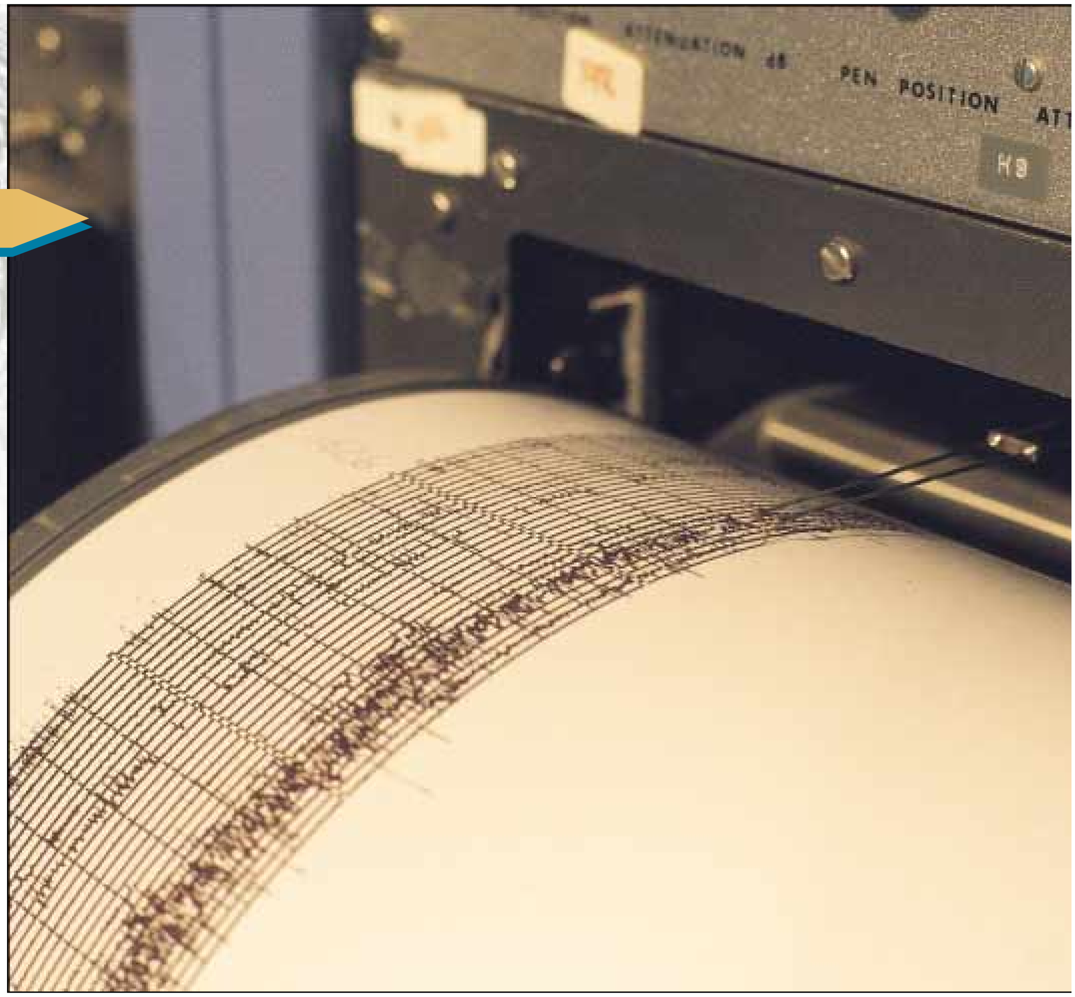
PUZZLER

A simple seismograph can be constructed with a spring-suspended pen that draws a line on a slowly unrolling strip of paper. The paper is mounted on a structure attached to the ground. During an earthquake, the pen remains nearly stationary while the paper shakes beneath it. How can a few jagged lines on a piece of paper allow scientists at a seismograph station to determine the distance to the origin of an earthquake? *(Ken M. Johns/Photo Researchers, Inc.)*



chapter

16 Wave Motion

Chapter Outline

Most of us experienced waves as children when we dropped a pebble into a pond. At the point where the pebble hits the water's surface, waves are created. These waves move outward from the creation point in expanding circles until they reach the shore. If you were to examine carefully the motion of a leaf floating on the disturbed water, you would see that the leaf moves up, down, and sideways about its original position but does not undergo any net displacement away from or toward the point where the pebble hit the water. The water molecules just beneath the leaf, as well as all the other water molecules on the pond's surface, behave in the same way. That is, the water *wave* moves from the point of origin to the shore, but the water is not carried with it.

An excerpt from a book by Einstein and Infeld gives the following remarks concerning wave phenomena:[1]

> A bit of gossip starting in Washington reaches New York [by word of mouth] very quickly, even though not a single individual who takes part in spreading it travels between these two cities. There are two quite different motions involved, that of the rumor, Washington to New York, and that of the persons who spread the rumor. The wind, passing over a field of grain, sets up a wave which spreads out across the whole field. Here again we must distinguish between the motion of the wave and the motion of the separate plants, which undergo only small oscillations... The particles constituting the medium perform only small vibrations, but the whole motion is that of a progressive wave. The essentially new thing here is that for the first time we consider the motion of something which is not matter, but energy propagated through matter.

The world is full of waves, the two main types being *mechanical* waves and *electromagnetic* waves. We have already mentioned examples of mechanical waves: sound waves, water waves, and "grain waves." In each case, some physical medium is being disturbed—in our three particular examples, air molecules, water molecules, and stalks of grain. Electromagnetic waves do not require a medium to propagate; some examples of electromagnetic waves are visible light, radio waves, television signals, and x-rays. Here, in Part 2 of this book, we study only mechanical waves.

The wave concept is abstract. When we observe what we call a water wave, what we see is a rearrangement of the water's surface. Without the water, there would be no wave. A wave traveling on a string would not exist without the string. Sound waves could not travel through air if there were no air molecules. With mechanical waves, what we interpret as a wave corresponds to the propagation of a disturbance through a medium.

Interference patterns produced by outward-spreading waves from many drops of liquid falling into a body of water.

[1] A. Einstein and L. Infeld, *The Evolution of Physics,* New York, Simon & Schuster, 1961. Excerpt from "What Is a Wave?"

The mechanical waves discussed in this chapter require (1) some source of disturbance, (2) a medium that can be disturbed, and (3) some physical connection through which adjacent portions of the medium can influence each other. We shall find that all waves carry energy. The amount of energy transmitted through a medium and the mechanism responsible for that transport of energy differ from case to case. For instance, the power of ocean waves during a storm is much greater than the power of sound waves generated by a single human voice.

16.1 BASIC VARIABLES OF WAVE MOTION

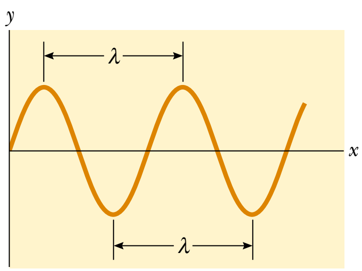


Figure 16.1 The wavelength λ of a wave is the distance between adjacent crests, adjacent troughs, or any other comparable adjacent identical points.

Imagine you are floating on a raft in a large lake. You slowly bob up and down as waves move past you. As you look out over the lake, you may be able to see the individual waves approaching. The point at which the displacement of the water from its normal level is highest is called the **crest** of the wave. The distance from one crest to the next is called the **wavelength** λ (Greek letter lambda). More generally, the wavelength is **the minimum distance between any two identical points (such as the crests) on adjacent waves,** as shown in Figure 16.1.

If you count the number of seconds between the arrivals of two adjacent waves, you are measuring the **period** T of the waves. In general, the period is **the time required for two identical points (such as the crests) of adjacent waves to pass by a point.**

The same information is more often given by the inverse of the period, which is called the **frequency** f. In general, the frequency of a periodic wave is **the number of crests (or troughs, or any other point on the wave) that pass a given point in a unit time interval.** The maximum displacement of a particle of the medium is called the **amplitude** A of the wave. For our water wave, this represents the highest distance of a water molecule above the undisturbed surface of the water as the wave passes by.

Waves travel with a specific speed, and this speed depends on the properties of the medium being disturbed. For instance, sound waves travel through room-temperature air with a speed of about 343 m/s (781 mi/h), whereas they travel through most solids with a speed greater than 343 m/s.

16.2 DIRECTION OF PARTICLE DISPLACEMENT

One way to demonstrate wave motion is to flick one end of a long rope that is under tension and has its opposite end fixed, as shown in Figure 16.2. In this manner, a single wave bump (called a *wave pulse*) is formed and travels along the rope with a definite speed. This type of disturbance is called a **traveling wave,** and Figure 16.2 represents four consecutive "snapshots" of the creation and propagation of the traveling wave. The rope is the medium through which the wave travels. Such a single pulse, in contrast to a train of pulses, has no frequency, no period, and no wavelength. However, the pulse does have definite amplitude and definite speed. As we shall see later, the properties of this particular medium that determine the speed of the wave are the tension in the rope and its mass per unit length. The shape of the wave pulse changes very little as it travels along the rope.[2]

As the wave pulse travels, each small segment of the rope, as it is disturbed, moves in a direction perpendicular to the wave motion. Figure 16.3 illustrates this

[2] Strictly speaking, the pulse changes shape and gradually spreads out during the motion. This effect is called *dispersion* and is common to many mechanical waves, as well as to electromagnetic waves. We do not consider dispersion in this chapter.

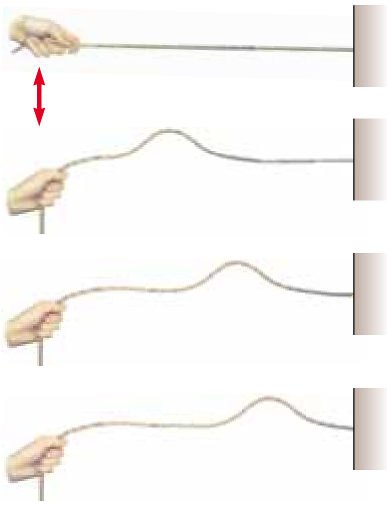


Figure 16.2 A wave pulse traveling down a stretched rope. The shape of the pulse is approximately unchanged as it travels along the rope.

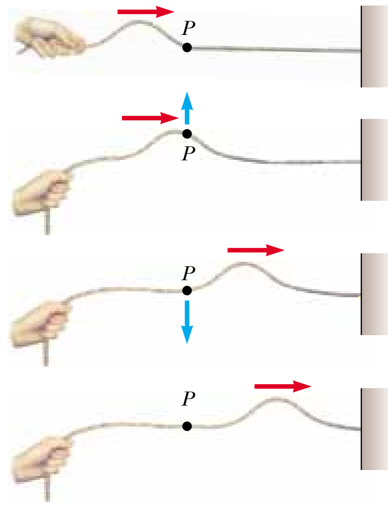


Figure 16.3 A pulse traveling on a stretched rope is a transverse wave. The direction of motion of any element *P* of the rope (blue arrows) is perpendicular to the direction of wave motion (red arrows).

point for one particular segment, labeled *P*. Note that no part of the rope ever moves in the direction of the wave.

A traveling wave that causes the particles of the disturbed medium to move perpendicular to the wave motion is called a **transverse wave.**

Transverse wave

Compare this with another type of wave—one moving down a long, stretched spring, as shown in Figure 16.4. The left end of the spring is pushed briefly to the right and then pulled briefly to the left. This movement creates a sudden compression of a region of the coils. The compressed region travels along the spring (to the right in Figure 16.4). The compressed region is followed by a region where the coils are extended. Notice that the direction of the displacement of the coils is *parallel* to the direction of propagation of the compressed region.

A traveling wave that causes the particles of the medium to move parallel to the direction of wave motion is called a **longitudinal wave.**

Longitudinal wave

Sound waves, which we shall discuss in Chapter 17, are another example of longitudinal waves. The disturbance in a sound wave is a series of high-pressure and low-pressure regions that travel through air or any other material medium.

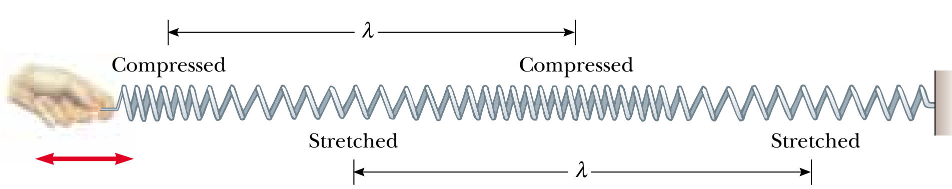


Figure 16.4 A longitudinal wave along a stretched spring. The displacement of the coils is in the direction of the wave motion. Each compressed region is followed by a stretched region.

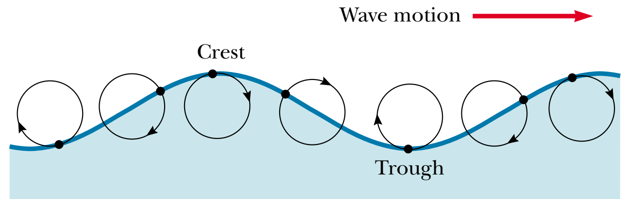


Figure 16.5 The motion of water molecules on the surface of deep water in which a wave is propagating is a combination of transverse and longitudinal displacements, with the result that molecules at the surface move in nearly circular paths. Each molecule is displaced both horizontally and vertically from its equilibrium position.

QuickLab

Make a "telephone" by poking a small hole in the bottom of two paper cups, threading a string through the holes, and tying knots in the ends of the string. If you speak into one cup while pulling the string taut, a friend can hear your voice in the other cup. What kind of wave is present in the string?

Some waves in nature exhibit a combination of transverse and longitudinal displacements. Surface water waves are a good example. When a water wave travels on the surface of deep water, water molecules at the surface move in nearly circular paths, as shown in Figure 16.5. Note that the disturbance has both transverse and longitudinal components. The transverse displacement is seen in Figure 16.5 as the variations in vertical position of the water molecules. The longitudinal displacement can be explained as follows: As the wave passes over the water's surface, water molecules at the crests move in the direction of propagation of the wave, whereas molecules at the troughs move in the direction opposite the propagation. Because the molecule at the labeled crest in Figure 16.5 will be at a trough after half a period, its movement in the direction of the propagation of the wave will be canceled by its movement in the opposite direction. This holds for every other water molecule disturbed by the wave. Thus, there is no net displacement of any water molecule during one complete cycle. Although the *molecules* experience no net displacement, the *wave* propagates along the surface of the water.

The three-dimensional waves that travel out from the point under the Earth's surface at which an earthquake occurs are of both types—transverse and longitudinal. The longitudinal waves are the faster of the two, traveling at speeds in the range of 7 to 8 km/s near the surface. These are called **P waves,** with "P" standing for *primary* because they travel faster than the transverse waves and arrive at a seismograph first. The slower transverse waves, called **S waves** (with "S" standing for *secondary*), travel through the Earth at 4 to 5 km/s near the surface. By recording the time interval between the arrival of these two sets of waves at a seismograph, the distance from the seismograph to the point of origin of the waves can be determined. A single such measurement establishes an imaginary sphere centered on the seismograph, with the radius of the sphere determined by the difference in arrival times of the P and S waves. The origin of the waves is located somewhere on that sphere. The imaginary spheres from three or more monitoring stations located far apart from each other intersect at one region of the Earth, and this region is where the earthquake occurred.

Quick Quiz 16.1

(a) In a long line of people waiting to buy tickets, the first person leaves and a pulse of motion occurs as people step forward to fill the gap. As each person steps forward, the gap moves through the line. Is the propagation of this gap transverse or longitudinal? (b) Consider the "wave" at a baseball game: people stand up and shout as the wave arrives at their location, and the resultant pulse moves around the stadium. Is this wave transverse or longitudinal?

16.3 ONE-DIMENSIONAL TRAVELING WAVES

Consider a wave pulse traveling to the right with constant speed v on a long, taut string, as shown in Figure 16.6. The pulse moves along the x axis (the axis of the string), and the transverse (vertical) displacement of the string (the medium) is measured along the y axis. Figure 16.6a represents the shape and position of the pulse at time $t = 0$. At this time, the shape of the pulse, whatever it may be, can be represented as $y = f(x)$. That is, y, which is the vertical position of any point on the string, is some definite function of x. The displacement y, sometimes called the *wave function,* depends on both x and t. For this reason, it is often written $y(x, t)$, which is read "y as a function of x and t." Consider a particular point P on the string, identified by a specific value of its x coordinate. Before the pulse arrives at P, the y coordinate of this point is zero. As the wave passes P, the y coordinate of this point increases, reaches a maximum, and then decreases to zero. Therefore, **the wave function y represents the y coordinate of any point P of the medium at any time t.**

Because its speed is v, the wave pulse travels to the right a distance vt in a time t (see Fig. 16.6b). If the shape of the pulse does not change with time, we can represent the wave function y for all times after $t = 0$. Measured in a stationary reference frame having its origin at O, the wave function is

$$y = f(x - vt) \tag{16.1}$$

Wave traveling to the right

If the wave pulse travels to the left, the string displacement is

$$y = f(x + vt) \tag{16.2}$$

Wave traveling to the left

For any given time t, the wave function y as a function of x defines a curve representing the shape of the pulse at this time. This curve is equivalent to a "snapshot" of the wave at this time. For a pulse that moves without changing shape, the speed of the pulse is the same as that of any feature along the pulse, such as the crest shown in Figure 16.6. To find the speed of the pulse, we can calculate how far the crest moves in a short time and then divide this distance by the time interval. To follow the motion of the crest, we must substitute some particular value, say x_0, in Equation 16.1 for $x - vt$. Regardless of how x and t change individually, we must require that $x - vt = x_0$ in order to stay with the crest. This expression therefore represents the equation of motion of the crest. At $t = 0$, the crest is at $x = x_0$; at a

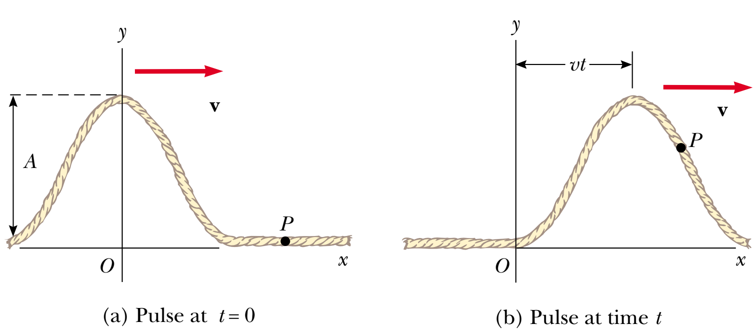


Figure 16.6 A one-dimensional wave pulse traveling to the right with a speed v. (a) At $t = 0$, the shape of the pulse is given by $y = f(x)$. (b) At some later time t, the shape remains unchanged and the vertical displacement of any point P of the medium is given by $y = f(x - vt)$.

time dt later, the crest is at $x = x_0 + v\,dt$. Therefore, in a time dt, the crest has moved a distance $dx = (x_0 + v\,dt) - x_0 = v\,dt$. Hence, the wave speed is

$$v = \frac{dx}{dt} \tag{16.3}$$

EXAMPLE 16.1 A Pulse Moving to the Right

A wave pulse moving to the right along the x axis is represented by the wave function

$$y(x, t) = \frac{2}{(x - 3.0t)^2 + 1}$$

where x and y are measured in centimeters and t is measured in seconds. Plot the wave function at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s.

Solution First, note that this function is of the form $y = f(x - vt)$. By inspection, we see that the wave speed is $v = 3.0$ cm/s. Furthermore, the wave amplitude (the maximum value of y) is given by $A = 2.0$ cm. (We find the maximum value of the function representing y by letting $x - 3.0t = 0$.) The wave function expressions are

$$y(x, 0) = \frac{2}{x^2 + 1} \qquad \text{at } t = 0$$

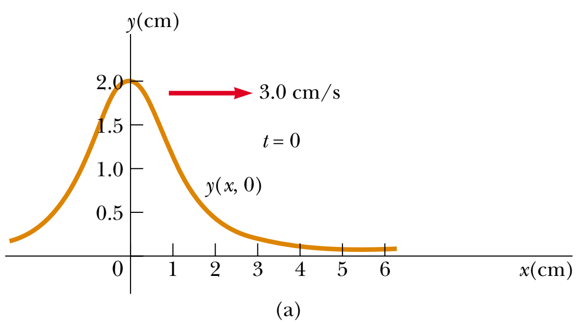
$$y(x, 1.0) = \frac{2}{(x - 3.0)^2 + 1} \qquad \text{at } t = 1.0 \text{ s}$$

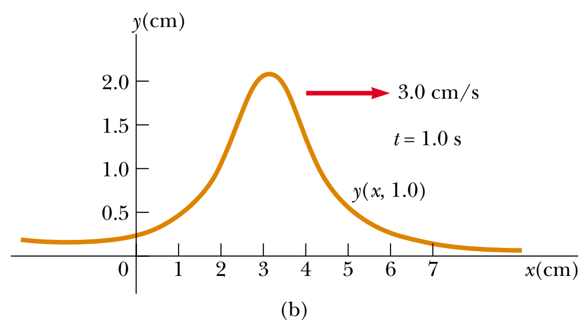
$$y(x, 2.0) = \frac{2}{(x - 6.0)^2 + 1} \qquad \text{at } t = 2.0 \text{ s}$$

We now use these expressions to plot the wave function versus x at these times. For example, let us evaluate $y(x, 0)$ at $x = 0.50$ cm:

$$y(0.50, 0) = \frac{2}{(0.50)^2 + 1} = 1.6 \text{ cm}$$

Likewise, at $x = 1.0$ cm, $y(1.0, 0) = 1.0$ cm, and at $x = 2.0$ cm, $y(2.0, 0) = 0.40$ cm. Continuing this procedure for other values of x yields the wave function shown in Figure 16.7a. In a similar manner, we obtain the graphs of $y(x, 1.0)$ and $y(x, 2.0)$, shown in Figure 16.7b and c, respectively. These snapshots show that the wave pulse moves to the right without changing its shape and that it has a constant speed of 3.0 cm/s.





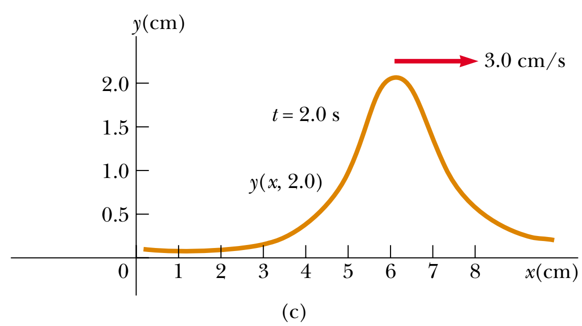


Figure 16.7 Graphs of the function $y(x, t) = 2/[(x - 3.0t)^2 + 1]$ at (a) $t = 0$, (b) $t = 1.0$ s, and (c) $t = 2.0$ s.

16.4 SUPERPOSITION AND INTERFERENCE

Many interesting wave phenomena in nature cannot be described by a single moving pulse. Instead, one must analyze complex waves in terms of a combination of many traveling waves. To analyze such wave combinations, one can make use of the **superposition principle:**

> If two or more traveling waves are moving through a medium, the resultant wave function at any point is the algebraic sum of the wave functions of the individual waves.

Linear waves obey the superposition principle

Waves that obey this principle are called *linear waves* and are generally characterized by small amplitudes. Waves that violate the superposition principle are called *nonlinear waves* and are often characterized by large amplitudes. In this book, we deal only with linear waves.

One consequence of the superposition principle is that **two traveling waves can pass through each other without being destroyed or even altered.** For instance, when two pebbles are thrown into a pond and hit the surface at different places, the expanding circular surface waves do not destroy each other but rather pass through each other. The complex pattern that is observed can be viewed as two independent sets of expanding circles. Likewise, when sound waves from two sources move through air, they pass through each other. The resulting sound that one hears at a given point is the resultant of the two disturbances.

Figure 16.8 is a pictorial representation of superposition. The wave function for the pulse moving to the right is y_1, and the wave function for the pulse moving

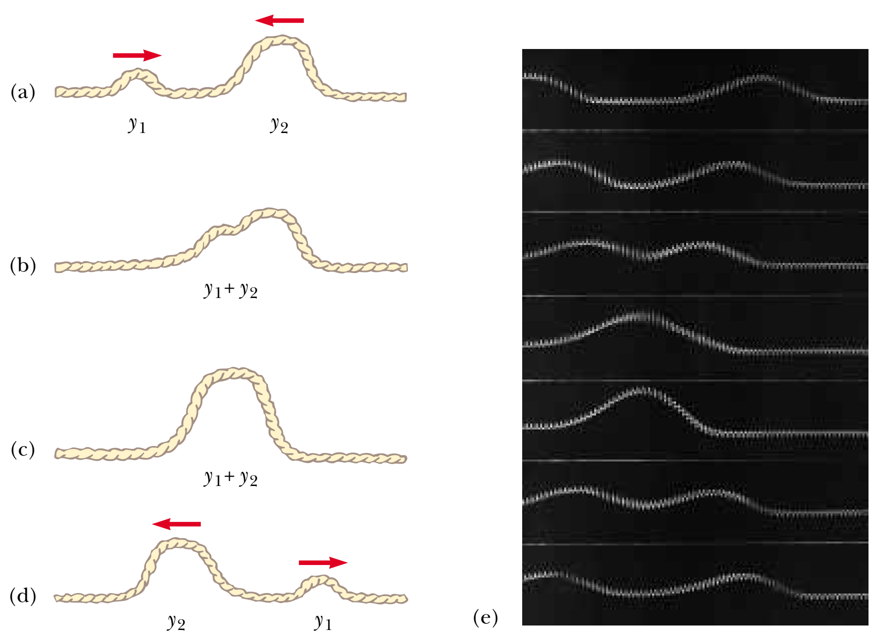


Figure 16.8 (a–d) Two wave pulses traveling on a stretched string in opposite directions pass through each other. When the pulses overlap, as shown in (b) and (c), the net displacement of the string equals the sum of the displacements produced by each pulse. Because each pulse displaces the string in the positive direction, we refer to the superposition of the two pulses as *constructive interference.* (e) Photograph of superposition of two equal, symmetric pulses traveling in opposite directions on a stretched spring.

Interference of water waves produced in a ripple tank. The sources of the waves are two objects that oscillate perpendicular to the surface of the tank.

to the left is y_2. The pulses have the same speed but different shapes. Each pulse is assumed to be symmetric, and the displacement of the medium is in the positive y direction for both pulses. (Note, however, that the superposition principle applies even when the two pulses are not symmetric.) When the waves begin to overlap (Fig. 16.8b), the wave function for the resulting complex wave is given by $y_1 + y_2$.

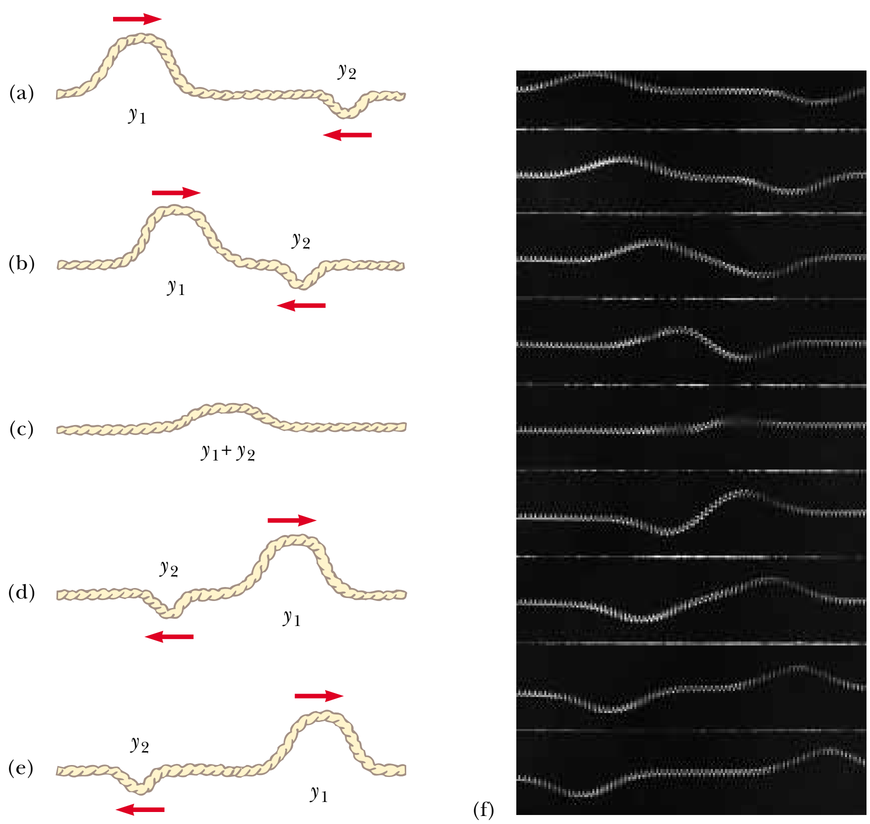


Figure 16.9 (a–e) Two wave pulses traveling in opposite directions and having displacements that are inverted relative to each other. When the two overlap in (c), their displacements partially cancel each other. (f) Photograph of superposition of two symmetric pulses traveling in opposite directions, where one pulse is inverted relative to the other.

When the crests of the pulses coincide (Fig. 16.8c), the resulting wave given by $y_1 + y_2$ is symmetric. The two pulses finally separate and continue moving in their original directions (Fig. 16.8d). Note that the pulse shapes remain unchanged, as if the two pulses had never met!

The combination of separate waves in the same region of space to produce a resultant wave is called **interference.** For the two pulses shown in Figure 16.8, the displacement of the medium is in the positive y direction for both pulses, and the resultant wave (created when the pulses overlap) exhibits a displacement greater than that of either individual pulse. Because the displacements caused by the two pulses are in the same direction, we refer to their superposition as **constructive interference.**

Now consider two pulses traveling in opposite directions on a taut string where one pulse is inverted relative to the other, as illustrated in Figure 16.9. In this case, when the pulses begin to overlap, the resultant wave is given by $y_1 + y_2$, but the values of the function y_2 are negative. Again, the two pulses pass through each other; however, because the displacements caused by the two pulses are in opposite directions, we refer to their superposition as **destructive interference.**

Quick Quiz 16.2

Two pulses are traveling toward each other at 10 cm/s on a long string, as shown in Figure 16.10. Sketch the shape of the string at $t = 0.6$ s.

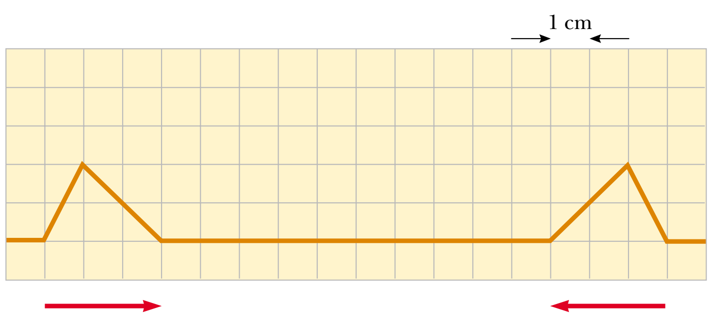


Figure 16.10 The pulses on this string are traveling at 10 cm/s.

16.5 THE SPEED OF WAVES ON STRINGS

In this section, we focus on determining the speed of a transverse pulse traveling on a taut string. Let us first conceptually argue the parameters that determine the speed. If a string under tension is pulled sideways and then released, the tension is responsible for accelerating a particular segment of the string back toward its equilibrium position. According to Newton's second law, the acceleration of the segment increases with increasing tension. If the segment returns to equilibrium more rapidly due to this increased acceleration, we would intuitively argue that the wave speed is greater. Thus, we expect the wave speed to increase with increasing tension.

Likewise, we can argue that the wave speed decreases if the mass per unit length of the string increases. This is because it is more difficult to accelerate a massive segment of the string than a light segment. If the tension in the string is T (not to be confused with the same symbol used for the period) and its mass per

The strings of this piano vary in both tension and mass per unit length. These differences in tension and density, in combination with the different lengths of the strings, allow the instrument to produce a wide range of sounds.

unit length is μ (Greek letter mu), then, as we shall show, the wave speed is

Speed of a wave on a stretched string

$$v = \sqrt{\frac{T}{\mu}} \tag{16.4}$$

First, let us verify that this expression is dimensionally correct. The dimensions of T are ML/T^2, and the dimensions of μ are M/L. Therefore, the dimensions of T/μ are L^2/T^2; hence, the dimensions of $\sqrt{T/\mu}$ are L/T—indeed, the dimensions of speed. No other combination of T and μ is dimensionally correct if we assume that they are the only variables relevant to the situation.

Now let us use a mechanical analysis to derive Equation 16.4. On our string under tension, consider a pulse moving to the right with a uniform speed v measured relative to a stationary frame of reference. Instead of staying in this reference frame, it is more convenient to choose as our reference frame one that moves along with the pulse with the same speed as the pulse, so that the pulse is at rest within the frame. This change of reference frame is permitted because Newton's laws are valid in either a stationary frame or one that moves with constant velocity. In our new reference frame, a given segment of the string initially to the right of the pulse moves to the left, rises up and follows the shape of the pulse, and then continues to move to the left. Figure 16.11a shows such a segment at the instant it is located at the top of the pulse.

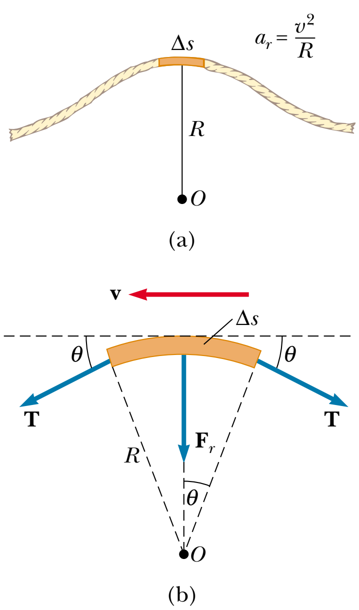


Figure 16.11 (a) To obtain the speed v of a wave on a stretched string, it is convenient to describe the motion of a small segment of the string in a moving frame of reference. (b) In the moving frame of reference, the small segment of length Δs moves to the left with speed v. The net force on the segment is in the radial direction because the horizontal components of the tension force cancel.

The small segment of the string of length Δs shown in Figure 16.11a, and magnified in Figure 16.11b, forms an approximate arc of a circle of radius R. In our moving frame of reference (which is moving to the right at a speed v along with the pulse), the shaded segment is moving to the left with a speed v. This segment has a centripetal acceleration equal to v^2/R, which is supplied by components of the tension $\mathbf{T}$ in the string. The force $\mathbf{T}$ acts on either side of the segment and tangent to the arc, as shown in Figure 16.11b. The horizontal components of $\mathbf{T}$ cancel, and each vertical component $T \sin \theta$ acts radially toward the center of the arc. Hence, the total radial force is $2T \sin \theta$. Because the segment is small, θ is small, and we can use the small-angle approximation $\sin \theta \approx \theta$. Therefore, the total radial force is

$$\sum F_r = 2T \sin \theta \approx 2T\theta$$

The segment has a mass $m = \mu \Delta s$. Because the segment forms part of a circle and subtends an angle 2θ at the center, $\Delta s = R(2\theta)$, and hence

$$m = \mu \Delta s = 2\mu R\theta$$

If we apply Newton's second law to this segment, the radial component of motion gives

$$\sum F_r = ma = \frac{mv^2}{R}$$

$$2T\theta = \frac{2\mu R\theta v^2}{R}$$

Solving for v gives Equation 16.4.

Notice that this derivation is based on the assumption that the pulse height is small relative to the length of the string. Using this assumption, we were able to use the approximation $\sin\theta \approx \theta$. Furthermore, the model assumes that the tension T is not affected by the presence of the pulse; thus, T is the same at all points on the string. Finally, this proof does *not* assume any particular shape for the pulse. Therefore, we conclude that a pulse of *any shape* travels along the string with speed $v = \sqrt{T/\mu}$ without any change in pulse shape.

EXAMPLE 16.2 The Speed of a Pulse on a Cord

A uniform cord has a mass of 0.300 kg and a length of 6.00 m (Fig. 16.12). The cord passes over a pulley and supports a 2.00-kg object. Find the speed of a pulse traveling along this cord.

Solution The tension T in the cord is equal to the weight of the suspended 2.00-kg mass:

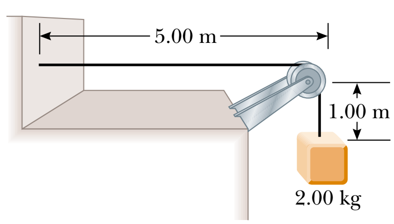


Figure 16.12 The tension T in the cord is maintained by the suspended object. The speed of any wave traveling along the cord is given by $v = \sqrt{T/\mu}$.

$$T = mg = (2.00\ \text{kg})(9.80\ \text{m/s}^2) = 19.6\ \text{N}$$

(This calculation of the tension neglects the small mass of the cord. Strictly speaking, the cord can never be exactly horizontal, and therefore the tension is not uniform.) The mass per unit length μ of the cord is

$$\mu = \frac{m}{\ell} = \frac{0.300\ \text{kg}}{6.00\ \text{m}} = 0.050\,0\ \text{kg/m}$$

Therefore, the wave speed is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{19.6\ \text{N}}{0.050\,0\ \text{kg/m}}} = \boxed{19.8\ \text{m/s}}$$

Exercise Find the time it takes the pulse to travel from the wall to the pulley.

Answer 0.253 s.

Quick Quiz 16.3

Suppose you create a pulse by moving the free end of a taut string up and down once with your hand. The string is attached at its other end to a distant wall. The pulse reaches the wall in a time t. Which of the following actions, taken by itself, decreases the time it takes the pulse to reach the wall? More than one choice may be correct.

(a) Moving your hand more quickly, but still only up and down once by the same amount.
(b) Moving your hand more slowly, but still only up and down once by the same amount.
(c) Moving your hand a greater distance up and down in the same amount of time.
(d) Moving your hand a lesser distance up and down in the same amount of time.
(e) Using a heavier string of the same length and under the same tension.
(f) Using a lighter string of the same length and under the same tension.
(g) Using a string of the same linear mass density but under decreased tension.
(h) Using a string of the same linear mass density but under increased tension.

16.6 REFLECTION AND TRANSMISSION

We have discussed traveling waves moving through a uniform medium. We now consider how a traveling wave is affected when it encounters a change in the medium. For example, consider a pulse traveling on a string that is rigidly attached to a support at one end (Fig. 16.13). When the pulse reaches the support, a severe change in the medium occurs—the string ends. The result of this change is that the wave undergoes **reflection**—that is, the pulse moves back along the string in the opposite direction.

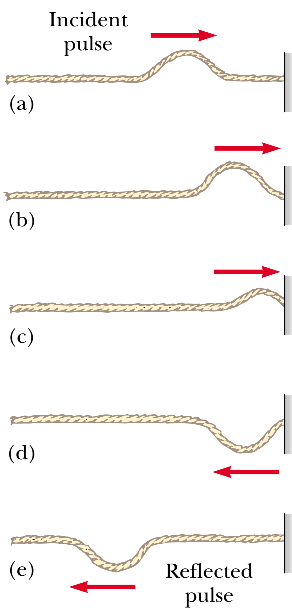


Figure 16.13 The reflection of a traveling wave pulse at the fixed end of a stretched string. The reflected pulse is inverted, but its shape is unchanged.

Note that the reflected pulse is inverted. This inversion can be explained as follows: When the pulse reaches the fixed end of the string, the string produces an upward force on the support. By Newton's third law, the support must exert an equal and opposite (downward) reaction force on the string. This downward force causes the pulse to invert upon reflection.

Now consider another case: this time, the pulse arrives at the end of a string that is free to move vertically, as shown in Figure 16.14. The tension at the free end is maintained because the string is tied to a ring of negligible mass that is free to slide vertically on a smooth post. Again, the pulse is reflected, but this time it is not inverted. When it reaches the post, the pulse exerts a force on the free end of the string, causing the ring to accelerate upward. The ring overshoots the height of the incoming pulse, and then the downward component of the tension force pulls the ring back down. This movement of the ring produces a reflected pulse that is not inverted and that has the same amplitude as the incoming pulse.

Finally, we may have a situation in which the boundary is intermediate between these two extremes. In this case, part of the incident pulse is reflected and part undergoes **transmission**—that is, some of the pulse passes through the boundary. For instance, suppose a light string is attached to a heavier string, as shown in Figure 16.15. When a pulse traveling on the light string reaches the boundary between the two, part of the pulse is reflected and inverted and part is transmitted to the heavier string. The reflected pulse is inverted for the same reasons described earlier in the case of the string rigidly attached to a support.

Note that the reflected pulse has a smaller amplitude than the incident pulse. In Section 16.8, we shall learn that the energy carried by a wave is related to its amplitude. Thus, according to the principle of the conservation of energy, when the pulse breaks up into a reflected pulse and a transmitted pulse at the boundary, the sum of the energies of these two pulses must equal the energy of the incident pulse. Because the reflected pulse contains only part of the energy of the incident pulse, its amplitude must be smaller.

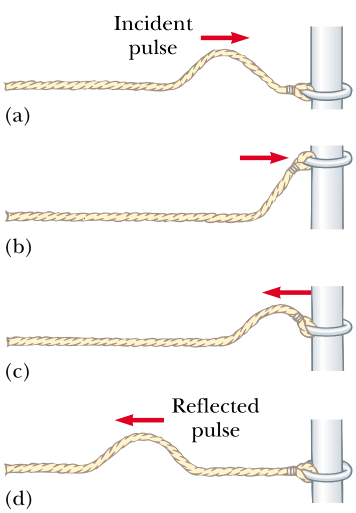


Figure 16.14 The reflection of a traveling wave pulse at the free end of a stretched string. The reflected pulse is not inverted.

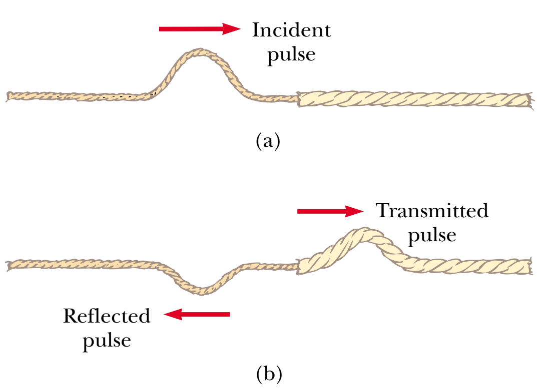


Figure 16.15 (a) A pulse traveling to the right on a light string attached to a heavier string. (b) Part of the incident pulse is reflected (and inverted), and part is transmitted to the heavier string.

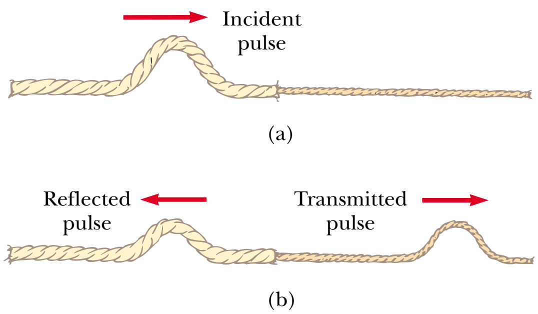


Figure 16.16 (a) A pulse traveling to the right on a heavy string attached to a lighter string. (b) The incident pulse is partially reflected and partially transmitted, and the reflected pulse is not inverted.

When a pulse traveling on a heavy string strikes the boundary between the heavy string and a lighter one, as shown in Figure 16.16, again part is reflected and part is transmitted. In this case, the reflected pulse is not inverted.

In either case, the relative heights of the reflected and transmitted pulses depend on the relative densities of the two strings. If the strings are identical, there is no discontinuity at the boundary and no reflection takes place.

According to Equation 16.4, the speed of a wave on a string increases as the mass per unit length of the string decreases. In other words, a pulse travels more slowly on a heavy string than on a light string if both are under the same tension. The following general rules apply to reflected waves: **When a wave pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is denser than A), the pulse is inverted upon reflection. When a wave pulse travels from medium A to medium B and $v_A < v_B$ (that is, when A is denser than B), the pulse is not inverted upon reflection.**

16.7 SINUSOIDAL WAVES

In this section, we introduce an important wave function whose shape is shown in Figure 16.17. The wave represented by this curve is called a **sinusoidal wave** because the curve is the same as that of the function sin θ plotted against θ. The sinusoidal wave is the simplest example of a periodic continuous wave and can be used to build more complex waves, as we shall see in Section 18.8. The red curve represents a snapshot of a traveling sinusoidal wave at $t = 0$, and the blue curve represents a snapshot of the wave at some later time t. At $t = 0$, the function describing the positions of the particles of the medium through which the sinusoidal wave is traveling can be written

$$y = A \sin\left(\frac{2\pi}{\lambda} x\right) \tag{16.5}$$

where the constant A represents the wave amplitude and the constant λ is the wavelength. Thus, we see that the position of a particle of the medium is the same whenever x is increased by an integral multiple of λ. If the wave moves to the right with a speed v, then the wave function at some later time t is

$$y = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] \tag{16.6}$$

That is, the traveling sinusoidal wave moves to the right a distance vt in the time t, as shown in Figure 16.17. Note that the wave function has the form $f(x - vt)$ and

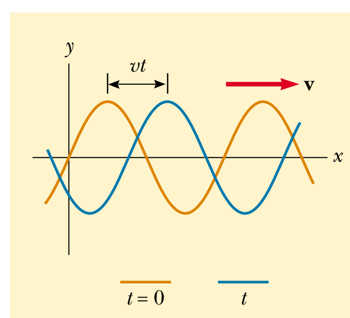


Figure 16.17 A one-dimensional sinusoidal wave traveling to the right with a speed v. The red curve represents a snapshot of the wave at $t = 0$, and the blue curve represents a snapshot at some later time t.

so represents a wave traveling to the right. If the wave were traveling to the left, the quantity $x - vt$ would be replaced by $x + vt$, as we learned when we developed Equations 16.1 and 16.2.

By definition, the wave travels a distance of one wavelength in one period T. Therefore, the wave speed, wavelength, and period are related by the expression

$$v = \frac{\lambda}{T} \quad \textbf{(16.7)}$$

Substituting this expression for v into Equation 16.6, we find that

$$y = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \quad \textbf{(16.8)}$$

This form of the wave function clearly shows the *periodic* nature of y. At any given time t (a snapshot of the wave), y has the *same* value at the positions x, $x + \lambda$, $x + 2\lambda$, and so on. Furthermore, at any given position x, the value of y is the same at times t, $t + T$, $t + 2T$, and so on.

We can express the wave function in a convenient form by defining two other quantities, the **angular wave number** k and the **angular frequency** ω:

Angular wave number

$$k \equiv \frac{2\pi}{\lambda} \quad \textbf{(16.9)}$$

Angular frequency

$$\omega \equiv \frac{2\pi}{T} \quad \textbf{(16.10)}$$

Using these definitions, we see that Equation 16.8 can be written in the more compact form

Wave function for a sinusoidal wave

$$y = A \sin(kx - \omega t) \quad \textbf{(16.11)}$$

The frequency of a sinusoidal wave is related to the period by the expression

Frequency

$$f = \frac{1}{T} \quad \textbf{(16.12)}$$

The most common unit for frequency, as we learned in Chapter 13, is second^{-1}, or **hertz** (Hz). The corresponding unit for T is seconds.

Using Equations 16.9, 16.10, and 16.12, we can express the wave speed v originally given in Equation 16.7 in the alternative forms

Speed of a sinusoidal wave

$$v = \frac{\omega}{k} \quad \textbf{(16.13)}$$

$$v = \lambda f \quad \textbf{(16.14)}$$

The wave function given by Equation 16.11 assumes that the vertical displacement y is zero at $x = 0$ and $t = 0$. This need not be the case. If it is not, we generally express the wave function in the form

General expression for a sinusoidal wave

$$y = A \sin(kx - \omega t + \phi) \quad \textbf{(16.15)}$$

where ϕ is the **phase constant,** just as we learned in our study of periodic motion in Chapter 13. This constant can be determined from the initial conditions.

EXAMPLE 16.3 A Traveling Sinusoidal Wave

A sinusoidal wave traveling in the positive x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical displacement of the medium at $t = 0$ and $x = 0$ is also 15.0 cm, as shown in Figure 16.18. (a) Find the angular wave number k, period T, angular frequency ω, and speed v of the wave.

Solution (a) Using Equations 16.9, 16.10, 16.12, and 16.14, we find the following:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{40.0 \text{ cm}} = \boxed{0.157 \text{ rad/cm}}$$

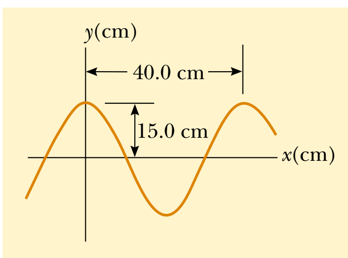


Figure 16.18 A sinusoidal wave of wavelength $\lambda = 40.0$ cm and amplitude $A = 15.0$ cm. The wave function can be written in the form $y = A\cos(kx - \omega t)$.

$$\omega = 2\pi f = 2\pi(8.00 \text{ s}^{-1}) = \boxed{50.3 \text{ rad/s}}$$

$$T = \frac{1}{f} = \frac{1}{8.00 \text{ s}^{-1}} = \boxed{0.125 \text{ s}}$$

$$v = \lambda f = (40.0 \text{ cm})(8.00 \text{ s}^{-1}) = \boxed{320 \text{ cm/s}}$$

(b) Determine the phase constant ϕ, and write a general expression for the wave function.

Solution Because $A = 15.0$ cm and because $y = 15.0$ cm at $x = 0$ and $t = 0$, substitution into Equation 16.15 gives

$$15.0 = (15.0)\sin\phi \qquad \text{or} \qquad \sin\phi = 1$$

We may take the principal value $\phi = \pi/2$ rad (or 90°). Hence, the wave function is of the form

$$y = A\sin\left(kx - \omega t + \frac{\pi}{2}\right) = A\cos(kx - \omega t)$$

By inspection, we can see that the wave function must have this form, noting that the cosine function has the same shape as the sine function displaced by 90°. Substituting the values for A, k, and ω into this expression, we obtain

$$\boxed{y = (15.0 \text{ cm})\cos(0.157x - 50.3t)}$$

Sinusoidal Waves on Strings

In Figure 16.2, we demonstrated how to create a pulse by jerking a taut string up and down once. To create a train of such pulses, normally referred to as a *wave train,* or just plain *wave,* we can replace the hand with an oscillating blade. If the wave consists of a train of identical cycles, whatever their shape, the relationships $f = 1/T$ and $v = f\lambda$ among speed, frequency, period, and wavelength hold true. We can make more definite statements about the wave function if the source of the waves vibrates in simple harmonic motion. Figure 16.19 represents snapshots of the wave created in this way at intervals of $T/4$. Note that because the end of the blade oscillates in simple harmonic motion, **each particle of the string, such as that at P, also oscillates vertically with simple harmonic motion.** This must be the case because each particle follows the simple harmonic motion of the blade. Therefore, every segment of the string can be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of oscillation of the blade.[3] Note that although each segment oscillates in the y direction, the wave travels in the x direction with a speed v. Of course, this is the definition of a transverse wave.

[3] In this arrangement, we are assuming that a string segment always oscillates in a vertical line. The tension in the string would vary if a segment were allowed to move sideways. Such motion would make the analysis very complex.

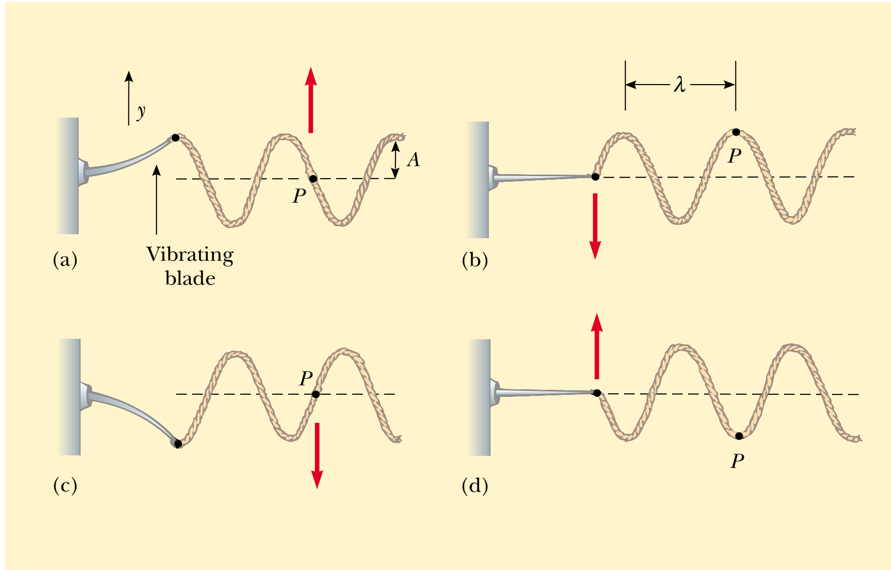


Figure 16.19 One method for producing a train of sinusoidal wave pulses on a string. The left end of the string is connected to a blade that is set into oscillation. Every segment of the string, such as the point P, oscillates with simple harmonic motion in the vertical direction.

If the wave at $t = 0$ is as described in Figure 16.19b, then the wave function can be written as

$$y = A\sin(kx - \omega t)$$

We can use this expression to describe the motion of any point on the string. The point P (or any other point on the string) moves only vertically, and so its x coordinate remains constant. Therefore, the **transverse speed** $\boldsymbol{v_y}$ (not to be confused with the wave speed v) and the **transverse acceleration** $\boldsymbol{a_y}$ are

$$v_y = \left.\frac{dy}{dt}\right]_{x\,=\,\text{constant}} = \frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t) \tag{16.16}$$

$$a_y = \left.\frac{dv_y}{dt}\right]_{x\,=\,\text{constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A\sin(kx - \omega t) \tag{16.17}$$

In these expressions, we must use partial derivatives (see Section 8.6) because y depends on both x and t. In the operation $\partial y/\partial t$, for example, we take a derivative with respect to t while holding x constant. The maximum values of the transverse speed and transverse acceleration are simply the absolute values of the coefficients of the cosine and sine functions:

$$v_{y,\,\max} = \omega A \tag{16.18}$$

$$a_{y,\,\max} = \omega^2 A \tag{16.19}$$

The transverse speed and transverse acceleration do not reach their maximum values simultaneously. The transverse speed reaches its maximum value (ωA) when $y = 0$, whereas the transverse acceleration reaches its maximum value ($\omega^2 A$) when $y = \pm A$. Finally, Equations 16.18 and 16.19 are identical in mathematical form to the corresponding equations for simple harmonic motion, Equations 13.10 and 13.11.

Quick Quiz 16.4

A sinusoidal wave is moving on a string. If you increase the frequency f of the wave, how do the transverse speed, wave speed, and wavelength change?

EXAMPLE 16.4 A Sinusoidally Driven String

The string shown in Figure 16.19 is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Determine the angular frequency ω and angular wave number k for this wave, and write an expression for the wave function.

Solution Using Equations 16.10, 16.12, and 16.13, we find that

$$\omega = \frac{2\pi}{T} = 2\pi f = 2\pi(5.00\text{ Hz}) = \boxed{31.4\text{ rad/s}}$$

$$k = \frac{\omega}{v} = \frac{31.4\text{ rad/s}}{20.0\text{ m/s}} = \boxed{1.57\text{ rad/m}}$$

Because $A = 12.0$ cm $= 0.120$ m, we have

$$\boxed{y = A\sin(kx - \omega t) = (0.120\text{ m})\sin(1.57x - 31.4t)}$$

Exercise Calculate the maximum values for the transverse speed and transverse acceleration of any point on the string.

Answer 3.77 m/s; 118 m/s^2.

16.8 RATE OF ENERGY TRANSFER BY SINUSOIDAL WAVES ON STRINGS

As waves propagate through a medium, they transport energy. We can easily demonstrate this by hanging an object on a stretched string and then sending a pulse down the string, as shown in Figure 16.20. When the pulse meets the suspended object, the object is momentarily displaced, as illustrated in Figure 16.20b. In the process, energy is transferred to the object because work must be done for it to move upward. This section examines the rate at which energy is transported along a string. We shall assume a one-dimensional sinusoidal wave in the calculation of the energy transferred.

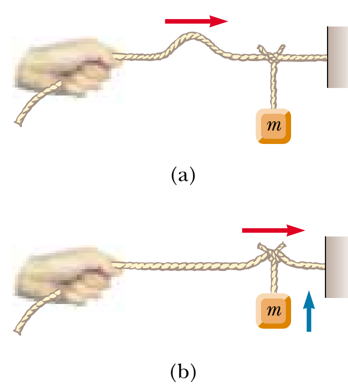


Figure 16.20 (a) A pulse traveling to the right on a stretched string on which an object has been suspended. (b) Energy is transmitted to the suspended object when the pulse arrives.

Consider a sinusoidal wave traveling on a string (Fig. 16.21). The source of the energy being transported by the wave is some external agent at the left end of the string; this agent does work in producing the oscillations. As the external agent performs work on the string, moving it up and down, energy enters the system of the string and propagates along its length. Let us focus our attention on a segment of the string of length Δx and mass Δm. Each such segment moves vertically with simple harmonic motion. Furthermore, all segments have the same angular frequency ω and the same amplitude A. As we found in Chapter 13, the elastic potential energy U associated with a particle in simple harmonic motion is $U = \frac{1}{2}ky^2$, where the simple harmonic motion is in the y direction. Using the relationship $\omega^2 = k/m$ developed in Equations 13.16 and 13.17, we can write this as

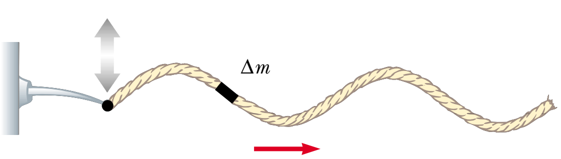


Figure 16.21 A sinusoidal wave traveling along the x axis on a stretched string. Every segment moves vertically, and every segment has the same total energy.

$U = \frac{1}{2}m\omega^2y^2$. If we apply this equation to the segment of mass Δm, we see that the potential energy of this segment is

$$\Delta U = \tfrac{1}{2}(\Delta m)\,\omega^2y^2$$

Because the mass per unit length of the string is $\mu = \Delta m/\Delta x$, we can express the potential energy of the segment as

$$\Delta U = \tfrac{1}{2}(\mu\Delta x)\,\omega^2y^2$$

As the length of the segment shrinks to zero, $\Delta x \rightarrow dx$, and this expression becomes a differential relationship:

$$dU = \tfrac{1}{2}(\mu dx)\,\omega^2y^2$$

We replace the general displacement y of the segment with the wave function for a sinusoidal wave:

$$dU = \tfrac{1}{2}\mu\omega^2[A\sin(kx - \omega t)]^2\ dx = \tfrac{1}{2}\mu\omega^2A^2\sin^2(kx - \omega t)\ dx$$

If we take a snapshot of the wave at time $t = 0$, then the potential energy in a given segment is

$$dU = \tfrac{1}{2}\mu\omega^2A^2\sin^2 kx\ dx$$

To obtain the total potential energy in one wavelength, we integrate this expression over all the string segments in one wavelength:

$$U_\lambda = \int dU = \int_0^\lambda \tfrac{1}{2}\mu\omega^2A^2\sin^2 kx\ dx = \tfrac{1}{2}\mu\omega^2A^2\int_0^\lambda \sin^2 kx\ dx$$
$$= \tfrac{1}{2}\mu\omega^2A^2\Big[\tfrac{1}{2}x - \tfrac{1}{4k}\sin 2\,kx\Big]_0^\lambda = \tfrac{1}{2}\mu\omega^2A^2(\tfrac{1}{2}\lambda) = \tfrac{1}{4}\mu\omega^2A^2\lambda$$

Because it is in motion, each segment of the string also has kinetic energy. When we use this procedure to analyze the total kinetic energy in one wavelength of the string, we obtain the same result:

$$K_\lambda = \int dK = \tfrac{1}{4}\mu\omega^2A^2\lambda$$

The total energy in one wavelength of the wave is the sum of the potential and kinetic energies:

$$E_\lambda = U_\lambda + K_\lambda = \tfrac{1}{2}\mu\omega^2A^2\lambda \qquad \textbf{(16.20)}$$

As the wave moves along the string, this amount of energy passes by a given point on the string during one period of the oscillation. Thus, the power, or rate of energy transfer, associated with the wave is

$$\mathcal{P} = \frac{E_\lambda}{\Delta t} = \frac{\frac{1}{2}\mu\omega^2A^2\lambda}{T} = \tfrac{1}{2}\mu\omega^2A^2\left(\frac{\lambda}{T}\right)$$

Power of a wave

$$\mathcal{P} = \tfrac{1}{2}\mu\omega^2\,A^2v \qquad \textbf{(16.21)}$$

This shows that the rate of energy transfer by a sinusoidal wave on a string is proportional to (a) the wave speed, (b) the square of the frequency, and (c) the square of the amplitude. In fact: **the rate of energy transfer in any sinusoidal wave is proportional to the square of the angular frequency and to the square of the amplitude.**

EXAMPLE 16.5 Power Supplied to a Vibrating String

A taut string for which $\mu = 5.00 \times 10^{-2}$ kg/m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

Solution The wave speed on the string is, from Equation 16.4,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80.0\ \text{N}}{5.00 \times 10^{-2}\ \text{kg/m}}} = 40.0\ \text{m/s}$$

Because $f = 60.0$ Hz, the angular frequency ω of the sinusoidal waves on the string has the value

$$\omega = 2\pi f = 2\pi(60.0\ \text{Hz}) = 377\ \text{s}^{-1}$$

Using these values in Equation 16.21 for the power, with $A = 6.00 \times 10^{-2}$ m, we obtain

$$\begin{aligned}\mathcal{P} &= \tfrac{1}{2}\mu\omega^2 A^2 v \\ &= \tfrac{1}{2}(5.00 \times 10^{-2}\ \text{kg/m})(377\ \text{s}^{-1})^2 \\ &= \times\ (6.00 \times 10^{-2}\ \text{m})^2(40.0\ \text{m/s}) \\ &= \boxed{512\ \text{W}}\end{aligned}$$

Optional Section

16.9 THE LINEAR WAVE EQUATION

In Section 16.3 we introduced the concept of the wave function to represent waves traveling on a string. All wave functions $y(x, t)$ represent solutions of an equation called the *linear wave equation*. This equation gives a complete description of the wave motion, and from it one can derive an expression for the wave speed. Furthermore, the linear wave equation is basic to many forms of wave motion. In this section, we derive this equation as applied to waves on strings.

Suppose a traveling wave is propagating along a string that is under a tension T. Let us consider one small string segment of length Δx (Fig. 16.22). The ends of the segment make small angles θ_A and θ_B with the x axis. The net force acting on the segment in the vertical direction is

$$\sum F_y = T\sin\theta_B - T\sin\theta_A = T(\sin\theta_B - \sin\theta_A)$$

Because the angles are small, we can use the small-angle approximation $\sin\theta \approx \tan\theta$ to express the net force as

$$\sum F_y \approx T(\tan\theta_B - \tan\theta_A)$$

However, the tangents of the angles at A and B are defined as the slopes of the string segment at these points. Because the slope of a curve is given by $\partial y/\partial x$, we have

$$\sum F_y \approx T\left[\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A\right] \tag{16.22}$$

We now apply Newton's second law to the segment, with the mass of the segment given by $m = \mu\Delta x$:

$$\sum F_y = ma_y = \mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right) \tag{16.23}$$

Combining Equation 16.22 with Equation 16.23, we obtain

$$\mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right) = T\left[\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A\right]$$

$$\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{(\partial y/\partial x)_B - (\partial y/\partial x)_A}{\Delta x} \tag{16.24}$$

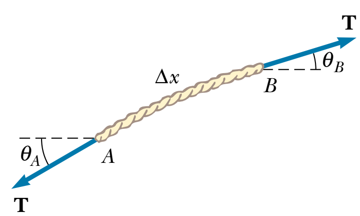


Figure 16.22 A segment of a string under tension T. The slopes at points A and B are given by $\tan\theta_A$ and $\tan\theta_B$, respectively.

The right side of this equation can be expressed in a different form if we note that the partial derivative of any function is defined as

$$\frac{\partial f}{\partial x} \equiv \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

If we associate $f(x + \Delta x)$ with $(\partial y/\partial x)_B$ and $f(x)$ with $(\partial y/\partial x)_A$, we see that, in the limit $\Delta x \to 0$, Equation 16.24 becomes

Linear wave equation

$$\frac{\mu}{T} \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} \quad \textbf{(16.25)}$$

This is the linear wave equation as it applies to waves on a string.

We now show that the sinusoidal wave function (Eq. 16.11) represents a solution of the linear wave equation. If we take the sinusoidal wave function to be of the form $y(x, t) = A \sin(kx - \omega t)$, then the appropriate derivatives are

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 A \sin(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 A \sin(kx - \omega t)$$

Substituting these expressions into Equation 16.25, we obtain

$$-\frac{\mu\omega^2}{T} \sin(kx - \omega t) = -k^2 \sin(kx - \omega t)$$

This equation must be true for all values of the variables x and t in order for the sinusoidal wave function to be a solution of the wave equation. Both sides of the equation depend on x and t through the same function $\sin(kx - \omega t)$. Because this function divides out, we do indeed have an identity, provided that

$$k^2 = \frac{\mu\omega^2}{T}$$

Using the relationship $v = \omega/k$ (Eq. 16.13) in this expression, we see that

$$v^2 = \frac{\omega^2}{k^2} = \frac{T}{\mu}$$

$$v = \sqrt{\frac{T}{\mu}}$$

which is Equation 16.4. This derivation represents another proof of the expression for the wave speed on a taut string.

The linear wave equation (Eq. 16.25) is often written in the form

Linear wave equation in general

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad \textbf{(16.26)}$$

This expression applies in general to various types of traveling waves. For waves on strings, y represents the vertical displacement of the string. For sound waves, y corresponds to displacement of air molecules from equilibrium or variations in either the pressure or the density of the gas through which the sound waves are propagating. In the case of electromagnetic waves, y corresponds to electric or magnetic field components.

We have shown that the sinusoidal wave function (Eq. 16.11) is one solution of the linear wave equation (Eq. 16.26). Although we do not prove it here, the linear wave equation is satisfied by *any* wave function having the form $y = f(x \pm vt)$. Furthermore, we have seen that the linear wave equation is a direct consequence of Newton's second law applied to any segment of the string.

SUMMARY

A **transverse wave** is one in which the particles of the medium move in a direction *perpendicular* to the direction of the wave velocity. An example is a wave on a taut string. A **longitudinal wave** is one in which the particles of the medium move in a direction *parallel* to the direction of the wave velocity. Sound waves in fluids are longitudinal. You should be able to identify examples of both types of waves.

Any one-dimensional wave traveling with a speed v in the x direction can be represented by a wave function of the form

$$y = f(x \pm vt) \qquad \textbf{(16.1, 16.2)}$$

where the positive sign applies to a wave traveling in the negative x direction and the negative sign applies to a wave traveling in the positive x direction. The shape of the wave at any instant in time (a snapshot of the wave) is obtained by holding t constant.

The **superposition principle** specifies that when two or more waves move through a medium, the resultant wave function equals the algebraic sum of the individual wave functions. When two waves combine in space, they interfere to produce a resultant wave. The **interference** may be **constructive** (when the individual displacements are in the same direction) or **destructive** (when the displacements are in opposite directions).

The **speed** of a wave traveling on a taut string of mass per unit length μ and tension T is

$$v = \sqrt{\frac{T}{\mu}} \qquad \textbf{(16.4)}$$

A wave is totally or partially reflected when it reaches the end of the medium in which it propagates or when it reaches a boundary where its speed changes discontinuously. If a wave pulse traveling on a string meets a fixed end, the pulse is reflected and inverted. If the pulse reaches a free end, it is reflected but not inverted.

The **wave function** for a one-dimensional sinusoidal wave traveling to the right can be expressed as

$$y = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] = A \sin(kx - \omega t) \qquad \textbf{(16.6, 16.11)}$$

where A is the **amplitude,** λ is the **wavelength,** k is the **angular wave number,** and ω is the **angular frequency.** If T is the **period** and f the **frequency,** v, k and ω can be written

$$v = \frac{\lambda}{T} = \lambda f \qquad \textbf{(16.7, 16.14)}$$

$$k \equiv \frac{2\pi}{\lambda} \qquad \textbf{(16.9)}$$

$$\omega \equiv \frac{2\pi}{T} = 2\pi f \qquad \textbf{(16.10, 16.12)}$$

You should know how to find the equation describing the motion of particles in a wave from a given set of physical parameters.

The **power** transmitted by a sinusoidal wave on a stretched string is

$$\mathcal{P} = \tfrac{1}{2}\mu\omega^2 A^2 v \qquad \textbf{(16.21)}$$

QUESTIONS

1. Why is a wave pulse traveling on a string considered a transverse wave?
2. How would you set up a longitudinal wave in a stretched spring? Would it be possible to set up a transverse wave in a spring?
3. By what factor would you have to increase the tension in a taut string to double the wave speed?
4. When traveling on a taut string, does a wave pulse always invert upon reflection? Explain.
5. Can two pulses traveling in opposite directions on the same string reflect from each other? Explain.
6. Does the vertical speed of a segment of a horizontal, taut string, through which a wave is traveling, depend on the wave speed?
7. If you were to shake one end of a taut rope periodically three times each second, what would be the period of the sinusoidal waves set up in the rope?
8. A vibrating source generates a sinusoidal wave on a string under constant tension. If the power delivered to the string is doubled, by what factor does the amplitude change? Does the wave speed change under these circumstances?
9. Consider a wave traveling on a taut rope. What is the difference, if any, between the speed of the wave and the speed of a small segment of the rope?
10. If a long rope is hung from a ceiling and waves are sent up the rope from its lower end, they do not ascend with constant speed. Explain.
11. What happens to the wavelength of a wave on a string when the frequency is doubled? Assume that the tension in the string remains the same.
12. What happens to the speed of a wave on a taut string when the frequency is doubled? Assume that the tension in the string remains the same.
13. How do transverse waves differ from longitudinal waves?
14. When all the strings on a guitar are stretched to the same tension, will the speed of a wave along the more massive bass strings be faster or slower than the speed of a wave on the lighter strings?
15. If you stretch a rubber hose and pluck it, you can observe a pulse traveling up and down the hose. What happens to the speed of the pulse if you stretch the hose more tightly? What happens to the speed if you fill the hose with water?
16. In a longitudinal wave in a spring, the coils move back and forth in the direction of wave motion. Does the speed of the wave depend on the maximum speed of each coil?
17. When two waves interfere, can the amplitude of the resultant wave be greater than either of the two original waves? Under what conditions?
18. A solid can transport both longitudinal waves and transverse waves, but a fluid can transport only longitudinal waves. Why?

PROBLEMS

1, **2**, **3** = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
= paired numerical/symbolic problems

Section 16.1 Basic Variables of Wave Motion

Section 16.2 Direction of Particle Displacement

Section 16.3 One-Dimensional Traveling Waves

1. At $t = 0$, a transverse wave pulse in a wire is described by the function

$$y = \frac{6}{x^2 + 3}$$

where x and y are in meters. Write the function $y(x, t)$ that describes this wave if it is traveling in the positive x direction with a speed of 4.50 m/s.

2. Two wave pulses A and B are moving in opposite directions along a taut string with a speed of 2.00 cm/s. The amplitude of A is twice the amplitude of B. The pulses are shown in Figure P16.2 at $t = 0$. Sketch the shape of the string at $t = 1$, 1.5, 2, 2.5, and 3 s.

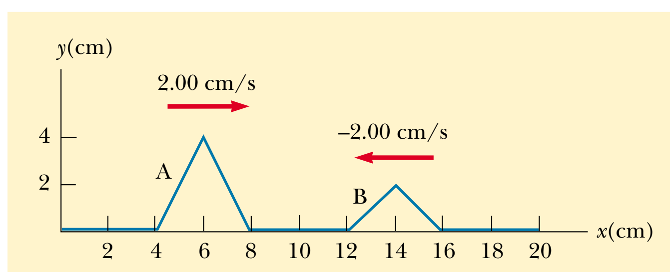


Figure P16.2

3. A wave moving along the x axis is described by

$$y(x, t) = 5.00e^{-(x+5.00t)^2}$$

where x is in meters and t is in seconds. Determine (a) the direction of the wave motion and (b) the speed of the wave.

4. Ocean waves with a crest-to-crest distance of 10.0 m can be described by the equation

$$y(x, t) = (0.800 \text{ m}) \sin[0.628(x - vt)]$$

where $v = 1.20$ m/s. (a) Sketch $y(x, t)$ at $t = 0$. (b) Sketch $y(x, t)$ at $t = 2.00$ s. Note how the entire wave form has shifted 2.40 m in the positive x direction in this time interval.

5. Two points, A and B, on the surface of the Earth are at the same longitude and 60.0° apart in latitude. Suppose that an earthquake at point A sends two waves toward point B. A transverse wave travels along the surface of the Earth at 4.50 km/s, and a longitudinal wave travels straight through the body of the Earth at 7.80 km/s. (a) Which wave arrives at point B first? (b) What is the time difference between the arrivals of the two waves at point B? Take the radius of the Earth to be 6 370 km.

6. A seismographic station receives S and P waves from an earthquake, 17.3 s apart. Suppose that the waves have traveled over the same path at speeds of 4.50 km/s and 7.80 km/s, respectively. Find the distance from the seismometer to the epicenter of the quake.

Section 16.4 Superposition and Interference

WEB **7.** Two sinusoidal waves in a string are defined by the functions

$$y_1 = (2.00 \text{ cm}) \sin(20.0x - 32.0t)$$

and

$$y_2 = (2.00 \text{ cm}) \sin(25.0x - 40.0t)$$

where y and x are in centimeters and t is in seconds. (a) What is the phase difference between these two waves at the point $x = 5.00$ cm at $t = 2.00$ s? (b) What is the positive x value closest to the origin for which the two phases differ by $\pm\pi$ at $t = 2.00$ s? (This is where the sum of the two waves is zero.)

8. Two waves in one string are described by the wave functions

$$y_1 = 3.0 \cos(4.0x - 1.6t)$$

and

$$y_2 = 4.0 \sin(5.0x - 2.0t)$$

where y and x are in centimeters and t is in seconds. Find the superposition of the waves $y_1 + y_2$ at the points (a) $x = 1.00$, $t = 1.00$; (b) $x = 1.00$, $t = 0.500$; (c) $x = 0.500$, $t = 0$. (Remember that the arguments of the trigonometric functions are in radians.)

9. Two pulses traveling on the same string are described by the functions

$$y_1 = \frac{5}{(3x - 4t)^2 + 2}$$

and

$$y_2 = \frac{-5}{(3x + 4t - 6)^2 + 2}$$

(a) In which direction does each pulse travel? (b) At what time do the two cancel? (c) At what point do the two waves always cancel?

Section 16.5 The Speed of Waves on Strings

10. A phone cord is 4.00 m long. The cord has a mass of 0.200 kg. A transverse wave pulse is produced by plucking one end of the taut cord. The pulse makes four trips down and back along the cord in 0.800 s. What is the tension in the cord?

11. Transverse waves with a speed of 50.0 m/s are to be produced in a taut string. A 5.00-m length of string with a total mass of 0.060 0 kg is used. What is the required tension?

12. A piano string having a mass per unit length 5.00×10^{-3} kg/m is under a tension of 1 350 N. Find the speed with which a wave travels on this string.

13. An astronaut on the Moon wishes to measure the local value of g by timing pulses traveling down a wire that has a large mass suspended from it. Assume that the wire has a mass of 4.00 g and a length of 1.60 m, and that a 3.00-kg mass is suspended from it. A pulse requires 36.1 ms to traverse the length of the wire. Calculate g_{Moon} from these data. (You may neglect the mass of the wire when calculating the tension in it.)

14. Transverse pulses travel with a speed of 200 m/s along a taut copper wire whose diameter is 1.50 mm. What is the tension in the wire? (The density of copper is 8.92 g/cm^3.)

15. Transverse waves travel with a speed of 20.0 m/s in a string under a tension of 6.00 N. What tension is required to produce a wave speed of 30.0 m/s in the same string?

16. A simple pendulum consists of a ball of mass M hanging from a uniform string of mass m and length L, with $m \ll M$. If the period of oscillation for the pendulum is T, determine the speed of a transverse wave in the string when the pendulum hangs at rest.

17. The elastic limit of a piece of steel wire is 2.70×10^9 Pa. What is the maximum speed at which transverse wave pulses can propagate along this wire before this stress is exceeded? (The density of steel is 7.86×10^3 kg/m^3.)

18. **Review Problem.** A light string with a mass per unit length of 8.00 g/m has its ends tied to two walls separated by a distance equal to three-fourths the length of the string (Fig. P16.18). An object of mass m is sus-

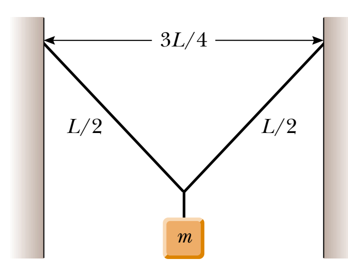


Figure P16.18

pended from the center of the string, putting a tension in the string. (a) Find an expression for the transverse wave speed in the string as a function of the hanging mass. (b) How much mass should be suspended from the string to produce a wave speed of 60.0 m/s?

19. **Review Problem.** A light string with a mass of 10.0 g and a length $L = 3.00$ m has its ends tied to two walls that are separated by the distance $D = 2.00$ m. Two objects, each with a mass $M = 2.00$ kg, are suspended from the string, as shown in Figure P16.19. If a wave pulse is sent from point A, how long does it take for it to travel to point B?

20. **Review Problem.** A light string of mass m and length L has its ends tied to two walls that are separated by the distance D. Two objects, each of mass M, are suspended from the string, as shown in Figure P16.19. If a wave pulse is sent from point A, how long does it take to travel to point B?

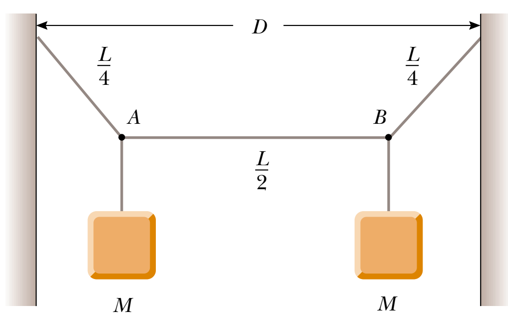


Figure P16.19 Problems 19 and 20.

WEB **21.** A 30.0-m steel wire and a 20.0-m copper wire, both with 1.00-mm diameters, are connected end to end and are stretched to a tension of 150 N. How long does it take a transverse wave to travel the entire length of the two wires?

Section 16.6 Reflection and Transmission

22. A series of pulses, each of amplitude 0.150 m, are sent down a string that is attached to a post at one end. The pulses are reflected at the post and travel back along the string without loss of amplitude. What is the displacement at a point on the string where two pulses are crossing (a) if the string is rigidly attached to the post? (b) if the end at which reflection occurs is free to slide up and down?

Section 16.7 Sinusoidal Waves

23. (a) Plot y versus t at $x = 0$ for a sinusoidal wave of the form $y = (15.0 \text{ cm}) \cos(0.157x - 50.3t)$, where x and y are in centimeters and t is in seconds. (b) Determine the period of vibration from this plot and compare your result with the value found in Example 16.3.

24. For a certain transverse wave, the distance between two successive crests is 1.20 m, and eight crests pass a given point along the direction of travel every 12.0 s. Calculate the wave speed.

25. A sinusoidal wave is traveling along a rope. The oscillator that generates the wave completes 40.0 vibrations in 30.0 s. Also, a given maximum travels 425 cm along the rope in 10.0 s. What is the wavelength?

26. Consider the sinusoidal wave of Example 16.3, with the wave function

$$y = (15.0 \text{ cm}) \cos(0.157x - 50.3t)$$

At a certain instant, let point A be at the origin and point B be the first point along the x axis where the wave is 60.0° out of phase with point A. What is the coordinate of point B?

27. When a particular wire is vibrating with a frequency of 4.00 Hz, a transverse wave of wavelength 60.0 cm is produced. Determine the speed of wave pulses along the wire.

28. A sinusoidal wave traveling in the $-x$ direction (to the left) has an amplitude of 20.0 cm, a wavelength of 35.0 cm, and a frequency of 12.0 Hz. The displacement of the wave at $t = 0$, $x = 0$ is $y = -3.00$ cm; at this same point, a particle of the medium has a positive velocity. (a) Sketch the wave at $t = 0$. (b) Find the angular wave number, period, angular frequency, and wave speed of the wave. (c) Write an expression for the wave function $y(x, t)$.

29. A sinusoidal wave train is described by the equation

$$y = (0.25 \text{ m}) \sin(0.30x - 40t)$$

where x and y are in meters and t is in seconds. Determine for this wave the (a) amplitude, (b) angular frequency, (c) angular wave number, (d) wavelength, (e) wave speed, and (f) direction of motion.

30. A transverse wave on a string is described by the expression

$$y = (0.120 \text{ m}) \sin(\pi x/8 + 4\pi t)$$

(a) Determine the transverse speed and acceleration of the string at $t = 0.200$ s for the point on the string located at $x = 1.60$ m. (b) What are the wavelength, period, and speed of propagation of this wave?

WEB **31.** (a) Write the expression for y as a function of x and t for a sinusoidal wave traveling along a rope in the *negative* x direction with the following characteristics: $A = 8.00$ cm, $\lambda = 80.0$ cm, $f = 3.00$ Hz, and $y(0, t) = 0$ at $t = 0$. (b) Write the expression for y as a function of x and t for the wave in part (a), assuming that $y(x, 0) = 0$ at the point $x = 10.0$ cm.

32. A transverse sinusoidal wave on a string has a period $T = 25.0$ ms and travels in the negative x direction with a speed of 30.0 m/s. At $t = 0$, a particle on the string at

$x = 0$ has a displacement of 2.00 cm and travels downward with a speed of 2.00 m/s. (a) What is the amplitude of the wave? (b) What is the initial phase angle? (c) What is the maximum transverse speed of the string? (d) Write the wave function for the wave.

33. A sinusoidal wave of wavelength 2.00 m and amplitude 0.100 m travels on a string with a speed of 1.00 m/s to the right. Initially, the left end of the string is at the origin. Find (a) the frequency and angular frequency, (b) the angular wave number, and (c) the wave function for this wave. Determine the equation of motion for (d) the left end of the string and (e) the point on the string at $x = 1.50$ m to the right of the left end. (f) What is the maximum speed of any point on the string?

34. A sinusoidal wave on a string is described by the equation

$$y = (0.51 \text{ cm}) \sin(kx - \omega t)$$

where $k = 3.10$ rad/cm and $\omega = 9.30$ rad/s. How far does a wave crest move in 10.0 s? Does it move in the positive or negative x direction?

35. A wave is described by $y = (2.00 \text{ cm}) \sin(kx - \omega t)$, where $k = 2.11$ rad/m, $\omega = 3.62$ rad/s, x is in meters, and t is in seconds. Determine the amplitude, wavelength, frequency, and speed of the wave.

36. A transverse traveling wave on a taut wire has an amplitude of 0.200 mm and a frequency of 500 Hz. It travels with a speed of 196 m/s. (a) Write an equation in SI units of the form $y = A \sin(kx - \omega t)$ for this wave. (b) The mass per unit length of this wire is 4.10 g/m. Find the tension in the wire.

37. A wave on a string is described by the wave function

$$y = (0.100 \text{ m}) \sin(0.50x - 20t)$$

(a) Show that a particle in the string at $x = 2.00$ m executes simple harmonic motion. (b) Determine the frequency of oscillation of this particular point.

Section 16.8 Rate of Energy Transfer by Sinusoidal Waves on Strings

38. A taut rope has a mass of 0.180 kg and a length of 3.60 m. What power must be supplied to the rope to generate sinusoidal waves having an amplitude of 0.100 m and a wavelength of 0.500 m and traveling with a speed of 30.0 m/s?

39. A two-dimensional water wave spreads in circular wave fronts. Show that the amplitude A at a distance r from the initial disturbance is proportional to $1/\sqrt{r}$. (*Hint:* Consider the energy carried by one outward-moving ripple.)

40. Transverse waves are being generated on a rope under constant tension. By what factor is the required power increased or decreased if (a) the length of the rope is doubled and the angular frequency remains constant, (b) the amplitude is doubled and the angular frequency is halved, (c) both the wavelength and the amplitude are doubled, and (d) both the length of the rope and the wavelength are halved?

WEB **41.** Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string that has a linear mass density of 4.00×10^{-2} kg/m. If the source can deliver a maximum power of 300 W and the string is under a tension of 100 N, what is the highest vibrational frequency at which the source can operate?

42. It is found that a 6.00-m segment of a long string contains four complete waves and has a mass of 180 g. The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley displacement of 15.0 cm. (The "peak-to-valley" distance is the vertical distance from the farthest positive displacement to the farthest negative displacement.) (a) Write the function that describes this wave traveling in the positive x direction. (b) Determine the power being supplied to the string.

43. A sinusoidal wave on a string is described by the equation

$$y = (0.15 \text{ m}) \sin(0.80x - 50t)$$

where x and y are in meters and t is in seconds. If the mass per unit length of this string is 12.0 g/m, determine (a) the speed of the wave, (b) the wavelength, (c) the frequency, and (d) the power transmitted to the wave.

44. A horizontal string can transmit a maximum power of $\mathcal{P}$ (without breaking) if a wave with amplitude A and angular frequency ω is traveling along it. To increase this maximum power, a student folds the string and uses the "double string" as a transmitter. Determine the maximum power that can be transmitted along the "double string," supposing that the tension is constant.

(Optional)

Section 16.9 The Linear Wave Equation

45. (a) Evaluate A in the scalar equality $(7 + 3)4 = A$. (b) Evaluate A, B, and C in the vector equality $7.00\mathbf{i} + 3.00\mathbf{k} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$. Explain how you arrive at your answers. (c) The functional equality or identity

$$A + B\cos(Cx + Dt + E) = (7.00 \text{ mm}) \cos(3x + 4t + 2)$$

is true for all values of the variables x and t, which are measured in meters and in seconds, respectively. Evaluate the constants A, B, C, D, and E. Explain how you arrive at your answers.

46. Show that the wave function $y = e^{b(x - vt)}$ is a solution of the wave equation (Eq. 16.26), where b is a constant.

47. Show that the wave function $y = \ln[b(x - vt)]$ is a solution to Equation 16.26, where b is a constant.

48. (a) Show that the function $y(x, t) = x^2 + v^2t^2$ is a solution to the wave equation. (b) Show that the function above can be written as $f(x + vt) + g(x - vt)$, and determine the functional forms for f and g. (c) Repeat parts (a) and (b) for the function $y(x, t) = \sin(x) \cos(vt)$.

ADDITIONAL PROBLEMS

49. The "wave" is a particular type of wave pulse that can sometimes be seen propagating through a large crowd gathered at a sporting arena to watch a soccer or American football match (Fig. P16.49). The particles of the medium are the spectators, with zero displacement corresponding to their being in the seated position and maximum displacement corresponding to their being in the standing position and raising their arms. When a large fraction of the spectators participate in the wave motion, a somewhat stable pulse shape can develop. The wave speed depends on people's reaction time, which is typically on the order of 0.1 s. Estimate the order of magnitude, in minutes, of the time required for such a wave pulse to make one circuit around a large sports stadium. State the quantities you measure or estimate and their values.

Figure P16.49

50. A traveling wave propagates according to the expression $y = (4.0 \text{ cm}) \sin(2.0x - 3.0t)$, where x is in centimeters and t is in seconds. Determine (a) the amplitude, (b) the wavelength, (c) the frequency, (d) the period, and (e) the direction of travel of the wave.

WEB **51.** The wave function for a traveling wave on a taut string is (in SI units)

$$y(x, t) = (0.350 \text{ m}) \sin(10\pi t - 3\pi x + \pi/4)$$

(a) What are the speed and direction of travel of the wave? (b) What is the vertical displacement of the string at $t = 0$, $x = 0.100$ m? (c) What are the wavelength and frequency of the wave? (d) What is the maximum magnitude of the transverse speed of the string?

52. Motion picture film is projected at 24.0 frames per second. Each frame is a photograph 19.0 mm in height. At what constant speed does the film pass into the projector?

53. **Review Problem.** A block of mass M, supported by a string, rests on an incline making an angle θ with the horizontal (Fig. P16.53). The string's length is L, and its mass is $m \ll M$. Derive an expression for the time it takes a transverse wave to travel from one end of the string to the other.

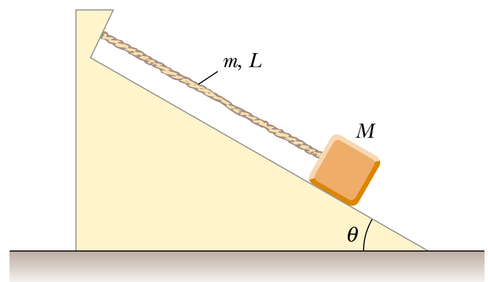


Figure P16.53

54. (a) Determine the speed of transverse waves on a string under a tension of 80.0 N if the string has a length of 2.00 m and a mass of 5.00 g. (b) Calculate the power required to generate these waves if they have a wavelength of 16.0 cm and an amplitude of 4.00 cm.

55. **Review Problem.** A 2.00-kg block hangs from a rubber cord. The block is supported so that the cord is not stretched. The unstretched length of the cord is 0.500 m, and its mass is 5.00 g. The "spring constant" for the cord is 100 N/m. The block is released and stops at the lowest point. (a) Determine the tension in the cord when the block is at this lowest point. (b) What is the length of the cord in this "stretched" position? (c) Find the speed of a transverse wave in the cord if the block is held in this lowest position.

56. **Review Problem.** A block of mass M hangs from a rubber cord. The block is supported so that the cord is not stretched. The unstretched length of the cord is L_0, and its mass is m, much less than M. The "spring constant" for the cord is k. The block is released and stops at the lowest point. (a) Determine the tension in the cord when the block is at this lowest point. (b) What is the length of the cord in this "stretched" position? (c) Find the speed of a transverse wave in the cord if the block is held in this lowest position.

57. A sinusoidal wave in a rope is described by the wave function

$$y = (0.20 \text{ m}) \sin(0.75\pi x + 18\pi t)$$

where x and y are in meters and t is in seconds. The rope has a linear mass density of 0.250 kg/m. If the tension in the rope is provided by an arrangement like the one illustrated in Figure 16.12, what is the value of the suspended mass?

58. A wire of density ρ is tapered so that its cross-sectional area varies with x, according to the equation

$$A = (1.0 \times 10^{-3}x + 0.010) \text{ cm}^2$$

(a) If the wire is subject to a tension T, derive a relationship for the speed of a wave as a function of position. (b) If the wire is aluminum and is subject to a tension of 24.0 N, determine the speed at the origin and at $x = 10.0$ m.

59. A rope of total mass m and length L is suspended vertically. Show that a transverse wave pulse travels the length of the rope in a time $t = 2\sqrt{L/g}$. (*Hint:* First find an expression for the wave speed at any point a distance x from the lower end by considering the tension in the rope as resulting from the weight of the segment below that point.)

60. If mass M is suspended from the bottom of the rope in Problem 59, (a) show that the time for a transverse wave to travel the length of the rope is

$$t = 2\sqrt{\frac{L}{mg}}\left[\sqrt{(M + m)} - \sqrt{M}\right]$$

(b) Show that this reduces to the result of Problem 59 when $M = 0$. (c) Show that for $m \ll M$, the expression in part (a) reduces to

$$t = \sqrt{\frac{mL}{Mg}}$$

61. It is stated in Problem 59 that a wave pulse travels from the bottom to the top of a rope of length L in a time $t = 2\sqrt{L/g}$. Use this result to answer the following questions. (It is *not* necessary to set up any new integrations.) (a) How long does it take for a wave pulse to travel halfway up the rope? (Give your answer as a fraction of the quantity $2\sqrt{L/g}$.) (b) A pulse starts traveling up the rope. How far has it traveled after a time $\sqrt{L/g}$?

62. Determine the speed and direction of propagation of each of the following sinusoidal waves, assuming that x is measured in meters and t in seconds:
(a) $y = 0.60 \cos(3.0x - 15t + 2)$
(b) $y = 0.40 \cos(3.0x + 15t - 2)$
(c) $y = 1.2 \sin(15t + 2.0x)$
(d) $y = 0.20 \sin(12t - x/2 + \pi)$

63. **Review Problem.** An aluminum wire under zero tension at room temperature is clamped at each end. The tension in the wire is increased by reducing the temperature, which results in a decrease in the wire's equilibrium length. What strain $(\Delta L/L)$ results in a transverse wave speed of 100 m/s? Take the cross-sectional area of the wire to be 5.00×10^{-6} m^2, the density of the material to be 2.70×10^3 kg/m^3, and Young's modulus to be 7.00×10^{10} N/m^2.

64. (a) Show that the speed of longitudinal waves along a spring of force constant k is $v = \sqrt{kL/\mu}$, where L is the unstretched length of the spring and μ is the mass per unit length. (b) A spring with a mass of 0.400 kg has an unstretched length of 2.00 m and a force constant of 100 N/m. Using the result you obtained in (a), determine the speed of longitudinal waves along this spring.

65. A string of length L consists of two sections: The left half has mass per unit length $\mu = \mu_0/2$, whereas the right half has a mass per unit length $\mu' = 3\mu = 3\mu_0/2$. Tension in the string is T_0. Notice from the data given that this string has the same total mass as a uniform string of length L and of mass per unit length μ_0.
(a) Find the speeds v and v' at which transverse wave pulses travel in the two sections. Express the speeds in terms of T_0 and μ_0, and also as multiples of the speed $v_0 = (T_0/\mu_0)^{1/2}$. (b) Find the time required for a pulse to travel from one end of the string to the other. Give your result as a multiple of $t_0 = L/v_0$.

66. A wave pulse traveling along a string of linear mass density μ is described by the relationship

$$y = [A_0 e^{-bx}] \sin(kx - \omega t)$$

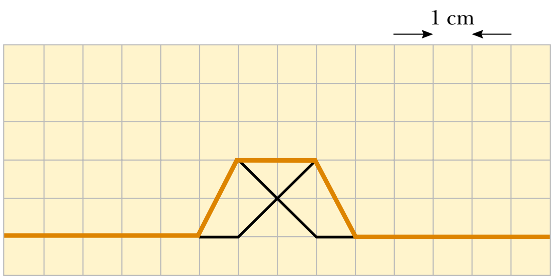
where the factor in brackets before the sine function is said to be the amplitude. (a) What is the power $\mathcal{P}(x)$ carried by this wave at a point x? (b) What is the power carried by this wave at the origin? (c) Compute the ratio $\mathcal{P}(x)/\mathcal{P}(0)$.

67. An earthquake on the ocean floor in the Gulf of Alaska produces a *tsunami* (sometimes called a "tidal wave") that reaches Hilo, Hawaii, 4 450 km away, in a time of 9 h 30 min. Tsunamis have enormous wavelengths (100–200 km), and the propagation speed of these waves is $v \approx \sqrt{gd}$, where d is the average depth of the water. From the information given, find the average wave speed and the average ocean depth between Alaska and Hawaii. (This method was used in 1856 to estimate the average depth of the Pacific Ocean long before soundings were made to obtain direct measurements.)

ANSWERS TO QUICK QUIZZES

16.1 (a) It is longitudinal because the disturbance (the shift of position) is parallel to the direction in which the wave travels. (b) It is transverse because the people stand up and sit down (vertical motion), whereas the wave moves either to the left or to the right (motion perpendicular to the disturbance).

16.2



16.3 Only answers (f) and (h) are correct. (a) and (b) affect the transverse speed of a particle of the string, but not the wave speed along the string. (c) and (d) change the amplitude. (e) and (g) increase the time by decreasing the wave speed.

16.4 The transverse speed increases because $v_{y,\,\max} = \omega A = 2\pi f A$. The wave speed does not change because it depends only on the tension and mass per length of the string, neither of which has been modified. The wavelength must decrease because the wave speed $v = \lambda f$ remains constant.