PUZZLER

The wristwatches worn by the people in this commercial jetliner properly record the passage of time as experienced by the travelers. Amazingly, however, the duration of the trip as measured by an Earth-bound observer is very slightly longer. How can high-speed travel affect something as regular as the ticking of a clock? *(© Larry Mulvehill/Photo Researchers, Inc.)*

chapter

39 Relativity

Chapter Outline

Most of our everyday experiences and observations have to do with objects that move at speeds much less than the speed of light. Newtonian mechanics was formulated to describe the motion of such objects, and this formalism is still very successful in describing a wide range of phenomena that occur at low speeds. It fails, however, when applied to particles whose speeds approach that of light.

Experimentally, the predictions of Newtonian theory can be tested at high speeds by accelerating electrons or other charged particles through a large electric potential difference. For example, it is possible to accelerate an electron to a speed of $0.99c$ (where c is the speed of light) by using a potential difference of several million volts. According to Newtonian mechanics, if the potential difference is increased by a factor of 4, the electron's kinetic energy is four times greater and its speed should double to $1.98c$. However, experiments show that the speed of the electron—as well as the speed of any other particle in the Universe—always remains less than the speed of light, regardless of the size of the accelerating voltage. Because it places no upper limit on speed, Newtonian mechanics is contrary to modern experimental results and is clearly a limited theory.

In 1905, at the age of only 26, Einstein published his special theory of relativity. Regarding the theory, Einstein wrote:

> The relativity theory arose from necessity, from serious and deep contradictions in the old theory from which there seemed no escape. The strength of the new theory lies in the consistency and simplicity with which it solves all these difficulties . . . . [1]

Although Einstein made many other important contributions to science, the special theory of relativity alone represents one of the greatest intellectual achievements of all time. With this theory, experimental observations can be correctly predicted over the range of speeds from $v = 0$ to speeds approaching the speed of light. At low speeds, Einstein's theory reduces to Newtonian mechanics as a limiting situation. It is important to recognize that Einstein was working on electromagnetism when he developed the special theory of relativity. He was convinced that Maxwell's equations were correct, and in order to reconcile them with one of his postulates, he was forced into the bizarre notion of assuming that space and time are not absolute.

This chapter gives an introduction to the special theory of relativity, with emphasis on some of its consequences. The special theory covers phenomena such as the slowing down of clocks and the contraction of lengths in moving reference frames as measured by a stationary observer. We also discuss the relativistic forms of momentum and energy, as well as some consequences of the famous mass–energy formula, $E = mc^2$.

In addition to its well-known and essential role in theoretical physics, the special theory of relativity has practical applications, including the design of nuclear power plants and modern global positioning system (GPS) units. These devices do not work if designed in accordance with nonrelativistic principles.

We shall have occasion to use relativity in some subsequent chapters of the extended version of this text, most often presenting only the outcome of relativistic effects.

[1] A. Einstein and L. Infeld, *The Evolution of Physics,* New York, Simon and Schuster, 1961.

39.1 THE PRINCIPLE OF GALILEAN RELATIVITY

To describe a physical event, it is necessary to establish a frame of reference. You should recall from Chapter 5 that Newton's laws are valid in all inertial frames of reference. Because an inertial frame is defined as one in which Newton's first law is valid, we can say that **an inertial frame of reference is one in which an object is observed to have no acceleration when no forces act on it.** Furthermore, any system moving with constant velocity with respect to an inertial system must also be an inertial system.

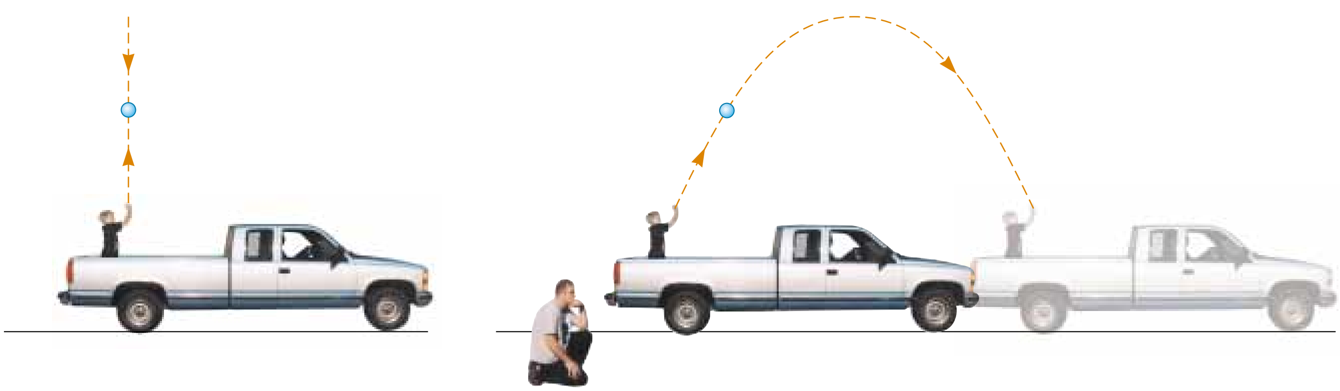
Inertial frame of reference

There is no preferred inertial reference frame. This means that the results of an experiment performed in a vehicle moving with uniform velocity will be identical to the results of the same experiment performed in a stationary vehicle. The formal statement of this result is called the **principle of Galilean relativity:**

The laws of mechanics must be the same in all inertial frames of reference.

Let us consider an observation that illustrates the equivalence of the laws of mechanics in different inertial frames. A pickup truck moves with a constant velocity, as shown in Figure 39.1a. If a passenger in the truck throws a ball straight up, and if air effects are neglected, the passenger observes that the ball moves in a vertical path. The motion of the ball appears to be precisely the same as if the ball were thrown by a person at rest on the Earth. The law of gravity and the equations of motion under constant acceleration are obeyed whether the truck is at rest or in uniform motion.

Now consider the same situation viewed by an observer at rest on the Earth. This stationary observer sees the path of the ball as a parabola, as illustrated in Figure 39.1b. Furthermore, according to this observer, the ball has a horizontal component of velocity equal to the velocity of the truck. Although the two observers disagree on certain aspects of the situation, they agree on the validity of Newton's laws and on such classical principles as conservation of energy and conservation of linear momentum. This agreement implies that no mechanical experiment can detect any difference between the two inertial frames. The only thing that can be detected is the relative motion of one frame with respect to the other. That is, the notion of absolute motion through space is meaningless, as is the notion of a preferred reference frame.



(a) (b)

Figure 39.1 (a) The observer in the truck sees the ball move in a vertical path when thrown upward. (b) The Earth observer sees the path of the ball as a parabola.

Quick Quiz 39.1

Which observer in Figure 39.1 is right about the ball's path?

Suppose that some physical phenomenon, which we call an *event,* occurs in an inertial system. The event's location and time of occurrence can be specified by the four coordinates (x, y, z, t). We would like to be able to transform these coordinates from one inertial system to another one moving with uniform relative velocity.

Consider two inertial systems S and S′ (Fig. 39.2). The system S′ moves with a constant velocity $\mathbf{v}$ along the xx' axes, where $\mathbf{v}$ is measured relative to S. We assume that an event occurs at the point P and that the origins of S and S′ coincide at $t = 0$. An observer in S describes the event with space–time coordinates (x, y, z, t), whereas an observer in S′ uses the coordinates (x', y', z', t') to describe the same event. As we see from Figure 39.2, the relationships between these various coordinates can be written

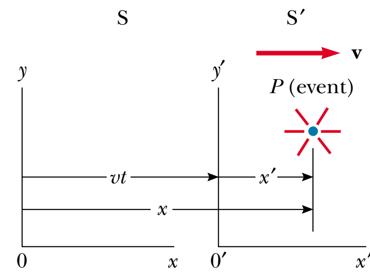


Figure 39.2 An event occurs at a point P. The event is seen by two observers in inertial frames S and S′, where S′ moves with a velocity $\mathbf{v}$ relative to S.

$$x' = x - vt$$
$$y' = y \qquad \textbf{(39.1)}$$
$$z' = z$$
$$t' = t$$

Galilean space–time transformation equations

These equations are the **Galilean space–time transformation equations.** Note that time is assumed to be the same in both inertial systems. That is, within the framework of classical mechanics, all clocks run at the same rate, regardless of their velocity, so that the time at which an event occurs for an observer in S is the same as the time for the same event in S′. Consequently, the time interval between two successive events should be the same for both observers. Although this assumption may seem obvious, it turns out to be incorrect in situations where v is comparable to the speed of light.

Now suppose that a particle moves a distance dx in a time interval dt as measured by an observer in S. It follows from Equations 39.1 that the corresponding distance dx' measured by an observer in S′ is $dx' = dx - v\,dt$, where frame S′ is moving with speed v relative to frame S. Because $dt = dt'$, we find that

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

or

$$u'_x = u_x - v \qquad \textbf{(39.2)}$$

Galilean velocity transformation equation

where u_x and u'_x are the x components of the velocity relative to S and S′, respectively. (We use the symbol $\mathbf{u}$ for particle velocity rather than $\mathbf{v}$, which is used for the relative velocity of two reference frames.) This is the **Galilean velocity transformation equation.** It is used in everyday observations and is consistent with our intuitive notion of time and space. As we shall soon see, however, it leads to serious contradictions when applied to electromagnetic waves.

Quick Quiz 39.2

Applying the Galilean velocity transformation equation, determine how fast (relative to the Earth) a baseball pitcher with a 90-mi/h fastball can throw a ball while standing in a boxcar moving at 110 mi/h.

The Speed of Light

It is quite natural to ask whether the principle of Galilean relativity also applies to electricity, magnetism, and optics. Experiments indicate that the answer is no. Recall from Chapter 34 that Maxwell showed that the speed of light in free space is $c = 3.00 \times 10^8$ m/s. Physicists of the late 1800s thought that light waves moved through a medium called the *ether* and that the speed of light was c only in a special, absolute frame at rest with respect to the ether. The Galilean velocity transformation equation was expected to hold in any frame moving at speed v relative to the absolute ether frame.

Because the existence of a preferred, absolute ether frame would show that light was similar to other classical waves and that Newtonian ideas of an absolute frame were true, considerable importance was attached to establishing the existence of the ether frame. Prior to the late 1800s, experiments involving light traveling in media moving at the highest laboratory speeds attainable at that time were not capable of detecting changes as small as $c \pm v$. Starting in about 1880, scientists decided to use the Earth as the moving frame in an attempt to improve their chances of detecting these small changes in the speed of light.

As observers fixed on the Earth, we can say that we are stationary and that the absolute ether frame containing the medium for light propagation moves past us with speed v. Determining the speed of light under these circumstances is just like determining the speed of an aircraft traveling in a moving air current, or wind; consequently, we speak of an "ether wind" blowing through our apparatus fixed to the Earth.

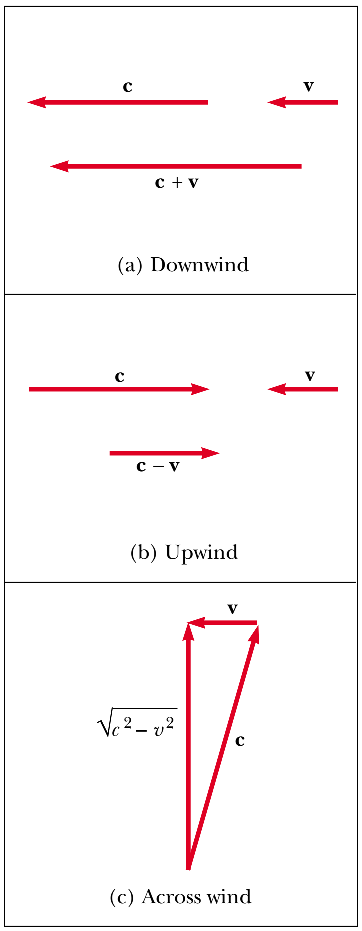


Figure 39.3 If the velocity of the ether wind relative to the Earth is **v** and the velocity of light relative to the ether is **c**, then the speed of light relative to the Earth is (a) $c + v$ in the downwind direction, (b) $c - v$ in the upwind direction, and (c) $(c^2 - v^2)^{1/2}$ in the direction perpendicular to the wind.

A direct method for detecting an ether wind would use an apparatus fixed to the Earth to measure the wind's influence on the speed of light. If v is the speed of the ether relative to the Earth, then the speed of light should have its maximum value, $c + v$, when propagating downwind, as shown in Figure 39.3a. Likewise, the speed of light should have its minimum value, $c - v$, when propagating upwind, as shown in Figure 39.3b, and an intermediate value, $(c^2 - v^2)^{1/2}$, in the direction perpendicular to the ether wind, as shown in Figure 39.3c. If the Sun is assumed to be at rest in the ether, then the velocity of the ether wind would be equal to the orbital velocity of the Earth around the Sun, which has a magnitude of approximately 3×10^4 m/s. Because $c = 3 \times 10^8$ m/s, it should be possible to detect a change in speed of about 1 part in 10^4 for measurements in the upwind or downwind directions. However, as we shall see in the next section, all attempts to detect such changes and establish the existence of the ether wind (and hence the absolute frame) proved futile! (You may want to return to Problem 40 in Chapter 4 to see a situation in which the Galilean velocity transformation equation does hold.)

If it is assumed that the laws of electricity and magnetism are the same in all inertial frames, a paradox concerning the speed of light immediately arises. We can understand this by recognizing that Maxwell's equations seem to imply that the speed of light always has the fixed value 3.00×10^8 m/s in all inertial frames, a result in direct contradiction to what is expected based on the Galilean velocity transformation equation. According to Galilean relativity, the speed of light should not be the same in all inertial frames.

For example, suppose a light pulse is sent out by an observer S′ standing in a boxcar moving with a velocity **v** relative to a stationary observer standing alongside the track (Fig. 39.4). The light pulse has a speed c relative to S′. According to Galilean relativity, the pulse speed relative to S should be $c + v$. This is in contradiction to Einstein's special theory of relativity, which, as we shall see, postulates that the speed of the pulse is the same for all observers.

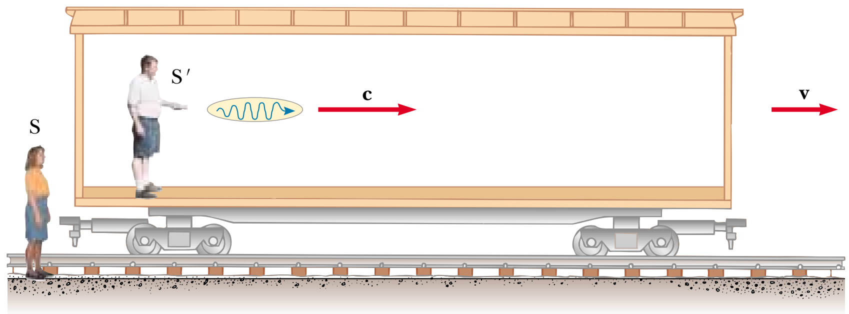


Figure 39.4 A pulse of light is sent out by a person in a moving boxcar. According to Galilean relativity, the speed of the pulse should be $c + v$ relative to a stationary observer.

To resolve this contradiction in theories, we must conclude that either (1) the laws of electricity and magnetism are not the same in all inertial frames or (2) the Galilean velocity transformation equation is incorrect. If we assume the first alternative, then a preferred reference frame in which the speed of light has the value c must exist and the measured speed must be greater or less than this value in any other reference frame, in accordance with the Galilean velocity transformation equation. If we assume the second alternative, then we are forced to abandon the notions of absolute time and absolute length that form the basis of the Galilean space–time transformation equations.

39.2 THE MICHELSON–MORLEY EXPERIMENT

The most famous experiment designed to detect small changes in the speed of light was first performed in 1881 by Albert A. Michelson (see Section 37.7) and later repeated under various conditions by Michelson and Edward W. Morley (1838–1923). We state at the outset that the outcome of the experiment contradicted the ether hypothesis.

The experiment was designed to determine the velocity of the Earth relative to that of the hypothetical ether. The experimental tool used was the Michelson interferometer, which was discussed in Section 37.7 and is shown again in Figure 39.5. Arm 2 is aligned along the direction of the Earth's motion through space. The Earth moving through the ether at speed v is equivalent to the ether flowing past the Earth in the opposite direction with speed v. This ether wind blowing in the direction opposite the direction of Earth's motion should cause the speed of light measured in the Earth frame to be $c - v$ as the light approaches mirror M_2 and $c + v$ after reflection, where c is the speed of light in the ether frame.

The two beams reflected from M_1 and M_2 recombine, and an interference pattern consisting of alternating dark and bright fringes is formed. The interference pattern was observed while the interferometer was rotated through an angle of 90°. This rotation supposedly would change the speed of the ether wind along the arms of the interferometer. The rotation should have caused the fringe pattern to shift slightly but measurably, but measurements failed to show any change in the interference pattern! The Michelson–Morley experiment was repeated at different times of the year when the ether wind was expected to change direction

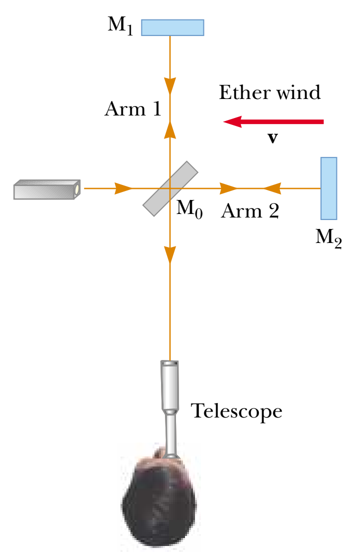
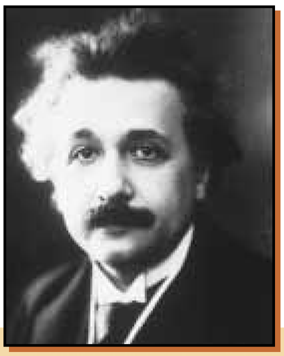


Figure 39.5 According to the ether wind theory, the speed of light should be $c - v$ as the beam approaches mirror M_2 and $c + v$ after reflection.

and magnitude, but the results were always the same: **no fringe shift of the magnitude required was ever observed.**[2]

The negative results of the Michelson–Morley experiment not only contradicted the ether hypothesis but also showed that it was impossible to measure the absolute velocity of the Earth with respect to the ether frame. However, as we shall see in the next section, Einstein offered a postulate for his special theory of relativity that places quite a different interpretation on these null results. In later years, when more was known about the nature of light, the idea of an ether that permeates all of space was relegated to the ash heap of worn-out concepts. **Light is now understood to be an electromagnetic wave, which requires no medium for its propagation.** As a result, the idea of an ether in which these waves could travel became unnecessary.



Albert Einstein (1879–1955)
Einstein, one of the greatest physicists of all times, was born in Ulm, Germany. In 1905, at the age of 26, he published four scientific papers that revolutionized physics. Two of these papers were concerned with what is now considered his most important contribution: the special theory of relativity.

In 1916, Einstein published his work on the general theory of relativity. The most dramatic prediction of this theory is the degree to which light is deflected by a gravitational field. Measurements made by astronomers on bright stars in the vicinity of the eclipsed Sun in 1919 confirmed Einstein's prediction, and as a result Einstein became a world celebrity.

Einstein was deeply disturbed by the development of quantum mechanics in the 1920s despite his own role as a scientific revolutionary. In particular, he could never accept the probabilistic view of events in nature that is a central feature of quantum theory. The last few decades of his life were devoted to an unsuccessful search for a unified theory that would combine gravitation and electromagnetism. *(AIP Niels Bohr Library)*

Optional Section

Details of the Michelson–Morley Experiment

To understand the outcome of the Michelson–Morley experiment, let us assume that the two arms of the interferometer in Figure 39.5 are of equal length L. We shall analyze the situation as if there were an ether wind, because that is what Michelson and Morley expected to find. As noted above, the speed of the light beam along arm 2 should be $c - v$ as the beam approaches M_2 and $c + v$ after the beam is reflected. Thus, the time of travel to the right is $L/(c - v)$, and the time of travel to the left is $L/(c + v)$. The total time needed for the round trip along arm 2 is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2Lc}{c^2 - v^2} = \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1}$$

Now consider the light beam traveling along arm 1, perpendicular to the ether wind. Because the speed of the beam relative to the Earth is $(c^2 - v^2)^{1/2}$ in this case (see Fig. 39.3), the time of travel for each half of the trip is $L/(c^2 - v^2)^{1/2}$, and the total time of travel for the round trip is

$$t_2 = \frac{2L}{(c^2 - v^2)^{1/2}} = \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

Thus, the time difference between the horizontal round trip (arm 2) and the vertical round trip (arm 1) is

$$\Delta t = t_1 - t_2 = \frac{2L}{c}\left[\left(1 - \frac{v^2}{c^2}\right)^{-1} - \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\right]$$

Because $v^2/c^2 \ll 1$, we can simplify this expression by using the following binomial expansion after dropping all terms higher than second order:

$$(1 - x)^n \approx 1 - nx \qquad \text{for } x \ll 1$$

In our case, $x = v^2/c^2$, and we find that

$$\Delta t = t_1 - t_2 \approx \frac{Lv^2}{c^3} \qquad \textbf{(39.3)}$$

This time difference between the two instants at which the reflected beams arrive at the viewing telescope gives rise to a phase difference between the beams,

[2] From an Earth observer's point of view, changes in the Earth's speed and direction of motion in the course of a year are viewed as ether wind shifts. Even if the speed of the Earth with respect to the ether were zero at some time, six months later the speed of the Earth would be 60 km/s with respect to the ether, and as a result a fringe shift should be noticed. No shift has ever been observed, however.

producing an interference pattern when they combine at the position of the telescope. A shift in the interference pattern should be detected when the interferometer is rotated through 90° in a horizontal plane, so that the two beams exchange roles. This results in a time difference twice that given by Equation 39.3. Thus, the path difference that corresponds to this time difference is

$$\Delta d = c(2\,\Delta t) = \frac{2Lv^2}{c^2}$$

Because a change in path length of one wavelength corresponds to a shift of one fringe, the corresponding fringe shift is equal to this path difference divided by the wavelength of the light:

$$\text{Shift} = \frac{2Lv^2}{\lambda c^2} \tag{39.4}$$

In the experiments by Michelson and Morley, each light beam was reflected by mirrors many times to give an effective path length L of approximately 11 m. Using this value and taking v to be equal to 3.0×10^4 m/s, the speed of the Earth around the Sun, we obtain a path difference of

$$\Delta d = \frac{2(11\text{ m})(3.0 \times 10^4\text{ m/s})^2}{(3.0 \times 10^8\text{ m/s})^2} = 2.2 \times 10^{-7}\text{ m}$$

This extra travel distance should produce a noticeable shift in the fringe pattern. Specifically, using 500-nm light, we expect a fringe shift for rotation through 90° of

$$\text{Shift} = \frac{\Delta d}{\lambda} = \frac{2.2 \times 10^{-7}\text{ m}}{5.0 \times 10^{-7}\text{ m}} \approx 0.44$$

The instrument used by Michelson and Morley could detect shifts as small as 0.01 fringe. However, **it detected no shift whatsoever in the fringe pattern.** Since then, the experiment has been repeated many times by different scientists under a wide variety of conditions, and no fringe shift has ever been detected. Thus, it was concluded that the motion of the Earth with respect to the postulated ether cannot be detected.

Many efforts were made to explain the null results of the Michelson–Morley experiment and to save the ether frame concept and the Galilean velocity transformation equation for light. All proposals resulting from these efforts have been shown to be wrong. No experiment in the history of physics received such valiant efforts to explain the absence of an expected result as did the Michelson–Morley experiment. The stage was set for Einstein, who solved the problem in 1905 with his special theory of relativity.

39.3 EINSTEIN'S PRINCIPLE OF RELATIVITY

In the previous section we noted the impossibility of measuring the speed of the ether with respect to the Earth and the failure of the Galilean velocity transformation equation in the case of light. Einstein proposed a theory that boldly removed these difficulties and at the same time completely altered our notion of space and time.[3] He based his special theory of relativity on two postulates:

[3] A. Einstein, "On the Electrodynamics of Moving Bodies," *Ann. Physik* 17:891, 1905. For an English translation of this article and other publications by Einstein, see the book by H. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, *The Principle of Relativity,* Dover, 1958.

The postulates of the special theory of relativity

1. **The principle of relativity:** The laws of physics must be the same in all inertial reference frames.
2. **The constancy of the speed of light:** The speed of light in vacuum has the same value, $c = 3.00 \times 10^8$ m/s, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

The first postulate asserts that *all* the laws of physics—those dealing with mechanics, electricity and magnetism, optics, thermodynamics, and so on—are the same in all reference frames moving with constant velocity relative to one another. This postulate is a sweeping generalization of the principle of Galilean relativity, which refers only to the laws of mechanics. From an experimental point of view, Einstein's principle of relativity means that any kind of experiment (measuring the speed of light, for example) performed in a laboratory at rest must give the same result when performed in a laboratory moving at a constant velocity past the first one. Hence, no preferred inertial reference frame exists, and it is impossible to detect absolute motion.

Note that postulate 2 is required by postulate 1: If the speed of light were not the same in all inertial frames, measurements of different speeds would make it possible to distinguish between inertial frames; as a result, a preferred, absolute frame could be identified, in contradiction to postulate 1.

Although the Michelson–Morley experiment was performed before Einstein published his work on relativity, it is not clear whether or not Einstein was aware of the details of the experiment. Nonetheless, the null result of the experiment can be readily understood within the framework of Einstein's theory. According to his principle of relativity, the premises of the Michelson–Morley experiment were incorrect. In the process of trying to explain the expected results, we stated that when light traveled against the ether wind its speed was $c - v$, in accordance with the Galilean velocity transformation equation. However, if the state of motion of the observer or of the source has no influence on the value found for the speed of light, one always measures the value to be c. Likewise, the light makes the return trip after reflection from the mirror at speed c, not at speed $c + v$. Thus, the motion of the Earth does not influence the fringe pattern observed in the Michelson–Morley experiment, and a null result should be expected.

If we accept Einstein's theory of relativity, we must conclude that relative motion is unimportant when measuring the speed of light. At the same time, we shall see that we must alter our common-sense notion of space and time and be prepared for some bizarre consequences. It may help as you read the pages ahead to keep in mind that our common-sense ideas are based on a lifetime of everyday experiences and not on observations of objects moving at hundreds of thousands of kilometers per second.

39.4 CONSEQUENCES OF THE SPECIAL THEORY OF RELATIVITY

Before we discuss the consequences of Einstein's special theory of relativity, we must first understand how an observer located in an inertial reference frame describes an event. As mentioned earlier, an event is an occurrence describable by three space coordinates and one time coordinate. Different observers in different inertial frames usually describe the same event with different coordinates.

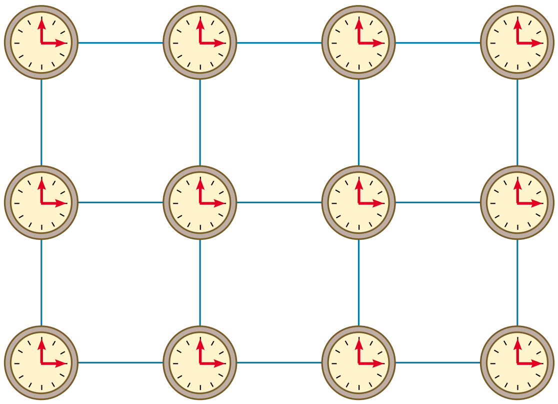


Figure 39.6 In studying relativity, we use a reference frame consisting of a coordinate grid and a set of synchronized clocks.

The reference frame used to describe an event consists of a coordinate grid and a set of synchronized clocks located at the grid intersections, as shown in Figure 39.6 in two dimensions. The clocks can be synchronized in many ways with the help of light signals. For example, suppose an observer is located at the origin with a master clock and sends out a pulse of light at $t = 0$. The pulse takes a time r/c to reach a clock located a distance r from the origin. Hence, this clock is synchronized with the master clock if this clock reads r/c at the instant the pulse reaches it. This procedure of synchronization assumes that the speed of light has the same value in all directions and in all inertial frames. Furthermore, the procedure concerns an event recorded by an observer in a specific inertial reference frame. An observer in some other inertial frame would assign different space–time coordinates to events being observed by using another coordinate grid and another array of clocks.

As we examine some of the consequences of relativity in the remainder of this section, we restrict our discussion to the concepts of simultaneity, time, and length, all three of which are quite different in relativistic mechanics from what they are in Newtonian mechanics. For example, in relativistic mechanics the distance between two points and the time interval between two events depend on the frame of reference in which they are measured. That is, **in relativistic mechanics there is no such thing as absolute length or absolute time.** Furthermore, **events at different locations that are observed to occur simultaneously in one frame are not observed to be simultaneous in another frame moving uniformly past the first.**

Simultaneity and the Relativity of Time

A basic premise of Newtonian mechanics is that a universal time scale exists that is the same for all observers. In fact, Newton wrote that "Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external." Thus, Newton and his followers simply took simultaneity for granted. In his special theory of relativity, Einstein abandoned this assumption.

Einstein devised the following thought experiment to illustrate this point. A boxcar moves with uniform velocity, and two lightning bolts strike its ends, as illustrated in Figure 39.7a, leaving marks on the boxcar and on the ground. The marks on the boxcar are labeled A' and B', and those on the ground are labeled A and B. An observer O' moving with the boxcar is midway between A' and B', and a ground observer O is midway between A and B. The events recorded by the observers are the striking of the boxcar by the two lightning bolts.

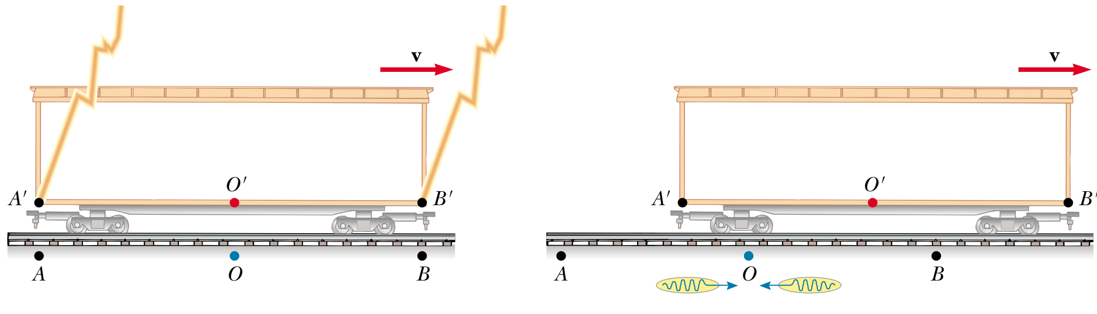


Figure 39.7 (a) Two lightning bolts strike the ends of a moving boxcar. (b) The events appear to be simultaneous to the stationary observer O, standing midway between A and B. The events do not appear to be simultaneous to observer O', who claims that the front of the car is struck before the rear. Note that in (b) the leftward-traveling light signal has already passed O' but the rightward-traveling signal has not yet reached O'.

The light signals recording the instant at which the two bolts strike reach observer O at the same time, as indicated in Figure 39.7b. This observer realizes that the signals have traveled at the same speed over equal distances, and so rightly concludes that the events at A and B occurred simultaneously. Now consider the same events as viewed by observer O'. By the time the signals have reached observer O, observer O' has moved as indicated in Figure 39.7b. Thus, the signal from B' has already swept past O', but the signal from A' has not yet reached O'. In other words, O' sees the signal from B' before seeing the signal from A'. According to Einstein, *the two observers must find that light travels at the same speed.* Therefore, observer O' concludes that the lightning strikes the front of the boxcar before it strikes the back.

This thought experiment clearly demonstrates that the two events that appear to be simultaneous to observer O do not appear to be simultaneous to observer O'. In other words,

two events that are simultaneous in one reference frame are in general not simultaneous in a second frame moving relative to the first. That is, simultaneity is not an absolute concept but rather one that depends on the state of motion of the observer.

Quick Quiz 39.3

Which observer in Figure 39.7 is correct?

The central point of relativity is this: Any inertial frame of reference can be used to describe events and do physics. **There is no preferred inertial frame of reference.** However, observers in different inertial frames always measure different time intervals with their clocks and different distances with their meter sticks. Nevertheless, all observers agree on the forms of the laws of physics in their respective frames because these laws must be the same for all observers in uniform motion. For example, the relationship $F = ma$ in a frame S has the same form $F' = ma'$ in a frame S′ that is moving at constant velocity relative to frame S. It is the alteration of time and space that allows the laws of physics (including Maxwell's equations) to be the same for all observers in uniform motion.

Time Dilation

We can illustrate the fact that observers in different inertial frames always measure different time intervals between a pair of events by considering a vehicle moving to the right with a speed v, as shown in Figure 39.8a. A mirror is fixed to the ceiling of the vehicle, and observer O' at rest in this system holds a laser a distance d below the mirror. At some instant, the laser emits a pulse of light directed toward the mirror (event 1), and at some later time after reflecting from the mirror, the pulse arrives back at the laser (event 2). Observer O' carries a clock C' and uses it to measure the time interval Δt_p between these two events. (The subscript p stands for *proper,* as we shall see in a moment.) Because the light pulse has a speed c, the time it takes the pulse to travel from O' to the mirror and back to O' is

$$\Delta t_p = \frac{\text{Distance traveled}}{\text{Speed}} = \frac{2d}{c} \tag{39.5}$$

This time interval Δt_p measured by O' requires only a single clock C' located at the same place as the laser in this frame.

Now consider the same pair of events as viewed by observer O in a second frame, as shown in Figure 39.8b. According to this observer, the mirror and laser are moving to the right with a speed v, and as a result the sequence of events appears entirely different. By the time the light from the laser reaches the mirror, the mirror has moved to the right a distance $v\,\Delta t/2$, where Δt is the time it takes the light to travel from O' to the mirror and back to O' as measured by O. In other words, O concludes that, because of the motion of the vehicle, if the light is to hit the mirror, it must leave the laser at an angle with respect to the vertical direction. Comparing Figure 39.8a and b, we see that the light must travel farther in (b) than in (a). (Note that neither observer "knows" that he or she is moving. Each is at rest in his or her own inertial frame.)

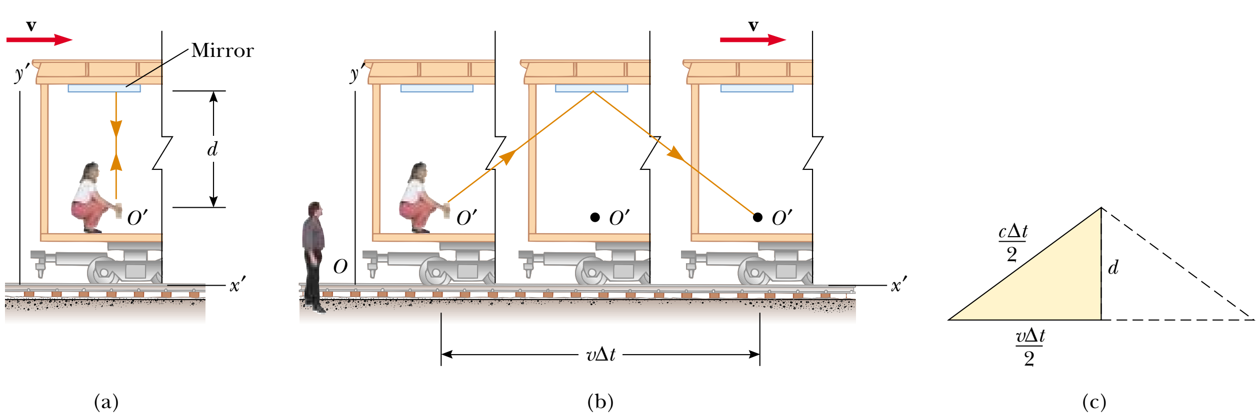


Figure 39.8 (a) A mirror is fixed to a moving vehicle, and a light pulse is sent out by observer O' at rest in the vehicle. (b) Relative to a stationary observer O standing alongside the vehicle, the mirror and O' move with a speed v. Note that what observer O measures for the distance the pulse travels is greater than $2d$. (c) The right triangle for calculating the relationship between Δt and Δt_p.

According to the second postulate of the special theory of relativity, both observers must measure c for the speed of light. Because the light travels farther in the frame of O, it follows that the time interval Δt measured by O is longer than the time interval Δt_p measured by O'. To obtain a relationship between these two time intervals, it is convenient to use the right triangle shown in Figure 39.8c. The Pythagorean theorem gives

$$\left(\frac{c\,\Delta t}{2}\right)^2 = \left(\frac{v\,\Delta t}{2}\right)^2 + d^2$$

Solving for Δt gives

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - \frac{v^2}{c^2}}} \tag{39.6}$$

Because $\Delta t_p = 2d/c$, we can express this result as

Time dilation

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\,\Delta t_p \tag{39.7}$$

where

$$\gamma = (1 - v^2/c^2)^{-1/2} \tag{39.8}$$

Because γ is always greater than unity, this result says that **the time interval Δt measured by an observer moving with respect to a clock is longer than the time interval Δt_p measured by an observer at rest with respect to the clock.** (That is, $\Delta t > \Delta t_p$.) This effect is known as **time dilation.** Figure 39.9 shows that as the velocity approaches the speed of light, γ increases dramatically. Note that for speeds less than one tenth the speed of light, γ is very nearly equal to unity.

The time interval Δt_p in Equations 39.5 and 39.7 is called the **proper time.** (In German, Einstein used the term *Eigenzeit,* which means "own-time.") In general, **proper time is the time interval between two events measured by an observer who sees the events occur at the same point in space.** Proper time is always the time measured with a single clock (clock C' in our case) at rest in the frame in which the events take place.

If a clock is moving with respect to you, it appears to fall behind (tick more slowly than) the clocks it is passing in the grid of synchronized clocks in your reference frame. Because the time interval $\gamma(2d/c)$, the interval between ticks of a moving

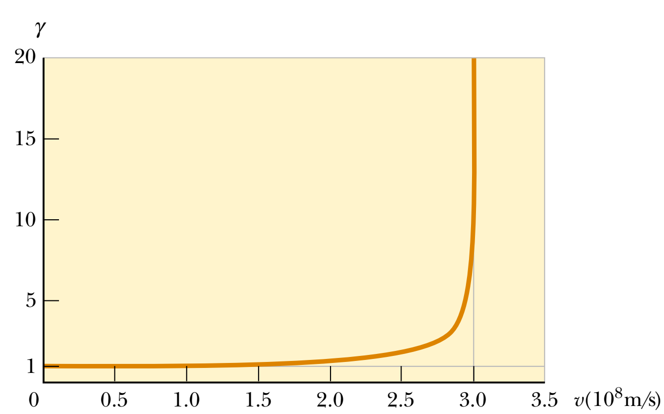


Figure 39.9 Graph of γ versus v. As the velocity approaches the speed of light, γ increases rapidly.

clock, is observed to be longer than $2d/c$, the time interval between ticks of an identical clock in your reference frame, it is often said that a moving clock runs more slowly than a clock in your reference frame by a factor γ. This is true for mechanical clocks as well as for the light clock just described. We can generalize this result by stating that all physical processes, including chemical and biological ones, slow down relative to a stationary clock when those processes occur in a moving frame. For example, the heartbeat of an astronaut moving through space would keep time with a clock inside the spaceship. Both the astronaut's clock and heartbeat would be slowed down relative to a stationary clock back on the Earth (although the astronaut would have no sensation of life slowing down in the spaceship).

Quick Quiz 39.4

A rocket has a clock built into its control panel. Use Figure 39.9 to determine approximately how fast the rocket must be moving before its clock appears to an Earth-bound observer to be ticking at one fifth the rate of a clock on the wall at Mission Control. What does an astronaut in the rocket observe?

Bizarre as it may seem, time dilation is a verifiable phenomenon. An experiment reported by Hafele and Keating provided direct evidence of time dilation.[4] Time intervals measured with four cesium atomic clocks in jet flight were compared with time intervals measured by Earth-based reference atomic clocks. In order to compare these results with theory, many factors had to be considered, including periods of acceleration and deceleration relative to the Earth, variations in direction of travel, and the fact that the gravitational field experienced by the flying clocks was weaker than that experienced by the Earth-based clock. The results were in good agreement with the predictions of the special theory of relativity and can be explained in terms of the relative motion between the Earth and the jet aircraft. In their paper, Hafele and Keating stated that "Relative to the atomic time scale of the U.S. Naval Observatory, the flying clocks lost 59 ± 10 ns during the eastward trip and gained 273 ± 7 ns during the westward trip . . . . These results provide an unambiguous empirical resolution of the famous clock paradox with macroscopic clocks."

Another interesting example of time dilation involves the observation of *muons*, unstable elementary particles that have a charge equal to that of the electron and a mass 207 times that of the electron. Muons can be produced by the collision of cosmic radiation with atoms high in the atmosphere. These particles have a lifetime of 2.2 μs when measured in a reference frame in which they are at rest or moving slowly. If we take 2.2 μs as the average lifetime of a muon and assume that its speed is close to the speed of light, we find that these particles travel only approximately 600 m before they decay (Fig. 39.10a). Hence, they cannot reach the Earth from the upper atmosphere where they are produced. However, experiments show that a large number of muons do reach the Earth. The phenomenon of time dilation explains this effect. Relative to an observer on the Earth, the muons have a lifetime equal to $\gamma\tau_p$, where $\tau_p = 2.2\ \mu$s is the lifetime in the frame traveling with the muons or the proper lifetime. For example, for a muon speed of $v = 0.99c$, $\gamma \approx 7.1$ and $\gamma\tau_p \approx 16\ \mu$s. Hence, the average distance traveled as measured by an observer on the Earth is $\gamma v \tau_p \approx 4\,800$ m, as indicated in Figure 39.10b.

In 1976, at the laboratory of the European Council for Nuclear Research

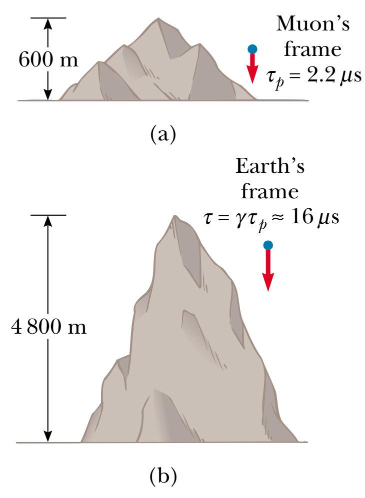


Figure 39.10 (a) Muons moving with a speed of $0.99c$ travel approximately 600 m as measured in the reference frame of the muons, where their lifetime is about 2.2 μs. (b) The muons travel approximately 4 800 m as measured by an observer on the Earth.

[4] J. C. Hafele and R. E. Keating, "Around the World Atomic Clocks: Relativistic Time Gains Observed," *Science,* 177:168, 1972.

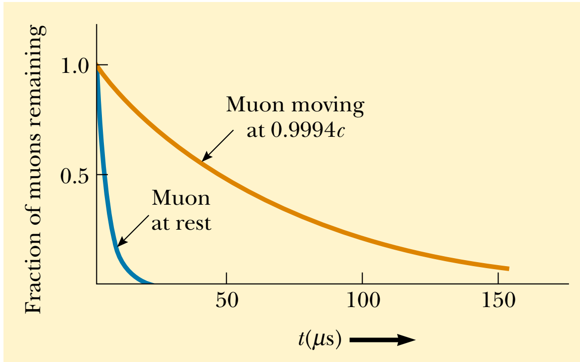


Figure 39.11 Decay curves for muons at rest and for muons traveling at a speed of $0.9994c$.

(CERN) in Geneva, muons injected into a large storage ring reached speeds of approximately $0.9994c$. Electrons produced by the decaying muons were detected by counters around the ring, enabling scientists to measure the decay rate and hence the muon lifetime. The lifetime of the moving muons was measured to be approximately 30 times as long as that of the stationary muon (Fig. 39.11), in agreement

EXAMPLE 39.1 What Is the Period of the Pendulum?

The period of a pendulum is measured to be 3.0 s in the reference frame of the pendulum. What is the period when measured by an observer moving at a speed of $0.95c$ relative to the pendulum?

Solution Instead of the observer moving at $0.95c$, we can take the equivalent point of view that the observer is at rest and the pendulum is moving at $0.95c$ past the stationary observer. Hence, the pendulum is an example of a moving clock.

The proper time is $\Delta t_p = 3.0$ s. Because a moving clock runs more slowly than a stationary clock by a factor γ, Equation 39.7 gives

$$\Delta t = \gamma \Delta t_p = \frac{1}{\sqrt{1 - \frac{(0.95c)^2}{c^2}}} \Delta t_p = \frac{1}{\sqrt{1 - 0.902}} \Delta t_p$$

$$= (3.2)(3.0 \text{ s}) = \boxed{9.6 \text{ s}}$$

That is, a moving pendulum takes longer to complete a period than a pendulum at rest does.

EXAMPLE 39.2 How Long Was Your Trip?

Suppose you are driving your car on a business trip and are traveling at 30 m/s. Your boss, who is waiting at your destination, expects the trip to take 5.0 h. When you arrive late, your excuse is that your car clock registered the passage of 5.0 h but that you were driving fast and so your clock ran more slowly than your boss's clock. If your car clock actually did indicate a 5.0-h trip, how much time passed on your boss's clock, which was at rest on the Earth?

Solution We begin by calculating γ from Equation 39.8:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(3 \times 10^1 \text{ m/s})^2}{(3 \times 10^8 \text{ m/s})^2}}} = \frac{1}{\sqrt{1 - 10^{-14}}}$$

If you try to determine this value on your calculator, you will probably get $\gamma = 1$. However, if we perform a binomial expansion, we can more precisely determine the value as

$$\gamma = (1 - 10^{-14})^{-1/2} \approx 1 + \tfrac{1}{2}(10^{-14}) = 1 + 5.0 \times 10^{-15}$$

This result indicates that at typical automobile speeds, γ is not much different from 1.

Applying Equation 39.7, we find Δt, the time interval measured by your boss, to be

$$\Delta t = \gamma \Delta t_p = (1 + 5.0 \times 10^{-15})(5.0 \text{ h})$$

$$= 5.0 \text{ h} + 2.5 \times 10^{-14} \text{ h} = \boxed{5.0 \text{ h} + 0.09 \text{ ns}}$$

Your boss's clock would be only 0.09 ns ahead of your car clock. You might want to try another excuse!

with the prediction of relativity to within two parts in a thousand.

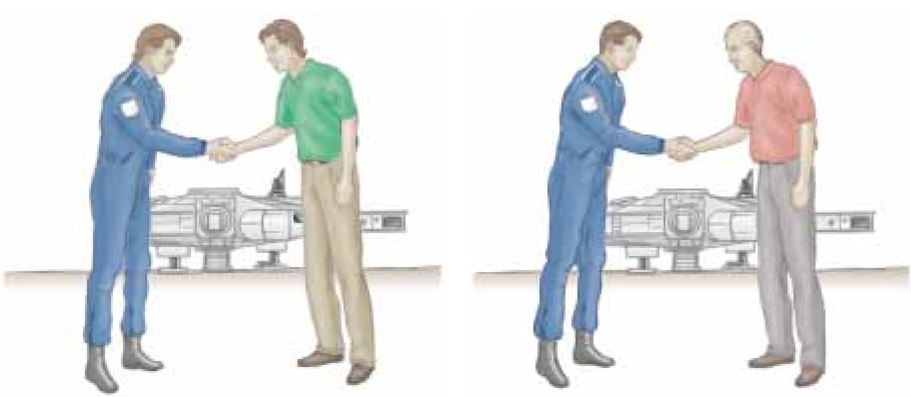
The Twins Paradox

An intriguing consequence of time dilation is the so-called *twins paradox* (Fig. 39.12). Consider an experiment involving a set of twins named Speedo and Goslo. When they are 20 yr old, Speedo, the more adventuresome of the two, sets out on an epic journey to Planet X, located 20 ly from the Earth. Furthermore, his spaceship is capable of reaching a speed of $0.95c$ relative to the inertial frame of his twin brother back home. After reaching Planet X, Speedo becomes homesick and immediately returns to the Earth at the same speed $0.95c$. Upon his return, Speedo is shocked to discover that Goslo has aged 42 yr and is now 62 yr old. Speedo, on the other hand, has aged only 13 yr.

At this point, it is fair to raise the following question—which twin is the traveler and which is really younger as a result of this experiment? From Goslo's frame of reference, he was at rest while his brother traveled at a high speed. But from Speedo's perspective, it is he who was at rest while Goslo was on the high-speed space journey. According to Speedo, he himself remained stationary while Goslo and the Earth raced away from him on a 6.5-yr journey and then headed back for another 6.5 yr. This leads to an apparent contradiction. Which twin has developed signs of excess aging?

To resolve this apparent paradox, recall that the special theory of relativity deals with inertial frames of reference moving relative to each other at uniform speed. However, the trip in our current problem is not symmetrical. Speedo, the space traveler, must experience a series of accelerations during his journey. As a result, his speed is not always uniform, and consequently he is not in an inertial frame. He cannot be regarded as always being at rest while Goslo is in uniform motion because to do so would be an incorrect application of the special theory of relativity. Therefore, there is no paradox. During each passing year noted by Goslo, slightly less than 4 months elapsed for Speedo.

The conclusion that Speedo is in a noninertial frame is inescapable. Each twin observes the other as accelerating, but it is Speedo that actually undergoes dynamical acceleration due to the real forces acting on him. The time required to accelerate and decelerate Speedo's spaceship may be made very small by using large rockets, so that Speedo can claim that he spends most of his time traveling to Planet X



(a) (b)

Figure 39.12 (a) As one twin leaves his brother on the Earth, both are the same age. (b) When Speedo returns from his journey to Planet X, he is younger than his twin Goslo.

at $0.95c$ in an inertial frame. However, Speedo must slow down, reverse his motion, and return to the Earth in an altogether different inertial frame. At the very best, Speedo is in two different inertial frames during his journey. Only Goslo, who is in a single inertial frame, can apply the simple time-dilation formula to Speedo's trip. Thus, Goslo finds that instead of aging 42 yr, Speedo ages only $(1 - v^2/c^2)^{1/2}(42 \text{ yr}) = 13$ yr. Conversely, Speedo spends 6.5 yr traveling to Planet X and 6.5 yr returning, for a total travel time of 13 yr, in agreement with our earlier statement.

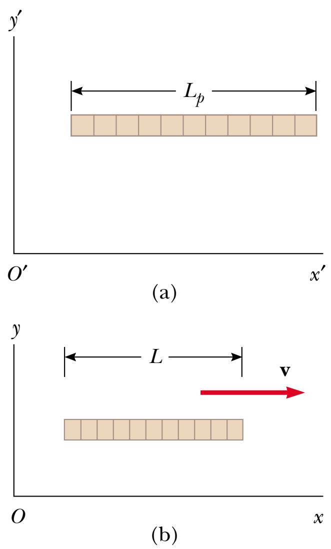


Figure 39.13 (a) A stick measured by an observer in a frame attached to the stick (that is, both have the same velocity) has its proper length L_p. (b) The stick measured by an observer in a frame in which the stick has a velocity **v** relative to the frame is shorter than its proper length L_p by a factor $(1 - v^2/c^2)^{1/2}$.

Quick Quiz 39.5

Suppose astronauts are paid according to the amount of time they spend traveling in space. After a long voyage traveling at a speed approaching c, would a crew rather be paid according to an Earth-based clock or their spaceship's clock?

Length Contraction

The measured distance between two points also depends on the frame of reference. **The proper length L_p of an object is the length measured by someone at rest relative to the object.** The length of an object measured by someone in a reference frame that is moving with respect to the object is always less than the proper length. This effect is known as **length contraction.**

Consider a spaceship traveling with a speed v from one star to another. There are two observers: one on the Earth and the other in the spaceship. The observer at rest on the Earth (and also assumed to be at rest with respect to the two stars) measures the distance between the stars to be the proper length L_p. According to this observer, the time it takes the spaceship to complete the voyage is $\Delta t = L_p/v$. Because of time dilation, the space traveler measures a smaller time of travel by the spaceship clock: $\Delta t_p = \Delta t/\gamma$. The space traveler claims to be at rest and sees the destination star moving toward the spaceship with speed v. Because the space traveler reaches the star in the time Δt_p, he or she concludes that the distance L between the stars is shorter than L_p. This distance measured by the space traveler is

$$L = v\,\Delta t_p = v\frac{\Delta t}{\gamma}$$

Because $L_p = v\,\Delta t$, we see that

Length contraction

$$L = \frac{L_p}{\gamma} = L_p\left(1 - \frac{v^2}{c^2}\right)^{1/2} \qquad \textbf{(39.9)}$$

If an object has a proper length L_p when it is at rest, then when it moves with speed v in a direction parallel to its length, it contracts to the length $L = L_p(1 - v^2/c^2)^{1/2} = L_p/\gamma$.

where $(1 - v^2/c^2)^{1/2}$ is a factor less than unity. This result may be interpreted as follows:

For example, suppose that a stick moves past a stationary Earth observer with speed v, as shown in Figure 39.13. The length of the stick as measured by an observer in a frame attached to the stick is the proper length L_p shown in Figure 39.13a. The length of the stick L measured by the Earth observer is shorter than L_p by the factor $(1 - v^2/c^2)^{1/2}$. Furthermore, length contraction is a symmetrical effect: If the stick is at rest on the Earth, an observer in a moving frame would measure its length to be shorter by the same factor $(1 - v^2/c^2)^{1/2}$. Note that **length contraction takes place only along the direction of motion.**

It is important to emphasize that proper length and proper time are measured in different reference frames. As an example of this point, let us return to the decaying muons moving at speeds close to the speed of light. An observer in the muon reference frame measures the proper lifetime (that is, the time interval τ_p), whereas an Earth-based observer measures a dilated lifetime. However, the Earth-based observer measures the proper height (the length L_p) of the mountain in Figure 39.10b. In the muon reference frame, this height is less than L_p, as the figure shows. Thus, in the muon frame, length contraction occurs but time dilation does not. In the Earth-based reference frame, time dilation occurs but length contraction does not. Thus, when calculations on the muon are performed in both

EXAMPLE 39.3 The Contraction of a Spaceship

A spaceship is measured to be 120.0 m long and 20.0 m in diameter while at rest relative to an observer. If this spaceship now flies by the observer with a speed of 0.99c, what length and diameter does the observer measure?

Solution From Equation 39.9, the length measured by the observer is

$$L = L_p\sqrt{1 - \frac{v^2}{c^2}} = (120.0 \text{ m})\sqrt{1 - \frac{(0.99c)^2}{c^2}} = \boxed{17 \text{ m}}$$

The diameter measured by the observer is still 20.0 m because the diameter is a dimension perpendicular to the motion and length contraction occurs only along the direction of motion.

Exercise If the ship moves past the observer with a speed of 0.100 0c, what length does the observer measure?

Answer 119.4 m.

EXAMPLE 39.4 How Long Was Your Car?

In Example 39.2, you were driving at 30 m/s and claimed that your clock was running more slowly than your boss's stationary clock. Although your statement was true, the time dilation was negligible. If your car is 4.3 m long when it is parked, how much shorter does it appear to a stationary roadside observer as you drive by at 30 m/s?

Solution The observer sees the horizontal length of the car to be contracted to a length

$$L = L_p\sqrt{1 - \frac{v^2}{c^2}} \approx L_p\left(1 - \frac{1}{2}\frac{v^2}{c^2}\right)$$

where we have again used the binomial expansion for the factor $\sqrt{1 - \frac{v^2}{c^2}}$. The roadside observer sees the car's length as having changed by an amount $L_p - L$:

$$L_p - L \approx \frac{L_p}{2}\left(\frac{v^2}{c^2}\right) = \left(\frac{4.3 \text{ m}}{2}\right)\left(\frac{3.0 \times 10^1 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}}\right)^2$$

$$= \boxed{2.2 \times 10^{-14} \text{ m}}$$

This is much smaller than the diameter of an atom!

EXAMPLE 39.5 A Voyage to Sirius

An astronaut takes a trip to Sirius, which is located a distance of 8 lightyears from the Earth. (Note that 1 lightyear (ly) is the distance light travels through free space in 1 yr.) The astronaut measures the time of the one-way journey to be 6 yr. If the spaceship moves at a constant speed of 0.8c, how can the 8-ly distance be reconciled with the 6-yr trip time measured by the astronaut?

Solution The 8 ly represents the proper length from the Earth to Sirius measured by an observer seeing both bodies nearly at rest. The astronaut sees Sirius approaching her at 0.8c but also sees the distance contracted to

$$\frac{8 \text{ ly}}{\gamma} = (8 \text{ ly})\sqrt{1 - \frac{v^2}{c^2}} = (8 \text{ ly})\sqrt{1 - \frac{(0.8c)^2}{c^2}} = 5 \text{ ly}$$

Thus, the travel time measured on her clock is

$$t = \frac{d}{v} = \frac{5 \text{ ly}}{0.8c} = 6 \text{ yr}$$

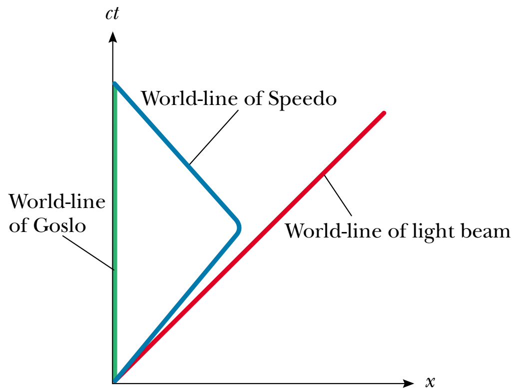


Figure 39.14 The twins paradox on a space–time graph. The twin who stays on the Earth has a world-line along the *t* axis. The path of the traveling twin through space–time is represented by a world-line that changes direction.

frames, the effect of "offsetting penalties" is seen, and the outcome of the experiment in one frame is the same as the outcome in the other frame!

Space–Time Graphs

It is sometimes helpful to make a *space–time graph,* in which time is the ordinate and displacement is the abscissa. The twins paradox is displayed in such a graph in Figure 39.14. A path through space–time is called a **world-line.** At the origin, the world-lines of Speedo and Goslo coincide because the twins are in the same location at the same time. After Speedo leaves on his trip, his world-line diverges from that of his brother. At their reunion, the two world-lines again come together. Note that Goslo's world-line is vertical, indicating no displacement from his original location. Also note that it would be impossible for Speedo to have a world-line that crossed the path of a light beam that left the Earth when he did. To do so would require him to have a speed greater than c.

World-lines for light beams are diagonal lines on space–time graphs, typically drawn at 45° to the right or left of vertical, depending on whether the light beam is traveling in the direction of increasing or decreasing x. These two world-lines means that all possible future events for Goslo and Speedo lie within two 45° lines extending from the origin. Either twin's presence at an event outside this "light cone" would require that twin to move at a speed greater than c, which, as we shall see in Section 39.5, is not possible. Also, the only past events that Goslo and Speedo could have experienced occurred within two similar 45° world-lines that approach the origin from below the x axis.

Quick Quiz 39.6

How is acceleration indicated on a space–time graph?

The Relativistic Doppler Effect

Another important consequence of time dilation is the shift in frequency found for light emitted by atoms in motion as opposed to light emitted by atoms at rest. This phenomenon, known as the Doppler effect, was introduced in Chapter 17 as it pertains to sound waves. In the case of sound, the motion of the source with respect to the medium of propagation can be distinguished from the motion of the observer with respect to the medium. Light waves must be analyzed differently, however, because they require no medium of propagation, and no method exists for distinguishing the motion of a light source from the motion of the observer.

If a light source and an observer approach each other with a relative speed v, the frequency f_{obs} measured by the observer is

$$f_{obs} = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} f_{source} \qquad (39.10)$$

where f_{source} is the frequency of the source measured in its rest frame. Note that this relativistic Doppler shift formula, unlike the Doppler shift formula for sound, depends only on the relative speed v of the source and observer and holds for relative speeds as great as c. As you might expect, the formula predicts that $f_{obs} > f_{source}$ when the source and observer approach each other. We obtain the expression for the case in which the source and observer recede from each other by replacing v with $-v$ in Equation 39.10.

The most spectacular and dramatic use of the relativistic Doppler effect is the measurement of shifts in the frequency of light emitted by a moving astronomical object such as a galaxy. Spectral lines normally found in the extreme violet region for galaxies at rest with respect to the Earth are shifted by about 100 nm toward the red end of the spectrum for distant galaxies—indicating that these galaxies are *receding* from us. The American astronomer Edwin Hubble (1889–1953) performed extensive measurements of this *red shift* to confirm that most galaxies are moving away from us, indicating that the Universe is expanding.

39.5 THE LORENTZ TRANSFORMATION EQUATIONS

We have seen that the Galilean transformation equations are not valid when v approaches the speed of light. In this section, we state the correct transformation equations that apply for all speeds in the range $0 \le v < c$.

Suppose that an event that occurs at some point P is reported by two observers, one at rest in a frame S and the other in a frame S′ that is moving to the right with speed v, as in Figure 39.15. The observer in S reports the event with space–time coordinates (x, y, z, t), and the observer in S′ reports the same event using the coordinates (x', y', z', t'). We would like to find a relationship between these coordinates that is valid for all speeds.

The equations that are valid from $v = 0$ to $v = c$ and enable us to transform coordinates from S to S′ are the **Lorentz transformation equations:**

Lorentz transformation equations for S → S′

$$x' = \gamma(x - vt)$$
$$y' = y$$

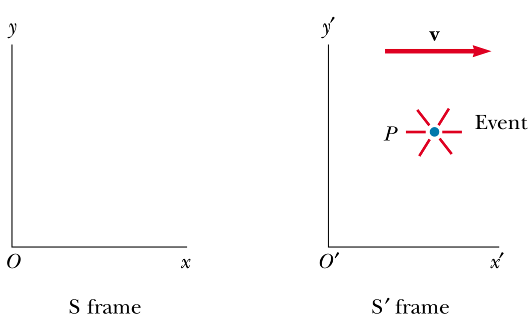


Figure 39.15 An event that occurs at some point P is observed by two persons, one at rest in the S frame and the other in the S′ frame, which is moving to the right with a speed v.

$$z' = z \tag{39.11}$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

These transformation equations were developed by Hendrik A. Lorentz (1853–1928) in 1890 in connection with electromagnetism. However, it was Einstein who recognized their physical significance and took the bold step of interpreting them within the framework of the special theory of relativity.

Note the difference between the Galilean and Lorentz time equations. In the Galilean case, $t = t'$, but in the Lorentz case the value for t' assigned to an event by an observer O' standing at the origin of the S′ frame in Figure 39.15 depends both on the time t and on the coordinate x as measured by an observer O standing in the S frame. This is consistent with the notion that an event is characterized by four space–time coordinates (x, y, z, t). In other words, in relativity, space and time are not separate concepts but rather are closely interwoven with each other.

If we wish to transform coordinates in the S′ frame to coordinates in the S frame, we simply replace v by $-v$ and interchange the primed and unprimed coordinates in Equations 39.11:

Inverse Lorentz transformation equations for S′ → S

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z' \tag{39.12}$$
$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

When $v \ll c$, the Lorentz transformation equations should reduce to the Galilean equations. To verify this, note that as v approaches zero, $v/c \ll 1$ and $v^2/c^2 \ll 1$; thus, $\gamma = 1$, and Equations 39.11 reduce to the Galilean space–time transformation equations:

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad t' = t$$

In many situations, we would like to know the difference in coordinates between two events or the time interval between two events as seen by observers O and O'. We can accomplish this by writing the Lorentz equations in a form suitable for describing pairs of events. From Equations 39.11 and 39.12, we can express the differences between the four variables x, x', t, and t' in the form

$$\left.\begin{aligned} \Delta x' &= \gamma(\Delta x - v\,\Delta t) \\ \Delta t' &= \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) \end{aligned}\right\} \text{S} \rightarrow \text{S}' \tag{39.13}$$

$$\left.\begin{aligned} \Delta x &= \gamma(\Delta x' + v\,\Delta t') \\ \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right) \end{aligned}\right\} \text{S}' \rightarrow \text{S} \tag{39.14}$$

[5] Although relative motion of the two frames along the x axis does not change the y and z coordinates of an object, it does change the y and z velocity components of an object moving in either frame, as we shall soon see.

EXAMPLE 39.6 Simultaneity and Time Dilation Revisited

Use the Lorentz transformation equations in difference form to show that (a) simultaneity is not an absolute concept and that (b) moving clocks run more slowly than stationary clocks.

Solution (a) Suppose that two events are simultaneous according to a moving observer O', such that $\Delta t' = 0$. From the expression for Δt given in Equation 39.14, we see that in this case the time interval Δt measured by a stationary observer O is $\Delta t = \gamma v \,\Delta x'/c^2$. That is, the time interval for the same two events as measured by O is nonzero, and so the events do not appear to be simultaneous to O.

(b) Suppose that observer O' finds that two events occur at the same place ($\Delta x' = 0$) but at different times ($\Delta t' \neq 0$). In this situation, the expression for Δt given in Equation 39.14 becomes $\Delta t = \gamma \,\Delta t'$. This is the equation for time dilation found earlier (Eq. 39.7), where $\Delta t' = \Delta t_p$ is the proper time measured by a clock located in the moving frame of observer O'.

Exercise Use the Lorentz transformation equations in difference form to confirm that $L = L_p/\gamma$ (Eq. 39.9).

where $\Delta x' = x_2' - x_1'$ and $\Delta t' = t_2' - t_1'$ are the differences measured by observer O' and $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$ are the differences measured by observer O. (We have not included the expressions for relating the y and z coordinates because they are unaffected by motion along the x direction.[5])

Derivation of the Lorentz Velocity Transformation Equation

Once again S is our stationary frame of reference, and S′ is our frame moving at a speed v relative to S. Suppose that an object has a speed u_x' measured in the S′ frame, where

$$u_x' = \frac{dx'}{dt'} \qquad \textbf{(39.15)}$$

Using Equation 39.11, we have

$$dx' = \gamma(dx - v\,dt)$$

$$dt' = \gamma\left(dt - \frac{v}{c^2}\,dx\right)$$

Substituting these values into Equation 39.15 gives

$$u_x' = \frac{dx'}{dt'} = \frac{dx - v\,dt}{dt - \dfrac{v}{c^2}\,dx} = \frac{\dfrac{dx}{dt} - v}{1 - \dfrac{v}{c^2}\dfrac{dx}{dt}}$$

But dx/dt is just the velocity component u_x of the object measured by an observer in S, and so this expression becomes

Lorentz velocity transformation equation for S → S′

$$u_x' = \frac{u_x - v}{1 - \dfrac{u_x v}{c^2}} \qquad \textbf{(39.16)}$$

If the object has velocity components along the y and z axes, the components as measured by an observer in S′ are

The speed of light is the speed limit of the Universe. It is the maximum possible speed for energy transfer and for information transfer. Any object with mass must move at a lower speed.

Lorentz velocity transformation equations for $S' \to S$

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} \quad \text{and} \quad u_z' = \frac{u_z}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} \tag{39.17}$$

Note that u_y' and u_z' do not contain the parameter v in the numerator because the relative velocity is along the x axis.

When u_x and v are both much smaller than c (the nonrelativistic case), the denominator of Equation 39.16 approaches unity, and so $u_x' \approx u_x - v$, which is the Galilean velocity transformation equation. In the other extreme, when $u_x = c$, Equation 39.16 becomes

$$u_x' = \frac{c - v}{1 - \frac{cv}{c^2}} = \frac{c\left(1 - \frac{v}{c}\right)}{1 - \frac{v}{c}} = c$$

From this result, we see that an object moving with a speed c relative to an observer in S also has a speed c relative to an observer in S′—independent of the relative motion of S and S′. Note that this conclusion is consistent with Einstein's second postulate—that the speed of light must be c relative to all inertial reference frames. Furthermore, the speed of an object can never exceed c. That is, the speed of light is the ultimate speed. We return to this point later when we consider the energy of a particle.

EXAMPLE 39.7 Relative Velocity of Spaceships

Two spaceships A and B are moving in opposite directions, as shown in Figure 39.16. An observer on the Earth measures the speed of ship A to be $0.750c$ and the speed of ship B to be $0.850c$. Find the velocity of ship B as observed by the crew on ship A.

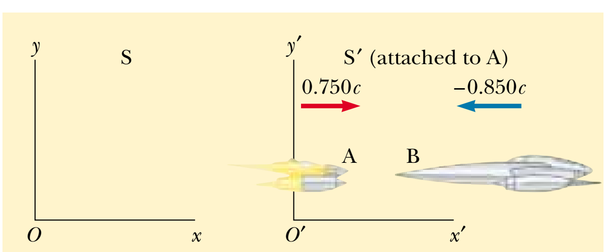


Figure 39.16 Two spaceships A and B move in opposite directions. The speed of B relative to A is *less* than c and is obtained from the relativistic velocity transformation equation.

Solution We can solve this problem by taking the S′ frame as being attached to ship A, so that $v = 0.750c$ relative to the Earth (the S frame). We can consider ship B as moving with a velocity $u_x = -0.850c$ relative to the Earth. Hence, we can obtain the velocity of ship B relative to ship A by using Equation 39.16:

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.977c$$

The negative sign indicates that ship B is moving in the negative x direction as observed by the crew on ship A. Note that the speed is less than c. That is, a body whose speed is less than c in one frame of reference must have a speed less than c in any other frame. (If the Galilean velocity transformation equation were used in this example, we would find that $u_x' = u_x - v = -0.850c - 0.750c = -1.60c$, which is impossible. The Galilean transformation equation does not work in relativistic situations.)

EXAMPLE 39.8 The Speeding Motorcycle

Imagine a motorcycle moving with a speed $0.80c$ past a stationary observer, as shown in Figure 39.17. If the rider tosses a ball in the forward direction with a speed of $0.70c$ relative to himself, what is the speed of the ball relative to the stationary observer?

Solution The speed of the motorcycle relative to the stationary observer is $v = 0.80c$. The speed of the ball in the frame of reference of the motorcyclist is $u_x' = 0.70c$. Therefore, the speed u_x of the ball relative to the stationary observer is

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} = \frac{0.70c + 0.80c}{1 + \frac{(0.70c)(0.80c)}{c^2}} = 0.96c$$

Exercise Suppose that the motorcyclist turns on the headlight so that a beam of light moves away from him with a speed c in the forward direction. What does the stationary observer measure for the speed of the light?

Answer c.

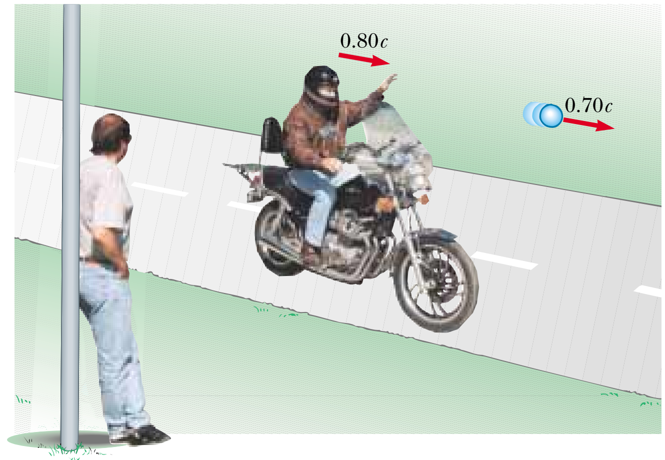


Figure 39.17 A motorcyclist moves past a stationary observer with a speed of $0.80c$ and throws a ball in the direction of motion with a speed of $0.70c$ relative to himself.

EXAMPLE 39.9 Relativistic Leaders of the Pack

Two motorcycle pack leaders named David and Emily are racing at relativistic speeds along perpendicular paths, as shown in Figure 39.18. How fast does Emily recede as seen by David over his right shoulder?

Solution Figure 39.18 represents the situation as seen by a police officer at rest in frame S, who observes the following:

David: $u_x = 0.75c$ $\quad u_y = 0$

Emily: $u_x = 0$ $\quad u_y = -0.90c$

To calculate Emily's speed of recession as seen by David, we take S′ to move along with David and then calculate u_x' and u_y' for Emily using Equations 39.16 and 39.17:

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{0 - 0.75c}{1 - \frac{(0)(0.75c)}{c^2}} = -0.75c$$

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} = \frac{\sqrt{1 - \frac{(0.75c)^2}{c^2}}\,(-0.90c)}{\left(1 - \frac{(0)(0.75c)}{c^2}\right)} = -0.60c$$

Thus, the speed of Emily as observed by David is

$$u' = \sqrt{(u_x')^2 + (u_y')^2} = \sqrt{(-0.75c)^2 + (-0.60c)^2} = 0.96c$$

Note that this speed is less than c, as required by the special theory of relativity.

Exercise Use the Galilean velocity transformation equation to calculate the classical speed of recession for Emily as observed by David.

Answer $1.2c$.

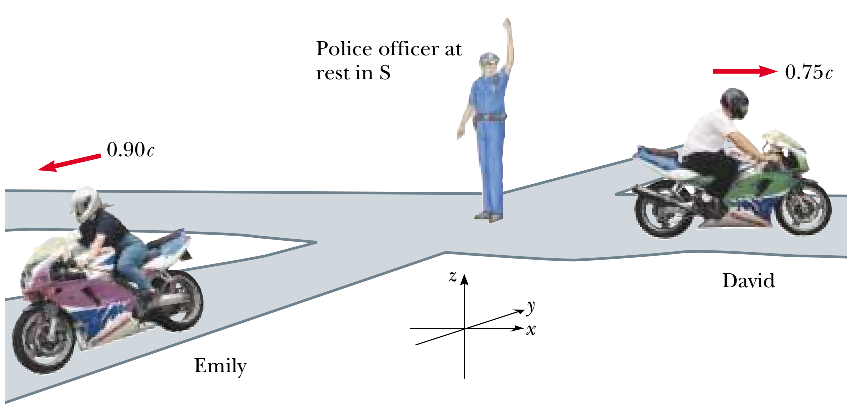


Figure 39.18 David moves to the east with a speed $0.75c$ relative to the police officer, and Emily travels south at a speed $0.90c$ relative to the officer.

To obtain u_x in terms of u'_x, we replace v by $-v$ in Equation 39.16 and interchange the roles of u_x and u'_x:

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} \tag{39.18}$$

39.6 RELATIVISTIC LINEAR MOMENTUM AND THE RELATIVISTIC FORM OF NEWTON'S LAWS

We have seen that in order to describe properly the motion of particles within the framework of the special theory of relativity, we must replace the Galilean transformation equations by the Lorentz transformation equations. Because the laws of physics must remain unchanged under the Lorentz transformation, we must generalize Newton's laws and the definitions of linear momentum and energy to conform to the Lorentz transformation equations and the principle of relativity. These generalized definitions should reduce to the classical (nonrelativistic) definitions for $v \ll c$.

First, recall that the law of conservation of linear momentum states that when two isolated objects collide, their combined total momentum remains constant. Suppose that the collision is described in a reference frame S in which linear momentum is conserved. If we calculate the velocities in a second reference frame S′ using the Lorentz velocity transformation equation and the classical definition of linear momentum, $\mathbf{p} = m\mathbf{u}$ (where $\mathbf{u}$ is the velocity of either object), we find that linear momentum is *not* conserved in S′. However, because the laws of physics are the same in all inertial frames, linear momentum must be conserved in all frames. In view of this condition and assuming that the Lorentz velocity transformation equation is correct, we must modify the definition of linear momentum to satisfy the following conditions:

Definition of relativistic linear momentum

- Linear momentum $\mathbf{p}$ must be conserved in all collisions.
- The relativistic value calculated for $\mathbf{p}$ must approach the classical value $m\mathbf{u}$ as $\mathbf{u}$ approaches zero.

For any particle, the correct relativistic equation for linear momentum that satisfies these conditions is

$$\mathbf{p} \equiv \frac{m\mathbf{u}}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma m\mathbf{u} \tag{39.19}$$

where $\mathbf{u}$ is the velocity of the particle and m is the mass of the particle. When u is much less than c, $\gamma = (1 - u^2/c^2)^{-1/2}$ approaches unity and $\mathbf{p}$ approaches $m\mathbf{u}$. Therefore, the relativistic equation for $\mathbf{p}$ does indeed reduce to the classical expression when u is much smaller than c.

The relativistic force $\mathbf{F}$ acting on a particle whose linear momentum is $\mathbf{p}$ is defined as

$$\mathbf{F} \equiv \frac{d\mathbf{p}}{dt} \tag{39.20}$$

where $\mathbf{p}$ is given by Equation 39.19. This expression, which is the relativistic form of Newton's second law, is reasonable because it preserves classical mechanics in

EXAMPLE 39.10 Linear Momentum of an Electron

An electron, which has a mass of 9.11×10^{-31} kg, moves with a speed of $0.750c$. Find its relativistic momentum and compare this value with the momentum calculated from the classical expression.

Solution Using Equation 39.19 with $u = 0.750c$, we have

$$p = \frac{m_e u}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$= \frac{(9.11 \times 10^{-31}\ \text{kg})(0.750 \times 3.00 \times 10^8\ \text{m/s})}{\sqrt{1 - \frac{(0.750c)^2}{c^2}}}$$

$$= \boxed{3.10 \times 10^{-22}\ \text{kg} \cdot \text{m/s}}$$

The (incorrect) classical expression gives

$$p_{\text{classical}} = m_e u = 2.05 \times 10^{-22}\ \text{kg} \cdot \text{m/s}$$

Hence, the correct relativistic result is 50% greater than the classical result!

the limit of low velocities and requires conservation of linear momentum for an isolated system ($\mathbf{F} = 0$) both relativistically and classically.

It is left as an end-of-chapter problem (Problem 63) to show that under relativistic conditions, the acceleration **a** of a particle decreases under the action of a constant force, in which case $a \propto (1 - u^2/c^2)^{3/2}$. From this formula, note that as the particle's speed approaches c, the acceleration caused by any finite force approaches zero. Hence, it is impossible to accelerate a particle from rest to a speed $u \geq c$.

39.7 RELATIVISTIC ENERGY

We have seen that the definition of linear momentum and the laws of motion require generalization to make them compatible with the principle of relativity. This implies that the definition of kinetic energy must also be modified.

To derive the relativistic form of the work–kinetic energy theorem, let us first use the definition of relativistic force, Equation 39.20, to determine the work done on a particle by a force F:

$$W = \int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} \frac{dp}{dt}\,dx \qquad \textbf{(39.21)}$$

for force and motion both directed along the x axis. In order to perform this integration and find the work done on the particle and the relativistic kinetic energy as a function of u, we first evaluate dp/dt:

$$\frac{dp}{dt} = \frac{d}{dt}\frac{mu}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{m(du/dt)}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}$$

Substituting this expression for dp/dt and $dx = u\,dt$ into Equation 39.21 gives

$$W = \int_0^t \frac{m(du/dt)u\,dt}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}} = m\int_0^u \frac{u}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}\,du$$

where we use the limits 0 and u in the rightmost integral because we have assumed that the particle is accelerated from rest to some final speed u. Evaluating the integral, we find that

Relativistic kinetic energy

$$W = \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} - mc^2 \quad \textbf{(39.22)}$$

Recall from Chapter 7 that the work done by a force acting on a particle equals the change in kinetic energy of the particle. Because of our assumption that the initial speed of the particle is zero, we know that the initial kinetic energy is zero. We therefore conclude that the work W is equivalent to the relativistic kinetic energy K:

$$K = \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} - mc^2 = \gamma mc^2 - mc^2 \quad \textbf{(39.23)}$$

This equation is routinely confirmed by experiments using high-energy particle accelerators.

At low speeds, where $u/c \ll 1$, Equation 39.23 should reduce to the classical expression $K = \frac{1}{2}mu^2$. We can check this by using the binomial expansion $(1 - x^2)^{-1/2} \approx 1 + \frac{1}{2}x^2 + \ldots$ for $x \ll 1$, where the higher-order powers of x are neglected in the expansion. In our case, $x = u/c$, so that

$$\frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} = \left(1 - \frac{u^2}{c^2}\right)^{-1/2} \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$$

Substituting this into Equation 39.23 gives

$$K \approx mc^2\left(1 + \frac{1}{2}\frac{u^2}{c^2}\right) - mc^2 = \frac{1}{2}mu^2$$

Definition of total energy

which is the classical expression for kinetic energy. A graph comparing the relativistic and nonrelativistic expressions is given in Figure 39.19. In the relativistic case, the particle speed never exceeds c, regardless of the kinetic energy. The two curves are in good agreement when $u \ll c$.

The constant term mc^2 in Equation 39.23, which is independent of the speed of the particle, is called the **rest energy** E_R of the particle (see Section 8.9). The term γmc^2, which does depend on the particle speed, is therefore the sum of the kinetic and rest energies. We define γmc^2 to be the **total energy** E:

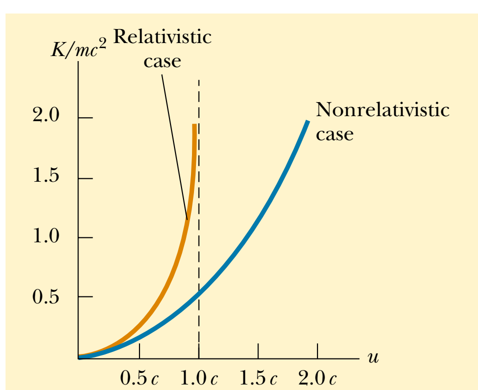


Figure 39.19 A graph comparing relativistic and nonrelativistic kinetic energy. The energies are plotted as a function of speed. In the relativistic case, u is always less than c.

Total energy = kinetic energy + rest energy

$$E = \gamma mc^2 = K + mc^2 \tag{39.24}$$

or

$$E = \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{39.25}$$

This is Einstein's famous equation about mass–energy equivalence.

The relationship $E = K + mc^2$ shows that **mass is a form of energy,** where c^2 in the rest energy term is just a constant conversion factor. This expression also shows that a small mass corresponds to an enormous amount of energy, a concept fundamental to nuclear and elementary-particle physics.

Energy–momentum relationship

In many situations, the linear momentum or energy of a particle is measured rather than its speed. It is therefore useful to have an expression relating the total energy E to the relativistic linear momentum p. This is accomplished by using the expressions $E = \gamma mc^2$ and $p = \gamma mu$. By squaring these equations and subtracting, we can eliminate u (Problem 39). The result, after some algebra, is[6]

$$E^2 = p^2c^2 + (mc^2)^2 \tag{39.26}$$

When the particle is at rest, $p = 0$ and so $E = E_R = mc^2$. For particles that have zero mass, such as photons, we set $m = 0$ in Equation 39.26 and see that

$$E = pc \tag{39.27}$$

This equation is an exact expression relating total energy and linear momentum for photons, which always travel at the speed of light.

Finally, note that because the mass m of a particle is independent of its motion, m must have the same value in all reference frames. For this reason, m is often called the **invariant mass.** On the other hand, because the total energy and linear momentum of a particle both depend on velocity, these quantities depend on the reference frame in which they are measured.

Because m is a constant, we conclude from Equation 39.26 that the quantity $E^2 - p^2c^2$ must have the same value in all reference frames. That is, $E^2 - p^2c^2$ is invariant under a Lorentz transformation. (Equations 39.26 and 39.27 do not make provision for potential energy.)

When we are dealing with subatomic particles, it is convenient to express their

Example 39.11 The Energy of a Speedy Electron

An electron in a television picture tube typically moves with a speed $u = 0.250c$. Find its total energy and kinetic energy in electron volts.

Solution Using the fact that the rest energy of the electron is 0.511 MeV together with Equation 39.25, we have

$$E = \frac{m_ec^2}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{0.511 \text{ MeV}}{\sqrt{1 - \frac{(0.250c)^2}{c^2}}}$$

$$= 1.03(0.511 \text{ MeV}) = \boxed{0.528 \text{ MeV}}$$

This is 3% greater than the rest energy.

We obtain the kinetic energy by subtracting the rest energy from the total energy:

$$K = E - m_ec^2 = 0.528 \text{ MeV} - 0.511 \text{ MeV} = \boxed{0.017 \text{ MeV}}$$

[6] One way to remember this relationship is to draw a right triangle having a hypotenuse of length E and legs of lengths pc and mc^2.

EXAMPLE 39.12 The Energy of a Speedy Proton

(a) Find the rest energy of a proton in electron volts.

Solution

$$E_R = m_pc^2 = (1.67 \times 10^{-27}\text{ kg})(3.00 \times 10^8\text{ m/s})^2$$
$$= (1.50 \times 10^{-10}\text{ J})(1.00\text{ eV}/1.60 \times 10^{-19}\text{ J})$$
$$= \boxed{938\text{ MeV}}$$

(b) If the total energy of a proton is three times its rest energy, with what speed is the proton moving?

Solution Equation 39.25 gives

$$E = 3m_pc^2 = \frac{m_pc^2}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$3 = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Solving for u gives

$$1 - \frac{u^2}{c^2} = \frac{1}{9}$$

$$\frac{u^2}{c^2} = \frac{8}{9}$$

$$u = \frac{\sqrt{8}}{3}c = \boxed{2.83 \times 10^8\text{ m/s}}$$

(c) Determine the kinetic energy of the proton in electron volts.

Solution From Equation 39.24,

$$K = E - m_pc^2 = 3m_pc^2 - m_pc^2 = 2m_pc^2$$

Because $m_pc^2 = 938$ MeV, $K = \boxed{1\,880\text{ MeV}}$

(d) What is the proton's momentum?

Solution We can use Equation 39.26 to calculate the momentum with $E = 3m_pc^2$:

$$E^2 = p^2c^2 + (m_pc^2)^2 = (3m_pc^2)^2$$
$$p^2c^2 = 9(m_pc^2)^2 - (m_pc^2)^2 = 8(m_pc^2)^2$$
$$p = \sqrt{8}\,\frac{m_pc^2}{c} = \sqrt{8}\,\frac{(938\text{ MeV})}{c} = \boxed{2\,650\text{ MeV}/c}$$

The unit of momentum is written MeV/c for convenience.

energy in electron volts because the particles are usually given this energy by acceleration through a potential difference. The conversion factor, as you recall from Equation 25.5, is

$$1\text{ eV} = 1.602 \times 10^{-19}\text{ J}$$

For example, the mass of an electron is 9.109×10^{-31} kg. Hence, the rest energy of the electron is

$$m_ec^2 = (9.109 \times 10^{-31}\text{ kg})(2.9979 \times 10^8\text{ m/s})^2 = 8.187 \times 10^{-14}\text{ J}$$
$$= (8.187 \times 10^{-14}\text{ J})(1\text{ eV}/1.602 \times 10^{-19}\text{ J}) = 0.5110\text{ MeV}$$

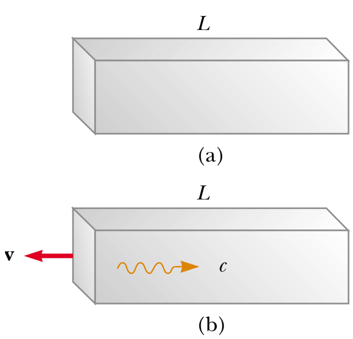


Figure 39.20 (a) A box of length L at rest. (b) When a light pulse directed to the right is emitted at the left end of the box, the box recoils to the left until the pulse strikes the right end.

39.8 EQUIVALENCE OF MASS AND ENERGY

To understand the equivalence of mass and energy, consider the following thought experiment proposed by Einstein in developing his famous equation $E = mc^2$. Imagine an isolated box of mass M_{box} and length L initially at rest, as shown in Figure 39.20a. Suppose that a pulse of light is emitted from the left side of the box, as depicted in Figure 39.20b. From Equation 39.27, we know that light of energy E carries linear momentum $p = E/c$. Hence, if momentum is to be conserved, the box must recoil to the left with a speed v. If it is assumed that the box is very massive, the recoil speed is much less than the speed of light, and conservation of momentum gives $M_{\text{box}}v = E/c$, or

$$v = \frac{E}{M_{\text{box}}c}$$

The time it takes the light pulse to move the length of the box is approximately $\Delta t = L/c$. In this time interval, the box moves a small distance Δx to the left, where

$$\Delta x = v\,\Delta t = \frac{EL}{M_{\text{box}}c^2}$$

The light then strikes the right end of the box and transfers its momentum to the box, causing the box to stop. With the box in its new position, its center of mass appears to have moved to the left. However, its center of mass cannot have moved because the box is an isolated system. Einstein resolved this perplexing situation by assuming that in addition to energy and momentum, light also carries mass. If M_{pulse} is the effective mass carried by the pulse of light and if the center of mass of the system (box plus pulse of light) is to remain fixed, then

$$M_{\text{pulse}}L = M_{\text{box}}\Delta x$$

Solving for M_{pulse}, and using the previous expression for Δx, we obtain

$$M_{\text{pulse}} = \frac{M_{\text{box}}\Delta x}{L} = \frac{M_{\text{box}}}{L}\,\frac{EL}{M_{\text{box}}c^2} = \frac{E}{c^2}$$

or

$$E = M_{\text{pulse}}c^2$$

Thus, Einstein reached the profound conclusion that "if a body gives off the energy E in the form of radiation, its mass diminishes by E/c^2, . . ."

Although we derived the relationship $E = mc^2$ for light energy, the equivalence of mass and energy is universal. Equation 39.24, $E = \gamma mc^2$, which represents the total energy of any particle, suggests that even when a particle is at rest ($\gamma = 1$) it still possesses enormous energy because it has mass. Probably the clearest experimental proof of the equivalence of mass and energy occurs in nuclear and elementary particle interactions, where large amounts of energy are released and the energy release is accompanied by a decrease in mass. Because energy and mass are related, we see that the laws of conservation of energy and conservation of mass are one and the same. Simply put, this law states that

the energy of a system of particles before interaction must equal the energy of the system after interaction, where energy of the ith particle is given by the expression

$$E_i = \frac{m_i c^2}{\sqrt{1 - \frac{u_i^2}{c^2}}} = \gamma m_i c^2$$

Conversion of mass–energy

The release of enormous quantities of energy from subatomic particles, accompanied by changes in their masses, is the basis of all nuclear reactions. In a conventional nuclear reactor, a uranium nucleus undergoes *fission,* a reaction that creates several lighter fragments having considerable kinetic energy. The combined mass of all the fragments is less than the mass of the parent uranium nucleus by an amount Δm. The corresponding energy Δmc^2 associated with this mass difference is exactly equal to the total kinetic energy of the fragments. This kinetic energy raises the temperature of water in the reactor, converting it to steam for the generation of electric power.

CONCEPTUAL EXAMPLE 39.13

Because mass is a measure of energy, can we conclude that the mass of a compressed spring is greater than the mass of the same spring when it is not compressed?

Solution Recall that when a spring of force constant k is compressed (or stretched) from its equilibrium position a distance x, it stores elastic potential energy $U = kx^2/2$. According to the special theory of relativity, any change in the total energy of a system is equivalent to a change in the mass of the system. Therefore, the mass of a compressed (or stretched) spring is greater than the mass of the spring in its equilibrium position by an amount U/c^2.

EXAMPLE 39.14 Binding Energy of the Deuteron

A deuteron, which is the nucleus of a deuterium atom, contains one proton and one neutron and has a mass of 2.013 553 u. This total deuteron mass is not equal to the sum of the masses of the proton and neutron. Calculate the mass difference and determine its energy equivalence, which is called the *binding energy* of the nucleus.

Solution Using atomic mass units (u), we have

$$m_p = \text{mass of proton} = 1.007\ 276\ \text{u}$$

$$m_n = \text{mass of neutron} = 1.008\ 665\ \text{u}$$

$$m_p + m_n = 2.015\ 941\ \text{u}$$

The mass difference Δm is therefore 0.002 388 u. By definition, 1 u $= 1.66 \times 10^{-27}$ kg, and therefore

$$\Delta m = 0.002\ 388\ \text{u} = \boxed{3.96 \times 10^{-30}\ \text{kg}}$$

Using $E = \Delta mc^2$, we find that the binding energy is

$$E = \Delta mc^2 = (3.96 \times 10^{-30}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2$$

$$= 3.56 \times 10^{-13}\ \text{J} = \boxed{2.23\ \text{MeV}}$$

Therefore, the minimum energy required to separate the proton from the neutron of the deuterium nucleus (the binding energy) is 2.23 MeV.

In the nuclear reaction called *fusion,* two atomic nuclei combine to form a single nucleus. The fusion reaction in which two deuterium nuclei fuse to form a helium nucleus is of major importance in current research and the development of controlled-fusion reactors. The decrease in mass that results from the creation of one helium nucleus from two deuterium nuclei is $\Delta m = 4.25 \times 10^{-29}$ kg. Hence, the corresponding excess energy that results from one fusion reaction is $\Delta mc^2 = 3.83 \times 10^{-12}\ \text{J} = 23.9$ MeV. To appreciate the magnitude of this result, note that if 1 g of deuterium is converted to helium, the energy released is about 10^{12} J! At the current cost of electrical energy, this quantity of energy would be worth about $70 000.

39.9 RELATIVITY AND ELECTROMAGNETISM

Consider two frames of reference S and S′ that are in relative motion, and assume that a single charge q is at rest in the S′ frame of reference. According to an ob-

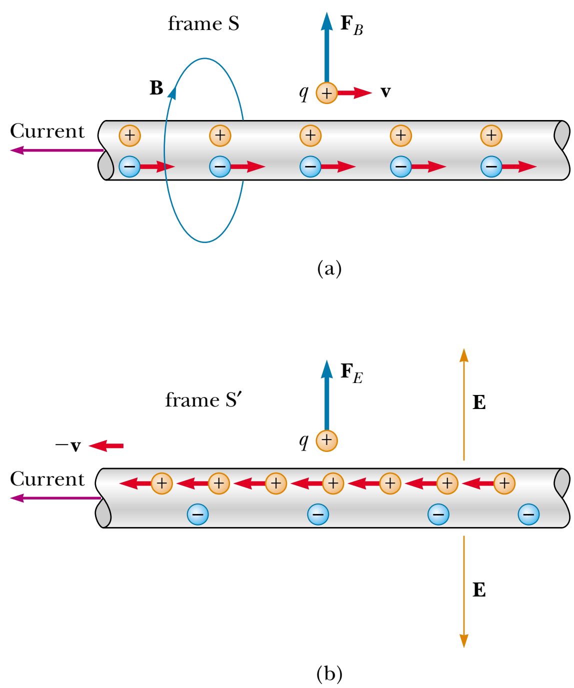


Figure 39.21 (a) In frame S, the positive charge q moves to the right with a velocity **v**, and the current-carrying wire is stationary. A magnetic field **B** surrounds the wire, and charge experiences a *magnetic* force directed away from the wire. (b) In frame S′, the wire moves to the left with a velocity −**v**, and the charge q is stationary. The wire creates an electric field **E**, and the charge experiences an *electric* force directed away from the wire.

server in this frame, an electric field surrounds the charge. However, an observer in frame S says that the charge is in motion and therefore measures both an electric field and a magnetic field. The magnetic field measured by the observer in frame S is created by the moving charge, which constitutes an electric current. In other words, electric and magnetic fields are viewed differently in frames of reference that are moving relative to each other. We now describe one situation that shows how an electric field in one frame of reference is viewed as a magnetic field in another frame of reference.

A positive test charge q is moving parallel to a current-carrying wire with velocity **v** relative to the wire in frame S, as shown in Figure 39.21a. We assume that the net charge on the wire is zero and that the electrons in the wire also move with velocity **v** in a straight line. The leftward current in the wire produces a magnetic field that forms circles around the wire and is directed into the page at the location of the moving test charge. Therefore, a magnetic force $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ directed away from the wire is exerted on the test charge. However, no electric force acts on the test charge because the net charge on the wire is zero when viewed in this frame.

Now consider the same situation as viewed from frame S′, where the test charge is at rest (Figure 39.21b). In this frame, the positive charges in the wire move to the left, the electrons in the wire are at rest, and the wire still carries a current. Because the test charge is not moving in this frame, $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} = 0$; there is no magnetic force exerted on the test charge when viewed in this frame. However, if a force is exerted on the test charge in frame S′, the frame of the wire, as described earlier, a force must be exerted on it in any other frame. What is the origin of this force in frame S, the frame of the test charge?

The answer to this question is provided by the special theory of relativity. When the situation is viewed in frame S, as in Figure 39.21a, the positive charges are at rest and the electrons in the wire move to the right with a velocity **v**. Because of length contraction, the electrons appear to be closer together than their proper separation. Because there is no net charge on the wire this contracted separation must equal the separation between the stationary positive charges. The situation is quite different when viewed in frame S′, shown in Figure 39.21b. In this frame, the positive charges appear closer together because of length contraction, and the electrons in the wire are at rest with a separation that is greater than that viewed in frame S. Therefore, there is a net positive charge on the wire when viewed in frame S′. This net positive charge produces an electric field pointing away from the wire toward the test charge, and so the test charge experiences an electric force directed away from the wire. Thus, what was viewed as a magnetic field (and a corresponding magnetic force) in the frame of the wire transforms into an electric field (and a corresponding electric force) in the frame of the test charge.

Optional Section

39.10 THE GENERAL THEORY OF RELATIVITY

Up to this point, we have sidestepped a curious puzzle. Mass has two seemingly different properties: a *gravitational attraction* for other masses and an *inertial* property that resists acceleration. To designate these two attributes, we use the subscripts g and i and write

Gravitational property $\quad F_g = m_g g$

Inertial property $\quad \Sigma F = m_i a$

The value for the gravitational constant G was chosen to make the magnitudes of m_g and m_i numerically equal. Regardless of how G is chosen, however, the strict proportionality of m_g and m_i has been established experimentally to an extremely high degree: a few parts in 10^{12}. Thus, it appears that gravitational mass and inertial mass may indeed be exactly proportional.

But why? They seem to involve two entirely different concepts: a force of mutual gravitational attraction between two masses, and the resistance of a single mass to being accelerated. This question, which puzzled Newton and many other physicists over the years, was answered when Einstein published his theory of gravitation, known as his *general theory of relativity,* in 1916. Because it is a mathematically complex theory, we offer merely a hint of its elegance and insight.

In Einstein's view, the remarkable coincidence that m_g and m_i seemed to be proportional to each other was evidence of an intimate and basic connection between the two concepts. He pointed out that no mechanical experiment (such as dropping a mass) could distinguish between the two situations illustrated in Figure 39.22a and b. In each case, the dropped briefcase undergoes a downward acceleration g relative to the floor.

Einstein carried this idea further and proposed that *no* experiment, mechanical or otherwise, could distinguish between the two cases. This extension to in-

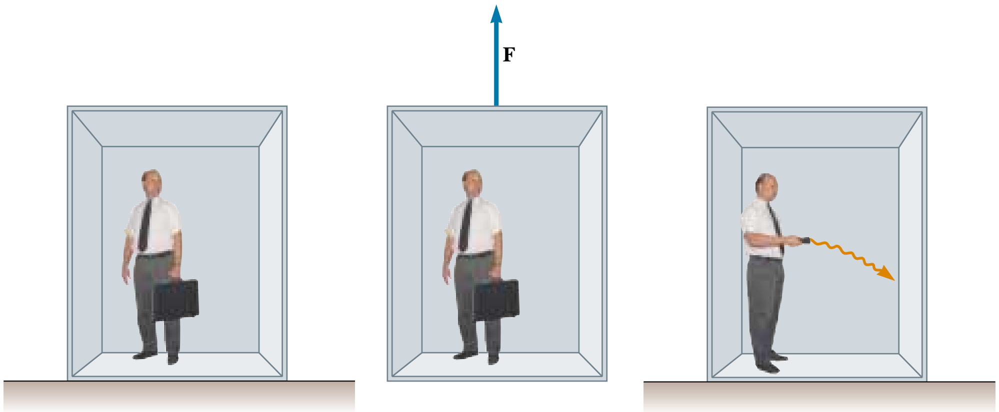


Figure 39.22 (a) The observer is at rest in a uniform gravitational field **g**. (b) The observer is in a region where gravity is negligible, but the frame of reference is accelerated by an external force **F** that produces an acceleration **g**. According to Einstein, the frames of reference in parts (a) and (b) are equivalent in every way. No local experiment can distinguish any difference between the two frames. (c) If parts (a) and (b) are truly equivalent, as Einstein proposed, then a ray of light should bend in a gravitational field.

clude all phenomena (not just mechanical ones) has interesting consequences. For example, suppose that a light pulse is sent horizontally across the elevator. During the time it takes the light to make the trip, the right wall of the elevator has accelerated upward. This causes the light to arrive at a location lower on the wall than the spot it would have hit if the elevator were not accelerating. Thus, in the frame of the elevator, the trajectory of the light pulse bends downward as the elevator accelerates upward to meet it. Because the accelerating elevator cannot be distinguished from a nonaccelerating one located in a gravitational field, Einstein proposed that a beam of light *should also be bent downward by a gravitational field,* as shown in Figure 39.22c. Experiments have verified the effect, although the bending is small. A laser aimed at the horizon falls less than 1 cm after traveling 6 000 km. (No such bending is predicted in Newton's theory of gravitation.)

The two postulates of Einstein's **general theory of relativity** are

- All the laws of nature have the same form for observers in any frame of reference, whether accelerated or not.

This Global Positioning System (GPS) unit incorporates relativistically corrected time calculations in its analysis of signals it receives from orbiting satellites. These corrections allow the unit to determine its position on the Earth's surface to within a few meters. If the corrections were not made, the location error would be about 1 km. *(Courtesy of Trimble Navigation Limited)*

- In the vicinity of any point, a gravitational field is equivalent to an accelerated frame of reference in the absence of gravitational effects. (This is the *principle of equivalence.*)

The second postulate implies that gravitational mass and inertial mass are completely equivalent, not just proportional. What were thought to be two different types of mass are actually identical.

One interesting effect predicted by the general theory is that time scales are altered by gravity. A clock in the presence of gravity runs more slowly than one located where gravity is negligible. Consequently, the frequencies of radiation emitted by atoms in the presence of a strong gravitational field are *red-shifted* to lower frequencies when compared with the same emissions in the presence of a weak field. This gravitational red shift has been detected in spectral lines emitted by atoms in massive stars. It has also been verified on the Earth by comparison of the frequencies of gamma rays (a high-energy form of electromagnetic radiation) emitted from nuclei separated vertically by about 20 m.

Quick Quiz 39.7

Two identical clocks are in the same house, one upstairs in a bedroom and the other downstairs in the kitchen. Which clock runs more slowly?

The second postulate suggests that a gravitational field may be "transformed away" at any point if we choose an appropriate accelerated frame of reference—a freely falling one. Einstein developed an ingenious method of describing the acceleration necessary to make the gravitational field "disappear." He specified a concept, the *curvature of space–time,* that describes the gravitational effect at every point. In fact, the curvature of space–time completely replaces Newton's gravitational theory. According to Einstein, there is no such thing as a gravitational force. Rather, the presence of a mass causes a curvature of space–time in the vicinity of the mass, and this curvature dictates the space–time path that all freely moving objects must follow. In 1979, John Wheeler summarized Einstein's general theory of relativity in a single sentence: "Space tells matter how to move and matter tells space how to curve."

Consider two travelers on the surface of the Earth walking directly toward the

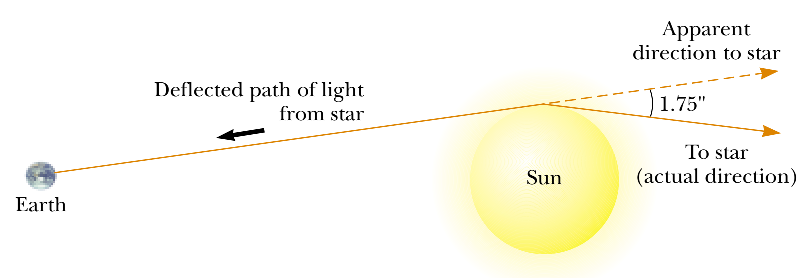
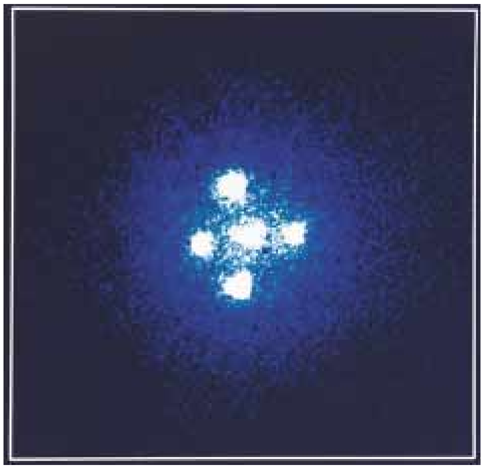


Figure 39.23 Deflection of starlight passing near the Sun. Because of this effect, the Sun or some other remote object can act as a *gravitational lens.* In his general theory of relativity, Einstein calculated that starlight just grazing the Sun's surface should be deflected by an angle of 1.75″.



Einstein's cross. The four bright spots are images of the same galaxy that have been bent around a massive object located between the galaxy and the Earth. The massive object acts like a lens, causing the rays of light that were diverging from the distant galaxy to converge on the Earth. (If the intervening massive object had a uniform mass distribution, we would see a bright ring instead of four spots.)

North Pole but from different starting locations. Even though both say they are walking due north, and thus should be on parallel paths, they see themselves getting closer and closer together, as if they were somehow attracted to each other. The curvature of the Earth causes this effect. In a similar way, what we are used to thinking of as the gravitational attraction between two masses is, in Einstein's view, two masses curving space–time and as a result moving toward each other, much like two bowling balls on a mattress rolling together.

One prediction of the general theory of relativity is that a light ray passing near the Sun should be deflected into the curved space–time created by the Sun's mass. This prediction was confirmed when astronomers detected the bending of starlight near the Sun during a total solar eclipse that occurred shortly after World War I (Fig. 39.23). When this discovery was announced, Einstein became an international celebrity.

If the concentration of mass becomes very great, as is believed to occur when a large star exhausts its nuclear fuel and collapses to a very small volume, a **black hole** may form. Here, the curvature of space–time is so extreme that, within a certain distance from the center of the black hole, all matter and light become trapped.

SUMMARY

The two basic postulates of the special theory of relativity are

- The laws of physics must be the same in all inertial reference frames.
- The speed of light in vacuum has the same value, $c = 3.00 \times 10^8$ m/s, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

Three consequences of the special theory of relativity are

- Events that are simultaneous for one observer are not simultaneous for another observer who is in motion relative to the first.
- Clocks in motion relative to an observer appear to be slowed down by a factor $\gamma = (1 - v^2/c^2)^{-1/2}$. This phenomenon is known as **time dilation.**
- The length of objects in motion appears to be contracted in the direction of

motion by a factor $1/\gamma = (1 - v^2/c^2)^{1/2}$. This phenomenon is known as **length contraction.**

To satisfy the postulates of special relativity, the Galilean transformation equations must be replaced by the **Lorentz transformation equations:**

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{v}{c^2}x\right) \end{aligned} \tag{39.11}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$.

The relativistic form of the **velocity transformation equation** is

$$u_x' = \frac{u_x - v}{1 - \dfrac{u_x v}{c^2}} \tag{39.16}$$

where u_x is the speed of an object as measured in the S frame and u_x' is its speed measured in the S′ frame.

The relativistic expression for the **linear momentum** of a particle moving with a velocity **u** is

$$\mathbf{p} \equiv \frac{m\mathbf{u}}{\sqrt{1 - \dfrac{u^2}{c^2}}} = \gamma m\mathbf{u} \tag{39.19}$$

The relativistic expression for the **kinetic energy** of a particle is

QUESTIONS

1. What two speed measurements do two observers in relative motion always agree on?
2. A spaceship in the shape of a sphere moves past an observer on the Earth with a speed $0.5c$. What shape does the observer see as the spaceship moves past?
3. An astronaut moves away from the Earth at a speed close to the speed of light. If an observer on Earth measures the astronaut's dimensions and pulse rate, what changes (if any) would the observer measure? Would the astronaut measure any changes about himself?
4. Two identical clocks are synchronized. One is then put in orbit directed eastward around the Earth while the other remains on Earth. Which clock runs slower? When the moving clock returns to Earth, are the two still synchronized?
5. Two lasers situated on a moving spacecraft are triggered simultaneously. An observer on the spacecraft claims to see the pulses of light simultaneously. What condition is necessary so that a second observer agrees?
6. When we say that a moving clock runs more slowly than a stationary one, does this imply that there is something physically unusual about the moving clock?
7. List some ways our day-to-day lives would change if the speed of light were only 50 m/s.
8. Give a physical argument that shows that it is impossible to accelerate an object of mass m to the speed of light, even if it has a continuous force acting on it.
9. It is said that Einstein, in his teenage years, asked the question, "What would I see in a mirror if I carried it in my hands and ran at the speed of light?" How would you answer this question?
10. Some distant star-like objects, called *quasars,* are receding from us at half the speed of light (or greater). What is the speed of the light we receive from these quasars?
11. How is it possible that photons of light, which have zero mass, have momentum?
12. With regard to reference frames, how does general relativity differ from special relativity?
13. Describe how the results of Example 39.7 would change if, instead of fast spaceships, two ordinary cars were approaching each other at highway speeds.
14. Two objects are identical except that one is hotter than the other. Compare how they respond to identical forces.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
☐ = paired numerical/symbolic problems

Section 39.1 The Principle of Galilean Relativity

1. A 2 000-kg car moving at 20.0 m/s collides and locks together with a 1 500-kg car at rest at a stop sign. Show that momentum is conserved in a reference frame moving at 10.0 m/s in the direction of the moving car.
2. A ball is thrown at 20.0 m/s inside a boxcar moving along the tracks at 40.0 m/s. What is the speed of the ball relative to the ground if the ball is thrown (a) forward? (b) backward? (c) out the side door?
3. In a laboratory frame of reference, an observer notes that Newton's second law is valid. Show that it is also valid for an observer moving at a constant speed, small compared with the speed of light, relative to the laboratory frame.
4. Show that Newton's second law is *not* valid in a reference frame moving past the laboratory frame of Problem 3 with a constant acceleration.

Section 39.2 The Michelson–Morley Experiment
Section 39.3 Einstein's Principle of Relativity
Section 39.4 Consequences of the Special Theory of Relativity

5. How fast must a meter stick be moving if its length is observed to shrink to 0.500 m?
6. At what speed does a clock have to move if it is to be seen to run at a rate that is one-half the rate of a clock at rest?
7. An astronaut is traveling in a space vehicle that has a speed of $0.500c$ relative to the Earth. The astronaut measures his pulse rate at 75.0 beats per minute. Signals generated by the astronaut's pulse are radioed to Earth when the vehicle is moving in a direction perpendicular to a line that connects the vehicle with an observer on the Earth. What pulse rate does the Earth observer measure? What would be the pulse rate if the speed of the space vehicle were increased to $0.990c$?
8. The proper length of one spaceship is three times that of another. The two spaceships are traveling in the same direction and, while both are passing overhead, an Earth observer measures the two spaceships to have the same length. If the slower spaceship is moving with a speed of $0.350c$, determine the speed of the faster spaceship.
9. An atomic clock moves at 1 000 km/h for 1 h as measured by an identical clock on Earth. How many nanoseconds slow will the moving clock be at the end of the 1-h interval?
10. If astronauts could travel at $v = 0.950c$, we on Earth would say it takes $(4.20/0.950) = 4.42$ yr to reach Alpha Centauri, 4.20 ly away. The astronauts disagree. (a) How much time passes on the astronauts' clocks? (b) What distance to Alpha Centauri do the astronauts measure?
11. WEB A spaceship with a proper length of 300 m takes 0.750 μs to pass an Earth observer. Determine the speed of this spaceship as measured by the Earth observer.
12. A spaceship of proper length L_p takes time t to pass an Earth observer. Determine the speed of this spaceship as measured by the Earth observer.
13. A muon formed high in the Earth's atmosphere travels at speed $v = 0.990c$ for a distance of 4.60 km before it decays into an electron, a neutrino, and an antineutrino $(\mu^- \rightarrow e^- + \nu + \bar{\nu})$. (a) How long does the muon live, as measured in its reference frame? (b) How far does the muon travel, as measured in its frame?
14. **Review Problem.** In 1962, when Mercury astronaut Scott Carpenter orbited the Earth 22 times, the press stated that for each orbit he aged 2 millionths of a second less than he would have had he remained on Earth. (a) Assuming that he was 160 km above the Earth in a circular orbit, determine the time difference between someone on Earth and the orbiting astronaut for the 22 orbits. You will need to use the approximation $\sqrt{1 - x} \approx 1 - x/2$ for small x. (b) Did the press report accurate information? Explain.
15. The pion has an average lifetime of 26.0 ns when at rest. In order for it to travel 10.0 m, how fast must it move?
16. For what value of v does $\gamma = 1.01$? Observe that for speeds less than this value, time dilation and length contraction are less-than-one-percent effects.
17. A friend passes by you in a spaceship traveling at a high speed. He tells you that his ship is 20.0 m long and that the identically constructed ship you are sitting in is 19.0 m long. According to your observations, (a) how long is your ship, (b) how long is your friend's ship, and (c) what is the speed of your friend's ship?
18. An interstellar space probe is launched from Earth. After a brief period of acceleration it moves with a constant velocity, 70.0% of the speed of light. Its nuclear-powered batteries supply the energy to keep its data transmitter active continuously. The batteries have a lifetime of 15.0 yr as measured in a rest frame. (a) How long do the batteries on the space probe last as measured by Mission Control on Earth? (b) How far is the probe from Earth when its batteries fail, as measured by Mission Control? (c) How far is the probe from Earth when its batteries fail, as measured by its built-in trip odometer? (d) For what total time after launch are data

received from the probe by Mission Control? Note that radio waves travel at the speed of light and fill the space between the probe and Earth at the time of battery failure.

19. **Review Problem.** An alien civilization occupies a brown dwarf, nearly stationary relative to the Sun, several lightyears away. The extraterrestrials have come to love original broadcasts of *The Ed Sullivan Show,* on our television channel 2, at carrier frequency 57.0 MHz. Their line of sight to us is in the plane of the Earth's orbit. Find the difference between the highest and lowest frequencies they receive due to the Earth's orbital motion around the Sun.

20. Police radar detects the speed of a car (Fig. P39.20) as follows: Microwaves of a precisely known frequency are broadcast toward the car. The moving car reflects the microwaves with a Doppler shift. The reflected waves are received and combined with an attenuated version of the transmitted wave. Beats occur between the two microwave signals. The beat frequency is measured. (a) For an electromagnetic wave reflected back to its source from a mirror approaching at speed v, show that the reflected wave has frequency

$$f = f_{\text{source}} \frac{c + v}{c - v}$$

where f_{source} is the source frequency. (b) When v is much less than c, the beat frequency is much less than the transmitted frequency. In this case, use the approximation $f + f_{\text{source}} \cong 2f_{\text{source}}$ and show that the beat frequency can be written as $f_b = 2v/\lambda$. (c) What beat frequency is measured for a car speed of 30.0 m/s if the microwaves have frequency 10.0 GHz? (d) If the beat frequency measurement is accurate to ± 5 Hz, how accurate is the velocity measurement?

Figure P39.20 *(Trent Steffler/David R. Frazier Photolibrary)*

21. *The red shift.* A light source recedes from an observer with a speed v_{source}, which is small compared with c. (a) Show that the fractional shift in the measured wavelength is given by the approximate expression

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v_{\text{source}}}{c}$$

This phenomenon is known as the red shift because the visible light is shifted toward the red. (b) Spectroscopic measurements of light at $\lambda = 397$ nm coming from a galaxy in Ursa Major reveal a red shift of 20.0 nm. What is the recessional speed of the galaxy?

Section 39.5 The Lorentz Transformation Equations

22. A spaceship travels at $0.750c$ relative to Earth. If the spaceship fires a small rocket in the forward direction, how fast (relative to the ship) must it be fired for it to travel at $0.950c$ relative to Earth?

WEB **23.** Two jets of material from the center of a radio galaxy fly away in opposite directions. Both jets move at $0.750c$ relative to the galaxy. Determine the speed of one jet relative to that of the other.

24. A moving rod is observed to have a length of 2.00 m, and to be oriented at an angle of 30.0° with respect to the direction of motion (Fig. P39.24). The rod has a speed of $0.995c$. (a) What is the proper length of the rod? (b) What is the orientation angle in the proper frame?

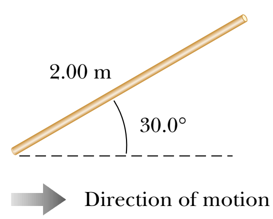


Figure P39.24

25. A Klingon space ship moves away from the Earth at a speed of $0.800c$ (Fig. P39.25). The starship *Enterprise* pursues at a speed of $0.900c$ relative to the Earth. Observers on Earth see the *Enterprise* overtaking the Klingon ship at a relative speed of $0.100c$. With what speed is the *Enterprise* overtaking the Klingon ship as seen by the crew of the *Enterprise*?

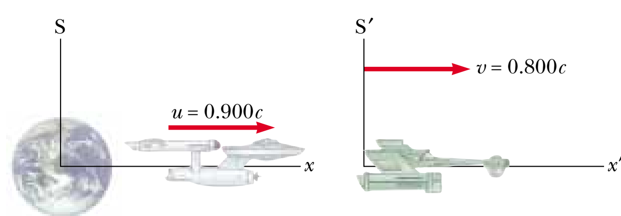


Figure P39.25

26. A red light flashes at position $x_R = 3.00$ m and time $t_R = 1.00 \times 10^{-9}$ s, and a blue light flashes at $x_B = 5.00$ m and $t_B = 9.00 \times 10^{-9}$ s (all values are measured in the S reference frame). Reference frame S′ has its origin at the same point as S at $t = t' = 0$; frame S′ moves constantly to the right. Both flashes are observed to occur at the same place in S′. (a) Find the relative velocity between S and S′. (b) Find the location of the two flashes in frame S′. (c) At what time does the red flash occur in the S′ frame?

Section 39.6 Relativistic Linear Momentum and the Relativistic Form of Newton's Laws

27. Calculate the momentum of an electron moving with a speed of (a) $0.010\,0c$, (b) $0.500c$, (c) $0.900c$.
28. The nonrelativistic expression for the momentum of a particle, $p = mu$, can be used if $u \ll c$. For what speed does the use of this formula yield an error in the momentum of (a) 1.00 percent and (b) 10.0 percent?
29. A golf ball travels with a speed of 90.0 m/s. By what fraction does its relativistic momentum p differ from its classical value mu? That is, find the ratio $(p - mu)/mu$.
30. Show that the speed of an object having momentum p and mass m is

$$u = \frac{c}{\sqrt{1 + (mc/p)^2}}$$

WEB 31. An unstable particle at rest breaks into two fragments of *unequal* mass. The mass of the lighter fragment is 2.50×10^{-28} kg, and that of the heavier fragment is 1.67×10^{-27} kg. If the lighter fragment has a speed of $0.893c$ after the breakup, what is the speed of the heavier fragment?

Section 39.7 Relativistic Energy

32. Determine the energy required to accelerate an electron (a) from $0.500c$ to $0.900c$ and (b) from $0.900c$ to $0.990c$.
33. Find the momentum of a proton in MeV/c units if its total energy is twice its rest energy.
34. Show that, for any object moving at less than one-tenth the speed of light, the relativistic kinetic energy agrees with the result of the classical equation $K = mu^2/2$ to within less than 1%. Thus, for most purposes, the classical equation is good enough to describe these objects, whose motion we call *nonrelativistic.*

WEB 35. A proton moves at $0.950c$. Calculate its (a) rest energy, (b) total energy, and (c) kinetic energy.

36. An electron has a kinetic energy five times greater than its rest energy. Find (a) its total energy and (b) its speed.
37. A cube of steel has a volume of 1.00 cm^3 and a mass of 8.00 g when at rest on the Earth. If this cube is now given a speed $u = 0.900c$, what is its density as measured by a stationary observer? Note that relativistic density is E_R/c^2V.
38. An unstable particle with a mass of 3.34×10^{-27} kg is initially at rest. The particle decays into two fragments that fly off with velocities of $0.987c$ and $-0.868c$. Find the masses of the fragments. (*Hint:* Conserve both mass–energy and momentum.)
39. Show that the energy–momentum relationship $E^2 = p^2c^2 + (mc^2)^2$ follows from the expressions $E = \gamma mc^2$ and $p = \gamma mu$.
40. A proton in a high-energy accelerator is given a kinetic energy of 50.0 GeV. Determine (a) its momentum and (b) its speed.
41. In a typical color television picture tube, the electrons are accelerated through a potential difference of 25 000 V. (a) What speed do the electrons have when they strike the screen? (b) What is their kinetic energy in joules?
42. Electrons are accelerated to an energy of 20.0 GeV in the 3.00-km-long Stanford Linear Accelerator. (a) What is the γ factor for the electrons? (b) What is their speed? (c) How long does the accelerator appear to them?
43. A pion at rest ($m_\pi = 270m_e$) decays to a muon ($m_\mu = 206m_e$) and an antineutrino ($m_{\bar{\nu}} \approx 0$). The reaction is written $\pi^- \rightarrow \mu^- + \bar{\nu}$. Find the kinetic energy of the muon and the antineutrino in electron volts. (*Hint:* Relativistic momentum is conserved.)

Section 39.8 Equivalence of Mass and Energy

44. Make an order-of-magnitude estimate of the ratio of mass increase to the original mass of a flag as you run it up a flagpole. In your solution explain what quantities you take as data and the values you estimate or measure for them.
45. When 1.00 g of hydrogen combines with 8.00 g of oxygen, 9.00 g of water is formed. During this chemical reaction, 2.86×10^5 J of energy is released. How much mass do the constituents of this reaction lose? Is the loss of mass likely to be detectable?
46. A spaceship of mass 1.00×10^6 kg is to be accelerated to $0.600c$. (a) How much energy does this require? (b) How many kilograms of matter would it take to provide this much energy?
47. In a nuclear power plant the fuel rods last 3 yr before they are replaced. If a plant with rated thermal power 1.00 GW operates at 80.0% capacity for the 3 yr, what is the loss of mass of the fuel?
48. A ^{57}Fe nucleus at rest emits a 14.0-keV photon. Use the conservation of energy and momentum to deduce the kinetic energy of the recoiling nucleus in electron volts. (Use $Mc^2 = 8.60 \times 10^{-9}$ J for the final state of the ^{57}Fe nucleus.)
49. The power output of the Sun is 3.77×10^{26} W. How much mass is converted to energy in the Sun each second?
50. A gamma ray (a high-energy photon of light) can produce an electron (e^-) and a positron (e^+) when

it enters the electric field of a heavy nucleus: $\gamma \rightarrow e^+ + e^-$. What minimum γ-ray energy is required to accomplish this task? (*Hint:* The masses of the electron and the positron are equal.)

Section 39.9 Relativity and Electromagnetism

51. As measured by observers in a reference frame S, a particle having charge q moves with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ and an electric field $\mathbf{E}$. The resulting force on the particle is then measured to be $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. Another observer moves along with the charged particle and also measures its charge to be q but measures the electric field to be $\mathbf{E}'$. If both observers are to measure the same force $\mathbf{F}$, show that $\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}$.

ADDITIONAL PROBLEMS

52. An electron has a speed of $0.750c$. Find the speed of a proton that has (a) the same kinetic energy as the electron; (b) the same momentum as the electron.

WEB 53. The cosmic rays of highest energy are protons, which have kinetic energy on the order of 10^{13} MeV. (a) How long would it take a proton of this energy to travel across the Milky Way galaxy, having a diameter of $\sim 10^5$ ly, as measured in the proton's frame? (b) From the point of view of the proton, how many kilometers across is the galaxy?

54. A spaceship moves away from the Earth at $0.500c$ and fires a shuttle craft in the forward direction at $0.500c$ relative to the ship. The pilot of the shuttle craft launches a probe at forward speed $0.500c$ relative to the shuttle craft. Determine (a) the speed of the shuttle craft relative to the Earth and (b) the speed of the probe relative to the Earth.

55. The net nuclear fusion reaction inside the Sun can be written as $4\,{}^1\text{H} \rightarrow {}^4\text{He} + \Delta E$. If the rest energy of each hydrogen atom is 938.78 MeV and the rest energy of the helium-4 atom is 3 728.4 MeV, what is the percentage of the starting mass that is released as energy?

56. An astronaut wishes to visit the Andromeda galaxy (2.00 million lightyears away), making a one-way trip that will take 30.0 yr in the spaceship's frame of reference. If his speed is constant, how fast must he travel relative to the Earth?

57. An alien spaceship traveling at $0.600c$ toward the Earth launches a landing craft with an advance guard of purchasing agents. The lander travels in the same direction with a velocity $0.800c$ relative to the spaceship. As observed on the Earth, the spaceship is 0.200 ly from the Earth when the lander is launched. (a) With what velocity is the lander observed to be approaching by observers on the Earth? (b) What is the distance to the Earth at the time of lander launch, as observed by the aliens? (c) How long does it take the lander to reach the Earth as observed by the aliens on the mother ship? (d) If the lander has a mass of 4.00×10^5 kg, what is its kinetic energy as observed in the Earth reference frame?

58. A physics professor on the Earth gives an exam to her students, who are on a rocket ship traveling at speed v relative to the Earth. The moment the ship passes the professor, she signals the start of the exam. She wishes her students to have time T_0 (rocket time) to complete the exam. Show that she should wait a time (Earth time) of

$$T = T_0\sqrt{\frac{1 - v/c}{1 + v/c}}$$

before sending a light signal telling them to stop. (*Hint:* Remember that it takes some time for the second light signal to travel from the professor to the students.)

59. Spaceship I, which contains students taking a physics exam, approaches the Earth with a speed of $0.600c$ (relative to the Earth), while spaceship II, which contains professors proctoring the exam, moves at $0.280c$ (relative to the Earth) directly toward the students. If the professors stop the exam after 50.0 min have passed on their clock, how long does the exam last as measured by (a) the students? (b) an observer on the Earth?

60. Energy reaches the upper atmosphere of the Earth from the Sun at the rate of 1.79×10^{17} W. If all of this energy were absorbed by the Earth and not re-emitted, how much would the mass of the Earth increase in 1 yr?

61. A supertrain (proper length, 100 m) travels at a speed of $0.950c$ as it passes through a tunnel (proper length, 50.0 m). As seen by a trackside observer, is the train ever completely within the tunnel? If so, with how much space to spare?

62. Imagine that the entire Sun collapses to a sphere of radius R_g such that the work required to remove a small mass m from the surface would be equal to its rest energy mc^2. This radius is called the *gravitational radius* for the Sun. Find R_g. (It is believed that the ultimate fate of very massive stars is to collapse beyond their gravitational radii into black holes.)

63. A charged particle moves along a straight line in a uniform electric field $\mathbf{E}$ with a speed of u. If the motion and the electric field are both in the x direction, (a) show that the acceleration of the charge q in the x direction is given by

$$a = \frac{du}{dt} = \frac{qE}{m}\left(1 - \frac{u^2}{c^2}\right)^{3/2}$$

(b) Discuss the significance of the dependence of the acceleration on the speed. (c) If the particle starts from rest at $x = 0$ at $t = 0$, how would you proceed to find the speed of the particle and its position after a time t has elapsed?

64. (a) Show that the Doppler shift $\Delta\lambda$ in the wavelength of light is described by the expression

$$\frac{\Delta\lambda}{\lambda} + 1 = \sqrt{\frac{c - v}{c + v}}$$

where λ is the source wavelength and v is the speed of relative approach between source and observer. (b) How fast would a motorist have to be going for a red light to appear green? Take 650 nm as a typical wavelength for red light, and one of 550 nm as typical for green.

65. A rocket moves toward a mirror at $0.800c$ relative to the reference frame S in Figure P39.65. The mirror is stationary relative to S. A light pulse emitted by the rocket travels toward the mirror and is reflected back to the rocket. The front of the rocket is 1.80×10^{12} m from the mirror (as measured by observers in S) at the moment the light pulse leaves the rocket. What is the total travel time of the pulse as measured by observers in (a) the S frame and (b) the front of the rocket?

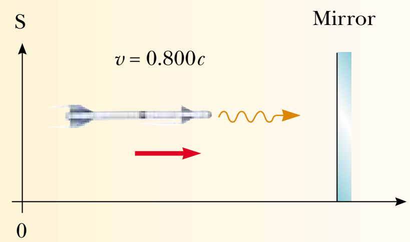


Figure P39.65 Problems 65 and 66.

66. An observer in a rocket moves toward a mirror at speed v relative to the reference frame labeled by S in Figure P39.65. The mirror is stationary with respect to S. A light pulse emitted by the rocket travels toward the mirror and is reflected back to the rocket. The front of the rocket is a distance d from the mirror (as measured by observers in S) at the moment the light pulse leaves the rocket. What is the total travel time of the pulse as measured by observers in (a) the S frame and (b) the front of the rocket?

67. Ted and Mary are playing a game of catch in frame S′, which is moving at $0.600c$, while Jim in frame S watches the action (Fig. P39.67). Ted throws the ball to Mary at $0.800c$ (according to Ted) and their separation (measured in S′) is 1.80×10^{12} m. (a) According to Mary, how fast is the ball moving? (b) According to Mary, how long does it take the ball to reach her? (c) According to Jim, how far apart are Ted and Mary, and how fast is the ball moving? (d) According to Jim, how long does it take the ball to reach Mary?

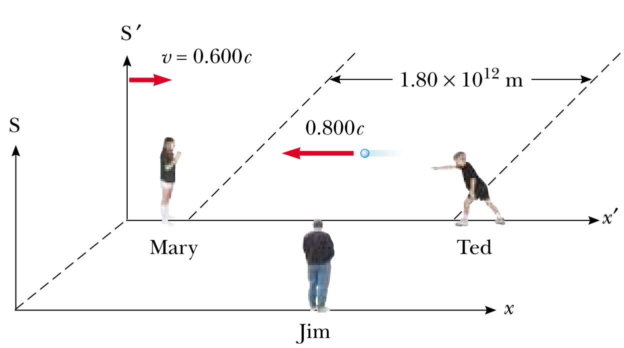


Figure P39.67

68. A rod of length L_0 moving with a speed v along the horizontal direction makes an angle θ_0 with respect to the x' axis. (a) Show that the length of the rod as measured by a stationary observer is $L = L_0[1 - (v^2/c^2)\cos^2\theta_0]^{1/2}$. (b) Show that the angle that the rod makes with the x axis is given by $\tan\theta = \gamma\tan\theta_0$. These results show that the rod is both contracted and rotated. (Take the lower end of the rod to be at the origin of the primed coordinate system.)

69. Consider two inertial reference frames S and S′, where S′ is moving to the right with a constant speed of $0.600c$ as measured by an observer in S. A stick of proper length 1.00 m moves to the left toward the origins of both S and S′, and the length of the stick is 50.0 cm as measured by an observer in S′. (a) Determine the speed of the stick as measured by observers in S and S′. (b) What is the length of the stick as measured by an observer in S?

70. Suppose our Sun is about to explode. In an effort to escape, we depart in a spaceship at $v = 0.800c$ and head toward the star Tau Ceti, 12.0 ly away. When we reach the midpoint of our journey from the Earth, we see our Sun explode and, unfortunately, at the same instant we see Tau Ceti explode as well. (a) In the spaceship's frame of reference, should we conclude that the two explosions occurred simultaneously? If not, which occurred first? (b) In a frame of reference in which the Sun and Tau Ceti are at rest, did they explode simultaneously? If not, which exploded first?

71. The light emitted by a galaxy shows a continuous distribution of wavelengths because the galaxy is composed of billions of different stars and other thermal emitters. Nevertheless, some narrow gaps occur in the continuous spectrum where light has been absorbed by cooler gases in the outer photospheres of normal stars. In particular, ionized calcium atoms at rest produce strong absorption at a wavelength of 394 nm. For a galaxy in the constellation Hydra, 2 billion lightyears away, this absorption line is shifted to 475 nm. How fast is the galaxy moving away from the Earth? (*Note:* The assumption that the recession speed is small compared with c, as made in Problem 21, is not a good approximation here.)

72. Prepare a graph of the relativistic kinetic energy and the classical kinetic energy, both as a function of speed, for an object with a mass of your choice. At what speed does the classical kinetic energy underestimate the relativistic value by 1 percent? By 5 percent? By 50 percent?

73. The total volume of water in the oceans is approximately 1.40×10^9 km^3. The density of sea water is 1 030 kg/m^3, and the specific heat of the water is 4 186 J/(kg · °C). Find the increase in mass of the oceans produced by an increase in temperature of 10.0°C.

ANSWERS TO QUICK QUIZZES

39.1 They both are because they can report only what they see. They agree that the person in the truck throws the ball up and then catches it a bit later.

39.2 It depends on the direction of the throw. Taking the direction in which the train is traveling as the positive x direction, use the values $u_x' = +90$ mi/h and $v = +110$ mi/h, with u_x in Equation 39.2 being the value you are looking for. If the pitcher throws the ball in the same direction as the train, a person at rest on the Earth sees the ball moving at 110 mi/h + 90 mi/h = 200 mi/h. If the pitcher throws in the opposite direction, the person on the Earth sees the ball moving in the same direction as the train but at only 110 mi/h − 90 mi/h = 20 mi/h.

39.3 Both are correct. Although the two observers reach different conclusions, each is correct in her or his own reference frame because the concept of simultaneity is not absolute.

39.4 About 2.9×10^8 m/s, because this is the speed at which $\gamma = 5$. For every 5 s ticking by on the Mission Control clock, the Earth-bound observer (with a powerful telescope!) sees the rocket clock ticking off 1 s. The astronaut sees her own clock operating at a normal rate. To her, Mission Control is moving away from her at a speed of 2.9×10^8 m/s, and she sees the Mission Control clock as running slow. Strange stuff, this relativity!

39.5 If their on-duty time is based on clocks that remain on the Earth, they will have larger paychecks. Less time will have passed for the astronauts in their frame of reference than for their employer back on the Earth.

39.6 By a curved line. This can be seen in the middle of Speedo's world-line in Figure 39.14, where he turns around and begins his trip home.

39.7 The downstairs clock runs more slowly because it is closer to the Earth and hence experiences a stronger gravitational field than the upstairs clock does.