PUZZLER

Jennifer is holding on to an electrically charged sphere that reaches an electric potential of about 100 000 V. The device that generates this high electric potential is called a *Van de Graaff generator.* What causes Jennifer's hair to stand on end like the needles of a porcupine? Why is she safe in this situation in view of the fact that 110 V from a wall outlet can kill you? *(Henry Leap and Jim Lehman)*

chapter

25 Electric Potential

Chapter Outline

The concept of potential energy was introduced in Chapter 8 in connection with such conservative forces as the force of gravity and the elastic force exerted by a spring. By using the law of conservation of energy, we were able to avoid working directly with forces when solving various problems in mechanics. In this chapter we see that the concept of potential energy is also of great value in the study of electricity. Because the electrostatic force given by Coulomb's law is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy. This idea enables us to define a scalar quantity known as *electric potential.* Because the electric potential at any point in an electric field is a scalar function, we can use it to describe electrostatic phenomena more simply than if we were to rely only on the concepts of the electric field and electric forces. In later chapters we shall see that the concept of electric potential is of great practical value.

25.1 POTENTIAL DIFFERENCE AND ELECTRIC POTENTIAL

11.8 When a test charge q_0 is placed in an electric field $\mathbf{E}$ created by some other charged object, the electric force acting on the test charge is $q_0\mathbf{E}$. (If the field is produced by more than one charged object, this force acting on the test charge is the vector sum of the individual forces exerted on it by the various other charged objects.) The force $q_0\mathbf{E}$ is conservative because the individual forces described by Coulomb's law are conservative. When the test charge is moved in the field by some external agent, the work done by the field on the charge is equal to the negative of the work done by the external agent causing the displacement. For an infinitesimal displacement $d\mathbf{s}$, the work done by the electric field on the charge is $\mathbf{F}\cdot d\mathbf{s} = q_0\mathbf{E}\cdot d\mathbf{s}$. As this amount of work is done by the field, the potential energy of the charge–field system is decreased by an amount $dU = -q_0\mathbf{E}\cdot d\mathbf{s}$. For a finite displacement of the charge from a point A to a point B, the change in potential energy of the system $\Delta U = U_B - U_A$ is

$$\Delta U = -q_0 \int_A^B \mathbf{E}\cdot d\mathbf{s} \qquad (25.1)$$

Change in potential energy

The integration is performed along the path that q_0 follows as it moves from A to B, and the integral is called either a *path integral* or a *line integral* (the two terms are synonymous). Because the force $q_0\mathbf{E}$ is conservative, **this line integral does not depend on the path taken from *A* to *B*.**

Quick Quiz 25.1

If the path between A and B does not make any difference in Equation 25.1, why don't we just use the expression $\Delta U = -q_0 Ed$, where d is the straight-line distance between A and B?

The potential energy per unit charge U/q_0 is independent of the value of q_0 and has a unique value at every point in an electric field. This quantity U/q_0 is called the **electric potential** (or simply the **potential**) V. Thus, the electric potential at any point in an electric field is

$$V = \frac{U}{q_0} \qquad (25.2)$$

The fact that potential energy is a scalar quantity means that electric potential also is a scalar quantity.

The **potential difference** $\Delta V = V_B - V_A$ between any two points A and B in an electric field is defined as the change in potential energy of the system divided by the test charge q_0:

Potential difference

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad \textbf{(25.3)}$$

Potential difference should not be confused with difference in potential energy. The potential difference is proportional to the change in potential energy, and we see from Equation 25.3 that the two are related by $\Delta U = q_0 \Delta V$.

Electric potential is a scalar characteristic of an electric field, independent of the charges that may be placed in the field. However, when we speak of potential energy, we are referring to the charge–field system. Because we are usually interested in knowing the electric potential at the location of a charge and the potential energy resulting from the interaction of the charge with the field, we follow the common convention of speaking of the potential energy as if it belonged to the charge.

Because the change in potential energy of a charge is the negative of the work done by the electric field on the charge (as noted in Equation 25.1), the potential difference ΔV between points A and B equals the work per unit charge that an external agent must perform to move a test charge from A to B without changing the kinetic energy of the test charge.

Just as with potential energy, only *differences* in electric potential are meaningful. To avoid having to work with potential differences, however, we often take the value of the electric potential to be zero at some convenient point in an electric field. This is what we do here: arbitrarily establish the electric potential to be zero at a point that is infinitely remote from the charges producing the field. Having made this choice, we can state that the **electric potential at an arbitrary point in an electric field equals the work required per unit charge to bring a positive test charge from infinity to that point.** Thus, if we take point A in Equation 25.3 to be at infinity, the electric potential at any point P is

$$V_P = -\int_\infty^P \mathbf{E} \cdot d\mathbf{s} \qquad \textbf{(25.4)}$$

In reality, V_P represents the potential difference ΔV between the point P and a point at infinity. (Eq. 25.4 is a special case of Eq. 25.3.)

Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a **volt** (V):

Definition of volt

$$1\ \mathrm{V} \equiv 1\ \frac{\mathrm{J}}{\mathrm{C}}$$

That is, 1 J of work must be done to move a 1-C charge through a potential difference of 1 V.

Equation 25.3 shows that potential difference also has units of electric field times distance. From this, it follows that the SI unit of electric field (N/C) can also be expressed in volts per meter:

$$1\ \frac{\mathrm{N}}{\mathrm{C}} = 1\ \frac{\mathrm{V}}{\mathrm{m}}$$

A unit of energy commonly used in atomic and nuclear physics is the **electron volt** (eV), which is defined as **the energy an electron (or proton) gains or loses by moving through a potential difference of 1 V.** Because 1 V = 1 J/C and because the fundamental charge is approximately 1.60×10^{-19} C, the electron volt is related to the joule as follows:

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ C}\cdot\text{V} = 1.60 \times 10^{-19} \text{ J} \qquad \textbf{(25.5)}$$

The electron volt

For instance, an electron in the beam of a typical television picture tube may have a speed of 3.5×10^7 m/s. This corresponds to a kinetic energy of 5.6×10^{-16} J, which is equivalent to 3.5×10^3 eV. Such an electron has to be accelerated from rest through a potential difference of 3.5 kV to reach this speed.

25.2 POTENTIAL DIFFERENCES IN A UNIFORM ELECTRIC FIELD

Equations 25.1 and 25.3 hold in all electric fields, whether uniform or varying, but they can be simplified for a uniform field. First, consider a uniform electric field directed along the negative y axis, as shown in Figure 25.1a. Let us calculate the potential difference between two points A and B separated by a distance d, where d is measured parallel to the field lines. Equation 25.3 gives

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\int_A^B E \cos 0^\circ \, ds = -\int_A^B E \, ds$$

Because E is constant, we can remove it from the integral sign; this gives

$$\Delta V = -E\int_A^B ds = -Ed \qquad \textbf{(25.6)}$$

Potential difference in a uniform electric field

The minus sign indicates that point B is at a lower electric potential than point A; that is, $V_B < V_A$. **Electric field lines always point in the direction of decreasing electric potential,** as shown in Figure 25.1a.

Now suppose that a test charge q_0 moves from A to B. We can calculate the change in its potential energy from Equations 25.3 and 25.6:

$$\Delta U = q_0 \, \Delta V = -q_0 E d \qquad \textbf{(25.7)}$$

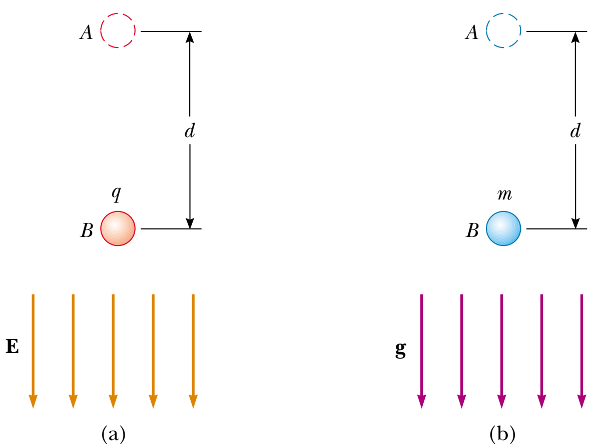


Figure 25.1 (a) When the electric field **E** is directed downward, point B is at a lower electric potential than point A. A positive test charge that moves from point A to point B loses electric potential energy. (b) A mass m moving downward in the direction of the gravitational field **g** loses gravitational potential energy.

QuickLab

It takes an electric field of about 30 000 V/cm to cause a spark in dry air. Shuffle across a rug and reach toward a doorknob. By estimating the length of the spark, determine the electric potential difference between your finger and the doorknob after shuffling your feet but before touching the knob. (If it is very humid on the day you attempt this, it may not work. Why?)

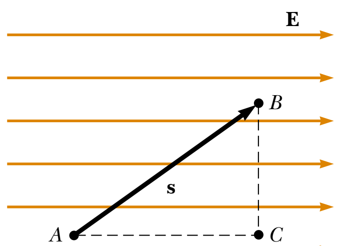


Figure 25.2 A uniform electric field directed along the positive x axis. Point B is at a lower electric potential than point A. Points B and C are at the *same* electric potential.

From this result, we see that if q_0 is positive, then ΔU is negative. We conclude that **a positive charge loses electric potential energy when it moves in the direction of the electric field.** This means that an electric field does work on a positive charge when the charge moves in the direction of the electric field. (This is analogous to the work done by the gravitational field on a falling mass, as shown in Figure 25.1b.) If a positive test charge is released from rest in this electric field, it experiences an electric force $q_0\mathbf{E}$ in the direction of $\mathbf{E}$ (downward in Fig. 25.1a). Therefore, it accelerates downward, gaining kinetic energy. **As the charged particle gains kinetic energy, it loses an equal amount of potential energy.**

If q_0 is negative, then ΔU is positive and the situation is reversed: **A negative charge gains electric potential energy when it moves in the direction of the electric field.** If a negative charge is released from rest in the field $\mathbf{E}$, it accelerates in a direction opposite the direction of the field.

Now consider the more general case of a charged particle that is free to move between any two points in a uniform electric field directed along the x axis, as shown in Figure 25.2. (In this situation, the charge is not being moved by an external agent as before.) If $\mathbf{s}$ represents the displacement vector between points A and B, Equation 25.3 gives

$$\Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\mathbf{E}\cdot\int_A^B d\mathbf{s} = -\mathbf{E}\cdot\mathbf{s} \qquad \textbf{(25.8)}$$

where again we are able to remove $\mathbf{E}$ from the integral because it is constant. The change in potential energy of the charge is

$$\Delta U = q_0\,\Delta V = -q_0\,\mathbf{E}\cdot\mathbf{s} \qquad \textbf{(25.9)}$$

11.9 Finally, we conclude from Equation 25.8 that all points in a plane perpendicular to a uniform electric field are at the same electric potential. We can see this in Figure 25.2, where the potential difference $V_B - V_A$ is equal to the potential difference $V_C - V_A$. (Prove this to yourself by working out the dot product $\mathbf{E}\cdot\mathbf{s}$ for $\mathbf{s}_{A\rightarrow B}$, where the angle θ between $\mathbf{E}$ and $\mathbf{s}$ is arbitrary as shown in Figure 25.2, and the dot product for $\mathbf{s}_{A\rightarrow C}$, where $\theta = 0$.) Therefore, $V_B = V_C$. **The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.**

An equipotential surface

Note that because $\Delta U = q_0\Delta V$, no work is done in moving a test charge between any two points on an equipotential surface. The equipotential surfaces of a uniform electric field consist of a family of planes that are all perpendicular to the field. Equipotential surfaces for fields with other symmetries are described in later sections.

Quick Quiz 25.2

The labeled points in Figure 25.3 are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves from A to B; from B to C; from C to D; from D to E.

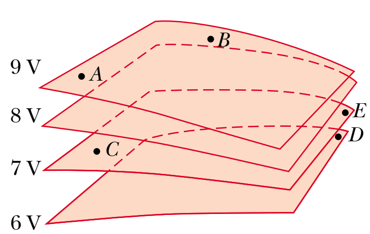


Figure 25.3 Four equipotential surfaces.

EXAMPLE 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge

A battery produces a specified potential difference between conductors attached to the battery terminals. A 12-V battery is connected between two parallel plates, as shown in Figure 25.4. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and if we do not consider points near the plate edges.) Find the magnitude of the electric field between the plates.

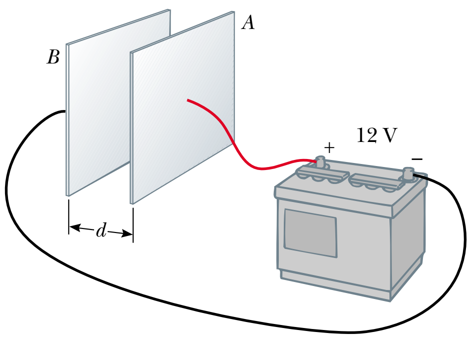


Figure 25.4 A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference ΔV divided by the plate separation d.

Solution The electric field is directed from the positive plate (A) to the negative one (B), and the positive plate is at a higher electric potential than the negative plate is. The potential difference between the plates must equal the potential difference between the battery terminals. We can understand this by noting that all points on a conductor in equilibrium are at the same electric potential[1]; no potential difference exists between a terminal and any portion of the plate to which it is connected. Therefore, the magnitude of the electric field between the plates is, from Equation 25.6,

$$E = \frac{|V_B - V_A|}{d} = \frac{12\text{ V}}{0.30 \times 10^{-2}\text{ m}} = \boxed{4.0 \times 10^3\text{ V/m}}$$

This configuration, which is shown in Figure 25.4 and called a *parallel-plate capacitor,* is examined in greater detail in Chapter 26.

EXAMPLE 25.2 Motion of a Proton in a Uniform Electric Field

A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m and is directed along the positive x axis (Fig. 25.5). The proton undergoes a displacement of 0.50 m in the direction of **E**. (a) Find the change in electric potential between points A and B.

Solution Because the proton (which, as you remember, carries a positive charge) moves in the direction of the field, we expect it to move to a position of lower electric potential.

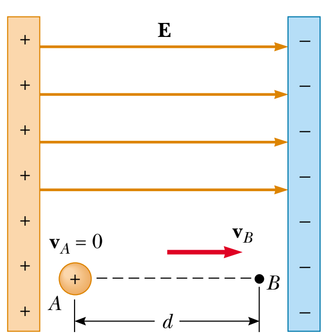


Figure 25.5 A proton accelerates from A to B in the direction of the electric field.

From Equation 25.6, we have

$$\Delta V = -Ed = -(8.0 \times 10^4\text{ V/m})(0.50\text{ m})$$
$$= \boxed{-4.0 \times 10^4\text{ V}}$$

(b) Find the change in potential energy of the proton for this displacement.

Solution

$$\Delta U = q_0\,\Delta V = e\,\Delta V$$
$$= (1.6 \times 10^{-19}\text{ C})(-4.0 \times 10^4\text{ V})$$
$$= \boxed{-6.4 \times 10^{-15}\text{ J}}$$

The negative sign means the potential energy of the proton decreases as it moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time loses electric potential energy (because energy is conserved).

Exercise Use the concept of conservation of energy to find the speed of the proton at point B.

Answer 2.77×10^6 m/s.

[1] The electric field vanishes within a conductor in electrostatic equilibrium; thus, the path integral $\int \mathbf{E} \cdot d\mathbf{s}$ between any two points in the conductor must be zero. A more complete discussion of this point is given in Section 25.6.

25.3 ELECTRIC POTENTIAL AND POTENTIAL ENERGY DUE TO POINT CHARGES

Consider an isolated positive point charge q. Recall that such a charge produces an electric field that is directed radially outward from the charge. To find the electric potential at a point located a distance r from the charge, we begin with the general expression for potential difference:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

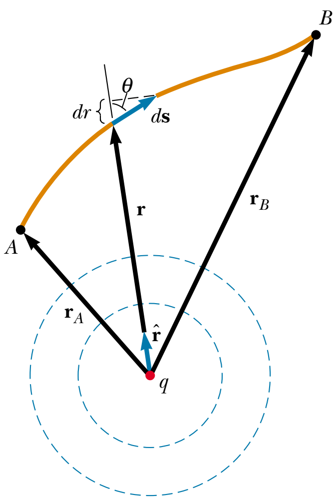


Figure 25.6 The potential difference between points A and B due to a point charge q depends *only* on the initial and final radial coordinates r_A and r_B. The two dashed circles represent cross-sections of spherical equipotential surfaces.

where A and B are the two arbitrary points shown in Figure 25.6. At any field point, the electric field due to the point charge is $\mathbf{E} = k_e q\hat{\mathbf{r}}/r^2$ (Eq. 23.4), where $\hat{\mathbf{r}}$ is a unit vector directed from the charge toward the field point. The quantity $\mathbf{E} \cdot d\mathbf{s}$ can be expressed as

$$\mathbf{E} \cdot d\mathbf{s} = k_e \frac{q}{r^2}\, \hat{\mathbf{r}} \cdot d\mathbf{s}$$

Because the magnitude of $\hat{\mathbf{r}}$ is 1, the dot product $\hat{\mathbf{r}} \cdot d\mathbf{s} = ds \cos\theta$, where θ is the angle between $\hat{\mathbf{r}}$ and $d\mathbf{s}$. Furthermore, $ds \cos\theta$ is the projection of $d\mathbf{s}$ onto $\mathbf{r}$; thus, $ds \cos\theta = dr$. That is, any displacement $d\mathbf{s}$ along the path from point A to point B produces a change dr in the magnitude of $\mathbf{r}$, the radial distance to the charge creating the field. Making these substitutions, we find that $\mathbf{E} \cdot d\mathbf{s} = (k_e q/r^2)\, dr$; hence, the expression for the potential difference becomes

$$V_B - V_A = -\int E_r\, dr = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r}\Bigg]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \qquad \textbf{(25.10)}$$

The integral of $\mathbf{E} \cdot d\mathbf{s}$ is *independent* of the path between points A and B—as it must be because the electric field of a point charge is conservative. Furthermore, Equation 25.10 expresses the important result that the potential difference between any two points A and B in a field created by a point charge depends only on the radial coordinates r_A and r_B. It is customary to choose the reference of electric potential to be zero at $r_A = \infty$. With this reference, the electric potential created by a point charge at any distance r from the charge is

Electric potential created by a point charge

$$V = k_e \frac{q}{r} \qquad \textbf{(25.11)}$$

Electric potential is graphed in Figure 25.7 as a function of r, the radial distance from a positive charge in the xy plane. Consider the following analogy to gravitational potential: Imagine trying to roll a marble toward the top of a hill shaped like Figure 25.7a. The gravitational force experienced by the marble is analogous to the repulsive force experienced by a positively charged object as it approaches another positively charged object. Similarly, the electric potential graph of the region surrounding a negative charge is analogous to a "hole" with respect to any approaching positively charged objects. A charged object must be infinitely distant from another charge before the surface is "flat" and has an electric potential of zero.

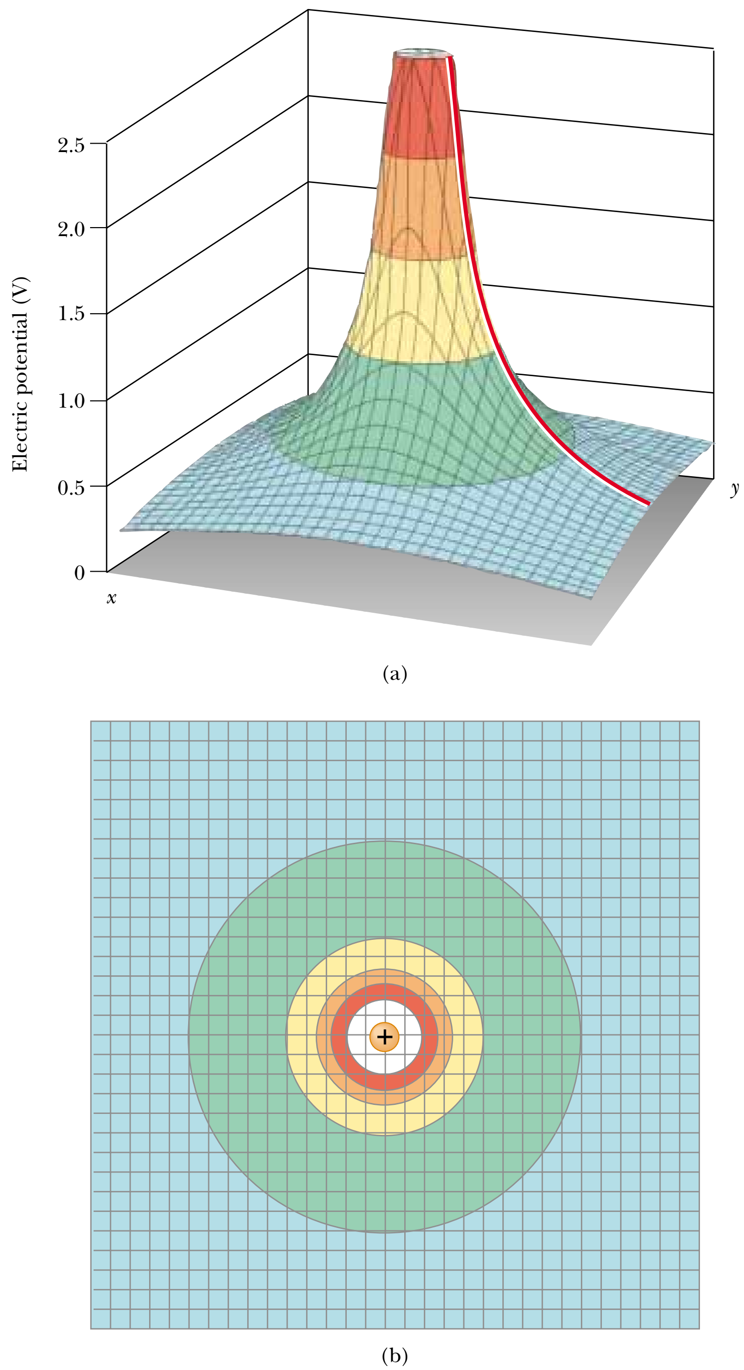


Figure 25.7 (a) The electric potential in the plane around a single positive charge is plotted on the vertical axis. (The electric potential function for a negative charge would look like a hole instead of a hill.) The red line shows the $1/r$ nature of the electric potential, as given by Equation 25.11. (b) View looking straight down the vertical axis of the graph in part (a), showing concentric circles where the electric potential is constant. These circles are cross sections of equipotential spheres having the charge at the center.

Quick Quiz 25.3

A spherical balloon contains a positively charged object at its center. As the balloon is inflated to a greater volume while the charged object remains at the center, does the electric potential at the surface of the balloon increase, decrease, or remain the same? How about the magnitude of the electric field? The electric flux?

We obtain the electric potential resulting from two or more point charges by applying the superposition principle. That is, the total electric potential at some point P due to several point charges is the sum of the potentials due to the individual charges. For a group of point charges, we can write the total electric potential at P in the form

Electric potential due to several point charges

$$V = k_e \sum_i \frac{q_i}{r_i} \qquad \textbf{(25.12)}$$

where the potential is again taken to be zero at infinity and r_i is the distance from the point P to the charge q_i. Note that the sum in Equation 25.12 is an algebraic sum of scalars rather than a vector sum (which we use to calculate the electric field of a group of charges). Thus, it is often much easier to evaluate V than to evaluate $\mathbf{E}$. The electric potential around a dipole is illustrated in Figure 25.8.

We now consider the potential energy of a system of two charged particles. If V_1 is the electric potential at a point P due to charge q_1, then the work an external agent must do to bring a second charge q_2 from infinity to P without acceleration is q_2V_1. By definition, this work equals the potential energy U of the two-particle system when the particles are separated by a distance r_{12} (Fig. 25.9). Therefore, we can express the potential energy as[2]

Electric potential energy due to two charges

$$U = k_e \frac{q_1 q_2}{r_{12}} \qquad \textbf{(25.13)}$$

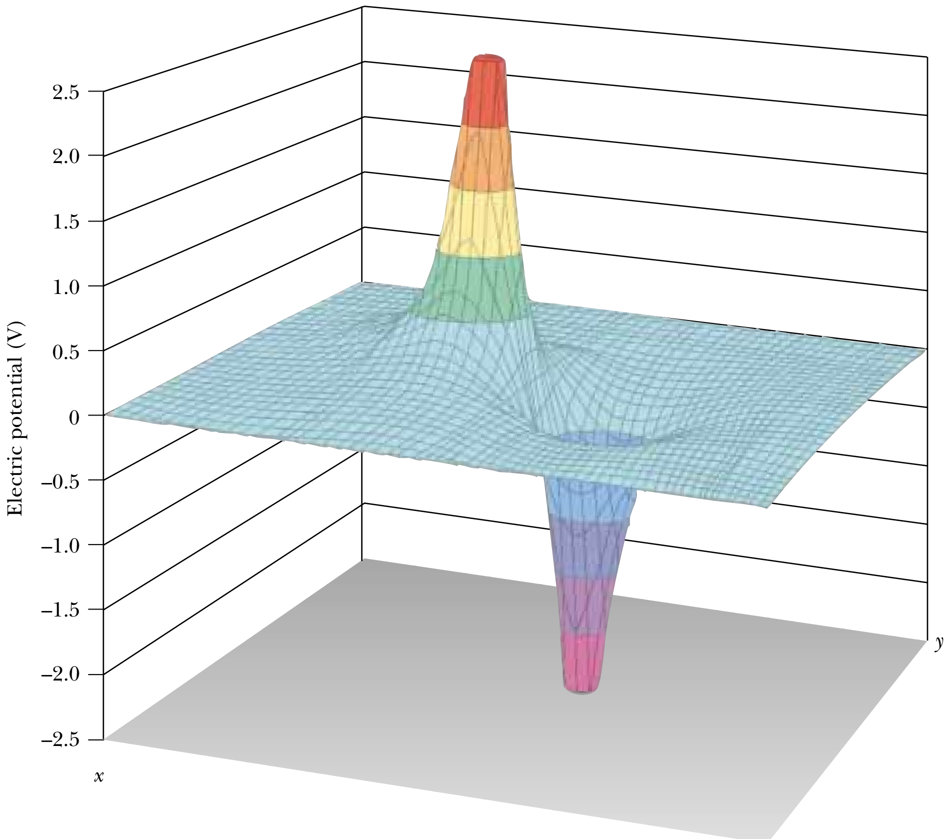
Note that if the charges are of the same sign, U is positive. This is consistent with the fact that positive work must be done by an external agent on the system to bring the two charges near one another (because like charges repel). If the charges are of opposite sign, U is negative; this means that negative work must be done against the attractive force between the unlike charges for them to be brought near each other.

If more than two charged particles are in the system, we can obtain the total potential energy by calculating U for every pair of charges and summing the terms algebraically. As an example, the total potential energy of the system of three charges shown in Figure 25.10 is

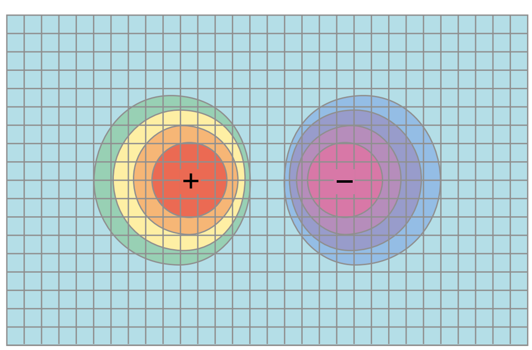
$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \qquad \textbf{(25.14)}$$

Physically, we can interpret this as follows: Imagine that q_1 is fixed at the position shown in Figure 25.10 but that q_2 and q_3 are at infinity. The work an external agent must do to bring q_2 from infinity to its position near q_1 is $k_e q_1 q_2/r_{12}$, which is the first term in Equation 25.14. The last two terms represent the work required to bring q_3 from infinity to its position near q_1 and q_2. (The result is independent of the order in which the charges are transported.)

[2] The expression for the electric potential energy of a system made up of two point charges, Equation 25.13, is of the *same* form as the equation for the gravitational potential energy of a system made up of two point masses, Gm_1m_2/r (see Chapter 14). The similarity is not surprising in view of the fact that both expressions are derived from an inverse-square force law.



(a)



(b)

Figure 25.8 (a) The electric potential in the plane containing a dipole. (b) Top view of the function graphed in part (a).

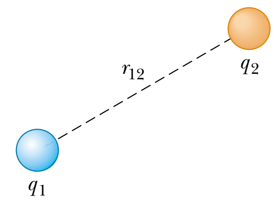


Figure 25. 9 If two point charges are separated by a distance r_{12}, the potential energy of the pair of charges is given by $k_e q_1 q_2 / r_{12}$.

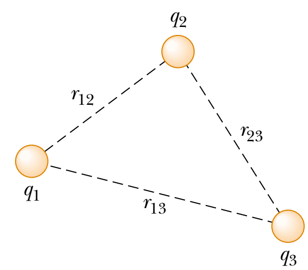


Figure 25.10 Three point charges are fixed at the positions shown. The potential energy of this system of charges is given by Equation 25.14.

EXAMPLE 25.3 The Electric Potential Due to Two Point Charges

A charge $q_1 = 2.00\ \mu C$ is located at the origin, and a charge $q_2 = -6.00\ \mu C$ is located at (0, 3.00) m, as shown in Figure 25.11a. (a) Find the total electric potential due to these charges at the point P, whose coordinates are (4.00, 0) m.

Solution For two charges, the sum in Equation 25.12 gives

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= 8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(\frac{2.00 \times 10^{-6}\ \text{C}}{4.00\ \text{m}} + \frac{-6.00 \times 10^{-6}\ \text{C}}{5.00\ \text{m}}\right)$$

$$= -6.29 \times 10^3\ \text{V}$$

(b) Find the change in potential energy of a 3.00-μC charge as it moves from infinity to point P (Fig. 25.11b).

Solution When the charge is at infinity, $U_i = 0$, and when the charge is at P, $U_f = q_3 V_P$; therefore,

$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \text{C})(-6.29 \times 10^3\ \text{V})$$

$$= -18.9 \times 10^{-3}\ \text{J}$$

Therefore, because $W = -\Delta U$, positive work would have to be done by an external agent to remove the charge from point P back to infinity.

Exercise Find the total potential energy of the system illustrated in Figure 25.11b.

Answer -5.48×10^{-2} J.

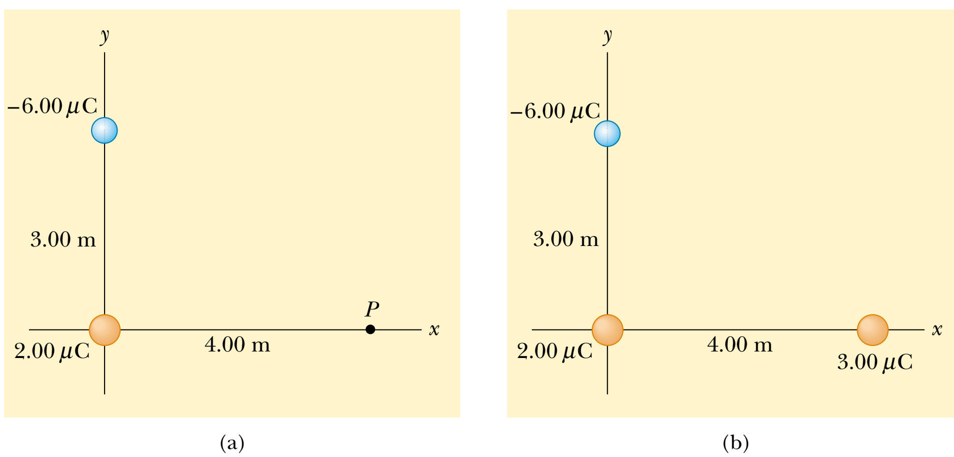


Figure 25.11 (a) The electric potential at P due to the two charges is the algebraic sum of the potentials due to the individual charges. (b) What is the potential energy of the three-charge system?

25.4 OBTAINING THE VALUE OF THE ELECTRIC FIELD FROM THE ELECTRIC POTENTIAL

The electric field $\mathbf{E}$ and the electric potential V are related as shown in Equation 25.3. We now show how to calculate the value of the electric field if the electric potential is known in a certain region.

From Equation 25.3 we can express the potential difference dV between two points a distance ds apart as

$$dV = -\mathbf{E}\cdot d\mathbf{s} \qquad \textbf{(25.15)}$$

If the electric field has only one component E_x, then $\mathbf{E}\cdot d\mathbf{s} = E_x\,dx$. Therefore, Equation 25.15 becomes $dV = -E_x\,dx$, or

$$E_x = -\frac{dV}{dx} \qquad \textbf{(25.16)}$$

That is, the magnitude of the electric field in the direction of some coordinate is equal to the negative of the derivative of the electric potential with respect to that coordinate. Recall from the discussion following Equation 25.8 that the electric potential does not change for any displacement perpendicular to an electric field. This is consistent with the notion, developed in Section 25.2, that equipotential surfaces are perpendicular to the field, as shown in Figure 25.12. A small positive charge placed at rest on an electric field line begins to move along the direction of **E** because that is the direction of the force exerted on the charge by the charge distribution creating the electric field (and hence is the direction of **a**). Because the charge starts with zero velocity, it moves in the direction of the change in velocity—that is, in the direction of **a**. In Figures 25.12a and 25.12b, a charge placed at rest in the field will move in a straight line because its acceleration vector is always parallel to its velocity vector. The magnitude of **v** increases, but its direction does not change. The situation is different in Figure 25.12c. A positive charge placed at some point near the dipole first moves in a direction parallel to **E** at that point. Because the direction of the electric field is different at different locations, however, the force acting on the charge changes direction, and **a** is no longer parallel to **v**. This causes the moving charge to change direction and speed, but it does not necessarily follow the electric field lines. Recall that it is not the velocity vector but rather the acceleration vector that is proportional to force.

If the charge distribution creating an electric field has spherical symmetry such that the volume charge density depends only on the radial distance r, then the electric field is radial. In this case, $\mathbf{E} \cdot d\mathbf{s} = E_r\,dr$, and thus we can express dV in the form $dV = -E_r\,dr$. Therefore,

$$E_r = -\frac{dV}{dr} \tag{25.17}$$

For example, the electric potential of a point charge is $V = k_e q/r$. Because V is a function of r only, the potential function has spherical symmetry. Applying Equation 25.17, we find that the electric field due to the point charge is $E_r = k_e q/r^2$, a familiar result. Note that the potential changes only in the radial direction, not in

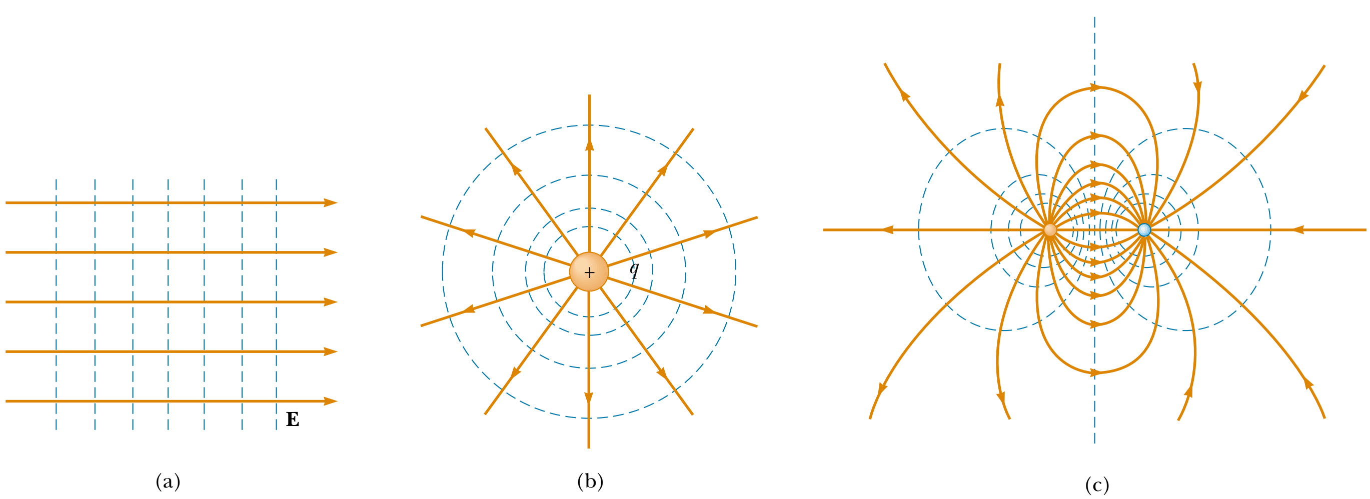


Figure 25.12 Equipotential surfaces (dashed blue lines) and electric field lines (red lines) for (a) a uniform electric field produced by an infinite sheet of charge, (b) a point charge, and (c) an electric dipole. In all cases, the equipotential surfaces are *perpendicular* to the electric field lines at every point. Compare these drawings with Figures 25.2, 25.7b, and 25.8b.

Equipotential surfaces are perpendicular to the electric field lines

any direction perpendicular to r. Thus, V (like E_r) is a function only of r. Again, this is consistent with the idea that **equipotential surfaces are perpendicular to field lines.** In this case the equipotential surfaces are a family of spheres concentric with the spherically symmetric charge distribution (Fig. 25.12b).

The equipotential surfaces for an electric dipole are sketched in Figure 25.12c. When a test charge undergoes a displacement $d\mathbf{s}$ along an equipotential surface, then $dV = 0$ because the potential is constant along an equipotential surface. From Equation 25.15, then, $dV = -\mathbf{E} \cdot d\mathbf{s} = 0$; thus, $\mathbf{E}$ must be perpendicular to the displacement along the equipotential surface. This shows that the equipotential surfaces must *always* be *perpendicular* to the electric field lines.

In general, the electric potential is a function of all three spatial coordinates. If $V(r)$ is given in terms of the cartesian coordinates, the electric field components E_x, E_y, and E_z can readily be found from $V(x, y, z)$ as the partial derivatives[3]

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

For example, if $V = 3x^2y + y^2 + yz$, then

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(3x^2y + y^2 + yz) = \frac{\partial}{\partial x}(3x^2y) = 3y\frac{d}{dx}(x^2) = 6xy$$

EXAMPLE 25.4 The Electric Potential Due to a Dipole

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$, as shown in Figure 25.13. The dipole is along the x axis and is centered at the origin. (a) Calculate the electric potential at point P.

Solution For point P in Figure 25.13,

$$V = k_e \sum \frac{q_i}{r_i} = k_e\left(\frac{q}{x-a} - \frac{q}{x+a}\right) = \boxed{\frac{2k_e qa}{x^2 - a^2}}$$

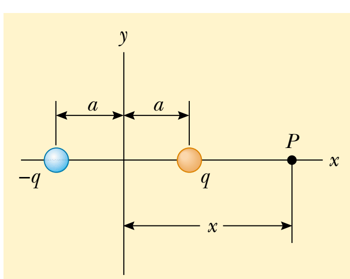


Figure 25.13 An electric dipole located on the x axis.

(How would this result change if point P happened to be located to the left of the negative charge?)

(b) Calculate V and E_x at a point far from the dipole.

Solution If point P is far from the dipole, such that $x \gg a$, then a^2 can be neglected in the term $x^2 - a^2$, and V becomes

$$V \approx \boxed{\frac{2k_e qa}{x^2}} \qquad (x \gg a)$$

Using Equation 25.16 and this result, we can calculate the electric field at a point far from the dipole:

$$E_x = -\frac{dV}{dx} = \boxed{\frac{4k_e qa}{x^3}} \qquad (x \gg a)$$

(c) Calculate V and E_x if point P is located anywhere between the two charges.

Solution

$$V = k_e \sum \frac{q_i}{r_i} = k_e\left(\frac{q}{a-x} - \frac{q}{x+a}\right) = -\frac{2k_e qx}{x^2 - a^2}$$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qx}{x^2 - a^2}\right) = 2k_e q\left(\frac{-x^2 - a^2}{(x^2 - a^2)^2}\right)$$

[3] In vector notation, $\mathbf{E}$ is often written

$$\mathbf{E} = -\nabla V = -\left(\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}\right)V$$

where ∇ is called the *gradient operator.*

We can check these results by considering the situation at the center of the dipole, where $x = 0$, $V = 0$, and $E_x = -2k_e q/a^2$.

Exercise Verify the electric field result in part (c) by calculating the sum of the individual electric field vectors at the origin due to the two charges.

25.5 ELECTRIC POTENTIAL DUE TO CONTINUOUS CHARGE DISTRIBUTIONS

We can calculate the electric potential due to a continuous charge distribution in two ways. If the charge distribution is known, we can start with Equation 25.11 for the electric potential of a point charge. We then consider the potential due to a small charge element dq, treating this element as a point charge (Fig. 25.14). The electric potential dV at some point P due to the charge element dq is

$$dV = k_e \frac{dq}{r} \qquad (25.18)$$

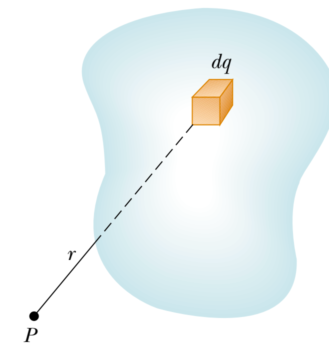


Figure 25.14 The electric potential at the point P due to a continuous charge distribution can be calculated by dividing the charged body into segments of charge dq and summing the electric potential contributions over all segments.

where r is the distance from the charge element to point P. To obtain the total potential at point P, we integrate Equation 25.18 to include contributions from all elements of the charge distribution. Because each element is, in general, a different distance from point P and because k_e is constant, we can express V as

$$V = k_e \int \frac{dq}{r} \qquad (25.19)$$

In effect, we have replaced the sum in Equation 25.12 with an integral. Note that this expression for V uses a particular reference: The electric potential is taken to be zero when point P is infinitely far from the charge distribution.

If the electric field is already known from other considerations, such as Gauss's law, we can calculate the electric potential due to a continuous charge distribution using Equation 25.3. If the charge distribution is highly symmetric, we first evaluate **E** at any point using Gauss's law and then substitute the value obtained into Equation 25.3 to determine the potential difference ΔV between any two points. We then choose the electric potential V to be zero at some convenient point.

We illustrate both methods with several examples.

EXAMPLE 25.5 Electric Potential Due to a Uniformly Charged Ring

(a) Find an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q.

Solution Let us orient the ring so that its plane is perpendicular to an x axis and its center is at the origin. We can then take point P to be at a distance x from the center of the ring, as shown in Figure 25.15. The charge element dq is at a distance $\sqrt{x^2 + a^2}$ from point P. Hence, we can express V as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

Because each element dq is at the same distance from point P, we can remove $\sqrt{x^2 + a^2}$ from the integral, and V reduces to

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{x^2 + a^2}}} \qquad (25.20)$$

The only variable in this expression for V is x. This is not surprising because our calculation is valid only for points along the x axis, where y and z are both zero.

(b) Find an expression for the magnitude of the electric field at point P.

Solution From symmetry, we see that along the x axis **E** can have only an x component. Therefore, we can use Equation 25.16:

$$E_x = -\frac{dV}{dx} = -k_eQ\frac{d}{dx}(x^2 + a^2)^{-1/2}$$
$$= -k_eQ(-\tfrac{1}{2})(x^2 + a^2)^{-3/2}(2x)$$
$$= \frac{k_eQx}{(x^2 + a^2)^{3/2}} \quad \textbf{(25.21)}$$

This result agrees with that obtained by direct integration (see Example 23.8). Note that $E_x = 0$ at $x = 0$ (the center of the ring). Could you have guessed this from Coulomb's law?

Exercise What is the electric potential at the center of the ring? What does the value of the field at the center tell you about the value of V at the center?

Answer $V = k_eQ/a$. Because $E_x = -dV/dx = 0$ at the center, V has either a maximum or minimum value; it is, in fact, a maximum.

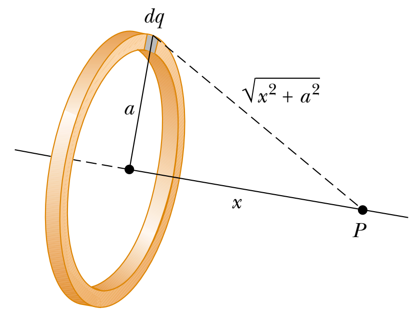


Figure 25.15 A uniformly charged ring of radius a lies in a plane perpendicular to the x axis. All segments dq of the ring are the same distance from any point P lying on the x axis.

EXAMPLE 25.6 Electric Potential Due to a Uniformly Charged Disk

Find (a) the electric potential and (b) the magnitude of the electric field along the perpendicular central axis of a uniformly charged disk of radius a and surface charge density σ.

Solution (a) Again, we choose the point P to be at a distance x from the center of the disk and take the plane of the disk to be perpendicular to the x axis. We can simplify the problem by dividing the disk into a series of charged rings. The electric potential of each ring is given by Equation 25.20. Consider one such ring of radius r and width dr, as indicated in Figure 25.16. The surface area of the ring is $dA = 2\pi r\,dr$; from the definition of surface charge density (see Section 23.5), we know that the charge on the ring is $dq = \sigma\,dA = \sigma 2\pi r\,dr$. Hence, the potential at the point P due to this ring is

$$dV = \frac{k_e\,dq}{\sqrt{r^2 + x^2}} = \frac{k_e\sigma 2\pi r\,dr}{\sqrt{r^2 + x^2}}$$

To find the *total* electric potential at P, we sum over all rings making up the disk. That is, we integrate dV from $r = 0$ to $r = a$:

$$V = \pi k_e\sigma\int_0^a \frac{2r\,dr}{\sqrt{r^2 + x^2}} = \pi k_e\sigma\int_0^a (r^2 + x^2)^{-1/2}\,2r\,dr$$

This integral is of the form $u^n\,du$ and has the value $u^{n+1}/(n+1)$, where $n = -\frac{1}{2}$ and $u = r^2 + x^2$. This gives

$$V = 2\pi k_e\sigma[(x^2 + a^2)^{1/2} - x] \quad \textbf{(25.22)}$$

(b) As in Example 25.5, we can find the electric field at any axial point from

$$E_x = -\frac{dV}{dx} = 2\pi k_e\sigma\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right) \quad \textbf{(25.23)}$$

The calculation of V and $\mathbf{E}$ for an arbitrary point off the axis is more difficult to perform, and we do not treat this situation in this text.

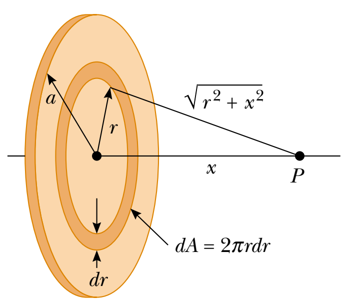


Figure 25.16 A uniformly charged disk of radius a lies in a plane perpendicular to the x axis. The calculation of the electric potential at any point P on the x axis is simplified by dividing the disk into many rings each of area $2\pi r\,dr$.

EXAMPLE 25.7 Electric Potential Due to a Finite Line of Charge

A rod of length ℓ located along the x axis has a total charge Q and a uniform linear charge density $\lambda = Q/\ell$. Find the electric potential at a point P located on the y axis a distance a from the origin (Fig. 25.17).

Solution The length element dx has a charge $dq = \lambda\, dx$. Because this element is a distance $r = \sqrt{x^2 + a^2}$ from point P, we can express the potential at point P due to this element as

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{x^2 + a^2}}$$

To obtain the total potential at P, we integrate this expression over the limits $x = 0$ to $x = \ell$. Noting that k_e and λ are constants, we find that

$$V = k_e \lambda \int_0^{\ell} \frac{dx}{\sqrt{x^2 + a^2}} = k_e \frac{Q}{\ell} \int_0^{\ell} \frac{dx}{\sqrt{x^2 + a^2}}$$

This integral has the following value (see Appendix B):

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

Evaluating V, we find that

$$V = \frac{k_e Q}{\ell} \ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right) \quad \textbf{(25.24)}$$

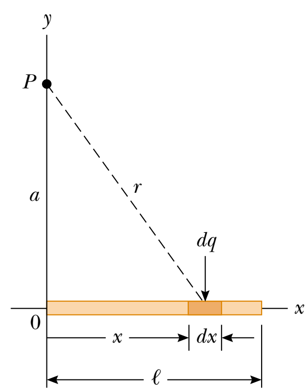


Figure 25.17 A uniform line charge of length ℓ located along the x axis. To calculate the electric potential at P, the line charge is divided into segments each of length dx and each carrying a charge $dq = \lambda\, dx$.

EXAMPLE 25.8 Electric Potential Due to a Uniformly Charged Sphere

An insulating solid sphere of radius R has a uniform positive volume charge density and total charge Q. (a) Find the electric potential at a point outside the sphere, that is, for $r > R$. Take the potential to be zero at $r = \infty$.

Solution In Example 24.5, we found that the magnitude of the electric field outside a uniformly charged sphere of radius R is

$$E_r = k_e \frac{Q}{r^2} \qquad (\text{for } r > R)$$

where the field is directed radially outward when Q is positive. In this case, to obtain the electric potential at an exterior point, such as B in Figure 25.18, we use Equation 25.4 and the expression for E_r given above:

$$V_B = -\int_{\infty}^{r} E_r\, dr = -k_e Q \int_{\infty}^{r} \frac{dr}{r^2}$$

$$V_B = k_e \frac{Q}{r} \qquad (\text{for } r > R)$$

Note that the result is identical to the expression for the electric potential due to a point charge (Eq. 25.11).

Because the potential must be continuous at $r = R$, we can use this expression to obtain the potential at the surface of the sphere. That is, the potential at a point such as C shown in Figure 25.18 is

$$V_C = k_e \frac{Q}{R} \qquad (\text{for } r = R)$$

(b) Find the potential at a point inside the sphere, that is, for $r < R$.

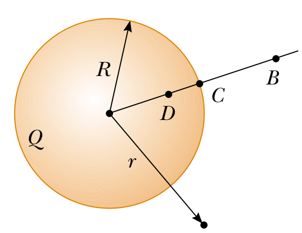


Figure 25.18 A uniformly charged insulating sphere of radius R and total charge Q. The electric potentials at points B and C are equivalent to those produced by a point charge Q located at the center of the sphere, but this is not true for point D.

Solution In Example 24.5 we found that the electric field inside an insulating uniformly charged sphere is

$$E_r = \frac{k_e Q}{R^3} r \qquad (\text{for } r < R)$$

We can use this result and Equation 25.3 to evaluate the potential difference $V_D - V_C$ at some interior point D:

$$V_D - V_C = -\int_R^r E_r\,dr = -\frac{k_e Q}{R^3}\int_R^r r\,dr = \frac{k_e Q}{2R^3}(R^2 - r^2)$$

Substituting $V_C = k_eQ/R$ into this expression and solving for V_D, we obtain

$$V_D = \frac{k_e Q}{2R}\left(3 - \frac{r^2}{R^2}\right) \qquad (\text{for } r < R) \qquad \textbf{(25.25)}$$

At $r = R$, this expression gives a result that agrees with that for the potential at the surface, that is, V_C. A plot of V versus r for this charge distribution is given in Figure 25.19.

Exercise What are the magnitude of the electric field and the electric potential at the center of the sphere?

Answer $E = 0$; $V_0 = 3k_eQ/2R$.

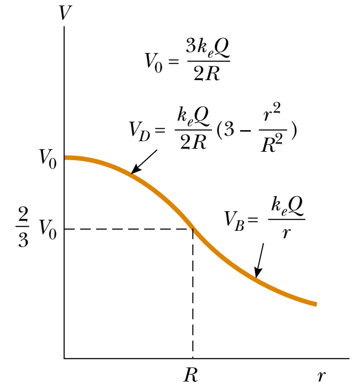


Figure 25.19 A plot of electric potential V versus distance r from the center of a uniformly charged insulating sphere of radius R. The curve for V_D inside the sphere is parabolic and joins smoothly with the curve for V_B outside the sphere, which is a hyperbola. The potential has a maximum value V_0 at the center of the sphere. We could make this graph three dimensional (similar to Figures 25.7a and 25.8a) by spinning it around the vertical axis.

25.6 ELECTRIC POTENTIAL DUE TO A CHARGED CONDUCTOR

In Section 24.4 we found that when a solid conductor in equilibrium carries a net charge, the charge resides on the outer surface of the conductor. Furthermore, we showed that the electric field just outside the conductor is perpendicular to the surface and that the field inside is zero.

We now show that **every point on the surface of a charged conductor in equilibrium is at the same electric potential.** Consider two points A and B on the surface of a charged conductor, as shown in Figure 25.20. Along a surface path connecting these points, $\mathbf{E}$ is always perpendicular to the displacement $d\mathbf{s}$; there-

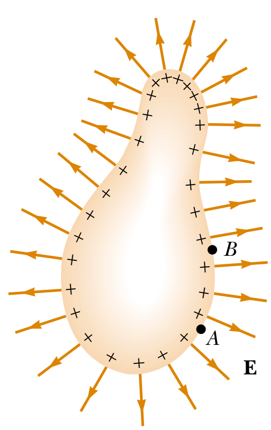


Figure 25.20 An arbitrarily shaped conductor carrying a positive charge. When the conductor is in electrostatic equilibrium, all of the charge resides at the surface, $\mathbf{E} = 0$ inside the conductor, and the direction of $\mathbf{E}$ just outside the conductor is perpendicular to the surface. The electric potential is constant inside the conductor and is equal to the potential at the surface. Note from the spacing of the plus signs that the surface charge density is nonuniform.

fore $\mathbf{E} \cdot d\mathbf{s} = 0$. Using this result and Equation 25.3, we conclude that the potential difference between A and B is necessarily zero:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

This result applies to any two points on the surface. Therefore, V is constant everywhere on the surface of a charged conductor in equilibrium. That is,

the surface of any charged conductor in electrostatic equilibrium is an equipotential surface. Furthermore, because the electric field is zero inside the conductor, we conclude from the relationship $E_r = -dV/dr$ that the electric potential is constant everywhere inside the conductor and equal to its value at the surface.

The surface of a charged conductor is an equipotential surface

Because this is true about the electric potential, no work is required to move a test charge from the interior of a charged conductor to its surface.

Consider a solid metal conducting sphere of radius R and total positive charge Q, as shown in Figure 25.21a. The electric field outside the sphere is k_eQ/r^2 and points radially outward. From Example 25.8, we know that the electric potential at the interior and surface of the sphere must be k_eQ/R relative to infinity. The potential outside the sphere is k_eQ/r. Figure 25.21b is a plot of the electric potential as a function of r, and Figure 25.21c shows how the electric field varies with r.

When a net charge is placed on a spherical conductor, the surface charge density is uniform, as indicated in Figure 25.21a. However, if the conductor is nonspherical, as in Figure 25.20, the surface charge density is high where the radius of curvature is small and the surface is convex (as noted in Section 24.4), and it is low where the radius of curvature is small and the surface is concave. Because the electric field just outside the conductor is proportional to the surface charge density, we see that **the electric field is large near convex points having small radii of curvature and reaches very high values at sharp points.**

Figure 25.22 shows the electric field lines around two spherical conductors: one carrying a net charge Q, and a larger one carrying zero net charge. In this case, the surface charge density is not uniform on either conductor. The sphere having zero net charge has negative charges induced on its side that faces the

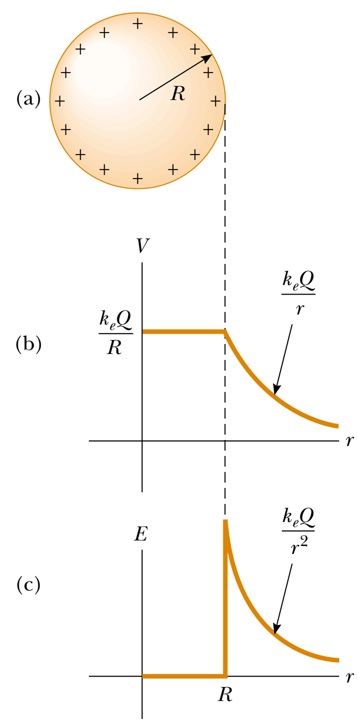
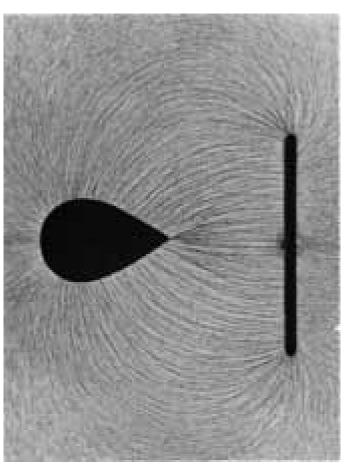


Figure 25.21 (a) The excess charge on a conducting sphere of radius R is uniformly distributed on its surface. (b) Electric potential versus distance r from the center of the charged conducting sphere. (c) Electric field magnitude versus distance r from the center of the charged conducting sphere.



Electric field pattern of a charged conducting plate placed near an oppositely charged pointed conductor. Small pieces of thread suspended in oil align with the electric field lines. The field surrounding the pointed conductor is most intense near the pointed end and at other places where the radius of curvature is small.

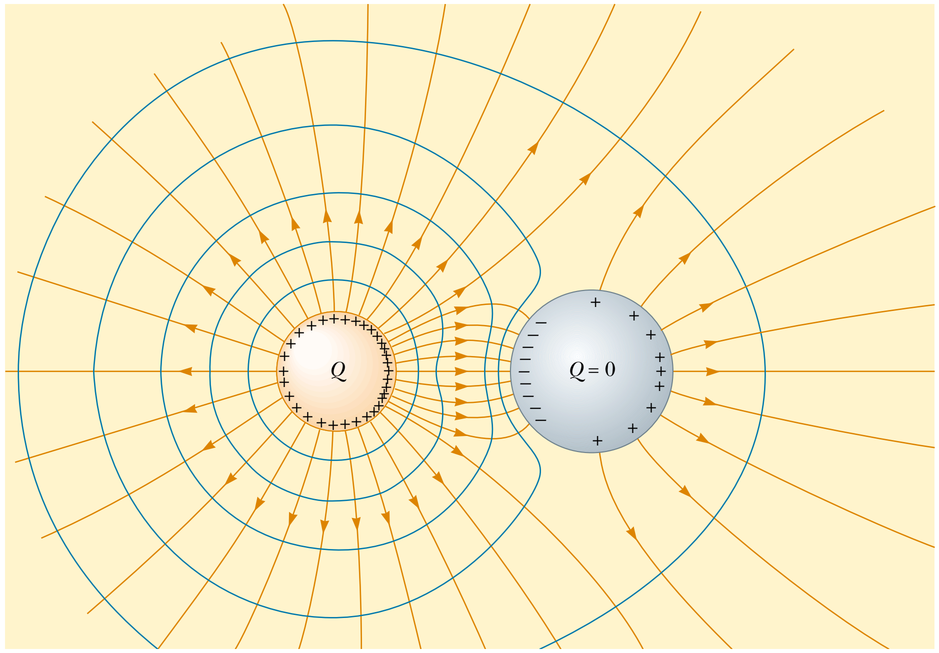


Figure 25.22 The electric field lines (in red) around two spherical conductors. The smaller sphere has a net charge Q, and the larger one has zero net charge. The blue curves are cross-sections of equipotential surfaces.

charged sphere and positive charges induced on its side opposite the charged sphere. The blue curves in the figure represent the cross-sections of the equipotential surfaces for this charge configuration. As usual, the field lines are perpendicular to the conducting surfaces at all points, and the equipotential surfaces are perpendicular to the field lines everywhere. Trying to move a positive charge in the region of these conductors would be like moving a marble on a hill that is flat on top (representing the conductor on the left) and has another flat area partway down the side of the hill (representing the conductor on the right).

EXAMPLE 25.9 Two Connected Charged Spheres

Two spherical conductors of radii r_1 and r_2 are separated by a distance much greater than the radius of either sphere. The spheres are connected by a conducting wire, as shown in Figure 25.23. The charges on the spheres in equilibrium are q_1 and q_2, respectively, and they are uniformly charged. Find the ratio of the magnitudes of the electric fields at the surfaces of the spheres.

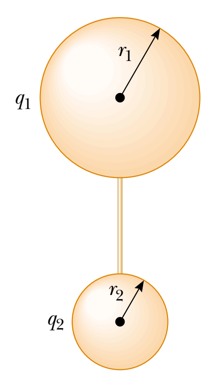


Figure 25.23 Two charged spherical conductors connected by a conducting wire. The spheres are at the *same* electric potential V.

Solution Because the spheres are connected by a conducting wire, they must both be at the same electric potential:

$$V = k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2}$$

Therefore, the ratio of charges is

$$(1) \qquad \frac{q_1}{q_2} = \frac{r_1}{r_2}$$

Because the spheres are very far apart and their surfaces uniformly charged, we can express the magnitude of the electric fields at their surfaces as

$$E_1 = k_e \frac{q_1}{{r_1}^2} \quad \text{and} \quad E_2 = k_e \frac{q_2}{{r_2}^2}$$

Taking the ratio of these two fields and making use of Equation (1), we find that

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

Hence, the field is more intense in the vicinity of the smaller sphere even though the electric potentials of both spheres are the same.

A Cavity Within a Conductor

Now consider a conductor of arbitrary shape containing a cavity as shown in Figure 25.24. Let us assume that no charges are inside the cavity. **In this case, the electric field inside the cavity must be zero** regardless of the charge distribution on the outside surface of the conductor. Furthermore, the field in the cavity is zero even if an electric field exists outside the conductor.

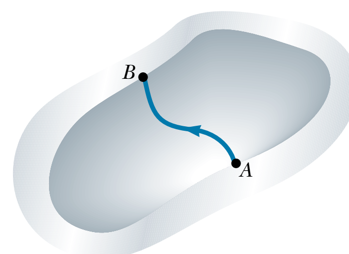


Figure 25.24 A conductor in electrostatic equilibrium containing a cavity. The electric field in the cavity is zero, regardless of the charge on the conductor.

To prove this point, we use the fact that every point on the conductor is at the same electric potential, and therefore any two points A and B on the surface of the cavity must be at the same potential. Now imagine that a field $\mathbf{E}$ exists in the cavity and evaluate the potential difference $V_B - V_A$ defined by Equation 25.3:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

If $\mathbf{E}$ is nonzero, we can always find a path between A and B for which $\mathbf{E} \cdot d\mathbf{s}$ is a positive number; thus, the integral must be positive. However, because $V_B - V_A = 0$, the integral of $\mathbf{E} \cdot d\mathbf{s}$ must be zero for all paths between any two points on the conductor, which implies that $\mathbf{E}$ is zero everywhere. This contradiction can be reconciled only if $\mathbf{E}$ is zero inside the cavity. Thus, we conclude that a cavity surrounded by conducting walls is a field-free region as long as no charges are inside the cavity.

Corona Discharge

A phenomenon known as **corona discharge** is often observed near a conductor such as a high-voltage power line. When the electric field in the vicinity of the conductor is sufficiently strong, electrons are stripped from air molecules. This causes the molecules to be ionized, thereby increasing the air's ability to conduct. The observed glow (or corona discharge) results from the recombination of free electrons with the ionized air molecules. If a conductor has an irregular shape, the electric field can be very high near sharp points or edges of the conductor; consequently, the ionization process and corona discharge are most likely to occur around such points.

Quick Quiz 25.4

(a) Is it possible for the magnitude of the electric field to be zero at a location where the electric potential is not zero? (b) Can the electric potential be zero where the electric field is nonzero?

Optional Section

25.7 THE MILLIKAN OIL-DROP EXPERIMENT

During the period from 1909 to 1913, Robert Millikan performed a brilliant set of experiments in which he measured e, the elementary charge on an electron, and demonstrated the quantized nature of this charge. His apparatus, diagrammed in Figure 25.25, contains two parallel metallic plates. Charged oil droplets from an atomizer are allowed to pass through a small hole in the upper plate. A horizontally directed light beam (not shown in the diagram) is used to illuminate the oil droplets, which are viewed through a telescope whose long axis is at right angles to the light beam. When the droplets are viewed in this manner, they appear as shining stars against a dark background, and the rate at which individual drops fall can be determined.[4]

Let us assume that a single drop having a mass m and carrying a charge q is being viewed and that its charge is negative. If no electric field is present between the plates, the two forces acting on the charge are the force of gravity $m\mathbf{g}$ acting downward and a viscous drag force $\mathbf{F}_D$ acting upward as indicated in Figure 25.26a. The drag force is proportional to the drop's speed. When the drop reaches its terminal speed v, the two forces balance each other ($mg = F_D$).

Now suppose that a battery connected to the plates sets up an electric field between the plates such that the upper plate is at the higher electric potential. In this case, a third force $q\mathbf{E}$ acts on the charged drop. Because q is negative and $\mathbf{E}$ is directed downward, this electric force is directed upward, as shown in Figure 25.26b. If this force is sufficiently great, the drop moves upward and the drag force $\mathbf{F}'_D$ acts downward. When the upward electric force $q\mathbf{E}$ balances the sum of the gravitational force and the downward drag force $\mathbf{F}'_D$, the drop reaches a new terminal speed v' in the upward direction.

With the field turned on, a drop moves slowly upward, typically at rates of hundredths of a centimeter per second. The rate of fall in the absence of a field is comparable. Hence, one can follow a single droplet for hours, alternately rising and falling, by simply turning the electric field on and off.

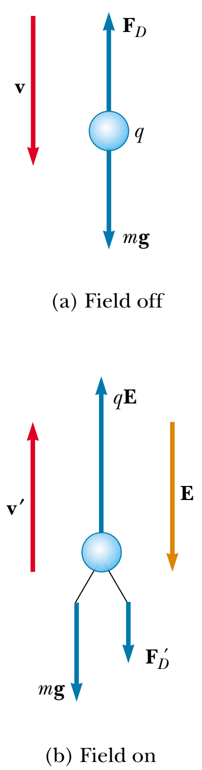


Figure 25.26 The forces acting on a negatively charged oil droplet in the Millikan experiment.

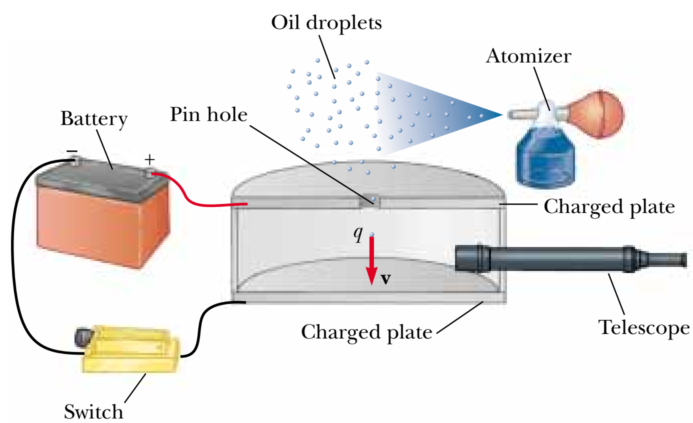


Figure 25.25 Schematic drawing of the Millikan oil-drop apparatus.

[4] At one time, the oil droplets were termed "Millikan's Shining Stars." Perhaps this description has lost its popularity because of the generations of physics students who have experienced hallucinations, near blindness, migraine headaches, and so forth, while repeating Millikan's experiment!

After recording measurements on thousands of droplets, Millikan and his coworkers found that all droplets, to within about 1% precision, had a charge equal to some integer multiple of the elementary charge e:

$$q = ne \qquad n = 0, -1, -2, -3, \ldots$$

where $e = 1.60 \times 10^{-19}$ C. Millikan's experiment yields conclusive evidence that charge is quantized. For this work, he was awarded the Nobel Prize in Physics in 1923.

Optional Section

25.8 APPLICATIONS OF ELECTROSTATICS

The practical application of electrostatics is represented by such devices as lightning rods and electrostatic precipitators and by such processes as xerography and the painting of automobiles. Scientific devices based on the principles of electrostatics include electrostatic generators, the field-ion microscope, and ion-drive rocket engines.

The Van de Graaff Generator

11.10 In Section 24.5 we described an experiment that demonstrates a method for transferring charge to a hollow conductor (the Faraday ice-pail experiment). When a charged conductor is placed in contact with the inside of a hollow conductor, all of the charge of the charged conductor is transferred to the hollow conductor. In principle, the charge on the hollow conductor and its electric potential can be increased without limit by repetition of the process.

In 1929 Robert J. Van de Graaff (1901–1967) used this principle to design and build an electrostatic generator. This type of generator is used extensively in nuclear physics research. A schematic representation of the generator is given in Figure 25.27. Charge is delivered continuously to a high-potential electrode by means of a moving belt of insulating material. The high-voltage electrode is a hollow conductor mounted on an insulating column. The belt is charged at point A by means of a corona discharge between comb-like metallic needles and a grounded grid. The needles are maintained at a positive electric potential of typically 10^4 V. The positive charge on the moving belt is transferred to the hollow conductor by a second comb of needles at point B. Because the electric field inside the hollow conductor is negligible, the positive charge on the belt is easily transferred to the conductor regardless of its potential. In practice, it is possible to increase the electric potential of the hollow conductor until electrical discharge occurs through the air. Because the "breakdown" electric field in air is about 3×10^6 V/m, a sphere 1 m in radius can be raised to a maximum potential of 3×10^6 V. The potential can be increased further by increasing the radius of the hollow conductor and by placing the entire system in a container filled with high-pressure gas.

Van de Graaff generators can produce potential differences as large as 20 million volts. Protons accelerated through such large potential differences receive enough energy to initiate nuclear reactions between themselves and various target nuclei. Smaller generators are often seen in science classrooms and museums. If a person insulated from the ground touches the sphere of a Van de Graaff generator, his or her body can be brought to a high electric potential. The hair acquires a net positive charge, and each strand is repelled by all the others. The result is a

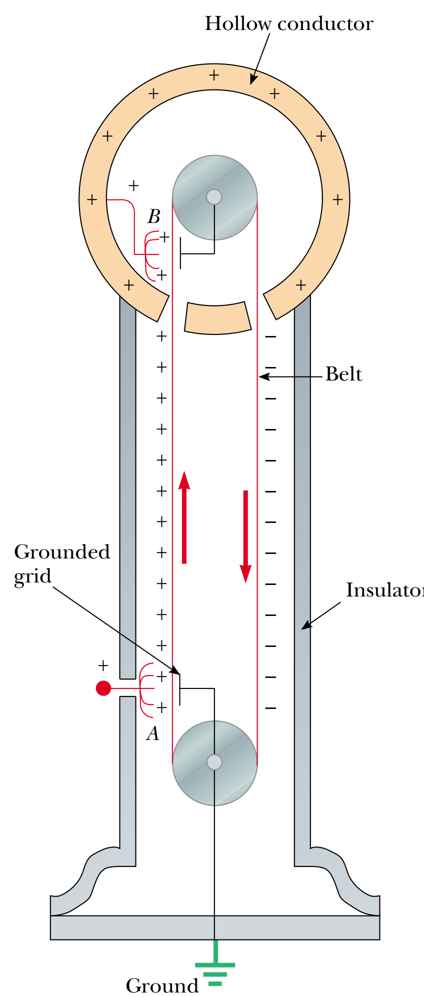


Figure 25.27 Schematic diagram of a Van de Graaff generator. Charge is transferred to the hollow conductor at the top by means of a moving belt. The charge is deposited on the belt at point A and transferred to the hollow conductor at point B.

scene such as that depicted in the photograph at the beginning of this chapter. In addition to being insulated from ground, the person holding the sphere is safe in this demonstration because the total charge on the sphere is very small (on the order of 1 μC). If this amount of charge accidentally passed from the sphere through the person to ground, the corresponding current would do no harm.

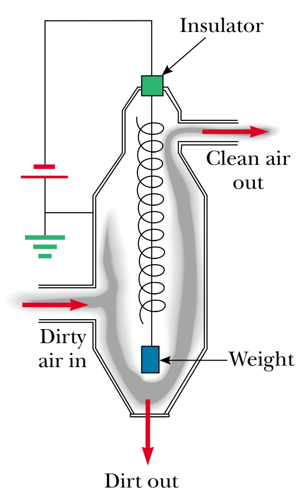
The Electrostatic Precipitator

QuickLab

Sprinkle some salt and pepper on an open dish and mix the two together. Now pull a comb through your hair several times and bring the comb to within 1 cm of the salt and pepper. What happens? How is what happens here related to the operation of an electrostatic precipitator?

One important application of electrical discharge in gases is the *electrostatic precipitator.* This device removes particulate matter from combustion gases, thereby reducing air pollution. Precipitators are especially useful in coal-burning power plants and in industrial operations that generate large quantities of smoke. Current systems are able to eliminate more than 99% of the ash from smoke.

Figure 25.28a shows a schematic diagram of an electrostatic precipitator. A high potential difference (typically 40 to 100 kV) is maintained between a wire running down the center of a duct and the walls of the duct, which are grounded. The wire is maintained at a negative electric potential with respect to the walls, so the electric field is directed toward the wire. The values of the field near the wire become high enough to cause a corona discharge around the wire; the discharge ionizes some air molecules to form positive ions, electrons, and such negative ions as O_2^-. The air to be cleaned enters the duct and moves near the wire. As the electrons and negative ions created by the discharge are accelerated toward the outer wall by the electric field, the dirt particles in the air become charged by collisions and ion capture. Because most of the charged dirt particles are negative, they too are drawn to the duct walls by the electric field. When the duct is periodically shaken, the particles break loose and are collected at the bottom.



(a)

(b)

(c)

Figure 25.28 (a) Schematic diagram of an electrostatic precipitator. The high negative electric potential maintained on the central coiled wire creates an electrical discharge in the vicinity of the wire. Compare the air pollution when the electrostatic precipitator is (b) operating and (c) turned off.

In addition to reducing the level of particulate matter in the atmosphere (compare Figs. 25.28b and c), the electrostatic precipitator recovers valuable materials in the form of metal oxides.

Xerography and Laser Printers

The basic idea of xerography[5] was developed by Chester Carlson, who was granted a patent for the xerographic process in 1940. The one feature of this process that makes it unique is the use of a photoconductive material to form an image. (A *photoconductor* is a material that is a poor electrical conductor in the dark but that becomes a good electrical conductor when exposed to light.)

The xerographic process is illustrated in Figure 25.29a to d. First, the surface of a plate or drum that has been coated with a thin film of photoconductive material (usually selenium or some compound of selenium) is given a positive electrostatic charge in the dark. An image of the page to be copied is then focused by a lens onto the charged surface. The photoconducting surface becomes conducting only in areas where light strikes it. In these areas, the light produces charge carriers in the photoconductor that move the positive charge off the drum. However, positive

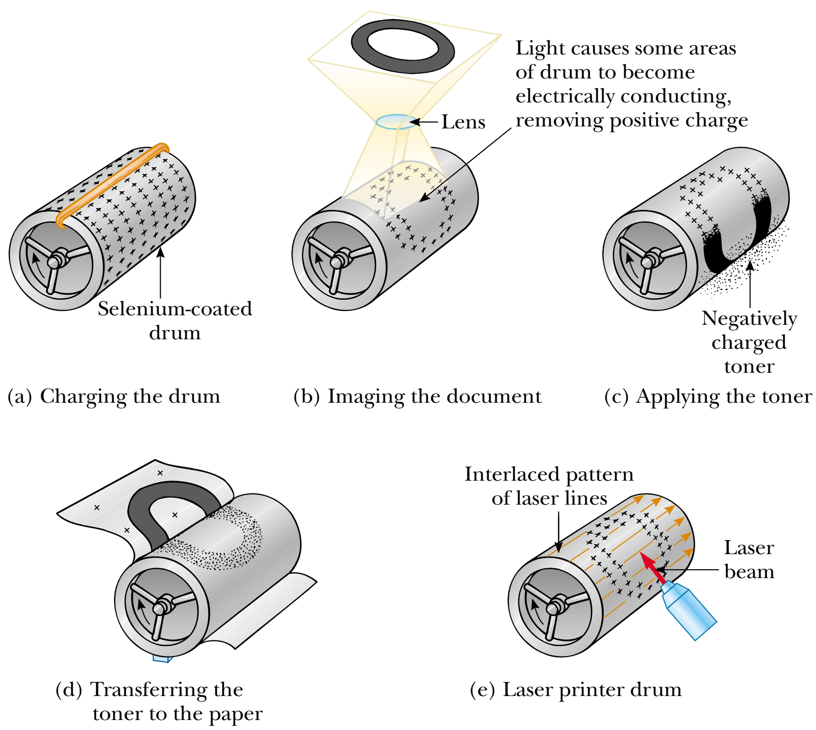


Figure 25.29 The xerographic process: (a) The photoconductive surface of the drum is positively charged. (b) Through the use of a light source and lens, an image is formed on the surface in the form of positive charges. (c) The surface containing the image is covered with a negatively charged powder, which adheres only to the image area. (d) A piece of paper is placed over the surface and given a positive charge. This transfers the image to the paper as the negatively charged powder particles migrate to the paper. The paper is then heat-treated to "fix" the powder. (e) A laser printer operates similarly except the image is produced by turning a laser beam on and off as it sweeps across the selenium-coated drum.

[5] The prefix *xero-* is from the Greek word meaning "dry." Note that no liquid ink is used anywhere in xerography.

charges remain on those areas of the photoconductor not exposed to light, leaving a latent image of the object in the form of a positive surface charge distribution.

Next, a negatively charged powder called a *toner* is dusted onto the photoconducting surface. The charged powder adheres only to those areas of the surface that contain the positively charged image. At this point, the image becomes visible. The toner (and hence the image) are then transferred to the surface of a sheet of positively charged paper.

Finally, the toner is "fixed" to the surface of the paper as the toner melts while passing through high-temperature rollers. This results in a permanent copy of the original.

A laser printer (Fig. 25.29e) operates by the same principle, with the exception that a computer-directed laser beam is used to illuminate the photoconductor instead of a lens.

SUMMARY

When a positive test charge q_0 is moved between points A and B in an electric field $\mathbf{E}$, the **change in the potential energy** is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad \textbf{(25.1)}$$

The **electric potential** $V = U/q_0$ is a scalar quantity and has units of joules per coulomb (J/C), where 1 J/C ≡ 1 V.

The **potential difference** ΔV between points A and B in an electric field $\mathbf{E}$ is defined as

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad \textbf{(25.3)}$$

The potential difference between two points A and B in a uniform electric field $\mathbf{E}$ is

$$\Delta V = -Ed \qquad \textbf{(25.6)}$$

where d is the magnitude of the displacement in the direction parallel to $\mathbf{E}$.

An **equipotential surface** is one on which all points are at the same electric potential. Equipotential surfaces are perpendicular to electric field lines.

If we define $V = 0$ at $r_A = \infty$, the electric potential due to a point charge at any distance r from the charge is

$$V = k_e \frac{q}{r} \qquad \textbf{(25.11)}$$

We can obtain the electric potential associated with a group of point charges by summing the potentials due to the individual charges.

The **potential energy associated with a pair of point charges** separated by a distance r_{12} is

$$U = k_e \frac{q_1 q_2}{r_{12}} \qquad \textbf{(25.13)}$$

This energy represents the work required to bring the charges from an infinite separation to the separation r_{12}. We obtain the potential energy of a distribution of point charges by summing terms like Equation 25.13 over all pairs of particles.

TABLE 25.1 Electric Potential Due to Various Charge Distributions

Charge Distribution	Electric Potential	Location
Uniformly charged ring of radius a	$V = k_e \dfrac{Q}{\sqrt{x^2 + a^2}}$	Along perpendicular central axis of ring, distance x from ring center
Uniformly charged disk of radius a	$V = 2\pi k_e \sigma[(x^2 + a^2)^{1/2} - x]$	Along perpendicular central axis of disk, distance x from disk center
Uniformly charged, *insulating* solid sphere of radius R and total charge Q	$V = k_e \dfrac{Q}{r}$ $V = \dfrac{k_e Q}{2R}\left(3 - \dfrac{r^2}{R^2}\right)$	$r \geq R$ $r < R$
Isolated *conducting* sphere of radius R and total charge Q	$V = k_e \dfrac{Q}{r}$ $V = k_e \dfrac{Q}{R}$	$r > R$ $r \leq R$

If we know the electric potential as a function of coordinates x, y, z, we can obtain the components of the electric field by taking the negative derivative of the electric potential with respect to the coordinates. For example, the x component of the electric field is

$$E_x = -\frac{dV}{dx} \tag{25.16}$$

The **electric potential due to a continuous charge distribution** is

$$V = k_e \int \frac{dq}{r} \tag{25.19}$$

Every point on the surface of a charged conductor in electrostatic equilibrium is at the same electric potential. The potential is constant everywhere inside the conductor and equal to its value at the surface.

Table 25.1 lists electric potentials due to several charge distributions.

Problem-Solving Hints

Calculating Electric Potential

- Remember that electric potential is a scalar quantity, so components need not be considered. Therefore, when using the superposition principle to evaluate the electric potential at a point due to a system of point charges, simply take the algebraic sum of the potentials due to the various charges. However, you must keep track of signs. The potential is positive for positive charges, and it is negative for negative charges.
- Just as with gravitational potential energy in mechanics, only *changes* in electric potential are significant; hence, the point where you choose the poten-

tial to be zero is arbitrary. When dealing with point charges or a charge distribution of finite size, we usually define $V = 0$ to be at a point infinitely far from the charges.

- You can evaluate the electric potential at some point P due to a continuous distribution of charge by dividing the charge distribution into infinitesimal elements of charge dq located at a distance r from P. Then, treat one charge element as a point charge, such that the potential at P due to the element is $dV = k_e dq/r$. Obtain the total potential at P by integrating dV over the entire charge distribution. In performing the integration for most problems, you must express dq and r in terms of a single variable. To simplify the integration, consider the geometry involved in the problem carefully. Review Examples 25.5 through 25.7 for guidance.
- Another method that you can use to obtain the electric potential due to a finite continuous charge distribution is to start with the definition of potential difference given by Equation 25.3. If you know or can easily obtain $\mathbf{E}$ (from Gauss's law), then you can evaluate the line integral of $\mathbf{E} \cdot d\mathbf{s}$. An example of this method is given in Example 25.8.
- Once you know the electric potential at a point, you can obtain the electric field at that point by remembering that the electric field component in a specified direction is equal to the negative of the derivative of the electric potential in that direction. Example 25.4 illustrates this procedure.

QUESTIONS

1. Distinguish between electric potential and electric potential energy.
2. A negative charge moves in the direction of a uniform electric field. Does the potential energy of the charge increase or decrease? Does it move to a position of higher or lower potential?
3. Give a physical explanation of the fact that the potential energy of a pair of like charges is positive whereas the potential energy of a pair of unlike charges is negative.
4. A uniform electric field is parallel to the x axis. In what direction can a charge be displaced in this field without any external work being done on the charge?
5. Explain why equipotential surfaces are always perpendicular to electric field lines.
6. Describe the equipotential surfaces for (a) an infinite line of charge and (b) a uniformly charged sphere.
7. Explain why, under static conditions, all points in a conductor must be at the same electric potential.
8. The electric field inside a hollow, uniformly charged sphere is zero. Does this imply that the potential is zero inside the sphere? Explain.
9. The potential of a point charge is defined to be zero at an infinite distance. Why can we not define the potential of an infinite line of charge to be zero at $r = \infty$?
10. Two charged conducting spheres of different radii are connected by a conducting wire, as shown in Figure 25.23. Which sphere has the greater charge density?
11. What determines the maximum potential to which the dome of a Van de Graaff generator can be raised?
12. Explain the origin of the glow sometimes observed around the cables of a high-voltage power line.
13. Why is it important to avoid sharp edges or points on conductors used in high-voltage equipment?
14. How would you shield an electronic circuit or laboratory from stray electric fields? Why does this work?
15. Why is it relatively safe to stay in an automobile with a metal body during a severe thunderstorm?
16. Walking across a carpet and then touching someone can result in a shock. Explain why this occurs.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
= paired numerical/symbolic problems

Section 25.1 Potential Difference and Electric Potential

1. How much work is done (by a battery, generator, or some other source of electrical energy) in moving Avogadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the potential is -5.00 V? (The potential in each case is measured relative to a common reference point.)
2. An ion accelerated through a potential difference of 115 V experiences an increase in kinetic energy of 7.37×10^{-17} J. Calculate the charge on the ion.
3. (a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V. (b) Calculate the speed of an electron that is accelerated through the same potential difference.
4. **Review Problem.** Through what potential difference would an electron need to be accelerated for it to achieve a speed of 40.0% of the speed of light, starting from rest? The speed of light is $c = 3.00 \times 10^8$ m/s; review Section 7.7.
5. What potential difference is needed to stop an electron having an initial speed of 4.20×10^5 m/s?

Section 25.2 Potential Differences in a Uniform Electric Field

6. A uniform electric field of magnitude 250 V/m is directed in the positive x direction. A $+12.0$-μC charge moves from the origin to the point $(x, y) =$ (20.0 cm, 50.0 cm). (a) What was the change in the potential energy of this charge? (b) Through what potential difference did the charge move?
7. The difference in potential between the accelerating plates of a TV set is about 25 000 V. If the distance between these plates is 1.50 cm, find the magnitude of the uniform electric field in this region.
8. Suppose an electron is released from rest in a uniform electric field whose magnitude is 5.90×10^3 V/m. (a) Through what potential difference will it have passed after moving 1.00 cm? (b) How fast will the electron be moving after it has traveled 1.00 cm?
9. WEB An electron moving parallel to the x axis has an initial speed of 3.70×10^6 m/s at the origin. Its speed is reduced to 1.40×10^5 m/s at the point $x = 2.00$ cm. Calculate the potential difference between the origin and that point. Which point is at the higher potential?
10. A uniform electric field of magnitude 325 V/m is directed in the *negative* y direction as shown in Figure P25.10. The coordinates of point A are $(-0.200, -0.300)$ m, and those of point B are (0.400, 0.500) m. Calculate the potential difference $V_B - V_A$, using the blue path.

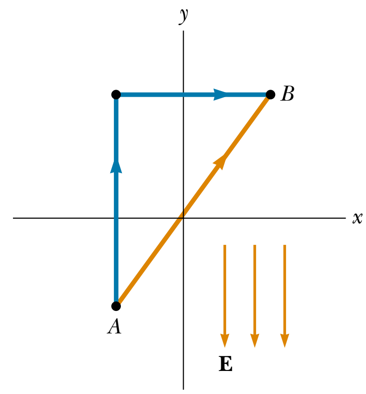


Figure P25.10

11. A 4.00-kg block carrying a charge $Q = 50.0\ \mu$C is connected to a spring for which $k = 100$ N/m. The block lies on a frictionless horizontal track, and the system is immersed in a uniform electric field of magnitude $E = 5.00 \times 10^5$ V/m, directed as shown in Figure P25.11. If the block is released from rest when the spring is unstretched (at $x = 0$), (a) by what maximum amount does the spring expand? (b) What is the equilibrium position of the block? (c) Show that the block's motion is simple harmonic, and determine its period. (d) Repeat part (a) if the coefficient of kinetic friction between block and surface is 0.200.
12. A block having mass m and charge Q is connected to a spring having constant k. The block lies on a frictionless horizontal track, and the system is immersed in a uniform electric field of magnitude E, directed as shown in Figure P25.11. If the block is released from rest when the spring is unstretched (at $x = 0$), (a) by what maximum amount does the spring expand? (b) What is the equilibrium position of the block? (c) Show that the block's motion is simple harmonic, and determine its period. (d) Repeat part (a) if the coefficient of kinetic friction between block and surface is μ_k.

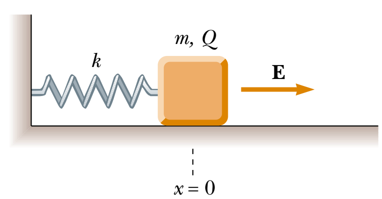


Figure P25.11 Problems 11 and 12.

13. On planet Tehar, the acceleration due to gravity is the same as that on Earth but there is also a strong downward electric field with the field being uniform close to the planet's surface. A 2.00-kg ball having a charge of 5.00 μC is thrown upward at a speed of 20.1 m/s and it hits the ground after an interval of 4.10 s. What is the potential difference between the starting point and the top point of the trajectory?

14. An insulating rod having linear charge density λ = 40.0 μC/m and linear mass density μ = 0.100 kg/m is released from rest in a uniform electric field E = 100 V/m directed perpendicular to the rod (Fig. P25.14). (a) Determine the speed of the rod after it has traveled 2.00 m. (b) How does your answer to part (a) change if the electric field is not perpendicular to the rod? Explain.

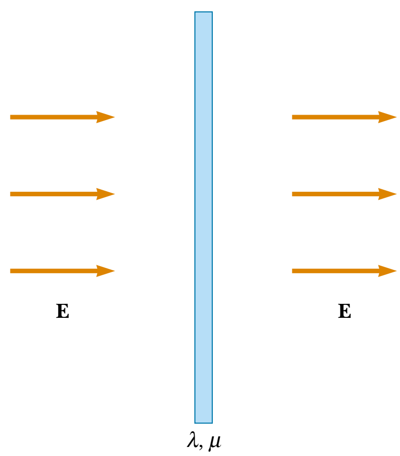


Figure P25.14

15. A particle having charge $q = +2.00$ μC and mass m = 0.010 0 kg is connected to a string that is L = 1.50 m long and is tied to the pivot point P in Figure P25.15. The particle, string, and pivot point all lie on a horizontal table. The particle is released from rest when the string makes an angle $\theta = 60.0°$ with a uniform electric field of magnitude E = 300 V/m. Determine the speed of the particle when the string is parallel to the electric field (point a in Fig. P25.15).

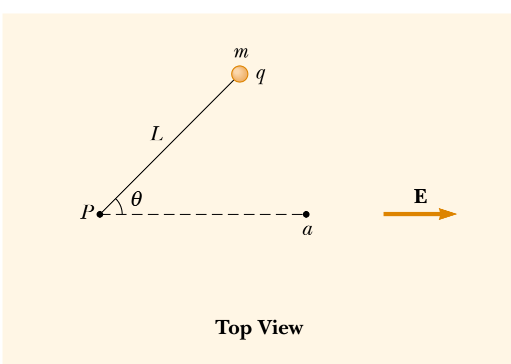


Figure P25.15

Section 25.3 Electric Potential and Potential Energy Due to Point Charges

Note: Unless stated otherwise, assume a reference level of potential $V = 0$ at $r = \infty$.

16. (a) Find the potential at a distance of 1.00 cm from a proton. (b) What is the potential difference between two points that are 1.00 cm and 2.00 cm from a proton? (c) Repeat parts (a) and (b) for an electron.

17. Given two 2.00-μC charges, as shown in Figure P25.17, and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin, (a) what is the net force exerted on q by the two 2.00-μC charges? (b) What is the electric field at the origin due to the two 2.00-μC charges? (c) What is the electric potential at the origin due to the two 2.00-μC charges?

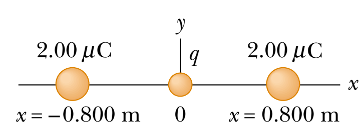


Figure P25.17

18. A charge $+q$ is at the origin. A charge $-2q$ is at x = 2.00 m on the x axis. For what finite value(s) of x is (a) the electric field zero? (b) the electric potential zero?

19. The Bohr model of the hydrogen atom states that the single electron can exist only in certain allowed orbits around the proton. The radius of each Bohr orbit is $r = n^2$ (0.052 9 nm) where n = 1, 2, 3, ... Calculate the electric potential energy of a hydrogen atom when the electron is in the (a) first allowed orbit, n = 1; (b) second allowed orbit, n = 2; and (c) when the electron has escaped from the atom ($r = \infty$). Express your answers in electron volts.

20. Two point charges $Q_1 = +5.00$ nC and $Q_2 = -3.00$ nC are separated by 35.0 cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?

21. The three charges in Figure P25.21 are at the vertices of an isosceles triangle. Calculate the electric potential at the midpoint of the base, taking q = 7.00 μC.

22. *Compare this problem with Problem 55 in Chapter 23.* Four identical point charges (q = +10.0 μC) are located on the corners of a rectangle, as shown in Figure P23.55. The dimensions of the rectangle are L = 60.0 cm and W = 15.0 cm. Calculate the electric potential energy of the charge at the lower left corner due to the other three charges.

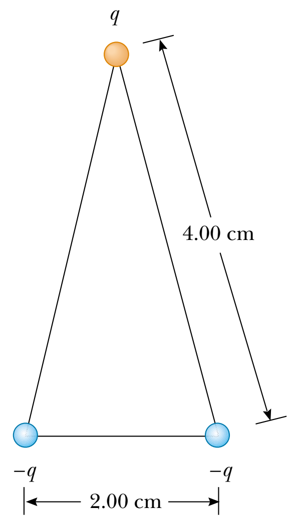


Figure P25.21

WEB **23.** Show that the amount of work required to assemble four identical point charges of magnitude Q at the corners of a square of side s is $5.41\,k_eQ^2/s$.

24. *Compare this problem with Problem 18 in Chapter 23.* Two point charges each of magnitude 2.00 μC are located on the x axis. One is at $x = 1.00$ m, and the other is at $x = -1.00$ m. (a) Determine the electric potential on the y axis at $y = 0.500$ m. (b) Calculate the electric potential energy of a third charge, of -3.00 μC, placed on the y axis at $y = 0.500$ m.

25. *Compare this problem with Problem 22 in Chapter 23.* Five equal negative point charges $-q$ are placed symmetrically around a circle of radius R. Calculate the electric potential at the center of the circle.

26. *Compare this problem with Problem 17 in Chapter 23.* Three equal positive charges q are at the corners of an equilateral triangle of side a, as shown in Figure P23.17. (a) At what point, if any, in the plane of the charges is the electric potential zero? (b) What is the electric potential at the point P due to the two charges at the base of the triangle?

27. **Review Problem.** Two insulating spheres having radii 0.300 cm and 0.500 cm, masses 0.100 kg and 0.700 kg, and charges -2.00 μC and 3.00 μC are released from rest when their centers are separated by 1.00 m. (a) How fast will each be moving when they collide? (*Hint:* Consider conservation of energy and linear momentum.) (b) If the spheres were conductors would the speeds be larger or smaller than those calculated in part (a)? Explain.

28. **Review Problem.** Two insulating spheres having radii r_1 and r_2, masses m_1 and m_2, and charges $-q_1$ and q_2 are released from rest when their centers are separated by a distance d. (a) How fast is each moving when they collide? (*Hint:* Consider conservation of energy and conservation of linear momentum.) (b) If the spheres were conductors, would the speeds be greater or less than those calculated in part (a)?

29. A small spherical object carries a charge of 8.00 nC. At what distance from the center of the object is the potential equal to 100 V? 50.0 V? 25.0 V? Is the spacing of the equipotentials proportional to the change in potential?

30. Two point charges of equal magnitude are located along the y axis equal distances above and below the x axis, as shown in Figure P25.30. (a) Plot a graph of the potential at points along the x axis over the interval $-3a < x < 3a$. You should plot the potential in units of k_eQ/a. (b) Let the charge located at $-a$ be negative and plot the potential along the y axis over the interval $-4a < y < 4a$.

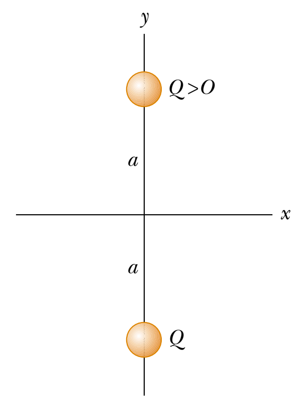


Figure P25.30

31. In Rutherford's famous scattering experiments that led to the planetary model of the atom, alpha particles (charge $+2e$, mass $= 6.64 \times 10^{-27}$ kg) were fired at a gold nucleus (charge $+79e$). An alpha particle, initially very far from the gold nucleus, is fired with a velocity of 2.00×10^7 m/s directly toward the center of the nucleus. How close does the alpha particle get to this center before turning around? Assume the gold nucleus remains stationary.

32. An electron starts from rest 3.00 cm from the center of a uniformly charged insulating sphere of radius 2.00 cm and total charge 1.00 nC. What is the speed of the electron when it reaches the surface of the sphere?

33. Calculate the energy required to assemble the array of charges shown in Figure P25.33, where $a = 0.200$ m, $b = 0.400$ m, and $q = 6.00$ μC.

34. Four identical particles each have charge q and mass m. They are released from rest at the vertices of a square of side L. How fast is each charge moving when their distance from the center of the square doubles?

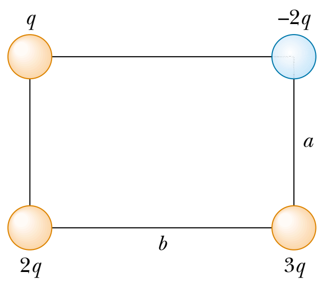


Figure P25.33

35. How much work is required to assemble eight identical point charges, each of magnitude q, at the corners of a cube of side s?

Section 25.4 Obtaining the Value of the Electric Field from the Electric Potential

36. The potential in a region between $x = 0$ and $x = 6.00$ m is $V = a + bx$ where $a = 10.0$ V and $b = -7.00$ V/m. Determine (a) the potential at $x = 0$, 3.00 m, and 6.00 m and (b) the magnitude and direction of the electric field at $x = 0$, 3.00 m, and 6.00 m.

WEB 37. Over a certain region of space, the electric potential is $V = 5x - 3x^2y + 2yz^2$. Find the expressions for the x, y, and z components of the electric field over this region. What is the magnitude of the field at the point P, which has coordinates $(1, 0, -2)$ m?

38. The electric potential inside a charged spherical conductor of radius R is given by $V = k_eQ/R$ and outside the conductor is given by $V = k_eQ/r$. Using $E_r = -dV/dr$, derive the electric field (a) inside and (b) outside this charge distribution.

39. It is shown in Example 25.7 that the potential at a point P a distance a above one end of a uniformly charged rod of length ℓ lying along the x axis is

$$V = \frac{k_eQ}{\ell}\ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right)$$

Use this result to derive an expression for the y component of the electric field at P. (*Hint:* Replace a with y.)

40. When an uncharged conducting sphere of radius a is placed at the origin of an xyz coordinate system that lies in an initially uniform electric field $\mathbf{E} = E_0\mathbf{k}$, the resulting electric potential is

$$V(x, y, z) = V_0 - E_0z + \frac{E_0a^3z}{(x^2 + y^2 + z^2)^{3/2}}$$

for points outside the sphere, where V_0 is the (constant) electric potential on the conductor. Use this equation to determine the x, y, and z components of the resulting electric field.

Section 25.5 Electric Potential Due to Continuous Charge Distributions

41. Consider a ring of radius R with the total charge Q spread uniformly over its perimeter. What is the potential difference between the point at the center of the ring and a point on its axis a distance $2R$ from the center?

42. *Compare this problem with Problem 33 in Chapter 23.* A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle, as shown in Figure P23.33. If the rod has a total charge of $-7.50\ \mu$C, find the electric potential at O, the center of the semicircle.

43. A rod of length L (Fig. P25.43) lies along the x axis with its left end at the origin and has a nonuniform charge density $\lambda = \alpha x$ (where α is a positive constant). (a) What are the units of α? (b) Calculate the electric potential at A.

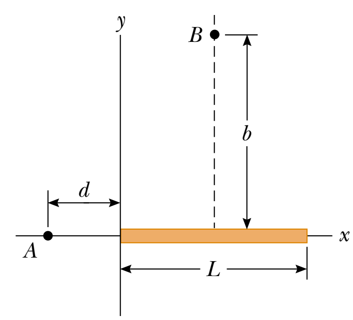


Figure P25.43 Problems 43 and 44.

44. For the arrangement described in the previous problem, calculate the electric potential at point B that lies on the perpendicular bisector of the rod a distance b above the x axis.

45. Calculate the electric potential at point P on the axis of the annulus shown in Figure P25.45, which has a uniform charge density σ.

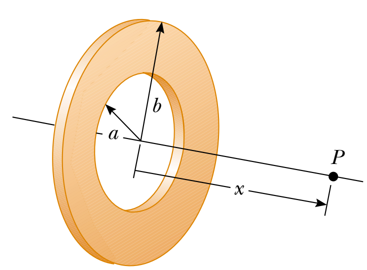


Figure P25.45

46. A wire of finite length that has a uniform linear charge density λ is bent into the shape shown in Figure P25.46. Find the electric potential at point O.

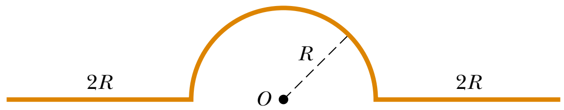


Figure P25.46

Section 25.6 Electric Potential Due to a Charged Conductor

47. How many electrons should be removed from an initially uncharged spherical conductor of radius 0.300 m to produce a potential of 7.50 kV at the surface?

48. Two charged spherical conductors are connected by a long conducting wire, and a charge of 20.0 μC is placed on the combination. (a) If one sphere has a radius of 4.00 cm and the other has a radius of 6.00 cm, what is the electric field near the surface of each sphere? (b) What is the electric potential of each sphere?

WEB **49.** A spherical conductor has a radius of 14.0 cm and charge of 26.0 μC. Calculate the electric field and the electric potential at (a) $r = 10.0$ cm, (b) $r = 20.0$ cm, and (c) $r = 14.0$ cm from the center.

50. Two concentric spherical conducting shells of radii $a = 0.400$ m and $b = 0.500$ m are connected by a thin wire, as shown in Figure P25.50. If a total charge $Q = 10.0$ μC is placed on the system, how much charge settles on each sphere?

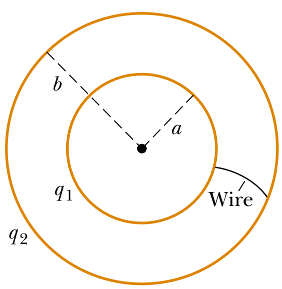


Figure P25.50

(Optional)
Section 25.7 The Millikan Oil-Drop Experiment

(Optional)
Section 25.8 Applications of Electrostatics

51. Consider a Van de Graaff generator with a 30.0-cm-diameter dome operating in dry air. (a) What is the maximum potential of the dome? (b) What is the maximum charge on the dome?

52. The spherical dome of a Van de Graaff generator can be raised to a maximum potential of 600 kV; then additional charge leaks off in sparks, by producing breakdown of the surrounding dry air. Determine (a) the charge on the dome and (b) the radius of the dome.

ADDITIONAL PROBLEMS

53. The liquid-drop model of the nucleus suggests that high-energy oscillations of certain nuclei can split the nucleus into two unequal fragments plus a few neutrons. The fragments acquire kinetic energy from their mutual Coulomb repulsion. Calculate the electric potential energy (in electron volts) of two spherical fragments from a uranium nucleus having the following charges and radii: $38e$ and 5.50×10^{-15} m; $54e$ and 6.20×10^{-15} m. Assume that the charge is distributed uniformly throughout the volume of each spherical fragment and that their surfaces are initially in contact at rest. (The electrons surrounding the nucleus can be neglected.)

54. On a dry winter day you scuff your leather-soled shoes across a carpet and get a shock when you extend the tip of one finger toward a metal doorknob. In a dark room you see a spark perhaps 5 mm long. Make order-of-magnitude estimates of (a) your electric potential and (b) the charge on your body before you touch the doorknob. Explain your reasoning.

55. The charge distribution shown in Figure P25.55 is referred to as a linear quadrupole. (a) Show that the potential at a point on the x axis where $x > a$ is

$$V = \frac{2k_eQa^2}{x^3 - xa^2}$$

(b) Show that the expression obtained in part (a) when $x \gg a$ reduces to

$$V = \frac{2k_eQa^2}{x^3}$$

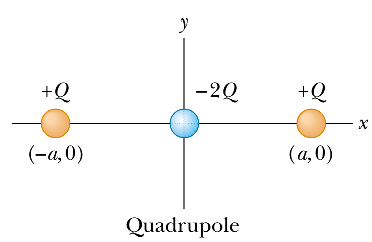


Figure P25.55

56. (a) Use the exact result from Problem 55 to find the electric field at any point along the axis of the linear quadrupole for $x > a$. (b) Evaluate E at $x = 3a$ if $a = 2.00$ mm and $Q = 3.00$ μC.

57. At a certain distance from a point charge, the magnitude of the electric field is 500 V/m and the electric potential is -3.00 kV. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

58. An electron is released from rest on the axis of a uniform positively charged ring, 0.100 m from the ring's center. If the linear charge density of the ring is $+0.100\ \mu\text{C/m}$ and the radius of the ring is 0.200 m, how fast will the electron be moving when it reaches the center of the ring?

59. (a) Consider a uniformly charged cylindrical shell having total charge Q, radius R, and height h. Determine the electrostatic potential at a point a distance d from the right side of the cylinder, as shown in Figure P25.59. (*Hint:* Use the result of Example 25.5 by treating the cylinder as a collection of ring charges.) (b) Use the result of Example 25.6 to solve the same problem for a solid cylinder.

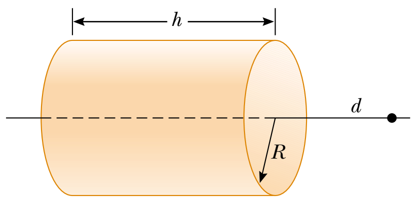


Figure P25.59

60. Two parallel plates having charges of equal magnitude but opposite sign are separated by 12.0 cm. Each plate has a surface charge density of 36.0 nC/m^2. A proton is released from rest at the positive plate. Determine (a) the potential difference between the plates, (b) the energy of the proton when it reaches the negative plate, (c) the speed of the proton just before it strikes the negative plate, (d) the acceleration of the proton, and (e) the force on the proton. (f) From the force, find the magnitude of the electric field and show that it is equal to that found from the charge densities on the plates.

61. Calculate the work that must be done to charge a spherical shell of radius R to a total charge Q.

62. A Geiger–Müller counter is a radiation detector that essentially consists of a hollow cylinder (the cathode) of inner radius r_a and a coaxial cylindrical wire (the anode) of radius r_b (Fig. P25.62). The charge per unit length on the anode is λ, while the charge per unit length on the cathode is $-\lambda$. (a) Show that the magnitude of the potential difference between the wire and the cylinder in the sensitive region of the detector is

$$\Delta V = 2k_e \lambda \ln\left(\frac{r_a}{r_b}\right)$$

(b) Show that the magnitude of the electric field over that region is given by

$$E = \frac{\Delta V}{\ln(r_a/r_b)}\left(\frac{1}{r}\right)$$

where r is the distance from the center of the anode to the point where the field is to be calculated.

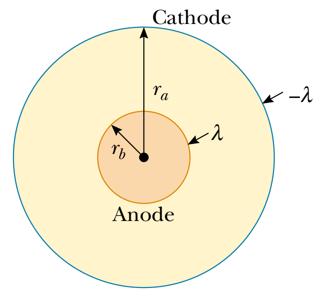


Figure P25.62

WEB **63.** From Gauss's law, the electric field set up by a uniform line of charge is

$$\mathbf{E} = \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector pointing radially away from the line and λ is the charge per unit length along the line. Derive an expression for the potential difference between $r = r_1$ and $r = r_2$.

64. A point charge q is located at $x = -R$, and a point charge $-2q$ is located at the origin. Prove that the equipotential surface that has zero potential is a sphere centered at $(-4R/3, 0, 0)$ and having a radius $r = 2R/3$.

65. Consider two thin, conducting, spherical shells as shown in cross-section in Figure P25.65. The inner shell has a radius $r_1 = 15.0$ cm and a charge of 10.0 nC. The outer shell has a radius $r_2 = 30.0$ cm and a charge of -15.0 nC. Find (a) the electric field **E** and (b) the electric potential V in regions A, B, and C, with $V = 0$ at $r = \infty$.

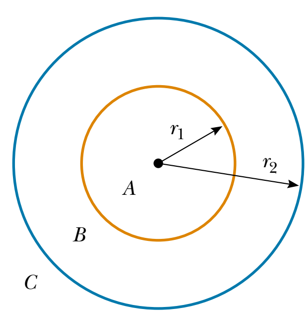


Figure P25.65

66. The x axis is the symmetry axis of a uniformly charged ring of radius R and charge Q (Fig. P25.66). A point charge Q of mass M is located at the center of the ring. When it is displaced slightly, the point charge accelerates along the x axis to infinity. Show that the ultimate speed of the point charge is

$$v = \left(\frac{2k_e Q^2}{MR}\right)^{1/2}$$

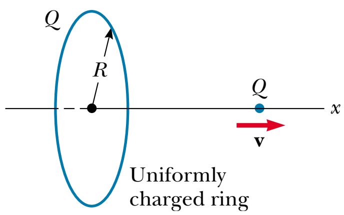


Figure P25.66

67. An infinite sheet of charge that has a surface charge density of 25.0 nC/m^2 lies in the yz plane, passes through the origin, and is at a potential of 1.00 kV at the point $y = 0$, $z = 0$. A long wire having a linear charge density of 80.0 nC/m lies parallel to the y axis and intersects the x axis at $x = 3.00$ m. (a) Determine, as a function of x, the potential along the x axis between wire and sheet. (b) What is the potential energy of a 2.00-nC charge placed at $x = 0.800$ m?

68. The thin, uniformly charged rod shown in Figure P25.68 has a linear charge density λ. Find an expression for the electric potential at P.

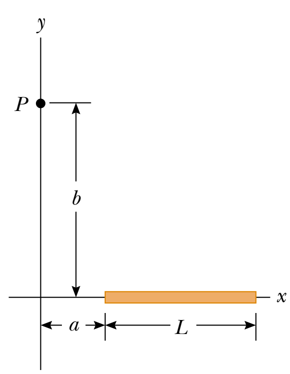


Figure P25.68

69. A dipole is located along the y axis as shown in Figure P25.69. (a) At a point P, which is far from the dipole ($r \gg a$), the electric potential is

$$V = k_e \frac{p \cos \theta}{r^2}$$

where $p = 2qa$. Calculate the radial component E_r and the perpendicular component E_θ of the associated electric field. Note that $E_\theta = -(1/r)(\partial V/\partial \theta)$. Do these results seem reasonable for $\theta = 90°$ and $0°$? for $r = 0$?

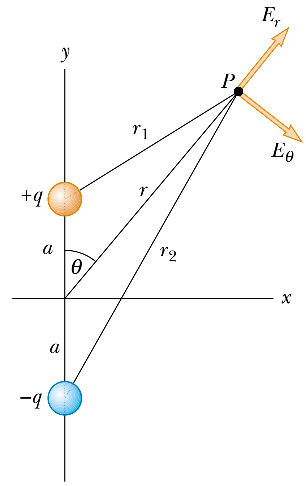


Figure P25.69

(b) For the dipole arrangement shown, express V in terms of cartesian coordinates using $r = (x^2 + y^2)^{1/2}$ and

$$\cos \theta = \frac{y}{(x^2 + y^2)^{1/2}}$$

Using these results and taking $r \gg a$, calculate the field components E_x and E_y.

70. Figure P25.70 shows several equipotential lines each labeled by its potential in volts. The distance between the lines of the square grid represents 1.00 cm. (a) Is the magnitude of the field bigger at A or at B? Why? (b) What is **E** at B? (c) Represent what the field looks like by drawing at least eight field lines.

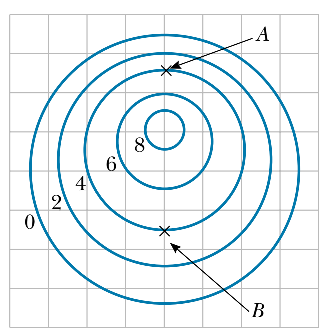


Figure P25.70

71. A disk of radius R has a nonuniform surface charge density $\sigma = Cr$, where C is a constant and r is measured from the center of the disk (Fig. P25.71). Find (by direct integration) the potential at P.

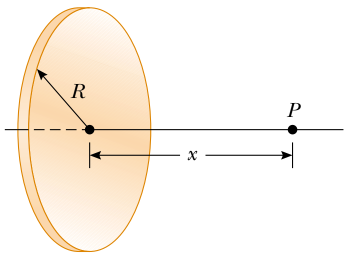


Figure P25.71

72. A solid sphere of radius R has a uniform charge density ρ and total charge Q. Derive an expression for its total electric potential energy. (*Hint:* Imagine that the sphere is constructed by adding successive layers of concentric shells of charge $dq = (4\pi r^2\, dr)\rho$ and use $dU = V\, dq$.)

73. The results of Problem 62 apply also to an electrostatic precipitator (see Figs. 25.28a and P25.62). An applied voltage $\Delta V = V_a - V_b = 50.0$ kV is to produce an electric field of magnitude 5.50 MV/m at the surface of the central wire. The outer cylindrical wall has uniform radius $r_a = 0.850$ m. (a) What should be the radius r_b of the central wire? You will need to solve a transcendental equation. (b) What is the magnitude of the electric field at the outer wall?

Answers to Quick Quizzes

25.1 We do if the electric field is uniform. (This is precisely what we do in the next section.) In general, however, an electric field changes from one place to another.

25.2 $B \rightarrow C$, $C \rightarrow D$, $A \rightarrow B$, $D \rightarrow E$. Moving from B to C decreases the electric potential by 2 V, so the electric field performs 2 J of work on each coulomb of charge that moves. Moving from C to D decreases the electric potential by 1 V, so 1 J of work is done by the field. It takes no work to move the charge from A to B because the electric potential does not change. Moving from D to E increases the electric potential by 1 V, and thus the field does -1 J of work, just as raising a mass to a higher elevation causes the gravitational field to do negative work on the mass.

25.3 The electric potential decreases in inverse proportion to the radius (see Eq. 25.11). The electric field magnitude decreases as the reciprocal of the radius squared (see Eq. 23.4). Because the surface area increases as r^2 while the electric field magnitude decreases as $1/r^2$, the electric flux through the surface remains constant (see Eq. 24.1).

25.4 (a) Yes. Consider four equal charges placed at the corners of a square. The electric potential graph for this situation is shown in the figure. At the center of the square, the electric field is zero because the individual fields from the four charges cancel, but the potential is not zero. This is also the situation inside a charged conductor. (b) Yes again. In Figure 25.8, for instance, the electric potential is zero at the center of the dipole, but the magnitude of the field at that point is not zero. (The two charges in a dipole are by definition of opposite sign; thus, the electric field lines created by the two charges extend from the positive to the negative charge and do not cancel anywhere.) This is the situation we presented in Example 25.4c, in which the equations we obtained give $V = 0$ and $E_x \neq 0$.