



## PUZZLER

Small “black boxes” like this one are commonly used to supply power to electronic devices such as CD players and tape players. Whereas these devices need only about 12 V to operate, wall outlets provide an output of 120 V. What do the black boxes do, and how do they work? (*George Semple*)

## chapter

# 33

## Alternating-Current Circuits

### Chapter Outline

- 33.1** ac Sources and Phasors
- 33.2** Resistors in an ac Circuit
- 33.3** Inductors in an ac Circuit
- 33.4** Capacitors in an ac Circuit
- 33.5** The *RLC* Series Circuit
- 33.6** Power in an ac Circuit
- 33.7** Resonance in a Series *RLC* Circuit
- 33.8** The Transformer and Power Transmission
- 33.9** (*Optional*) Rectifiers and Filters

In this chapter we describe alternating-current (ac) circuits. Every time we turn on a television set, a stereo, or any of a multitude of other electrical appliances, we are calling on alternating currents to provide the power to operate them. We begin our study by investigating the characteristics of simple series circuits that contain resistors, inductors, and capacitors and that are driven by a sinusoidal voltage. We shall find that the maximum alternating current in each element is proportional to the maximum alternating voltage across the element. We shall also find that when the applied voltage is sinusoidal, the current in each element is sinusoidal, too, but not necessarily in phase with the applied voltage. We conclude the chapter with two sections concerning transformers, power transmission, and  $RC$  filters.

### 33.1 AC SOURCES AND PHASORS

An ac circuit consists of circuit elements and a generator that provides the alternating current. As you recall from Section 31.5, the basic principle of the ac generator is a direct consequence of Faraday's law of induction. When a conducting loop is rotated in a magnetic field at constant angular frequency  $\omega$ , a sinusoidal voltage (emf) is induced in the loop. This instantaneous voltage  $\Delta v$  is

$$\Delta v = \Delta V_{\max} \sin \omega t$$


where  $\Delta V_{\max}$  is the maximum output voltage of the ac generator, or the **voltage amplitude**. From Equation 13.6, the angular frequency is

$$\omega = 2\pi f = \frac{2\pi}{T}$$

where  $f$  is the frequency of the generator (the voltage source) and  $T$  is the period. The generator determines the frequency of the current in any circuit connected to the generator. Because the output voltage of an ac generator varies sinusoidally with time, the voltage is positive during one half of the cycle and negative during the other half. Likewise, the current in any circuit driven by an ac generator is an alternating current that also varies sinusoidally with time. Commercial electric-power plants in the United States use a frequency of 60 Hz, which corresponds to an angular frequency of 377 rad/s.

The primary aim of this chapter can be summarized as follows: If an ac generator is connected to a series circuit containing resistors, inductors, and capacitors, we want to know the amplitude and time characteristics of the alternating current. To simplify our analysis of circuits containing two or more elements, we use graphical constructions called *phasor diagrams*. In these constructions, alternating (sinusoidal) quantities, such as current and voltage, are represented by rotating vectors called **phasors**. The length of the phasor represents the amplitude (maximum value) of the quantity, and the projection of the phasor onto the vertical axis represents the instantaneous value of the quantity. As we shall see, a phasor diagram greatly simplifies matters when we must combine several sinusoidally varying currents or voltages that have different phases.

### 33.2 RESISTORS IN AN AC CIRCUIT

Consider a simple ac circuit consisting of a resistor and an ac generator , as shown in Figure 33.1. At any instant, the algebraic sum of the voltages around a

closed loop in a circuit must be zero (Kirchhoff's loop rule). Therefore,  $\Delta v - \Delta v_R = 0$ , or<sup>1</sup>

$$\Delta v = \Delta v_R = \Delta V_{\max} \sin \omega t \quad (33.1)$$

where  $\Delta v_R$  is the **instantaneous voltage across the resistor**. Therefore, the instantaneous current in the resistor is

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{\max}}{R} \sin \omega t = I_{\max} \sin \omega t \quad (33.2)$$

where  $I_{\max}$  is the maximum current:

$$I_{\max} = \frac{\Delta V_{\max}}{R}$$

From Equations 33.1 and 33.2, we see that the instantaneous voltage across the resistor is

$$\Delta v_R = I_{\max} R \sin \omega t \quad (33.3)$$

Let us discuss the current-versus-time curve shown in Figure 33.2a. At point  $a$ , the current has a maximum value in one direction, arbitrarily called the positive direction. Between points  $a$  and  $b$ , the current is decreasing in magnitude but is still in the positive direction. At  $b$ , the current is momentarily zero; it then begins to increase in the negative direction between points  $b$  and  $c$ . At  $c$ , the current has reached its maximum value in the negative direction.

The current and voltage are in step with each other because they vary identically with time. Because  $i_R$  and  $\Delta v_R$  both vary as  $\sin \omega t$  and reach their maximum values at the same time, as shown in Figure 33.2a, they are said to be **in phase**. Thus we can say that, for a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor.

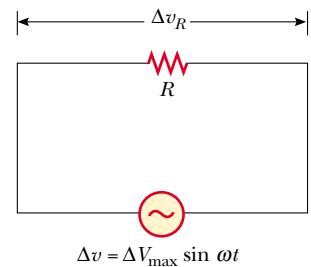
A *phasor diagram* is used to represent current–voltage phase relationships. The lengths of the arrows correspond to  $\Delta V_{\max}$  and  $I_{\max}$ . The projections of the phasor arrows onto the vertical axis give  $\Delta v_R$  and  $i_R$  values. As we showed in Section 13.5, if the phasor arrow is imagined to rotate steadily with angular speed  $\omega$ , its vertical-axis component oscillates sinusoidally in time. In the case of the single-loop resistive circuit of Figure 33.1, the current and voltage phasors lie along the same line, as in Figure 33.2b, because  $i_R$  and  $\Delta v_R$  are in phase.

Note that **the average value of the current over one cycle is zero**. That is, the current is maintained in the positive direction for the same amount of time and at the same magnitude as it is maintained in the negative direction. However, the direction of the current has no effect on the behavior of the resistor. We can understand this by realizing that collisions between electrons and the fixed atoms of the resistor result in an increase in the temperature of the resistor. Although this temperature increase depends on the magnitude of the current, it is independent of the direction of the current.

We can make this discussion quantitative by recalling that the rate at which electrical energy is converted to internal energy in a resistor is the power  $\mathcal{P} = i^2 R$ , where  $i$  is the instantaneous current in the resistor. Because this rate is proportional to the square of the current, it makes no difference whether the current is direct or alternating—that is, whether the sign associated with the current is positive or negative. However, the temperature increase produced by an alternating

Maximum current in a resistor

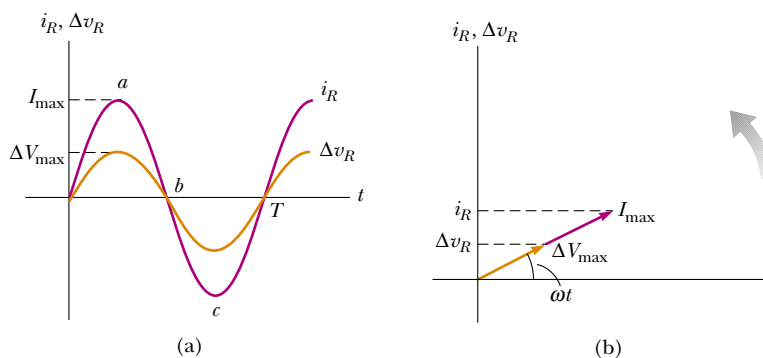
The current in a resistor is in phase with the voltage



**Figure 33.1** A circuit consisting of a resistor of resistance  $R$  connected to an ac generator, designated by the symbol



<sup>1</sup> The lowercase symbols  $v$  and  $i$  are used to indicate the instantaneous values of the voltage and the current.



**Figure 33.2** (a) Plots of the instantaneous current  $i_R$  and instantaneous voltage  $\Delta v_R$  across a resistor as functions of time. The current is in phase with the voltage, which means that the current is zero when the voltage is zero, maximum when the voltage is maximum, and minimum when the voltage is minimum. At time  $t = T$ , one cycle of the time-varying voltage and current has been completed. (b) Phasor diagram for the resistive circuit showing that the current is in phase with the voltage.

current having a maximum value  $I_{\max}$  is not the same as that produced by a direct current equal to  $I_{\max}$ . This is because the alternating current is at this maximum value for only an instant during each cycle (Fig. 33.3a). What is of importance in an ac circuit is an average value of current, referred to as the **rms current**. As we learned in Section 21.1, the notation *rms* stands for *root mean square*, which in this case means the square root of the mean (average) value of the square of the current:  $I_{\text{rms}} = \sqrt{\overline{i^2}}$ . Because  $i^2$  varies as  $\sin^2 \omega t$  and because the average value of  $i^2$  is  $\frac{1}{2} I_{\max}^2$  (see Fig. 33.3b), the rms current is<sup>2</sup>

rms current

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max} \quad (33.4)$$

This equation states that an alternating current whose maximum value is 2.00 A delivers to a resistor the same power as a direct current that has a value of  $(0.707)(2.00 \text{ A}) = 1.41 \text{ A}$ . Thus, we can say that the average power delivered to a resistor that carries an alternating current is

Average power delivered to a resistor

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R$$

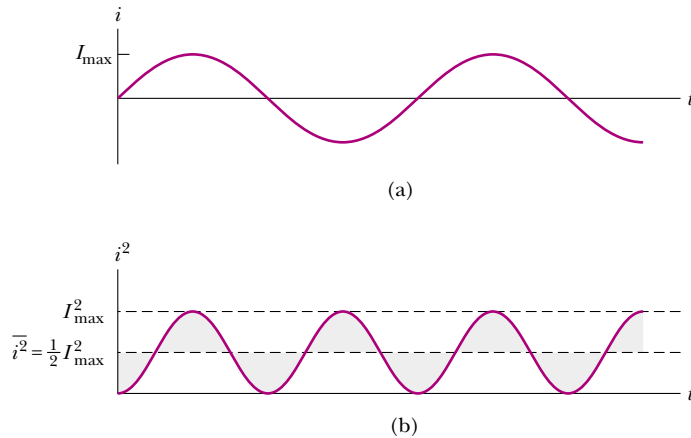
<sup>2</sup> That the square root of the average value of  $i^2$  is equal to  $I_{\max}/\sqrt{2}$  can be shown as follows: The current in the circuit varies with time according to the expression  $i = I_{\max} \sin \omega t$ , so  $i^2 = I_{\max}^2 \sin^2 \omega t$ . Therefore, we can find the average value of  $i^2$  by calculating the average value of  $\sin^2 \omega t$ . A graph of  $\cos^2 \omega t$  versus time is identical to a graph of  $\sin^2 \omega t$  versus time, except that the points are shifted on the time axis. Thus, the time average of  $\sin^2 \omega t$  is equal to the time average of  $\cos^2 \omega t$  when taken over one or more complete cycles. That is,

$$(\sin^2 \omega t)_{\text{av}} = (\cos^2 \omega t)_{\text{av}}$$

Using this fact and the trigonometric identity  $\sin^2 \theta + \cos^2 \theta = 1$ , we obtain

$$\begin{aligned} (\sin^2 \omega t)_{\text{av}} + (\cos^2 \omega t)_{\text{av}} &= 2(\sin^2 \omega t)_{\text{av}} = 1 \\ (\sin^2 \omega t)_{\text{av}} &= \frac{1}{2} \end{aligned}$$

When we substitute this result in the expression  $i^2 = I_{\max}^2 \sin^2 \omega t$ , we obtain  $(i^2)_{\text{av}} = \overline{i^2} = I_{\text{rms}}^2 = I_{\max}^2/2$ , or  $I_{\text{rms}} = I_{\max}/\sqrt{2}$ . The factor  $1/\sqrt{2}$  is valid only for sinusoidally varying currents. Other waveforms, such as sawtooth variations, have different factors.



**Figure 33.3** (a) Graph of the current in a resistor as a function of time. (b) Graph of the current squared in a resistor as a function of time. Notice that the gray shaded regions *under* the curve and *above* the dashed line for  $I_{\max}^2/2$  have the same area as the gray shaded regions *above* the curve and *below* the dashed line for  $I_{\max}^2/2$ . Thus, the average value of  $i^2$  is  $I_{\max}^2/2$ .

Alternating voltage also is best discussed in terms of rms voltage, and the relationship is identical to that for current:

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = 0.707 \Delta V_{\text{max}} \quad (33.5)$$

rms voltage

When we speak of measuring a 120-V alternating voltage from an electrical outlet, we are referring to an rms voltage of 120 V. A quick calculation using Equation 33.5 shows that such an alternating voltage has a maximum value of about 170 V. One reason we use rms values when discussing alternating currents and voltages in this chapter is that ac ammeters and voltmeters are designed to read rms values. Furthermore, with rms values, many of the equations we use have the same form as their direct-current counterparts.

### Quick Quiz 33.1

Which of the following statements might be true for a resistor connected to an ac generator? (a)  $\mathcal{P}_{\text{av}} = 0$  and  $i_{\text{av}} = 0$ ; (b)  $\mathcal{P}_{\text{av}} = 0$  and  $i_{\text{av}} > 0$ ; (c)  $\mathcal{P}_{\text{av}} > 0$  and  $i_{\text{av}} = 0$ ; (d)  $\mathcal{P}_{\text{av}} > 0$  and  $i_{\text{av}} > 0$ .

### EXAMPLE 33.1 What Is the rms Current?

The voltage output of a generator is given by  $\Delta v = (200 \text{ V})\sin \omega t$ . Find the rms current in the circuit when this generator is connected to a  $100\text{-}\Omega$  resistor.

**Solution** Comparing this expression for voltage output with the general form  $\Delta v = \Delta V_{\text{max}} \sin \omega t$ , we see that  $\Delta V_{\text{max}} = 200 \text{ V}$ . Thus, the rms voltage is

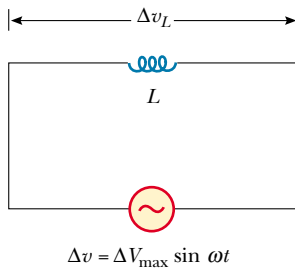
$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = \frac{200 \text{ V}}{\sqrt{2}} = 141 \text{ V}$$

Therefore,

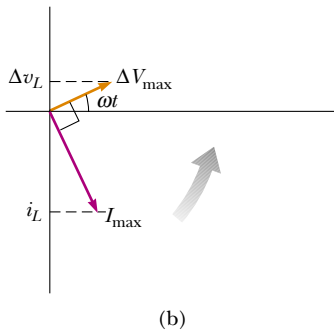
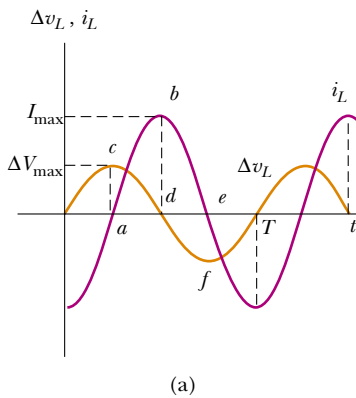
$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = \frac{141 \text{ V}}{100 \text{ }\Omega} = 1.41 \text{ A}$$

**Exercise** Find the maximum current in the circuit.

**Answer** 2.00 A.



**Figure 33.4** A circuit consisting of an inductor of inductance  $L$  connected to an ac generator.



**Figure 33.5** (a) Plots of the instantaneous current  $i_L$  and instantaneous voltage  $\Delta v_L$  across an inductor as functions of time. The current lags behind the voltage by  $90^\circ$ . (b) Phasor diagram for the inductive circuit, showing that the current lags behind the voltage by  $90^\circ$ .

The current in an inductor lags the voltage by  $90^\circ$

### 33.3 INDUCTORS IN AN AC CIRCUIT

Now consider an ac circuit consisting only of an inductor connected to the terminals of an ac generator, as shown in Figure 33.4. If  $\Delta v_L = \mathcal{E}_L = -L(di/dt)$  is the self-induced instantaneous voltage across the inductor (see Eq. 32.1), then Kirchhoff's loop rule applied to this circuit gives  $\Delta v + \Delta v_L = 0$ , or

$$\Delta v - L \frac{di}{dt} = 0$$

When we substitute  $\Delta V_{\max} \sin \omega t$  for  $\Delta v$  and rearrange, we obtain

$$L \frac{di}{dt} = \Delta V_{\max} \sin \omega t \quad (33.6)$$

Solving this equation for  $di$ , we find that

$$di = \frac{\Delta V_{\max}}{L} \sin \omega t dt$$

Integrating this expression<sup>3</sup> gives the instantaneous current in the inductor as a function of time:

$$i_L = \frac{\Delta V_{\max}}{L} \int \sin \omega t dt = -\frac{\Delta V_{\max}}{\omega L} \cos \omega t \quad (33.7)$$

When we use the trigonometric identity  $\cos \omega t = -\sin(\omega t - \pi/2)$ , we can express Equation 33.7 as

$$i_L = \frac{\Delta V_{\max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \quad (33.8)$$

Comparing this result with Equation 33.6, we see that the instantaneous current  $i_L$  in the inductor and the instantaneous voltage  $\Delta v_L$  across the inductor are out of phase by  $(\pi/2)$  rad =  $90^\circ$ .

In general, inductors in an ac circuit produce a current that is out of phase with the ac voltage. A plot of voltage and current versus time is provided in Figure 33.5a. At point  $a$ , the current begins to increase in the positive direction. At this instant the rate of change of current is at a maximum, and thus the voltage across the inductor is also at a maximum. As the current increases between points  $a$  and  $b$ ,  $di/dt$  (the slope of the current curve) gradually decreases until it reaches zero at point  $b$ . As a result, the voltage across the inductor is decreasing during this same time interval, as the curve segment between  $c$  and  $d$  indicates. Immediately after point  $b$ , the current begins to decrease, although it still has the same direction it had during the previous quarter cycle (from  $a$  to  $b$ ). As the current decreases to zero (from  $b$  to  $e$ ), a voltage is again induced in the inductor ( $d$  to  $f$ ), but the polarity of this voltage is opposite that of the voltage induced between  $c$  and  $d$  (because back emfs are always directed to oppose the change in the current). Note that the voltage reaches its maximum value one quarter of a period before the current reaches its maximum value. Thus, we see that

for a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by  $90^\circ$  (one-quarter cycle in time).

<sup>3</sup> We neglect the constant of integration here because it depends on the initial conditions, which are not important for this situation.

The phasor diagram for the inductive circuit of Figure 33.4 is shown in Figure 33.5b.

From Equation 33.7 we see that the current in an inductive circuit reaches its maximum value when  $\cos \omega t = -1$ :

$$I_{\max} = \frac{\Delta V_{\max}}{\omega L} = \frac{\Delta V_{\max}}{X_L} \quad (33.9)$$

Maximum current in an inductor

where the quantity  $X_L$ , called the **inductive reactance**, is

$$X_L = \omega L \quad (33.10)$$

Inductive reactance

Equation 33.9 indicates that, for a given applied voltage, the maximum current decreases as the inductive reactance increases. The expression for the rms current in an inductor is similar to Equation 33.9, with  $I_{\max}$  replaced by  $I_{\text{rms}}$  and  $\Delta V_{\max}$  replaced by  $\Delta V_{\text{rms}}$ .

Inductive reactance, like resistance, has units of ohms. However, unlike resistance, reactance depends on frequency as well as on the characteristics of the inductor. Note that the reactance of an inductor in an ac circuit increases as the frequency of the current increases. This is because at higher frequencies, the instantaneous current must change more rapidly than it does at the lower frequencies; this causes an increase in the maximum induced emf associated with a given maximum current.

Using Equations 33.6 and 33.9, we find that the instantaneous voltage across the inductor is

$$\Delta v_L = -L \frac{di}{dt} = -\Delta V_{\max} \sin \omega t = -I_{\max} X_L \sin \omega t \quad (33.11)$$

### CONCEPTUAL EXAMPLE 33.2

Figure 33.6 shows a circuit consisting of a series combination of an alternating voltage source, a switch, an inductor, and a lightbulb. The switch is thrown closed, and the circuit is allowed to come to equilibrium so that the lightbulb glows steadily. An iron rod is then inserted into the interior of the inductor. What happens to the brightness of the lightbulb, and why?

**Solution** The bulb gets dimmer. As the rod is inserted, the inductance increases because the magnetic field inside the inductor increases. According to Equation 33.10, this increase in  $L$  means that the inductive reactance of the inductor also increases. The voltage across the inductor increases while the voltage across the lightbulb decreases. With less

voltage across it, the lightbulb glows more dimly. In theatrical productions of the early 20th century, this method was used to dim the lights in the theater gradually.

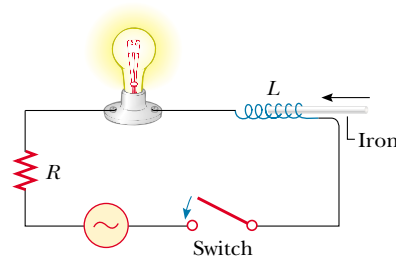


Figure 33.6

### EXAMPLE 33.3 A Purely Inductive ac Circuit

In a purely inductive ac circuit (see Fig. 33.4),  $L = 25.0$  mH and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

**Solution** Equation 33.10 gives

$$X_L = \omega L = 2\pi fL = 2\pi(60.0 \text{ Hz})(25.0 \times 10^{-3} \text{ H}) = 9.42 \Omega$$

From a modified version of Equation 33.9, the rms current is

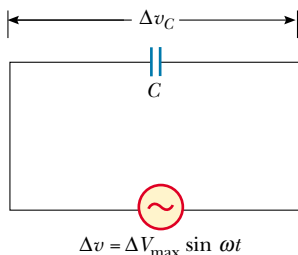
$$I_{\text{rms}} = \frac{\Delta V_{L,\text{rms}}}{X_L} = \frac{150 \text{ V}}{9.42 \Omega} = 15.9 \text{ A}$$



**Exercise** Calculate the inductive reactance and rms current in the circuit if the frequency is 6.00 kHz.

**Answer** 942  $\Omega$ , 0.159 A.

**Exercise** Show that inductive reactance has SI units of ohms.



**Figure 33.7** A circuit consisting of a capacitor of capacitance  $C$  connected to an ac generator.

### 33.4 CAPACITORS IN AN AC CIRCUIT

Figure 33.7 shows an ac circuit consisting of a capacitor connected across the terminals of an ac generator. Kirchhoff's loop rule applied to this circuit gives  $\Delta v - \Delta v_C = 0$ , or

$$\Delta v = \Delta v_C = \Delta V_{\max} \sin \omega t \quad (33.12)$$

where  $\Delta v_C$  is the instantaneous voltage across the capacitor. We know from the definition of capacitance that  $C = q/\Delta v_C$ ; hence, Equation 33.12 gives

$$q = C \Delta V_{\max} \sin \omega t \quad (33.13)$$

where  $q$  is the instantaneous charge on the capacitor. Because  $i = dq/dt$ , differentiating Equation 33.13 gives the instantaneous current in the circuit:

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{\max} \cos \omega t \quad (33.14)$$

Using the trigonometric identity

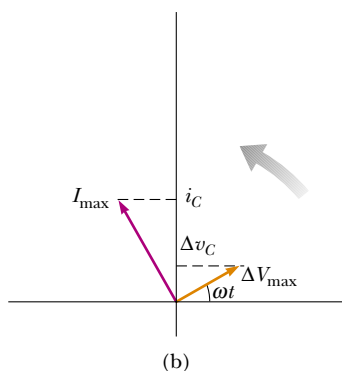
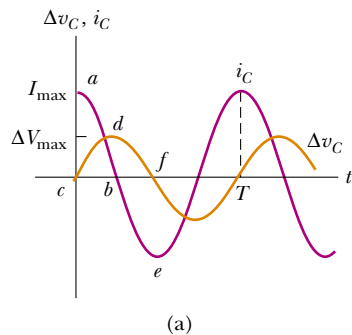
$$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

we can express Equation 33.14 in the alternative form

$$i_C = \omega C \Delta V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right) \quad (33.15)$$

Comparing this expression with Equation 33.12, we see that the current is  $\pi/2$  rad =  $90^\circ$  out of phase with the voltage across the capacitor. A plot of current and voltage versus time (Fig. 33.8a) shows that the current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.

Looking more closely, we see that the segment of the current curve from  $a$  to  $b$  indicates that the current starts out at a relatively high value. We can understand this by recognizing that there is no charge on the capacitor at  $t = 0$ ; as a consequence, nothing in the circuit except the resistance of the wires can hinder the flow of charge at this instant. However, the current decreases as the voltage across the capacitor increases (from  $c$  to  $d$  on the voltage curve), and the capacitor is charging. When the voltage is at point  $d$ , the current reverses and begins to increase in the opposite direction (from  $b$  to  $e$  on the current curve). During this time, the voltage across the capacitor decreases from  $d$  to  $f$  because the plates are now losing the charge they accumulated earlier. During the second half of the cycle, the current is initially at its maximum value in the opposite direction (point  $e$ ) and then decreases as the voltage across the capacitor builds up. The phasor diagram in Figure 33.8b also shows that



**Figure 33.8** (a) Plots of the instantaneous current  $i_C$  and instantaneous voltage  $\Delta v_C$  across a capacitor as functions of time. The voltage lags behind the current by  $90^\circ$ . (b) Phasor diagram for the capacitive circuit, showing that the current leads the voltage by  $90^\circ$ .

for a sinusoidally applied voltage, the current in a capacitor always leads the voltage across the capacitor by  $90^\circ$ .



From Equation 33.14, we see that the current in the circuit reaches its maximum value when  $\cos \omega t = 1$ :

$$I_{\max} = \omega C \Delta V_{\max} = \frac{\Delta V_{\max}}{X_C} \quad (33.16)$$

where  $X_C$  is called the **capacitive reactance**:

$$X_C = \frac{1}{\omega C} \quad (33.17)$$

Capacitive reactance

Note that capacitive reactance also has units of ohms.

The rms current is given by an expression similar to Equation 33.16, with  $I_{\max}$  replaced by  $I_{\text{rms}}$  and  $\Delta V_{\max}$  replaced by  $\Delta V_{\text{rms}}$ .

Combining Equations 33.12 and 33.16, we can express the instantaneous voltage across the capacitor as

$$\Delta v_C = \Delta V_{\max} \sin \omega t = I_{\max} X_C \sin \omega t \quad (33.18)$$

Equations 33.16 and 33.17 indicate that as the frequency of the voltage source increases, the capacitive reactance decreases and therefore the maximum current increases. Again, note that the frequency of the current is determined by the frequency of the voltage source driving the circuit. As the frequency approaches zero, the capacitive reactance approaches infinity, and hence the current approaches zero. This makes sense because the circuit approaches direct-current conditions as  $\omega$  approaches 0.

### EXAMPLE 33.4 A Purely Capacitive ac Circuit

An  $8.00\text{-}\mu\text{F}$  capacitor is connected to the terminals of a  $60.0\text{-Hz}$  ac generator whose rms voltage is  $150\text{ V}$ . Find the capacitive reactance and the rms current in the circuit.

**Solution** Using Equation 33.17 and the fact that  $\omega = 2\pi f = 377\text{ s}^{-1}$  gives

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\text{ s}^{-1})(8.00 \times 10^{-6}\text{ F})} = 332\ \Omega$$

Hence, from a modified Equation 33.16, the rms current is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{150\text{ V}}{332\ \Omega} = 0.452\text{ A}$$

**Exercise** If the frequency is doubled, what happens to the capacitive reactance and the current?

**Answer**  $X_C$  is halved, and  $I_{\text{rms}}$  is doubled.

## 33.5 THE RLC SERIES CIRCUIT

13.7 Figure 33.9a shows a circuit that contains a resistor, an inductor, and a capacitor connected in series across an alternating-voltage source. As before, we assume that the applied voltage varies sinusoidally with time. It is convenient to assume that the instantaneous applied voltage is given by

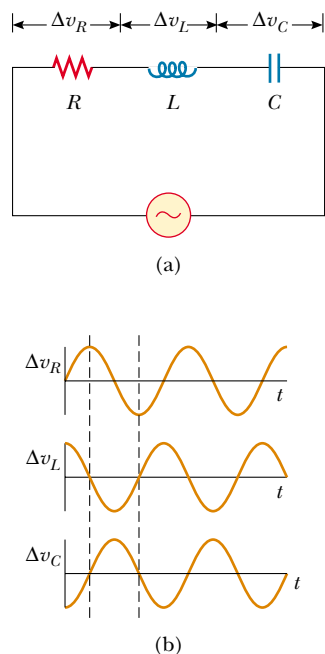
$$\Delta v = \Delta V_{\max} \sin \omega t$$

while the current varies as

$$i = I_{\max} \sin(\omega t - \phi)$$

where  $\phi$  is the **phase angle** between the current and the applied voltage. Our aim

Phase angle  $\phi$



**Figure 33.9** (a) A series circuit consisting of a resistor, an inductor, and a capacitor connected to an ac generator. (b) Phase relationships for instantaneous voltages in the series  $RLC$  circuit.

is to determine  $\phi$  and  $I_{\max}$ . Figure 33.9b shows the voltage versus time across each element in the circuit and their phase relationships.

To solve this problem, we must analyze the phasor diagram for this circuit. First, we note that because the elements are in series, the current everywhere in the circuit must be the same at any instant. That is, **the current at all points in a series ac circuit has the same amplitude and phase**. Therefore, as we found in the preceding sections, the voltage across each element has a different amplitude and phase, as summarized in Figure 33.10. In particular, the voltage across the resistor is in phase with the current, the voltage across the inductor leads the current by  $90^\circ$ , and the voltage across the capacitor lags behind the current by  $90^\circ$ . Using these phase relationships, we can express the instantaneous voltages across the three elements as

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t \quad (33.19)$$

$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t \quad (33.20)$$

$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t \quad (33.21)$$

where  $\Delta V_R$ ,  $\Delta V_L$ , and  $\Delta V_C$  are the maximum voltage values across the elements:

$$\Delta V_R = I_{\max} R \quad \Delta V_L = I_{\max} X_L \quad \Delta V_C = I_{\max} X_C$$

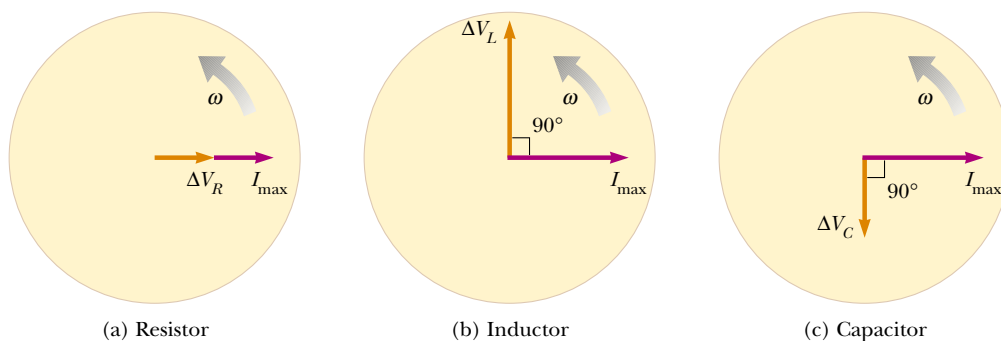
At this point, we could proceed by noting that the instantaneous voltage  $\Delta v$  across the three elements equals the sum

$$\Delta v = \Delta v_R + \Delta v_L + \Delta v_C$$

### Quick Quiz 33.2

For the circuit of Figure 33.9a, is the voltage of the ac source equal to (a) the sum of the maximum voltages across the elements, (b) the sum of the instantaneous voltages across the elements, or (c) the sum of the rms voltages across the elements?

Although this analytical approach is correct, it is simpler to obtain the sum by examining the phasor diagram. Because the current at any instant is the same in all



**Figure 33.10** Phase relationships between the voltage and current phasors for (a) a resistor, (b) an inductor, and (c) a capacitor connected in series.

elements, we can obtain a phasor diagram for the circuit. We combine the three phasor pairs shown in Figure 33.10 to obtain Figure 33.11a, in which a single phasor  $I_{\max}$  is used to represent the current in each element. To obtain the vector sum of the three voltage phasors in Figure 33.11a, we redraw the phasor diagram as in Figure 33.11b. From this diagram, we see that the vector sum of the voltage amplitudes  $\Delta V_R$ ,  $\Delta V_L$ , and  $\Delta V_C$  equals a phasor whose length is the maximum applied voltage  $\Delta V_{\max}$ , where the phasor  $\Delta V_{\max}$  makes an angle  $\phi$  with the current phasor  $I_{\max}$ . Note that the voltage phasors  $\Delta V_L$  and  $\Delta V_C$  are in opposite directions along the same line, and hence we can construct the difference phasor  $\Delta V_L - \Delta V_C$ , which is perpendicular to the phasor  $\Delta V_R$ . From either one of the right triangles in Figure 33.11b, we see that

$$\begin{aligned}\Delta V_{\max} &= \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{\max}R)^2 + (I_{\max}X_L - I_{\max}X_C)^2} \\ \Delta V_{\max} &= I_{\max} \sqrt{R^2 + (X_L - X_C)^2}\end{aligned}\quad (33.22)$$

Therefore, we can express the maximum current as

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The **impedance**  $Z$  of the circuit is defined as

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}\quad (33.23)$$

where impedance also has units of ohms. Therefore, we can write Equation 33.22 in the form

$$\Delta V_{\max} = I_{\max}Z\quad (33.24)$$

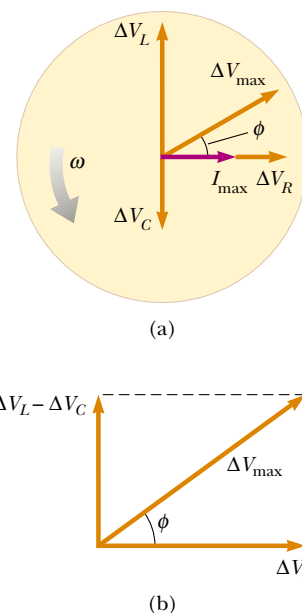
We can regard Equation 33.24 as the ac equivalent of Equation 27.8, which defined *resistance* in a dc circuit as the ratio of the voltage across a conductor to the current in that conductor. Note that the impedance and therefore the current in an ac circuit depend upon the resistance, the inductance, the capacitance, and the frequency (because the reactances are frequency-dependent).

By removing the common factor  $I_{\max}$  from each phasor in Figure 33.11a, we can construct the *impedance triangle* shown in Figure 33.12. From this phasor diagram we find that the phase angle  $\phi$  between the current and the voltage is

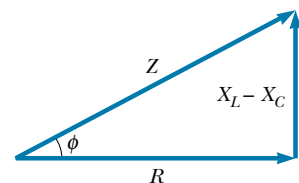
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)\quad (33.25)$$

Also, from Figure 33.12, we see that  $\cos \phi = R/Z$ . When  $X_L > X_C$  (which occurs at high frequencies), the phase angle is positive, signifying that the current lags behind the applied voltage, as in Figure 33.11a. When  $X_L < X_C$ , the phase angle is negative, signifying that the current leads the applied voltage. When  $X_L = X_C$ , the phase angle is zero. In this case, the impedance equals the resistance and the current has its maximum value, given by  $\Delta V_{\max}/R$ . The frequency at which this occurs is called the *resonance frequency*; it is described further in Section 33.7.

Table 33.1 gives impedance values and phase angles for various series circuits containing different combinations of elements.

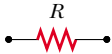

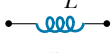
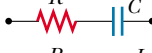

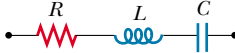


**Figure 33.11** (a) Phasor diagram for the series RLC circuit shown in Figure 33.9a. The phasor  $\Delta V_R$  is in phase with the current phasor  $I_{\max}$ , the phasor  $\Delta V_L$  leads  $I_{\max}$  by  $90^\circ$ , and the phasor  $\Delta V_C$  lags  $I_{\max}$  by  $90^\circ$ . The total voltage  $\Delta V_{\max}$  makes an angle  $\phi$  with  $I_{\max}$ . (b) Simplified version of the phasor diagram shown in (a).



**Figure 33.12** An impedance triangle for a series RLC circuit gives the relationship  $Z = \sqrt{R^2 + (X_L - X_C)^2}$ .

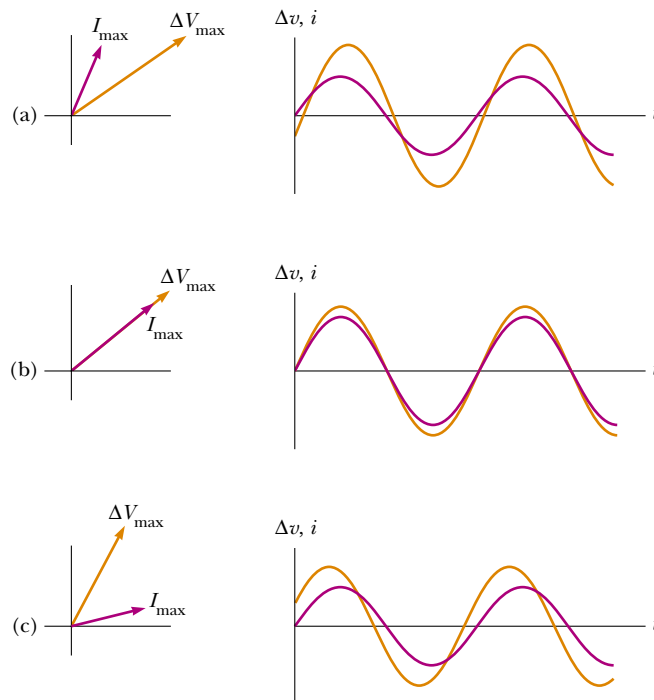
**TABLE 33.1** Impedance Values and Phase Angles for Various Circuit-Element Combinations<sup>a</sup>

Circuit Elements	Impedance $Z$	Phase Angle $\phi$
	$R$	$0^\circ$
	$X_C$	$-90^\circ$
	$X_L$	$+90^\circ$
	$\sqrt{R^2 + X_C^2}$	Negative, between $-90^\circ$ and $0^\circ$
	$\sqrt{R^2 + X_L^2}$	Positive, between $0^\circ$ and $90^\circ$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

<sup>a</sup> In each case, an ac voltage (not shown) is applied across the elements.

**Quick Quiz 33.3**

Label each part of Figure 33.13 as being  $X_L > X_C$ ,  $X_L = X_C$ , or  $X_L < X_C$ .

**Figure 33.13****EXAMPLE 33.5** Finding  $L$  from a Phasor Diagram

In a series  $RLC$  circuit, the applied voltage has a maximum value of 120 V and oscillates at a frequency of 60.0 Hz. The

circuit contains an inductor whose inductance can be varied, a 200- $\Omega$  resistor, and a 4.00- $\mu\text{F}$  capacitor. What value of  $L$

should an engineer analyzing the circuit choose such that the voltage across the capacitor lags the applied voltage by  $30.0^\circ$ ?

**Solution** The phase relationships for the drops in voltage across the elements are shown in Figure 33.14. From the figure we see that the phase angle is  $\phi = -60.0^\circ$ . This is because the phasors representing  $I_{\max}$  and  $\Delta V_R$  are in the same direction (they are in phase). From Equation 33.25, we find that

$$X_L = X_C + R \tan \phi$$

Substituting Equations 33.10 and 33.17 (with  $\omega = 2\pi f$ ) into this expression gives

$$2\pi fL = \frac{1}{2\pi fC} + R \tan \phi$$

$$L = \frac{1}{2\pi f} \left[ \frac{1}{2\pi fC} + R \tan \phi \right]$$

Substituting the given values into the equation gives  $L =$

0.84 H.

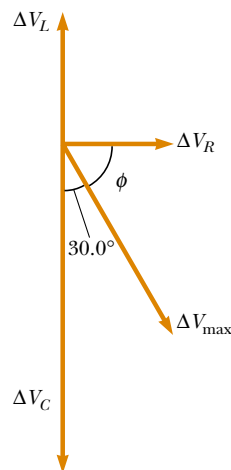


Figure 33.14

### EXAMPLE 33.6 Analyzing a Series RLC Circuit

A series RLC ac circuit has  $R = 425 \Omega$ ,  $L = 1.25 \text{ H}$ ,  $C = 3.50 \mu\text{F}$ ,  $\omega = 377 \text{ s}^{-1}$ , and  $\Delta V_{\max} = 150 \text{ V}$ . (a) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

**Solution** The reactances are  $X_L = \omega L = 471 \Omega$  and

$X_C = 1/\omega C = 758 \Omega$ . The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(425 \Omega)^2 + (471 \Omega - 758 \Omega)^2} = 513 \Omega$$

(b) Find the maximum current in the circuit.

**Solution**

$$I_{\max} = \frac{V_{\max}}{Z} = \frac{150 \text{ V}}{513 \Omega} = 0.292 \text{ A}$$

(c) Find the phase angle between the current and voltage.

**Solution**

$$\phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right) = \tan^{-1} \left( \frac{471 \Omega - 758 \Omega}{425 \Omega} \right)$$

$$= -34.0^\circ$$

Because the circuit is more capacitive than inductive,  $\phi$  is negative and the current leads the applied voltage.

(d) Find both the maximum and the instantaneous voltage across each element.

**Solution** The maximum voltages are

$$\Delta V_R = I_{\max} R = (0.292 \text{ A})(425 \Omega) = 124 \text{ V}$$

$$\Delta V_L = I_{\max} X_L = (0.292 \text{ A})(471 \Omega) = 138 \text{ V}$$

$$\Delta V_C = I_{\max} X_C = (0.292 \text{ A})(758 \Omega) = 221 \text{ V}$$

Using Equations 33.19, 33.20, and 33.21, we find that we can write the instantaneous voltages across the three elements as

$$\Delta v_R = (124 \text{ V}) \sin 377t$$

$$\Delta v_L = (138 \text{ V}) \cos 377t$$

$$\Delta v_C = (-221 \text{ V}) \cos 377t$$

**Comments** The sum of the maximum voltages across the elements is  $\Delta V_R + \Delta V_L + \Delta V_C = 483 \text{ V}$ . Note that this sum is much greater than the maximum voltage of the generator, 150 V. As we saw in Quick Quiz 33.2, the sum of the maximum voltages is a meaningless quantity because when sinusoidally varying quantities are added, *both their amplitudes and their phases* must be taken into account. We know that the

maximum voltages across the various elements occur at different times. That is, the voltages must be added in a way that takes account of the different phases. When this is done, Equation 33.22 is satisfied. You should verify this result.

**Exercise** Construct a phasor diagram to scale, showing the voltages across the elements and the applied voltage. From your diagram, verify that the phase angle is  $-34.0^\circ$ .

### 33.6 POWER IN AN AC CIRCUIT

**No power losses are associated with pure capacitors and pure inductors in an ac circuit.** To see why this is true, let us first analyze the power in an ac circuit containing only a generator and a capacitor.

When the current begins to increase in one direction in an ac circuit, charge begins to accumulate on the capacitor, and a voltage drop appears across it. When this voltage drop reaches its maximum value, the energy stored in the capacitor is  $\frac{1}{2}C(\Delta V_{\max})^2$ . However, this energy storage is only momentary. The capacitor is charged and discharged twice during each cycle: Charge is delivered to the capacitor during two quarters of the cycle and is returned to the voltage source during the remaining two quarters. Therefore, **the average power supplied by the source is zero.** In other words, **no power losses occur in a capacitor in an ac circuit.**

Similarly, the voltage source must do work against the back emf of the inductor. When the current reaches its maximum value, the energy stored in the inductor is a maximum and is given by  $\frac{1}{2}LI_{\max}^2$ . When the current begins to decrease in the circuit, this stored energy is returned to the source as the inductor attempts to maintain the current in the circuit.

In Example 28.1 we found that the power delivered by a battery to a dc circuit is equal to the product of the current and the emf of the battery. Likewise, the instantaneous power delivered by an ac generator to a circuit is the product of the generator current and the applied voltage. For the *RLC* circuit shown in Figure 33.9a, we can express the instantaneous power  $\mathcal{P}$  as

$$\begin{aligned}\mathcal{P} &= i \Delta v = I_{\max} \sin(\omega t - \phi) \Delta V_{\max} \sin \omega t \\ &= I_{\max} \Delta V_{\max} \sin \omega t \sin(\omega t - \phi)\end{aligned}\quad (33.26)$$

Clearly, this result is a complicated function of time and therefore is not very useful from a practical viewpoint. What is generally of interest is the average power over one or more cycles. Such an average can be computed by first using the trigonometric identity  $\sin(\omega t - \phi) = \sin \omega t \cos \phi - \cos \omega t \sin \phi$ . Substituting this into Equation 33.26 gives

$$\mathcal{P} = I_{\max} \Delta V_{\max} \sin^2 \omega t \cos \phi - I_{\max} \Delta V_{\max} \sin \omega t \cos \omega t \sin \phi \quad (33.27)$$

We now take the time average of  $\mathcal{P}$  over one or more cycles, noting that  $I_{\max}$ ,  $\Delta V_{\max}$ ,  $\phi$ , and  $\omega$  are all constants. The time average of the first term on the right in Equation 33.27 involves the average value of  $\sin^2 \omega t$ , which is  $\frac{1}{2}$  (as shown in footnote 2). The time average of the second term on the right is identically zero because  $\sin \omega t \cos \omega t = \frac{1}{2} \sin 2\omega t$ , and the average value of  $\sin 2\omega t$  is zero. Therefore, we can express the **average power**  $\mathcal{P}_{\text{av}}$  as

$$\mathcal{P}_{\text{av}} = \frac{1}{2} I_{\max} \Delta V_{\max} \cos \phi \quad (33.28)$$

It is convenient to express the average power in terms of the rms current and rms voltage defined by Equations 33.4 and 33.5:

$$\mathcal{P}_{\text{av}} = I_{\text{rms}} \Delta V_{\text{rms}} \cos \phi \quad (33.29)$$

where the quantity  $\cos \phi$  is called the **power factor**. By inspecting Figure 33.11b, we see that the maximum voltage drop across the resistor is given by  $\Delta V_R = \Delta V_{\max} \cos \phi = I_{\max} R$ . Using Equation 33.5 and the fact that  $\cos \phi = I_{\max} R / \Delta V_{\max}$ , we find that we can express  $\mathcal{P}_{\text{av}}$  as

$$\mathcal{P}_{\text{av}} = I_{\text{rms}} \Delta V_{\text{rms}} \cos \phi = I_{\text{rms}} \left( \frac{\Delta V_{\max}}{\sqrt{2}} \right) \frac{I_{\max} R}{\Delta V_{\max}} = I_{\text{rms}} \frac{I_{\max} R}{\sqrt{2}}$$

After making the substitution  $I_{\max} = \sqrt{2} I_{\text{rms}}$  from Equation 33.4, we have

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R \quad (33.30)$$

Average power delivered to an RLC circuit

In words, the **average power delivered by the generator is converted to internal energy in the resistor**, just as in the case of a dc circuit. **No power loss occurs in an ideal inductor or capacitor.** When the load is purely resistive, then  $\phi = 0$ ,  $\cos \phi = 1$ , and from Equation 33.29 we see that

$$\mathcal{P}_{\text{av}} = I_{\text{rms}} \Delta V_{\text{rms}}$$

Equation 33.29 shows that the power delivered by an ac source to any circuit depends on the phase, and this result has many interesting applications. For example, a factory that uses large motors in machines, generators, or transformers has a large inductive load (because of all the windings). To deliver greater power to such devices in the factory without using excessively high voltages, technicians introduce capacitance in the circuits to shift the phase.

### EXAMPLE 33.7 Average Power in an RLC Series Circuit

Calculate the average power delivered to the series RLC circuit described in Example 33.6.

**Solution** First, let us calculate the rms voltage and rms current, using the values of  $\Delta V_{\max}$  and  $I_{\max}$  from Example 33.6:

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\max}}{\sqrt{2}} = \frac{150 \text{ V}}{\sqrt{2}} = 106 \text{ V}$$

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = \frac{0.292 \text{ A}}{\sqrt{2}} = 0.206 \text{ A}$$

Because  $\phi = -34.0^\circ$ , the power factor,  $\cos \phi$ , is 0.829; hence, the average power delivered is

$$\begin{aligned} \mathcal{P}_{\text{av}} &= I_{\text{rms}} \Delta V_{\text{rms}} \cos \phi = (0.206 \text{ A})(106 \text{ V})(0.829) \\ &= 18.1 \text{ W} \end{aligned}$$

We can obtain the same result using Equation 33.30.

## 33.7 RESONANCE IN A SERIES RLC CIRCUIT

A series RLC circuit is said to be **in resonance** when the current has its maximum value. In general, the rms current can be written

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} \quad (33.31)$$

where  $Z$  is the impedance. Substituting the expression for  $Z$  from Equation 33.23 into 33.31 gives

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad (33.32)$$



Because the impedance depends on the frequency of the source, the current in the  $RLC$  circuit also depends on the frequency. The frequency  $\omega_0$  at which  $X_L - X_C = 0$  is called the **resonance frequency** of the circuit. To find  $\omega_0$ , we use the condition  $X_L = X_C$ , from which we obtain  $\omega_0 L = 1/\omega_0 C$ , or

Resonance frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (33.33)$$

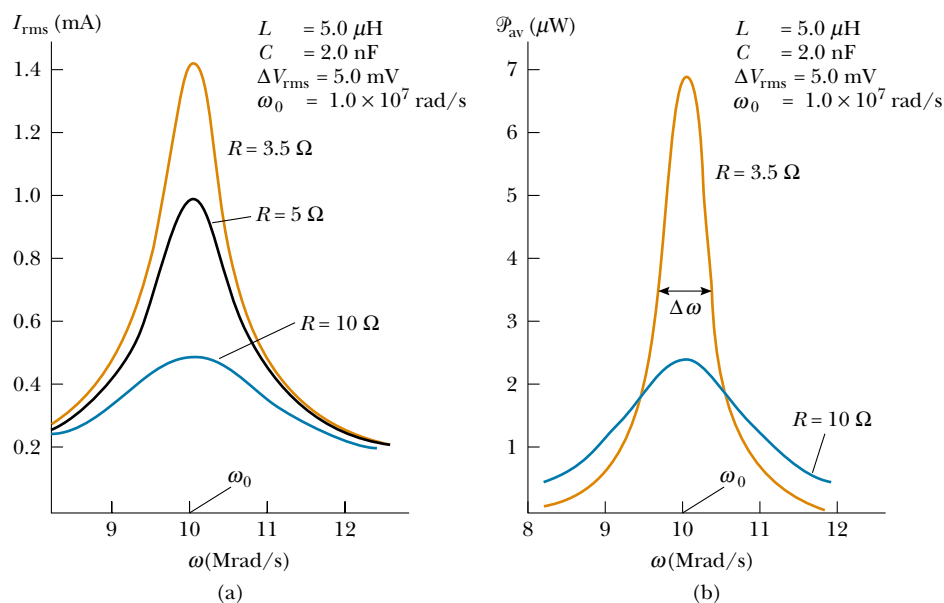
Note that this frequency also corresponds to the natural frequency of oscillation of an  $LC$  circuit (see Section 32.5). Therefore, the current in a series  $RLC$  circuit reaches its maximum value when the frequency of the applied voltage matches the natural oscillator frequency—which depends only on  $L$  and  $C$ . Furthermore, at this frequency the current is in phase with the applied voltage.

### Quick Quiz 33.4

What is the impedance of a series  $RLC$  circuit at resonance? What is the current in the circuit at resonance?

A plot of rms current versus frequency for a series  $RLC$  circuit is shown in Figure 33.15a. The data assume a constant  $\Delta V_{\text{rms}} = 5.0$  mV, that  $L = 5.0$   $\mu\text{H}$ , and that  $C = 2.0$  nF. The three curves correspond to three values of  $R$ . Note that in each case the current reaches its maximum value at the resonance frequency  $\omega_0$ . Furthermore, the curves become narrower and taller as the resistance decreases.

By inspecting Equation 33.32, we must conclude that, when  $R = 0$ , the current becomes infinite at resonance. Although the equation predicts this, real circuits always have some resistance, which limits the value of the current.



**Figure 33.15** (a) The rms current versus frequency for a series  $RLC$  circuit, for three values of  $R$ . The current reaches its maximum value at the resonance frequency  $\omega_0$ . (b) Average power versus frequency for the series  $RLC$  circuit, for two values of  $R$ .

It is also interesting to calculate the average power as a function of frequency for a series RLC circuit. Using Equations 33.30, 33.31, and 33.23, we find that

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R = \frac{(\Delta V_{\text{rms}})^2}{Z^2} R = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + (X_L - X_C)^2} \quad (33.34)$$

Because  $X_L = \omega L$ ,  $X_C = 1/\omega C$ , and  $\omega_0^2 = 1/LC$ , we can express the term  $(X_L - X_C)^2$  as

$$(X_L - X_C)^2 = \left( \omega L - \frac{1}{\omega C} \right)^2 = \frac{L^2}{\omega^2} (\omega^2 - \omega_0^2)^2$$

Using this result in Equation 33.34 gives

$$\mathcal{P}_{\text{av}} = \frac{(\Delta V_{\text{rms}})^2 R \omega^2}{R^2 \omega^2 + L^2 (\omega^2 - \omega_0^2)^2} \quad (33.35)$$

This expression shows that at resonance, when  $\omega = \omega_0$ , **the average power is a maximum** and has the value  $(\Delta V_{\text{rms}})^2/R$ . Figure 33.15b is a plot of average power versus frequency for two values of  $R$  in a series RLC circuit. As the resistance is made smaller, the curve becomes sharper in the vicinity of the resonance frequency. This curve sharpness is usually described by a dimensionless parameter known as the **quality factor**, denoted by  $Q$ :<sup>4</sup>

$$Q = \frac{\omega_0}{\Delta\omega}$$

where  $\Delta\omega$  is the width of the curve measured between the two values of  $\omega$  for which  $\mathcal{P}_{\text{av}}$  has half its maximum value, called the *half-power points* (see Fig. 33.15b.) It is left as a problem (Problem 70) to show that the width at the half-power points has the value  $\Delta\omega = R/L$ , so

$$Q = \frac{\omega_0 L}{R} \quad (33.36)$$

The curves plotted in Figure 33.16 show that a high- $Q$  circuit responds to only a very narrow range of frequencies, whereas a low- $Q$  circuit can detect a much broader range of frequencies. Typical values of  $Q$  in electronic circuits range from 10 to 100.

The receiving circuit of a radio is an important application of a resonant circuit. One tunes the radio to a particular station (which transmits a specific electromagnetic wave or signal) by varying a capacitor, which changes the resonant frequency of the receiving circuit. When the resonance frequency of the circuit matches that of the incoming electromagnetic wave, the current in the receiving circuit increases. This signal caused by the incoming wave is then amplified and fed to a speaker. Because many signals are often present over a range of frequencies, it is important to design a high- $Q$  circuit to eliminate unwanted signals. In this manner, stations whose frequencies are near but not equal to the resonance frequency give signals at the receiver that are negligibly small relative to the signal that matches the resonance frequency.

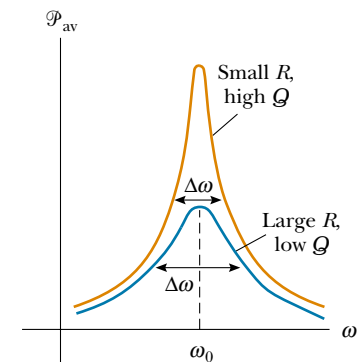
<sup>4</sup> The quality factor is also defined as the ratio  $2\pi E/\Delta E$ , where  $E$  is the energy stored in the oscillating system and  $\Delta E$  is the energy lost per cycle of oscillation. The quality factor for a mechanical system can also be defined, as noted in Section 13.7.

Average power as a function of frequency in an RLC circuit

Quality factor

### QuickLab

Tune a radio to your favorite station. Can you determine what the product of  $LC$  must be for the radio's tuning circuitry?



**Figure 33.16** Average power versus frequency for a series RLC circuit. The width  $\Delta\omega$  of each curve is measured between the two points where the power is half its maximum value. The power is a maximum at the resonance frequency  $\omega_0$ .

**Quick Quiz 33.5**

An airport metal detector (Fig. 33.17) is essentially a resonant circuit. The portal you step through is an inductor (a large loop of conducting wire) that is part of the circuit. The frequency of the circuit is tuned to the resonant frequency of the circuit when there is no metal in the inductor. Any metal on your body increases the effective inductance of the loop and changes the current in it. If you want the detector to be able to detect a small metallic object, should the circuit have a high quality factor or a low one?



**Figure 33.17** When you pass through a metal detector, you become part of a resonant circuit. As you step through the detector, the inductance of the circuit changes, and thus the current in the circuit changes. (Terry Qing/FPG International)

**EXAMPLE 33.8** A Resonating Series *RLC* Circuit

Consider a series *RLC* circuit for which  $R = 150 \, \Omega$ ,  $L = 20.0 \, \text{mH}$ ,  $\Delta V_{\text{rms}} = 20.0 \, \text{V}$ , and  $\omega = 5000 \, \text{s}^{-1}$ . Determine the value of the capacitance for which the current is a maximum.

**Solution** The current has its maximum value at the resonance frequency  $\omega_0$ , which should be made to match the “driving” frequency of  $5000 \, \text{s}^{-1}$ :

$$\omega_0 = 5.00 \times 10^3 \, \text{s}^{-1} = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(25.0 \times 10^6 \, \text{s}^{-2})(20.0 \times 10^{-3} \, \text{H})} = 2.00 \, \mu\text{F}$$

**Exercise** Calculate the maximum value of the rms current in the circuit as the frequency is varied.

**Answer** 0.133 A.

**33.8 THE TRANSFORMER AND POWER TRANSMISSION**

When electric power is transmitted over great distances, it is economical to use a high voltage and a low current to minimize the  $I^2R$  loss in the transmission lines.

Consequently, 350-kV lines are common, and in many areas even higher-voltage (765-kV) lines are under construction. At the receiving end of such lines, the consumer requires power at a low voltage (for safety and for efficiency in design). Therefore, a device is required that can change the alternating voltage and current without causing appreciable changes in the power delivered. The ac transformer is that device.

In its simplest form, the **ac transformer** consists of two coils of wire wound around a core of iron, as illustrated in Figure 33.18. The coil on the left, which is connected to the input alternating voltage source and has  $N_1$  turns, is called the *primary winding* (or the *primary*). The coil on the right, consisting of  $N_2$  turns and connected to a load resistor  $R$ , is called the *secondary winding* (or the *secondary*). The purpose of the iron core is to increase the magnetic flux through the coil and to provide a medium in which nearly all the flux through one coil passes through the other coil. Eddy current losses are reduced by using a laminated core. Iron is used as the core material because it is a soft ferromagnetic substance and hence reduces hysteresis losses. Transformation of energy to internal energy in the finite resistance of the coil wires is usually quite small. Typical transformers have power efficiencies from 90% to 99%. In the discussion that follows, we assume an *ideal transformer*, one in which the energy losses in the windings and core are zero.

First, let us consider what happens in the primary circuit when the switch in the secondary circuit is open. If we assume that the resistance of the primary is negligible relative to its inductive reactance, then the primary circuit is equivalent to a simple circuit consisting of an inductor connected to an ac generator. Because the current is  $90^\circ$  out of phase with the voltage, the power factor  $\cos \phi$  is zero, and hence the average power delivered from the generator to the primary circuit is zero. Faraday's law states that the voltage  $\Delta V_1$  across the primary is

$$\Delta V_1 = -N_1 \frac{d\Phi_B}{dt} \quad (33.37)$$

where  $\Phi_B$  is the magnetic flux through each turn. If we assume that all magnetic field lines remain within the iron core, the flux through each turn of the primary equals the flux through each turn of the secondary. Hence, the voltage across the secondary is

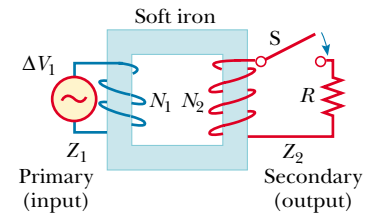
$$\Delta V_2 = -N_2 \frac{d\Phi_B}{dt} \quad (33.38)$$

Solving Equation 33.37 for  $d\Phi_B/dt$  and substituting the result into Equation 33.38, we find that

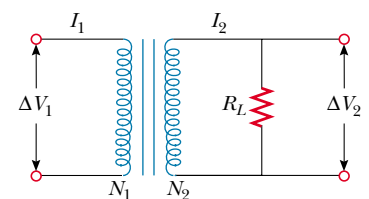
$$\Delta V_2 = \frac{N_2}{N_1} \Delta V_1 \quad (33.39)$$

When  $N_2 > N_1$ , the output voltage  $\Delta V_2$  exceeds the input voltage  $\Delta V_1$ . This setup is referred to as a *step-up transformer*. When  $N_2 < N_1$ , the output voltage is less than the input voltage, and we have a *step-down transformer*.

When the switch in the secondary circuit is thrown closed, a current  $I_2$  is induced in the secondary. If the load in the secondary circuit is a pure resistance, the induced current is in phase with the induced voltage. The power supplied to the secondary circuit must be provided by the ac generator connected to the primary circuit, as shown in Figure 33.19. In an ideal transformer, where there are no losses, the power  $I_1 \Delta V_1$  supplied by the generator is equal to the power  $I_2 \Delta V_2$  in



**Figure 33.18** An ideal transformer consists of two coils wound on the same iron core. An alternating voltage  $\Delta V_1$  is applied to the primary coil, and the output voltage  $\Delta V_2$  is across the resistor of resistance  $R$ .



**Figure 33.19** Circuit diagram for a transformer.



This cylindrical step-down transformer drops the voltage from 4 000 V to 220 V for delivery to a group of residences. (George Semple)



**Nikola Tesla (1856–1943)** Tesla was born in Croatia but spent most of his professional life as an inventor in the United States. He was a key figure in the development of alternating-current electricity, high-voltage transformers, and the transport of electric power via ac transmission lines. Tesla's viewpoint was at odds with the ideas of Thomas Edison, who committed himself to the use of direct current in power transmission. Tesla's ac approach won out. (UPI/Bettmann)

the secondary circuit. That is,

$$I_1 \Delta V_1 = I_2 \Delta V_2 \quad (33.40)$$

The value of the load resistance  $R_L$  determines the value of the secondary current because  $I_2 = \Delta V_2/R_L$ . Furthermore, the current in the primary is  $I_1 = \Delta V_1/R_{\text{eq}}$ , where

$$R_{\text{eq}} = \left( \frac{N_1}{N_2} \right)^2 R_L \quad (33.41)$$

is the equivalent resistance of the load resistance when viewed from the primary side. From this analysis we see that a transformer may be used to match resistances between the primary circuit and the load. In this manner, maximum power transfer can be achieved between a given power source and the load resistance. For example, a transformer connected between the 1-k $\Omega$  output of an audio amplifier and an 8- $\Omega$  speaker ensures that as much of the audio signal as possible is transferred into the speaker. In stereo terminology, this is called *impedance matching*.

We can now also understand why transformers are useful for transmitting power over long distances. Because the generator voltage is stepped up, the current in the transmission line is reduced, and hence  $I^2R$  losses are reduced. In practice, the voltage is stepped up to around 230 000 V at the generating station, stepped down to around 20 000 V at a distributing station, then to 4 000 V for delivery to residential areas, and finally to 120–240 V at the customer's site. The power is supplied by a three-wire cable. In the United States, two of these wires are "hot," with voltages of 120 V with respect to a common ground wire. Home appliances operating on 120 V are connected in parallel between one of the hot wires and ground. Larger appliances, such as electric stoves and clothes dryers, require 240 V. This is obtained across the two hot wires, which are 180° out of phase so that the voltage difference between them is 240 V.

There is a practical upper limit to the voltages that can be used in transmission lines. Excessive voltages could ionize the air surrounding the transmission lines, which could result in a conducting path to ground or to other objects in the vicinity. This, of course, would present a serious hazard to any living creatures. For this reason, a long string of insulators is used to keep high-voltage wires away from their supporting metal towers. Other insulators are used to maintain separation between wires.



**Figure 33.20** The primary winding in this transformer is directly attached to the prongs of the plug. The secondary winding is connected to the wire on the right, which runs to an electronic device. Many of these power-supply transformers also convert alternating current to direct current. (George Semple)

Many common household electronic devices require low voltages to operate properly. A small transformer that plugs directly into the wall, like the one illustrated in the photograph at the beginning of this chapter, can provide the proper voltage. Figure 33.20 shows the two windings wrapped around a common iron core that is found inside all these little “black boxes.” This particular transformer converts the 120-V ac in the wall socket to 12.5-V ac. (Can you determine the ratio of the numbers of turns in the two coils?) Some black boxes also make use of diodes to convert the alternating current to direct current (see Section 33.9).

**web**

For information on how small transformers and hundreds of other everyday devices operate, visit <http://www.howstuffworks.com>

**EXAMPLE 33.9** The Economics of ac Power

An electricity-generating station needs to deliver 20 MW of power to a city 1.0 km away. (a) If the resistance of the wires is  $2.0\ \Omega$  and the electricity costs about  $10\text{¢/kWh}$ , estimate what it costs the utility company to send the power to the city for one day. A common voltage for commercial power generators is 22 kV, but a step-up transformer is used to boost the voltage to 230 kV before transmission.

**Solution** The power losses in the transmission line are the result of the resistance of the line. We can determine the loss from Equation 27.23,  $\mathcal{P} = I^2R$ . Because this is an estimate, we can use dc equations and calculate  $I$  from Equation 27.22:

$$I = \frac{\mathcal{P}}{\Delta V} = \frac{20 \times 10^6\ \text{W}}{230 \times 10^3\ \text{V}} = 87\ \text{A}$$

Therefore,

$$\mathcal{P} = I^2R = (87\ \text{A})^2(2.0\ \Omega) = 15\ \text{kW}$$

Over the course of a day, the energy loss due to the resistance of the wires is  $(15\ \text{kW})(24\ \text{h}) = 360\ \text{kWh}$ , at a cost of **\$36**.

(b) Repeat the calculation for the situation in which the power plant delivers the electricity at its original voltage of 22 kV.

**Solution**

$$I = \frac{\mathcal{P}}{\Delta V} = \frac{20 \times 10^6\ \text{W}}{22 \times 10^3\ \text{V}} = 910\ \text{A}$$

$$\mathcal{P} = I^2R = (910\ \text{A})^2(2.0\ \Omega) = 1.7 \times 10^3\ \text{kW}$$


$$\text{Cost per day} = (1.7 \times 10^3\ \text{kW})(24\ \text{h})(\$0.10/\text{kWh})$$

$$= \mathbf{\$4\ 100}$$

The tremendous savings that are possible through the use of transformers and high-voltage transmission lines, along with the efficiency of using alternating current to operate motors, led to the universal adoption of alternating current instead of direct current for commercial power grids.

*Optional Section***33.9** RECTIFIERS AND FILTERS

Portable electronic devices such as radios and compact disc (CD) players are often powered by direct current supplied by batteries. Many devices come with ac–dc converters that provide a readily available direct-current source if the batteries are low. Such a converter contains a transformer that steps the voltage down from 120 V to typically 9 V and a circuit that converts alternating current to direct current. The process of converting alternating current to direct current is called **rectification**, and the converting device is called a **rectifier**.

The most important element in a rectifier circuit is a **diode**, a circuit element that conducts current in one direction but not the other. Most diodes used in modern electronics are semiconductor devices. The circuit symbol for a diode is  , where the arrow indicates the direction of the current through the diode. A diode has low resistance to current in one direction (the direction of the arrow) and high resistance to current in the opposite direction. We can understand how a diode rectifies a current by considering Figure 33.21a, which shows a



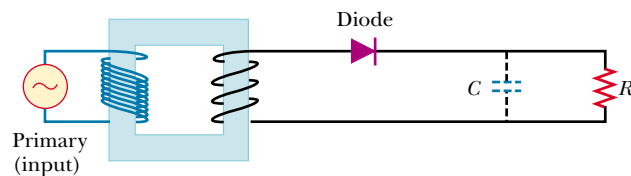
diode and a resistor connected to the secondary of a transformer. The transformer reduces the voltage from 120-V ac to the lower voltage that is needed for the device having a resistance  $R$  (the load resistance). Because current can pass through the diode in only one direction, the alternating current in the load resistor is reduced to the form shown by the solid curve in Figure 33.21b. The diode conducts current only when the side of the symbol containing the arrowhead has a positive potential relative to the other side. In this situation, the diode acts as a *half-wave rectifier* because current is present in the circuit during only half of each cycle.

When a capacitor is added to the circuit, as shown by the dashed lines and the capacitor symbol in Figure 33.21a, the circuit is a simple dc power supply. The time variation in the current in the load resistor (the dashed curve in Fig. 33.21b) is close to being zero, as determined by the  $RC$  time constant of the circuit.

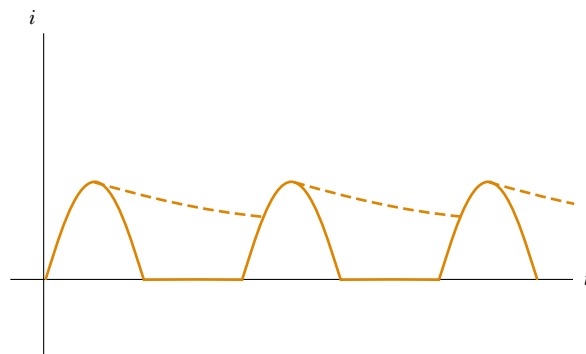
The  $RC$  circuit in Figure 33.21a is one example of a **filter circuit**, which is used to smooth out or eliminate a time-varying signal. For example, radios are usually powered by a 60-Hz alternating voltage. After rectification, the voltage still contains a small ac component at 60 Hz (sometimes called *ripple*), which must be filtered. By “filtered,” we mean that the 60-Hz ripple must be reduced to a value much less than that of the audio signal to be amplified, because without filtering, the resulting audio signal includes an annoying hum at 60 Hz.

To understand how a filter works, let us consider the simple series  $RC$  circuit shown in Figure 33.22a. The input voltage is across the two elements and is represented by  $\Delta V_{\max} \sin \omega t$ . Because we are interested only in maximum values, we can use Equation 33.24, taking  $X_L = 0$  and substituting  $X_C = 1/\omega C$ . This shows that the maximum input voltage is related to the maximum current by

$$\Delta V_{\text{in}} = I_{\text{max}} Z = I_{\text{max}} \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$



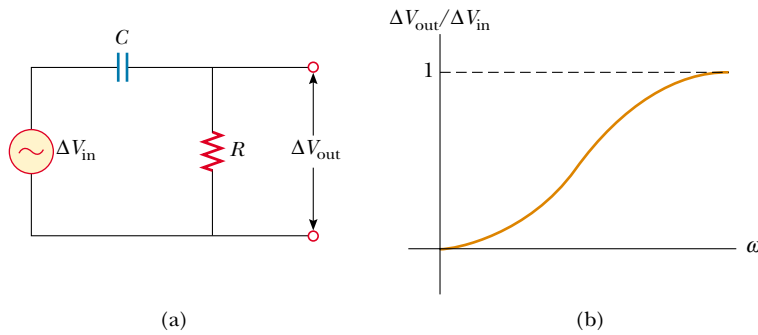
(a)



(b)

**Figure 33.21** (a) A half-wave rectifier with an optional filter capacitor. (b) Current versus time in the resistor. The solid curve represents the current with no filter capacitor, and the dashed curve is the current when the circuit includes the capacitor.





**Figure 33.22** (a) A simple  $RC$  high-pass filter. (b) Ratio of output voltage to input voltage for an  $RC$  high-pass filter as a function of the angular frequency of the circuit.

If the voltage across the resistor is considered to be the output voltage, then the maximum output voltage is

$$\Delta V_{\text{out}} = I_{\text{max}}R$$

Therefore, the ratio of the output voltage to the input voltage is

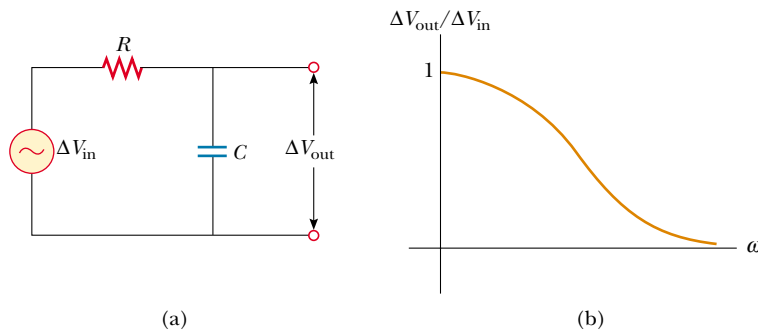
$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \quad (33.42)$$

High-pass filter

A plot of this ratio as a function of angular frequency (see Fig. 33.22b) shows that at low frequencies  $\Delta V_{\text{out}}$  is much smaller than  $\Delta V_{\text{in}}$ , whereas at high frequencies the two voltages are equal. Because the circuit preferentially passes signals of higher frequency while blocking low-frequency signals, the circuit is called an  $RC$  high-pass filter. Physically, a high-pass filter works because a capacitor “blocks out” direct current and ac current at low frequencies.

Now let us consider the circuit shown in Figure 33.23a, where the output voltage is taken across the capacitor. In this case, the maximum voltage equals the voltage across the capacitor. Because the impedance across the capacitor is  $X_C = 1/\omega C$ , we have

$$\Delta V_{\text{out}} = I_{\text{max}}X_C = \frac{I_{\text{max}}}{\omega C}$$



**Figure 33.23** (a) A simple  $RC$  low-pass filter. (b) Ratio of output voltage to input voltage for an  $RC$  low-pass filter as a function of the angular frequency of the circuit.

Low-pass filter

Therefore, the ratio of the output voltage to the input voltage is

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{1/\omega C}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \quad (33.43)$$

This ratio, plotted as a function of  $\omega$  in Figure 33.23b, shows that in this case the circuit preferentially passes signals of low frequency. Hence, the circuit is called an *RC* low-pass filter.

You may be familiar with crossover networks, which are an important part of the speaker systems for high-fidelity audio systems. These networks utilize low-pass filters to direct low frequencies to a special type of speaker, the “woofer,” which is designed to reproduce the low notes accurately. The high frequencies are sent to the “tweeter” speaker.

### Quick Quiz 33.6

Suppose you are designing a high-fidelity system containing both large loudspeakers (woofers) and small loudspeakers (tweeters). (a) What circuit element would you place in series with a woofer, which passes low-frequency signals? (b) What circuit element would you place in series with a tweeter, which passes high-frequency signals?

### SUMMARY

If an ac circuit consists of a generator and a resistor, the current is in phase with the voltage. That is, the current and voltage reach their maximum values at the same time.

The **rms current** and **rms voltage** in an ac circuit in which the voltages and current vary sinusoidally are given by the expressions

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = 0.707I_{\text{max}} \quad (33.4)$$

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = 0.707\Delta V_{\text{max}} \quad (33.5)$$

where  $I_{\text{max}}$  and  $\Delta V_{\text{max}}$  are the maximum values.

If an ac circuit consists of a generator and an inductor, the current lags behind the voltage by  $90^\circ$ . That is, the voltage reaches its maximum value one quarter of a period before the current reaches its maximum value.

If an ac circuit consists of a generator and a capacitor, the current leads the voltage by  $90^\circ$ . That is, the current reaches its maximum value one quarter of a period before the voltage reaches its maximum value.

In ac circuits that contain inductors and capacitors, it is useful to define the **inductive reactance**  $X_L$  and the **capacitive reactance**  $X_C$  as

$$X_L = \omega L \quad (33.10)$$

$$X_C = \frac{1}{\omega C} \quad (33.17)$$

where  $\omega$  is the angular frequency of the ac generator. The SI unit of reactance is the ohm.

The **impedance**  $Z$  of an  $RLC$  series ac circuit, which also has the ohm as its unit, is

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \quad (33.23)$$

This expression illustrates that we cannot simply add the resistance and reactances in a circuit. We must account for the fact that the applied voltage and current are out of phase, with the **phase angle**  $\phi$  between the current and voltage being

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \quad (33.25)$$

The sign of  $\phi$  can be positive or negative, depending on whether  $X_L$  is greater or less than  $X_C$ . The phase angle is zero when  $X_L = X_C$ .

The **average power** delivered by the generator in an  $RLC$  ac circuit is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}} \Delta V_{\text{rms}} \cos \phi \quad (33.29)$$

An equivalent expression for the average power is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R \quad (33.30)$$

The average power delivered by the generator results in increasing internal energy in the resistor. No power loss occurs in an ideal inductor or capacitor.

The rms current in a series  $RLC$  circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad (33.32)$$

A series  $RLC$  circuit is in resonance when the inductive reactance equals the capacitive reactance. When this condition is met, the current given by Equation 33.32 reaches its maximum value. When  $X_L = X_C$  in a circuit, the **resonance frequency**  $\omega_0$  of the circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (33.33)$$

The current in a series  $RLC$  circuit reaches its maximum value when the frequency of the generator equals  $\omega_0$ —that is, when the “driving” frequency matches the resonance frequency.

Transformers allow for easy changes in alternating voltage. Because energy (and therefore power) are conserved, we can write

$$I_1 \Delta V_1 = I_2 \Delta V_2 \quad (33.40)$$

to relate the currents and voltages in the primary and secondary windings of a transformer.

## QUESTIONS

- Fluorescent lights flicker on and off 120 times every second. Explain what causes this. Why can't you see it happening?
- Why does a capacitor act as a short circuit at high frequencies? Why does it act as an open circuit at low frequencies?
- Explain how the acronyms in the mnemonic “ELI the ICE man” can be used to recall whether current leads voltage or voltage leads current in  $RLC$  circuits. (Note that “E” represents voltage.)
- Why is the sum of the maximum voltages across the elements in a series  $RLC$  circuit usually greater than the maximum applied voltage? Doesn't this violate Kirchhoff's second rule?
- Does the phase angle depend on frequency? What is the phase angle when the inductive reactance equals the capacitive reactance?
- Energy is delivered to a series  $RLC$  circuit by a generator. This energy appears as internal energy in the resistor. What is the source of this energy?

- Explain why the average power delivered to an  $RLC$  circuit by the generator depends on the phase between the current and the applied voltage.
- A particular experiment requires a beam of light of very stable intensity. Why would an ac voltage be unsuitable for powering the light source?
- Consider a series  $RLC$  circuit in which  $R$  is an incandescent lamp,  $C$  is some fixed capacitor, and  $L$  is a variable inductance. The source is 120-V ac. Explain why the lamp glows brightly for some values of  $L$  and does not glow at all for other values.
- What determines the maximum voltage that can be used on a transmission line?
- Will a transformer operate if a battery is used for the input voltage across the primary? Explain.
- How can the average value of a current be zero and yet the square root of the average squared current not be zero?
- What is the time average of the “square-wave” voltage shown in Figure Q33.13? What is its rms voltage?

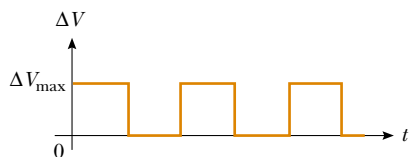


Figure Q33.13

- Explain how the quality factor is related to the response characteristics of a radio receiver. Which variable most strongly determines the quality factor?

- Why are the primary and secondary windings of a transformer wrapped on an iron core that passes through both coils?
- With reference to Figure Q33.16, explain why the capacitor prevents a dc signal from passing between circuits A and B, yet allows an ac signal to pass from circuit A to circuit B. (The circuits are said to be capacitively coupled.)

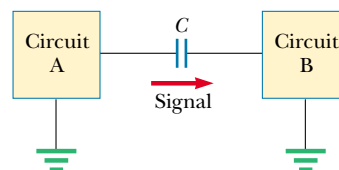


Figure Q33.16

- With reference to Figure Q33.17, if  $C$  is made sufficiently large, an ac signal passes from circuit A to ground rather than from circuit A to circuit B. Hence, the capacitor acts as a filter. Explain.

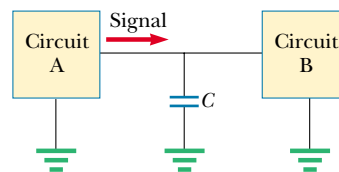


Figure Q33.17

## PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*  
 WEB = solution posted at <http://www.saunderscollege.com/physics/> = Computer useful in solving problem = Interactive Physics  
 □ = paired numerical/symbolic problems

Note: Assume that all ac voltages and currents are sinusoidal unless stated otherwise.

### Section 33.1 ac Sources and Phasors

### Section 33.2 Resistors in an ac Circuit

- The rms output voltage of an ac generator is 200 V, and the operating frequency is 100 Hz. Write the equation giving the output voltage as a function of time.
- (a) What is the resistance of a lightbulb that uses an average power of 75.0 W when connected to a 60.0-Hz power source having a maximum voltage of 170 V?  
(b) What is the resistance of a 100-W bulb?
- An ac power supply produces a maximum voltage  $\Delta V_{\max} = 100$  V. This power supply is connected to a  $24.0\text{-}\Omega$  resistor, and the current and resistor voltage are measured with an ideal ac ammeter and voltmeter, as shown in Figure P33.3. What does each meter read?

Note that an ideal ammeter has zero resistance and that an ideal voltmeter has infinite resistance.

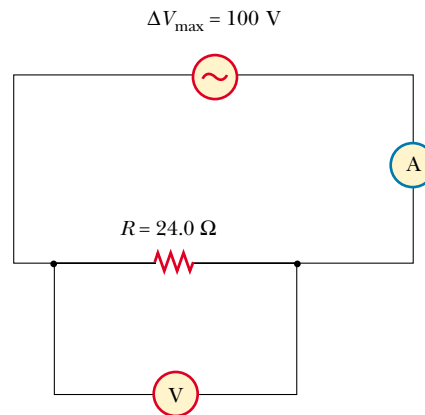


Figure P33.3

4. In the simple ac circuit shown in Figure 33.1,  $R = 70.0 \Omega$  and  $\Delta v = \Delta V_{\max} \sin \omega t$ . (a) If  $\Delta v_R = 0.250 \Delta V_{\max}$  for the first time at  $t = 0.0100$  s, what is the angular frequency of the generator? (b) What is the next value of  $t$  for which  $\Delta v_R = 0.250 \Delta V_{\max}$ ?
5. The current in the circuit shown in Figure 33.1 equals 60.0% of the peak current at  $t = 7.00$  ms. What is the smallest frequency of the generator that gives this current?
6. Figure P33.6 shows three lamps connected to a 120-V ac (rms) household supply voltage. Lamps 1 and 2 have 150-W bulbs; lamp 3 has a 100-W bulb. Find the rms current and the resistance of each bulb.

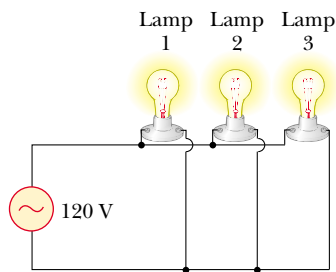


Figure P33.6

7. An audio amplifier, represented by the ac source and resistor in Figure P33.7, delivers to the speaker alternating voltage at audio frequencies. If the source voltage has an amplitude of 15.0 V,  $R = 8.20 \Omega$ , and the speaker is equivalent to a resistance of  $10.4 \Omega$ , what time-averaged power is transferred to it?

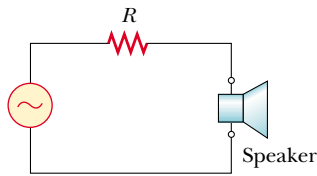


Figure P33.7

### Section 33.3 Inductors in an ac Circuit

8. An inductor is connected to a 20.0-Hz power supply that produces a 50.0-V rms voltage. What inductance is needed to keep the instantaneous current in the circuit below 80.0 mA?
9. In a purely inductive ac circuit, such as that shown in Figure 33.4,  $\Delta V_{\max} = 100$  V. (a) If the maximum current is 7.50 A at 50.0 Hz, what is the inductance  $L$ ? (b) At what angular frequency  $\omega$  is the maximum current 2.50 A?
10. An inductor has a  $54.0\text{-}\Omega$  reactance at 60.0 Hz. What is the maximum current when this inductor is connected to a 50.0-Hz source that produces a 100-V rms voltage?

- WEB 11. For the circuit shown in Figure 33.4,  $\Delta V_{\max} = 80.0$  V,  $\omega = 65.0\pi$  rad/s, and  $L = 70.0$  mH. Calculate the current in the inductor at  $t = 15.5$  ms.
12. A 20.0-mH inductor is connected to a standard outlet ( $\Delta V_{\text{rms}} = 120$  V,  $f = 60.0$  Hz). Determine the energy stored in the inductor at  $t = (1/180)$  s, assuming that this energy is zero at  $t = 0$ .
13. **Review Problem.** Determine the maximum magnetic flux through an inductor connected to a standard outlet ( $\Delta V_{\text{rms}} = 120$  V,  $f = 60.0$  Hz).

### Section 33.4 Capacitors in an ac Circuit

14. (a) For what frequencies does a  $22.0\text{-}\mu\text{F}$  capacitor have a reactance below  $175 \Omega$ ? (b) Over this same frequency range, what is the reactance of a  $44.0\text{-}\mu\text{F}$  capacitor?
15. What maximum current is delivered by a  $2.20\text{-}\mu\text{F}$  capacitor when it is connected across (a) a North American outlet having  $\Delta V_{\text{rms}} = 120$  V and  $f = 60.0$  Hz? (b) a European outlet having  $\Delta V_{\text{rms}} = 240$  V and  $f = 50.0$  Hz?
16. A capacitor  $C$  is connected to a power supply that operates at a frequency  $f$  and produces an rms voltage  $\Delta V$ . What is the maximum charge that appears on either of the capacitor plates?
17. What maximum current is delivered by an ac generator with  $\Delta V_{\max} = 48.0$  V and  $f = 90.0$  Hz when it is connected across a  $3.70\text{-}\mu\text{F}$  capacitor?
18. A  $1.00\text{-mF}$  capacitor is connected to a standard outlet ( $\Delta V_{\text{rms}} = 120$  V,  $f = 60.0$  Hz). Determine the current in the capacitor at  $t = (1/180)$  s, assuming that at  $t = 0$  the energy stored in the capacitor is zero.

### Section 33.5 The RLC Series Circuit

19. An inductor ( $L = 400$  mH), a capacitor ( $C = 4.43 \mu\text{F}$ ), and a resistor ( $R = 500 \Omega$ ) are connected in series. A 50.0-Hz ac generator produces a peak current of 250 mA in the circuit. (a) Calculate the required peak voltage  $\Delta V_{\max}$ . (b) Determine the phase angle by which the current leads or lags the applied voltage.
20. At what frequency does the inductive reactance of a  $57.0\text{-}\mu\text{H}$  inductor equal the capacitive reactance of a  $57.0\text{-}\mu\text{F}$  capacitor?
21. A series ac circuit contains the following components:  $R = 150 \Omega$ ,  $L = 250$  mH,  $C = 2.00 \mu\text{F}$ , and a generator with  $\Delta V_{\max} = 210$  V operating at 50.0 Hz. Calculate the (a) inductive reactance, (b) capacitive reactance, (c) impedance, (d) maximum current, and (e) phase angle between current and generator voltage.
22. A sinusoidal voltage  $\Delta v(t) = (40.0 \text{ V}) \sin(100t)$  is applied to a series RLC circuit with  $L = 160$  mH,  $C = 99.0 \mu\text{F}$ , and  $R = 68.0 \Omega$ . (a) What is the impedance of the circuit? (b) What is the maximum current? (c) Determine the numerical values for  $I_{\max}$ ,  $\omega$ , and  $\phi$  in the equation  $i(t) = I_{\max} \sin(\omega t - \phi)$ .
- WEB 23. An RLC circuit consists of a  $150\text{-}\Omega$  resistor, a  $21.0\text{-}\mu\text{F}$  capacitor, and a  $460\text{-mH}$  inductor, connected in series with a 120-V, 60.0-Hz power supply. (a) What is the

phase angle between the current and the applied voltage? (b) Which reaches its maximum earlier, the current or the voltage?

24. A person is working near the secondary of a transformer, as shown in Figure P33.24. The primary voltage is 120 V at 60.0 Hz. The capacitance  $C_s$ , which is the stray capacitance between the person's hand and the secondary winding, is 20.0 pF. Assuming that the person has a body resistance to ground  $R_b = 50.0$  k $\Omega$ , determine the rms voltage across the body. (*Hint*: Redraw the circuit with the secondary of the transformer as a simple ac source.)

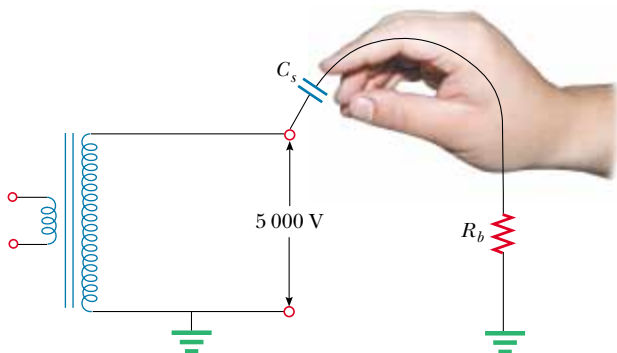


Figure P33.24

25. An ac source with  $\Delta V_{\text{max}} = 150$  V and  $f = 50.0$  Hz is connected between points  $a$  and  $d$  in Figure P33.25. Calculate the maximum voltages between points (a)  $a$  and  $b$ , (b)  $b$  and  $c$ , (c)  $c$  and  $d$ , and (d)  $b$  and  $d$ .

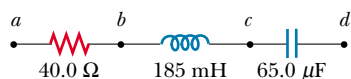


Figure P33.25 Problems 25 and 64.

26. Draw to scale a phasor diagram showing  $Z$ ,  $X_L$ ,  $X_C$ , and  $\phi$  for an ac series circuit for which  $R = 300$   $\Omega$ ,  $C = 11.0$   $\mu\text{F}$ ,  $L = 0.200$  H, and  $f = (500/\pi)$  Hz.
27. A coil of resistance 35.0  $\Omega$  and inductance 20.5 H is in series with a capacitor and a 200-V (rms), 100-Hz source. The rms current in the circuit is 4.00 A. (a) Calculate the capacitance in the circuit. (b) What is  $\Delta V_{\text{rms}}$  across the coil?

### Section 33.6 Power in an ac Circuit

28. The voltage source in Figure P33.28 has an output  $\Delta V_{\text{rms}} = 100$  V at  $\omega = 1000$  rad/s. Determine (a) the current in the circuit and (b) the power supplied by the source. (c) Show that the power delivered to the resistor is equal to the power supplied by the source.

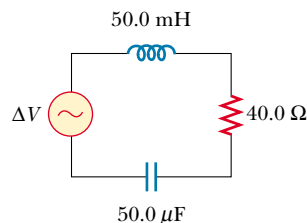


Figure P33.28

- WEB 29. An ac voltage of the form  $\Delta v = (100 \text{ V}) \sin(1000t)$  is applied to a series  $RLC$  circuit. If  $R = 400$   $\Omega$ ,  $C = 5.00$   $\mu\text{F}$ , and  $L = 0.500$  H, what is the average power delivered to the circuit?
30. A series  $RLC$  circuit has a resistance of 45.0  $\Omega$  and an impedance of 75.0  $\Omega$ . What average power is delivered to this circuit when  $\Delta V_{\text{rms}} = 210$  V?
31. In a certain series  $RLC$  circuit,  $I_{\text{rms}} = 9.00$  A,  $\Delta V_{\text{rms}} = 180$  V, and the current leads the voltage by 37.0°. (a) What is the total resistance of the circuit? (b) What is the reactance of the circuit ( $X_L - X_C$ )?
32. Suppose you manage a factory that uses many electric motors. The motors create a large inductive load to the electric power line, as well as a resistive load. The electric company builds an extra-heavy distribution line to supply you with a component of current that is 90° out of phase with the voltage, as well as with current in phase with the voltage. The electric company charges you an extra fee for “reactive volt-amps” in addition to the amount you pay for the energy you use. You can avoid the extra fee by installing a capacitor between the power line and your factory. The following problem models this solution.
- In an  $LR$  circuit, a 120-V (rms), 60.0-Hz source is in series with a 25.0-mH inductor and a 20.0- $\Omega$  resistor. What are (a) the rms current and (b) the power factor? (c) What capacitor must be added in series to make the power factor 1? (d) To what value can the supply voltage be reduced if the power supplied is to be the same as that provided before installation of the capacitor?

**33. Review Problem.** Over a distance of 100 km, power of 100 MW is to be transmitted at 50.0 kV with only 1.00% loss. Copper wire of what diameter should be used for each of the two conductors of the transmission line? Assume that the current density in the conductors is uniform.

**34. Review Problem.** Suppose power  $\mathcal{P}$  is to be transmitted over a distance  $d$  at a voltage  $\Delta V$ , with only 1.00% loss. Copper wire of what diameter should be used for each of the two conductors of the transmission line? Assume that the current density in the conductors is uniform.

35. A diode is a device that allows current to pass in only one direction (the direction indicated by the arrowhead in its circuit-diagram symbol). Find, in terms of  $\Delta V$  and



$R$ , the average power delivered to the diode circuit shown in Figure P33.35.

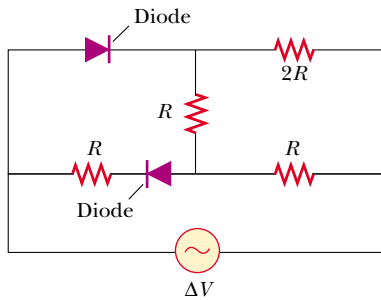


Figure P33.35

### Section 33.7 Resonance in a Series RLC Circuit

36. The tuning circuit of an AM radio contains an  $LC$  combination. The inductance is  $0.200\text{ mH}$ , and the capacitor is variable, so the circuit can resonate at any frequency between  $550\text{ kHz}$  and  $1\,650\text{ kHz}$ . Find the range of values required for  $C$ .
37. An  $RLC$  circuit is used in a radio to tune in to an FM station broadcasting at  $99.7\text{ MHz}$ . The resistance in the circuit is  $12.0\ \Omega$ , and the inductance is  $1.40\ \mu\text{H}$ . What capacitance should be used?
38. A series  $RLC$  circuit has the following values:  $L = 20.0\text{ mH}$ ,  $C = 100\text{ nF}$ ,  $R = 20.0\ \Omega$ , and  $\Delta V_{\text{max}} = 100\text{ V}$ , with  $\Delta v = \Delta V_{\text{max}} \sin \omega t$ . Find (a) the resonant frequency, (b) the amplitude of the current at the resonant frequency, (c) the  $Q$  of the circuit, and (d) the amplitude of the voltage across the inductor at resonance.
39. A  $10.0\text{-}\Omega$  resistor, a  $10.0\text{-mH}$  inductor, and a  $100\text{-}\mu\text{F}$  capacitor are connected in series to a  $50.0\text{-V}$  (rms) source having variable frequency. What is the energy delivered to the circuit during one period if the operating frequency is twice the resonance frequency?
40. A resistor  $R$ , an inductor  $L$ , and a capacitor  $C$  are connected in series to an ac source of rms voltage  $\Delta V$  and variable frequency. What is the energy delivered to the circuit during one period if the operating frequency is twice the resonance frequency?
41. Compute the quality factor for the circuits described in Problems 22 and 23. Which circuit has the sharper resonance?

### Section 33.8 The Transformer and Power Transmission

42. A step-down transformer is used for recharging the batteries of portable devices such as tape players. The turns ratio inside the transformer is  $13:1$ , and it is used with  $120\text{-V}$  (rms) household service. If a particular ideal transformer draws  $0.350\text{ A}$  from the house outlet, what (a) voltage and (b) current are supplied to a tape player from the transformer? (c) How much power is delivered?

43. A transformer has  $N_1 = 350$  turns and  $N_2 = 2\,000$  turns. If the input voltage is  $\Delta v(t) = (170\text{ V}) \cos \omega t$ , what rms voltage is developed across the secondary coil?
44. A step-up transformer is designed to have an output voltage of  $2\,200\text{ V}$  (rms) when the primary is connected across a  $110\text{-V}$  (rms) source. (a) If there are  $80$  turns on the primary winding, how many turns are required on the secondary? (b) If a load resistor across the secondary draws a current of  $1.50\text{ A}$ , what is the current in the primary under ideal conditions? (c) If the transformer actually has an efficiency of  $95.0\%$ , what is the current in the primary when the secondary current is  $1.20\text{ A}$ ?
45. In the transformer shown in Figure P33.45, the load resistor is  $50.0\ \Omega$ . The turns ratio  $N_1:N_2$  is  $5:2$ , and the source voltage is  $80.0\text{ V}$  (rms). If a voltmeter across the load measures  $25.0\text{ V}$  (rms), what is the source resistance  $R_s$ ?

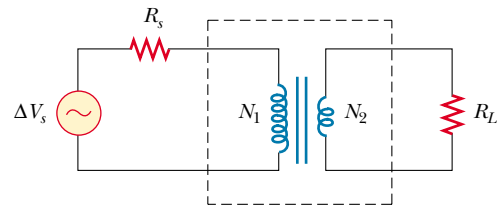


Figure P33.45

46. The secondary voltage of an ignition transformer in a furnace is  $10.0\text{ kV}$ . When the primary operates at an rms voltage of  $120\text{ V}$ , the primary impedance is  $24.0\ \Omega$  and the transformer is  $90.0\%$  efficient. (a) What turns ratio is required? What are (b) the current in the secondary and (c) the impedance in the secondary?
47. A transmission line that has a resistance per unit length of  $4.50 \times 10^{-4}\ \Omega/\text{m}$  is to be used to transmit  $5.00\text{ MW}$  over  $400\text{ mi}$  ( $6.44 \times 10^5\text{ m}$ ). The output voltage of the generator is  $4.50\text{ kV}$ . (a) What is the power loss if a transformer is used to step up the voltage to  $500\text{ kV}$ ? (b) What fraction of the input power is lost to the line under these circumstances? (c) What difficulties would be encountered on attempting to transmit the  $5.00\text{ MW}$  at the generator voltage of  $4.50\text{ kV}$ ?

(Optional)

### Section 33.9 Rectifiers and Filters

48. The  $RC$  low-pass filter shown in Figure 33.23 has a resistance  $R = 90.0\ \Omega$  and a capacitance  $C = 8.00\text{ nF}$ . Calculate the gain ( $\Delta V_{\text{out}}/\Delta V_{\text{in}}$ ) for input frequencies of (a)  $600\text{ Hz}$  and (b)  $600\text{ kHz}$ .
- WEB 49. The  $RC$  high-pass filter shown in Figure 33.22 has a resistance  $R = 0.500\ \Omega$ . (a) What capacitance gives an output signal that has one-half the amplitude of a  $300\text{-Hz}$  input signal? (b) What is the gain ( $\Delta V_{\text{out}}/\Delta V_{\text{in}}$ ) for a  $600\text{-Hz}$  signal?



50. The circuit in Figure P33.50 represents a high-pass filter in which the inductor has internal resistance. What is the source frequency if the output voltage  $\Delta V_2$  is one-half the input voltage?

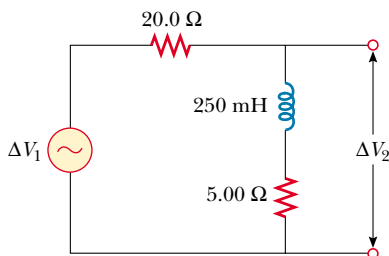


Figure P33.50

51. The resistor in Figure P33.51 represents the midrange speaker in a three-speaker system. Assume that its resistance is constant at  $8.00\ \Omega$ . The source represents an audio amplifier producing signals of uniform amplitude  $\Delta V_{\text{in}} = 10.0\ \text{V}$  at all audio frequencies. The inductor and capacitor are to function as a bandpass filter with  $\Delta V_{\text{out}}/\Delta V_{\text{in}} = \frac{1}{2}$  at 200 Hz and at 4 000 Hz. (a) Determine the required values of  $L$  and  $C$ . (b) Find the maximum value of the gain ratio  $\Delta V_{\text{out}}/\Delta V_{\text{in}}$ . (c) Find the frequency  $f_0$  at which the gain ratio has its maximum value. (d) Find the phase shift between  $\Delta V_{\text{in}}$  and  $\Delta V_{\text{out}}$  at 200 Hz, at  $f_0$ , and at 4 000 Hz. (e) Find the average power transferred to the speaker at 200 Hz, at  $f_0$ , and at 4 000 Hz. (f) Treating the filter as a resonant circuit, find its quality factor.

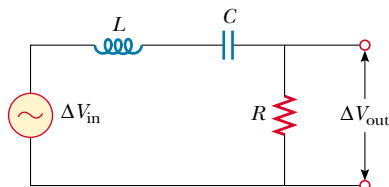


Figure P33.51

52. Show that two successive high-pass filters having the same values of  $R$  and  $C$  give a combined gain

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{1}{1 + (1/\omega RC)^2}$$

53. Consider a low-pass filter followed by a high-pass filter, as shown in Figure P33.53. If  $R = 1\ 000\ \Omega$  and  $C = 0.050\ 0\ \mu\text{F}$ , determine  $\Delta V_{\text{out}}/\Delta V_{\text{in}}$  for a 2.00-kHz input frequency.

### ADDITIONAL PROBLEMS

54. Show that the rms value for the sawtooth voltage shown in Figure P33.54 is  $\Delta V_{\text{max}}/\sqrt{3}$ .

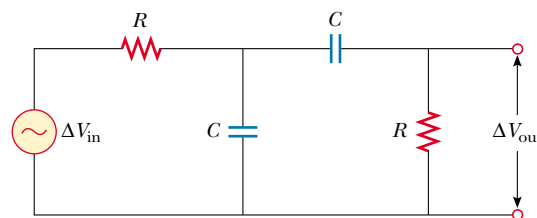


Figure P33.53

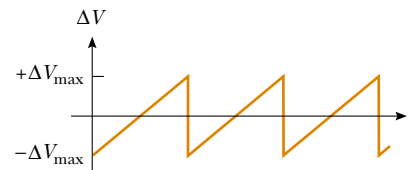


Figure P33.54

55. A series  $RLC$  circuit consists of an  $8.00\text{-}\Omega$  resistor, a  $5.00\text{-}\mu\text{F}$  capacitor, and a  $50.0\text{-mH}$  inductor. A variable frequency source applies an emf of  $400\ \text{V}$  (rms) across the combination. Determine the power delivered to the circuit when the frequency is equal to one-half the resonance frequency.
56. To determine the inductance of a coil used in a research project, a student first connects the coil to a  $12.0\text{-V}$  battery and measures a current of  $0.630\ \text{A}$ . The student then connects the coil to a  $24.0\text{-V}$  (rms),  $60.0\text{-Hz}$  generator and measures an rms current of  $0.570\ \text{A}$ . What is the inductance?
57. In Figure P33.57, find the current delivered by the  $45.0\text{-V}$  (rms) power supply (a) when the frequency is very large and (b) when the frequency is very small.

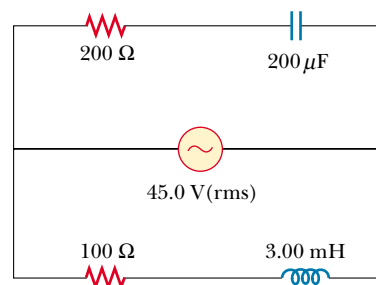
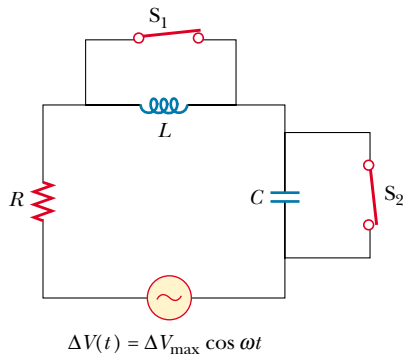


Figure P33.57

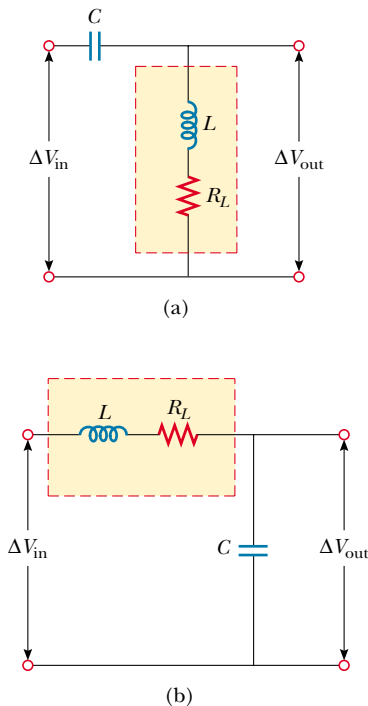
58. In the circuit shown in Figure P33.58, assume that all parameters except  $C$  are given. (a) Find the current as a function of time. (b) Find the power delivered to the circuit. (c) Find the current as a function of time after *only* switch 1 is opened. (d) After switch 2 is *also* opened, the current and voltage are in phase. Find the capacitance  $C$ . (e) Find the impedance of the circuit when both switches are open. (f) Find the maximum

energy stored in the capacitor during oscillations. (g) Find the maximum energy stored in the inductor during oscillations. (h) Now the frequency of the voltage source is doubled. Find the phase difference between the current and the voltage. (i) Find the frequency that makes the inductive reactance one-half the capacitive reactance.



**Figure P33.58**

59. As an alternative to the  $RC$  filters described in Section 33.9,  $LC$  filters are used as both high- and low-pass filters. However, all real inductors have resistance, as indicated in Figure P33.59, and this resistance must be taken into account. (a) Determine which circuit in Figure P33.59 is the high-pass filter and which is the low-pass filter. (b) Derive the output/input ratio for each



**Figure P33.59**

circuit, following the procedure used for the  $RC$  filters in Section 33.9.

60. An  $80.0\text{-}\Omega$  resistor and a  $200\text{-mH}$  inductor are connected in *parallel* across a  $100\text{-V}$  (rms),  $60.0\text{-Hz}$  source. (a) What is the rms current in the resistor? (b) By what angle does the total current lead or lag behind the voltage?
61. Make an order-of-magnitude estimate of the electric current that the electric company delivers to a town from a remote generating station. State the data that you measure or estimate. If you wish, you may consider a suburban bedroom community of 20 000 people.
62. A voltage  $\Delta v = (100\text{ V}) \sin \omega t$  (in SI units) is applied across a series combination of a  $2.00\text{-H}$  inductor, a  $10.0\text{-}\mu\text{F}$  capacitor, and a  $10.0\text{-}\Omega$  resistor. (a) Determine the angular frequency  $\omega_0$  at which the power delivered to the resistor is a maximum. (b) Calculate the power at that frequency. (c) Determine the two angular frequencies  $\omega_1$  and  $\omega_2$  at which the power delivered is one-half the maximum value. [The  $Q$  of the circuit is approximately  $\omega_0/(\omega_2 - \omega_1)$ .]
63. Consider a series  $RLC$  circuit having the following circuit parameters:  $R = 200\ \Omega$ ,  $L = 663\ \text{mH}$ , and  $C = 26.5\ \mu\text{F}$ . The applied voltage has an amplitude of  $50.0\ \text{V}$  and a frequency of  $60.0\ \text{Hz}$ . Find the following: (a) the current  $I_{\max}$ , including its phase constant  $\phi$  relative to the applied voltage  $\Delta v$ ; (b) the voltage  $\Delta V_R$  across the resistor and its phase relative to the current; (c) the voltage  $\Delta V_C$  across the capacitor and its phase relative to the current; and (d) the voltage  $\Delta V_L$  across the inductor and its phase relative to the current.
64. A power supply with  $\Delta V_{\text{rms}} = 120\ \text{V}$  is connected between points  $a$  and  $d$  in Figure P33.25. At what frequency will it deliver a power of  $250\ \text{W}$ ?
65. Example 28.2 showed that maximum power is transferred when the internal resistance of a dc source is equal to the resistance of the load. A transformer may be used to provide maximum power transfer between two ac circuits that have different impedances. (a) Show that the ratio of turns  $N_1/N_2$  needed to meet this condition is

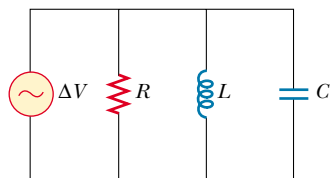
$$\frac{N_1}{N_2} = \sqrt{\frac{Z_1}{Z_2}}$$

- (b) Suppose you want to use a transformer as an impedance-matching device between an audio amplifier that has an output impedance of  $8.00\ \text{k}\Omega$  and a speaker that has an input impedance of  $8.00\ \Omega$ . What should your  $N_1/N_2$  ratio be?
66. Figure P33.66a shows a parallel  $RLC$  circuit, and the corresponding phasor diagram is provided in Figure P33.66b. The instantaneous voltages and rms voltages across the three circuit elements are the same, and each is in phase with the current through the resistor. The currents in  $C$  and  $L$  lead or lag behind the current in the resistor, as shown in Figure P33.66b. (a) Show that the rms current delivered by the source is

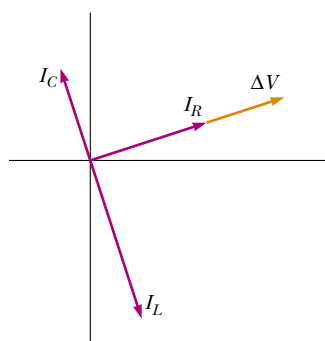
$$I_{\text{rms}} = \Delta V_{\text{rms}} \left[ \frac{1}{R^2} + \left( \omega C - \frac{1}{\omega L} \right)^2 \right]^{1/2}$$

(b) Show that the phase angle  $\phi$  between  $\Delta V_{\text{rms}}$  and  $I_{\text{rms}}$  is

$$\tan \phi = R \left( \frac{1}{X_C} - \frac{1}{X_L} \right)$$



(a)



(b)

Figure P33.66

67. An  $80.0\text{-}\Omega$  resistor, a  $200\text{-mH}$  inductor, and a  $0.150\text{-}\mu\text{F}$  capacitor are connected *in parallel* across a  $120\text{-V}$  (rms) source operating at  $374\text{ rad/s}$ . (a) What is the resonant frequency of the circuit? (b) Calculate the rms current in the resistor, the inductor, and the capacitor. (c) What

rms current is delivered by the source? (d) Is the current leading or lagging behind the voltage? By what angle?

68. Consider the phase-shifter circuit shown in Figure P33.68. The input voltage is described by the expression  $\Delta v = (10.0\text{ V}) \sin 200t$  (in SI units). Assuming that  $L = 500\text{ mH}$ , find (a) the value of  $R$  such that the output voltage lags behind the input voltage by  $30.0^\circ$  and (b) the amplitude of the output voltage.

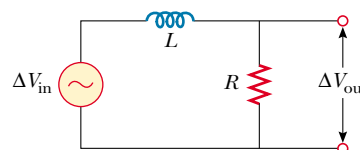


Figure P33.68

69. A series  $RLC$  circuit is operating at  $2\,000\text{ Hz}$ . At this frequency,  $X_L = X_C = 1\,884\ \Omega$ . The resistance of the circuit is  $40.0\ \Omega$ . (a) Prepare a table showing the values of  $X_L$ ,  $X_C$ , and  $Z$  for  $f = 300, 600, 800, 1\,000, 1\,500, 2\,000, 3\,000, 4\,000, 6\,000,$  and  $10\,000\text{ Hz}$ . (b) Plot on the same set of axes  $X_L$ ,  $X_C$ , and  $Z$  as functions of  $\ln f$ .
70. A series  $RLC$  circuit in which  $R = 1.00\ \Omega$ ,  $L = 1.00\text{ mH}$ , and  $C = 1.00\text{ nF}$  is connected to an ac generator delivering  $1.00\text{ V}$  (rms). Make a precise graph of the power delivered to the circuit as a function of the frequency, and verify that the full width of the resonance peak at half-maximum is  $R/2\pi L$ .
71. Suppose the high-pass filter shown in Figure 33.22 has  $R = 1\,000\ \Omega$  and  $C = 0.050\,0\ \mu\text{F}$ . (a) At what frequency does  $\Delta V_{\text{out}}/\Delta V_{\text{in}} = \frac{1}{2}$ ? (b) Plot  $\log_{10}(\Delta V_{\text{out}}/\Delta V_{\text{in}})$  versus  $\log_{10}(f)$  over the frequency range from  $1\text{ Hz}$  to  $1\text{ MHz}$ . (This log-log plot of gain versus frequency is known as a *Bode plot*.)

## ANSWERS TO QUICK QUIZZES

- 33.1 (c)  $\mathcal{P}_{\text{av}} > 0$  and  $i_{\text{av}} = 0$ . The average power is proportional to the rms current—which, as Figure 33.3 shows, is nonzero even though the average current is zero. Condition (a) is valid only for an open circuit, and conditions (b) and (d) can never be true because  $i_{\text{av}} = 0$  for ac circuits even though  $i_{\text{rms}} > 0$ .
- 33.2 (b) Sum of instantaneous voltages across elements. Choices (a) and (c) are incorrect because the unaligned sine curves in Figure 33.9b mean that the voltages are out of phase, so we cannot simply add the maximum (or rms) voltages across the elements. (In other words,  $\Delta V \neq \Delta V_R + \Delta V_L + \Delta V_C$  even though it is true that  $\Delta v = \Delta v_R + \Delta v_L + \Delta v_C$ .)
- 33.3 (a)  $X_L < X_C$ . (b)  $X_L = X_C$ . (c)  $X_L > X_C$ .
- 33.4 Equation 33.23 indicates that at resonance (when  $X_L = X_C$ ) the impedance is due strictly to the resistor,  $Z = R$ . At resonance, the current is given by the expression  $I_{\text{rms}} = \Delta V_{\text{rms}}/R$ .
- 33.5 High. The higher the quality factor, the more sensitive the detector. As you can see from Figure 33.15a, when  $Q = \omega_0/\Delta\omega$  is high, as it is in the  $R = 3.5\ \Omega$  case, a slight change in the resonance frequency (as might happen when a small piece of metal passes through the portal) causes a large change in current that can be detected easily.
- 33.6 (a) An inductor. The current in an inductive circuit decreases with increasing frequency (see Eq. 33.9). Thus, an inductor connected in series with a woofer blocks high-frequency signals and passes low-frequency signals. (b) A capacitor. The current in a capacitive circuit decreases with decreasing frequency (see Eq. 33.16). When a capacitor is connected in series with a tweeter, the capacitor blocks low-frequency signals and passes high-frequency signals.