.00-kg mass is removed, (a) how far will the spring stretch if a 1.50-kg mass is hung on it and (b) how much work must an external agent do to stretch the same spring 4.00 cm from its unstretched position?

20. An archer pulls her bow string back 0.400 m by exerting a force that increases uniformly from zero to 230 N. (a) What is the equivalent spring constant of the bow? (b) How much work is done by the archer in pulling the bow?

21. A 6 000-kg freight car rolls along rails with negligible friction. The car is brought to rest by a combination of two coiled springs, as illustrated in Figure P7.21. Both springs obey Hooke's law with $k_1 = 1\,600$ N/m and $k_2 = 3\,400$ N/m. After the first spring compresses a distance of 30.0 cm, the second spring (acting with the first) increases the force so that additional compression occurs, as shown in the graph. If the car is brought to rest 50.0 cm after first contacting the two-spring system, find the car's initial speed.

22. A 100-g bullet is fired from a rifle having a barrel 0.600 m long. Assuming the origin is placed where the bullet begins to move, the force (in newtons) exerted on the bullet by the expanding gas is $15\,000 + 10\,000x - 25\,000x^2$, where x is in meters. (a) Determine the work done by the gas on the bullet as the bullet travels the length of the barrel. (b) If the barrel is 1.00 m long, how much work is done and how does this value compare with the work calculated in part (a)?

23. If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm.

24. If it takes work W to stretch a Hooke's-law spring a distance d from its unstressed length, determine the extra work required to stretch it an additional distance d.

25. A small mass m is pulled to the top of a frictionless half-cylinder (of radius R) by a cord that passes over the top of the cylinder, as illustrated in Figure P7.25. (a) If the mass moves at a constant speed, show that $F = mg\cos\theta$. (*Hint:* If the mass moves at a constant speed, the component of its acceleration tangent to the cylinder must be zero at all times.) (b) By directly integrating $W = \int \mathbf{F} \cdot d\mathbf{s}$, find the work done in moving the mass at constant speed from the bottom to the top of the half-

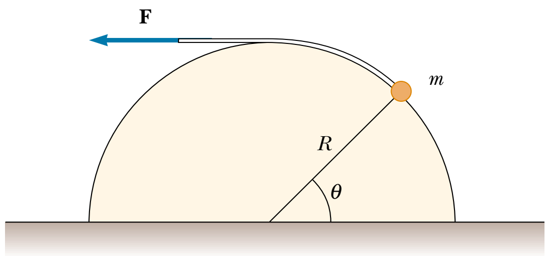


Figure P7.25

cylinder. Here $d\mathbf{s}$ represents an incremental displacement of the small mass.

26. Express the unit of the force constant of a spring in terms of the basic units meter, kilogram, and second.

Section 7.4 Kinetic Energy and the Work–Kinetic Energy Theorem

27. A 0.600-kg particle has a speed of 2.00 m/s at point A and kinetic energy of 7.50 J at point B. What is (a) its kinetic energy at A? (b) its speed at B? (c) the total work done on the particle as it moves from A to B?

28. A 0.300-kg ball has a speed of 15.0 m/s. (a) What is its kinetic energy? (b) If its speed were doubled, what would be its kinetic energy?

29. A 3.00-kg mass has an initial velocity $\mathbf{v}_i = (6.00\mathbf{i} - 2.00\mathbf{j})$ m/s. (a) What is its kinetic energy at this time? (b) Find the total work done on the object if its velocity changes to $(8.00\mathbf{i} + 4.00\mathbf{j})$ m/s. (*Hint:* Remember that $v^2 = \mathbf{v}\cdot\mathbf{v}$.)

30. A mechanic pushes a 2 500-kg car, moving it from rest and making it accelerate from rest to a speed v. He does 5 000 J of work in the process. During this time, the car moves 25.0 m. If friction between the car and the road is negligible, (a) what is the final speed v of the car? (b) What constant horizontal force did he exert on the car?

31. A mechanic pushes a car of mass m, doing work W in making it accelerate from rest. If friction between the car and the road is negligible, (a) what is the final speed of the car? During the time the mechanic pushes the car, the car moves a distance d. (b) What constant horizontal force did the mechanic exert on the car?

32. A 4.00-kg particle is subject to a total force that varies with position, as shown in Figure P7.17. The particle starts from rest at $x = 0$. What is its speed at (a) $x = 5.00$ m, (b) $x = 10.0$ m, (c) $x = 15.0$ m?

33. A 40.0-kg box initially at rest is pushed 5.00 m along a rough, horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between the box and the floor is 0.300, find (a) the work done by the applied force, (b) the energy loss due to friction, (c) the work done by the normal force, (d) the work done by gravity, (e) the change in kinetic energy of the box, and (f) the final speed of the box.

34. You can think of the work–kinetic energy theorem as a second theory of motion, parallel to Newton's laws in describing how outside influences affect the motion of an object. In this problem, work out parts (a) and (b) separately from parts (c) and (d) to compare the predictions of the two theories. In a rifle barrel, a 15.0-g bullet is accelerated from rest to a speed of 780 m/s. (a) Find the work that is done on the bullet. (b) If the rifle barrel is 72.0 cm long, find the magnitude of the average total force that acted on it, as $F = W/(d \cos \theta)$. (c) Find the constant acceleration of a bullet that starts from rest and gains a speed of 780 m/s over a distance of 72.0 cm. (d) Find the total force that acted on it as $\Sigma F = ma$.

35. A crate of mass 10.0 kg is pulled up a rough incline with an initial speed of 1.50 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20.0° with the horizontal. The coefficient of kinetic friction is 0.400, and the crate is pulled 5.00 m. (a) How much work is done by gravity? (b) How much energy is lost because of friction? (c) How much work is done by the 100-N force? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after it has been pulled 5.00 m?

36. A block of mass 12.0 kg slides from rest down a frictionless 35.0° incline and is stopped by a strong spring with $k = 3.00 \times 10^4$ N/m. The block slides 3.00 m from the point of release to the point where it comes to rest against the spring. When the block comes to rest, how far has the spring been compressed?

WEB 37. A sled of mass m is given a kick on a frozen pond. The kick imparts to it an initial speed $v_i = 2.00$ m/s. The coefficient of kinetic friction between the sled and the ice is $\mu_k = 0.100$. Utilizing energy considerations, find the distance the sled moves before it stops.

38. A picture tube in a certain television set is 36.0 cm long. The electrical force accelerates an electron in the tube from rest to 1.00% of the speed of light over this distance. Determine (a) the kinetic energy of the electron as it strikes the screen at the end of the tube, (b) the magnitude of the average electrical force acting on the electron over this distance, (c) the magnitude of the average acceleration of the electron over this distance, and (d) the time of flight.

39. A bullet with a mass of 5.00 g and a speed of 600 m/s penetrates a tree to a depth of 4.00 cm. (a) Use work and energy considerations to find the average frictional force that stops the bullet. (b) Assuming that the frictional force is constant, determine how much time elapsed between the moment the bullet entered the tree and the moment it stopped.

40. An Atwood's machine (see Fig. 5.15) supports masses of 0.200 kg and 0.300 kg. The masses are held at rest beside each other and then released. Neglecting friction, what is the speed of each mass the instant it has moved 0.400 m?

41. A 2.00-kg block is attached to a spring of force constant 500 N/m, as shown in Figure 7.10. The block is pulled 5.00 cm to the right of equilibrium and is then released from rest. Find the speed of the block as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between the block and the surface is 0.350.

Section 7.5 Power

42. Make an order-of-magnitude estimate of the power a car engine contributes to speeding up the car to highway speed. For concreteness, consider your own car (if you use one). In your solution, state the physical quantities you take as data and the values you measure or estimate for them. The mass of the vehicle is given in the owner's manual. If you do not wish to consider a car, think about a bus or truck for which you specify the necessary physical quantities.

WEB 43. A 700-N Marine in basic training climbs a 10.0-m vertical rope at a constant speed in 8.00 s. What is his power output?

44. If a certain horse can maintain 1.00 hp of output for 2.00 h, how many 70.0-kg bundles of shingles can the horse hoist (using some pulley arrangement) to the roof of a house 8.00 m tall, assuming 70.0% efficiency?

45. A certain automobile engine delivers 2.24×10^4 W (30.0 hp) to its wheels when moving at a constant speed of 27.0 m/s ($\approx$ 60 mi/h). What is the resistive force acting on the automobile at that speed?

46. A skier of mass 70.0 kg is pulled up a slope by a motor-driven cable. (a) How much work is required for him to be pulled a distance of 60.0 m up a 30.0° slope (assumed frictionless) at a constant speed of 2.00 m/s? (b) A motor of what power is required to perform this task?

47. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s. (a) What is the average power of the elevator motor during this period? (b) How does this power compare with its power when it moves at its cruising speed?

48. An energy-efficient lightbulb, taking in 28.0 W of power, can produce the same level of brightness as a conventional bulb operating at 100-W power. The lifetime of the energy-efficient bulb is 10 000 h and its purchase price is \$17.0, whereas the conventional bulb has a lifetime of 750 h and costs \$0.420 per bulb. Determine the total savings obtained through the use of one energy-efficient bulb over its lifetime as opposed to the use of conventional bulbs over the same time period. Assume an energy cost of \$0.080 0 per kilowatt hour.

(Optional)

Section 7.6 Energy and the Automobile

49. A compact car of mass 900 kg has an overall motor efficiency of 15.0%. (That is, 15.0% of the energy supplied by the fuel is delivered to the wheels of the car.) (a) If burning 1 gal of gasoline supplies 1.34×10^8 J of energy, find the amount of gasoline used by the car in accelerating from rest to 55.0 mi/h. Here you may ignore the effects of air resistance and rolling resistance. (b) How many such accelerations will 1 gal provide? (c) The mileage claimed for the car is 38.0 mi/gal at 55 mi/h. What power is delivered to the wheels (to overcome frictional effects) when the car is driven at this speed?

50. Suppose the empty car described in Table 7.2 has a fuel economy of 6.40 km/L (15 mi/gal) when traveling at 26.8 m/s (60 mi/h). Assuming constant efficiency, determine the fuel economy of the car if the total mass of the passengers and the driver is 350 kg.

51. When an air conditioner is added to the car described in Problem 50, the additional output power required to operate the air conditioner is 1.54 kW. If the fuel economy of the car is 6.40 km/L without the air conditioner, what is it when the air conditioner is operating?

(Optional)

Section 7.7 Kinetic Energy at High Speeds

52. An electron moves with a speed of $0.995c$. (a) What is its kinetic energy? (b) If you use the classical expression to calculate its kinetic energy, what percentage error results?

53. A proton in a high-energy accelerator moves with a speed of $c/2$. Using the work–kinetic energy theorem, find the work required to increase its speed to (a) $0.750c$ and (b) $0.995c$.

54. Find the kinetic energy of a 78.0-kg spacecraft launched out of the Solar System with a speed of 106 km/s using (a) the classical equation $K = \frac{1}{2}mv^2$ and (b) the relativistic equation.

ADDITIONAL PROBLEMS

55. A baseball outfielder throws a 0.150-kg baseball at a speed of 40.0 m/s and an initial angle of 30.0°. What is the kinetic energy of the baseball at the highest point of the trajectory?

56. While running, a person dissipates about 0.600 J of mechanical energy per step per kilogram of body mass. If a 60.0-kg runner dissipates a power of 70.0 W during a race, how fast is the person running? Assume a running step is 1.50 m in length.

57. A particle of mass m moves with a constant acceleration $\mathbf{a}$. If the initial position vector and velocity of the particle are $\mathbf{r}_i$ and $\mathbf{v}_i$, respectively, use energy arguments to show that its speed v_f at any time satisfies the equation

$$v_f{}^2 = v_i{}^2 + 2\mathbf{a} \cdot (\mathbf{r}_f - \mathbf{r}_i)$$

where $\mathbf{r}_f$ is the position vector of the particle at that same time.

58. The direction of an arbitrary vector $\mathbf{A}$ can be completely specified with the angles α, β, and γ that the vec-

tor makes with the x, y, and z axes, respectively. If $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$, (a) find expressions for $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ (known as *direction cosines*) and (b) show that these angles satisfy the relation $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. (*Hint:* Take the scalar product of **A** with **i**, **j**, and **k** separately.)

59. A 4.00-kg particle moves along the x axis. Its position varies with time according to $x = t + 2.0t^3$, where x is in meters and t is in seconds. Find (a) the kinetic energy at any time t, (b) the acceleration of the particle and the force acting on it at time t, (c) the power being delivered to the particle at time t, and (d) the work done on the particle in the interval $t = 0$ to $t = 2.00$ s.

60. A traveler at an airport takes an escalator up one floor (Fig. P7.60). The moving staircase would itself carry him upward with vertical velocity component v between entry and exit points separated by height h. However, while the escalator is moving, the hurried traveler climbs the steps of the escalator at a rate of n steps/s. Assume that the height of each step is h_s. (a) Determine the amount of work done by the traveler during his escalator ride, given that his mass is m. (b) Determine the work the escalator motor does on this person.

Figure P7.60 (©Ron Chapple/FPG)

61. When a certain spring is stretched beyond its proportional limit, the restoring force satisfies the equation $F = -kx + \beta x^3$. If $k = 10.0$ N/m and $\beta = 100$ N/m^3, calculate the work done by this force when the spring is stretched 0.100 m.

62. In a control system, an accelerometer consists of a 4.70-g mass sliding on a low-friction horizontal rail. A low-mass spring attaches the mass to a flange at one end of the rail. When subject to a steady acceleration of $0.800g$, the mass is to assume a location 0.500 cm away from its equilibrium position. Find the stiffness constant required for the spring.

63. A 2 100-kg pile driver is used to drive a steel I-beam into the ground. The pile driver falls 5.00 m before coming into contact with the beam, and it drives the beam 12.0 cm into the ground before coming to rest. Using energy considerations, calculate the average force the beam exerts on the pile driver while the pile driver is brought to rest.

64. A cyclist and her bicycle have a combined mass of 75.0 kg. She coasts down a road inclined at 2.00° with the horizontal at 4.00 m/s and down a road inclined at 4.00° at 8.00 m/s. She then holds on to a moving vehicle and coasts on a level road. What power must the vehicle expend to maintain her speed at 3.00 m/s? Assume that the force of air resistance is proportional to her speed and that other frictional forces remain constant. (*Warning:* You must *not* attempt this dangerous maneuver.)

65. A single constant force **F** acts on a particle of mass m. The particle starts at rest at $t = 0$. (a) Show that the instantaneous power delivered by the force at any time t is $(F^2/m)t$. (b) If $F = 20.0$ N and $m = 5.00$ kg, what is the power delivered at $t = 3.00$ s?

66. A particle is attached between two identical springs on a horizontal frictionless table. Both springs have spring constant k and are initially unstressed. (a) If the particle is pulled a distance x along a direction perpendicular to the initial configuration of the springs, as in Figure P7.66, show that the force exerted on the particle by the springs is

$$\mathbf{F} = -2kx\left(1 - \frac{L}{\sqrt{x^2 + L^2}}\right)\mathbf{i}$$

(b) Determine the amount of work done by this force in moving the particle from $x = A$ to $x = 0$.

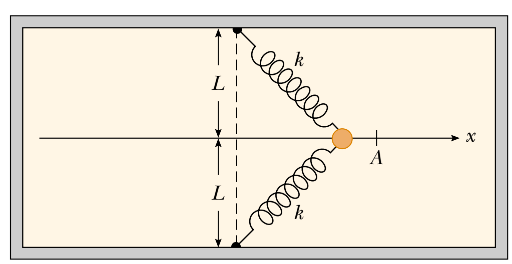


Figure P7.66

67. **Review Problem.** Two constant forces act on a 5.00-kg object moving in the xy plane, as shown in Figure P7.67. Force $\mathbf{F}_1$ is 25.0 N at 35.0°, while $\mathbf{F}_2 = 42.0$ N at 150°. At time $t = 0$, the object is at the origin and has velocity $(4.0\mathbf{i} + 2.5\mathbf{j})$ m/s. (a) Express the two forces in unit–vector notation. Use unit–vector notation for your other answers. (b) Find the total force on the object. (c) Find the object's acceleration. Now, considering the instant $t = 3.00$ s, (d) find the object's velocity, (e) its location, (f) its kinetic energy from $\frac{1}{2}mv_f^{\,2}$, and (g) its kinetic energy from $\frac{1}{2}mv_i^{\,2} + \Sigma\mathbf{F}\cdot\mathbf{d}$.

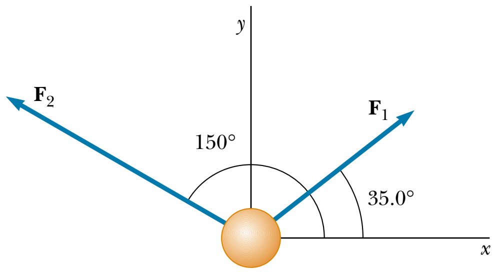


Figure P7.67

68. When different weights are hung on a spring, the spring stretches to different lengths as shown in the following table. (a) Make a graph of the applied force versus the extension of the spring. By least-squares fitting, determine the straight line that best fits the data. (You may not want to use all the data points.) (b) From the slope of the best-fit line, find the spring constant k. (c) If the spring is extended to 105 mm, what force does it exert on the suspended weight?

F (N)	2.0	4.0	6.0	8.0	10	12	14	16	18
L (mm)	15	32	49	64	79	98	112	126	149

69. A 200-g block is pressed against a spring of force constant 1.40 kN/m until the block compresses the spring 10.0 cm. The spring rests at the bottom of a ramp inclined at 60.0° to the horizontal. Using energy considerations, determine how far up the incline the block moves before it stops (a) if there is no friction between the block and the ramp and (b) if the coefficient of kinetic friction is 0.400.

70. A 0.400-kg particle slides around a horizontal track. The track has a smooth, vertical outer wall forming a circle with a radius of 1.50 m. The particle is given an initial speed of 8.00 m/s. After one revolution, its speed has dropped to 6.00 m/s because of friction with the rough floor of the track. (a) Find the energy loss due to friction in one revolution. (b) Calculate the coefficient of kinetic friction. (c) What is the total number of revolutions the particle makes before stopping?

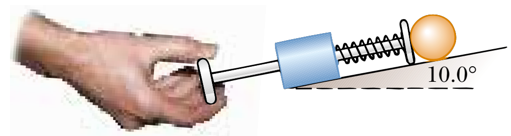


Figure P7.71

WEB **71.** The ball launcher in a pinball machine has a spring that has a force constant of 1.20 N/cm (Fig. P7.71). The surface on which the ball moves is inclined 10.0° with respect to the horizontal. If the spring is initially compressed 5.00 cm, find the launching speed of a 100-g ball when the plunger is released. Friction and the mass of the plunger are negligible.

72. In diatomic molecules, the constituent atoms exert attractive forces on each other at great distances and repulsive forces at short distances. For many molecules, the Lennard–Jones law is a good approximation to the magnitude of these forces:

$$F = F_0\left[2\left(\frac{\sigma}{r}\right)^{13} - \left(\frac{\sigma}{r}\right)^{7}\right]$$

where r is the center-to-center distance between the atoms in the molecule, σ is a length parameter, and F_0 is the force when $r = \sigma$. For an oxygen molecule, $F_0 = 9.60 \times 10^{-11}$ N and $\sigma = 3.50 \times 10^{-10}$ m. Determine the work done by this force if the atoms are pulled apart from $r = 4.00 \times 10^{-10}$ m to $r = 9.00 \times 10^{-10}$ m.

73. A horizontal string is attached to a 0.250-kg mass lying on a rough, horizontal table. The string passes over a light, frictionless pulley, and a 0.400-kg mass is then attached to its free end. The coefficient of sliding friction between the 0.250-kg mass and the table is 0.200. Using the work–kinetic energy theorem, determine (a) the speed of the masses after each has moved 20.0 m from rest and (b) the mass that must be added to the 0.250-kg mass so that, given an initial velocity, the masses continue to move at a constant speed. (c) What mass must be removed from the 0.400-kg mass so that the same outcome as in part (b) is achieved?

74. Suppose a car is modeled as a cylinder moving with a speed v, as in Figure P7.74. In a time Δt, a column of air

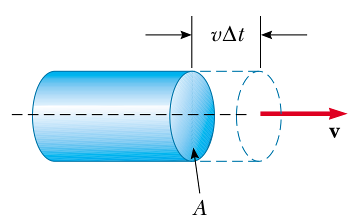


Figure P7.74

of mass Δm must be moved a distance $v\,\Delta t$ and, hence, must be given a kinetic energy $\frac{1}{2}(\Delta m)v^2$. Using this model, show that the power loss due to air resistance is $\frac{1}{2}\rho A v^3$ and that the resistive force is $\frac{1}{2}\rho A v^2$, where ρ is the density of air.

75. A particle moves along the x axis from $x = 12.8$ m to $x = 23.7$ m under the influence of a force

$$F = \frac{375}{x^3 + 3.75\,x}$$

where F is in newtons and x is in meters. Using numerical integration, determine the total work done by this force during this displacement. Your result should be accurate to within 2%.

76. More than 2 300 years ago the Greek teacher Aristotle wrote the first book called *Physics*. The following passage, rephrased with more precise terminology, is from the end of the book's Section Eta:

Let $\mathcal{P}$ be the power of an agent causing motion; w, the thing moved; d, the distance covered; and t, the time taken. Then (1) a power equal to $\mathcal{P}$ will in a period of time equal to t move $w/2$ a distance $2d$; or (2) it will move $w/2$ the given distance d in time $t/2$. Also, if (3) the given power $\mathcal{P}$ moves the given object w a distance $d/2$ in time $t/2$, then (4) $\mathcal{P}/2$ will move $w/2$ the given distance d in the given time t.

(a) Show that Aristotle's proportions are included in the equation $\mathcal{P}t = bwd$, where b is a proportionality constant. (b) Show that our theory of motion includes this part of Aristotle's theory as one special case. In particular, describe a situation in which it is true, derive the equation representing Aristotle's proportions, and determine the proportionality constant.

Answers to Quick Quizzes

7.1 No. The force does no work on the object because the force is pointed toward the center of the circle and is therefore perpendicular to the motion.

7.2 (a) Assuming the person lifts with a force of magnitude mg, the weight of the box, the work he does during the vertical displacement is mgh because the force is in the direction of the displacement. The work he does during the horizontal displacement is zero because now the force he exerts on the box is perpendicular to the displacement. The net work he does is $mgh + 0 = mgh$. (b) The work done by the gravitational force on the box as the box is displaced vertically is $-mgh$ because the direction of this force is opposite the direction of the displacement. The work done by the gravitational force is zero during the horizontal displacement because now the direction of this force is perpendicular to the direction of the displacement. The net work done by the gravitational force $-mgh + 0 = -mgh$. The total work done on the box is $+mgh - mgh = 0$.

7.3 No. For example, consider the two vectors $\mathbf{A} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{B} = 2\mathbf{i} - \mathbf{j}$. Their dot product is $\mathbf{A}\cdot\mathbf{B} = 8$, yet both vectors have negative y components.

7.4 Force divided by displacement, which in SI units is newtons per meter (N/m).

7.5 Yes, whenever the frictional force has a component along the direction of motion. Consider a crate sitting on the bed of a truck as the truck accelerates to the east. The static friction force exerted on the crate by the truck acts to the east to give the crate the same acceleration as the truck (assuming that the crate does not slip). Because the crate accelerates, its kinetic energy must increase.

7.6 Because the two vehicles perform the same amount of work, the areas under the two graphs are equal. However, the graph for the low-power truck extends over a longer time interval and does not extend as high on the $\mathcal{P}$ axis as the graph for the sports car does.