

23. (a) The first minimum in the diffraction pattern is at an angular position θ , measured from the center of the pattern, such that $\sin \theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the antenna. If f is the frequency, then the wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{220 \times 10^9 \text{ Hz}} = 1.36 \times 10^{-3} \text{ m} .$$

Thus

$$\theta = \sin^{-1} \left(\frac{1.22\lambda}{d} \right) = \sin^{-1} \left(\frac{1.22(1.36 \times 10^{-3} \text{ m})}{55.0 \times 10^{-2} \text{ m}} \right) = 3.02 \times 10^{-3} \text{ rad} .$$

The angular width of the central maximum is twice this, or $6.04 \times 10^{-3} \text{ rad}$ (0.346°).

- (b) Now $\lambda = 1.6 \text{ cm}$ and $d = 2.3 \text{ m}$, so

$$\theta = \sin^{-1} \left(\frac{1.22(1.6 \times 10^{-2} \text{ m})}{2.3 \text{ m}} \right) = 8.5 \times 10^{-3} \text{ rad} .$$

The angular width of the central maximum is $1.7 \times 10^{-2} \text{ rad}$ (0.97°).