PUZZLER

The marks in the pavement are part of a sensor that controls the traffic lights at this intersection. What are these marks, and how do they detect when a car is waiting at the light? *(© David R. Frazier)*

chapter

32 Inductance

Chapter Outline

32.1 Self-Inductance

32.2 *RL* Circuits

32.3 Energy in a Magnetic Field

32.4 Mutual Inductance

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32.6 *(Optional)* The *RLC* Circuit

In Chapter 31, we saw that emfs and currents are induced in a circuit when the magnetic flux through the area enclosed by the circuit changes with time. This electromagnetic induction has some practical consequences, which we describe in this chapter. First, we describe an effect known as *self-induction,* in which a time-varying current in a circuit produces in the circuit an induced emf that opposes the emf that initially set up the time-varying current. Self-induction is the basis of the *inductor,* an electrical element that has an important role in circuits that use time-varying currents. We discuss the energy stored in the magnetic field of an inductor and the energy density associated with the magnetic field.

Next, we study how an emf is induced in a circuit as a result of a changing magnetic flux produced by a second circuit; this is the basic principle of *mutual induction.* Finally, we examine the characteristics of circuits that contain inductors, resistors, and capacitors in various combinations.

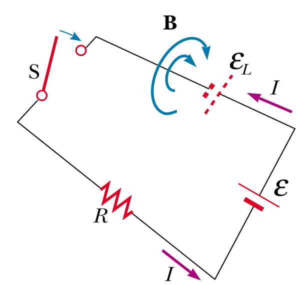


Figure 32.1 After the switch is thrown closed, the current produces a magnetic flux through the area enclosed by the loop. As the current increases toward its equilibrium value, this magnetic flux changes in time and induces an emf in the loop. The battery symbol drawn with dashed lines represents the self-induced emf.

32.1 SELF-INDUCTANCE

In this chapter, we need to distinguish carefully between emfs and currents that are caused by batteries or other sources and those that are induced by changing magnetic fields. We use the adjective *source* (as in the terms *source emf* and *source current*) to describe the parameters associated with a physical source, and we use the adjective *induced* to describe those emfs and currents caused by a changing magnetic field.

Consider a circuit consisting of a switch, a resistor, and a source of emf, as shown in Figure 32.1. When the switch is thrown to its closed position, the source current does not immediately jump from zero to its maximum value $\mathcal{E}/R$. Faraday's law of electromagnetic induction (Eq. 31.1) can be used to describe this effect as follows: As the source current increases with time, the magnetic flux through the circuit loop due to this current also increases with time. This increasing flux creates an induced emf in the circuit. The direction of the induced emf is such that it would cause an induced current in the loop (if a current were not already flowing in the loop), which would establish a magnetic field that would oppose the change in the source magnetic field. Thus, the direction of the induced emf is opposite the direction of the source emf; this results in a gradual rather than instantaneous increase in the source current to its final equilibrium value. This effect is called *self-induction* because the changing flux through the circuit and the resultant induced emf arise from the circuit itself. The emf $\mathcal{E}_L$ set up in this case is called a **self-induced emf.** It is also often called a **back emf.**

As a second example of self-induction, consider Figure 32.2, which shows a coil wound on a cylindrical iron core. (A practical device would have several hun-

Joseph Henry **(1797–1878)** Henry, an American physicist, became the first director of the Smithsonian Institution and first president of the Academy of Natural Science. He improved the design of the electromagnet and constructed one of the first motors. He also discovered the phenomenon of self-induction but failed to publish his findings. The unit of inductance, the henry, is named in his honor. *(North Wind Picture Archives)*

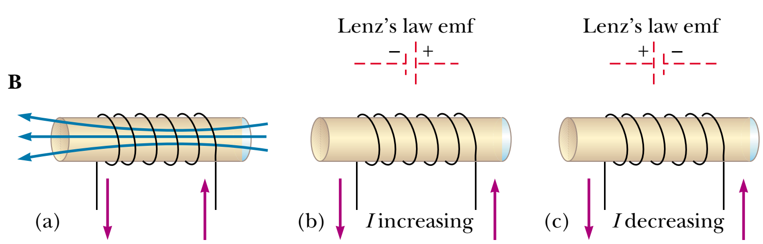


Figure 32.2 (a) A current in the coil produces a magnetic field directed to the left. (b) If the current increases, the increasing magnetic flux creates an induced emf having the polarity shown by the dashed battery. (c) The polarity of the induced emf reverses if the current decreases.

dred turns.) Assume that the source current in the coil either increases or decreases with time. When the source current is in the direction shown, a magnetic field directed from right to left is set up inside the coil, as seen in Figure 32.2a. As the source current changes with time, the magnetic flux through the coil also changes and induces an emf in the coil. From Lenz's law, the polarity of this induced emf must be such that it opposes the change in the magnetic field from the source current. If the source current is increasing, the polarity of the induced emf is as pictured in Figure 32.2b, and if the source current is decreasing, the polarity of the induced emf is as shown in Figure 32.2c.

To obtain a quantitative description of self-induction, we recall from Faraday's law that the induced emf is equal to the negative time rate of change of the magnetic flux. The magnetic flux is proportional to the magnetic field due to the source current, which in turn is proportional to the source current in the circuit. Therefore, **a self-induced emf $\mathcal{E}_L$ is always proportional to the time rate of change of the source current.** For a closely spaced coil of N turns (a toroid or an ideal solenoid) carrying a source current I, we find that

Self-induced emf

$$\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} \tag{32.1}$$

where L is a proportionality constant—called the **inductance** of the coil—that depends on the geometry of the circuit and other physical characteristics. From this expression, we see that the inductance of a coil containing N turns is

Inductance of an N-turn coil

$$L = \frac{N\Phi_B}{I} \tag{32.2}$$

where it is assumed that the same flux passes through each turn. Later, we shall use this equation to calculate the inductance of some special circuit geometries.

From Equation 32.1, we can also write the inductance as the ratio

Inductance

$$L = -\frac{\mathcal{E}_L}{dI/dt} \tag{32.3}$$

Just as resistance is a measure of the opposition to current ($R = \Delta V/I$), inductance is a measure of the opposition to a *change* in current.

The SI unit of inductance is the **henry** (H), which, as we can see from Equation 32.3, is 1 volt-second per ampere:

$$1\ \text{H} = 1\ \frac{\text{V}\cdot\text{s}}{\text{A}}$$

That the inductance of a device depends on its geometry is analogous to the capacitance of a capacitor depending on the geometry of its plates, as we found in Chapter 26. Inductance calculations can be quite difficult to perform for complicated geometries; however, the following examples involve simple situations for which inductances are easily evaluated.

EXAMPLE 32.1 Inductance of a Solenoid

Find the inductance of a uniformly wound solenoid having N turns and length ℓ. Assume that ℓ is much longer than the radius of the windings and that the core of the solenoid is air.

Solution We can assume that the interior magnetic field due to the source current is uniform and given by Equation 30.17:

$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I$$

where $n = N/\ell$ is the number of turns per unit length. The magnetic flux through each turn is

$$\Phi_B = BA = \mu_0 \frac{NA}{\ell} I$$

where A is the cross-sectional area of the solenoid. Using this expression and Equation 32.2, we find that

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{\ell} \quad \textbf{(32.4)}$$

This result shows that L depends on geometry and is proportional to the square of the number of turns. Because $N = n\ell$, we can also express the result in the form

$$L = \mu_0 \frac{(n\ell)^2}{\ell} A = \mu_0 n^2 A\ell = \mu_0 n^2 V \quad \textbf{(32.5)}$$

where $V = A\ell$ is the volume of the solenoid.

Exercise What would happen to the inductance if a ferromagnetic material were placed inside the solenoid? 13.6

Answer The inductance would increase. For a given current, the magnetic flux is now much greater because of the increase in the field originating from the magnetization of the ferromagnetic material. For example, if the material has a magnetic permeability of $500\mu_0$, the inductance would increase by a factor of 500.

The fact that various materials in the vicinity of a coil can substantially alter the coil's inductance is used to great advantage by traffic engineers. A flat, horizontal coil made of numerous loops of wire is placed in a shallow groove cut into the pavement of the lane approaching an intersection. (See the photograph at the beginning of this chapter.) These loops are attached to circuitry that measures inductance. When an automobile passes over the loops, the change in inductance caused by the large amount of iron passing over the loops is used to control the lights at the intersection.

EXAMPLE 32.2 Calculating Inductance and emf

(a) Calculate the inductance of an air-core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm^2.

Solution Using Equation 32.4, we obtain

$$\begin{aligned} L &= \frac{\mu_0 N^2 A}{\ell} \\ &= (4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}) \frac{(300)^2(4.00 \times 10^{-4}\ \mathrm{m^2})}{25.0 \times 10^{-2}\ \mathrm{m}} \\ &= 1.81 \times 10^{-4}\ \mathrm{T\cdot m^2/A} = 0.181\ \mathrm{mH} \end{aligned}$$

(b) Calculate the self-induced emf in the solenoid if the current through it is decreasing at the rate of 50.0 A/s.

Solution Using Equation 32.1 and given that $dI/dt = -50.0$ A/s, we obtain

$$\begin{aligned} \mathcal{E}_L &= -L\frac{dI}{dt} = -(1.81 \times 10^{-4}\ \mathrm{H})(-50.0\ \mathrm{A/s}) \\ &= 9.05\ \mathrm{mV} \end{aligned}$$

32.2 RL CIRCUITS

13.6 If a circuit contains a coil, such as a solenoid, the self-inductance of the coil prevents the current in the circuit from increasing or decreasing instantaneously. A circuit element that has a large self-inductance is called an **inductor** and has the circuit symbol —ʘʘʘ—. We always assume that the self-inductance of the remainder of a circuit is negligible compared with that of the inductor. Keep in mind, however, that even a circuit without a coil has some self-inductance that can affect the behavior of the circuit.

Because the inductance of the inductor results in a back emf, **an inductor in a circuit opposes changes in the current through that circuit.** If the battery voltage in the circuit is increased so that the current rises, the inductor opposes this change, and the rise is not instantaneous. If the battery voltage is decreased, the presence of the inductor results in a slow drop in the current rather than an immediate drop. Thus, the inductor causes the circuit to be "sluggish" as it reacts to changes in the voltage.

Quick Quiz 32.1

A switch controls the current in a circuit that has a large inductance. Is a spark more likely to be produced at the switch when the switch is being closed or when it is being opened, or doesn't it matter?

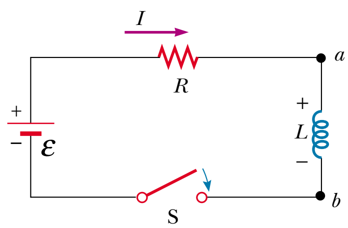


Figure 32.3 A series *RL* circuit. As the current increases toward its maximum value, an emf that opposes the increasing current is induced in the inductor.

Consider the circuit shown in Figure 32.3, in which the battery has negligible internal resistance. This is an ***RL* circuit** because the elements connected to the battery are a resistor and an inductor. Suppose that the switch S is thrown closed at $t = 0$. The current in the circuit begins to increase, and a back emf that opposes the increasing current is induced in the inductor. The back emf is, from Equation 32.1,

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

Because the current is increasing, dI/dt is positive; thus, $\mathcal{E}_L$ is negative. This negative value reflects the decrease in electric potential that occurs in going from a to b across the inductor, as indicated by the positive and negative signs in Figure 32.3.

With this in mind, we can apply Kirchhoff's loop rule to this circuit, traversing the circuit in the clockwise direction:

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0 \quad \textbf{(32.6)}$$

where IR is the voltage drop across the resistor. (We developed Kirchhoff's rules for circuits with steady currents, but we can apply them to a circuit in which the current is changing if we imagine them to represent the circuit at one *instant* of time.) We must now look for a solution to this differential equation, which is similar to that for the *RC* circuit (see Section 28.4).

A mathematical solution of Equation 32.6 represents the current in the circuit as a function of time. To find this solution, we change variables for convenience, letting $x = \dfrac{\mathcal{E}}{R} - I$, so that $dx = -dI$. With these substitutions, we can write Equation 32.6 as

$$x + \frac{L}{R}\frac{dx}{dt} = 0$$

$$\frac{dx}{x} = -\frac{R}{L}\,dt$$

Integrating this last expression, we have

$$\ln\frac{x}{x_0} = -\frac{R}{L}\,t$$

where we take the integrating constant to be $-\ln x_0$ and x_0 is the value of x at time $t = 0$. Taking the antilogarithm of this result, we obtain

$$x = x_0 e^{-Rt/L}$$

Because $I = 0$ at $t = 0$, we note from the definition of x that $x_0 = \mathcal{E}/R$. Hence, this last expression is equivalent to

$$\frac{\mathcal{E}}{R} - I = \frac{\mathcal{E}}{R}\, e^{-Rt/L}$$

$$I = \frac{\mathcal{E}}{R}\,(1 - e^{-Rt/L})$$

This expression shows the effect of the inductor. The current does not increase instantly to its final equilibrium value when the switch is closed but instead increases according to an exponential function. If we remove the inductance in the circuit, which we can do by letting L approach zero, the exponential term becomes zero and we see that there is no time dependence of the current in this case—the current increases instantaneously to its final equilibrium value in the absence of the inductance.

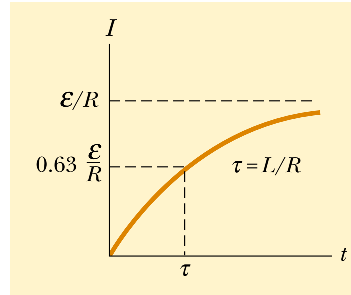


Figure 32.4 Plot of the current versus time for the *RL* circuit shown in Figure 32.3. The switch is thrown closed at $t = 0$, and the current increases toward its maximum value $\mathcal{E}/R$. The time constant τ is the time it takes I to reach 63% of its maximum value.

We can also write this expression as

$$I = \frac{\mathcal{E}}{R}\,(1 - e^{-t/\tau}) \qquad \textbf{(32.7)}$$

where the constant τ is the **time constant** of the *RL* circuit:

Time constant of an *RL* circuit

$$\tau = L/R \qquad \textbf{(32.8)}$$

Physically, τ is the time it takes the current in the circuit to reach $(1 - e^{-1}) = 0.63$ of its final value $\mathcal{E}/R$. The time constant is a useful parameter for comparing the time responses of various circuits.

Figure 32.4 shows a graph of the current versus time in the *RL* circuit. Note that the equilibrium value of the current, which occurs as t approaches infinity, is $\mathcal{E}/R$. We can see this by setting dI/dt equal to zero in Equation 32.6 and solving for the current I. (At equilibrium, the change in the current is zero.) Thus, we see that the current initially increases very rapidly and then gradually approaches the equilibrium value $\mathcal{E}/R$ as t approaches infinity.

Let us also investigate the time rate of change of the current in the circuit. Taking the first time derivative of Equation 32.7, we have

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L}\, e^{-t/\tau} \qquad \textbf{(32.9)}$$

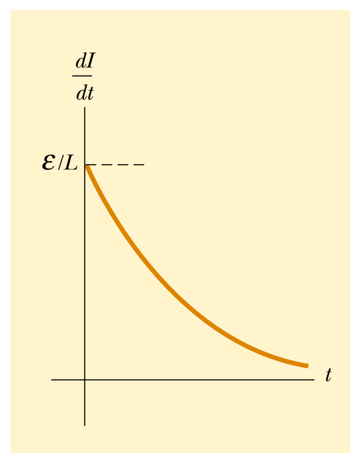


Figure 32.5 Plot of dI/dt versus time for the *RL* circuit shown in Figure 32.3. The time rate of change of current is a maximum at $t = 0$, which is the instant at which the switch is thrown closed. The rate decreases exponentially with time as I increases toward its maximum value.

From this result, we see that the time rate of change of the current is a maximum (equal to $\mathcal{E}/L$) at $t = 0$ and falls off exponentially to zero as t approaches infinity (Fig. 32.5).

Now let us consider the *RL* circuit shown in Figure 32.6. The circuit contains two switches that operate such that when one is closed, the other is opened. Suppose that S_1 has been closed for a length of time sufficient to allow the current to reach its equilibrium value $\mathcal{E}/R$. In this situation, the circuit is described completely by the outer loop in Figure 32.6. If S_2 is closed at the instant at which S_1 is opened, the circuit changes so that it is described completely by just the upper loop in Figure 32.6. The lower loop no longer influences the behavior of the circuit. Thus, we have a circuit with no battery ($\mathcal{E} = 0$). If we apply Kirchhoff's loop rule to the upper loop at the instant the switches are thrown, we obtain

$$IR + L\frac{dI}{dt} = 0$$

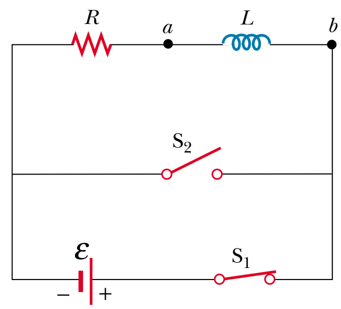


Figure 32.6 An *RL* circuit containing two switches. When S_1 is closed and S_2 open as shown, the battery is in the circuit. At the instant S_2 is closed, S_1 is opened, and the battery is no longer part of the circuit.

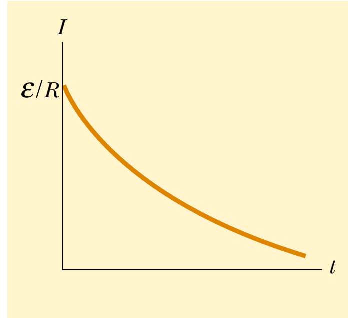


Figure 32.7 Current versus time for the upper loop of the circuit shown in Figure 32.6. For $t < 0$, S_1 is closed and S_2 is open. At $t = 0$, S_2 is closed as S_1 is opened, and the current has its maximum value $\mathcal{E}/R$.

It is left as a problem (Problem 18) to show that the solution of this differential equation is

$$I = \frac{\mathcal{E}}{R} e^{-t/\tau} = I_0 e^{-t/\tau} \tag{32.10}$$

where $\mathcal{E}$ is the emf of the battery and $I_0 = \mathcal{E}/R$ is the current at $t = 0$, the instant at which S_2 is closed as S_1 is opened.

If no inductor were present in the circuit, the current would immediately decrease to zero if the battery were removed. When the inductor is present, it acts to oppose the decrease in the current and to maintain the current. A graph of the current in the circuit versus time (Fig. 32.7) shows that the current is continuously decreasing with time. Note that the slope dI/dt is always negative and has its maximum value at $t = 0$. The negative slope signifies that $\mathcal{E}_L = -L\,(dI/dt)$ is now positive; that is, point *a* in Figure 32.6 is at a lower electric potential than point *b*.

Quick Quiz 32.2

Two circuits like the one shown in Figure 32.6 are identical except for the value of L. In circuit A the inductance of the inductor is L_A, and in circuit B it is L_B. Switch S_1 is thrown closed at $t = 0$, while switch S_2 remains open. At $t = 10$ s, switch S_1 is opened and switch S_2 is closed. The resulting time rates of change for the two currents are as graphed in Figure 32.8. If we assume that the time constant of each circuit is much less than 10 s, which of the following is true? (a) $L_A > L_B$; (b) $L_A < L_B$; (c) not enough information to tell.

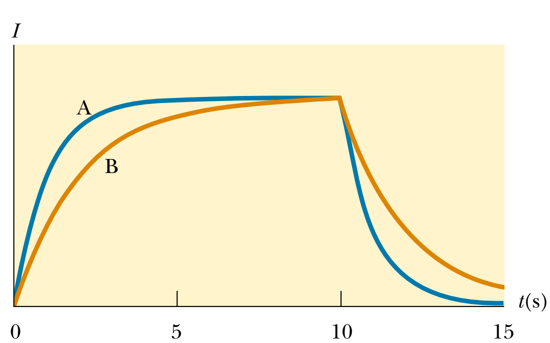


Figure 32.8

EXAMPLE 32.3 Time Constant of an *RL* Circuit

The switch in Figure 32.9a is thrown closed at $t = 0$. (a) Find the time constant of the circuit.

Solution The time constant is given by Equation 32.8:

$$\tau = \frac{L}{R} = \frac{30.0 \times 10^{-3}\ \text{H}}{6.00\ \Omega} = \boxed{5.00\ \text{ms}}$$

(b) Calculate the current in the circuit at $t = 2.00$ ms.

Solution Using Equation 32.7 for the current as a function of time (with t and τ in milliseconds), we find that at $t = 2.00$ ms

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) = \frac{12.0\ \text{V}}{6.00\ \Omega}(1 - e^{-0.400}) = \boxed{0.659\ \text{A}}$$

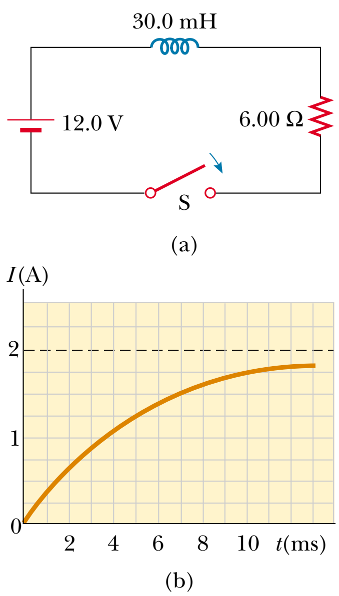


Figure 32.9 (a) The switch in this *RL* circuit is thrown closed at $t = 0$. (b) A graph of the current versus time for the circuit in part (a).

A plot of Equation 32.7 for this circuit is given in Figure 32.9b.

(c) Compare the potential difference across the resistor with that across the inductor.

Solution At the instant the switch is closed, there is no current and thus no potential difference across the resistor. At this instant, the battery voltage appears entirely across the inductor in the form of a back emf of 12.0 V as the inductor tries to maintain the zero-current condition. (The left end of the inductor is at a higher electric potential than the right end.) As time passes, the emf across the inductor decreases and the current through the resistor (and hence the potential difference across it) increases. The sum of the two potential differences at all times is 12.0 V, as shown in Figure 32.10.

Exercise Calculate the current in the circuit and the voltage across the resistor after a time interval equal to one time constant has elapsed.

Answer 1.26 A, 7.56 V.

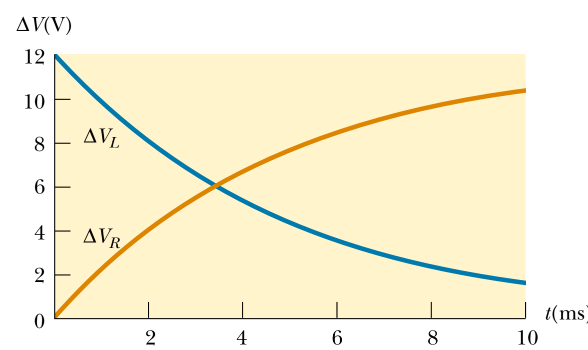


Figure 32.10 The sum of the potential differences across the resistor and inductor in Figure 32.9a is 12.0 V (the battery emf) at all times.

32.3 ENERGY IN A MAGNETIC FIELD

13.6

Because the emf induced in an inductor prevents a battery from establishing an instantaneous current, the battery must do work against the inductor to create a current. Part of the energy supplied by the battery appears as internal energy in the resistor, while the remaining energy is stored in the magnetic field of the inductor. If we multiply each term in Equation 32.6 by I and rearrange the expression, we have

$$I\mathcal{E} = I^2R + LI\frac{dI}{dt} \qquad \textbf{(32.11)}$$

This expression indicates that the rate at which energy is supplied by the battery ($I\mathcal{E}$) equals the sum of the rate at which energy is delivered to the resistor, I^2R, and the rate at which energy is stored in the inductor, $LI(dI/dt)$. Thus, Equation 32.11 is simply an expression of energy conservation. If we let U denote the energy stored in the inductor at any time, then we can write the rate dU/dt at which energy is stored as

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

To find the total energy stored in the inductor, we can rewrite this expression as $dU = LI\,dI$ and integrate:

$$U = \int dU = \int_0^I LI\,dI = L\int_0^I I\,dI$$

Energy stored in an inductor

$$U = \tfrac{1}{2}LI^2 \qquad \textbf{(32.12)}$$

where L is constant and has been removed from the integral. This expression represents the energy stored in the magnetic field of the inductor when the current is I. Note that this equation is similar in form to Equation 26.11 for the energy stored in the electric field of a capacitor, $U = Q^2/2C$. In either case, we see that energy is required to establish a field.

We can also determine the energy density of a magnetic field. For simplicity, consider a solenoid whose inductance is given by Equation 32.5:

$$L = \mu_0 n^2 A\ell$$

The magnetic field of a solenoid is given by Equation 30.17:

$$B = \mu_0 nI$$

Substituting the expression for L and $I = B/\mu_0 n$ into Equation 32.12 gives

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}\mu_0 n^2 A\ell\left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0}A\ell \qquad \textbf{(32.13)}$$

Because $A\ell$ is the volume of the solenoid, the energy stored per unit volume in the magnetic field surrounding the inductor is

Magnetic energy density

$$u_B = \frac{U}{A\ell} = \frac{B^2}{2\mu_0} \qquad \textbf{(32.14)}$$

Although this expression was derived for the special case of a solenoid, it is valid for any region of space in which a magnetic field exists. Note that Equation 32.14 is similar in form to Equation 26.13 for the energy per unit volume stored in an electric field, $u_E = \frac{1}{2}\epsilon_0 E^2$. In both cases, the energy density is proportional to the square of the magnitude of the field.

EXAMPLE 32.4 What Happens to the Energy in the Inductor?

Consider once again the RL circuit shown in Figure 32.6, in which switch S_2 is closed at the instant S_1 is opened (at $t = 0$). Recall that the current in the upper loop decays exponentially with time according to the expression $I = I_0 e^{-t/\tau}$, where $I_0 = \mathcal{E}/R$ is the initial current in the circuit and $\tau = L/R$ is the time constant. Show that all the energy initially stored in the magnetic field of the inductor appears as internal energy in the resistor as the current decays to zero.

Solution The rate dU/dt at which energy is delivered to the resistor (which is the power) is equal to I^2R, where I is the instantaneous current:

$$\frac{dU}{dt} = I^2R = (I_0e^{-Rt/L})^2R = I_0{}^2Re^{-2Rt/L}$$

To find the total energy delivered to the resistor, we solve for dU and integrate this expression over the limits $t = 0$ to $t \rightarrow \infty$ (the upper limit is infinity because it takes an infinite amount of time for the current to reach zero):

$$(1) \qquad U = \int_0^\infty I_0{}^2Re^{-2Rt/L}dt = I_0{}^2R\int_0^\infty e^{-2Rt/L}dt$$

The value of the definite integral is $L/2R$ (this is left for the student to show in the exercise at the end of this example), and so U becomes

$$U = I_0{}^2R\left(\frac{L}{2R}\right) = \frac{1}{2}LI_0{}^2$$

Note that this is equal to the initial energy stored in the magnetic field of the inductor, given by Equation 32.13, as we set out to prove.

Exercise Show that the integral on the right-hand side of Equation (1) has the value $L/2R$.

EXAMPLE 32.5 The Coaxial Cable

Coaxial cables are often used to connect electrical devices, such as your stereo system and a loudspeaker. Model a long coaxial cable as consisting of two thin concentric cylindrical conducting shells of radii a and b and length ℓ, as shown in Figure 32.11. The conducting shells carry the same current I in opposite directions. Imagine that the inner conductor carries current to a device and that the outer one acts as a return path carrying the current back to the source. (a) Calculate the self-inductance L of this cable.

Solution To obtain L, we must know the magnetic flux through any cross-section in the region between the two shells, such as the light blue rectangle in Figure 32.11. Ampère's law (see Section 30.3) tells us that the magnetic field in the region between the shells is $B = \mu_0I/2\pi r$, where r is measured from the common center of the shells. The magnetic field is zero outside the outer shell ($r > b$) because the net current through the area enclosed by a circular path surrounding the cable is zero, and hence from Ampère's law, $\oint \mathbf{B} \cdot d\mathbf{s} = 0$. The magnetic field is zero inside the inner shell because the shell is hollow and no current is present within a radius $r < a$.

The magnetic field is perpendicular to the light blue rectangle of length ℓ and width $b - a$, the cross-section of interest. Because the magnetic field varies with radial position across this rectangle, we must use calculus to find the total magnetic flux. Dividing this rectangle into strips of width dr, such as the dark blue strip in Figure 32.11, we see that the area of each strip is $\ell\,dr$ and that the flux through each strip is $B\,dA = B\ell\,dr$. Hence, we find the total flux through the entire cross-section by integrating:

$$\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 I}{2\pi r}\,\ell\,dr = \frac{\mu_0 I\ell}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I\ell}{2\pi}\ln\left(\frac{b}{a}\right)$$

Using this result, we find that the self-inductance of the cable is

$$L = \frac{\Phi_B}{I} = \boxed{\frac{\mu_0\ell}{2\pi}\ln\left(\frac{b}{a}\right)}$$

(b) Calculate the total energy stored in the magnetic field of the cable.

Solution Using Equation 32.12 and the results to part (a) gives

$$U = \tfrac{1}{2}LI^2 = \boxed{\frac{\mu_0\ell I^2}{4\pi}\ln\left(\frac{b}{a}\right)}$$

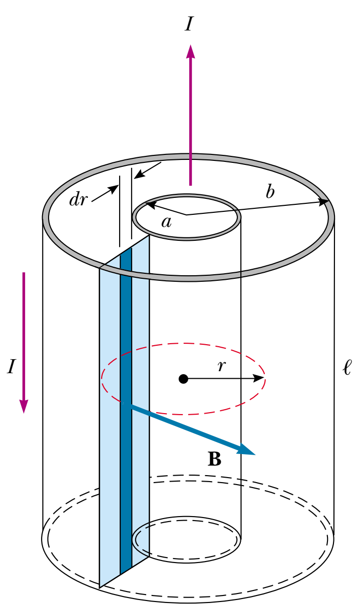


Figure 32.11 Section of a long coaxial cable. The inner and outer conductors carry equal currents in opposite directions.

32.4 MUTUAL INDUCTANCE

Very often, the magnetic flux through the area enclosed by a circuit varies with time because of time-varying currents in nearby circuits. This condition induces an emf through a process known as *mutual induction,* so called because it depends on the interaction of two circuits.

Consider the two closely wound coils of wire shown in cross-sectional view in Figure 32.12. The current I_1 in coil 1, which has N_1 turns, creates magnetic field lines, some of which pass through coil 2, which has N_2 turns. The magnetic flux caused by the current in coil 1 and passing through coil 2 is represented by Φ_{12}. In analogy to Equation 32.2, we define the **mutual inductance** M_{12} of coil 2 with respect to coil 1:

Definition of mutual inductance

$$M_{12} \equiv \frac{N_2 \Phi_{12}}{I_1} \tag{32.15}$$

Quick Quiz 32.3

Referring to Figure 32.12, tell what happens to M_{12} (a) if coil 1 is brought closer to coil 2 and (b) if coil 1 is rotated so that it lies in the plane of the page.

Quick Quiz 32.3 demonstrates that mutual inductance depends on the geometry of both circuits and on their orientation with respect to each other. As the circuit separation distance increases, the mutual inductance decreases because the flux linking the circuits decreases.

If the current I_1 varies with time, we see from Faraday's law and Equation 32.15 that the emf induced by coil 1 in coil 2 is

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{12}}{dt} = -N_2 \frac{d}{dt}\left(\frac{M_{12} I_1}{N_2}\right) = -M_{12}\frac{dI_1}{dt} \tag{32.16}$$

In the preceding discussion, we assumed that the source current is in coil 1. We can also imagine a source current I_2 in coil 2. The preceding discussion can be repeated to show that there is a mutual inductance M_{21}. If the current I_2 varies with time, the emf induced by coil 2 in coil 1 is

$$\mathcal{E}_1 = -M_{21}\frac{dI_2}{dt} \tag{32.17}$$

In mutual induction, the emf induced in one coil is always proportional to the rate at which the current in the other coil is changing. Although the

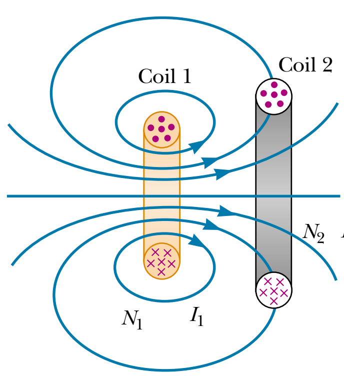


Figure 32.12 A cross-sectional view of two adjacent coils. A current in coil 1 sets up a magnetic flux, part of which passes through coil 2.

proportionality constants M_{12} and M_{21} appear to have different values, it can be shown that they are equal. Thus, with $M_{12} = M_{21} = M$, Equations 32.16 and 32.17 become

$$\mathcal{E}_2 = -M\frac{dI_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M\frac{dI_2}{dt}$$

These two equations are similar in form to Equation 32.1 for the self-induced emf $\mathcal{E} = -L(dI/dt)$. The unit of mutual inductance is the henry.

QuickLab

Tune in a relatively weak station on a radio. Now slowly rotate the radio about a vertical axis through its center. What happens to the reception? Can you explain this in terms of the mutual induction of the station's broadcast antenna and your radio's antenna?

Quick Quiz 32.4

(a) Can you have mutual inductance without self-inductance? (b) How about self-inductance without mutual inductance?

EXAMPLE 32.6 "Wireless" Battery Charger

An electric toothbrush has a base designed to hold the toothbrush handle when not in use. As shown in Figure 32.13a, the handle has a cylindrical hole that fits loosely over a matching cylinder on the base. When the handle is placed on the base, a changing current in a solenoid inside the base cylinder induces a current in a coil inside the handle. This induced current charges the battery in the handle.

We can model the base as a solenoid of length ℓ with N_B turns (Fig. 32.13b), carrying a source current I, and having a cross-sectional area A. The handle coil contains N_H turns. Find the mutual inductance of the system.

Solution Because the base solenoid carries a source current I, the magnetic field in its interior is

$$B = \frac{\mu_0 N_B I}{\ell}$$

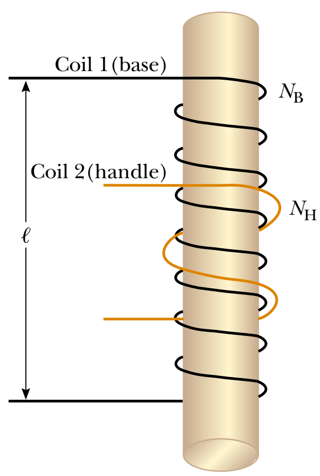
Because the magnetic flux Φ_{BH} through the handle's coil caused by the magnetic field of the base coil is BA, the mutual inductance is

$$M = \frac{N_H \Phi_{BH}}{I} = \frac{N_H BA}{I} = \mu_0 \frac{N_H N_B A}{\ell}$$

Exercise Calculate the mutual inductance of two solenoids with $N_B = 1\,500$ turns, $A = 1.0 \times 10^{-4}\ \text{m}^2$, $\ell = 0.02$ m, and $N_H = 800$ turns.

Answer 7.5 mH.

(a)



(b)

Figure 32.13 (a) This electric toothbrush uses the mutual induction of solenoids as part of its battery-charging system. (b) A coil of N_H turns wrapped around the center of a solenoid of N_B turns.

32.5 OSCILLATIONS IN AN *LC* CIRCUIT

13.7 When a capacitor is connected to an inductor as illustrated in Figure 32.14, the combination is an ***LC* circuit.** If the capacitor is initially charged and the switch is then closed, both the current in the circuit and the charge on the capacitor oscillate between maximum positive and negative values. If the resistance of the circuit is zero, no energy is transformed to internal energy. In the following analysis, we neglect the resistance in the circuit. We also assume an idealized situation in which energy is not radiated away from the circuit. We shall discuss this radiation in Chapter 34, but we neglect it for now. With these idealizations—zero resistance and no radiation—the oscillations in the circuit persist indefinitely.

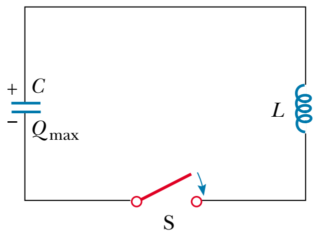


Figure 32.14 A simple *LC* circuit. The capacitor has an initial charge Q_{max}, and the switch is thrown closed at $t = 0$.

Assume that the capacitor has an initial charge Q_{max} (the maximum charge) and that the switch is thrown closed at $t = 0$. Let us look at what happens from an energy viewpoint.

When the capacitor is fully charged, the energy U in the circuit is stored in the electric field of the capacitor and is equal to $Q_{max}^2/2C$ (Eq. 26.11). At this time, the current in the circuit is zero, and thus no energy is stored in the inductor. After the switch is thrown closed, the rate at which charges leave or enter the capacitor plates (which is also the rate at which the charge on the capacitor changes) is equal to the current in the circuit. As the capacitor begins to discharge after the switch is closed, the energy stored in its electric field decreases. The discharge of the capacitor represents a current in the circuit, and hence some energy is now stored in the magnetic field of the inductor. Thus, energy is transferred from the electric field of the capacitor to the magnetic field of the inductor. When the capacitor is fully discharged, it stores no energy. At this time, the current reaches its maximum value, and all of the energy is stored in the inductor. The current continues in the same direction, decreasing in magnitude, with the capacitor eventually becoming fully charged again but with the polarity of its plates now opposite the initial polarity. This is followed by another discharge until the circuit returns to its original state of maximum charge Q_{max} and the plate polarity shown in Figure 32.14. The energy continues to oscillate between inductor and capacitor.

The oscillations of the *LC* circuit are an electromagnetic analog to the mechanical oscillations of a block–spring system, which we studied in Chapter 13. Much of what we discussed is applicable to *LC* oscillations. For example, we investigated the effect of driving a mechanical oscillator with an external force, which leads to the phenomenon of *resonance.* We observe the same phenomenon in the *LC* circuit. For example, a radio tuner has an *LC* circuit with a natural frequency, which we determine as follows: When the circuit is driven by the electromagnetic oscillations of a radio signal detected by the antenna, the tuner circuit responds with a large amplitude of electrical oscillation only for the station frequency that matches the natural frequency. Thus, only the signal from one station is passed on to the amplifier, even though signals from all stations are driving the circuit at the same time. When you turn the knob on the radio tuner to change the station, you are changing the natural frequency of the circuit so that it will exhibit a resonance response to a different driving frequency.

A graphical description of the energy transfer between the inductor and the capacitor in an *LC* circuit is shown in Figure 32.15. The right side of the figure shows the analogous energy transfer in the oscillating block–spring system studied in Chapter 13. In each case, the situation is shown at intervals of one-fourth the period of oscillation T. The potential energy $\frac{1}{2}kx^2$ stored in a stretched spring is analogous to the electric potential energy $Q_{max}^2/2C$ stored in the capacitor. The kinetic energy $\frac{1}{2}mv^2$ of the moving block is analogous to the magnetic energy $\frac{1}{2}LI^2$

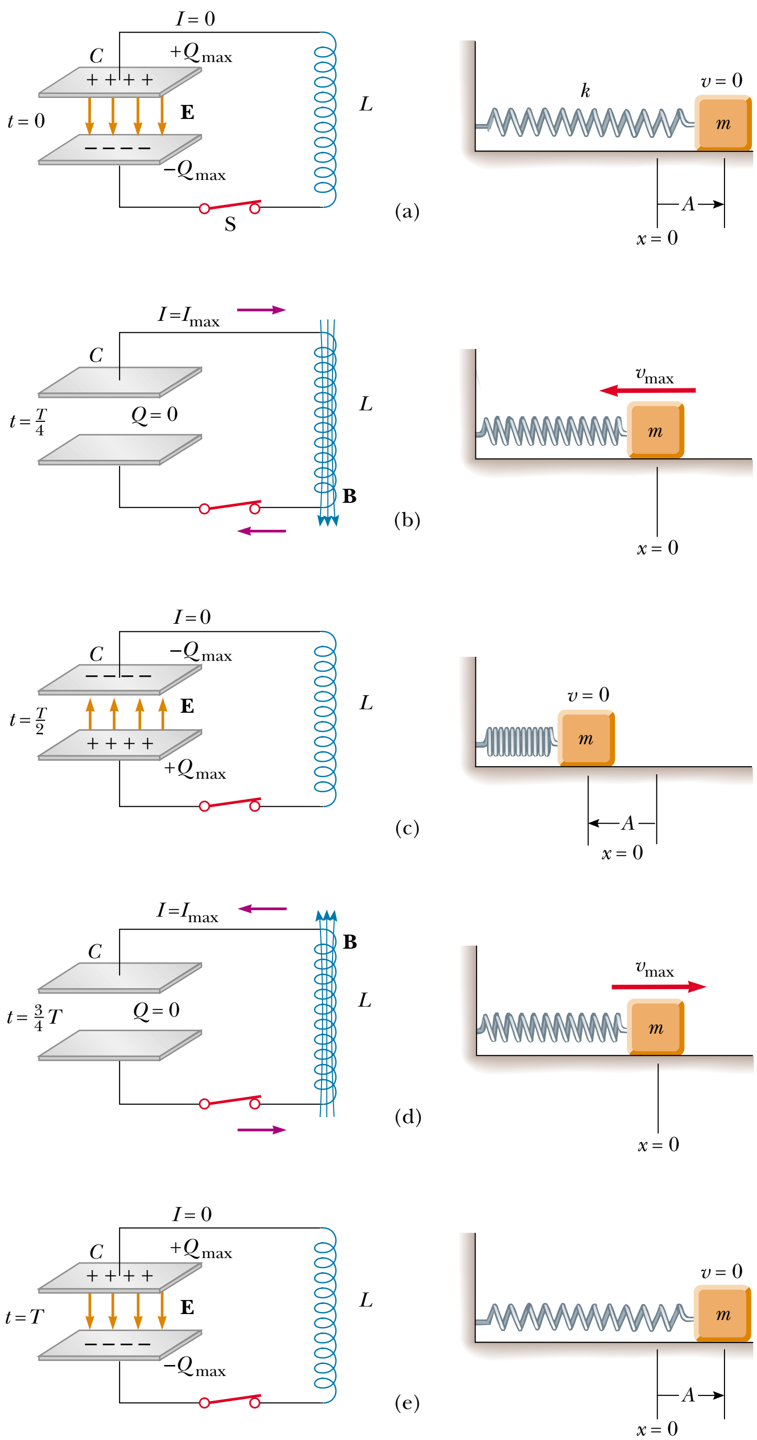


Figure 32.15 Energy transfer in a resistanceless, non-radiating *LC* circuit. The capacitor has a charge Q_{max} at $t = 0$, the instant at which the switch is thrown closed. The mechanical analog of this circuit is a block–spring system.

stored in the inductor, which requires the presence of moving charges. In Figure 32.15a, all of the energy is stored as electric potential energy in the capacitor at $t = 0$. In Figure 32.15b, which is one fourth of a period later, all of the energy is stored as magnetic energy $\frac{1}{2}LI_{\text{max}}^2$ in the inductor, where I_{max} is the maximum current in the circuit. In Figure 32.15c, the energy in the LC circuit is stored completely in the capacitor, with the polarity of the plates now opposite what it was in Figure 32.15a. In parts d and e the system returns to the initial configuration over the second half of the cycle. At times other than those shown in the figure, part of the energy is stored in the electric field of the capacitor and part is stored in the magnetic field of the inductor. In the analogous mechanical oscillation, part of the energy is potential energy in the spring and part is kinetic energy of the block.

Let us consider some arbitrary time t after the switch is closed, so that the capacitor has a charge $Q < Q_{\text{max}}$ and the current is $I < I_{\text{max}}$. At this time, both elements store energy, but the sum of the two energies must equal the total initial energy U stored in the fully charged capacitor at $t = 0$:

Total energy stored in an LC circuit

$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{1}{2}LI^2 \qquad \textbf{(32.18)}$$

Because we have assumed the circuit resistance to be zero, no energy is transformed to internal energy, and hence *the total energy must remain constant in time.* This means that $dU/dt = 0$. Therefore, by differentiating Equation 32.18 with respect to time while noting that Q and I vary with time, we obtain

The total energy in an ideal LC circuit remains constant; $dU/dt = 0$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C} + \frac{1}{2}LI^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0 \qquad \textbf{(32.19)}$$

We can reduce this to a differential equation in one variable by remembering that the current in the circuit is equal to the rate at which the charge on the capacitor changes: $I = dQ/dt$. From this, it follows that $dI/dt = d^2Q/dt^2$. Substitution of these relationships into Equation 32.19 gives

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q \qquad \textbf{(32.20)}$$

We can solve for Q by noting that this expression is of the same form as the analogous Equations 13.16 and 13.17 for a block–spring system:

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

where k is the spring constant, m is the mass of the block, and $\omega = \sqrt{k/m}$. The solution of this equation has the general form

$$x = A\cos(\omega t + \phi)$$

where ω is the angular frequency of the simple harmonic motion, A is the amplitude of motion (the maximum value of x), and ϕ is the phase constant; the values of A and ϕ depend on the initial conditions. Because it is of the same form as the differential equation of the simple harmonic oscillator, we see that Equation 32.20 has the solution

Charge versus time for an ideal LC circuit

$$Q = Q_{\text{max}}\cos(\omega t + \phi) \qquad \textbf{(32.21)}$$

where Q_{max} is the maximum charge of the capacitor and the angular frequency ω is

$$\omega = \frac{1}{\sqrt{LC}} \qquad (32.22)$$

Angular frequency of oscillation

Note that the angular frequency of the oscillations depends solely on the inductance and capacitance of the circuit. This is the *natural frequency* of oscillation of the *LC* circuit.

Because Q varies sinusoidally, the current in the circuit also varies sinusoidally. We can easily show this by differentiating Equation 32.21 with respect to time:

$$I = \frac{dQ}{dt} = -\omega Q_{max} \sin(\omega t + \phi) \qquad (32.23)$$

Current versus time for an ideal *LC* current

To determine the value of the phase angle ϕ, we examine the initial conditions, which in our situation require that at $t = 0$, $I = 0$ and $Q = Q_{max}$. Setting $I = 0$ at $t = 0$ in Equation 32.23, we have

$$0 = -\omega Q_{max} \sin \phi$$

which shows that $\phi = 0$. This value for ϕ also is consistent with Equation 32.21 and with the condition that $Q = Q_{max}$ at $t = 0$. Therefore, in our case, the expressions for Q and I are

$$Q = Q_{max} \cos \omega t \qquad (32.24)$$

$$I = -\omega Q_{max} \sin \omega t = -I_{max} \sin \omega t \qquad (32.25)$$

Graphs of Q versus t and I versus t are shown in Figure 32.16. Note that the charge on the capacitor oscillates between the extreme values Q_{max} and $-Q_{max}$, and that the current oscillates between I_{max} and $-I_{max}$. Furthermore, the current is 90° out of phase with the charge. That is, when the charge is a maximum, the current is zero, and when the charge is zero, the current has its maximum value.

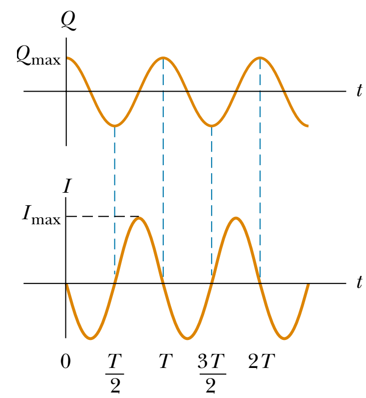


Figure 32.16 Graphs of charge versus time and current versus time for a resistanceless, nonradiating *LC* circuit. Note that Q and I are 90° out of phase with each other.

Quick Quiz 32.5

What is the relationship between the amplitudes of the two curves in Figure 32.16?

Let us return to the energy discussion of the *LC* circuit. Substituting Equations 32.24 and 32.25 in Equation 32.18, we find that the total energy is

$$U = U_C + U_L = \frac{Q_{max}^2}{2C} \cos^2 \omega t + \frac{LI_{max}^2}{2} \sin^2 \omega t \qquad (32.26)$$

This expression contains all of the features described qualitatively at the beginning of this section. It shows that the energy of the *LC* circuit continuously oscillates between energy stored in the electric field of the capacitor and energy stored in the magnetic field of the inductor. When the energy stored in the capacitor has its maximum value $Q_{max}^2/2C$, the energy stored in the inductor is zero. When the energy stored in the inductor has its maximum value $\frac{1}{2}LI_{max}^2$, the energy stored in the capacitor is zero.

Plots of the time variations of U_C and U_L are shown in Figure 32.17. The sum $U_C + U_L$ is a constant and equal to the total energy $Q_{max}^2/2C$ or $LI_{max}^2/2$. Analytical verification of this is straightforward. The amplitudes of the two graphs in Figure 32.17 must be equal because the maximum energy stored in the capacitor

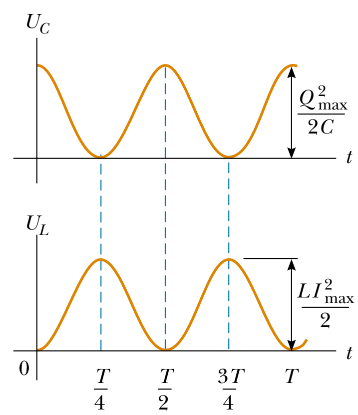


Figure 32.17 Plots of U_C versus t and U_L versus t for a resistanceless, nonradiating *LC* circuit. The sum of the two curves is a constant and equal to the total energy stored in the circuit.

(when $I = 0$) must equal the maximum energy stored in the inductor (when $Q = 0$). This is mathematically expressed as

$$\frac{Q_{\text{max}}^2}{2C} = \frac{LI_{\text{max}}^2}{2}$$

Using this expression in Equation 32.26 for the total energy gives

$$U = \frac{Q_{\text{max}}^2}{2C}(\cos^2 \omega t + \sin^2 \omega t) = \frac{Q_{\text{max}}^2}{2C} \quad \textbf{(32.27)}$$

because $\cos^2 \omega t + \sin^2 \omega t = 1$.

In our idealized situation, the oscillations in the circuit persist indefinitely; however, we remember that the total energy U of the circuit remains constant only if energy transfers and transformations are neglected. In actual circuits, there is always some resistance, and hence energy is transformed to internal energy. We mentioned at the beginning of this section that we are also ignoring radiation from the circuit. In reality, radiation is inevitable in this type of circuit, and the total energy in the circuit continuously decreases as a result of this process.

EXAMPLE 32.7 An Oscillatory *LC* Circuit

In Figure 32.18, the capacitor is initially charged when switch S_1 is open and S_2 is closed. Switch S_1 is then thrown closed at the same instant that S_2 is opened, so that the capacitor is connected directly across the inductor. (a) Find the frequency of oscillation of the circuit.

Solution Using Equation 32.22 gives for the frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi[(2.81 \times 10^{-3}\text{ H})(9.00 \times 10^{-12}\text{ F})]^{1/2}}$$

$$= \boxed{1.00 \times 10^6\text{ Hz}}$$

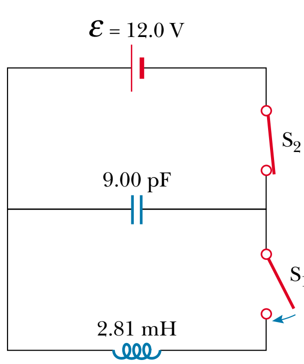


Figure 32.18 First the capacitor is fully charged with the switch S_1 open and S_2 closed. Then, S_1 is thrown closed at the same time that S_2 is thrown open.

(b) What are the maximum values of charge on the capacitor and current in the circuit?

Solution The initial charge on the capacitor equals the maximum charge, and because $C = Q/\mathcal{E}$, we have

$$Q_{\text{max}} = C\mathcal{E} = (9.00 \times 10^{-12}\text{ F})(12.0\text{ V}) = \boxed{1.08 \times 10^{-10}\text{ C}}$$

From Equation 32.25, we can see how the maximum current is related to the maximum charge:

$$I_{\text{max}} = \omega Q_{\text{max}} = 2\pi f Q_{\text{max}}$$

$$= (2\pi \times 10^6\text{ s}^{-1})(1.08 \times 10^{-10}\text{ C})$$

$$= \boxed{6.79 \times 10^{-4}\text{ A}}$$

(c) Determine the charge and current as functions of time.

Solution Equations 32.24 and 32.25 give the following expressions for the time variation of Q and I:

$$Q = Q_{\text{max}} \cos \omega t$$

$$= \boxed{(1.08 \times 10^{-10}\text{ C}) \cos[(2\pi \times 10^6\text{ rad/s})t]}$$

$$I = -I_{\text{max}} \sin \omega t$$

$$= \boxed{(-6.79 \times 10^{-4}\text{ A}) \sin[(2\pi \times 10^6\text{ rad/s})t]}$$

Exercise What is the total energy stored in the circuit?

Answer 6.48×10^{-10} J.

Optional Section

32.6 THE *RLC* CIRCUIT

13.7 We now turn our attention to a more realistic circuit consisting of an inductor, a capacitor, and a resistor connected in series, as shown in Figure 32.19. We let the resistance of the resistor represent all of the resistance in the circuit. We assume that the capacitor has an initial charge Q_{max} before the switch is closed. Once the switch is thrown closed and a current is established, the total energy stored in the capacitor and inductor at any time is given, as before, by Equation 32.18. However, the total energy is no longer constant, as it was in the *LC* circuit, because the resistor causes transformation to internal energy. Because the rate of energy transformation to internal energy within a resistor is I^2R, we have

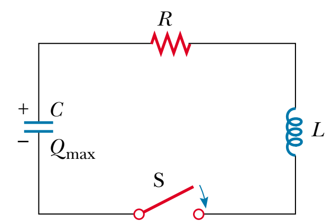


Figure 32.19 A series *RLC* circuit. The capacitor has a charge Q_{max} at $t = 0$, the instant at which the switch is thrown closed.

$$\frac{dU}{dt} = -I^2R$$

where the negative sign signifies that the energy U of the circuit is decreasing in time. Substituting this result into Equation 32.19 gives

$$LI\frac{dI}{dt} + \frac{Q}{C}\frac{dQ}{dt} = -I^2R \tag{32.28}$$

To convert this equation into a form that allows us to compare the electrical oscillations with their mechanical analog, we first use the fact that $I = dQ/dt$ and move all terms to the left-hand side to obtain

$$LI\frac{d^2Q}{dt^2} + \frac{Q}{C}I + I^2R = 0$$

Now we divide through by I:

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} + IR = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \tag{32.29}$$

The *RLC* circuit is analogous to the damped harmonic oscillator discussed in Section 13.6 and illustrated in Figure 32.20. The equation of motion for this mechanical system is, from Equation 13.32,

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \tag{32.30}$$

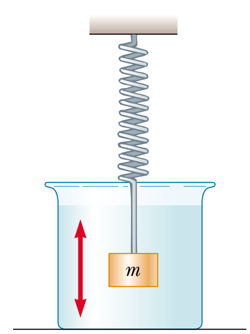


Figure 32.20 A block–spring system moving in a viscous medium with damped harmonic motion is analogous to an *RLC* circuit.

Comparing Equations 32.29 and 32.30, we see that Q corresponds to the position x of the block at any instant, L to the mass m of the block, R to the damping coefficient b, and C to $1/k$, where k is the force constant of the spring. These and other relationships are listed in Table 32.1.

Because the analytical solution of Equation 32.29 is cumbersome, we give only a qualitative description of the circuit behavior. In the simplest case, when $R = 0$, Equation 32.29 reduces to that of a simple *LC* circuit, as expected, and the charge and the current oscillate sinusoidally in time. This is equivalent to removal of all damping in the mechanical oscillator.

When R is small, a situation analogous to light damping in the mechanical oscillator, the solution of Equation 32.29 is

$$Q = Q_{max}e^{-Rt/2L}\cos\omega_d t \tag{32.31}$$

TABLE 32.1 Analogies Between Electrical and Mechanical Systems

Electric Circuit		One-Dimensional Mechanical System
Charge	$Q \leftrightarrow x$	Displacement
Current	$I \leftrightarrow v_x$	Velocity
Potential difference	$\Delta V \leftrightarrow F_x$	Force
Resistance	$R \leftrightarrow b$	Viscous damping coefficient
Capacitance	$C \leftrightarrow 1/k$	(k = spring constant)
Inductance	$L \leftrightarrow m$	Mass
Current = time derivative of charge	$I = \frac{dQ}{dt} \leftrightarrow v_x = \frac{dx}{dt}$	Velocity = time derivative of position
Rate of change of current = second time derivative of charge	$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	Acceleration = second time derivative of position
Energy in inductor	$U_L = \frac{1}{2}LI^2 \leftrightarrow K = \frac{1}{2}mv^2$	Kinetic energy of moving mass
Energy in capacitor	$U_C = \frac{1}{2}\frac{Q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2$	Potential energy stored in a spring
Rate of energy loss due to resistance	$I^2R \leftrightarrow bv^2$	Rate of energy loss due to friction
RLC circuit	$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$	Damped mass on a spring

where

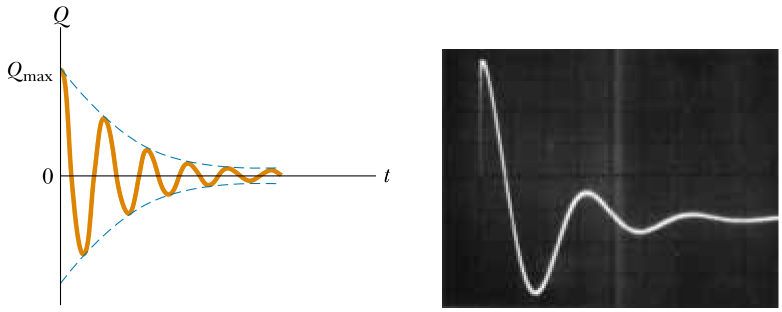
$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2} \qquad \textbf{(32.32)}$$

is the angular frequency at which the circuit oscillates. That is, the value of the charge on the capacitor undergoes a damped harmonic oscillation in analogy with a mass–spring system moving in a viscous medium. From Equation 32.32, we see that, when $R \ll \sqrt{4L/C}$ (so that the second term in the brackets is much smaller than the first), the frequency ω_d of the damped oscillator is close to that of the undamped oscillator, $1/\sqrt{LC}$. Because $I = dQ/dt$, it follows that the current also undergoes damped harmonic oscillation. A plot of the charge versus time for the damped oscillator is shown in Figure 32.21a. Note that the maximum value of Q decreases after each oscillation, just as the amplitude of a damped block–spring system decreases in time.

Quick Quiz 32.6

Figure 32.21a has two dashed blue lines that form an "envelope" around the curve. What is the equation for the upper dashed line?

When we consider larger values of R, we find that the oscillations damp out more rapidly; in fact, there exists a critical resistance value $R_c = \sqrt{4L/C}$ above which no oscillations occur. A system with $R = R_c$ is said to be *critically damped.* When R exceeds R_c, the system is said to be *overdamped* (Fig. 32.22).



(a) (b)

Figure 32.21 (a) Charge versus time for a damped *RLC* circuit. The charge decays in this way when $R \ll \sqrt{4L/C}$. The *Q*-versus-*t* curve represents a plot of Equation 32.31. (b) Oscilloscope pattern showing the decay in the oscillations of an *RLC* circuit. The parameters used were $R = 75\ \Omega$, $L = 10$ mH, and $C = 0.19\ \mu$F.

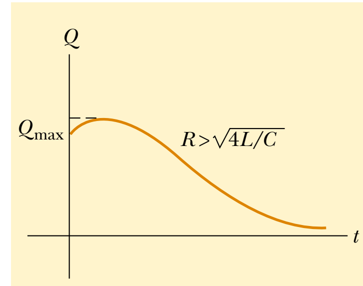


Figure 32.22 Plot of *Q* versus *t* for an overdamped *RLC* circuit, which occurs for values of $R > \sqrt{4L/C}$.

SUMMARY

When the current in a coil changes with time, an emf is induced in the coil according to Faraday's law. The **self-induced emf** is

$$\mathcal{E}_L = -L\frac{dI}{dt} \tag{32.1}$$

where L is the **inductance** of the coil. Inductance is a measure of how much opposition an electrical device offers to a change in current passing through the device. Inductance has the SI unit of **henry** (H), where $1\ \text{H} = 1\ \text{V}\cdot\text{s/A}$.

The inductance of any coil is

$$L = \frac{N\Phi_B}{I} \tag{32.2}$$

where Φ_B is the magnetic flux through the coil and N is the total number of turns. The inductance of a device depends on its geometry. For example, the inductance of an air-core solenoid is

$$L = \frac{\mu_0 N^2 A}{\ell} \tag{32.4}$$

where A is the cross-sectional area, and ℓ is the length of the solenoid.

If a resistor and inductor are connected in series to a battery of emf $\mathcal{E}$, and if a switch in the circuit is thrown closed at $t = 0$, then the current in the circuit varies in time according to the expression

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) \tag{32.7}$$

where $\tau = L/R$ is the time constant of the *RL* circuit. That is, the current increases to an equilibrium value of $\mathcal{E}/R$ after a time that is long compared with τ. If the battery in the circuit is replaced by a resistanceless wire, the current decays exponentially with time according to the expression

$$I = \frac{\mathcal{E}}{R}e^{-t/\tau} \tag{32.10}$$

where $\mathcal{E}/R$ is the initial current in the circuit.

The energy stored in the magnetic field of an inductor carrying a current I is

$$U = \tfrac{1}{2}LI^2 \qquad \textbf{(32.12)}$$

This energy is the magnetic counterpart to the energy stored in the electric field of a charged capacitor.

The energy density at a point where the magnetic field is B is

$$u_B = \frac{B^2}{2\mu_0} \qquad \textbf{(32.14)}$$

The mutual inductance of a system of two coils is given by

$$M_{12} = \frac{N_2\Phi_{12}}{I_1} = M_{21} = \frac{N_1\Phi_{21}}{I_2} = M \qquad \textbf{(32.15)}$$

This mutual inductance allows us to relate the induced emf in a coil to the changing source current in a nearby coil using the relationships

$$\boldsymbol{\varepsilon}_2 = -M_{12}\frac{dI_1}{dt} \quad \text{and} \quad \boldsymbol{\varepsilon}_1 = -M_{21}\frac{dI_2}{dt} \qquad \textbf{(32.16, 32.17)}$$

In an LC circuit that has zero resistance and does not radiate electromagnetically (an idealization), the values of the charge on the capacitor and the current in the circuit vary in time according to the expressions

$$Q = Q_{\text{max}}\cos(\omega t + \phi) \qquad \textbf{(32.21)}$$

$$I = \frac{dQ}{dt} = -\omega Q_{\text{max}}\sin(\omega t + \phi) \qquad \textbf{(32.23)}$$

where Q_{max} is the maximum charge on the capacitor, ϕ is a phase constant, and ω is the angular frequency of oscillation:

$$\omega = \frac{1}{\sqrt{LC}} \qquad \textbf{(32.22)}$$

The energy in an LC circuit continuously transfers between energy stored in the capacitor and energy stored in the inductor. The total energy of the LC circuit at any time t is

$$U = U_C + U_L = \frac{Q_{\text{max}}^2}{2C}\cos^2\omega t + \frac{LI_{\text{max}}^2}{2}\sin^2\omega t \qquad \textbf{(32.26)}$$

At $t = 0$, all of the energy is stored in the electric field of the capacitor ($U = Q_{\text{max}}^2/2C$). Eventually, all of this energy is transferred to the inductor ($U = LI_{\text{max}}^2/2$). However, the total energy remains constant because energy transformations are neglected in the ideal LC circuit.

QUESTIONS

1. Why is the induced emf that appears in an inductor called a "counter" or "back" emf?
2. The current in a circuit containing a coil, resistor, and battery reaches a constant value. Does the coil have an inductance? Does the coil affect the value of the current?
3. What parameters affect the inductance of a coil? Does the inductance of a coil depend on the current in the coil?
4. How can a long piece of wire be wound on a spool so that the wire has a negligible self-inductance?
5. A long, fine wire is wound as a solenoid with a self-inductance L. If it is connected across the terminals of a battery, how does the maximum current depend on L?
6. For the series RL circuit shown in Figure Q32.6, can the back emf ever be greater than the battery emf? Explain.

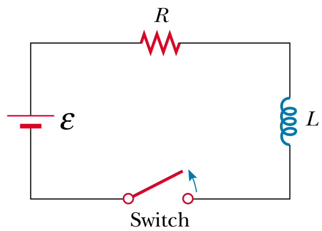


Figure Q32.6

7. Consider this thesis: "Joseph Henry, America's first professional physicist, changed the view of the Universe during a school vacation at the Albany Academy in 1830. Before that time, one could think of the Universe as consisting of just one thing: matter. In Henry's experiment, after a battery is removed from a coil, the energy that keeps the current flowing for a while does not belong to any piece of matter. This energy belongs to the magnetic field surrounding the coil. With Henry's discovery of self-induction, Nature forced us to admit that the Universe consists of fields as well as matter." What in your view constitutes the Universe? Argue for your answer.
8. Discuss the similarities and differences between the energy stored in the electric field of a charged capacitor and the energy stored in the magnetic field of a current-carrying coil.
9. What is the inductance of two inductors connected in series? Does it matter if they are solenoids or toroids?
10. The centers of two circular loops are separated by a fixed distance. For what relative orientation of the loops is their mutual inductance a maximum? a minimum? Explain.
11. Two solenoids are connected in series so that each carries the same current at any instant. Is mutual induction present? Explain.
12. In the *LC* circuit shown in Figure 32.15, the charge on the capacitor is sometimes zero, even though current is in the circuit. How is this possible?
13. If the resistance of the wires in an *LC* circuit were not zero, would the oscillations persist? Explain.
14. How can you tell whether an *RLC* circuit is overdamped or underdamped?
15. What is the significance of critical damping in an *RLC* circuit?
16. Can an object exert a force on itself? When a coil induces an emf in itself, does it exert a force on itself?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
☐ = paired numerical/symbolic problems

Section 32.1 Self-Inductance

1. A coil has an inductance of 3.00 mH, and the current through it changes from 0.200 A to 1.50 A in a time of 0.200 s. Find the magnitude of the average induced emf in the coil during this time.
2. A coiled telephone cord forms a spiral with 70 turns, a diameter of 1.30 cm, and an unstretched length of 60.0 cm. Determine the self-inductance of one conductor in the unstretched cord.
3. A 2.00-H inductor carries a steady current of 0.500 A. When the switch in the circuit is thrown open, the current is effectively zero in 10.0 ms. What is the average induced emf in the inductor during this time?
4. A small air-core solenoid has a length of 4.00 cm and a radius of 0.250 cm. If the inductance is to be 0.060 0 mH, how many turns per centimeter are required?
5. Calculate the magnetic flux through the area enclosed by a 300-turn, 7.20-mH coil when the current in the coil is 10.0 mA.
6. The current in a solenoid is increasing at a rate of 10.0 A/s. The cross-sectional area of the solenoid is π cm^2, and there are 300 turns on its 15.0-cm length. What is the induced emf opposing the increasing current?

WEB 7. A 10.0-mH inductor carries a current $I = I_{max} \sin \omega t$, with $I_{max} = 5.00$ A and $\omega/2\pi = 60.0$ Hz. What is the back emf as a function of time?
8. An emf of 24.0 mV is induced in a 500-turn coil at an instant when the current is 4.00 A and is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil?
9. An inductor in the form of a solenoid contains 420 turns, is 16.0 cm in length, and has a cross-sectional area of 3.00 cm^2. What uniform rate of decrease of current through the inductor induces an emf of 175 μV?
10. An inductor in the form of a solenoid contains N turns, has length ℓ, and has cross-sectional area A. What uniform rate of decrease of current through the inductor induces an emf $\mathcal{E}$?
11. The current in a 90.0-mH inductor changes with time as $I = t^2 - 6.00t$ (in SI units). Find the magnitude of the induced emf at (a) $t = 1.00$ s and (b) $t = 4.00$ s. (c) At what time is the emf zero?
12. A 40.0-mA current is carried by a uniformly wound air-core solenoid with 450 turns, a 15.0-mm diameter, and 12.0-cm length. Compute (a) the magnetic field inside the solenoid, (b) the magnetic flux through each turn,

and (c) the inductance of the solenoid. (d) Which of these quantities depends on the current?

13. A solenoid has 120 turns uniformly wrapped around a wooden core, which has a diameter of 10.0 mm and a length of 9.00 cm. (a) Calculate the inductance of the solenoid. (b) The wooden core is replaced with a soft iron rod that has the same dimensions but a magnetic permeability $\mu_m = 800\mu_0$. What is the new inductance?

14. A toroid has a major radius R and a minor radius r, and it is tightly wound with N turns of wire, as shown in Figure P32.14. If $R \gg r$, the magnetic field within the region of the torus, of cross-sectional area $A = \pi r^2$, is essentially that of a long solenoid that has been bent into a large circle of radius R. Using the uniform field of a long solenoid, show that the self-inductance of such a toroid is approximately

$$L \cong \mu_0 N^2 A/2\pi R$$

(An exact expression for the inductance of a toroid with a rectangular cross-section is derived in Problem 64.)

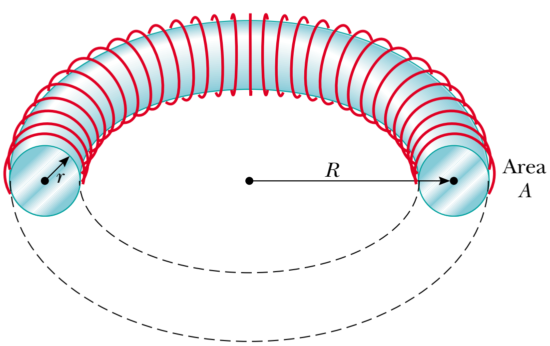


Figure P32.14

15. An emf self-induced in a solenoid of inductance L changes in time as $\mathcal{E} = \mathcal{E}_0 e^{-kt}$. Find the total charge that passes through the solenoid, if the charge is finite.

Section 32.2 *RL* Circuits

16. Calculate the resistance in an *RL* circuit in which $L = 2.50$ H and the current increases to 90.0% of its final value in 3.00 s.

17. A 12.0-V battery is connected into a series circuit containing a 10.0-Ω resistor and a 2.00-H inductor. How long will it take the current to reach (a) 50.0% and (b) 90.0% of its final value?

18. Show that $I = I_0 e^{-t/\tau}$ is a solution of the differential equation

$$IR + L\frac{dI}{dt} = 0$$

where $\tau = L/R$ and I_0 is the current at $t = 0$.

19. Consider the circuit in Figure P32.19, taking $\mathcal{E} = 6.00$ V, $L = 8.00$ mH, and $R = 4.00\ \Omega$. (a) What is the inductive time constant of the circuit? (b) Calculate the current in the circuit 250 μs after the switch is closed. (c) What is the value of the final steady-state current? (d) How long does it take the current to reach 80.0% of its maximum value?

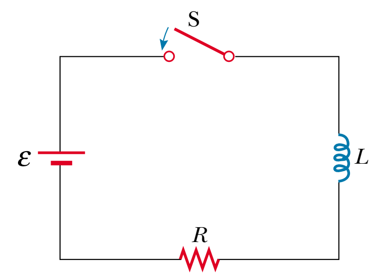


Figure P32.19 Problems 19, 20, 21, and 24.

20. In the circuit shown in Figure P32.19, let $L = 7.00$ H, $R = 9.00\ \Omega$, and $\mathcal{E} = 120$ V. What is the self-induced emf 0.200 s after the switch is closed?

WEB **21.** For the *RL* circuit shown in Figure P32.19, let $L = 3.00$ H, $R = 8.00\ \Omega$, and $\mathcal{E} = 36.0$ V. (a) Calculate the ratio of the potential difference across the resistor to that across the inductor when $I = 2.00$ A. (b) Calculate the voltage across the inductor when $I = 4.50$ A.

22. A 12.0-V battery is connected in series with a resistor and an inductor. The circuit has a time constant of 500 μs, and the maximum current is 200 mA. What is the value of the inductance?

23. An inductor that has an inductance of 15.0 H and a resistance of 30.0 Ω is connected across a 100-V battery. What is the rate of increase of the current (a) at $t = 0$ and (b) at $t = 1.50$ s?

24. When the switch in Figure P32.19 is thrown closed, the current takes 3.00 ms to reach 98.0% of its final value. If $R = 10.0\ \Omega$, what is the inductance?

25. The switch in Figure P32.25 is closed at time $t = 0$. Find the current in the inductor and the current through the switch as functions of time thereafter.

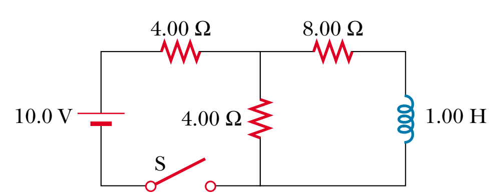


Figure P32.25

26. A series *RL* circuit with $L = 3.00$ H and a series *RC* circuit with $C = 3.00$ μF have equal time constants. If the two circuits contain the same resistance R, (a) what is the value of R and (b) what is the time constant?

27. A current pulse is fed to the partial circuit shown in Figure P32.27. The current begins at zero, then becomes 10.0 A between $t = 0$ and $t = 200\ \mu s$, and then is zero once again. Determine the current in the inductor as a function of time.

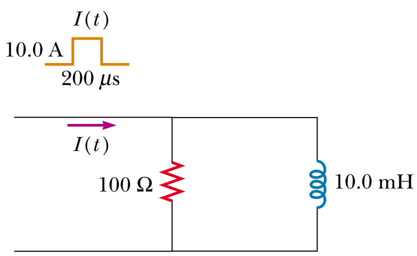


Figure P32.27

28. One application of an *RL* circuit is the generation of time-varying high voltage from a low-voltage source, as shown in Figure P32.28. (a) What is the current in the circuit a long time after the switch has been in position A? (b) Now the switch is thrown quickly from A to B. Compute the initial voltage across each resistor and the inductor. (c) How much time elapses before the voltage across the inductor drops to 12.0 V?

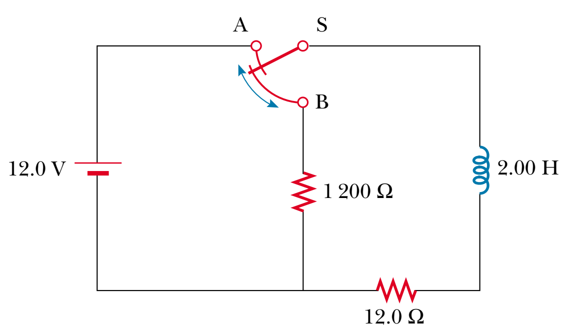


Figure P32.28

WEB **29.** A 140-mH inductor and a 4.90-Ω resistor are connected with a switch to a 6.00-V battery, as shown in Figure P32.29. (a) If the switch is thrown to the left (connecting the battery), how much time elapses before the current reaches 220 mA? (b) What is the current in the inductor 10.0 s after the switch is closed? (c) Now the switch is quickly thrown from A to B. How much time elapses before the current falls to 160 mA?

30. Consider two ideal inductors, L_1 and L_2, that have *zero* internal resistance and are far apart, so that their magnetic fields do not influence each other. (a) If these inductors are connected in series, show that they are equivalent to a single ideal inductor having $L_{eq} = L_1 + L_2$. (b) If these same two inductors are connected in parallel, show that they are equivalent to a single ideal inductor having $1/L_{eq} = 1/L_1 + 1/L_2$. (c) Now consider two inductors L_1 and L_2 that have *nonzero* internal resistances R_1 and R_2, respectively. Assume that they are still far apart so that their magnetic fields do not influence each other. If these inductors are connected in series, show that they are equivalent to a single inductor having $L_{eq} = L_1 + L_2$ and $R_{eq} = R_1 + R_2$. (d) If these same inductors are now connected in parallel, is it necessarily true that they are equivalent to a single ideal inductor having $1/L_{eq} = 1/L_1 + 1/L_2$ and $1/R_{eq} = 1/R_1 + 1/R_2$? Explain your answer.

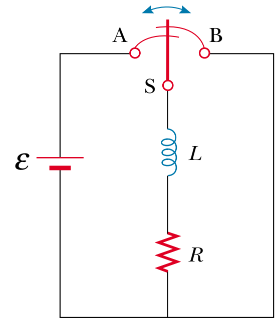


Figure P32.29

Section 32.3 Energy in a Magnetic Field

31. Calculate the energy associated with the magnetic field of a 200-turn solenoid in which a current of 1.75 A produces a flux of $3.70 \times 10^{-4}\ \text{T} \cdot \text{m}^2$ in each turn.

32. The magnetic field inside a superconducting solenoid is 4.50 T. The solenoid has an inner diameter of 6.20 cm and a length of 26.0 cm. Determine (a) the magnetic energy density in the field and (b) the energy stored in the magnetic field within the solenoid.

33. An air-core solenoid with 68 turns is 8.00 cm long and has a diameter of 1.20 cm. How much energy is stored in its magnetic field when it carries a current of 0.770 A?

34. At $t = 0$, an emf of 500 V is applied to a coil that has an inductance of 0.800 H and a resistance of 30.0 Ω. (a) Find the energy stored in the magnetic field when the current reaches half its maximum value. (b) After the emf is connected, how long does it take the current to reach this value?

WEB **35.** On a clear day there is a 100-V/m vertical electric field near the Earth's surface. At the same place, the Earth's magnetic field has a magnitude of 0.500×10^{-4} T. Compute the energy densities of the two fields.

36. An *RL* circuit in which $L = 4.00$ H and $R = 5.00\ \Omega$ is connected to a 22.0-V battery at $t = 0$. (a) What energy is stored in the inductor when the current is 0.500 A? (b) At what rate is energy being stored in the inductor when $I = 1.00$ A? (c) What power is being delivered to the circuit by the battery when $I = 0.500$ A?

37. A 10.0-V battery, a 5.00-Ω resistor, and a 10.0-H inductor are connected in series. After the current in the circuit has reached its maximum value, calculate (a) the power being supplied by the battery, (b) the power being delivered to the resistor, (c) the power being delivered to the inductor, and (d) the energy stored in the magnetic field of the inductor.

38. A uniform electric field with a magnitude of 680 kV/m throughout a cylindrical volume results in a total energy of 3.40 μJ. What magnetic field over this same region stores the same total energy?

39. Assume that the magnitude of the magnetic field outside a sphere of radius R is $B = B_0(R/r)^2$, where B_0 is a constant. Determine the total energy stored in the magnetic field outside the sphere and evaluate your result for $B_0 = 5.00 \times 10^{-5}$ T and $R = 6.00 \times 10^6$ m, values appropriate for the Earth's magnetic field.

Section 32.4 Mutual Inductance

40. Two coils are close to each other. The first coil carries a time-varying current given by
$I(t) = (5.00 \text{ A})\, e^{-0.025\,0t} \sin(377t)$. At $t = 0.800$ s, the voltage measured across the second coil is -3.20 V. What is the mutual inductance of the coils?

41. Two coils, held in fixed positions, have a mutual inductance of 100 μH. What is the peak voltage in one when a sinusoidal current given by
$I(t) = (10.0 \text{ A}) \sin(1\,000t)$ flows in the other?

42. An emf of 96.0 mV is induced in the windings of a coil when the current in a nearby coil is increasing at the rate of 1.20 A/s. What is the mutual inductance of the two coils?

43. Two solenoids A and B, spaced close to each other and sharing the same cylindrical axis, have 400 and 700 turns, respectively. A current of 3.50 A in coil A produces an average flux of 300 $\mu\text{T}\cdot\text{m}^2$ through each turn of A and a flux of 90.0 $\mu\text{T}\cdot\text{m}^2$ through each turn of B. (a) Calculate the mutual inductance of the two solenoids. (b) What is the self-inductance of A? (c) What emf is induced in B when the current in A increases at the rate of 0.500 A/s?

44. A 70-turn solenoid is 5.00 cm long and 1.00 cm in diameter and carries a 2.00-A current. A single loop of wire, 3.00 cm in diameter, is held so that the plane of the loop is perpendicular to the long axis of the solenoid, as illustrated in Figure P31.18 (page 1004). What is the mutual inductance of the two if the plane of the loop passes through the solenoid 2.50 cm from one end?

45. Two single-turn circular loops of wire have radii R and r, with $R \gg r$. The loops lie in the same plane and are concentric. (a) Show that the mutual inductance of the pair is $M = \mu_0 \pi r^2/2R$. (*Hint:* Assume that the larger loop carries a current I and compute the resulting flux through the smaller loop.) (b) Evaluate M for $r = 2.00$ cm and $R = 20.0$ cm.

46. On a printed circuit board, a relatively long straight conductor and a conducting rectangular loop lie in the same plane, as shown in Figure P31.9 (page 1003). If $h = 0.400$ mm, $w = 1.30$ mm, and $L = 2.70$ mm, what is their mutual inductance?

47. Two inductors having self-inductances L_1 and L_2 are connected in parallel, as shown in Figure P32.47a. The mutual inductance between the two inductors is M. Determine the equivalent self-inductance L_{eq} for the system (Fig. P32.47b).

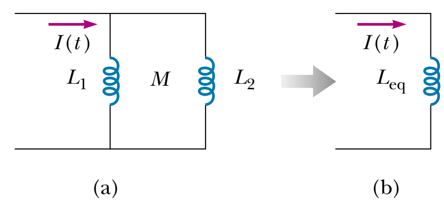


Figure P32.47

Section 32.5 Oscillations in an *LC* Circuit

48. A 1.00-μF capacitor is charged by a 40.0-V power supply. The fully-charged capacitor is then discharged through a 10.0-mH inductor. Find the maximum current in the resulting oscillations.

49. An LC circuit consists of a 20.0-mH inductor and a 0.500-μF capacitor. If the maximum instantaneous current is 0.100 A, what is the greatest potential difference across the capacitor?

50. In the circuit shown in Figure P32.50, $\mathcal{E} = 50.0$ V, $R = 250\ \Omega$, and $C = 0.500\ \mu$F. The switch S is closed for a long time, and no voltage is measured across the capacitor. After the switch is opened, the voltage across the capacitor reaches a maximum value of 150 V. What is the inductance L?

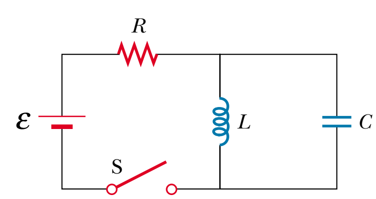


Figure P32.50

51. A fixed inductance $L = 1.05\ \mu$H is used in series with a variable capacitor in the tuning section of a radio. What capacitance tunes the circuit to the signal from a station broadcasting at 6.30 MHz?

52. Calculate the inductance of an LC circuit that oscillates at 120 Hz when the capacitance is 8.00 μF.

53. An LC circuit like the one shown in Figure 32.14 contains an 82.0-mH inductor and a 17.0-μF capacitor that initially carries a 180-μC charge. The switch is thrown closed at $t = 0$. (a) Find the frequency (in hertz) of the resulting oscillations. At $t = 1.00$ ms, find (b) the charge on the capacitor and (c) the current in the circuit.

54. The switch in Figure P32.54 is connected to point a for a long time. After the switch is thrown to point b, what are (a) the frequency of oscillation of the LC circuit, (b) the maximum charge that appears on the capacitor, (c) the maximum current in the inductor, and (d) the total energy the circuit possesses at $t = 3.00$ s?

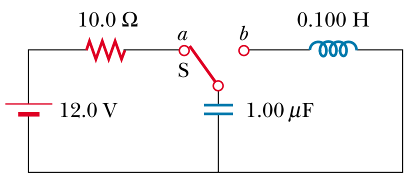


Figure P32.54

WEB **55.** An LC circuit like that illustrated in Figure 32.14 consists of a 3.30-H inductor and an 840-pF capacitor, initially carrying a 105-μC charge. At $t = 0$ the switch is thrown closed. Compute the following quantities at $t = 2.00$ ms: (a) the energy stored in the capacitor; (b) the energy stored in the inductor; (c) the total energy in the circuit.

(Optional)
Section 32.6 The *RLC* Circuit

56. In Figure 32.19, let $R = 7.60\ \Omega$, $L = 2.20$ mH, and $C = 1.80\ \mu$F. (a) Calculate the frequency of the damped oscillation of the circuit. (b) What is the critical resistance?

57. Consider an LC circuit in which $L = 500$ mH and $C = 0.100\ \mu$F. (a) What is the resonant frequency ω_0? (b) If a resistance of 1.00 kΩ is introduced into this circuit, what is the frequency of the (damped) oscillations? (c) What is the percent difference between the two frequencies?

58. Show that Equation 32.29 in the text is Kirchhoff's loop rule as applied to Figure 32.19.

59. Electrical oscillations are initiated in a series circuit containing a capacitance C, inductance L, and resistance R. (a) If $R \ll \sqrt{4L/C}$ (weak damping), how much time elapses before the amplitude of the current oscillation falls off to 50.0% of its initial value? (b) How long does it take the energy to decrease to 50.0% of its initial value?

ADDITIONAL PROBLEMS

60. Initially, the capacitor in a series LC circuit is charged. A switch is closed, allowing the capacitor to discharge, and after time t the energy stored in the capacitor is one-fourth its initial value. Determine L if C is known.

61. A 1.00-mH inductor and a 1.00-μF capacitor are connected in series. The current in the circuit is described by $I = 20.0t$, where t is in seconds and I is in amperes. The capacitor initially has no charge. Determine (a) the voltage across the inductor as a function of time, (b) the voltage across the capacitor as a function of time, and (c) the time when the energy stored in the capacitor first exceeds that in the inductor.

62. An inductor having inductance L and a capacitor having capacitance C are connected in series. The current in the circuit increases linearly in time as described by $I = Kt$. The capacitor is initially uncharged. Determine (a) the voltage across the inductor as a function of time, (b) the voltage across the capacitor as a function of time, and (c) the time when the energy stored in the capacitor first exceeds that in the inductor.

63. A capacitor in a series LC circuit has an initial charge Q and is being discharged. Find, in terms of L and C, the flux through each of the N turns in the coil, when the charge on the capacitor is $Q/2$.

64. The toroid in Figure P32.64 consists of N turns and has a rectangular cross-section. Its inner and outer radii are a and b, respectively. (a) Show that

$$L = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{b}{a}$$

(b) Using this result, compute the self-inductance of a 500-turn toroid for which $a = 10.0$ cm, $b = 12.0$ cm, and $h = 1.00$ cm. (c) In Problem 14, an approximate formula for the inductance of a toroid with $R \gg r$ was derived. To get a feel for the accuracy of that result, use the expression in Problem 14 to compute the approximate inductance of the toroid described in part (b). Compare the result with the answer to part (b).

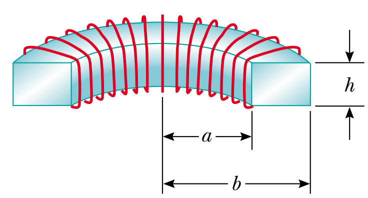


Figure P32.64

65. (a) A flat circular coil does not really produce a uniform magnetic field in the area it encloses, but estimate the self-inductance of a flat circular coil, with radius R and N turns, by supposing that the field at its center is uniform over its area. (b) A circuit on a laboratory table consists of a 1.5-V battery, a 270-Ω resistor, a switch, and three 30-cm-long cords connecting them. Suppose that the circuit is arranged to be circular. Think of it as a flat coil with one turn. Compute the order of magnitude of its self-inductance and (c) of the time constant describing how fast the current increases when you close the switch.

66. A soft iron rod ($\mu_m = 800\ \mu_0$) is used as the core of a solenoid. The rod has a diameter of 24.0 mm and is

10.0 cm long. A 10.0-m piece of 22-gauge copper wire (diameter = 0.644 mm) is wrapped around the rod in a single uniform layer, except for a 10.0-cm length at each end, which is to be used for connections. (a) How many turns of this wire can wrap around the rod? (*Hint:* The diameter of the wire adds to the diameter of the rod in determining the circumference of each turn. Also, the wire spirals diagonally along the surface of the rod.) (b) What is the resistance of this inductor? (c) What is its inductance?

67. A wire of nonmagnetic material with radius R carries current uniformly distributed over its cross-section. If the total current carried by the wire is I, show that the magnetic energy per unit length inside the wire is $\mu_0 I^2/16\pi$.

68. An 820-turn wire coil of resistance 24.0 Ω is placed around a 12 500-turn solenoid, 7.00 cm long, as shown in Figure P32.68. Both coil and solenoid have cross-sectional areas of 1.00×10^{-4} m^2. (a) How long does it take the solenoid current to reach 63.2 percent of its maximum value? Determine (b) the average back emf caused by the self-inductance of the solenoid during this interval, (c) the average rate of change in magnetic flux through the coil during this interval, and (d) the magnitude of the average induced current in the coil.

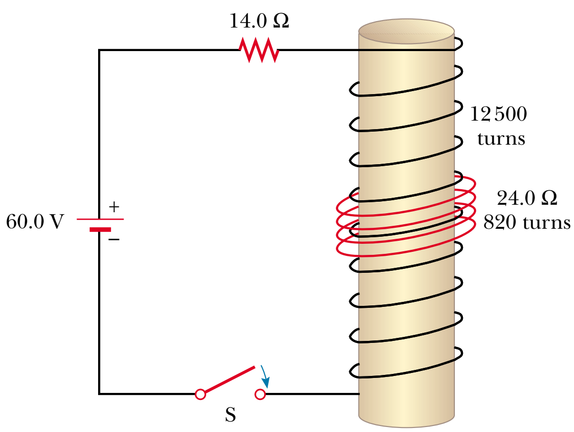


Figure P32.68

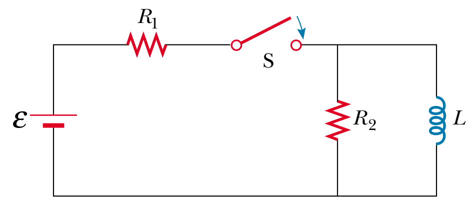


Figure P32.69 Problems 69 and 70.

69. At $t = 0$, the switch in Figure P32.69 is thrown closed. Using Kirchhoff's laws for the instantaneous currents and voltages in this two-loop circuit, show that the current in the inductor is

$$I(t) = \frac{\mathcal{E}}{R_1}\left[1 - e^{-(R'/L)t}\right]$$

where $R' = R_1R_2/(R_1 + R_2)$.

70. In Figure P32.69, take $\mathcal{E} = 6.00$ V, $R_1 = 5.00$ Ω, and $R_2 = 1.00$ Ω. The inductor has negligible resistance. When the switch is thrown open after having been closed for a long time, the current in the inductor drops to 0.250 A in 0.150 s. What is the inductance of the inductor?

71. In Figure P32.71, the switch is closed for $t < 0$, and steady-state conditions are established. The switch is thrown open at $t = 0$. (a) Find the initial voltage $\mathcal{E}_0$ across L just after $t = 0$. Which end of the coil is at the higher potential: a or b? (b) Make freehand graphs of the currents in R_1 and in R_2 as a function of time, treating the steady-state directions as positive. Show values before and after $t = 0$. (c) How long after $t = 0$ does the current in R_2 have the value 2.00 mA?

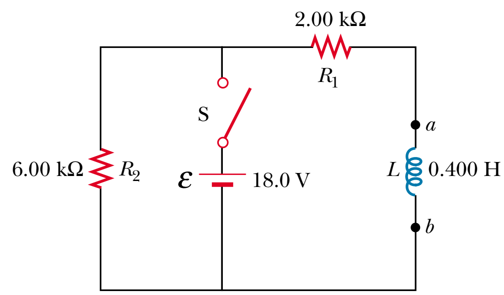


Figure P32.71

72. The switch in Figure P32.72 is thrown closed at $t = 0$. Before the switch is closed, the capacitor is uncharged, and all currents are zero. Determine the currents in L, C, and R and the potential differences across L, C, and R (a) the instant after the switch is closed and (b) long after it is closed.

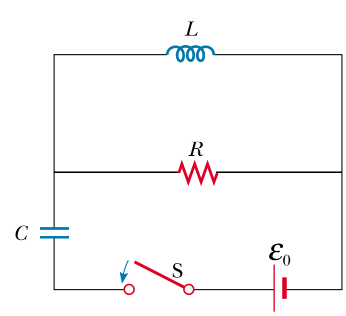


Figure P32.72

73. To prevent damage from arcing in an electric motor, a discharge resistor is sometimes placed in parallel with the armature. If the motor is suddenly unplugged while running, this resistor limits the voltage that appears across the armature coils. Consider a 12.0-V dc motor with an armature that has a resistance of 7.50 Ω and an inductance of 450 mH. Assume that the back emf in the armature coils is 10.0 V when the motor is running at normal speed. (The equivalent circuit for the armature is shown in Fig. P32.73.) Calculate the maximum resistance R that limits the voltage across the armature to 80.0 V when the motor is unplugged.

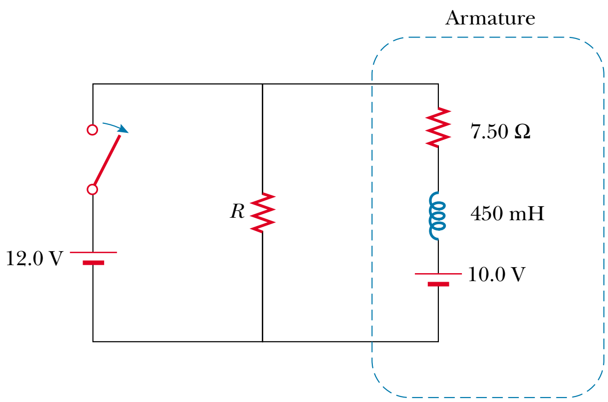


Figure P32.73

74. An air-core solenoid 0.500 m in length contains 1 000 turns and has a cross-sectional area of 1.00 cm². (a) If end effects are neglected, what is the self-inductance? (b) A secondary winding wrapped around the center of the solenoid has 100 turns. What is the mutual inductance? (c) The secondary winding carries a constant current of 1.00 A, and the solenoid is connected to a load of 1.00 kΩ. The constant current is suddenly stopped. How much charge flows through the load resistor?

75. The lead-in wires from a television antenna are often constructed in the form of two parallel wires (Fig. P32.75). (a) Why does this configuration of conductors have an inductance? (b) What constitutes the flux loop for this configuration? (c) Neglecting any magnetic flux inside the wires, show that the inductance of a length x of this type of lead-in is

$$L = \frac{\mu_0 x}{\pi} \ln\left(\frac{w - a}{a}\right)$$

where a is the radius of the wires and w is their center-to-center separation.

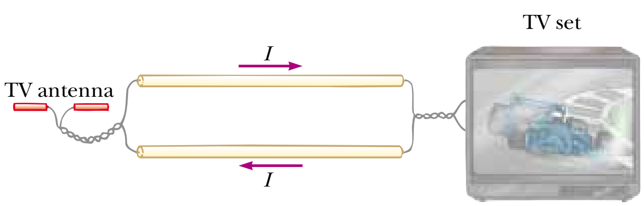


Figure P32.75

Note: Problems 76 through 79 require the application of ideas from this chapter and earlier chapters to some properties of superconductors, which were introduced in Section 27.5.

76. **Review Problem.** *The resistance of a superconductor.* In an experiment carried out by S. C. Collins between 1955 and 1958, a current was maintained in a superconducting lead ring for 2.50 yr with no observed loss. If the inductance of the ring was 3.14×10^{-8} H and the sensitivity of the experiment was 1 part in 10^9, what was the maximum resistance of the ring? (*Hint:* Treat this as a decaying current in an *RL* circuit, and recall that $e^{-x} \cong 1 - x$ for small x.)

77. **Review Problem.** A novel method of storing electrical energy has been proposed. A huge underground superconducting coil, 1.00 km in diameter, would be fabricated. It would carry a maximum current of 50.0 kA through each winding of a 150-turn Nb_3Sn solenoid. (a) If the inductance of this huge coil were 50.0 H, what would be the total energy stored? (b) What would be the compressive force per meter length acting between two adjacent windings 0.250 m apart?

78. **Review Problem.** *Superconducting Power Transmission.* The use of superconductors has been proposed for the manufacture of power transmission lines. A single coaxial cable (Fig. P32.78) could carry 1.00×10^3 MW (the output of a large power plant) at 200 kV, dc, over a distance of 1 000 km without loss. An inner wire with a radius of 2.00 cm, made from the superconductor Nb_3Sn, carries the current I in one direction. A surrounding superconducting cylinder, of radius 5.00 cm, would carry the return current I. In such a system, what is the magnetic field (a) at the surface of the inner conductor and (b) at the inner surface of the outer conductor? (c) How much energy would be stored in the space between the conductors in a 1 000-km superconducting line? (d) What is the pressure exerted on the outer conductor?

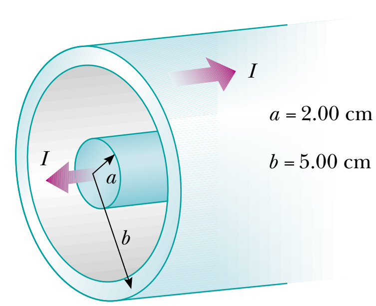


Figure P32.78

79. Review Problem. *The Meissner Effect.* Compare this problem with Problem 63 in Chapter 26 on the force attracting a perfect dielectric into a strong electric field. A fundamental property of a Type I superconducting material is *perfect diamagnetism,* or demonstration of the *Meissner effect,* illustrated in the photograph on page 855 and again in Figure 30.34, and described as follows: The superconducting material has $\mathbf{B} = 0$ everywhere inside it. If a sample of the material is placed into an externally produced magnetic field, or if it is cooled to become superconducting while it is in a magnetic field, electric currents appear on the surface of the sample. The currents have precisely the strength and orientation required to make the total magnetic field zero throughout the interior of the sample. The following problem will help you to understand the magnetic force that can then act on the superconducting sample.

Consider a vertical solenoid with a length of 120 cm and a diameter of 2.50 cm consisting of 1 400 turns of copper wire carrying a counterclockwise current of 2.00 A, as shown in Figure P32.79a. (a) Find the magnetic field in the vacuum inside the solenoid. (b) Find the energy density of the magnetic field, and note that the units J/m^3 of energy density are the same as the units $\text{N/m}^2 (=\text{Pa})$ of pressure. (c) A superconducting bar 2.20 cm in diameter is inserted partway into the solenoid. Its upper end is far outside the solenoid, where the magnetic field is small. The lower end of the bar is deep inside the solenoid. Identify the direction required for the current on the curved surface of the bar so that the total magnetic field is zero within the bar. The field created by the supercurrents is sketched in Figure P32.79b, and the total field is sketched in Figure P32.79c. (d) The field of the solenoid exerts a force on the current in the superconductor. Identify the direction of the force on the bar. (e) Calculate the magnitude of the force by multiplying the energy density of the solenoid field by the area of the bottom end of the superconducting bar.

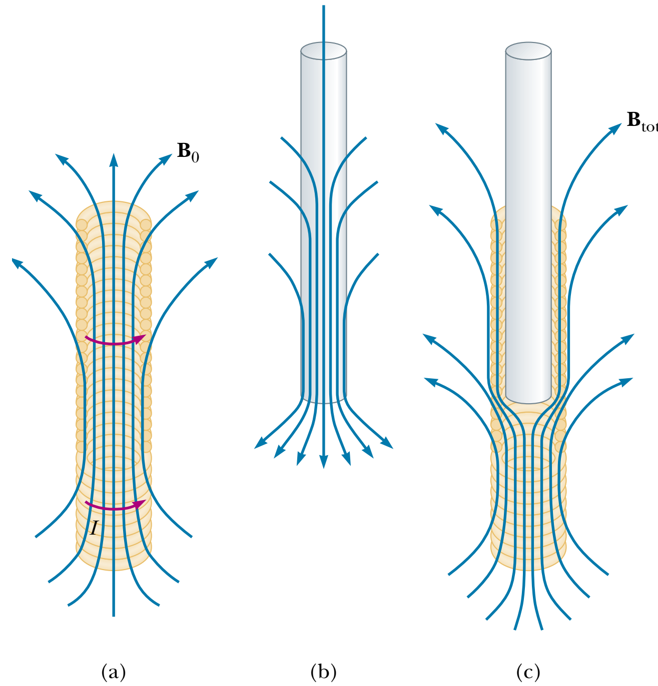


Figure P32.79

Answers to Quick Quizzes

32.1 When it is being opened. When the switch is initially open, there is no current in the circuit; when the switch is then closed, the inductor tends to maintain the no-current condition, and as a result there is very little chance of sparking. When the switch is initially closed, there is current in the circuit; when the switch is then opened, the current decreases. An induced emf is set up across the inductor, and this emf tends to maintain the original current. Sparking can occur as the current bridges the air gap between the poles of the switch.

32.2 (b). Figure 32.8 shows that circuit B has the greater time constant because in this circuit it takes longer for the current to reach its maximum value and then longer for this current to decrease to zero after switch S_2 is closed. Equation 32.8 indicates that, for equal resistances R_A and R_B, the condition $\tau_B > \tau_A$ means that $L_A < L_B$.

32.3 (a) M_{12} increases because the magnetic flux through coil 2 increases. (b) M_{12} decreases because rotation of coil 1 decreases its flux through coil 2.

32.4 (a) No. Mutual inductance requires a system of coils, and each coil has self-inductance. (b) Yes. A single coil has self-inductance but no mutual inductance because it does not interact with any other coils.

32.5 From Equation 32.25, $I_{\text{max}} = \omega Q_{\text{max}}$. Thus, the amplitude of the I-t graph is ω times the amplitude of the Q-t graph.

32.6 Equation 32.31 without the cosine factor. The dashed lines represent the positive and negative amplitudes (maximum values) for each oscillation period, and it is the $Q = Q_{\text{max}}e^{-Rt/2L}$ part of Equation 32.31 that gives the value of the ever-decreasing amplitude.